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Abstract

Aid for trade increases a recipient's public services, which lower its import and export transport costs. Formulating a two-country endogenous growth model, we obtain two main results. First, a permanent increase in the donor's aid/GDP ratio raises the steady-state growth rate as well as both countries' long-run fractions and cost shares of imported varieties if and only if it lowers the product of transport costs. Second, under a plausible condition, there exists a unique interior growth-maximizing aid/GDP ratio. These results are robust to alternative specifications for congestion and stock-flow nature of public goods.

Keywords: Aid for trade, Global growth, Public services, Transport cost, Endogenous growth
JEL classification: F35; F43

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1 Introduction

The international development community is starting to expand Aid for Trade as a way to improve aid effectiveness. Based on the notion that many developing countries lack enough money to facilitate international trade which contributes to faster economic growth, the WTO's 2005 Hong Kong Ministerial Conference declared that: "Aid for Trade should aim to help developing countries, particularly LDCs, to build the supply-side capacity and trade-related infrastructure that they need to assist them to implement and benefit from WTO Agreements and more broadly to expand their trade." (WTO, 2005) According to the third monitoring report jointly released by OECD and WTO (2011), aid for trade from bilateral and multilateral donors amounted to USD 40.1 billion in 2009 at 2009 constant prices, increasing by about 60% from the 2002-2005 baseline average of USD 25.1 billion. Of total aid for trade in 2009, economic infrastructure (e.g., transport and storage, communications, and energy) accounted for the majority (51.2%), followed by productive capacity building (45.3%) and trade policy and regulations and trade-related adjustment (3.5%). Understanding the mechanism through which aid for trade affects economic growth of both recipient and donor countries is essential for more effective implementation of the emerging agenda. The purpose of this paper is to provide a theoretical framework to examine under what circumstances aid for trade raises global growth.

Although there have been many studies on foreign aid and economic growth since the beginning of this century, little attention has been paid to international trade in goods and services as a transmission mechanism. On the empirical side, some researchers find that the aid/GDP ratio positively affects the growth rate of real GDP per capita if the former is interacted with an index of good policies (e.g., Burnside and Dollar, 2000), aid squared is included (e.g., Hansen and Tarp, 2001), or aid is interacted with a climate variable (e.g., Dalgaard et al., 2004), but they do not give theoretical explanations for their specifications.[1] On the theoretical side, Chatterjee et al. (2003) and Chatterjee and Turnovsky (2007) construct small-country, one-good endogenous growth models with international borrowing to show that aid for public investment enhancing the productivity of private capital always raises the long-run growth rate.[2] Since their models deal only with transactions in financial assets, they overlook trade costs in which governments play another important role. Naito (2013) develops a small-country Heckscher-Ohlin model of endogenous growth, and first demonstrate that aid for public investment reducing the import transport cost can either raise or lower the long-run growth rate depending on the factor intensity ranking. However, his model does not consider the possibility that a change in a country's growth rate affects its terms of trade, which may counteract, or even reverse, the direct growth effect.[3] This paper presents a first attempt to analyze the relationship between aid for trade and economic growth in a large-country model.

We formulate a two-country endogenous growth model with endogenous trade status. Acemoglu and Ventura (2002) build a multi-country AK model of endogenous growth, where countries trade only differentiated intermediate goods. The most striking aspect of the Acemoglu-Ventura model is that all countries converge to a common growth rate, so that the world income distribution is stable in the steady state, even without diminishing returns to capital nor international knowledge spillovers. This is because faster growth in one country worsens its terms of trade, which in turn pulls down its growth rate but pushes up that of

[1] Using longer time horizons and correcting for the endogeneity bias, Rajan and Subramanian (2008) question the robustness of these empirical results.

[2] In Chatterjee et al. (2003) and Chatterjee and Turnovsky (2007), the growth rate of consumption is endogenous in spite of their small-country assumption because they postulate that the interest rate of the small borrowing country is increasing in its debt/capital ratio.

[3] Acemoglu and Ventura (2002) provide empirical evidence that a 1 percentage point rise in a country's growth rate of GDP lowers its growth rate of terms of trade by around 0.6 percentage points.

the other.[4] However, since the intermediate goods are either exogenously or endogenously differentiated, their model cannot express the evolution of trade status (i.e., whether each country imports, does not trade, or exports each good). Naito (2012a) incorporates the continuum-good Ricardian framework of Dornbusch, Fischer, and Samuelson (1977) (henceforth DFS) into the two-country version of Acemoglu and Ventura (2002) to see the effects of unilateral reduction in an exogenous trade cost on the paths of growth rates, fractions of imported varieties, and welfare of both the liberalizing and partner countries. In this type of model, we can show that each country's fraction and cost share of imported varieties always move in the same direction as its growth rate, so policy shocks such as aid for trade can bring about rich interactions among countries' growth rates, terms of trade, and fractions and cost shares of imported varieties over time.

Endogenizing transport costs is central to our analysis. In recent years, many researchers examine, both qualitatively and quantitatively, the welfare effects of various forms of trade liberalization (e.g., Eaton and Kortum, 2002; Alvarez and Lucas, 2007; Arkolakis et al., 2012). This requires estimating countries' trade costs (e.g., Anderson and van Wincoop, 2003, 2004; Hummels, 2007; Waugh, 2010; Hummels and Schaur, 2012; Novy, 2013) It is natural to think that bilateral trade costs depend on infrastructure of both destination and source countries, which can be affected by aid for trade. For example, OECD and WTO (2011, p. 125–126) report that, after a rehabilitation project of inter-capital road between Kyrgyz Republic and Kazakhstan financed by the Asian Development Bank and European Development Bank, Kyrgyz Republic's exports to Kazakhstan increased by 160%. More generally, using a panel of 100 developing countries during 2002-2007, Calì and te Velde (2011) find that aid to economic infrastructure significantly increases recipients' exports. The inverse relationship between infrastructure and transport costs is introduced theoretically by Bougheas et al. (1999) and confirmed empirically by Limão and Venables (2001). In particular, Limão and Venables (2001) estimate that the elasticities of the bilateral CIF/FOB price ratio as a transport cost with respect to (the inverse of) infrastructure of destination and source countries are 0.34 and 0.66, respectively. This indicates that a country's import transport cost depends not only on its own infrastructure but also, and sometimes even more, on infrastructure of its trading partner.[5] We follow such specification of transport costs, where each country's public services are financed by its domestic income tax and net transfers. In line with the classic Barro (1990) model of government and growth, the public services are regarded as a flow rather than a stock variable, and they are subject to congestion caused by the real GDP as in Barro and Sala-i-Martin (2004, p. 223) because it especially applies to transportation facilities we are considering.[6] Under this specification, aid for trade increases a recipient's public services, which lower its import and export transport costs.[7]

We obtain the following main results. First, a permanent increase in the donor's aid/GDP ratio raises the steady-state growth rate as well as both countries' long-run fractions and cost shares of imported varieties if and only if it lowers the product of transport costs. An increase in aid for trade increases the recipient's public services, and the resulting falls in the transport costs create upward pressure on growth of both countries. On the other hand, it decreases the donor's public services, which impose downward pressure on growth of both countries through rises in the transport costs. After a period of terms-of-trade movement, global growth rises if and only if the world as a whole becomes more open as a result of aid for trade. In

[4] The terms-of-trade effect does not work as a force for growth convergence in R&D-based models (e.g., Feenstra, 1996) or Heckscher-Ohlin-type models (e.g., Bond et al., 2003).

[5] Waugh (2010) demonstrates that allowing for exporter fixed effects in his trade cost function can explain trade data better than importer fixed effects, thereby supporting the importance of exporters' characteristics.

[6] In section 5, we also study alternative models with congestion caused by capital (e.g., Barro and Sala-i-Martin, 1992) and with public capital stock (e.g., Futagami et al., 1993) to check the robustness of our results in the original model.

[7] Aid for trade could also contribute to productive capacity building in the form of technological improvements in private sectors. We do not consider this case in order to focus on the role of economic infrastructure.

contrast to Chatterjee et al. (2003) and Chatterjee and Turnovsky (2007), aid for productive government spending can either raise or lower the long-run growth rate. Second, under a plausible condition, there exists a unique interior growth-maximizing aid/GDP ratio. This is because a country's transport cost gets less and less sensitive to each country's public services as the latter increases. Since a gradual increase in aid for trade continues to increase the recipient's public services but to decrease those of the donor, the positive growth effects decrease and are eventually outweighed by the increasing negative ones. Our model implies an inverted U-shaped relationship between aid and growth reported by Hansen and Tarp (2001). We also show that the growth-maximizing aid/GDP ratio is increasing in an indicator of good governance for the recipient. This supports Burnside and Dollar's (2000) hypothesis that the effectiveness of aid in enhancing recipients' growth is conditional on their good policies. Third, the growth-enhancing permanent increase in the donor's aid/GDP ratio is also Pareto-improving if the subjective discount rate of the country whose growth rate may fall in the initial period is sufficiently low. When an increase in aid for trade raises global growth, we can say that the path of the growth rate of at least one of the two countries is higher than the original steady-state growth rate for all periods, and so is its welfare. However, when the initial terms of trade is far from its long-run value, the path of the growth rate of the other country may be lower than the original one in the early stage of transition. Consequently, the latter's welfare rises if it is sufficiently patient that the long-run gain should outweigh the short-run loss. Lahiri and Raimondos-Møller (1997), Lahiri et al. (2002), Mun and Nakagawa (2008), and Naito (2012b) show that tying aid to reductions in trade costs can be Pareto-improving in their static multi-country models. Our third result contributes to this literature by emphasizing the importance of faster global growth. Finally, these results are robust to alternative specifications for congestion and stock-flow nature of public goods. More specifically, our main results are valid even under congestion caused by capital (e.g., Barro and Sala-i-Martin, 1992) and under public capital stock (e.g., Futagami et al., 1993). Our arguments are directly applicable to the real-world situations where aid for trade affects trade-related infrastructure as a stock.

The rest of this paper is organized as follows. Section 2 sets up the model with exogenous transport costs and obtains preliminary results. Section 3 extends the model to include aid for trade. Section 4 examines the growth and welfare effects of aid for trade. Section 5 provides some extensions. Section 6 concludes.

2 The model with exogenous transport costs

2.1 Preferences, technologies, and endowments

Our benchmark model with exogenous transport costs is based on Naito (2012a), who combines Acemoglu and Ventura (2002) with DFS (1977). Consider a two-country world economy. In each country $j(=1,2)$, there is a representative household whose overall utility, or national welfare, is given by:

$$U_j = \int_0^\infty u_{jt} \exp(-\rho_j t) dt,$$
$$u_{jt} = \ln C_{jt},$$

where t is time (omitted whenever no confusion arises); u_{jt} is the instantaneous utility in period t; ρ_j is the subjective discount rate; and C_{jt} is consumption in period t. The production function of a representative final good firm is:

$$Y_j = Z_j(\int_0^1 x_j(i)^{(\sigma_j-1)/\sigma_j} di)^{\sigma_j/(\sigma_j-1)}; \sigma_j > 1,$$

where Y_j is the output of the final good for consumption or investment; Z_j is the productivity in the final good sector; $x_j(i)$ is the input of variety $i(\in [0,1])$ of intermediate good; and σ_j is the elasticity of substitution between any two varieties. The production function of a representative intermediate good firm producing variety i is:

$$x(i) = K^x(i)/a_j(i),$$

where $x(i)$ is the output of variety i; $K^x(i)$ is the input of capital in producing variety i; and $a_j(i)$ is the unit capital requirement for variety i. The unit capital requirements are distributed as:

$$A(i) \equiv a_2(i)/a_1(i); A' < 0.$$

We assume that varieties are indexed in descending order of relative capital productivity in country 1 to country 2. We also assume that each country's unit capital requirement for the most productive variety is close to zero (i.e., $a_1(0) \approx 0, a_2(1) \approx 0$). This will ensure interiority of the cutoff varieties, which will be introduced later.

Turning to international trade, only the intermediate goods are tradable, but subject to iceberg transport costs: the representative final good firm in country j must buy $\tau_j(> 1)$ units of a variety from the representative intermediate good firm producing the variety in country $j'(\neq j)$ to obtain one unit of it at home because $\tau_j - 1$ units melt away in transit. Put the other way around, an exporter in country j' has to ship τ_j units of a variety at home to deliver one unit of it to country j. Therefore, τ_j is seen either as the import transport cost factor for country j, or the export transport cost factor for country j', relative to the domestic transport cost factor normalized to unity. The transport costs are treated as exogenous at first, but they will be endogenized when we deal with aid for trade.

Finally, country j is endowed with the capital stock K_{j0} as the initial condition. For all $t > 0$, however, K_{jt} is determined endogenously as a result of investment.

