Comparative Advantage, Monopolistic Competition, and Heterogeneous Firms in a Ricardian Model with a Continuum of Sectors

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Abstract

Why does the fraction of firms that export vary with countries' comparative advantage? To address this question, I develop a general-equilibrium Ricardian model of North-South trade in which both institutional quality and firm heterogeneity play a key role in determining international trade flows. Because of contractual frictions that vary across countries and sectors, North with better institutions produces and exports relatively more in sectors where production is more institutionally dependent. In addition, institution-induced comparative advantage makes it relatively easier for Northern heterogeneous firms to incur export costs in more contract-dependent sectors, thereby leading to a higher exporters' percentage.

Keywords: Comparative advantage, Firm heterogeneity, Endogenous relative wage

JEL classification: D23, F12, F14, L33, O43

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1 Introduction

A growing body of empirical evidence using firm-level data has extensively revealed that the extent to which firms participate in exporting varies systematically across countries and sectors. These works have found that, in developed countries, the percentage of firms that export tends to be substantially higher in sectors where production technology is more complex and customized (such as chemical products), and this percentage steadily declines as sectors’ production requires simpler and more generic technology (such as textile products). In developing countries, on the other hand, the opposite patterns are typically observed: the ratio of exporting firms to overall firms tends to be notably higher (resp. lower) in simpler (resp. more complex) sectors. At the same time, these studies have also documented that exporting occurs in every major manufacturing sector of both developed and developing countries: even in strong comparative disadvantage sectors, a small fraction of firms do export.¹

Why does the fraction of firms that export vary with countries’ comparative advantage? To address this question, combining the recent empirical finding quantified by Levchenko (2007) and Nunn (2007) with firm heterogeneity of Melitz (2003), I develop a general-equilibrium Ricardian model of North-South trade in which both institutional quality and firm heterogeneity play a key role in determining international trade flows. Following Levchenko’s and Nunn’s finding, the model assumes that each country is different in terms of contracting institutions, and each sector is different in terms of contract intensity. Furthermore, each firm is different in terms of its productivity à la Melitz. These three-dimensional differences in country, sector, and firm characteristics endogenously pin down patterns of specialization and trade in equilibrium.

To investigate the role of country and sector characteristics, I build on the concept of “partial contractibility,” originally developed by Acemoglu, Antràs, and Helpman (2007). I consider an environment in which North has better institutions for partially ex-ante contractible activities than South, whereas customized sectors make use of relationship-specific investments intensively more than generic sectors. Because of these contractual frictions that vary across countries and sectors, aggregate output differences emerge. In contrast to Acemoglu et al., I do not explicitly examine the interaction between contractual incompleteness and technological complementarity by simply assuming that production in customized (generic) sectors is more (less) dependent on institutions. Instead, I extend their framework by allowing countries’ institutional quality and sectors’ institutional dependency to obey the Ricardian law of comparative advantage. This elaboration makes it possible to capture the aggregate relationship between country and sector characteristics neatly in a way such

¹See Bernard, Eaton, Jensen, and Kortum (2003) and Bernard, Jensen, Redding, and Schott (2007) for the United States, Tomiura (2007) for Japan, and Lu (2011) for China, respectively. For example, Bernard et al. (2007, Table 2) report that, as of 2002, 36 percent (8 percent) of U.S. firms export in a chemical manufacturing sector (an apparel manufacturing sector), whereby a more customized sector tends to exhibit a higher percentage of exporters across 21 sectors. Conversely, Lu (2011, Figure 1) shows that, as of 2005, around 60 percent (less than 20 percent) of Chinese firms export in cloth and fur manufacturing sectors (a chemical manufacturing sector), indicating that there exists a clear negative relationship between export participation and the capital-labor ratio across 29 sectors. Finally, Bernard et al. (2003, 2007), Tomiura (2007), and Lu (2011) all document that there exist some exporting firms in every manufacturing sector.
that North with better institutions produces and exports relatively more in sectors where production is more institutionally dependent.\textsuperscript{2}

To formalize the higher tendency to export participation in comparative advantage sectors, I incorporate firm-level differences in productivity. Since exporting requires fixed export costs that less productive firms cannot cover, only a small fraction of firms are able to export. The variation in this fraction is further reinforced by institution-induced comparative advantage in my setup, because Northern (Southern) firms are relatively better at producing in more (less) contract-dependent sectors, which in turn softens the relative burden of incurring export costs. As a result, compared to comparative disadvantage sectors of a counterpart country, relatively less productive firms can export in a country’s comparative advantage sectors. This mechanism explains why the stronger each country’s comparative advantage is, the smaller the productivity cutoff for exporting becomes, thereby leading to a higher exporters’ percentage.

Following the new literature on institutions and trade, I employ contracting institutions – more specifically contract enforcement – (rather than the classical determinants of international trade such as capital or (un)skilled labor) to rationalize the stylized fact of export participation. There are at least three reasons for this. First, countries’ abilities to enforce written contracts can have quantitatively larger impacts on comparative advantage than countries’ factor endowments. For instance, Nunn (2007) estimates that “contract enforcement explains more of the global pattern of trade than countries’ endowments of physical capital and skilled labor combined.” Second, as rigorously demonstrated by Costinot (2009a), this empirical evidence is appropriately captured by (Ricardian) institutional differences with log-supermodularity in country and sector characteristics.\textsuperscript{3}

In this specification, the characteristics of firms in terms of their productivity are an independent factor of the variation in export participation as discussed above. Finally, a Ricardian view of institutional differences can give a complementary explanation for Bernard, Redding, and Schott’s (2007) factor-endowment-driven comparative advantage theory. Although the main result is strikingly similar, I show that some phenomena (e.g., home-market effects) are better understood through a lens of Ricardian sources of comparative advantage.

In this paper, I do not attempt to explain why North has better institutions than South, why customized sectors depend heavily on institutions more than generic sectors, or why some firms are more productive than others. Taking these country, sector, and firm characteristics as given,

\textsuperscript{2}There is mounting evidence on the link between countries’ institutions and sectors’ types that affects the pattern of trade. For instance, devising a measure of input customization (the share of a sector’s inputs that are not sold in organized exchanges), Nunn (2007) shows that countries with better contract enforcement export relatively more in sectors for which relationship-specific investments are more important. Similarly, Manova (2008) finds evidence that trade liberalizations induce countries with better financial systems to export relatively more in sectors for which financial requirements are more important. For the other related papers in this literature, see Costinot (2009a).

\textsuperscript{3}As far as log-supermodularity in country and sector characteristics is central, the modeling of technological and institutional differences is isomorphic (Costinot, 2009a). This is because a country with better institutions faces less severe underinvestment and has a bigger cost advantage in production, which is a key presumption in Levchenko (2007) and Nunn (2007). Although I interpret institutions as a country’s contractibility to alleviate contractual frictions between firms and suppliers as in Acemoglu et al. (2007), I admit that this channel is at best one of potential sources of institution-induced comparative advantage and more nuanced studies along the line of Chor (2010) and Nunn and Trefler (2014) are necessary.
I instead set out to explore how contracting institutions and heterogeneous firms jointly shape an endogenous pattern of trade. By so doing, the model shows that North with better institutions gains a comparative advantage in contract-dependent sectors and is a net exporter of customized products in intra-industry trade. South with worse contracting institutions, on the other hand, is shown to be a net exporter of generic products. Moreover, within sectors in bilateral trade flows, the fraction of exporters is monotonically increasing in countries’ comparative advantage strength. These results, both of which are consistent with firm-level empirical research, hold even if North has an absolute advantage in institutional quality in any sector, and there exists no technological difference (in terms of firm productivity distributions) between the two countries.

This paper is closely related to two branches of the recent literature of international trade. The first is an emerging literature on institutions and trade (e.g., Antràs, 2005; Acemoglu et al., 2007; Costinot, 2009b). These papers show that, even in the absence of inherent technological differences, cross-country institutional differences can endogenously generate comparative advantage, which is at the heart of my model as well. In this strand of the papers, however, all firms are generally treated as identical and therefore every firm is able to export everywhere. In the real world, a large proportion of firms do not export even in strong comparative advantage sectors. The current paper demonstrates that not only is comparative advantage endogenously induced by institutions, but the fraction of exporters is higher in stronger comparative advantage sectors, as suggested by the existing evidence.

Another branch of the related literature is the so-called heterogeneous-firm model of trade, especially developed by the seminal work of Melitz (2003). While the Melitz model is successful in explaining the exporters’ behaviors among developed countries (North-North trade), recent empirical evidence has pointed out that this model is less suitable for the study of bilateral trade flows between different countries (North-South trade), as exemplified by Lu (2011) who analyzes Chinese firm-level manufacturing data. A number of papers – among others, Demidova (2008), Falvey, Greenaway, and Yu (2005), Fan, Lai, and Qi (2011), and Okubo (2009) – incorporate the asymmetry of countries in this setting. My approach differs from these papers, because I focus on the role of wage differentials in North-South trade, and because most results hold without specifying any parameterization of firm productivity distributions. More importantly, none of these papers sheds new light on the interplay between institutions and comparative advantage. Although I restrict the analysis only to an open economy and abstract from welfare implications for expositional simplicity, it is straightforward to extend the current setup to see the impact of trade on inter-/intra-sectoral resource allocations and welfare gains from trade.

Finally, this paper is also related to the heterogeneous-firm literature on factor-proportions theory, 

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4 Acemoglu et al. (2007) introduce firm heterogeneity in the degree of complementarity among inputs, but all products are assumed to be freely traded and hence all firms export in their model.

5 Wage differentials are one of the most prominent factors that have triggered large trade flows among dissimilar countries in the past two decades. For instance, noting that in 2006 for the first time the United States did more trade in manufactured goods with developing countries than developed countries, Krugman (2008) asserts that this is largely due to the wage differentials between the U.S. and developing countries: China’s and Mexico’s wages are respectively only 4 percent and 13 percent of the U.S. level.
especially to Bernard et al. (2007) as argued above. Using Helpman-Krugman’s (1985) two-factor model, they provide a rich framework for analyzing distributional consequences from trade, a feature missing in this Ricardian one-factor model. Their analysis, however, primarily applies to the situation in which two countries are not too different, and numerical simulations are required for outside factor-price-equalization regions. In contrast, it is possible in the current paper to analytically examine trade patterns between any two countries of arbitrary country size (with endogenous wage differentials) by sacrificing distributional issues via trade liberalization. A further distinction of this paper is in addressing Krugman’s (1980) home-market effect. I show that, due to selection into domestic and export markets that varies with comparative advantage, the home-market effect works oppositely for the extensive/intensive margins of specialization and those of trade between North and South.

2 Setup

Consider a world composing of two large countries, North and South, \( i \in \{N, S\} \). For notational simplicity, country superscript \( i \) is dropped unless needed in this section.

**Demand** Each country is populated by a mass \( L \) of identical consumers who devote their income into differentiated goods of a continuum of sectors over an interval \([0, 1]\). The preferences of a representative consumer are Cobb-Douglas across sectors and C.E.S. Dixit-Stiglitz within sectors:

\[
U = \int_0^1 \lambda_z \ln Q_z dz,
\]

where

\[
Q_z = \left[ \int_{v \in V_z} q_z(v)^{\frac{\sigma-1}{\sigma}} dv \right]^{\frac{\sigma}{\sigma - 1}},
\]

is aggregate consumption of varieties in sector \( z \). \( V_z \) is the mass of available goods within the sector, which potentially includes both domestic and foreign varieties. Given this aggregate good \( Q_z \), its dual aggregate price is given by

\[
P_z = \left[ \int_{v \in V_z} p_z(v)^{1-\sigma} dv \right]^{\frac{1}{1-\sigma}}.
\]

\( \lambda_z \) denotes a *constant* share of expenditure spent on sector \( z \), which is *identical* between the two countries. Letting \( R_z = P_z Q_z \) and \( Y = wL \) respectively denote aggregate expenditure in sector \( z \) and aggregate labor income in the economy, \( \lambda_z \) is defined as

\[
\lambda_z = \frac{P_z Q_z}{Y} = \frac{R_z}{wL}, \quad \int_0^1 \lambda_z dz = 1.
\]

Thus, the sum of aggregate sector expenditure equals aggregate labor income \( (\int_0^1 R_z dz = wL) \). Letting \( X_z = \lambda_z Y \) denote labor income spent on sector \( z \), the above preferences generate demand
functions for each differentiated variety in sector $z$:  

$$q_z(v) = A_z p_z(v)^{-\sigma}, \quad A_z = X_z P_z^{\sigma-1},$$

where $A_z$ is the index of aggregate market demand. In the following, I focus on a particular sector and drop sector subscript $z$ from relevant variables.

Before proceeding further, it is important to note that there is no homogeneous-good sector with nontrade costs, and wage rates $w$ cannot be normalized between North and South. This structure of the preferences is similar to that of Krugman (1980), and more recently to that of Antràs (2005) and Okubo (2009). Note also that while the elasticity of substitution between any two varieties within a sector is assumed to be greater than one ($\sigma > 1$), the elasticity of substitution between any varieties across sectors is unity. The unit elasticity of substitution implies that firm behavior in each sector can be analyzed independently.

**Production** There is a continuum of firms that produce a different variety $v$ in each sector. Labor is the only factor of production to produce a variety and firms face a perfectly elastic supply of labor at each country size $L$. Since labor is completely mobile across sectors but immobile across countries as in conventional Ricardian models, a wage rate $w$ is the same across sectors within a country but is different across countries.

Following Krugman (1980) and Melitz (2003), firm technology is summarized in a linear cost function of output $q$:

$$l = \begin{cases} 
    f_d + \frac{q}{\theta(\varphi, z, \mu)} = f_d + \frac{q}{\varphi \mu(z)} & \text{if domestic production,} \\
    f_x + \frac{\tau q}{\theta(\varphi, z, \mu)} = f_x + \frac{\tau q}{\varphi \mu(z)} & \text{if exporting,} 
\end{cases}$$

where $\theta(\cdot, \cdot, \cdot)$ is labor productivity, $f_d$ is a fixed cost for domestic production, $f_x$ is a fixed cost for exporting, and $\tau (\geq 1)$ is a iceberg transport cost. These costs are identical across countries and sectors.

A few points are in order for this specification. First, labor productivity $\theta(\cdot, \cdot, \cdot)$ depends on three factors: (i) firm-specific $\varphi$; (ii) sector-specific $z$; and (iii) country-specific $\mu$. In Melitz (2003), he considers symmetric countries, implying that a country-specific factor $\mu$ is ignorable. He also focuses on one sector within each country, leading a sector-specific factor $z$ to be absent from his analysis. Therefore, only a firm-specific factor $\varphi$ is important in the Melitz model. In the current model, by contrast, since the two countries are asymmetric and there is a continuum of sectors, the three factors jointly affect labor productivity. It follows from this cost function that the country-specific factor $\mu(\cdot) \in (0, 1)$ affects firms’ variable costs only (leaving fixed costs identical) and labor productivity is

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6By excluding a homogeneous-good sector, it is possible to explicitly investigate the role of the relative wage or “factoral terms of trade” (Matsuyama, 2008) in comparative advantage, which is an orthodox practice in Ricardian models. While introducing a homogeneous good à la Helpman and Krugman (1985) would help to simplify the analysis, empirical evidence suggests that the bulk of recent trade flows cannot be captured without a terms-of-trade effect between developed and developing countries as emphasized in Introduction.
greater if $\mu(\cdot)$ is closer to one. I assume that $\mu(\cdot)$ is related to a country’s ability to enforce written contracts between firms and suppliers (as will be shown in the next subsection) and is referred to as “partial contractibility” in this paper.

Second, I adopt a reduced form of labor productivity: $\theta(\varphi, z, \mu) = \varphi \mu(z)$. While this form is used for simplicity, one can justify this simplification from Costinot’s (2009a) log-supermodular argument. He defines Ricardian technological differences as labor productivity that satisfies $\theta(\varphi, z, \mu) = f(\varphi)/a(z, \mu)$, where $a(\cdot, \cdot)(>1)$ is the unit labor requirement (defined as the inverse of labor productivity), and shows that Ricardian sources of comparative advantage hold if $1/a(\cdot, \cdot)$ is log-supermodular (i.e., $\frac{\partial^2}{\partial z \partial \mu} \ln \frac{1}{a(z, \mu)} \geq 0 \Leftrightarrow \frac{\partial^2}{\partial z \partial \mu} \ln a(z, \mu) \leq 0$), or equivalently

$$\frac{a(z^2, \mu^1)}{a(z^2, \mu^2)} \geq \frac{a(z^1, \mu^1)}{a(z^1, \mu^2)},$$

for $z^1 \geq z^2$, $\mu^1 \geq \mu^2$, $a(z^1, \mu^2) \neq 0$ and $a(z^2, \mu^2) \neq 0$. My specification is restricted relative to Costinot’s in that $\theta(\varphi, z, \mu) = f(\varphi)/a(z, \mu) = \varphi \mu(z)$. In addition to applying this reduced form, I further assume that North has partial contractibility strictly superior to South in any sector. Noting the inverse relationship between $\mu(\cdot)$ and $a(\cdot, \cdot)$, log-supermodularity in terms of $\mu(\cdot)$ is given by

$$1 < \frac{\mu^N(z)}{\mu^S(z)} < \frac{\mu^N(z')}{\mu^S(z')} < \infty,$$

for $z > z'$, $\mu^N > \mu^S$, $\mu^S(z) \neq 0$ and $\mu^S(z') \neq 0$. Thus, not only does $\mu(z) = 1/a(z, \mu)$ satisfy log-supermodularity (or Ricardo’s classic inequality), but North has an absolute advantage in $\mu(z)$ in any sector.

Figure 1 depicts $\mu^i(z)$ satisfying the above inequalities with the additional assumptions that
\( \mu^N(1) = \mu^S(1) = 1 \) and \( \mu^S(0) = 0 \). As is clear from the figure, log-supermodularity means that the gap between \( \mu^N \) and \( \mu^S \) is gradually larger as sectors’ production becomes more customized in this framework. An economic interpretation of this figure is as follows. In a generic sector, the severe holdup problem is less likely irrespective of institutional quality because the production does not rely heavily on relationship-specific investments. As a result, the gap between \( \mu^N \) and \( \mu^S \) is relatively smaller and \( \mu^i \) is closer to one in a more generic sector. In a customized sector, on the contrary, producers are more likely to suffer from the holdup problem and production efficiency is relatively more sensitive to institutional quality. Although \( \mu^i \) is significantly less than one for both countries, superior contractibility gives North a relatively bigger cost advantage in a more customized sector. As formally established by Costinot (2009a), this “relatively more” property – which lies at the core of neoclassical trade theory and is also the pivotal element in the empirical evidence reported by Levchenko (2007) and Nunn (2007) – is elegantly captured by log-supermodularity.

Finally, in this decomposition \( \theta(\varphi, z, \mu) = \varphi \mu(z) \), I refer to a country’s distribution \( G(\varphi) \) of firms’ productivity draws \( \varphi \) as “technologies” as in Melitz (2003), whereas a country’s partial contractibility \( \mu(z) \) on firms’ relationship-specific investments as “institutions” as in Levchenko (2007). Furthermore, I restrict attention to environments in which all firms have access to the same technologies across countries and sectors, i.e., \( G^i(\varphi) = G(\varphi) \) for \( i \in \{N, S\} \) and \( z \in [0, 1] \). Consequently, there are no technological differences across countries and institutional differences solely give rise to countries’ comparative advantage. In reality, technological and institutional differences coexist and these two differences are not precisely separable. This distinction is not crucial for the analysis below and the following definition is made for the sake of convenience.

**Definition 1** *Technologies* are a country’s distribution \( G(\varphi) \) of firms’ productivity draws \( \varphi \), which is identical across countries and sectors. *Institutions* are a country’s partial contractibility \( \mu(z) \) on firms’ relationship-specific investments, which varies across countries and sectors.

Each firm chooses its price to maximize profits \( \pi = px - wl \) for domestic production and exporting. Solving profit-maximizing problem yields the following first-order conditions:

\[
\begin{align*}
p(\varphi) &= \frac{\sigma}{\sigma - 1} \frac{w}{\varphi \mu}, \\
q(\varphi) &= A \left( \frac{\sigma - 1}{\sigma} \frac{\varphi \mu}{w} \right)^{\sigma}, \\
r(\varphi) &= p(\varphi)q(\varphi) = A \left( \frac{\sigma - 1}{\sigma} \frac{\varphi \mu}{w} \right)^{\sigma-1}, \\
\pi(\varphi) &= \frac{r(\varphi)}{\sigma} - wf = B \left( \frac{\mu}{w} \right)^{\sigma-1} \varphi^{\sigma-1} - wf,
\end{align*}
\]

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This means that, if the firm productivity distribution is Pareto, \( G(\varphi) = 1 - (\varphi_{min}/\varphi)^k \), both the shape and scale parameters, \( k \) and \( \varphi_{min} \), are identical across countries and sectors. While these parameters are more likely to vary with country and sector characteristics in evidence (Tybout, 2000), the modeling of sector-variant distributions could come at the cost of obscuring Ricardian sources of comparative advantage if \( G^i(\varphi) \) is log-supermodular in sector and firm characteristics (Costinot, 2009a).

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where
\[ B = \frac{(\sigma - 1)^{\sigma - 1}}{\sigma^\sigma} \] \[ A = \frac{(\sigma - 1)^{\sigma - 1}}{\sigma^\sigma} X P^{\sigma - 1}, \]
is aggregate market demand. For analytical simplicity, I assume that the variable trade cost is zero \((\tau = 1)\) and thus \( p = p_d = p_x \). Section 5 shows that the main result qualitatively holds even with the variable trade cost \((\tau > 1)\), and the supplementary note offers a detailed analysis that incorporates \(\tau\). It is also assumed that the fixed trade cost is higher than the fixed production cost \((f_x > f_d)\) and, under this assumption, only a subset of firms are able to export even in the absence of \(\tau\).

While the above first-order conditions are similar to those in the existing literature, two features of the current setup are worth emphasizing. First, wages \(w\) cannot be normalized between the two countries, since they are asymmetric and there is no freely tradable homogeneous-good sector in this model. As noted earlier, this assumption is made to examine the role of endogenous factorial terms of trade in the Ricardian model. Second, institutional quality \(\mu\) enters into these conditions. It is immediate to see that the pricing rule is higher and the output level is lower in a more customized sector due to the holdup problem.

**Holdup Problem** So far, I have not explicitly explored how a country’s institutional quality \(\mu\) is related to the enforcement of contracts between firms and input suppliers, and thus to the holdup problem in relationship-specific investments. As originally proposed by Grossman and Hart (1986), the holdup problem occurs because the parties cannot specify every unforeseeable contingency into an initial contract ex ante, and they have to renegotiate the contract ex post. In what follows, building on seminal work of Antràs (2003, 2005) and Antràs and Helpman (2004), I show that institutional quality \(\mu\) plays a qualitatively similar (but distinct) role with incomplete contracting.

Suppose that, while letting perfect institutions \((\mu = 1)\) prevail in any sector of both countries, production of final goods now requires intermediate inputs which each firm cannot manufacture by itself. To produce a variety, every firm asks a domestic input supplier to provide a specialized input. This input is relationship-specific in the sense that it has a higher value within the parties and a third party (such as courts of law) cannot distinguish its true value. Because no enforceable contract will be signed ex ante in such a circumstance, the firm and its supplier have to bargain over the surplus after production takes place. Let \(\beta\) and \(1 - \beta\) denote the firm’s and its supplier’s ex-post bargaining power. Then, the firm’s profit is \(\pi_F = \beta pq + T\) and the supplier’s profit is \(\pi_S = (1 - \beta) pq - w l - T\), where \(T\) is a transfer from the supplier to the firm. This transfer works to make the supplier break-even and the firm’s ex-post profit is \(\pi = \pi_F + \pi_S = pq - w l\) in a subgame-perfect Nash equilibrium. The supplier chooses its input level to maximize \(\pi_S\), so the first-order conditions are

\[ p(\varphi) = \frac{\sigma}{\sigma - 1} \frac{w}{\varphi(1 - \beta)}, \quad q(\varphi) = A \left( \frac{\sigma - 1}{\sigma} \frac{\varphi(1 - \beta)}{w} \right)^{\sigma}. \]

Comparing the two pricing rules reveals \(\mu = 1 - \beta\) in equilibrium, and the distribution of bargaining power between the firm and its supplier is directly related to institutional comparative advantage.

To see this in more detail, imagine what happens if the agents were able to sign complete contracts.
In such an environment, the input level is ex ante verifiable and the supplier could directly bargain over the profit rather than the revenue. Then, the firm’s profit is $\pi_F^* = \beta \pi^* + T$ and the supplier’s profit is $\pi_S^* = (1 - \beta)\pi^* - w l - T$, where $\pi^* = \pi_F^* + \pi_S^*$ is the joint profit under complete contracting. The supplier would choose its input level to maximize $\pi_S^*$, so the first-order conditions are

$$p^*(\varphi) = \frac{\sigma}{\sigma - 1} \frac{w}{\varphi}, \quad q^*(\varphi) = A \left( \frac{\sigma - 1}{\sigma} \frac{\varphi}{w} \right)^\sigma.$$

Evidently, the pricing rule is $1/(1-\beta)$ times higher under incomplete contracting because the supplier receives only a fraction of the marginal return to its investment for specialized input, leading the input level to be $(1-\beta)^\sigma$ times lower. Given this interpretation, the distribution of ex-post bargaining power has a direct impact on comparative advantage through ex-ante efficiency of specialized input production, i.e., the holdup problem. In particular, across sectors, a fraction of ex-ante contractible activities would be relatively smaller in a more customized sector and the supplier’s holdup problem is relatively severer in such a sector (Acemoglu et al., 2007). Furthermore, across countries, South would have absolutely worse legal institutions in enforcing contracts and the supplier’s holdup problem in South is absolutely severer than North (Antrás and Helpman, 2004). This is a theoretical justification for log-supermodularity in institutional quality $\mu$, with North having an absolute advantage in it.

The previous literature on organizations and trade typically assumes that the degree of the holdup problem in relationship-specific investments is the same across sectors, but varies across organizational forms (i.e., vertical integration and outsourcing). Instead, I allow this degree to vary across sectors, while abstracting from the firm boundaries. The model is developed below, keeping in mind the similarity between imperfect institutions and incomplete contracts.

**Firm Behavior** The current paper analyzes a static version of the Melitz (2003) model. To enter a sector in country $i \in \{N, S\}$, firms bear a fixed cost of entry $f_e$, measured in country $i$’s labor units. Upon paying this fixed cost, firms draw their productivity level $\varphi$ from a known distribution $G_i^i(\varphi) = G(\varphi)$. After observing this productivity level, each firm decides whether to exit or not. If the firm chooses to produce, it bears additional fixed costs $f_d$ for domestic production and $f_x$ for exporting, as described before. An entering firm in country $i$ would then immediately exit if $\pi_d^i < 0$, or would produce and serve its domestic market if $\pi_d^i \geq 0$. Moreover, among domestic firms, only the most productive firms would earn $\pi_x^i \geq 0$ and serve the foreign market in $j$ as exporters under the assumption $f_x > f_d$.

While this firm behavior is similar across countries and sectors, the productivity cutoffs for domestic production and exporting would vary by reflecting countries’ comparative advantage. Regarding exporting participation, if $\pi_x^i \geq 0$ and $\pi_x^j \geq 0$ for some firms in $i \neq j \in \{N, S\}$, well-known two-way (intra-industry) trade occurs in this sector: trade occurs even in the same sector because products are differentiated and consumers are strictly better off by importing products unavailable in the domestic market. If $\pi_x^i \geq 0$ for some firms and $\pi_x^j < 0$ for any firm, on the contrary, one-way (inter-industry) trade occurs in this sector, whereby exporting from $i$ to $j$ takes place.
3 Partial Equilibrium

In this section, I first explore partial equilibrium in which some important variables are exogenously given. The next section embeds this analysis in a general-equilibrium setting.

To see an equilibrium of sector \( z \), consider country \( j \)'s market where competition occurs between domestic firms in \( j \) and exporters from \( i \). From the first-order conditions in the previous section, the profit functions of these firms are respectively given by\(^8\)

\[
\pi^j_d = B^j \left( \frac{\mu^j(z)}{w^j} \right)^{\sigma-1} \varphi^{\sigma-1} - w^j f_d, \quad \pi^j_x = B^j \left( \frac{\mu^j(z)}{w^j} \right)^{\sigma-1} \varphi^{\sigma-1} - w^j f_x.
\]

Notice that, since these firms compete in \( j \)'s market, aggregate demand \( B^j \) is common for both profit functions. \( \pi^j_d \) and \( \pi^j_x \) are measured by different wage rates and contractibility levels, however, because exporters from \( i \) have to use domestic labor and institutions to produce own variety. If there exist multinational enterprises that directly employ local labor and have internal contractibility within the firm boundaries, this argument is no longer true. See Section 5 for the possibility of foreign affiliate production in the current setup.