2.2 Optimization and equilibrium

In country j, the representative household maximizes its overall utility, subject to the flow budget constraint:

$$p_{jt}^Y(C_{jt} + \dot{K}_{jt} + \delta_j K_{jt}) = r_{jt}K_{jt}, \tag{1}$$

with $\{p_{jt}^Y, r_{jt}\}_{t=0}^{\infty}$ and K_{j0} given, where p_j^Y is the price of the final good; δ_j is the depreciation rate of capital; r_j is the rental rate of capital; and a dot over a variable represents time differentiation (e.g., $\dot{K}_{jt} \equiv dK_{jt}/dt$). Dynamic optimization yields the Euler equation $\dot{C}_{jt}/C_{jt} = r_{jt}/p_{jt}^Y - \delta_j - \rho_j$ and the transversality condition $\lim_{s\to\infty} \exp(-\int_0^s (r_{jv}/p_{jv}^Y - \delta_j)dv)K_{js} = 0$. Moreover, integrating Eq. (1) from $s = t$ to infinity, and using the Euler equation and the transversality condition, we obtain the consumption function $C_{jt} = \rho_j K_{jt}$. This means that consumption and capital always grow at the same (but not necessarily constant) rate given by the Euler equation:

$$\dot{C}_{jt}/C_{jt} = \dot{K}_{jt}/K_{jt} = r_{jt}/p_{jt}^Y - \delta_j - \rho_j \forall t \in [0, \infty). \tag{2}$$

In the final good sector, the representative firm maximizes its profit $\Pi_j^Y = p_j^Y Y_j - \int_0^1 p_j(i)x_j(i)di$, subject to its production function, with p_j^Y and $\{p_j(i)\}_{i=0}^1$ given, where $p_j(i)$ is the demand price of variety i. Cost minimization implies:

$$\int_0^1 p_j(i)x_j(i)di = P_jY_j; P_j(\{p_j(i)\}_{i=0}^1) \equiv Z_j^{-1}(\int_0^1 p_j(i)^{1-\sigma_j}di)^{1/(1-\sigma_j)}, \tag{3}$$

where $P_j(\cdot)$ is the unit cost function, or the price index of the intermediate goods. Substituting Eq. (3) back into the profit definition, we obtain $\Pi_j^Y = p_j^Y Y_j - P_jY_j$. If the profit-maximizing output of the final good is positive, which is true in equilibrium, then its price must be equal to its marginal cost:

$$p_j^Y = P_j. \tag{4}$$

This also means that the maximized profit is zero.

In the intermediate good sector, the representative firm producing variety i maximizes its profit $\Pi^x(i) = p(i)x(i) - r_jK^x(i)$, subject to $K^x(i) = a_j(i)x(i)$ from its production function, with $p(i)$ and r_j given, where $p(i)$ is the supply price of variety i. The first-order condition for profit maximization, allowing for the possibility of inaction, is:

$$p(i) - r_ja_j(i) \leq 0, x(i) \geq 0, x(i)(p(i) - r_ja_j(i)) = 0.$$

The representative final good firm in country 1 buys variety i_1 domestically if and only if doing so is cheaper than importing it from abroad: $r_1a_1(i_1) \leq \tau_1r_2a_2(i_1)$, or $r_1/(\tau_1r_2) \leq A(i_1)$. Considering the productivity distribution, varieties with small i are actually produced in country 1:

$$p(i_1) = r_1a_1(i_1), i_1 \in [0, I_1], \tag{5}$$

where the cutoff variety I_1 is determined by:

$$r_1/(\tau_1r_2) = A(I_1) \Leftrightarrow I_1 = A^{-1}(r_1/(\tau_1r_2)) \equiv I_1(\tau_1r_2/r_1); I_1' > 0. \tag{6}$$

Similarly, the representative final good firm in country 2 buys variety i_2 domestically if and only if $\tau_2r_1a_1(i_2) \geq r_2a_2(i_2)$, or $\tau_2r_1/r_2 \geq A(i_2)$. Then varieties with large i are actually produced in country 2:

$$p(i_2) = r_2a_2(i_2), i_2 \in [I_2, 1]; \tag{7}$$

$$\tau_2r_1/r_2 = A(I_2) \Leftrightarrow I_2 = A^{-1}(\tau_2r_1/r_2) \equiv I_2(\tau_2r_1/r_2); I_2' < 0. \tag{8}$$

The first important observation from Eqs. (5) to (8) is that $I_2 < I_1$ for all r_1/r_2. The two cutoffs partition the unit interval into three subsets, $[0, I_2], [I_2, I_1]$, and $[I_1, 1]$, with their measures $I_2, I_1 - I_2$, and $1 - I_1$, respectively. For country 1, relatively the most productive varieties in $[0, I_2]$ are exported to country 2, the mediocre varieties in $[I_2, I_1]$ are sold only domestically, and relatively the least productive varieties in $[I_1, 1]$ are entirely imported from country 2. Similarly, country 2 exports relatively the most productive varieties in $[I_1, 1]$, sells the mediocre varieties in $[I_2, I_1]$ only domestically, and entirely imports relatively the least productive varieties in $[0, I_2]$. The measure I_2 of the set $[0, I_2]$ represents country 2's fraction of imported varieties, or country 1's fraction of exported varieties. Similarly, the measure $1 - I_1$ of the set $[I_1, 1]$ shows country 1's fraction of imported varieties, or country 2's fraction of exported varieties. The remaining measure $I_1 - I_2$ of the set $[I_2, I_1]$ indicates each country's fraction of nontraded varieties. The second observation is that country j's fraction of imported varieties is decreasing in $\tau_j r_{j'}/r_j$. Intuitively, a fall in country j's import transport cost and/or a rise in its relative rental rate, with the latter indicating an improvement in its terms of trade $p(i_j)/p(i_{j'}) = (r_j/r_{j'})a_j(i_j)/a_{j'}(i_{j'})$, make some domestic suppliers replaced by foreign ones.

The demand prices are related to the supply prices in the following ways:

$$p_j(i_j) = p(i_j), \tag{9}$$

$$p_j(i_{j'}) = \tau_j p(i_{j'}), j' \neq j. \tag{10}$$

For each exported or nontraded variety, its demand price just equals its supply price. For each imported variety, however, the representative final good firm has to pay τ_j times its supply price to import one unit of it.

The market-clearing conditions for the final good, capital, and the intermediate goods are, respectively:

$$Y_j = C_j + \dot{K}_j + \delta_j K_j, j = 1, 2, \tag{11}$$

$$K_1 = \int_0^{I_1} K^x(i_1) di_1, \tag{12}$$

$$K_2 = \int_{I_2}^{1} K^x(i_2) di_2,$$

$$x(i_1) = x_1(i_1) + \tau_2 x_2(i_1), i_1 \in [0, I_2], \tag{13}$$

$$x(i_1) = x_1(i_1), i_1 \in [I_2, I_1], \tag{14}$$

$$x(i_2) = x_2(i_2) + \tau_1 x_1(i_2), i_2 \in [I_1, 1],$$

$$x(i_2) = x_2(i_2), i_2 \in [I_2, I_1].$$

Eqs. (13) and (14) are the market-clearing conditions for country 1's exported and nontraded varieties, respectively. The second term in the right-hand side of Eq. (13) shows the quantity the representative final good firm in country 2 buys to import $x_2(i_1)$ units of variety i_1.

Finally, Eqs. (1), (3), (4), (5), (7), (9), and (10) imply Warlas' law. Therefore, any one of the eight types of market-clearing conditions is redundant, and the corresponding good or factor can be used as the numeraire.

2.3 Dynamic system

Let capital in country 2 be the numeraire: $r_2 \equiv 1$. As shown in Appendix A, our model is reduced to the following two-dimensional autonomous dynamic system:

$$\dot{\kappa} = \kappa(\gamma_1(\tau_1/r_1) - \gamma_2(\tau_2 r_1)); \kappa \equiv K_1/K_2, \tag{15}$$

$$\gamma_1(\tau_1/r_1) \equiv 1/Q_1(\tau_1/r_1, 1) - \delta_1 - \rho_1,$$

$$\gamma_2(\tau_2 r_1) \equiv 1/Q_2(\tau_2 r_1, 1) - \delta_2 - \rho_2,$$

$$\kappa = (\beta_2(\tau_2 r_1)/\beta_1(\tau_1/r_1))/r_1, \tag{16}$$

where $\kappa, \gamma_j(\cdot), Q_j(\cdot)$, and $\beta_j(\cdot)$ are the relative supply of capital in country 1 to country 2, country j's growth rate (rewritten from Eq. (2)), country j's simplified intermediate good price index (rewritten from Eq. (3)), and country j's cost share of imported varieties, respectively (see Appendix A for precise functional forms of $Q_j(\cdot)$ and $\beta_j(\cdot)$). Eq. (15) simply states that κ grows by the difference between the growth rates of the two countries $\gamma_1(\cdot)$ and $\gamma_2(\cdot)$. Eq. (16) requires that the relative rental rate of capital in country 1 to country 2 r_1 should equalize its relative supply and relative demand. For all $t \in [0, \infty)$, with κ_t predetermined and with τ_1 and τ_2 exogenous, Eq. (16) determines r_{1t}, and then Eq. (15) determines $\dot{\kappa}_t$.

Before studying the dynamics, we summarize how our key variables are related to $\tau_j r_{j'}/r_j$:

Lemma 1 *Country j's fraction of imported varieties (i.e., $1 - I_1$ and I_2 for countries 1 and 2, respectively), cost share of imported varieties β_j, and growth rate γ_j are decreasing in $\tau_j r_{j'}/r_j$.*

Proof. See Appendix A. ■

An important implication of this lemma is that the former three endogenous variables are always positively correlated. Once we find that one of the three increases due to a fall in $\tau_j r_{j'}/r_j$, we can immediately say that the other two increase as well.

We define a steady state as a situation in which all variables grow at constant rates. From Eqs. (15) and (16), a steady state is determined by:

$$0 = \gamma_1(\tau_1/r_1^*) - \gamma_2(\tau_2 r_1^*),$$

$$\kappa^* = (\beta_2(\tau_2 r_1^*)/\beta_1(\tau_1/r_1^*))/r_1^*,$$

where an asterisk represents a steady state. The former equation determines r_1^*. For that r_1^*, the latter equation determines κ^*. The following lemma summarizes the properties of our dynamic system:

Lemma 2 *In the model with exogenous transport costs, there exists a unique steady state which is globally stable.*

Proof. See Appendix B. ■

Based on our well-behaved dynamic system, we see the effect of changes in transport costs on the steady-state growth rate. Differentiating $\gamma_j = r_j/p_j^Y - \delta_j - \rho_j$ from Eq. (2), we obtain $d\gamma_j = (r_j/p_j^Y)(dr_j/r_j - dp_j^Y/p_j^Y) = (\gamma_j + \delta_j + \rho_j)(dr_j/r_j - dp_j^Y/p_j^Y)$. With Eq. (4) and the logarithmically differentiated form of $Q_j(\cdot)$ (i.e., Eq. (A.5) in Appendix A), this is rewritten as:

$$d\gamma_1 = \Gamma_1 \beta_1 (dr_1/r_1 - d\tau_1/\tau_1), \tag{17}$$

$$d\gamma_2 = -\Gamma_2 \beta_2 (d\tau_2/\tau_2 + dr_1/r_1); \Gamma_j \equiv \gamma_j + \delta_j + \rho_j. \tag{18}$$

Using Eqs. (17) and (18) to solve $d\gamma_1^* = d\gamma_2^*$ for dr_1^*/r_1^*, and substituting the latter back into either Eq. (17) or (18), we obtain:

$$d\gamma_1^* = d\gamma_2^* = -[\Gamma_1^*\beta_1^*\Gamma_2^*\beta_2^*/(\Gamma_1^*\beta_1^* + \Gamma_2^*\beta_2^*)](d\tau_1/\tau_1 + d\tau_2/\tau_2). \tag{19}$$

Lemma 3 *The steady-state growth rate rises if and only if the product of transport costs falls.*

When the world as a whole becomes more open in terms of the product of transport costs, we observe accelerated global growth. Moreover, from Lemma 1, both countries import and export larger fractions of varieties, and enjoy greater import shares. Note that these benefits can be realized even if the import transport cost of one country rises; indeed they are if the import transport cost of the other country falls at a larger rate.

3 The model with aid for trade

In this section, we formulate a full-fledged model of aid for trade. Suppose that aid for trade increases a recipient's public services, which lower its import and export transport costs. This reflects the fact that over a half of aid for trade goes to economic infrastructure such as transport and storage, communications, and energy (e.g., OECD and WTO, 2011), and such infrastructure contributes to lower transport costs (e.g., Bougheas et al., 1999; Limão and Venables, 2001). We first make some modifications to the model in the previous section, and then characterize the steady state and transitional dynamics of the complete model.

3.1 Dynamic system

In each country, the government imposes an income tax of an exogenous and constant rate $t_j (\in (0,1))$ on the representative household. Then the flow budget constraint of the representative household is modified to:

$$p_{jt}^Y(C_{jt} + \dot{K}_{jt} + \delta_j K_{jt}) = (1 - t_j)r_{jt}K_{jt}. \tag{20}$$

The growth rate is now given by:

$$\dot{C}_{jt}/C_{jt} = \dot{K}_{jt}/K_{jt} = (1 - t_j)r_{jt}/p_{jt}^Y - \delta_j - \rho_j \forall t \in [0, \infty). \tag{21}$$

Since all firms take all prices as given, their behavior does not change with endogenous transport costs.