To compare these two profit functions graphically, they are drawn in \( (\varphi^{\sigma-1}, \pi) \) space with slope \( B \left( \frac{\mu}{w} \right)^{\sigma-1} \) and intercept \(-wf\). Then, \( \pi^j_x \) is steeper (flatter) than \( \pi^j_d \) if and only if

\[
\frac{\mu^i(z)}{w^i} > \frac{\mu^j(z)}{w^j} \iff \begin{cases} 
\mu(z) > \omega & \text{if } i = N, \\
\mu(z) < \omega & \text{if } i = S,
\end{cases}
\]

where \( \mu(z) = \mu^N(z)/\mu^S(z) = a(z, \mu^S)/a(z, \mu^N) \) and \( \omega = w^N/w^S \) respectively denote the relative contractibility (or relative labor requirement) and the relative wage in North. Following standard Ricardian models, I say that North (South) has an institutional comparative advantage in sector \( z \) if it has a large (small) relative labor productivity in partial contractibility \( \mu(z) \), and/or a small (large) relative wage \( \omega \), i.e., \( \mu(z) > \omega \) (\( \mu(z) < \omega \)). Under this definition, country \( i \)'s institutional comparative advantage is identified as the sectors where the slope of exporters \( \pi^j_x \) is steeper than that of domestic firms \( \pi^j_d \).

**Definition 2** North (South) has an institutional comparative advantage in sector \( z \) if \( \mu(z) > \omega \) (\( \mu(z) < \omega \)), or equivalently if the slope of the \( \pi^N_z \) (\( \pi^S_z \)) is steeper than that of the \( \pi^N_d \) (\( \pi^S_d \)).

Since this definition indicates neither North nor South has a comparative advantage in a sector of \( \mu(z) = \omega \) where \( \pi^i_d \) and \( \pi^i_x \) are parallel, I first derive the condition under which \( \mu(z) \) and \( \omega \) are equal. From Figure 1, the ratio of contractibility \( \mu(z) = \mu^N(z)/\mu^S(z) \) has to satisfy \( \mu(z) > 1 \), \( \mu'(z) < 0 \), \( \mu''(z) > 0 \), \( \lim_{z \to -1} \mu(z) = 1 \), and \( \lim_{z \to \infty} \mu(z) = \infty \), where the first condition stems from absolute advantage of North and the third stems from log-supermodularity of \( \mu^i(z) \). The relative wage in

\(^8\)Following the literature (e.g. Melitz and Redding, 2014), I assume that all production costs, including the fixed export cost \( f_x \), are measured in terms of a source country labor.
North $\omega$, on the other hand, should be the same for all sectors $z \in [0, 1]$ because labor is completely mobile across sectors within a country in the Ricardian model. This suggests that, if $\omega$ is greater than one, these two curves intersect at a unique cutoff $\tilde{z} = \mu^{-1}(\omega)$ such that: (i) $z \in [0, \tilde{z}) \Leftrightarrow \mu(z) > \omega$; (ii) $z = \tilde{z} \Leftrightarrow \mu(z) = \omega$; and (iii) $z \in (\tilde{z}, 1] \Leftrightarrow \mu(z) < \omega$. It is then immediate from Definition 2 that North (South) has an institutional comparative advantage in relatively customized (generic) sectors $z \in [0, \tilde{z}) \ (z \in (\tilde{z}, 1])$.

Next I consider the equilibrium in the cutoff sector $\tilde{z}$ as a benchmark. The left panel of Figure 2 depicts the profit functions in Northern market $(\pi^N_d, \pi^S_d)$ under the condition that $w^S f_x > w^N f_d \Leftrightarrow \omega < \frac{f_x}{f_d}$. This condition ensures that Southern exporters bear the higher fixed cost (measured by the local labor wage) than Northern domestic firms. Following the same line of reasoning, the right panel depicts the profit functions in Southern market $(\pi^N_x, \pi^S_x)$ under the condition that $w^N f_x > w^S f_d \Leftrightarrow \omega > \frac{f_x}{f_d}$. Figure 2 shows that Northern (Southern) firms with productivity above $\tilde{\varphi}_d^N$ ($\tilde{\varphi}_x^S$) export to South (North) and hence two-way trade occurs in the cutoff sector $\tilde{z}$.

This argument helps understand what happens in sectors other than the cutoff sector $\tilde{z}$. In sectors where North has a comparative advantage (i.e., $z \in [0, \tilde{z})$), for example, it follows from $\mu(z) > \omega$ that the profit functions of Northern firms are steeper relative to those of Southern firms in both domestic and export markets of Figure 2. This implies that, as country $i$’s comparative advantage is stronger, the productivity cutoffs, $\tilde{\varphi}_d^i$ and $\tilde{\varphi}_x^i$, become relatively smaller than the counterpart cutoffs in country $j$ and less productive firms are more likely to find it profitable to operate in these sectors. At the same time, the opposite is true for country $j$: $\tilde{\varphi}_d^j$ and $\tilde{\varphi}_x^j$ become relatively larger and less productive firms are more likely to exit these sectors. Moreover, two-way trade would occur in any sector $z \in [0, 1]$ as long as the slopes of $\pi^N_x$ and $\pi^S_x$ are positive, i.e., $B (\frac{\mu}{w})^{\sigma-1} > 0$.

Notice in Figure 2 that, due to wage differentials, domestic firms in $j$ might bear the higher fixed cost than foreign exporters from $i$ in $j$’s market. Because it is empirically well-known that $f_x$ is huge
in any manufacturing sector. I hereafter assume the following condition for the fixed costs which will be shown to be necessarily held in the general-equilibrium setting where $\omega$ is endogenous.

**Assumption 1** $\frac{f_d}{f_x} < \omega < \frac{f_x}{f_d}$.

While the existence of the unique cutoff sector $\bar{z}$ in the absence of variable trade cost is reminiscent of Dornbusch, Fischer, and Samuelson’s (1977) Ricardian model with a continuum of goods, there exist three noteworthy distinctions between the current paper and theirs. First, since they analyze perfect competition with homogeneous goods, complete specialization (or inter-industry trade) occurs below/above the cutoff $\bar{z}$; in contrast, this paper studies monopolistic competition with differentiated goods and incomplete specialization (or intra-industry trade) can occur in all manufacturing sectors. Secondly, the mass and size of domestic firms and exporters are both indeterminate and irrelevant in their neoclassical trade model, but it is endogenously determined in the current framework in which the mass of varieties exported is only a subset of the mass of varieties produced in the home market. Finally, this paper’s focus is on North-South trade where the difference in economic development plays a prominent role in shaping countries’ comparative advantage. The result – that a less developed country nevertheless exports differentiated goods in customized sectors – seems to be consistent with recent trade flows (see, e.g., Krugman, 2008).

**Proposition 1**

(i) If $\omega > 1$, there exists a unique cutoff sector $\bar{z} \in (0, 1)$ such that North (South) has an institutional comparative advantage in sectors $z \in (0, \bar{z})$ ($z \in (\bar{z}, 1]$).

(ii) Two-way trade can occur in any sector $z \in [0, 1]$.  

It is important to emphasize that this partial-equilibrium analysis cannot clarify the interplay among key variables of the model. To see this, it is useful to go back to Figure 2. The figure depicts $\bar{\varphi}_d^j < \bar{\varphi}_x^i$ in the cutoff sector $\bar{z}$, indicating that foreign exporters from $i$ are more productive than domestic firms in $j$. This outcome is not wholly surprising because foreign exporters are assumed to incur the higher fixed cost under Assumption 1, and it can be easily formalized by using a partial-equilibrium framework. However, $\bar{\varphi}_d^i$ and $\bar{\varphi}_d^j$ or $\bar{\varphi}_x^i$ and $\bar{\varphi}_x^j$ are not comparable. Obviously, $\bar{\varphi}_d^N$ and $\bar{\varphi}_x^S$ ($\bar{\varphi}_x^N$ and $\bar{\varphi}_x^S$) are determined at which $\pi_d^N = 0$ and $\pi_d^S = 0$ ($\pi_x^N = 0$ and $\pi_x^S = 0$), but these variables depend on the aggregate market demand $B^i$ as well as the wage rate $w^i$, both of which are exogenous in partial equilibrium. Also, Proposition 1(i) requires that $\omega$ should be greater than

---

9Das, Roberts, and Tybout (2007) econometrically estimate the average costs of foreign market entry among three Colombian manufacturing sectors (leather products, knitted fabrics, and basic chemicals), and find that these fixed costs are (i) large enough to cause export hysteresis and (ii) remarkably similar across these sectors: the average export costs range from $412,000 (in U.S. dollars) for knitted fabrics to $430,000 for leather products.

10The idea of bringing firm heterogeneity into the Dornbusch et al. (1977) model is not entirely new, although the models’ setups in the existing literature are less general than the current one. For instance, Okubo (2009) uses a specific distribution of firm productivity levels, whereas Fan et al. (2011) abstract from endogenous wage differentials between countries by introducing the homogeneous-good sector.
one if North-South trade is to occur, but it is not clear whether this holds or not. In this sense, the above partial-equilibrium setting is restricted, and a general-equilibrium approach is necessary to endogenize these variables.

4 General Equilibrium

In this section, the partial-equilibrium model is embedded into a general-equilibrium framework to examine the interaction among the key variables and to see endogenous patterns of specialization and trade.

General-Equilibrium Setup This subsection outlines several equilibrium conditions that play a central role in characterizing the endogenous variables in general equilibrium. In the subsequent subsections, I solve this general-equilibrium model with some restrictions on the exogenous variables.

First of all, a zero profit condition must hold for the cutoff firms in both domestic and export markets, which is respectively achieved by setting \( \bar{\phi}_d^i = \inf\{\phi: \pi_d^i(\phi) > 0\} \) and \( \bar{\phi}_x^i = \inf\{\phi: \pi_x^i(\phi) > 0\} \) in the static model. This condition is referred to as a zero cutoff profit (ZCP) condition:

\[
\pi_d^i(\bar{\phi}_d^i) = 0 \iff B_i^d \left( \frac{\mu_i}{w_i} \right)^{\sigma-1} (\bar{\phi}_d^i)^{\sigma-1} = w_i f_d, \quad (ZCP_d^i)
\]

\[
\pi_x^i(\bar{\phi}_x^i) = 0 \iff B_j^x \left( \frac{\mu_i}{w_i} \right)^{\sigma-1} (\bar{\phi}_x^i)^{\sigma-1} = w_i f_x, \quad (ZCP_x^i)
\]

for \( i \neq j \in \{N, S\} \). In this condition, aggregate market demand of exporters \( B_j^i \) should be different from that of domestic firms \( B_i^j \) because exporters from country \( i \) have to face aggregate market demand in country \( j \).

Secondly, a free entry (FE) condition must be satisfied. Since potential entrants are ex ante identical in the current model, this condition is defined as

\[
\int_{\bar{\phi}_d^i}^{\infty} \pi_d^i(\phi)dG(\phi) + \int_{\bar{\phi}_x^i}^{\infty} \pi_x^i(\phi)dG(\phi) = w_i f_e, \quad (FE_i^i)
\]

where the first and second terms in the left-hand side respectively denote the expected operating profits from domestic production and exporting earned by potential entrants. The sum of these expected profits has to be equal to the fixed entry cost \( w_i f_e \).

Finally, a labor market clearing (LMC) condition must be taken into account:

\[
\int_0^1 M_e^i \int_{\bar{\phi}_d^i}^{\infty} l_d^i(\phi)dG(\phi)d\phi + \int_0^1 M_e^i \int_{\bar{\phi}_x^i}^{\infty} l_x^i(\phi)dG(\phi)d\phi + \int_0^1 M_e^i f_e dz = L_i, \quad (LMC_i^i)
\]

where \( M_e^i \) denotes the mass of potential entrants in \( i \). In this equation, the first and second terms in the left-hand side are the sum of expected labor demands used for domestic production and exporting by potential entrants, whereas the third term is expected labor demands used for investment by
potential entrants. Note that $l_i^z(\varphi)$ is summed up over sectors of the economy as a whole because two-way trade can occur in any sector $z \in [0, 1]$ as seen in Proposition 1(ii). The sum of these expected labor demands has to be equal to the fixed labor supply $L^i$.

Now, it is possible to endogenize the important variables in general equilibrium. Since there are the eight equations (the ZCP, FE, and LMC conditions that must hold in North and South), these conditions provide implicit solutions for the following eight unknowns:

$$\tilde{\varphi}^N_d, \tilde{\varphi}^S_d, \tilde{\varphi}^N_x, \tilde{\varphi}^S_x, B^N, B^S, w^N, w^S,$$

where the LMC condition in South can be omitted by Walras’ law, thereby normalizing $w^S = 1$ as a numéraire. (The mass of potential entrants $M^i_e$ can be written as a function of these eight unknowns as will be shown later.)

Relative Equilibrium Conditions This subsection sets forth the characterization of the eight unknowns from the eight equilibrium conditions. It is challenging, however, to solve a full general equilibrium model with asymmetric countries. In particular, without specifying the functional form of the firm size distribution $G(\varphi)$, explicit solutions of these unknowns cannot be obtained. In the following, instead of obtaining the exact values of each of them, the main focus is devoted to characterizing the relative terms of these unknowns.

Recall from Proposition 1(i) that North-South trade occurs only if the relative wage in North $\omega$ is greater than one. Although this relative wage is endogenously determined in the model, suppose first that $\omega > 1$ in equilibrium. In other words, the LMC conditions are left out from the model as if it were partial-equilibrium. As will be clear, this inequality must be true in general equilibrium because I assume that North has an absolute advantage in partial contractibility $\mu$ in any sector. This means that the marginal product of labor is higher (on average across heterogeneous firms) in North, and thus wages have to be greater in North than South if trade is to occur between the two countries. This intuition will be confirmed later by integrating the LMC conditions.

Under the circumstance, I first examine the sectoral difference in the relative productivity cutoffs $(\varphi_d = \varphi^N_d / \varphi^S_d, \varphi_x = \varphi^N_x / \varphi^S_x)$ and the relative market demand $(B = B^N / B^S)$ by focusing on the ZCP and FE conditions. To do this, using the ZCP conditions, rewrite the FE condition as

$$f_dJ(\tilde{\varphi}^i_d) + f_xJ(\tilde{\varphi}^i_x) = f_e,$$

where $J(\tilde{\varphi}) = \int_0^\infty \left( (\varphi / \tilde{\varphi})^{\sigma - 1} - 1 \right) dG(\varphi)$. $J(\cdot)$ is monotonically decreasing with $\lim_{\varphi \to 0} J(\tilde{\varphi}) = \infty$ and $\lim_{\tilde{\varphi} \to \infty} J(\tilde{\varphi}) = 0$. Since the equality of this condition must hold in any sector, changes in $z$ shift the productivity cutoffs in opposite directions, and thus $\tilde{\varphi}^i_x / \tilde{\varphi}^i_d$ must be strictly increasing or decreasing in $z$. Note that these changes in $z$ affect $\varphi$’s and $B$’s while they have no impact on $w$’s as wages are independent of $z$ (as long as labor is completely mobile across sectors), and the equilibrium analysis here does not rely on the exogenous $\omega$ assumption. Moreover, dividing the ZCP condition of domestic production by that of exporting for each country, the relative market demand $B = B^N / B^S$
Figure 3 – Market demand and productivity cutoffs

is given by

$$B = \begin{cases} \left( \frac{\bar{\psi}^N}{\bar{\psi}_d} \right)^{\sigma^{-1}} \frac{f_d}{f_x} & \text{if } i = N, \\ \left( \frac{\bar{\psi}^S}{\bar{\psi}_x} \right)^{\sigma^{-1}} \frac{f_x}{f_d} & \text{if } i = S. \end{cases}$$

Using the property of the FE condition derived above, it can be shown that $B$ is strictly increasing in $z$ (see Appendix). The intuition behind this result is explained by recalling that $B$ is proportional to the relative aggregate price ($P = P^N/P^S$). If $z$ is close to one, the institutional differential is almost negligible whereas there exists the wage differential (i.e., $\omega > 1$). South is thus able to produce goods relatively cheaply, thereby leading to $P^N > P^S$ and $B > 1$ in the neighborhood of $z = 1$. If $z$ is close to zero, on the other hand, the institutional differential is sufficiently large to dominate the wage differential (due to log-supermodularity), resulting in $P^N < P^S$ and $B < 1$ in the neighborhood of $z = 0$. Roughly speaking, this intuition mirrors the idea that country $i$ has a comparative advantage in sectors where the aggregate price $P^i$ is relatively lower than $P^j$.\(^{11}\) The first quadrant of Figure 3 depicts this relationship in $(z, B)$ space.

Next, dividing the ZCP condition of North by the corresponding condition of South, the relative productivity cutoffs $\bar{\varphi}_d = \frac{\bar{\varphi}^N}{\bar{\varphi}_d}$ and $\bar{\varphi}_x = \frac{\bar{\varphi}^S}{\bar{\varphi}_x}$ satisfy the following relative ZCP condition:

$$\bar{\varphi}_d = \left( \frac{\omega}{B} \right)^{\frac{1}{\sigma-1}} \frac{\omega}{\mu}, \quad \bar{\varphi}_x = \left( B\omega \right)^{\frac{1}{\sigma-1}} \frac{\omega}{\mu}, \quad (RZCP)$$

where all variables are represented by the relative terms in North. It is easy to show that $\bar{\varphi}_d$ decreases with $B$ while $\bar{\varphi}_x$ increases with $B$, and that

$$\bar{\varphi}_x \geq \bar{\varphi}_d \iff B \geq 1.$$

This equality holds in the cutoff sector $z = \bar{z}$ because $\bar{\varphi}_d$ and $\bar{\varphi}_x$ are equal if and only if $\pi^i_d$ and $\pi^i_x$ are parallel in that sector (see Figure 2). Thus, the relative market demand and relative productivity

\(^{11}\)This statement is not precise because the aggregate price $P^i$ includes prices of both domestic and foreign varieties in the presence of two-way trade. Given log-supermodularity, this would hold in a closed-economy version of the model.
cutoffs respectively satisfy $B(\bar{z}) = 1$ and $\varphi_d(\bar{z}) = \varphi_x(\bar{z}) = \omega^{1/(\sigma - 1)}$ (using $\mu(\bar{z}) = \omega$) in the cutoff sector. The second quadrant of Figure 3 depicts this relationship in $(B, \bar{\varphi})$ space.

Finally, combining the first and second quadrants, Figure 3 highlights the sectoral difference among the six endogenous variables (represented in the relative terms) that are derived from the ZCP and FE conditions: in any sector $z \in [0, 1]$, the relative market demand $B$ is determined in the first quadrant, and the relative productivity cutoffs $\bar{\varphi}_d$ and $\bar{\varphi}_x$ are subsequently determined in the second quadrant. It is immediately seen that

$$0 \leq z < \bar{z} \iff B < 1 \iff \bar{\varphi}_d > \omega^{1/(\sigma - 1)} > \bar{\varphi}_x,$$

$$z = \bar{z} \iff B = 1 \iff \bar{\varphi}_d = \omega^{1/(\sigma - 1)} = \bar{\varphi}_x,$$

$$\bar{z} < z \leq 1 \iff B > 1 \iff \bar{\varphi}_d < \omega^{1/(\sigma - 1)} < \bar{\varphi}_x.$$

In this relationship, either $\bar{\varphi}_d$ or $\bar{\varphi}_x$ is necessarily greater than one under the condition that the relative wage $\omega$ is greater than one. For instance, $\bar{\varphi}_d$ is greater than one in sectors where North has a comparative advantage $z \in [0, \bar{z})$; however whether $\bar{\varphi}_x$ is greater than one or not is indeterminate in the current setup.

Based on this observation, Figure 4 illustrates the relationship among the productivity cutoffs in the comparative advantage sectors of North (left panel) and South (right panel). While the figure shows that $\bar{\varphi}_x = \bar{\varphi}_x^N / \bar{\varphi}_x^S > 1$ (left panel) and $\bar{\varphi}_d = \bar{\varphi}_d^N / \bar{\varphi}_d^S > 1$ (right panel), these might not hold in some sectors. Regardless of whether or not they are greater than one, the gap between $\bar{\varphi}_x^i$ and $\bar{\varphi}_d^i$ is narrower than the gap between $\bar{\varphi}_x^j$ and $\bar{\varphi}_d^j$ in country $i$’s comparative advantage sectors. In addition, this former (latter) gap becomes smaller (bigger) as country $i$’s comparative advantage is stronger. More formally, it follows from the ZCP conditions that

$$\frac{\bar{\varphi}_x^N}{\bar{\varphi}_d^N} = \left( \frac{B f_x}{f_d} \right)^{\frac{1}{\sigma - 1}}, \quad \frac{\bar{\varphi}_x^S}{\bar{\varphi}_d^S} = \left( \frac{1}{B} \frac{f_x}{f_d} \right)^{\frac{1}{\sigma - 1}}.$$

Since $B$ is strictly increasing in $z$, $\bar{\varphi}_x^N / \bar{\varphi}_d^N$ ($\bar{\varphi}_x^S / \bar{\varphi}_d^S$) is strictly increasing (decreasing) in $z$: as country $i$’s comparative advantage is stronger, the productivity cutoff for exporting $\bar{\varphi}_x^i$ is closer to that for domestic production $\bar{\varphi}_d^i$ and a larger portion of firms are able to enter the foreign market. Hence the

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12 Noting that $\mu(z) = a(z, \mu^N)/a(z, \mu^S)$ is the relative unit labor requirement and is decreasing in $z$, comparative advantage of North (South) is said to be “stronger” if $z$ is closer to zero (one) in this model.
model predicts a higher ratio of exporting firms to overall surviving firms in sectors where countries are relatively more productive.

The above equations also indicate that the productivity cutoff for exporting is bigger than that of domestic production ($\bar{\varphi}_i^x > \bar{\varphi}_i^d$) in the comparative disadvantage sectors for $i \in \{N, S\}$. In the comparative advantage sectors, the usual outcome ($\bar{\varphi}_i^x > \bar{\varphi}_i^d$) occurs in both countries if

$$\frac{f_d}{f_x} < B < \frac{f_x}{f_d},$$

whereas the “perverse” outcome ($\bar{\varphi}_d^i > \bar{\varphi}_x^i$) occurs if

$$\begin{cases} B < \frac{f_d}{f_x} & \text{if } i = N, \\ \frac{f_x}{f_d} < B & \text{if } i = S. \end{cases}$$

$\bar{\varphi}_d^i > \bar{\varphi}_x^i$ implies that, among surviving firms in $i$, less productive firms serve only the foreign market in $j$, while more productive firms serve both the foreign market in $j$ and the home market in $i$. Clearly, this occurs in sectors where countries’ comparative advantage (measured by $B$) is strong enough relative to the fixed-cost ratio between $f_d$ and $f_x$.

**Proposition 2**

(i) The gap between $\bar{\varphi}_x^i$ and $\bar{\varphi}_d^i$ is narrower than $\bar{\varphi}_x^j$ and $\bar{\varphi}_d^j$ in country $i$’s comparative advantage sectors and this gap is monotonically decreasing in its comparative advantage strength.

(ii) While $\bar{\varphi}_x^i$ is always greater than $\bar{\varphi}_d^i$ in country $i$’s comparative disadvantage sectors, this might not hold in extremely strong comparative advantage sectors.

These two findings fit well with recent empirical research. The first finding – among domestic firms, more firms export in stronger comparative advantage sectors – is consistent with evidence that was reviewed in Introduction. The logic of this result comes from the interplay between the Ricardian productivity difference and relative burden of fixed export costs: log-supermodularity in country and sector characteristics allows relatively less productive firms to incur the fixed export cost relatively more easily in comparative advantage sectors. Strictly speaking, this observation is not satisfactory since several important questions cannot be addressed without the mass of varieties produced and exported. For example, how does countries’ comparative advantage affect the extensive and intensive margins? Does a larger country size lead to a higher export participation ratio in any sector? Later I investigate what determines the mass of varieties to answer these questions.

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13 From $f_x > f_d$, comparative disadvantage sectors of North, for example, must satisfy

$$\bar{z} < z \leq 1 \iff B > 1 \implies \bar{\varphi}_x^N > \bar{\varphi}_d^N.$$ 

(Note also that Assumption 1 has an influence on the relationship between $\bar{\varphi}_d^i$ and $\bar{\varphi}_x^i$ for $i \neq j \in \{N, S\}$.)
The second finding is also in keeping with Lu’s (2011) empirical evidence that manufacturing exporters in China are typically less productive than domestic firms in labor-intensive sectors, but exporters are more productive in capital-intensive sectors. To rationalize this evidence, Lu develops a Heckscher-Ohlin model with heterogeneous firms, emphasizing that allowing factor intensity to vary across sectors is crucial for the Melitz model of North-South trade. Although my theoretical focus is apparently different from hers, the central message is surprisingly similar: exporters can be less productive than domestic firms in comparative advantage sectors, whereas exporters are always more productive in comparative disadvantage sectors. The rationale for this result is as follows. In comparative disadvantage sectors of country \( i \), by definition, firms in country \( j \) are relatively better at producing than those in \( i \). Thus, exporters from \( i \) must be sufficiently productive, not only because they have to cover the fixed export cost, but also because they have to compete with more efficient foreign rivals in the export market. In comparative advantage sectors of \( i \), on the other hand, firms in \( j \) are relatively poorer at producing than those in \( i \), which makes relatively less productive firms in \( i \) find it profitable to enter the export market. This is the reason why the partitioning of firms by export status might not be induced only with the condition \( f_x > f_d \) in comparative advantage sectors. (This holds even with variable trade cost as far as \( \tau^{\sigma-1} f_x > f_d \); see supplementary note.)

If \( \bar{\varphi}_d > \bar{\varphi}_x \) were true, all surviving firms in \( i \) could export, which is not supported by evidence.\(^{14}\) This result should be interpreted as meaning that aggregate productivity premia of exporters relative to domestic firms are smaller in stronger comparative advantage sectors. I exclude the possibility \( \bar{\varphi}_d > \bar{\varphi}_x \) in the following analysis by assuming that \( f_x \) is large enough to satisfy (1) in any sector.

**Full General Equilibrium** The previous subsection provides implicit solutions of firm selection \((\bar{\varphi}_d, \bar{\varphi}_x)\) and aggregate market demand \((B^i)\) for a given wage rate \((w^i)\). Now that each sector’s equilibrium is characterized by these six endogenous variables, this subsection explores full general-equilibrium interactions by explicitly incorporating the LMC conditions in the model. Here I show that the relative wage in North \((\omega = w^N/w^S)\) is necessarily greater than one, a sufficient condition of North-South trade required in Proposition 1(i).

Recall from the profit-maximization problem in Section 2 that labor demand of a firm with productivity \( \varphi \) is given by

\[
\begin{align*}
    l_d^i(\varphi) &= f_d + \frac{\sigma - 1}{\sigma} r_d^i(\varphi), \\
    l_x^i(\varphi) &= f_x + \frac{\sigma - 1}{\sigma} r_x^i(\varphi).
\end{align*}
\]

Substituting these values into the LMC condition and using the FE condition, the previous LMC condition is simplified as follows (see Appendix):

\[
\int_0^1 \frac{R^i_z dz}{w^i} = L^i,
\]

where \( \int_0^1 R^i_z dz = \int_0^1 P^i_z Q^i_z dz \) is aggregate expenditure in country \( i \). Consequently, each country’s

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\(^{14}\) In Lu’s (2011) dataset, \( \bar{\varphi}_d > \bar{\varphi}_x \) occurs because it includes Chinese exporters involved in processing trade. If firms engage in final-good trade only as in the current model, this possibility would not exist.
wage is determined by the equality between aggregate expenditure \( \int_0^1 R^N_x(z)dz \) and aggregate payments to labor \( (w^i L^i) \) as in usual general-equilibrium trade models.

By Walras’ law, I can focus on the LMC condition in North. To derive the relative wage \( \omega \), however, it is easiest to use the balance-of-payments (BOP) condition that is equivalent with the above LMC condition in the current model where two-way trade can occur in any sector (see Proposition 1(ii)):

\[
\int_0^1 R^N_x(z)dz = \int_0^1 R^S_x(z)dz, \tag{BOP}
\]

where \( R^N_x = M^i \int_{\bar{\varphi}_a}^{\infty} r^i_x(\varphi)dG(\varphi) \) is aggregate sector exports. This equivalence stems from that aggregate expenditure in North is the sum of expenditure spent on domestic products and imports that can occur in any sector.

laborers to be allocated to each country’s comparative advantage sectors in trade equilibrium and thus increasing function of \( \tilde{\varphi} \) where

\[
\int^1_0 \int^\infty_{\bar{\varphi}_a} r^i_x(\varphi)dG(\varphi)\]

is an expensive, thereby ensuring some Southern labor demands in equilibrium. Figure 5 depicts \( \xi(z, \omega) \) space.\(^{15}\)

This equation simply indicates that North runs a trade surplus in sectors \( z \in [0, \tilde{z}) \) and a trade deficit in sectors \( z \in (\tilde{z}, 1] \). In equilibrium, the relative wage is adjusted so that aggregate trade surpluses are offset by aggregate trade deficits.