Let country 1 and country 2 be the donor and recipient countries, respectively. Following Barro (1990), the government budget constraints of the donor and recipient are expressed as:

$$p_1^Y G_1 = t_1 r_1 K_1 - m r_1 K_1; m \in [0, t_1), \tag{22}$$

$$p_2^Y (G_2 + C_2^G) = t_2 r_2 K_2 + m r_1 K_1. \tag{23}$$

where G_j is country j's flow of total public services; m is the donor's aid/GDP ratio, which is assumed to be exogenous and constant; and C_2^G is the recipient's government consumption. Eqs. (22) and (23) state that each government spends its income tax revenue and net transfer to buy the final good. One difference between Eqs. (22) and (23) is that the recipient government may use the final good not just for productive public services but also for unproductive government consumption. This reflects the idea that some aid might be wasted in the recipient country.[8] More specifically, the recipient government's expenditure for productive public services is given by:

$$p_2^Y G_2 = t_2 r_2 K_2 + m\theta r_2 K_2; \theta \in [0, r_1\kappa]. \tag{24}$$

Eq. (24) says that the recipient government spends $m\theta r_2 K_2$ out of mr_1K_1, in addition to its income tax revenue, for its public services. The exogenous and constant parameter θ is an indicator of good governance for the recipient. The larger θ is, the more fraction of aid $m\theta r_2 K_2/(mr_1K_1) = \theta/(r_1\kappa)$ is allocated to public services.[9] When θ reaches its upper bound $r_1\kappa$, the whole amount of aid is spent for public services. In the worst case where $\theta = 0$, the whole amount of aid is spent for government consumption. In general, the expenditure for unproductive government consumption is obtained by subtracting Eq. (24) from Eq. (23):

$$p_2^Y C_2^G = m(r_1K_1 - \theta r_2 K_2).$$

Country j's import transport cost function is specified in the following general way:

$$\begin{aligned} \tau_j(g_j, g_{j'}) &= (g_j/\overline{g}_j)^{-\mu_j}(g_{j'}/\overline{g}_{j'})^{-\chi_{j'}}; \\ g_j &\equiv G_j/(r_jK_j/p_j^Y) = p_j^Y G_j/(r_jK_j), g_j \in (0, \overline{g}_j), \mu_j > 0, \chi_{j'} > 0, \end{aligned} \tag{25}$$

where g_j is country j's effective public services, which is given by the ratio of country j's total public services to its real GDP; and $\overline{g}_j$ is the upper bound of g_j, interpreted as other factors affecting the transport cost such as geography. Several points should be noted about this specification. First, the assumption that a transport cost depends on the effective public services rather than the total ones captures congestion caused by the real GDP (e.g., Barro and Sala-i-Martin, 2004, p. 223): for a given level of total public services, an increase in total activities using them decreases the effective level of public services available to each user. This is especially plausible to transportation facilities we are considering. Second, the parameters μ_j and $\chi_{j'}$ measure the elasticities of country j's import transport cost with respect to the effective public services of country j and country j', respectively.[10] This means that a 1% increase in the recipient's effective public services not only lowers its own import transport cost by μ_2%, but also lowers the donor's import transport cost, or the recipient's own export transport cost, by χ_2%. Limão and Venables (2001) estimate that $\mu_j = 0.34$ and $\chi_{j'} = 0.66$, suggesting that a country's effective public services can affect its export transport cost by more than its import transport cost. Third, increases in public services lower each country's import transport cost factor relative to its domestic transport cost factor. Although better infrastructure is in fact likely to reduce both international and domestic transport costs, we are considering that the former reduction is greater than the latter. As indirect evidence, Novy (2013) finds that his micro-founded measure of bilateral trade costs relative to domestic ones for the average of thirteen OECD countries steadily decreases from the tariff equivalent of 144% in 1970 to 94% in 2000.

The market-clearing conditions for the final goods in the donor and recipient countries are given by,

[8] We assume that C_2^G does not affect preferences, technologies, or endowments of any country. When C_2^G positively affects the recipient's utility function as in Barro (1990, section V), aid tends to raise its welfare more than the present case.

[9] Although θ is not directly observable, we can calculate it by multiplying the fraction of aid allocated to public services $\theta/(r_1\kappa)$ by the relative GDP of the donor to the recipient $r_1\kappa$. It will be calibrated at the end of section 4.1.

[10] Naito (2012b) only considers the case where $\mu_j = 1$ and $\chi_{j'} = 0$ in his static DFS model of aid for trade with a Cobb-Douglas final good production function.

respectively:

$$Y_1 = C_1 + \dot{K}_1 + \delta_1 K_1 + G_1, \tag{26}$$

$$Y_2 = C_2 + \dot{K}_2 + \delta_2 K_2 + G_2 + C_2^G. \tag{27}$$

From Eqs. (3), (4), (5), (7), (9), (10), (20), (22), and (23), we obtain Walras' law as before. Having revised our model, we derive the dynamic system as follows:[11]

$$\dot{\kappa} = \kappa(\gamma_1(\tau_1/r_1) - \gamma_2(\tau_2 r_1)); \tag{28}$$

$$\gamma_1(\tau_1/r_1) \equiv (1-t_1)/Q_1(\tau_1/r_1, 1) - \delta_1 - \rho_1,$$

$$\gamma_2(\tau_2 r_1) \equiv (1-t_2)/Q_2(\tau_2 r_1, 1) - \delta_2 - \rho_2,$$

$$\kappa = \lambda(m, \beta_1(\tau_1/r_1), \beta_2(\tau_2 r_1))/r_1; \tag{29}$$

$$\lambda(m, \beta_1(\tau_1/r_1), \beta_2(\tau_2 r_1)) \equiv \beta_2(\tau_2 r_1)/[m(1-\beta_1(\tau_1/r_1) - \beta_2(\tau_2 r_1)) + \beta_1(\tau_1/r_1)].$$

Eqs. (28) and (29) look similar to Eqs. (15) and (16), respectively. However, the transport costs are endogenously determined in the present model. From Eqs. (22) and (24), the effective public services are simply given by:

$$g_1 = t_1 - m \equiv g_1(m), \tag{30}$$

$$g_2 = t_2 + m\theta \equiv g_2(m). \tag{31}$$

Eqs. (28) and (29), together with Eqs. (25), (30), and (31), constitute the dynamic system.[12] For all $t \in [0, \infty)$, with κ_t predetermined and with m exogenous, Eq. (29) determines r_{1t}, and then Eq. (28) determines $\dot{\kappa}_t$.

3.2 Steady state and transitional dynamics

Substituting $\tau_1 = \tau_1(g_1(m), g_2(m))$ and $\tau_2 = \tau_2(g_2(m), g_1(m))$ into Eqs. (28) and (29), and setting $\dot{\kappa} = 0$, a steady state is determined by:

$$0 = \gamma_1(\tau_1(g_1(m), g_2(m))/r_1^*) - \gamma_2(\tau_2(g_2(m), g_1(m))r_1^*),$$

$$\kappa^* = \lambda(m, \beta_1(\tau_1(g_1(m), g_2(m))/r_1^*), \beta_2(\tau_2(g_2(m), g_1(m))r_1^*))/r_1^*.$$

Since Eqs. (25), (30), and (31) imply that the public services and hence the transport costs are determined only by the donor's aid/GDP ratio, we can treat the endogenous transport costs as if they were exogenous.

[11] Eq. (28) is obtained in the same way as Eq. (15), except that Eq. (2) is replaced by Eq. (21). To derive Eq. (29), we first rewrite Eq. (12) using Eqs. (4), (13), (14), (20), (22), (23), (26), (27), (A.3), and (A.4) to obtain $\beta_1(r_1K_1 - mr_1K_1) = \beta_2(r_2K_2 + mr_1K_1) - mr_1K_1$. This also shows the balanced current account condition for country 1. This equation is rewritten as Eq. (29).

[12] Eq. (29) implies that the upper bound of θ in Eq. (24) is equal to $\lambda(\cdot)$. For a richer country to be a donor, we must have $\lambda(\cdot) > 1$. The more productive the donor is relative to the recipient (i.e., the higher $A(i)$ is overall), the larger I_1 and I_2 are (from Eqs. (6) and (8)), and hence the more likely $\lambda(\cdot) > 1$ is to be satisfied (from Eqs. (29), (A.3) and (A.4)).

Moreover, noting Lemma 1 and that λ is decreasing in β_1 but increasing in β_2, the right-hand side of Eq. (29) is decreasing in r_1 as in Eq. (16). Applying the proof of Lemma 2 directly, we immediately obtain the following proposition:

Proposition 1 *In the model with aid for trade, there exists a unique steady state which is globally stable.*

The dynamics of our model can be graphically explained in Fig. 1, where r_1 and γ_j are measured on the horizontal and vertical axes, respectively. Since a rise in r_1 means the improved terms of trade in the donor country, curve $\gamma_1(\tau_1(g_1(m), g_2(m))/r_1)$ is upward-sloping. By the same token, curve $\gamma_2(\tau_2(g_2(m), g_1(m))r_1)$ is downward-sloping. The steady state is found at point A: (r_1^*, γ^*), the intersection of the two curves. Suppose that the donor is relatively large in the initial period. Then, due to a low value of r_1 satisfying the capital market-clearing condition (29), the recipient with relatively high terms of trade starts to grow faster than the donor. Since this decreases κ and thus raises r_1, the recipient's growth rate falls while the donor's growth rate rises along curve $\gamma_2(\tau_2(g_2(m), g_1(m))r_1)$ and curve $\gamma_1(\tau_1(g_1(m), g_2(m))/r_1)$, respectively. This process continues until the growth rates of these countries are equalized in the steady state.

4 Effects of aid for trade

4.1 Growth effects

Suppose that the world economy is originally in the steady state, and that the donor government increases its aid/GDP ratio permanently from m to m'. We know from Eqs. (30) and (31) that it decreases the donor's effective public services g_1 but increases those of the recipient g_2. Since these changes in the effective public services have the opposing effects on the donor's import transport cost τ_1 from Eq. (25), it is generally unclear if the donor's growth rate γ_1 falls or rises, with its rental rate r_1 given. The same applies to the recipient's growth rate γ_2.

We see two special cases graphically. Fig. 2 considers the case where both χ_1 and χ_2 are close to zero, meaning that an increase in a country's effective public services lowers only its own import transport cost. The original steady state point A is at the intersection of two dashed curves $\gamma_1(\tau_1(g_1(m), g_2(m))/r_1)$ and $\gamma_2(\tau_2(g_2(m), g_1(m))r_1)$. A rise in τ_1 caused by an increase in m shifts curve $\gamma_1(\tau_1(g_1(m), g_2(m))/r_1)$ down to solid curve $\gamma_1(\tau_1(g_1(m'), g_2(m'))/r_1)$, whereas a fall in τ_2 shifts curve $\gamma_2(\tau_2(g_2(m), g_1(m))r_1)$ up to solid curve $\gamma_2(\tau_2(g_2(m'), g_1(m'))r_1)$. In the long run, the world economy reaches point D: $(r_1^{*\prime}, \gamma^{*\prime})$, the intersection of the two solid curves, which is to the right of and above point A. Things become more complicated in the short run, however, because we cannot tell how r_1 changes in the initial period. An increase in m, on one hand, increases the relative demand for capital in country 1 through a decrease in β_1 and an increase in β_2. On the other hand, it decreases that relative demand depending on whether $1-\beta_1-\beta_2 > 0$. If $1-\beta_1-\beta_2 > 0$, then the direction of change in r_1 in the initial period is ambiguous. In the case described in Fig. 2, r_1 rises slightly from $r_{10} = r_1^*$ to r_{10}'. Then the recipient jumps up to point B while the donor jumps down to point C in the initial period. Since the recipient grows faster than the donor, the former's terms of trade starts to deteriorate. This continues to pull down the recipient's growth rate but to push up that of the donor along the corresponding solid curves until they are equalized at point D.

Fig. 3 illustrates the other extreme case where both μ_1 and μ_2 are close to zero, so that an increase in a country's effective public services lowers only the other country's import transport cost. Since an increase in m lowers τ_1 but raises τ_2, curve $\gamma_1(\tau_1(g_1(m), g_2(m))/r_1)$ now shifts up to solid curve $\gamma_1(\tau_1(g_1(m'), g_2(m'))/r_1)$

whereas curve $\gamma_2(\tau_2(g_2(m), g_1(m))r_1)$ shifts down to solid curve $\gamma_2(\tau_2(g_2(m'), g_1(m'))r_1)$. Somewhat paradoxically, an increase in aid for trade increases the donor's growth potential but decreases that of the recipient. In the long run, the world economy arrives at point D, which is now to the left of and above point A. In the short run, we cannot still tell if r_1 rises or falls in the initial period, but it is more likely to fall due to an increase in β_1 and a decrease in β_2. When r_1 falls slightly from $r_{10} = r_1^*$ to r'_{10}, as indicated in Fig. 3, the recipient jumps down to point B while the donor jumps up to point C in the initial period. After that, the donor's terms of trade continues to deteriorate until the growth rates are equalized at point D.

In both Fig. 2 and Fig. 3, a permanent increase in the donor's aid/GDP ratio raises the steady-state growth rate. This is not always true because one of the two growth curves shifts up whereas the other shifts down in those cases. More generally, as suggested in the first paragraph of this section, both curves $\gamma_1(\cdot)$ and $\gamma_2(\cdot)$ can shift in either direction, and so can the steady-state growth rate. To see precisely under what conditions aid for trade raises or lowers global growth, we proceed to the analytical method.