It is then straightforward to derive the relative wage \( \omega \). Arranging the BOP condition in net exports shown above and using \( \lambda_z = P^i \xi^i/Y^i \) defined in Section 2, \( \omega \) can be explicitly solved as a function of \( \tilde{z} \):

\[
\omega \equiv \xi(\tilde{z}) = \frac{\int_0^{\tilde{z}} \left( \frac{L^S}{L^N} - \lambda_z \right) dz}{L \left[ \int_0^{\tilde{z}} \left( \frac{L^S}{L^N} - \lambda_z \right) dz \right]}
\]

where \( L = L^N/L^S \) and \( L^i/L^j \) is a share of labor allocated to sector \( z \). It is easily verified that \( \xi \) is an increasing function of \( \tilde{z} \) satisfying \( \lim_{\tilde{z} \to 0} \xi(\tilde{z}) = 0 \) and \( \lim_{\tilde{z} \to 1} \xi(\tilde{z}) = \infty \) (see Appendix). Intuitively, these properties follow from the fact that \( \xi \) summarizes the LMC condition in each country. If \( \tilde{z} \) is higher for a given \( \omega \), there are more labor demands in North and Southern production is less likely. For North-South trade to occur, therefore, \( \xi \) must be increasing in \( \tilde{z} \) so that Northern labor is more expensive, thereby ensuring some Southern labor demands in equilibrium. Figure 5 depicts \( \xi \) curve in \( (z, \omega) \) space.\(^{15}\)

\(^{15}\)Although \( \xi \) curve has some similarity with that in Dornbusch et al. (1977), their neoclassical model allows all laborers to be allocated to each country’s comparative advantage sectors in trade equilibrium and thus \( \int_0^{\tilde{z}} L^N \xi^i dz = \int_0^{\tilde{z}} L^S \xi^i dz = 1 \) in the equation of \( \xi \) curve. In contrast, this is not true in the current model due to incomplete specialization that can occur in any sector.
The other condition that pins down $\bar{z} = \bar{z}$ and $\omega$ is the relative partial contractibility $\mu = \mu^N / \mu^S$. Among its properties, $\mu > 1$ (absolute advantage of North) and $\mu'' > 0$ (log-supermodularity) are of particular importance. Figure 5 depicts $\mu$ curve in the same space and the intersection of $\xi$ and $\mu$ curves shows that $\omega$ is greater than one. These two curves intersect at $(\bar{z}, \omega)$ because $\omega = \omega(B(z), \varphi_d(z), \varphi_x(z))$ is expressed in terms of $\omega$ in $z = \bar{z}$, i.e., $B(\bar{z}) = 1$ and $\varphi_d(\bar{z}) = \varphi_x(\bar{z}) = \omega^{1/(\sigma-1)}$, implying also that $\omega$ is endogenous. This $\omega$ in turn leads to endogenous solutions of $B$, $\varphi_d$, and $\varphi_x$, which completes the characterization of the eight unknowns in equilibrium.

**Proposition 3**

(i) North (South) is a net exporter of customized (generic) sectors $z \in [0, \bar{z})$ ($z \in (\bar{z}, 1]$) in two-way trade.

(ii) The relative wage in North is greater than one.$^{16}$

It is worthwhile to stress that the above channel for the relative wage is similar to that developed by Antràs (2005), who shows (with representative firms) that, irrespective of the relative country size $L = L^N / L^S$, better contracting environments in North lead to the higher relative wage in general equilibrium. As in his model, the equilibrium outcome $\omega > 1$ directly reflects that North has superior partial contractibility which helps mitigate the serious holdup problem. This in turn gives rise to better production efficiency and overall higher productivity (i.e., wage) in North.

---

$^{16}$While one constraint of Assumption 1 ($\omega > f_d/f_x$) is satisfied as long as $f_x > f_d$, the other constraint is written as

$$\int_0^\bar{z} \lambda_z dz < \frac{1 - \int_0^1 \frac{L^S}{L^N} dz + \frac{L}{L^N} \int_0^1 \frac{L^N}{L^S} dz}{1 + \frac{L}{L^N}}.$$  

To see whether $\omega < f_x/f_d$ internally holds, it is enough to check whether the above inequality holds for a sufficiently large $f_x/f_d$ that the model goes back to autarkic equilibrium. In this case, it is approximated to $\int_0^\bar{z} \lambda_z dz < \int_0^1 \frac{L^N}{L^S} dz$, which is consistent with the current setup where North has a comparative advantage in sectors $z \in [0, \bar{z})$. 

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**Comparative Statics** In this subsection, I examine comparative statics with respect to the relative country size $L$ in a single unified framework for the analysis of home-market effects on the extensive and intensive margins that will be addressed in the next subsection.

Figure 6 illustrates a general-equilibrium interaction between good and factor markets by integrating the previous analysis. To explore effects of an increase in $L$, I compare two equilibria in the figure by adding primes (′) to all variables and functions with thin lines for a new equilibrium. The first and second quadrants come from Figure 3, whereas the fourth quadrant comes from Figure 5. The third quadrant depicts the relationship between $\omega$ and $\bar{\phi}$’s, which is derived from the RZCP conditions: $\bar{\phi}_d = \frac{1}{\mu B} \frac{\omega^\sigma}{(\sigma - 1)}$, $\bar{\phi}_x = \frac{B}{\mu} \frac{\omega^\sigma}{(\sigma - 1)}$. It is clear that these conditions are increasing in $\omega$, with $\bar{\phi}_d(\bar{z}) = \bar{\phi}_x(\bar{z}) = \omega^{1/(\sigma - 1)}$ and $z \geq \bar{z} \Leftrightarrow B \geq 1 \Leftrightarrow \xi \geq \omega \Leftrightarrow \bar{\phi}_d \geq \bar{\phi}_x$. The figure shows that:

- In the fourth quadrant, $\xi$ curve shifts to the right (from the LMC/BOP conditions) while keeping $\mu$ curve unchanged, leading to $\omega > \omega'$ and $\bar{z} < \bar{z}'$.
- In the first quadrant, this decreases $B$ for any $z \in [0, 1]$ because $B(\bar{z}') = 1$ must be true in a new equilibrium and $B$ is increasing in $z$ (from the ZCP/FE conditions).
- In the second quadrant, $\bar{\phi}_d$ curve shifts inward while $\bar{\phi}_x$ curve shifts upward because $B = \frac{\omega^\sigma}{(\sigma - 1)} \bar{\phi}_d^{-1} = \frac{\mu}{\omega^\sigma/(\sigma - 1)} \bar{\phi}_x$ (from the RZCP condition) and $\omega$ is decreasing in $L$ (from the fourth quadrant).
• In the third quadrant, both curves shift inward because $\bar{\varphi}_d = \frac{1}{\mu B} \omega^{\sigma/(\sigma - 1)}$, $\bar{\varphi}_x = \frac{B}{\mu} \omega^{\sigma/(\sigma - 1)}$ (from the RZCP condition) and $B$ is decreasing in $L$ (from the first quadrant).

As is clear, the endogenous variables respond to changes in the exogenous variable $L$ through the relative wage $\omega$. Note from Figures 3 and 4 that when $\omega$ is sufficiently large, it is more likely that $\bar{\varphi}_d > 1$ and $\bar{\varphi}_x > 1$, and the higher aggregate productivity of domestic firms and exporters in North is partially reflected by the higher relative wage. Building on this view, the comparative statics here suggest that an increase in $L$ decreases the Ricardian productivity advantage as well as the firm-level productivity advantage in North by lowering $\omega$, although it increases the range of sectors over which North has a comparative advantage and is a net exporter of customized products.

One of key insights arising from the comparative statics is that, even with the C.E.S. preferences, country size does affect firm selection ($\bar{\varphi}_d^i$, $\bar{\varphi}_x^i$) and aggregate market demand ($B^i$) through the relative wage $\omega$ in this asymmetric-country model with the Ricardian productivity difference. In a symmetric-country model or a model with a homogeneous-good sector, by contrast, country size has impacts only on the mass of potential entrants without affecting the firm-level variables.

**Margins of Specialization and Trade** Up until now the analysis has mainly dealt with the characterization of the eight equilibrium unknowns. It has not addressed the mass of varieties domestically produced and exported in each sector, which is also endogenously determined in equilibrium. To calculate the mass of varieties explicitly, I hereafter assume that firm productivity $\varphi$ follows a Pareto distribution:

$$G(\varphi) = 1 - \left( \frac{\varphi_{\min}}{\varphi} \right)^k, \quad \varphi \geq \varphi_{\min} > 0,$$

where both the shape and scale parameters, $k$ and $\varphi_{\min}$, are identical across countries and sectors under Definition 1. The purpose of this subsection is to investigate the interactions among comparative advantage, country size and the extensive/intensive margins.

In what follows, I first derive the mass of potential entrants in a sector of country $i$, which is denoted by $M_{ie}^i$. Applying the Pareto distribution to the aggregate price in each country $P^i$, the mass of potential entrants is expressed as

$$M_{ie}^i = \frac{1}{\sigma} \left( \frac{w^i}{\mu^i} \right)^{\sigma - 1} \frac{X^i}{B^i} \left( \frac{\bar{\varphi}_d^i}{\bar{\varphi}_x^i} \right)^{k - \sigma + 1} - \frac{X^j}{B^j} \left( \frac{\bar{\varphi}_d^j}{\bar{\varphi}_x^j} \right)^{k - \sigma + 1} \frac{1}{\Delta},$$

where implicit solutions for the eight unknowns are already given in the preceding analysis. Note that if two-way trade is to occur in any sector as suggested in Proposition 1(ii), $M_{ie}^i$ should be strictly positive for any range of parameters. It directly follows that $M_{ie}^i > 0$ if and only if

$$X^{\frac{\sigma - 1}{k}} \left( \frac{f_d}{f_x} \right)^{k - \sigma + 1} \frac{1}{k} < B < X^{\frac{\sigma - 1}{k}} \left( \frac{f_x}{f_d} \right)^{k - \sigma + 1} \frac{1}{k},$$

(2)

$$\Delta = \left( \frac{\varphi_{\min}^d}{\varphi_{\min}^x} \right)^{k - \sigma + 1} - \left( \frac{\bar{\varphi}_d}{\bar{\varphi}_x} \right)^{k - \sigma + 1},$$

which is positive under condition (1) and $k - \sigma + 1 > 0$. 

---

\( \Delta \) = \left( \frac{\varphi_{\min}^d}{\varphi_{\min}^x} \right)^{k - \sigma + 1} - \left( \frac{\bar{\varphi}_d}{\bar{\varphi}_x} \right)^{k - \sigma + 1}, \) which is positive under condition (1) and $k - \sigma + 1 > 0$. 

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where \( X = X^N/X^S = \omega L \). Although the restriction on \( B \) in (2) has different implications from that in (1) – i.e. condition (1) by which \( \bar{\phi}_d > \bar{\phi}_x \) versus condition (2) by which \( M_e > 0 \), the range of (1) is certainly greater than the range of (2) in the current setup (see Appendix). Accordingly, only with (1) does one-way trade take place in sectors where countries’ comparative advantage is extremely strong. To perform a consistent analysis (especially for the LMC/BOP conditions), I henceforth assume that the fixed export cost is sufficiently large to satisfy (2) in any sector, but the main analysis would essentially go through even if two types of trade coexist in the model.

I now examine the effects of comparative advantage and country size on the extensive and intensive margins. Let aggregate sector exports \( R^i_x \) decompose into

\[
R^i_x = M^i_e \int_{\bar{\phi}_d}^{\infty} \bar{r}^i_x(\varphi) dG(\varphi)
= [1 - G(\bar{\phi}_d)] M^i_e \times \frac{1}{[1 - G(\bar{\phi}_d)]} \int_{\bar{\phi}_x}^{\infty} r^i_x(\varphi) dG(\varphi)
= M^i_{ex} \times \bar{r}^i_x,
\]

where \( M^i_{ex} \) is the mass of firms that actually engage in exporting (extensive margin), and \( \bar{r}^i_x \) is average exports across heterogeneous firms (intensive margin). Similarly, aggregate domestic sales are decomposed into \( R^i_d = M^i_{ed} \times \bar{r}^i_d \). Under the Pareto distribution, these margins are expressed as

\[
M^i_{ed} = \left( \frac{\varphi_{min}}{\bar{\varphi}_d^i} \right)^k M^i_e, \quad M^i_{ex} = \left( \frac{\varphi_{min}}{\bar{\varphi}_x^i} \right)^k M^i_e,
\]

\[
\bar{r}^i_d = \frac{k \sigma}{k - \sigma + 1} w^i f_d, \quad \bar{r}^i_x = \frac{k \sigma}{k - \sigma + 1} w^i x.
\]

It is obvious that the intensive margins \((\bar{r}^i_d, \bar{r}^i_x)\) are independent of sector index \( z \) and thus comparative advantage; due to log-supermodularity, however, aggregate domestic sales and exports \((R^i_d, R^i_x)\) are both increasing in it, suggesting that comparative advantage increases these aggregates solely through the extensive margins \((M^i_{ed}, M^i_{ex})\). In other words, comparative advantage has two opposing effects on the intensive margins. First, stronger comparative advantage allows all firms to be relatively better at producing and makes each firm’s revenue \((\bar{r}^i_d, \bar{r}^i_x)\) higher, which increases the intensive margins. Second, stronger comparative advantage allows less productive firms to enter into domestic and export markets and makes the productivity cutoffs \((\bar{\varphi}_d^i, \bar{\varphi}_x^i)\) smaller as seen in Figure 2, which decreases the intensive margins. Under the Pareto distribution, these two effects are exactly offset, leaving the intensive margins independent of comparative advantage strength.

As in the previous subsections, the current paper’s primary interest lies in the analysis of the relative terms, rather than the absolute terms. From the equations of the absolute margins, the relative extensive and intensive margins in North are given by

\[
M_{ed} = \frac{M^N_{ed}}{M^S_{ed}} = \frac{M_e}{(\bar{\varphi}_d)^k}, \quad M_{ex} = \frac{M^N_{ex}}{M^S_{ex}} = \frac{M_e}{(\bar{\varphi}_x)^k}, \quad \frac{\bar{r}^N_d}{\bar{r}^S_d} = \frac{\bar{r}^N_x}{\bar{r}^S_x} = \omega > 1,
\]

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where $M_e = M^N_e/M^S_e$. Hence, regardless of country size, the intensive margins are always higher in North reflecting the absolute advantage assumption. It also follows that the relative extensive margins of specialization and trade are increasing in $L$ due to the home-market effect ($M^N_{ed}/M^S_{ed} > L^N/L^S$, $M^N_{ex}/M^S_{ex} > L^N/L^S$) whereas the relative intensive margins are decreasing in it due to the lower relative wage (see Figure 6).\textsuperscript{18} This effect of country size on the two margins should be opposite in order to restore the trade balance.

To make this point clear, Figure 7 depicts $M_{ed}$ and $M_{ex}$ curves in $(z, M)$ space. Using $M^i_e$ derived above, these two curves are shown to be downward-sloping with $M^i_{ex}(z) < M^i_{ed}(z) < 0$ for $z \in [0, 1]$ and their intersection is at \((\bar{z}, \frac{XF-1}{M^i_{ex}})\) where $F = (fx/fd)[(k-\sigma+1)/(\sigma-1)]$. Let $\bar{z}_d$ and $\bar{z}_x$ respectively denote the cutoffs at which $M_{ed}(\bar{z}_d) = 1$ and $M_{ex}(\bar{z}_x) = 1$ with $\bar{z}_d < \bar{z}_x < \bar{z}$. These $\bar{z}_d$ and $\bar{z}_x$ are the cutoffs below which North produces and exports relatively more varieties than South (extensive margin), while $\bar{z}$ is the cutoff below which North is a net exporter of customized products, i.e., $R^N_x - R^S_x = M^N_{ex}\bar{z}^N_x - M^S_{ex}\bar{z}^S_x > 0$ for $z \in [0, \bar{z}]$. The relationship between the extensive/intensive margins of trade and aggregate sector exports across sectors are summarized in Table 1. It is important to note that the vertical intersection of the two curves in Figure 7 is smaller than unity for any $L$: indeed, if $\frac{XF-1}{M^i_{ex}} > 1$, not only is the intensive margin of trade $\bar{r}_x^i$ but the extensive margin of trade $M^i_{ex}$ is also higher for North in some comparative advantage sectors of South $z \in (\bar{z}, 1]$, contradicting the previous argument that trade is balanced in the cutoff sector $\bar{z}$. Simple inspection also reveals that the vertical intersection between the two curves is increasing in $L$, and thus this intersection becomes closer to $(\bar{z}_x, 1)$ with the increase in $L$, whereby the extensive margin of trade is higher while the intensive margin of trade is lower as claimed above. (This logic also applies for the extensive/intensive margins of specialization.)

\textsuperscript{18}In contrast to Krugman (1980) who emphasizes the role of the variable trade cost in the home-market effect, the fixed trade cost plays a qualitatively similar role in the current paper. The separate supplementary note develops a more general model in which firms incur both variable and fixed trade costs, and shows that the extensive and intensive margins are exactly the same as the above even in such a model.
Table 1 – Margins of trade and aggregate sector exports

<table>
<thead>
<tr>
<th>Sector</th>
<th>Extensive margin</th>
<th>Intensive margin</th>
<th>Aggregate sector exports</th>
</tr>
</thead>
<tbody>
<tr>
<td>$z \in [0, \bar{z}_x)$</td>
<td>$M^N_x &gt; M^S_x$</td>
<td>$\bar{r}^N_x &gt; \bar{r}^S_x$</td>
<td>$M^N_x \bar{r}^N_x &gt; M^S_x \bar{r}^S_x$</td>
</tr>
<tr>
<td>$z \in (\bar{z}_x, \bar{z})$</td>
<td>$M^N_x &lt; M^S_x$</td>
<td>$\bar{r}^N_x &gt; \bar{r}^S_x$</td>
<td>$M^N_x \bar{r}^N_x &gt; M^S_x \bar{r}^S_x$</td>
</tr>
<tr>
<td>$z \in (\bar{z}, 1]$</td>
<td>$M^N_x &lt; M^S_x$</td>
<td>$\bar{r}^N_x &gt; \bar{r}^S_x$</td>
<td>$M^N_x \bar{r}^N_x &lt; M^S_x \bar{r}^S_x$</td>
</tr>
</tbody>
</table>

Regarding the relative intensive margin of specialization and trade in each country,

$$\frac{\bar{r}^N_x}{\bar{r}^N_d} = \frac{\bar{r}^S_x}{\bar{r}^S_d} = \frac{f_x}{f_d} > 1,$$

suggesting that the intensive margin of exporters is higher than that of nonexporters in any sector of the two countries because exporters are on average more productive and earn higher average revenue than nonexporters. On the other hand, the relative extensive margin of specialization and trade in each country is given by

$$\frac{M^I_x}{M^I_d} = \left(\frac{\bar{\varphi}^I_d}{\bar{\varphi}^I_x}\right)^k = \begin{cases} \left(\frac{1}{B} \frac{f_d}{f_x}\right)^{\frac{k}{\sigma - 1}} & \text{if } i = N, \\ \left(B \frac{f_d}{f_x}\right)^{\frac{k}{\sigma - 1}} & \text{if } i = S, \end{cases}$$

which is between zero and unity under condition (1) or (2). Comparative statics on this equation reveal that $M^I_x/M^I_d$ is increasing in comparative advantage strength ($1/B$ for North and $B$ for South) and the fixed-cost ratio ($f_d/f_x$), and is decreasing in the degree of firms’ productivity dispersion ($k/(\sigma - 1)$). Further, this ratio is increasing (decreasing) in the relative country size $L$ for North (South) because $B$ decreases with $L$ (recall the comparative statics in Figure 6). The intuition behind the last result is as follows. While an increase in $L$ increases the extensive margins in North due to the home-market effect, it decreases the Ricardian productivity advantage of North through the lower relative wage $\omega$, which decreases the intensive margins there. This decrease in the intensive margins is bigger for $\bar{r}^N_x$ than $\bar{r}^N_d$ ($\frac{\partial \bar{r}^N_x}{\partial L} = \frac{f_x}{f_d} > 1$): a decrease of the Ricardian productivity advantage is more serious for exporters because they have to incur the higher fixed cost. To restore the trade balance, an increase in the extensive margins is bigger for $M^N_x$ than $M^N_d$ and thus $M^N_x/M^N_d$ is increasing in $L$. Noting that an increase in $L$ works oppositely for the extensive and intensive margins in South, this intuition also explains why $M^S_x/M^S_d$ is decreasing in $L$.

**Proposition 4**

(i) The volume of domestic sales and exports increases with comparative advantage strength solely through the extensive margins.
(ii) The export participation ratio is increasing (decreasing) in the relative country size for North (South).

5 Discussions

In this section, I briefly argue two extensions of the basic model: variable trade costs and multinational firms. The separate supplementary note offers a more detailed analysis that incorporates the variable trade cost.

Transport Costs The model has assumed that only the fixed trade cost \( f_x \) exists and firms can export without incurring the variable trade cost \( \tau \). If this cost is incorporated into the previous setting, the profit functions in Northern market are given by

\[
\pi_d^N = B^N \left( \frac{\mu^N(z)}{w^N} \right) \varphi^N - w^N f_d, \quad \pi_x^S = B^N \left( \frac{\mu^S(z)}{\tau w^S} \right) \varphi^S - w^S f_x,
\]

and \( \pi_x^S \) is steeper (flatter) than \( \pi_d^N \) if and only if

\[
\frac{\mu^S(z)}{\tau w^S} > \frac{\mu^N(z)}{w^N} \iff \omega > \tau \mu(z).\]

Similarly, from the profit functions in Southern market, \( \pi_x^N \) is steeper (flatter) than \( \pi_d^S \) if and only if

\[
\frac{\mu^N(z)}{\tau w^S} > \frac{\mu^S(z)}{w^S} \iff \mu(z) > \tau \omega.\]

In the presence of variable trade cost \( \tau \), the cutoff sector \( \bar{z} \) is no longer identical between North and South. Instead, there exist two distinct cutoff sectors, namely \( \bar{z}^N \) and \( \bar{z}^S \), such that North (South) has an institutional comparative advantage in \( z \in [0, \bar{z}^N] \) (\( z \in (\bar{z}^S, 1] \)), where \( \bar{z}^N = \mu^{-1}(\tau \omega) \) and \( \bar{z}^S = \mu^{-1}(\omega/\tau) \) (see Definition 2). While this result bears a resemblance to that in Dornbusch et al. (1977), Figure 2 shows that the variable trade cost \( \tau \) does not allow nontraded sectors to exist in \( z \in [\bar{z}^N, \bar{z}^S] \) in the current model, i.e., two-way trade can occur in any sector.\(^{19}\) Moreover, the general-equilibrium approach is not qualitatively affected by \( \tau \) because: (i) changes in \( z \) still shift \( \varphi_d^i \) and \( \varphi_x^i \) in opposite directions, implying that the relative market demand \( B = B^N / B^S \) increases with \( z \); (ii) the relative productivity cutoffs \( \bar{\varphi}_d = \bar{\varphi}_d^N / \bar{\varphi}_d^S \) and \( \bar{\varphi}_x = \bar{\varphi}_x^N / \bar{\varphi}_x^S \) are exactly the same as those examined in Section 4; and (iii) the variable trade cost does not affect a mechanism through which each country’s wage is determined by the equality between aggregate expenditure and aggregate payments to labor, giving rise to \( \omega > 1 \) under the absolute advantage assumption. These observations jointly suggest that, even in the presence of variable trade cost \( \tau \), there should arise a general-equilibrium interplay between good and factor markets that is similar to Figure 6. Therefore, although each absolute term of the eight unknowns is affected by \( \tau \), the key characterizations represented in the relative terms

\(^{19}\)Just as (2) is required for two-way trade to occur in any sector, so (2’) below is needed in this extension.
generally continue to hold.

One of interesting implications of this extension is that the introduction of $\tau$ alters the relationship between (1) and (2). Since the relative productivity cutoffs $\bar{\varphi}_i^x/\bar{\varphi}_d$ in this setting for each country are respectively given by

$$\frac{\bar{\varphi}_N}{\bar{\varphi}_d} = \tau \left( B \frac{f_x}{f_d} \right)^{\frac{1}{\sigma-1}}, \quad \frac{\bar{\varphi}_S}{\bar{\varphi}_d} = \tau \left( \frac{1}{B} \frac{f_x}{f_d} \right)^{\frac{1}{\sigma-1}},$$

the usual outcome ($\bar{\varphi}_N^x > \bar{\varphi}_d^i$) occurs in any sector of both countries if

$$\left( \frac{1}{\tau} \right)^{\frac{1}{\sigma-1}} \frac{f_d}{f_x} < B < \tau^{\frac{1}{\sigma-1}} \frac{f_x}{f_d}. \quad (1')$$

On the other hand, two-way trade ($M_{iex} > 0$) occurs in any sector of both countries if

$$X^{\frac{k-\sigma+1}{k}} \left( \frac{1}{\tau} \right)^{\frac{1}{\sigma-1}} \frac{f_d}{f_x} < B < X^{\frac{k-\sigma+1}{k}} \tau^{\frac{1}{\sigma-1}} \left( \frac{f_x}{f_d} \right)^{\frac{k-\sigma+1}{k}}. \quad (2')$$

Evidently, (1') and (2') converge to (1) and (2) as $\tau \to 1$. While (1) certainly subsumes (2) without $\tau$, whether the range of (1') is greater than the range of (2') depends on trade cost parameters. (It is not possible to directly compare (1) and (1') or (2) and (2') because $B$ is a function of $\tau$.)

**Multinational Enterprises** The main analysis has assumed that exporting is only one option for serving a foreign market. In the real world, firm sales by multinational enterprises have been growing faster than exporting, and foreign direct investment (FDI) plays a key role in a country where the market system is less developed.\(^{20}\) As initially raised by Williamson (1985), this issue is of particular importance when contracts are better enforced within the firm boundaries. In the international context, this means that Northern firms are able to respond to poor contract enforcement by FDI (while employing local workers in South), thereby replacing weak external governance with an internal principal-agent relationship. It also implies that the firm’s choice between intra-firm and arm’s-length trade is affected by contractibility. In fact, Bernard, Jensen, Redding, and Schott (2010) find empirical evidence suggesting that the intra-firm fraction of the U.S. imports is significantly higher for products for which contractibility is more difficult. Because North has a comparative advantage in contract-dependent sectors in the model, it is worth investigating the role played by multinational firms in the presence of partial contractibility.

To analyze export versus FDI in the previous environment, suppose that

$$1 < \frac{\mu_N(z)}{\mu^M(z)} < \frac{\mu^N(z')}{\mu^M(z')} < \infty, \quad 1 < \frac{\mu_M(z)}{\mu^S(z)} < \frac{\mu^M(z')}{\mu^S(z')} < \infty,$$

\(^{20}\)For simplicity, I have confined attention to the case where imperfect institutions are related to the production side only. As surveyed by Dixit (2011), these frictions are also important in other aspects, such as the distribution channel of exported products in foreign countries, and multinationals can alleviate this problem by internalization.
for $z > z', \mu^N > \mu^M > \mu^S, \mu^M(z) \neq 0, \mu^M(z') \neq 0, \mu^S(z) \neq 0$ and $\mu^S(z') \neq 0$. $\mu^M$ denotes partial contractibility within the boundaries of multinational firms, lying between $\mu^N$ and $\mu^S$ for $z \in [0, 1]$ in Figure 1. These inequalities mean that multinational enterprises have strictly better contract enforcement within the firm boundaries than Southern domestic firms, but their contractibility is lower than Northern domestic firms because they have to be at least partially affected by Southern governance structure, such as courts of law and social norms (Dixit, 2011): Northern (Southern) firms receive negative (positive) feedback from becoming multinationals on contractibility. At the same time, Northern multinationals are able to exploit wage differentials, while Southern multinationals have to pay the higher wage. This creates a new tradeoff between wage and contractibility that has been missing in the export-versus-FDI literature. As a result, the profit functions of Northern and Southern multinationals,

$$\pi^N_m = B^S \left( \frac{\mu^M(z)}{w^S} \right)^{\sigma-1} \varphi^{\sigma-1} - w^S f_m,$$

$$\pi^S_m = B^N \left( \frac{\mu^M(z)}{w^N} \right)^{\sigma-1} \varphi^{\sigma-1} - w^N f_m,$$

are respectively steeper (flatter) than those of Northern and Southern exporters, $\pi^x_N$ and $\pi^x_S$, if and only if

$$\frac{\mu^M(z)}{w^S} \leq \frac{\mu^N(z)}{w^N} \iff \omega \geq \tilde{\mu}(z), \quad \frac{\mu^M(z)}{w^N} \leq \frac{\mu^S(z)}{w^S} \iff \hat{\mu}(z) \geq \omega,$$

where $\tilde{\mu}(z) = \mu^N(z)/\mu^M(z)$ and $\hat{\mu}(z) = \mu^M(z)/\mu^S(z)$. Thus, FDI undertaken by Northern (Southern) firms is more likely to emerge in equilibrium if and only if endogenous wage differentials are sufficiently large (small) relative to exogenous contractibility differentials.