Differentiating $\gamma_j = (1-t_j)r_j/p_j^Y - \delta_j - \rho_j$ from Eq. (21), we obtain $d\gamma_j = [(1-t_j)r_j/p_j^Y](dr_j/r_j - dp_j^Y/p_j^Y) = \Gamma_j(dr_j/r_j - dp_j^Y/p_j^Y)$. This means that we can still use Eqs. (17) and (18) even with the income taxes. Using Eqs. (25), (30), and (31) to eliminate the rates of changes in the transport costs, Eqs. (17) and (18) are rewritten as:

$$d\gamma_1 = \Gamma_1\beta_1\{dr_1/r_1 - [\mu_1/(t_1-m) - \chi_2\theta/(t_2+m\theta)]dm\}, \tag{32}$$

$$d\gamma_2 = -\Gamma_2\beta_2\{[\chi_1/(t_1-m) - \mu_2\theta/(t_2+m\theta)]dm + dr_1/r_1\}. \tag{33}$$

Using Eqs. (32) and (33) to solve $d\gamma_1^* = d\gamma_2^*$ for dr_1^*/r_1^*, we obtain:

$$\begin{aligned} dr_1^*/r_1^* &= [E^*/(\Gamma_1^*\beta_1^* + \Gamma_2^*\beta_2^*)]dm; \\ E^* &\equiv \Gamma_1^*\beta_1^*[\mu_1/(t_1-m) - \chi_2\theta/(t_2+m\theta)] - \Gamma_2^*\beta_2^*[\chi_1/(t_1-m) - \mu_2\theta/(t_2+m\theta)]. \end{aligned} \tag{34}$$

We can use Eq. (34) to interpret the direction of change in r_1^* in Fig. 2 and Fig. 3. In Fig. 2, where both χ_1 and χ_2 are close to zero, an increase in m makes the recipient grow faster than the donor, which in turn improves the latter's terms of trade. Just the opposite occurs in Fig. 3, where both μ_1 and μ_2 are close to zero. Substituting Eq. (34) back into either Eqs. (32) or (33), we obtain:

$$\begin{aligned} d\gamma_1^* = d\gamma_2^* &= [\Gamma_1^*\beta_1^*\Gamma_2^*\beta_2^*/(\Gamma_1^*\beta_1^* + \Gamma_2^*\beta_2^*)]F(m)dm; \\ F(m) &\equiv \theta(\mu_2+\chi_2)/(t_2+m\theta) - (\mu_1+\chi_1)/(t_1-m). \end{aligned} \tag{35}$$

To interpret Eq. (35), we look back to the derivation of Eqs. (32) and (33) to obtain:

$$\begin{aligned} d\tau_1/\tau_1 + d\tau_2/\tau_2 &= [\mu_1/(t_1-m) - \chi_2\theta/(t_2+m\theta)]dm + [\chi_1/(t_1-m) - \mu_2\theta/(t_2+m\theta)]dm \\ &= -F(m)dm. \end{aligned} \tag{36}$$

Substituting $F(m)dm$ from Eq. (36) back into Eq. (35), we find that Eq. (19) and Lemma 3 continue to hold in the model with aid for trade. A marginal increase in the donor's aid/GDP ratio increases the recipient's effective public services at the rate of $\theta/(t_2 + m\theta)$, but decreases those of the donor at the rate of $1/(t_1 - m)$. The former lowers the recipient's import and export transport costs at the rates of $\mu_2\theta/(t_2 + m\theta)$ and $\chi_2\theta/(t_2 + m\theta)$, whereas the latter raises the donor's import and export transport costs at the rates of $\mu_1/(t_1 - m)$ and $\chi_1/(t_1 - m)$, respectively. The aid increase raises global growth when the former two negative effects on the transport costs are stronger in total than the latter two positive effects.

Proposition 2 *A permanent increase in the donor's aid/GDP ratio raises the steady-state growth rate if and only if it lowers the product of transport costs.*

It should be noted from Lemma 1 that, whenever an increase in aid for trade lowers the product of transport costs, it not only raises the steady-state growth rate, but it also increases both countries' long-run fractions and cost shares of imported varieties. Since a country's fraction of imported varieties is the other country's fraction of exported varieties, the aid-induced growth acceleration is accompanied by increased exported varieties for both the donor and recipient countries.

We next study the relationship between m and γ_j^*. Eq. (35) implies that the sign of the slope of the graph $\gamma_j^*(m)$ against m corresponds to the sign of $F(m)$, which contains no endogenous variable. Since $F(m)$ is monotonically decreasing in m, and approaches negative infinity as m approaches its upper bound t_1, there exists $\widehat{m} \in (0, t_1)$ such that $F(\widehat{m}) = 0, F(m) > 0 \forall m \in [0, \widehat{m})$, and $F(m) < 0 \forall m \in (\widehat{m}, t_1)$ if and only if $F(0) = \theta(\mu_2 + \chi_2)/t_2 - (\mu_1 + \chi_1)/t_1 > 0$, or $t_2/t_1 < \theta(\mu_2 + \chi_2)/(\mu_1 + \chi_1)$. From these considerations, we obtain the following proposition:

Proposition 3 *There exists a unique $\widehat{m} \in (0, t_1)$ which maximizes the steady-state growth rate if and only if $t_2/t_1 < \theta(\mu_2 + \chi_2)/(\mu_1 + \chi_1)$. The growth-maximizing aid/GDP ratio $\widehat{m}$ is solved as:*

$$\widehat{m} = [\theta(\mu_2 + \chi_2)t_1 - (\mu_1 + \chi_1)t_2]/[\theta(\mu_1 + \chi_1 + \mu_2 + \chi_2)]. \tag{37}$$

Eqs. (30), (31), and (36) imply that, as the donor's aid/GDP ratio continues to increase, the recipient's effective public services get larger and larger whereas those of the donor get smaller and smaller, and so it becomes more and more difficult for the product of transport costs to fall. This creates an inverted U-shaped relationship between aid for trade and global growth. Eq. (37) shows that the growth-maximizing aid/GDP ratio is larger: (i) the smaller the recipient's income tax rate is relative to the donor's; (ii) the larger the recipient's transport cost elasticities are relative to the donor's; and (iii) the larger the recipient's governance indicator is.[13] The last result is particularly interesting because it is consistent with Burnside and Dollar's (2000) finding that aid is more effective for growth in recipient countries with better policies.

From Eq. (37), we can easily calculate the growth-maximizing aid/GDP ratio using real-world data. We regard an aggregate of high-income countries in the World Development Indicators as the donor, and an aggregate of low- and middle-income countries as the recipient. The income tax rates are given by $t_1 = 0.255$ and $t_2 = 0.171$, which come from the annual averages of the revenue/GDP ratios (excluding grants) during 2001-2010 from the World Development Indicators. The transport cost elasticities are set as $\mu_1 = \mu_2 = 0.34$ and $\chi_1 = \chi_2 = 0.66$ from Limão and Venables (2001). To calibrate θ, we have to multiply the fraction of aid allocated to public services $\theta/(r_1\kappa)$ by the relative GDP of the donor to the recipient $r_1\kappa$. We use 0.326, the share of aid for trade in total sector allocable ODA averaged over eight years during 2002-2009 from OECD and WTO (2011), as the fraction of aid allocated to public services. The relative GDP (at constant 2005 US dollars) of 3.99 is calculated as its ten-year average during 2001-2010 from the World Development

[13](iii) is verified by $\partial\widehat{m}/\partial\theta = (\mu_1 + \chi_1)t_2/[\theta^2(\mu_1 + \chi_1 + \mu_2 + \chi_2)] > 0$.

Indicators. Accordingly, θ is calibrated as $\theta = 0.326 \times 3.99 = 1.301$. Then $\widehat{m}$ is calculated as $\widehat{m} = 0.0615$, or 6.15%. This is much larger than 0.32%, the actual ODA/GNI ratio of total DAC countries in 2010 according to OECD (2012). This suggests that there is indeed enough room for further aid for trade to raise global growth.

4.2 Welfare effects

Since the welfare of country j is the discounted sum of its instantaneous utility $\ln C_{jt} = \ln \rho_j + \ln K_{jt} = \ln \rho_j + \ln K_{j0} + \int_0^t \gamma_{js} ds$, the welfare effects of an increase in aid for trade can be seen by tracing the changes in the growth rates. We first consider the cases where a permanent increase in the donor's aid/GDP ratio from period zero onward raises the steady-state growth rate as in Fig. 2 and Fig. 3. Then the path of the growth rate of at least one of the two countries is higher than point A, the original steady state, for all $s \in [0, \infty)$ (i.e., $\gamma'_{2s} > \gamma^*$ for $r'_{10} < r_1^{*\prime}$; $\gamma'_{1s} > \gamma^*$ for $r'_{10} > r_1^{*\prime}$), meaning that its welfare rises. If r'_{10} is far lower or higher than $r_1^{*\prime}$, however, the path of the growth rate of the other country may be lower than point A in the early periods. In Fig. 2, for example, the donor country starts to grow at point C, which is below the horizontal dashed line passing through point A, although that country goes above that line later on as it moves up and to the right along curve $\gamma_1(\tau_1(g_1(m'), g_2(m', r_1))/r_1)$. Therefore, its welfare also rises as long as the representative household in that country is sufficiently patient (i.e., has a sufficiently low subjective discount rate). In this case, an increase in aid for trade is Pareto-improving. The same is true in Fig. 3, where the recipient country experiences slower growth in the early periods.

What if a permanent increase in the donor's aid/GDP ratio does not raise the steady-state growth rate? Drawing diagrams similar to Fig. 2 and Fig. 3, point D now becomes lower than point A. Since the path of the growth rate of at least one of the two countries is lower than point A for all $s \in [0, \infty)$ (i.e., $\gamma'_{1s} < \gamma^*$ for $r'_{10} < r_1^{*\prime}$; $\gamma'_{2s} < \gamma^*$ for $r'_{10} > r_1^{*\prime}$), its welfare falls. This means that an increase in aid for trade cannot be Pareto-improving then. The following proposition summarizes conditions for Pareto-improving aid for trade:

Proposition 4 .

1. *A permanent increase in the donor's aid/GDP ratio raises the welfare of both countries only if it raises the steady-state growth rate.*

2. *A permanent increase in the donor's aid/GDP ratio raises the welfare of both countries if it raises the steady-state growth rate, and if the subjective discount rate of the country whose growth rate may fall in the initial period is sufficiently low.*

The first statement gives a necessary condition for Pareto-improving aid for trade, requiring that an increase in aid for trade should alleviate the distortions in terms of the product of transport costs. The second statement provides a sufficient condition, ensuring that a country's possible short-run welfare loss is outweighed by its long-run welfare gain.

5 Extensions

In this section, we extend our model in two directions. First, congestion is caused by the capital stock instead of the real GDP (e.g., Barro and Sala-i-Martin, 1992). Second, each public good is modeled as a stock rather than a flow (e.g., Futagami et al., 1993). It turns out that our main results hold under these extensions.

5.1 Congestion caused by capital

Suppose that, in Eq. (25), country j's effective public services are alternatively given by:

$$g_j \equiv G_j/K_j.$$

This means that, given G_j, an increase in K_j (instead of r_jK_j/p_j^Y) decreases g_j. Then Eqs. (22) and (24) are rewritten as:

$$g_1 = (t_1 - m)r_1/p_1^Y = (t_1 - m)/Q_1(\tau_1(g_1, g_2)/r_1, 1), \tag{38}$$

$$g_2 = (t_2 + m\theta)r_2/p_2^Y = (t_2 + m\theta)/Q_2(\tau_2(g_2, g_1)r_1, 1), \tag{39}$$

where Eqs. (25) and (A.7) are used. One difficulty with Eqs. (38) and (39), compared with Eqs. (30) and (31), is the presence of the gross rate of return to capital $r_j/p_j^Y = 1/Q_j(\tau_j(g_j, g_{j'})r_{j'}/r_j, 1)$, which in turn depends on g_1, g_2, and r_1. To proceed, we assume that Eqs. (38) and (39) can be solved for g_1 and g_2 as functions of m and r_1: $g_1 = g_1(m, r_1), g_2 = g_2(m, r_1)$. Eqs. (28) and (29), together with Eq. (25), $g_1 = g_1(m, r_1)$, and $g_2 = g_2(m, r_1)$, constitute the dynamic system. A steady state is determined by (see Appendix E for the stability condition):

$$0 = \gamma_1(\tau_1(g_1(m, r_1^*), g_2(m, r_1^*))/r_1^*) - \gamma_2(\tau_2(g_2(m, r_1^*), g_1(m, r_1^*))r_1^*), \tag{40}$$

$$\kappa^* = \lambda(m, \beta_1(\tau_1(g_1(m, r_1^*), g_2(m, r_1^*))/r_1^*), \beta_2(\tau_2(g_2(m, r_1^*), g_1(m, r_1^*))r_1^*))/r_1^*.$$

To see the growth effects of aid for trade, we first solve Eqs. (38) and (39) for the rates of changes in g_1 and g_2 as follows:

$$dg_1/g_1 = M_1^f dm + R_1^f dr_1/r_1, \tag{41}$$

$$dg_2/g_2 = M_2^f dm + R_2^f dr_1/r_1, \tag{42}$$

where M_1^f, R_1^f, M_2^f, and R_2^f are defined in Appendix C, and a superscript f stands for a "flow" of public good. As Eqs. (38) and (39) indicate, the effects of changes in m and r_1 on g_1 and g_2 are generally ambiguous. An increase in m directly decreases g_1 but increases g_2. Not only that, these first-step changes in g_1 and g_2 indirectly decrease g_2 but increase g_1 through changes in the transport costs. Likewise, a rise in r_1 directly increases g_1 but decreases g_2, which indirectly increases g_2 but decreases g_1. Once Eqs. (41) and (42) are obtained, we can use them together with Eqs. (17), (18), (25), and (40) to derive:

$$d\gamma_1^* = d\gamma_2^* = [\Gamma_1^*\beta_1^*\Gamma_2^*\beta_2^*/(D^{f*}|H^{f*}|)]F(m)dm, \tag{43}$$

where $F(m)$ is exactly the same as that defined in Eq. (35). The only difference between Eq. (43) and Eq. (35) is the denominator in the brackets. In Eq. (43), D^{f*} is the denominator of the expression for

$(dr_1^*/r_1^*)/dm$, and $|H^{f*}|$ is the determinant of the coefficient matrix for the logarithmically differentiated forms of Eqs. (38) and (39) (see Appendix C for definitions). We can show that:

$$D^{f*}|H^{f*}| = \Gamma_1^*\beta_1^*[1 - \beta_2^*(\mu_2 + \chi_2)] + \Gamma_2^*\beta_2^*[1 - \beta_1^*(\mu_1 + \chi_1)].$$

This is positive if $\mu_j + \chi_j < 1/\beta_j^* \forall j$.[14] This condition implies that the direct effects of a change in r_1 on g_1 and g_2 dominate the indirect effects, so that R_1^f and R_2^f have the expected signs: $R_1^f > 0, R_2^f < 0$. Under this mild condition, Proposition 3 holds as is. Moreover, Appendix C shows that Proposition 2 is also true.

5.2 Public capital stock

Suppose that country j's effective public services are specified in the same way as section 5.1:

$$g_j \equiv G_j/K_j,$$

but now G_j is country j's public capital stock. To express accumulation of public capital, we replace Eqs. (22), (23), (24), (26), and (27) with:

$$p_1^Y(\dot{G}_1 + \delta_1 G_1) = t_1 r_1 K_1 - m r_1 K_1; m \in [0, t_1), \tag{44}$$
$$p_2^Y(\dot{G}_2 + \delta_2 G_2 + C_2^G) = t_2 r_2 K_2 + m r_1 K_1, \tag{45}$$
$$p_2^Y(\dot{G}_2 + \delta_2 G_2) = t_2 r_2 K_2 + m\theta r_2 K_2; \theta \in [0, r_1\kappa], \tag{46}$$
$$Y_1 = C_1 + \dot{K}_1 + \delta_1 K_1 + \dot{G}_1 + \delta_1 G_1, \tag{47}$$
$$Y_2 = C_2 + \dot{K}_2 + \delta_2 K_2 + \dot{G}_2 + \delta_2 G_2 + C_2^G. \tag{48}$$

In Eq. (44), for example, the donor government uses its net revenue to buy the final good for public investment. We assume that each country's public capital depreciates at the same rate as its private capital.

In deriving the dynamic system, we first realize that Eqs. (28) and (29) continue to hold.[15] For dynamics of $g_j = G_j/K_j$, we just rewrite $\dot{g}_j/g_j = \dot{G}_j/G_j - \dot{K}_j/K_j$ using Eqs. (28), (44), (46), and (A.7) to obtain:

$$\dot{g}_1/g_1 = [(t_1 - m)/g_1]r_1/p_1^Y - \delta_1 - \gamma_1(\tau_1/r_1) = (t_1 - m)/(g_1 Q_1(\tau_1/r_1, 1)) - \delta_1 - \gamma_1(\tau_1/r_1), \tag{49}$$
$$\dot{g}_2/g_2 = [(t_2 + m\theta)/g_2]r_2/p_2^Y - \delta_2 - \gamma_2(\tau_2 r_1) = (t_2 + m\theta)/(g_2 Q_2(\tau_2 r_1, 1)) - \delta_2 - \gamma_2(\tau_2 r_1). \tag{50}$$

Eqs. (28), (29), (49), and (50), together with Eq. (25), constitute the dynamic system. A steady state is determined by (see Appendix E for the stability condition):

[14] The estimates of $\mu_j = 0.34$ and $\chi_j = 0.66$ by Limão and Venables (2001) automatically satisfy this condition because $1/\beta_j^* > 1$.
[15] Eq. (29) is obtained from Eqs. (4), (12), (13), (14), (20), (44), (45), (47), (48), (A.3), and (A.4).

$$0 = \gamma_1(\tau_1(g_1^*, g_2^*)/r_1^*) - \gamma_2(\tau_2(g_2^*, g_1^*)r_1^*), \tag{51}$$

$$\kappa^* = \lambda(m, \beta_1(\tau_1(g_1^*, g_2^*)/r_1^*), \beta_2(\tau_2(g_2^*, g_1^*)r_1^*))/r_1^*,$$

$$0 = (t_1 - m)/(g_1^* Q_1(\tau_1(g_1^*, g_2^*)/r_1^*, 1)) - \delta_1 - \gamma_1(\tau_1(g_1^*, g_2^*)/r_1^*), \tag{52}$$

$$0 = (t_2 + m\theta)/(g_2^* Q_2(\tau_2(g_2^*, g_1^*)r_1^*, 1)) - \delta_2 - \gamma_2(\tau_2(g_2^*, g_1^*)r_1^*). \tag{53}$$

As far as the steady state is concerned, we can derive the growth effect of aid for trade in the same way as section 5.1. More specifically, we first solve Eqs. (52) and (53) for g_1 and g_2 as functions of m and r_1, then substitute the result into Eq. (51) to solve for r_1^* as a function of m, and finally substitute the result back into $\gamma_1(\cdot)$ to obtain γ_1^* as a function of m. The first stage results in:

$$dg_1/g_1 = M_1^s dm + R_1^s dr_1/r_1, \tag{54}$$

$$dg_2/g_2 = M_2^s dm + R_2^s dr_1/r_1, \tag{55}$$

where M_1^s, R_1^s, M_2^s, and R_2^s are defined in Appendix D, and a superscript s stands for a "stock" of public good. Then, from Eqs. (17), (18), (25), (51), (54), and (55), we obtain:

$$d\gamma_1^* = d\gamma_2^* = [\Gamma_1^* \beta_1^* \Gamma_2^* \beta_2^* (\gamma_1^* + \delta_1)(\gamma_2^* + \delta_2)/(D^{s*}|H^{s*}|)]F(m)dm, \tag{56}$$

where $F(m)$ is the same as that defined in Eq. (35) once again, and D^{s*} and $|H^{s*}|$ defined in Appendix D can be interpreted in a similar way to D^{f*} and $|H^{f*}|$ in section 5.1. Since Appendix D shows that $D^{s*}|H^{s*}|$ is always positive, Proposition 3 is still true. Finally, it is verified in Appendix D that Proposition 2 also holds.

We have shown that our main results are robust to two extensions. In particular, Eqs. (35), (43), and (56) imply that the growth-maximizing aid/GDP ratio (37) stays the same for all three specifications. This is because, in both extended models, the indirect effects of an increase in m on g_1 and g_2 through changes in the transport costs are cancelled out, whereas the direct effects interpreted right after Eq. (36) in section 4.1 work, in affecting the steady-state growth rate.[16] Our logic behind the growth effects of aid for trade based on the original model is solid in broader contexts.

6 Concluding remarks

Our results have some policy implications. First, aid for trade does not always raise global growth. This is because it decreases the donor's public services, and the resulting rises in the transport costs partly weaken the growth potentials of both countries. To achieve faster global growth, we should check if aid for trade makes the world more open as a whole. Second, there may exist an inverted U-shaped relationship between aid for trade and global growth. Hansen and Tarp (2001) find in their growth regressions that the coefficient on aid squared is negative and so large that the turning point after which the aid-growth relationship becomes negative is within the sample range. Our model provides sound theoretical support for their empirical specifications. Third, not only the recipient but also the donor countries may gain from aid

[16] Appendixes C and D show that, in both extended models, the total growth effect can be decomposed into the term including $F(m)$ and the remainder term. Since the latter equals zero, only the former survives.

for trade. The existence of distortions due to transport costs is responsible for Pareto improvement. This serves as further justification for appropriate expansion of the aid for trade initiatives.

There are some directions for future search. First, although we obtain simple and powerful results about the long-run growth effects of aid for trade analytically, its short-run effects are unclear due to the ambiguity of the short-run change in the relative rental rate. To describe the behavior of endogenous variables during the transition, it will be informative to conduct numerical experiments based on our models. By doing so, we will also be able to calculate the exact welfare changes caused by aid for trade. Second, to focus on the effects of a change in the donor's aid/GDP ratio, we assume that the recipient's governance indicator as well as each country's income tax rate is exogenous and constant. However, if the donor government becomes more generous in giving aid, the recipient government might feel more tempted to line its own pockets. Then the donor will also choose its aid/GDP ratio, taking the recipient's reaction into account. It will be interesting to formulate such a game-theoretic model of aid, governance, and growth.

Appendix A. Derivation of Eqs. (15) and (16)

Here we derive the dynamic system of the model with exogenous transport costs. From Eqs. (5), (7), (9), and (10), the demand prices are given by:

$$
\begin{aligned}
p_1(i_1) &= p(i_1) = r_1 a_1(i_1), i_1 \in [0, I_1], \\
p_1(i_2) &= \tau_1 p(i_2) = \tau_1 r_2 a_2(i_2), i_2 \in [I_1, 1], \\
p_2(i_2) &= p(i_2) = r_2 a_2(i_2), i_2 \in [I_2, 1], \\
p_2(i_1) &= \tau_2 p(i_1) = \tau_2 r_1 a_1(i_1), i_1 \in [0, I_2].
\end{aligned}
$$

Then the intermediate good price indexes (3) for the two countries are rewritten as:

$$
\begin{aligned}
P_1 &= Z_1^{-1}[(\tau_1 r_2)^{1-\sigma_1} \int_{I_1}^{1} a_2(i_2)^{1-\sigma_1} di_2 + r_1^{1-\sigma_1} \int_0^{I_1} a_1(i_1)^{1-\sigma_1} di_1]^{1/(1-\sigma_1)} \\
&\equiv \widetilde{Q}_1(\tau_1 r_2, r_1, I_1), \quad (A.1) \\
P_2 &= Z_2^{-1}[(\tau_2 r_1)^{1-\sigma_2} \int_0^{I_2} a_1(i_1)^{1-\sigma_2} di_1 + r_2^{1-\sigma_2} \int_{I_2}^{1} a_2(i_2)^{1-\sigma_2} di_2]^{1/(1-\sigma_2)} \\
&\equiv \widetilde{Q}_2(\tau_2 r_1, r_2, I_2). \quad (A.2)
\end{aligned}
$$

Country j's simplified intermediate good price index function, Eq. (A.1) or (A.2), is increasing and homogeneous of degree one in $(\tau_j r_{j'}, r_j)$, with its cutoff variety I_j given. Substituting Eqs. (6) and (8) into Eqs. (A.1) and (A.2), we define $\widetilde{Q}_1(\tau_1 r_2, r_1, I_1(\tau_1 r_2/r_1)) \equiv Q_1(\tau_1 r_2, r_1)$ and $\widetilde{Q}_2(\tau_2 r_1, r_2, I_2(\tau_2 r_1/r_2)) \equiv Q_2(\tau_2 r_1, r_2)$, respectively. Next, deriving the demand for an imported variety $x_j(i_{j'}) = (\partial P_j/\partial p_j(i_{j'}))Y_j = Z_j^{\sigma_j - 1} P_j^{\sigma_j} p_j(i_{j'})^{-\sigma_j} Y_j$ from Eq. (3), and using Eqs. (A.1) and (A.2), the shares of imported varieties in the total cost of the final good for the two countries are calculated as:

$$\int_{I_1}^{1} p_1(i_2)x_1(i_2)di_2/(P_1Y_1) = (Z_1Q_1(1, r_1/(\tau_1 r_2)))^{\sigma_1-1}\int_{I_1}^{1} a_2(i_2)^{1-\sigma_1}di_2$$
$$\equiv \widetilde{\beta}_1(\tau_1 r_2/r_1, I_1) \in (0,1), \tag{A.3}$$
$$\int_{0}^{I_2} p_2(i_1)x_2(i_1)di_1/(P_2Y_2) = (Z_2Q_2(1, r_2/(\tau_2 r_1)))^{\sigma_2-1}\int_{0}^{I_2} a_1(i_1)^{1-\sigma_2}di_1$$
$$\equiv \widetilde{\beta}_2(\tau_2 r_1/r_2, I_2) \in (0,1). \tag{A.4}$$

Country j's import share function, Eq. (A.3) or (A.4), serving as a natural indicator of openness, is decreasing in $\tau_j r_{j'}/r_j$, with I_j given. Substituting Eqs. (6) and (8) into Eqs. (A.3) and (A.4), we define $\widetilde{\beta}_1(\tau_1 r_2/r_1, I_1(\tau_1 r_2/r_1)) \equiv \beta_1(\tau_1 r_2/r_1)$ and $\widetilde{\beta}_2(\tau_2 r_1/r_2, I_2(\tau_2 r_1/r_2)) \equiv \beta_2(\tau_2 r_1/r_2)$, respectively.