Under these circumstances, how does the existence of multinational firms affect the patterns of specialization and trade? Is there any systematic relationship between the relative export/FDI flows and countries’ comparative advantage? If so, what impacts does it have on wage inequality between North and South? Although these questions have vital implications for consequences of globalization, I leave this extension for my future work.
Appendix

A.1 Proof of the Market Demand

I show that the relative market demand $B = B^N / B^S$ increases with $z$. Taking the logarithm of the ZCP conditions and differentiating them with respect to $z$ gives

$$\frac{B^{N'}}{B^N} + (\sigma - 1) \frac{\mu^{N'}}{\mu^N} + (\sigma - 1) \frac{\varphi_d^{N'}}{\varphi_d^N} = 0,$$

(A.1)

$$\frac{B^{S'}}{B^S} + (\sigma - 1) \frac{\mu^{S'}}{\mu^S} + (\sigma - 1) \frac{\varphi_d^{S'}}{\varphi_d^S} = 0,$$

(A.2)

$$\frac{B'}{B^S} + (\sigma - 1) \frac{\mu'}{\mu^S} + (\sigma - 1) \frac{\varphi_d'}{\varphi_d^S} = 0,$$

(A.3)

$$\frac{B^{N'}}{B^N} + (\sigma - 1) \frac{\mu^{N'}}{\mu^N} + (\sigma - 1) \frac{\varphi_d^{N'}}{\varphi_d^N} = 0.$$

(A.4)

Further, differentiating the FE condition with respect to $z$ and rearranging,

$$\varphi_d^{N'} = -C^N \varphi_d^N,$$

(A.5)

$$\varphi_d^{S'} = -C^S \varphi_d^S,$$

(A.6)

where $C^i = f_d J'(\varphi_d^i)/f_x J'(\varphi_x^i) > 0$. Note that (A.1)–(A.6) are six equations with six unknowns $(\varphi_d^{N'}, \varphi_d^{S'}, \varphi_x^{N'}, \varphi_x^{S'}, B^N, B^S)$. Substituting (A.5) and (A.6) respectively into (A.3) and (A.4), and subtracting (A.2) and (A.1) respectively from these yields

$$\frac{C^N \varphi_d^{N'}}{\varphi_d^N} + \frac{\varphi_d^{S'}}{\varphi_d^S} = \frac{\mu'}{\mu}, \quad -\frac{\varphi_d^{N'}}{\varphi_d^N} - \frac{C^S \varphi_d^{S'}}{\varphi_d^S} = \frac{\mu'}{\mu},$$

where $\mu'/\mu = \mu^{N'}/\mu^N - \mu^{S'}/\mu^S < 0$. These are two equations with two unknowns $(\varphi_d^{N'}, \varphi_d^{S'})$, which can be solved for

$$\varphi_d^{N'} = \frac{\mu'}{\mu} \left( \frac{1}{\varphi_d^N} + \frac{C^S}{\varphi_d^S} \right), \quad \varphi_d^{S'} = -\frac{\mu'}{\mu} \left( \frac{1}{\varphi_d^S} + \frac{C^N}{\varphi_d^N} \right),$$

where

$$\Xi = \frac{1}{\varphi_d^S \varphi_d^N} \left( \frac{\varphi_d^{N'} \varphi_d^{S'}}{\varphi_d^N \varphi_d^S} C^S - 1 \right).$$

From the ZCP conditions and $C^i$ defined above, $\Xi$ is positive if and only if

$$\frac{J'(\varphi_d^N) J'(\varphi_d^S)}{J'(\varphi_x^N) J'(\varphi_x^S)} > \left( \frac{f_x}{f_d} \right)^{\frac{2 \sigma}{\sigma - 1}}.$$

(Assuming a Pareto distribution, for example, the left-hand side is $(f_x/f_d)^{2(k+1)/(\sigma-1)}$ and this holds if $k - \sigma + 1 > 0$). Under this condition, $\varphi_d^{N'} < 0, \varphi_d^{S'} > 0$ and from (A.5) and (A.6) $\varphi_x^{N'} > 0, \varphi_x^{S'} < 0$. From the relative market demand $B = B^N / B^S$ in the main text, these then imply that $B' > 0$. □
A.2 Proofs of the LMC and BOP Conditions

A.2.1 Proof of the LMC Condition

I show that the LMC condition is simplified as

$$\int_0^1 \frac{R^i dz}{w^i} = L^i.$$ 

From the LMC condition, aggregate labor demand in a particular sector of $i$ is given by

$$M^i_c \int_{\bar{\varphi}_d^i}^{\infty} l^i_d(\varphi)dG(\varphi) + M^i_e \int_{\bar{\varphi}_e^i}^{\infty} l^i_x(\varphi)dG(\varphi) + M^i_e f_e,$$

where the first two terms are aggregate labor demands for production and the third is aggregate labor demand for investment by potential entrants in this sector. Using the optimal labor demand of heterogeneous firms, the former aggregate labor demands are

$$M^i_c \int_{\bar{\varphi}_d^i}^{\infty} l^i_d(\varphi)dG(\varphi) + M^i_e \int_{\bar{\varphi}_e^i}^{\infty} l^i_x(\varphi)dG(\varphi) = M^i_c \left[1 - G(\bar{\varphi}_d^i)\right] f_d + \frac{\sigma - 1}{\sigma} \int_{\bar{\varphi}_d^i}^{\infty} r^i_d(\varphi)dG(\varphi) + M^i_e \left[1 - G(\bar{\varphi}_e^i)\right] f_x + \frac{\sigma - 1}{\sigma} \int_{\bar{\varphi}_e^i}^{\infty} r^i_x(\varphi)dG(\varphi),$$

where $\Pi^i$ denotes aggregate sector profit. On the other hand, the latter aggregate labor demands are

$$M^i_e f_e = \frac{M^i_e}{w^i} \left\{ \frac{1}{\sigma} \int_{\bar{\varphi}_d^i}^{\infty} r^i_d(\varphi)dG(\varphi) - \left[1 - G(\bar{\varphi}_d^i)\right] w^i f_d + \frac{1}{\sigma} \int_{\bar{\varphi}_e^i}^{\infty} r^i_x(\varphi)dG(\varphi) - \left[1 - G(\bar{\varphi}_e^i)\right] w^i f_x \right\}$$

$$= \frac{\Pi^i}{w^i},$$

which is derived from the FE condition:

$$f_e = \int_{\bar{\varphi}_d^i}^{\infty} \frac{\pi^i_d(\varphi)}{w^i} dG(\varphi) + \int_{\bar{\varphi}_e^i}^{\infty} \frac{\pi^i_x(\varphi)}{w^i} dG(\varphi)$$

$$= \frac{1}{w^i} \left\{ \int_{\bar{\varphi}_d^i}^{\infty} \frac{r^i_d(\varphi)}{\sigma} dG(\varphi) - \left[1 - G(\bar{\varphi}_d^i)\right] w^i f_d + \int_{\bar{\varphi}_e^i}^{\infty} \frac{r^i_x(\varphi)}{\sigma} dG(\varphi) - \left[1 - G(\bar{\varphi}_e^i)\right] w^i f_x \right\}.$$ 

Summing up these two kinds of aggregate labor demands gives

$$M^i_c \int_{\bar{\varphi}_d^i}^{\infty} l^i_d(\varphi)dG(\varphi) + M^i_e \int_{\bar{\varphi}_e^i}^{\infty} l^i_x(\varphi)dG(\varphi) + M^i_e f_e = \frac{M^i_c}{w^i} \left\{ \int_{\bar{\varphi}_d^i}^{\infty} r^i_d(\varphi)dG(\varphi) + \int_{\bar{\varphi}_e^i}^{\infty} r^i_x(\varphi)dG(\varphi) \right\}$$

$$= \frac{R^i}{w^i}.$$
Finally, integrating the above aggregate sector labor demands over the interval \([0,1]\) completes the proof. \(\square\)

### A.2.2 Proof of the BOP Condition

I first show that the BOP condition is written as

\[
\int_{0}^{\tilde{z}} (w^N L^N_z - R^N_z) \, dz = \int_{\tilde{z}}^{1} (w^S L^S_z - R^S_z) \, dz.
\]

From the BOP condition \((\int_{0}^{1} R^N_x(z) \, dz = \int_{0}^{1} R^S_x(z) \, dz)\) and log-supermodularity, there exists a unique cutoff \(\tilde{z}\) such that

\[
\int_{0}^{\tilde{z}} \left[ M^N_i \int_{\varphi_x^N}^{\infty} r^N_x(\varphi) dG(\varphi) - M^S_i \int_{\varphi_x^S}^{\tilde{z}} r^S_x(\varphi) dG(\varphi) \right] \, dz = \int_{\tilde{z}}^{1} \left[ M^N_i \int_{\varphi_x^N}^{\infty} r^N_x(\varphi) dG(\varphi) - M^S_i \int_{\varphi_x^S}^{\tilde{z}} r^S_x(\varphi) dG(\varphi) \right] \, dz.
\]

Note that for \(i \neq j \in \{N, S\}\), the terms in the square brackets are

\[
R^i_x(z) - R^i_x(z) = M^i_x \int_{\varphi_x^i}^{\infty} r^i_x(\varphi) dG(\varphi) - M^i_x \int_{\varphi_x^i}^{\infty} r^i_x(\varphi) dG(\varphi)
\]

which is positive (negative) if \(z \in [0, \tilde{z})\) and negative (positive) if \(z \in (\tilde{z}, 1]\) for \(i = N\) \((i = S)\). The proof immediately follows from the above.

Next, I show that the above BOP condition is written as

\[
\omega \equiv \xi(\tilde{z}) = \frac{\int_{\tilde{z}}^{1} \left( \frac{L^S_z}{L^N_z} - \lambda_z \right) \, dz}{\int_{0}^{\tilde{z}} \left( \frac{L^N_z}{L^N_z} - \lambda_z \right) \, dz}.
\]

By manipulating the BOP condition,

\[
\begin{aligned}
\int_{0}^{\tilde{z}} (w^N L^N_z - R^N_z) \, dz &= \int_{\tilde{z}}^{1} (w^S L^S_z - R^S_z) \, dz \\
\iff \int_{0}^{\tilde{z}} \left( \frac{L^N_z}{L^N_z} - \frac{R^N_z}{w^N L^N_z} \right) \, dz &= \int_{\tilde{z}}^{1} \left( \frac{w^S L^S_z}{w^S L^S_z} - \frac{w^S L^S_z}{w^N L^N_z} - \frac{R^S_z}{w^S L^S_z} \frac{w^S L^S_z}{w^N L^N_z} \right) \, dz \\
\iff \int_{0}^{\tilde{z}} \left( \frac{L^N_z}{L^N_z} - \lambda_z \right) \, dz &= \frac{1}{\omega L} \int_{\tilde{z}}^{1} \left( \frac{L^S_z}{L^S_z} - \lambda_z \right) \, dz,
\end{aligned}
\]

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where the second equation comes from dividing both sides of the first equation by $w^N L^N$, and the third comes from $\lambda_z = \lambda^N_z = \lambda^S_z = \frac{R_i}{w^T}$, solving the last equation for $\omega$ completes the proof.

Finally, I show that the above BOP condition satisfies

$$\xi'(\tilde{z}) > 0, \quad \lim_{\tilde{z} \to 0} \xi(\tilde{z}) = 0, \quad \lim_{\tilde{z} \to 1} \xi(\tilde{z}) = \infty.$$ 

Let $\xi_1 = \int_{\tilde{z}}^1 \left(\frac{L^S_z}{L^S} - \lambda_z\right) dz$ and $\xi_2 = \int_0^{\tilde{z}} \left(\frac{L^N_z}{L^N} - \lambda_z\right) dz$ denote the numerator and denominator of $\xi(\tilde{z})$. Differentiating these terms with respect to $\tilde{z}$ yields

$$\frac{d\xi_1}{d\tilde{z}} = \left(-\frac{L^S_z}{L^S} + \lambda_z\right) + \int_{\tilde{z}}^1 \frac{\partial}{\partial \tilde{z}} \left(\frac{L^S_z}{L^S} - \lambda_z\right) dz,$$

$$\frac{d\xi_2}{d\tilde{z}} = \left(\frac{L^N_z}{L^N} - \lambda_z\right) + \int_0^{\tilde{z}} \frac{\partial}{\partial \tilde{z}} \left(\frac{L^N_z}{L^N} - \lambda_z\right) dz.$$

The first term in the right-hand side captures the marginal effect of $\tilde{z}$ on the volume of trade across sectors, which is zero because exports from North and South are by definition exactly the same in sector $\tilde{z}$, i.e., $\frac{L^N_z}{L^N} - \lambda_z = \frac{L^S_z}{L^S} - \lambda_z = 0$. The second term captures the marginal effect of $\tilde{z}$ within sectors, which is $\int_{\tilde{z}}^1 \frac{\partial}{\partial \tilde{z}} \left(\frac{L^S_z}{L^S} - \lambda_z\right) dz > 0$ and $\int_0^{\tilde{z}} \frac{\partial}{\partial \tilde{z}} \left(\frac{L^N_z}{L^N} - \lambda_z\right) dz < 0$ because log-supermodularity increases the within-sectoral volume of trade in North (South) as $\tilde{z}$ is close to zero (one). Combined with this observation, the desired result follows from noting that both $\xi_1$ and $\xi_2$ are positive for any $\tilde{z}$.

\[\square\]

### A.3 Proofs of the Extensive and Intensive Margins

#### A.3.1 Derivation of the Extensive Margins

I first show the derivation of $M^i_e$. From the C.E.S. preferences, the aggregate price in a sector of country $i$ is

$$(P_i)^{1-\sigma} = \int_{v \in V} p^{1-\sigma}(v) dv$$

$$= M^i_e \int_{\phi^i_d}^{\infty} \left(\frac{\sigma}{\sigma - 1} \frac{w^i}{\varphi^i d}\right)^{1-\sigma} dG(\varphi) + M^i_j \int_{\varphi^j_d}^{\infty} \left(\frac{\sigma}{\sigma - 1} \frac{w^j}{\varphi^j d}\right)^{1-\sigma} dG(\varphi),$$