For reference, we examine the properties of $Q_j(\cdot)$ and $\beta_j(\cdot)$. First, logarithmically differentiating Eqs. (A.1) and (A.2), and using Eqs. (6), (8), (A.3), and (A.4), we obtain:

$$dQ_j/Q_j = \beta_j(d\tau_j/\tau_j + dr_{j'}/r_{j'}) + (1-\beta_j)dr_j/r_j. \tag{A.5}$$

It turns out that each country's price index is independent of its cutoff variety. Second, logarithmically differentiating Eqs. (A.3) and (A.4), noting that $Q_j(1, r_j/(\tau_j r_{j'})) = Q_j(\tau_j r_{j'}, r_j)/(\tau_j r_{j'})$, and using Eqs. (6), (8), and (A.5), we obtain:

$$d\beta_j/\beta_j = -B_j(d\tau_j/\tau_j + dr_{j'}/r_{j'} - dr_j/r_j); \tag{A.6}$$
$$B_1 \equiv (\sigma_1-1)(1-\beta_1) - (I_1 a_2(I_1)^{1-\sigma_1}/\int_{I_1}^{1} a_2(i_2)^{1-\sigma_1}di_2)A/(A'I_1) > 0,$$
$$B_2 \equiv (\sigma_2-1)(1-\beta_2) - (I_2 a_1(I_2)^{1-\sigma_2}/\int_{0}^{I_2} a_1(i_1)^{1-\sigma_2}di_1)A/(A'I_2) > 0.$$

In the definition of the elasticity B_j, the first term captures the change in the demand for the existing imported varieties, whereas the second term indicates the change in the set of imported varieties. These changes are called adjustments at the intensive and extensive margins, respectively.

We now turn to country j's growth rate (2), which is just the difference between its gross rate of return to capital r_j/p_j^Y and the sum of its depreciation and subjective discount rates $\delta_j + \rho_j$. From Eqs. (4), (A.1), and (A.2), we obtain:

$$r_j/p_j^Y = r_j/Q_j(\tau_j r_{j'}, r_j) = 1/Q_j(\tau_j r_{j'}/r_j, 1). \tag{A.7}$$

This means that country j's growth rate is decreasing in $\tau_j r_{j'}/r_j$. From Eqs. (6), (8), (A.6), and (A.7), we obtain Lemma 1.

Finally, we derive the dynamic system. Eq. (15) is obtained from Eqs. (2), (A.7), and the definition of κ. Eq. (16) is equivalent to $\beta_1 r_1 K_1 = \beta_2 r_2 K_2$, which in turn is derived by rewriting country 1's capital market-clearing condition (12) using Eqs. (1), (4), (11), (13), (14), (A.3), and (A.4).

Appendix B. Proof of Lemma 2

We first show the existence and uniqueness of r_1^*. From Eqs. (6), (8), (A.1), and (A.2), we have:

$$
\begin{aligned}
Q_1(\tau_1/r_1,1) &= Z_1^{-1}[(\tau_1/r_1)^{1-\sigma_1}\int_{I_1(\tau_1/r_1)}^{1} a_2(i_2)^{1-\sigma_1}di_2 + \int_0^{I_1(\tau_1/r_1)} a_1(i_1)^{1-\sigma_1}di_1]^{1/(1-\sigma_1)},\\
Q_2(\tau_2 r_1,1) &= Z_2^{-1}[(\tau_2 r_1)^{1-\sigma_2}\int_0^{I_2(\tau_2 r_1)} a_1(i_1)^{1-\sigma_2}di_1 + \int_{I_2(\tau_2 r_1)}^{1} a_2(i_2)^{1-\sigma_2}di_2]^{1/(1-\sigma_2)}.
\end{aligned}
$$

Since these imply that $\lim_{r_1\to 0} Q_1(\tau_1/r_1,1) = Z_1^{-1}(\int_0^1 a_1(i_1)^{1-\sigma_1}di_1)^{1/(1-\sigma_1)} > 0, \lim_{r_1\to\infty} Q_1(\tau_1/r_1,1) = 0, \lim_{r_1\to 0} Q_2(\tau_2 r_1,1) = 0$, and $\lim_{r_1\to\infty} Q_2(\tau_2 r_1,1) = Z_2^{-1}(\int_0^1 a_2(i_2)^{1-\sigma_2}di_2)^{1/(1-\sigma_2)} > 0$, we have $\lim_{r_1\to 0}(\gamma_1(\tau_1/r_1) - \gamma_2(\tau_2 r_1)) = -\infty < 0$ and $\lim_{r_1\to\infty}(\gamma_1(\tau_1/r_1) - \gamma_2(\tau_2 r_1)) = \infty > 0$. Hence, from the intermediate value theorem, there exists $r_1^* \in (0,\infty)$ such that $0 = \gamma_1(\tau_1/r_1^*) - \gamma_2(\tau_2 r_1^*)$ holds. Moreover, since $\gamma_1(\tau_1/r_1)$ is increasing in r_1 whereas $\gamma_2(\tau_2 r_1)$ is decreasing in r_1 from Lemma 1, such r_1^* is unique. On the other hand, Since $\beta_1(\tau_1/r_1)$ is increasing in r_1 whereas $\beta_2(\tau_2 r_1)$ is decreasing in r_1 from Lemma 1, $\beta_2(\tau_2 r_1)/\beta_1(\tau_1/r_1)$ is decreasing in r_1, and so is $(\beta_2/\beta_1)/r_1$. Hence, $\kappa^* = (\beta_2^*/\beta_1^*)/r_1^*$ is also unique.

Turning to transitional dynamics, $\dot{\kappa}$ is increasing in r_1 from Eq. (15), whereas r_1 is decreasing in κ from Eq. (16). Therefore, $\dot{\kappa} > 0$ if and only if $\kappa < \kappa^*$, ensuring the global stability.

Appendix C. Derivations of results in section 5.1

Logarithmically differentiating Eqs. (38) and (39), and using Eqs. (4), (25), and (A.5), we obtain:

$$
H^f \begin{bmatrix} dg_1/g_1 \\ dg_2/g_2 \end{bmatrix} = \begin{bmatrix} -1/(t_1-m) \\ \theta/(t_2+m\theta) \end{bmatrix} dm + \begin{bmatrix} \beta_1 \\ -\beta_2 \end{bmatrix} dr_1/r_1; \tag{C.1}
$$

$$
\begin{aligned}
H^f &\equiv \begin{bmatrix} h_{11}^f & h_{12}^f \\ h_{21}^f & h_{22}^f \end{bmatrix},\\
h_{11}^f &\equiv 1-\beta_1\mu_1,\\
h_{12}^f &\equiv -\beta_1\chi_2 < 0,\\
h_{21}^f &\equiv -\beta_2\chi_1 < 0,\\
h_{22}^f &\equiv 1-\beta_2\mu_2.
\end{aligned}
$$

The determinant of the coefficient matrix H^f is:

$$
\begin{aligned}
|H^f| &= h_{11}^f h_{22}^f - h_{12}^f h_{21}^f\\
&= (1-\beta_1\mu_1)(1-\beta_2\mu_2) - \beta_1\chi_2\beta_2\chi_1\\
&= 1-\beta_2\mu_2-\beta_1\mu_1+\beta_1\beta_2(\mu_1\mu_2-\chi_2\chi_1).
\end{aligned}
$$

Solving Eq. (C.1) for dg_1/g_1 and dg_2/g_2 gives Eqs. (41) and (42), where:

$$
\begin{aligned}
M_1^f &\equiv (\partial g_1/g_1)/\partial m = (1/|H^f|)\{-[1/(t_1-m)](1-\beta_2\mu_2)+\beta_1\chi_2\theta/(t_2+m\theta)\},\\
M_2^f &\equiv (\partial g_2/g_2)/\partial m = (1/|H^f|)\{(1-\beta_1\mu_1)\theta/(t_2+m\theta)-[1/(t_1-m)]\beta_2\chi_1\},\\
R_1^f &\equiv (\partial g_1/g_1)/(\partial r_1/r_1) = (1/|H^f|)\beta_1[1-\beta_2(\mu_2+\chi_2)],\\
R_2^f &\equiv (\partial g_2/g_2)/(\partial r_1/r_1) = -(1/|H^f|)\beta_2[1-\beta_1(\mu_1+\chi_1)].
\end{aligned}
$$

Next, using Eqs. (25), (41), and (42), Eqs. (17) and (18) are rewritten as:

$$
d\gamma_1 = \Gamma_1\beta_1[(1+\mu_1R_1^f+\chi_2R_2^f)dr_1/r_1+(\mu_1M_1^f+\chi_2M_2^f)dm], \tag{C.2}
$$

$$
d\gamma_2 = -\Gamma_2\beta_2[-(\chi_1M_1^f+\mu_2M_2^f)dm+(1-\chi_1R_1^f-\mu_2R_2^f)dr_1/r_1]; \tag{C.3}
$$

$$
\begin{aligned}
1+\mu_1R_1^f+\chi_2R_2^f &= (1/|H^f|)[1-\beta_2(\mu_2+\chi_2)] = R_1^f/\beta_1,\\
1-\chi_1R_1^f-\mu_2R_2^f &= (1/|H^f|)[1-\beta_1(\mu_1+\chi_1)] = -R_2^f/\beta_2,\\
\mu_1M_1^f+\chi_2M_2^f &= (1/|H^f|)[-\mu_1/(t_1-m)+\chi_2\theta/(t_2+m\theta)\\
&\quad+\beta_2(\mu_1\mu_2-\chi_2\chi_1)/(t_1-m)],\\
\chi_1M_1^s+\mu_2M_2^s &= (1/|H^f|)[-\chi_1/(t_1-m)+\mu_2\theta/(t_2+m\theta)\\
&\quad-\beta_1(\mu_2\mu_1-\chi_1\chi_2)\theta/(t_2+m\theta)].
\end{aligned}
$$

Substituting Eqs. (C.2) and (C.3) into the totally differentiated form of Eq. (40): $0 = d\gamma_1^* - d\gamma_2^*$, and solving it for dr_1^*/r_1^*, we obtain:

$$
\begin{aligned}
dr_1^*/r_1^* &= (E^{f*}/D^{f*})dm; \qquad \text{(C.4)}\\
D^{f*} &\equiv \Gamma_1^*\beta_1^*(1+\mu_1R_1^{f*}+\chi_2R_2^{f*})+\Gamma_2^*\beta_2^*(1-\chi_1R_1^{f*}-\mu_2R_2^{f*}),\\
E^{f*} &\equiv -\Gamma_1^*\beta_1^*(\mu_1M_1^{f*}+\chi_2M_2^{f*})+\Gamma_2^*\beta_2^*(\chi_1M_1^{f*}+\mu_2M_2^{f*}).
\end{aligned}
$$

Substituting Eq. (C.4) back into the expression in the brackets of Eq. (C.2) divided by dm, we have:

$$
\begin{aligned}
&(1+\mu_1R_1^{f*}+\chi_2R_2^{f*})(dr_1^*/r_1^*)/dm+\mu_1M_1^{f*}+\chi_2M_2^{f*}\\
&= (\Gamma_2^*\beta_2^*/D^{f*})[(1/|H^{f*}|)F(m)+O^{f*}];\\
O^{f*} &\equiv -(1/|H^{f*}|)\beta_1^*(\mu_2\mu_1-\chi_1\chi_2)\theta/(t_2+m\theta)+(1/|H^{f*}|)\beta_2^*(\mu_1\mu_2-\chi_2\chi_1)/(t_1-m)\\
&\quad+(\mu_1R_1^{f*}+\chi_2R_2^{f*})(\chi_1M_1^{f*}+\mu_2M_2^{f*})-(\chi_1R_1^{f*}+\mu_2R_2^{f*})(\mu_1M_1^{f*}+\chi_2M_2^{f*}).
\end{aligned}
$$

Since we can show that $O^{f*} = 0$, we obtain Eq. (43). If $\mu_j+\chi_j < 1/\beta_j^*\forall j$, then we have $D^{f*}|H^{f*}| = \Gamma_1^*\beta_1^*[1-\beta_2^*(\mu_2+\chi_2)]+\Gamma_2^*\beta_2^*[1-\beta_1^*(\mu_1+\chi_1)] > 0$. Moreover, since $|H^{f*}| = (1-\beta_1^*\mu_1)(1-\beta_2^*\mu_2)-\beta_1^*\chi_2\beta_2^*\chi_1 > \beta_1^*\chi_1\beta_2^*\chi_2-\beta_1^*\chi_2\beta_2^*\chi_1 = 0$, we have $D^{f*} > 0, R_1^{f*} > 0$, and $R_2^{f*} < 0$.

Finally, from Eqs. (25), (41), (42), and (C.4), we obtain:

$$d\tau_1^*/\tau_1^* + d\tau_2^*/\tau_2^* = -[(\Gamma_1^*\beta_1^* + \Gamma_2^*\beta_2^*)/(D^{f*}|H^{f*}|)]F(m)dm. \quad \text{(C.5)}$$

Eqs. (43) and (C.5) imply that Proposition 2 is true.

Appendix D. Derivations of results in section 5.2

Logarithmically differentiating Eqs. (52) and (53), or:

$$\begin{aligned}
[(t_1 - m)/g_1]r_1/p_1^Y &= \gamma_1 + \delta_1, \\
[(t_2 + m\theta)/g_2]r_2/p_2^Y &= \gamma_2 + \delta_2,
\end{aligned}$$

and using Eqs. (4), (17), (18), (25), and (A.5), we obtain:

$$H^s \begin{bmatrix} dg_1/g_1 \\ dg_2/g_2 \end{bmatrix} = \begin{bmatrix} -(\gamma_1 + \delta_1)/(t_1 - m) \\ (\gamma_2 + \delta_2)\theta/(t_2 + m\theta) \end{bmatrix} dm + \begin{bmatrix} -\rho_1\beta_1 \\ \rho_2\beta_2 \end{bmatrix} dr_1/r_1; \quad \text{(D.1)}$$

$$\begin{aligned}
H^s &\equiv \begin{bmatrix} h_{11}^s & h_{12}^s \\ h_{21}^s & h_{22}^s \end{bmatrix}, \\
h_{11}^s &\equiv \gamma_1 + \delta_1 + \rho_1\beta_1\mu_1 > 0, \\
h_{12}^s &\equiv \rho_1\beta_1\chi_2 > 0, \\
h_{21}^s &\equiv \rho_2\beta_2\chi_1 > 0, \\
h_{22}^s &\equiv \gamma_2 + \delta_2 + \rho_2\beta_2\mu_2 > 0.
\end{aligned}$$

The determinant of the coefficient matrix is:

$$\begin{aligned}
|H^s| &= h_{11}^s h_{22}^s - h_{12}^s h_{21}^s \\
&= (\gamma_1 + \delta_1 + \rho_1\beta_1\mu_1)(\gamma_2 + \delta_2 + \rho_2\beta_2\mu_2) - \rho_1\beta_1\chi_2\rho_2\beta_2\chi_1 \\
&= (\gamma_1 + \delta_1)(\gamma_2 + \delta_2 + \rho_2\beta_2\mu_2) + \rho_1\beta_1\mu_1(\gamma_2 + \delta_2) + \rho_1\beta_1\rho_2\beta_2(\mu_1\mu_2 - \chi_2\chi_1) \\
&= (\gamma_2 + \delta_2)(\gamma_1 + \delta_1 + \rho_1\beta_1\mu_1) + \rho_2\beta_2\mu_2(\gamma_1 + \delta_1) + \rho_2\beta_2\rho_1\beta_1(\mu_2\mu_1 - \chi_1\chi_2).
\end{aligned}$$

Eq. (D.1) is solved for Eqs. (54) and (55), where:

$$\begin{aligned}
M_1^s &\equiv (\partial g_1/g_1)/\partial m = (1/|H^s|)\{-[(\gamma_1 + \delta_1)/(t_1 - m)]h_{22}^s - h_{12}^s(\gamma_2 + \delta_2)\theta/(t_2 + m\theta)\}, \\
M_2^s &\equiv (\partial g_2/g_2)/\partial m = (1/|H^s|)\{h_{11}^s(\gamma_2 + \delta_2)\theta/(t_2 + m\theta) + [(\gamma_1 + \delta_1)/(t_1 - m)]h_{21}^s\}, \\
R_1^s &\equiv (\partial g_1/g_1)/(\partial r_1/r_1) = (1/|H^s|)(-\rho_1\beta_1 h_{22}^s - h_{12}^s\rho_2\beta_2), \\
R_2^s &\equiv (\partial g_2/g_2)/(\partial r_1/r_1) = (1/|H^s|)(h_{11}^s\rho_2\beta_2 + \rho_1\beta_1 h_{21}^s).
\end{aligned}$$

Using Eqs. (25), (54), and (55), Eqs. (17) and (18) are rewritten as:

$$d\gamma_1 = \Gamma_1\beta_1[(1+\mu_1 R_1^s + \chi_2 R_2^s)dr_1/r_1 + (\mu_1 M_1^s + \chi_2 M_2^s)dm], \tag{D.2}$$

$$d\gamma_2 = -\Gamma_2\beta_2[-(\chi_1 M_1^s + \mu_2 M_2^s)dm + (1-\chi_1 R_1^s - \mu_2 R_2^s)dr_1/r_1]; \tag{D.3}$$

$$\begin{aligned}
1+\mu_1 R_1^s + \chi_2 R_2^s &= (1/|H^s|)(\gamma_1+\delta_1)[\gamma_2+\delta_2+\rho_2\beta_2(\mu_2+\chi_2)],\\
1-\chi_1 R_1^s - \mu_2 R_2^s &= (1/|H^s|)(\gamma_2+\delta_2)[\gamma_1+\delta_1+\rho_1\beta_1(\mu_1+\chi_1)],\\
\mu_1 M_1^s + \chi_2 M_2^s &= (1/|H^s|)\{-[(\gamma_1+\delta_1)/(t_1-m)]\mu_1(\gamma_2+\delta_2)\\
&\quad + [(\gamma_2+\delta_2)\theta/(t_2+m\theta)]\chi_2(\gamma_1+\delta_1)\\
&\quad - [(\gamma_1+\delta_1)/(t_1-m)]\rho_2\beta_2(\mu_1\mu_2-\chi_2\chi_1)\},\\
\chi_1 M_1^s + \mu_2 M_2^s &= (1/|H^s|)\{-[(\gamma_1+\delta_1)/(t_1-m)]\chi_1(\gamma_2+\delta_2)\\
&\quad + [(\gamma_2+\delta_2)\theta/(t_2+m\theta)]\mu_2(\gamma_1+\delta_1)\\
&\quad + [(\gamma_2+\delta_2)\theta/(t_2+m\theta)]\rho_1\beta_1(\mu_2\mu_1-\chi_1\chi_2)\}.
\end{aligned}$$

From Eqs. (51), (D.2), and (D.3), dr_1^*/r_1^* is solved as:

$$\begin{aligned}
&dr_1^*/r_1^* = (E^{s*}/D^{s*})dm; \qquad \text{(D.4)}\\
&\quad D^{s*} \equiv \Gamma_1^*\beta_1^*(1+\mu_1 R_1^{s*}+\chi_2 R_2^{s*}) + \Gamma_2^*\beta_2^*(1-\chi_1 R_1^{s*}-\mu_2 R_2^{s*}),\\
&\quad E^{s*} \equiv -\Gamma_1^*\beta_1^*(\mu_1 M_1^{s*}+\chi_2 M_2^{s*}) + \Gamma_2^*\beta_2^*(\chi_1 M_1^{s*}+\mu_2 M_2^{s*}).
\end{aligned}$$

Substituting Eq. (D.4) back into the expression in the brackets of Eq. (D.2) divided by dm, we have:

$$\begin{aligned}
&(1+\mu_1 R_1^{s*}+\chi_2 R_2^{s*})(dr_1^*/r_1^*)/dm + \mu_1 M_1^{s*} + \chi_2 M_2^{s*}\\
&= (\Gamma_2^*\beta_2^*/D^{s*})\{[(\gamma_1^*+\delta_1)(\gamma_2^*+\delta_2)/|H^{s*}|]F(m) + O^{s*}\};\\
O^{s*} &\equiv (1/|H^{s*}|)[(\gamma_2^*+\delta_2)\theta/(t_2+m\theta)]\rho_1\beta_1^*(\mu_2\mu_1-\chi_1\chi_2)\\
&- (1/|H^{s*}|)[(\gamma_1^*+\delta_1)/(t_1-m)]\rho_2\beta_2^*(\mu_1\mu_2-\chi_2\chi_1)\\
&+ (\mu_1 R_1^{s*}+\chi_2 R_2^{s*})(\chi_1 M_1^{s*}+\mu_2 M_2^{s*}) - (\chi_1 R_1^{s*}+\mu_2 R_2^{s*})(\mu_1 M_1^{s*}+\chi_2 M_2^{s*}).
\end{aligned}$$

Since it can be verified that $O^{s*}=0$, we obtain Eq. (56), where:

$$D^{s*}|H^{s*}| = \Gamma_1^*\beta_1^*(\gamma_1^*+\delta_1)[\gamma_2^*+\delta_2+\rho_2\beta_2^*(\mu_2+\chi_2)] + \Gamma_2^*\beta_2^*(\gamma_2^*+\delta_2)[\gamma_1^*+\delta_1+\rho_1\beta_1^*(\mu_1+\chi_1)] > 0.$$

Finally, from Eqs. (25), (54), (55), and (D.4), we obtain:

$$d\tau_1^*/\tau_1^* + d\tau_2^*/\tau_2^* = -[(\Gamma_1^*\beta_1^*+\Gamma_2^*\beta_2^*)(\gamma_1^*+\delta_1)(\gamma_2^*+\delta_2)/(D^{s*}|H^{s*}|)]F(m)dm. \tag{D.5}$$

Eqs. (56) and (D.5) imply that Proposition 2 holds.

Appendix E. Stability conditions for the extended models

The model with congestion caused by capital

The model consists of Eqs. (28) and (29), together with Eqs. (25), (38), and (39). First, linearizing Eq. (28) around the steady state, and using Eqs. (40), (C.2), and (C.3), we obtain:

$$d(\ln \kappa - \ln \kappa^*)/dt = D^{f*}(\ln r_1 - \ln r_1^*). \tag{E.1}$$

Second, linearizing Eq. (29) around the steady state, and using Eqs. (25), (41), (42), and (A.6), we have:

$$\begin{aligned}
&\ln r_1 - \ln r_1^* = -(1/\Omega^*)(\ln \kappa - \ln \kappa^*); \\
&\qquad \Omega^* \equiv 1 - \Lambda_1^* B_1^* + \Lambda_2^* B_2^* - (\Lambda_1^* B_1^* \mu_1 + \Lambda_2^* B_2^* \chi_1) R_1^{f*} - (\Lambda_1^* B_1^* \chi_2 + \Lambda_2^* B_2^* \mu_2) R_2^{f*}, \\
&\qquad \Lambda_1^* \equiv \partial \ln \lambda^* / \partial \ln \beta_1^* = -(1-m)\beta_1^*/[m(1-\beta_1^*-\beta_2^*)+\beta_1^*] < 0, \\
&\qquad \Lambda_2^* \equiv \partial \ln \lambda^* / \partial \ln \beta_2^* = [m(1-\beta_1^*)+\beta_1^*]/[m(1-\beta_1^*-\beta_2^*)+\beta_1^*] > 0.
\end{aligned} \tag{E.2}$$

Combining Eqs. (E.1) and (E.2), we obtain:

$$d(\ln \kappa - \ln \kappa^*)/dt = -(D^{f*}/\Omega^*)(\ln \kappa - \ln \kappa^*). \tag{E.3}$$

Assuming that $\mu_j + \chi_j < 1/\beta_j^* \forall j \Rightarrow D^{f*} > 0, R_1^{f*} > 0, R_2^{f*} < 0$ as in Appendix C, Eq. (E.3) is stable around the steady state if and only if:

$$\Omega^* > 0.$$

This is true unless χ_1 and χ_2 are too high.

The model with public capital stock

The model consists of Eqs. (28), (29), (49), and (50), together with Eq. (25). First, linearizing Eq. (28) around the steady state, and using Eqs. (17), (18), (25), and (51), we obtain:

$$\begin{aligned}
d(\ln \kappa - \ln \kappa^*)/dt &= (\Gamma_1^* \beta_1^* + \Gamma_2^* \beta_2^*)(\ln r_1 - \ln r_1^*) \\
&\quad + (\Gamma_1^* \beta_1^* \mu_1 - \Gamma_2^* \beta_2^* \chi_1)(\ln g_1 - \ln g_1^*) + (\Gamma_1^* \beta_1^* \chi_2 - \Gamma_2^* \beta_2^* \mu_2)(\ln g_2 - \ln g_2^*).
\end{aligned} \tag{E.4}$$

Next, linearizing Eq. (29) around the steady state, and using Eqs. (25) and (A.6), we have:

$$\begin{aligned}
&\ln r_1 - \ln r_1^* = S_\kappa^*(\ln \kappa - \ln \kappa^*) + S_1^*(\ln g_1 - \ln g_1^*) + S_2^*(\ln g_2 - \ln g_2^*); \\
&\qquad S_\kappa^* \equiv -1/(1 - \Lambda_1^* B_1^* + \Lambda_2^* B_2^*) < 0, \\
&\qquad S_1^* \equiv (\Lambda_1^* B_1^* \mu_1 + \Lambda_2^* B_2^* \chi_1)/(1 - \Lambda_1^* B_1^* + \Lambda_2^* B_2^*), \\
&\qquad S_2^* \equiv (\Lambda_1^* B_1^* \chi_2 + \Lambda_2^* B_2^* \mu_2)/(1 - \Lambda_1^* B_1^* + \Lambda_2^* B_2^*).
\end{aligned} \tag{E.5}$$

Moreover, linearizing Eqs. (49) and (50) around the steady state, and using Eqs. (17), (18), (25), (52), (53), and (A.5), we obtain:

$$d(\ln g_1 - \ln g_1^*)/dt = -\rho_1\beta_1^*(\ln r_1 - \ln r_1^*) - h_{11}^{s*}(\ln g_1 - \ln g_1^*) - h_{12}^{s*}(\ln g_2 - \ln g_2^*), \tag{E.6}$$

$$d(\ln g_2 - \ln g_2^*)/dt = \rho_2\beta_2^*(\ln r_1 - \ln r_1^*) - h_{21}^{s*}(\ln g_1 - \ln g_1^*) - h_{22}^{s*}(\ln g_2 - \ln g_2^*). \tag{E.7}$$