Since $B^i = \frac{(\sigma-1)^{\sigma-1}}{\sigma} X^i (P_i)^{\sigma-1}$, this aggregate price is also written as $(P_i)^{1-\sigma} = \frac{(\sigma-1)^{\sigma-1}}{\sigma} X^i$. Combining these two relationships gives

$$M^i_e \int_{\phi^i_d}^{\infty} \left(\frac{\sigma}{\sigma - 1} \frac{w^i}{\varphi^i d}\right)^{1-\sigma} dG(\varphi) + M^i_j \int_{\varphi^j_d}^{\infty} \left(\frac{\sigma}{\sigma - 1} \frac{w^j}{\varphi^j d}\right)^{1-\sigma} dG(\varphi) = \frac{(\sigma-1)^{\sigma-1}}{\sigma} X^i B^i.$$
These two equations, \(((P^N)^{1-\sigma}, (P^S)^{1-\sigma})\), are the two systems with the two unknowns \((M^N_e, M^S_e)\). Rewriting these equations in a matrix form,

\[
\begin{bmatrix}
\left(\frac{\sigma}{\sigma-1} \frac{w^N}{\mu^N}\right)^{1-\sigma} [V(\infty) - V(\tilde{\varphi}^N_d)] & \left(\frac{\sigma}{\sigma-1} \frac{w^S}{\mu^S}\right)^{1-\sigma} [V(\infty) - V(\tilde{\varphi}^S_d)] \\
\left(\frac{\sigma}{\sigma-1} \frac{w^N}{\mu^N}\right)^{1-\sigma} [V(\infty) - V(\tilde{\varphi}^N_x)] & \left(\frac{\sigma}{\sigma-1} \frac{w^S}{\mu^S}\right)^{1-\sigma} [V(\infty) - V(\tilde{\varphi}^S_x)]
\end{bmatrix}
\begin{bmatrix}
M^N_e \\
M^S_e
\end{bmatrix} = \begin{bmatrix}
\left(\frac{\sigma}{\sigma-1} \frac{w^N}{\mu^N}\right)^{1-\sigma} [V(\infty) - V(\tilde{\varphi}^N_d)] - \left(\frac{\sigma}{\sigma-1} \frac{w^S}{\mu^S}\right)^{1-\sigma} [V(\infty) - V(\tilde{\varphi}^S_d)] \\
\left(\frac{\sigma}{\sigma-1} \frac{w^N}{\mu^N}\right)^{1-\sigma} [V(\infty) - V(\tilde{\varphi}^N_x)] - \left(\frac{\sigma}{\sigma-1} \frac{w^S}{\mu^S}\right)^{1-\sigma} [V(\infty) - V(\tilde{\varphi}^S_x)]
\end{bmatrix} \cdot \begin{bmatrix}
\left(\frac{\sigma}{\sigma-1} \frac{w^N}{\mu^N}\right)^{1-\sigma} \frac{\bar{X} - \bar{\varphi}^N}{\lambda^N} \\
\left(\frac{\sigma}{\sigma-1} \frac{w^S}{\mu^S}\right)^{1-\sigma} \frac{\bar{X} - \bar{\varphi}^S}{\lambda^S}
\end{bmatrix},
\]

where \(V(\varphi) = \int_0^\varphi y^{\sigma-1}dG(y)\). Applying Cramer’s rule,

\[
M^i_e = \frac{1}{\sigma} \left(\frac{w^i}{\mu^i}\right)^{1-\sigma} \frac{\bar{X} - \bar{\varphi}^N_d}{\lambda^N} - \frac{\bar{X} - \bar{\varphi}^S_d}{\lambda^S},
\]

where \(\Delta = [V(\infty) - V(\tilde{\varphi}^N_d)][V(\infty) - V(\tilde{\varphi}^S_d)] - [V(\infty) - V(\tilde{\varphi}^N_x)][V(\infty) - V(\tilde{\varphi}^S_x)] > 0\). Note that this holds for a general distribution function \(G(\cdot)\).

Next, I derive condition (2) when \(G(\cdot)\) is Pareto. Since \(V(\infty) - V(\varphi) = \frac{k \varphi_{\min}^{\sigma k - \sigma + 1}}{k - \sigma + 1} \varphi^{k - \sigma + 1}\) under Pareto, substituting this value into the above \(M^i_e\) gives

\[
M^i_e = \frac{1}{\sigma} \left(\frac{w^i}{\mu^i}\right)^{1-\sigma} \frac{\bar{X} - \bar{\varphi}^N_d}{\lambda^N} - \frac{\bar{X} - \bar{\varphi}^S_d}{\lambda^S},
\]

where \(\Delta = \left(\frac{k \varphi_{\min}^{\sigma k - \sigma + 1}}{k - \sigma + 1}\right) \left[\left(\frac{1}{\varphi_d^{\sigma k - \sigma + 1}}\right)^{k - \sigma + 1} - \left(\frac{1}{\varphi_x^{\sigma k - \sigma + 1}}\right)^{k - \sigma + 1}\right] > 0\); and \(k - \sigma + 1 > 0\) comes from a finite variance of the truncated Pareto distribution \(V(\varphi)\). Then, \(M^i_e > 0\) if and only if \(\frac{\bar{X} - \bar{\varphi}^N_d}{\lambda^N} > \frac{\bar{X} - \bar{\varphi}^S_d}{\lambda^S}\) or

\[
\left(\frac{\varphi_x}{\varphi_d}\right)^{k - \sigma + 1} < \frac{B}{X} < \left(\frac{\varphi_x^{\sigma k - \sigma + 1}}{\varphi_x^{\sigma k - \sigma + 1}}\right)^{k - \sigma + 1} \iff \left(\frac{1}{f_d \bar{f}}\right)^{k - \sigma + 1} < \frac{B}{X} < \left(\frac{1}{f_x \bar{f}}\right)^{k - \sigma + 1}.
\]

Arranging this inequality gives condition (2).

Finally, I show that the range of (1) subsumes that of (2). Comparing these two conditions, the above statement holds if and only if

\[
\frac{f_d}{f_x} < X = \omega L < \frac{f_x}{f_d},
\]

Substituting \(\omega L = \int_0^\omega \left(\frac{L^S}{L^N} - \lambda_\sigma\right)dz\) (the BOP condition) into the above inequality yields

\[
\frac{1 + \int_0^\varphi \frac{\varphi S N N}{\varphi N - \lambda_\sigma}dz - \frac{\int_0^\varphi \varphi S N N}{\varphi N - \lambda_\sigma}dz}{1 + \int_0^\varphi \frac{\varphi S N N}{\varphi N - \lambda_\sigma}dz} \equiv \Gamma_1 < \left(\frac{\int_0^\varphi \varphi S N N}{\varphi N - \lambda_\sigma}dz - \int_0^\varphi \varphi S N N}{\varphi N - \lambda_\sigma}dz\right) \equiv \Gamma_2.
\]

Note that \(\Gamma_1 > 1 - \int_0^\varphi \frac{\varphi S N N}{\varphi N - \lambda_\sigma}dz\) and \(\Gamma_2 < \left(1 + \frac{f_d}{f_x}\right)\int_0^\varphi \frac{\varphi S N N}{\varphi N - \lambda_\sigma}dz\) because the numerator and denominator
A.3.3 Derivation of the Extensive Margins under Pareto

By following the similar steps, it is easily confirmed that \( \bar{I} \) first show that the relative extensive margins are given by

\[
I = \frac{k\sigma}{k - \sigma + 1} w^i f_x, \quad I = \frac{k\sigma}{k - \sigma + 1} w^j f_d.
\]

By definition,

\[
\bar{I} = \frac{1}{1 - G(\bar{\varphi})} \int_{\bar{\varphi}}^{\infty} v^i(\varphi) dG(\varphi)
\]

\[
= \frac{1}{1 - G(\bar{\varphi})} B^i \sigma \left( \frac{\mu^i}{w^i} \right)^{\sigma - 1} \left[ V(\infty) - V(\bar{\varphi}) \right]
\]

\[
= \frac{k}{k - \sigma + 1} B^i \sigma \left( \frac{\mu^i}{w^i} \right)^{\sigma - 1} \left( \frac{k\sigma}{k - \sigma + 1} \bar{\varphi} \right)^{\sigma - 1}
\]

\[
= \frac{k}{k - \sigma + 1} B^i \sigma \left( \frac{\mu^i}{w^i} \right)^{\sigma - 1} \left[ 1 - w^i f_x \right]
\]

\[
= \frac{k\sigma}{k - \sigma + 1} w^i f_x.
\]

By following the similar steps, it is easily confirmed that \( \bar{I} = \frac{k\sigma}{k - \sigma + 1} w^j f_d. \)

A.3.2 Derivation of the Intensive Margins under Pareto

I show that the intensive margins under Pareto are given by

\[
\bar{I} = \frac{k\sigma}{k - \sigma + 1} w^i f_x, \quad \bar{I} = \frac{k\sigma}{k - \sigma + 1} w^j f_d.
\]

By definition,

\[
\bar{I} = \frac{1}{1 - G(\bar{\varphi})} \int_{\bar{\varphi}}^{\infty} v^i(\varphi) dG(\varphi)
\]

\[
= \frac{1}{1 - G(\bar{\varphi})} B^i \sigma \left( \frac{\mu^i}{w^i} \right)^{\sigma - 1} \left[ V(\infty) - V(\bar{\varphi}) \right]
\]

\[
= \frac{k}{k - \sigma + 1} B^i \sigma \left( \frac{\mu^i}{w^i} \right)^{\sigma - 1} \left( \frac{k\sigma}{k - \sigma + 1} \bar{\varphi} \right)^{\sigma - 1}
\]

\[
= \frac{k}{k - \sigma + 1} B^i \sigma \left( \frac{\mu^i}{w^i} \right)^{\sigma - 1} \left[ 1 - w^i f_x \right]
\]

\[
= \frac{k\sigma}{k - \sigma + 1} w^i f_x.
\]

By following the similar steps, it is easily confirmed that \( \bar{I} = \frac{k\sigma}{k - \sigma + 1} w^j f_d. \)

A.3.3 Derivation of the Extensive Margins under Pareto

I first show that the relative extensive margins are given by

\[
M_{ed} = \frac{M_e}{\langle \bar{\varphi} \rangle^k} = \frac{1}{\omega} \frac{X - B^\frac{k}{\sigma - 1} F^{-1}}{1 - X B^{-\frac{1}{\sigma - 1}} F^{-1}}, \quad M_{ex} = \frac{M_e}{\langle \bar{\varphi} \rangle^k} = \frac{1}{\omega} \frac{X B^{-\frac{k}{\sigma - 1}} F^{-1}}{B^{-\frac{1}{\sigma - 1}} F - X},
\]

where \( X = X^N/X^S = \omega L \) and \( F = (f_x/f_d)^{(k - \sigma + 1)/(\sigma - 1)} > 1. \) From \( M_{ei} \) under Pareto, \( M_e = M_{e^N}/M_{e^S} \) is given by

\[
M_e = \frac{X^N}{B^N} \frac{1}{\langle \bar{\varphi} \rangle^k} - \frac{X^S}{B^S} \frac{1}{\langle \bar{\varphi} \rangle^k} - \frac{X^N}{B^N} \frac{1}{\langle \bar{\varphi} \rangle^k}.
\]

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Using the ZCP conditions, this equation can be written as

\[
M_e = \left( \frac{\omega}{\mu} \right)^{\sigma-1} (\bar{\varphi}_d)^{k-\sigma+1} \frac{X - B^k F^{-1}}{B(1 - XB^{-\frac{k}{\sigma-1}} F^{-1})}
\]

\[
= \left( \frac{\omega}{\mu} \right)^{\sigma-1} (\bar{\varphi}_x)^{k-\sigma+1} \frac{(XB^{-\frac{k}{\sigma-1}} F - 1)B}{B^k F^{-1} F - X}
\]

Dividing the first and second equalities respectively by \((\bar{\varphi}_d)^k\) and \((\bar{\varphi}_x)^k\),

\[
M_{ed} = \left( \frac{\omega}{\mu} \frac{1}{\bar{\varphi}_d} \right)^{\sigma-1} \frac{X - B^k F^{-1}}{B(1 - XB^{-\frac{k}{\sigma-1}} F^{-1})}, \quad M_{ex} = \left( \frac{\omega}{\mu} \frac{1}{\bar{\varphi}_x} \right)^{\sigma-1} \frac{(XB^{-\frac{k}{\sigma-1}} F - 1)B}{B^k F^{-1} F - X},
\]

where the values in the underbraces come from the RZCP conditions. Arranging these equations gives the result.

Next, I show that these masses increase more than proportionally to an increase in country size, i.e. \(M_{ed}^N/M_{ed}^S > L^N/L^S\) and \(M_{ex}^N/M_{ex}^S > L^N/L^S\). Using \(M_{ed}\) derived above, it suffices for \(M_{ed}^N/M_{ed}^S > L^N/L^S\) to see that the following inequality holds:

\[
M_{ed} > L = \frac{X}{\omega} \iff B < X \frac{\sigma-1}{k},
\]

which is true under condition (2). Similarly, for \(M_{ex}^N/M_{ex}^S > L^N/L^S\),

\[
M_{ex} > L \iff B < \left( \frac{(X^2 - 1) + \sqrt{(X^2 - 1)^2 + (2XF)^2}}{2XF} \right)^{\frac{\sigma-1}{k}} \equiv \Phi.
\]

Since \(\Phi < X^{(\sigma-1)/k}(f_x/f_d)^{(k-\sigma+1)/k}\), this inequality is also true under condition (2).

Finally, I show that \(M'_{ex}(z) < M'_{ed}(z) < 0\) for \(z \in [0,1]\) and their intersection is at \((\bar{z}, \frac{XF-1}{\omega(F-X)})\) in Figure 7. The first relationship holds from simple inspection of the above expressions of \(M_{ex}\) and \(M_{ed}\), because only \(B\) is a function of \(z\) with \(B'(z) > 0\) and \(k - \sigma + 1 > 0\). Regarding the intersection of the two curves, it follows from noting that \(\bar{\varphi}_x(\bar{z}) = \bar{\varphi}_d(\bar{z}) = \omega^{1/(\sigma-1)}\) and \(M_{e}(\bar{z}) = \omega^{k-\sigma+1} \frac{XF-1}{F-X} \). □
References