Finally, substituting Eq. (E.5) into Eqs. (E.4), (E.6), and (E.7), the linearized dynamic system is given by:

$$\begin{bmatrix} d(\ln\kappa - \ln\kappa^*)/dt \\ d(\ln g_1 - \ln g_1^*)/dt \\ d(\ln g_2 - \ln g_2^*)/dt \end{bmatrix} = J^{s*} \begin{bmatrix} \ln\kappa - \ln\kappa^* \\ \ln g_1 - \ln g_1^* \\ \ln g_2 - \ln g_2^* \end{bmatrix}; J^{s*} \equiv \begin{bmatrix} j_{11}^{s*} & j_{12}^{s*} & j_{13}^{s*} \\ j_{21}^{s*} & j_{22}^{s*} & j_{23}^{s*} \\ j_{31}^{s*} & j_{32}^{s*} & j_{33}^{s*} \end{bmatrix}, \tag{E.8}$$

$$\begin{aligned}
j_{11}^{s*} &\equiv (\Gamma_1^*\beta_1^* + \Gamma_2^*\beta_2^*)S_\kappa^* < 0, \\
j_{12}^{s*} &\equiv \Gamma_1^*\beta_1^*\mu_1 - \Gamma_2^*\beta_2^*\chi_1 + (\Gamma_1^*\beta_1^* + \Gamma_2^*\beta_2^*)S_1^*, \\
j_{13}^{s*} &\equiv \Gamma_1^*\beta_1^*\chi_2 - \Gamma_2^*\beta_2^*\mu_2 + (\Gamma_1^*\beta_1^* + \Gamma_2^*\beta_2^*)S_2^*, \\
j_{21}^{s*} &\equiv -\rho_1\beta_1^*S_\kappa^* > 0, \\
j_{22}^{s*} &\equiv -h_{11}^{s*} - \rho_1\beta_1^*S_1^*, \\
j_{23}^{s*} &\equiv -h_{12}^{s*} - \rho_1\beta_1^*S_2^*, \\
j_{31}^{s*} &\equiv \rho_2\beta_2^*S_\kappa^* < 0, \\
j_{32}^{s*} &\equiv -h_{21}^{s*} + \rho_2\beta_2^*S_1^*, \\
j_{33}^{s*} &\equiv -h_{22}^{s*} + \rho_2\beta_2^*S_2^*.
\end{aligned}$$

The characteristic polynomial associated with the Jacobian matrix J^{s*} is:

$$\begin{aligned}
&\varphi(J^{s*}) \\
&\equiv \det(zI - J^{s*}) \\
&= \begin{vmatrix} z - j_{11}^{s*} & -j_{12}^{s*} & -j_{13}^{s*} \\ -j_{21}^{s*} & z - j_{22}^{s*} & -j_{23}^{s*} \\ -j_{31}^{s*} & -j_{32}^{s*} & z - j_{33}^{s*} \end{vmatrix} \\
&= z^3 + J_1^{s*}z^2 + J_2^{s*}z + J_3^{s*}; \\
J_1^{s*} &\equiv -(j_{11}^{s*} + j_{22}^{s*} + j_{33}^{s*}), \\
J_2^{s*} &\equiv j_{22}^{s*}j_{33}^{s*} - j_{23}^{s*}j_{32}^{s*} + j_{33}^{s*}j_{11}^{s*} - j_{31}^{s*}j_{13}^{s*} + j_{11}^{s*}j_{22}^{s*} - j_{12}^{s*}j_{21}^{s*}, \\
J_3^{s*} &\equiv -(j_{11}^{s*}j_{22}^{s*}j_{33}^{s*} + j_{12}^{s*}j_{23}^{s*}j_{31}^{s*} + j_{13}^{s*}j_{21}^{s*}j_{32}^{s*} - j_{13}^{s*}j_{22}^{s*}j_{31}^{s*} - j_{12}^{s*}j_{21}^{s*}j_{33}^{s*} - j_{11}^{s*}j_{23}^{s*}j_{32}^{s*}).
\end{aligned}$$

Noting that κ, g_1, and g_2 are all state variables, Eq. (E.8) is stable around the steady state if and only if all eigenvalues of the characteristic equation $\varphi(J^{s*}) = 0$ have negative real parts, which is true if and only if (e.g., Chiang and Wainwright, 2005, p. 542):

$$J_1^{s*} > 0, J_2^{s*} > 0, J_3^{s*} > 0, J_1^{s*} J_2^{s*} - J_3^{s*} > 0.$$

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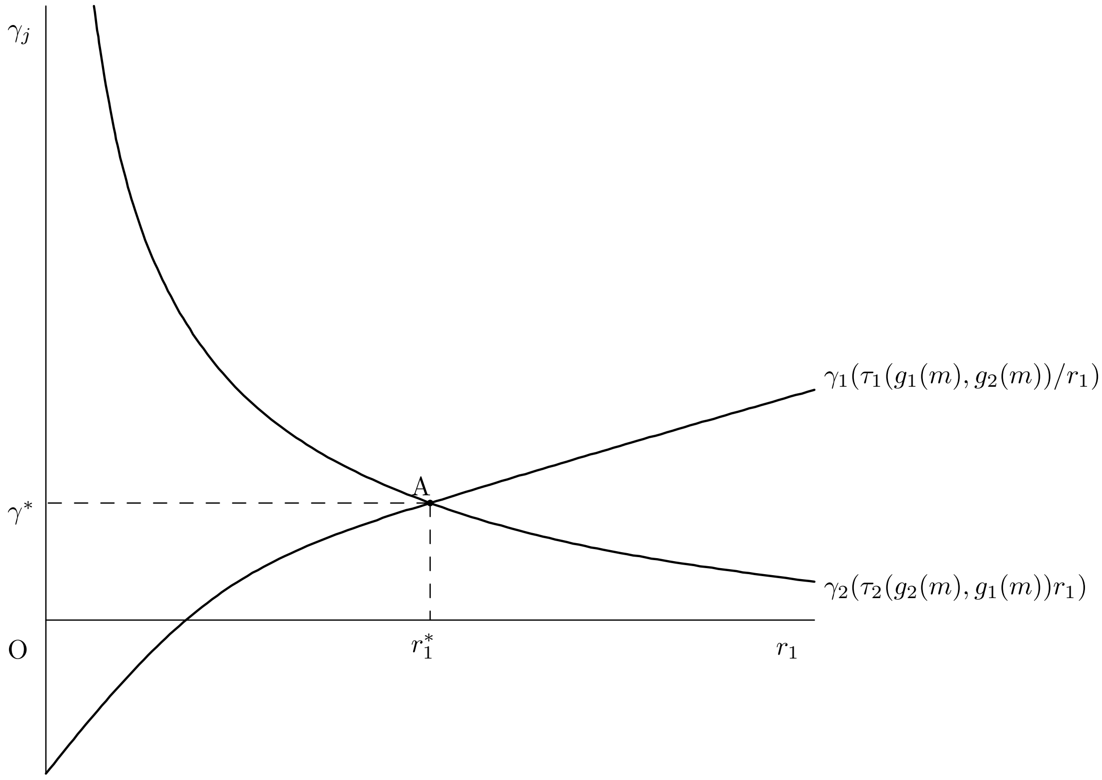

Fig. 1. Steady state of the model with aid for trade.

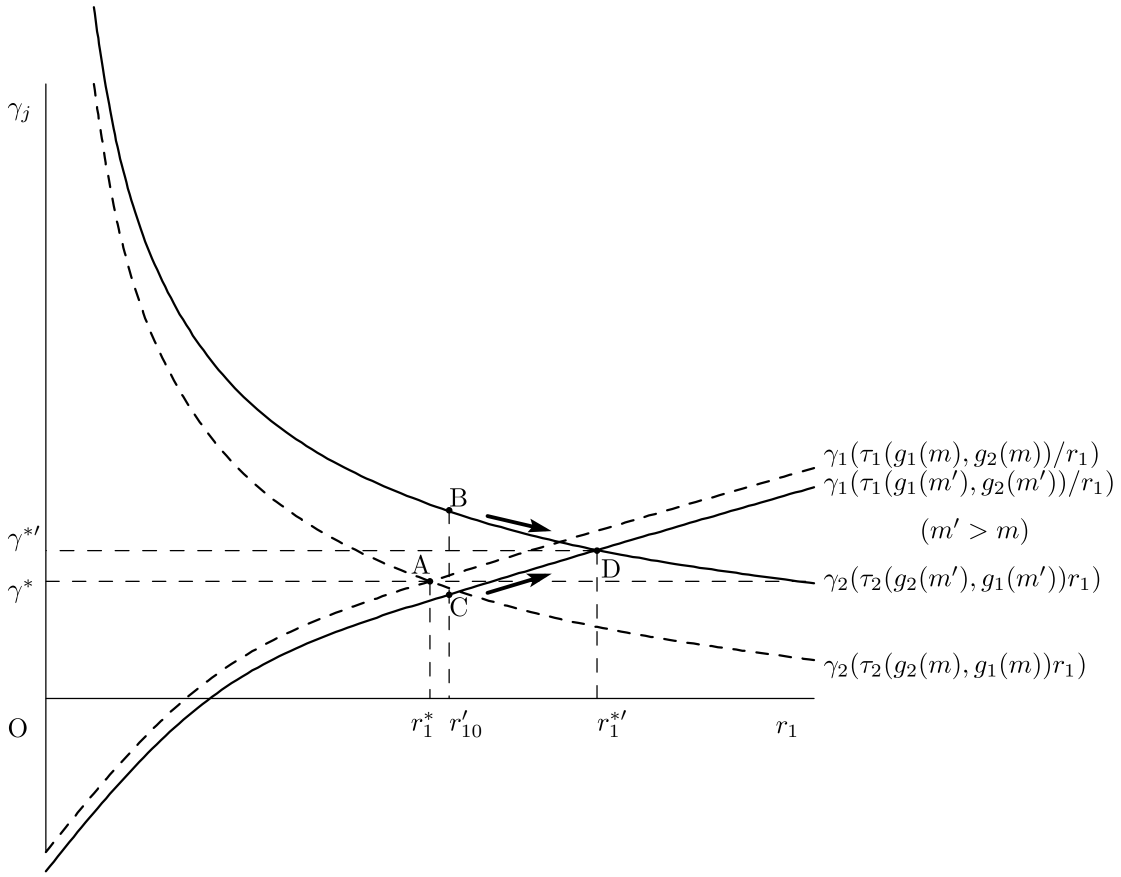

Fig. 2. The case where an increase in aid for trade raises global growth: $\chi_1 = \chi_2 \to 0$.

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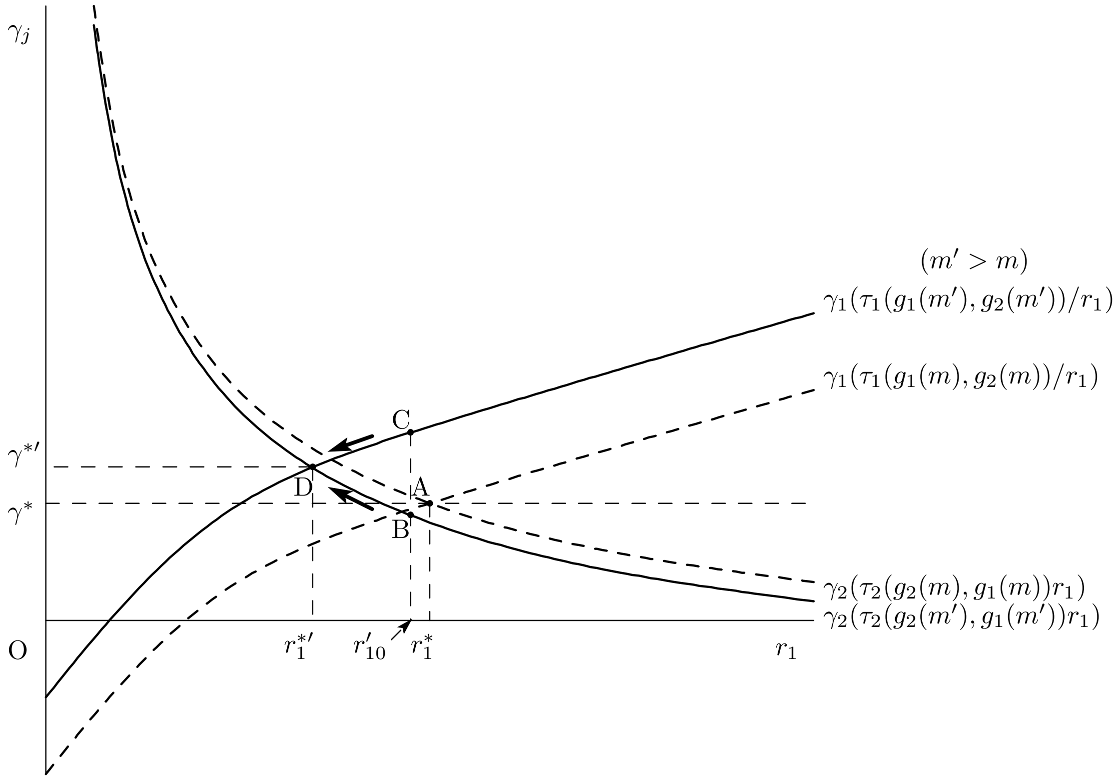

Fig. 3. The case where an increase in aid for trade raises global growth: $\mu_1 = \mu_2 \to 0$.