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Country Size, Technology, and Ricardian Comparative Advantage

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Abstract

We develop a Ricardian model with heterogeneous firms in which country size and technology play a crucial role in the firm-level variables. We show that a country with larger size and higher technology exhibits higher productivity and lower price-cost margins even under assumptions of C.E.S. preferences and monopolistic competition. Welfare is higher in this country, not only due to increased product variety but also due to increased competition in a domestic market. We also show that country size and technology impact critically on the intensive margin as well as the extensive margin in the gravity equation.

Keywords: Ricardian comparative advantage, Country size, Technology, Heterogeneous firms
JEL classification Numbers: F12, F14

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1 Introduction

Old trade theory based on comparative advantage has regained empirical relevance. In contrast to the latter half of the twentieth century where the bulk of world trade was dominated between similar industrial countries, recent years have witnessed rapidly rising trade between developed and less developed countries with lower wages, especially China. In this dissimilar-dissimilar trade, the two different types of countries not only exchange different goods across sectors by engaging in horizontal specialization, but also exchange similar goods within sectors by engaging in vertical differentiation. For instance, Schott (2008) finds empirical evidence that, while China's export bundle increasingly and disproportionately resembles that of the most developed countries in the OECD between 1972 and 2001, Chinese exports are less sophisticated and sell for a substantial discount relative to OECD varieties within narrowly defined products.

This paper develops a general-equilibrium Ricardian model with heterogeneous firms in which country size and technology play a crucial role in the firm-level variables. Following Dornbusch, Fischer, and Samuelson (1977) (henceforth DFS), we assume that productivity levels of two countries vary systematically across a continuum of sectors, where its equilibrium determines the relative wage and trade structure. Moreover, productivity levels of firms producing differentiated varieties under monopolistic competition are drawn idiosyncratically from a fixed distribution a la Melitz (2003). The interplay of these two-dimensional productivity differences helps explain vertical differentiation as well as horizontal specialization of trade between dissimilar countries in a single unified framework, shedding new light on the role of country size and technology in the selection of entry into domestic and export markets in the Ricardian model.

We show that a country with larger size and higher technology exhibits higher productivity and lower price-cost margins even under assumptions of C.E.S. preferences and monopolistic competition. Consider, for example, the impact of country size. As in DFS, an increase in country size expands the range of sectors over which a growing country has a comparative advantage by reducing its relative wage. In our Ricardian model with heterogeneous firms, the lower relative wage reduces the price-cost margins (defined as the price *minus* the cost) in a growing country relative to another country, even though the markups (defined as the price *over* the cost) are constant due to C.E.S. preferences. As a result, country size increases the degree of competition in a domestic market and raises the productivity cutoff of domestic production, which is a sufficient statistic for welfare. Welfare is higher in a large country, not only due to increased product variety but also due to increased competition in a domestic market. This stands in sharp contrast to the standard heterogeneous-firm model with C.E.S. preferences (e.g., Melitz, 2003) in which all the firm-level variables are independent of country size. Thus, our Ricardian model can overcome the drawback of pro-competitive effects of trade that typically arise in C.E.S. preferences and monopolistic competition, while preserving the usefulness of the workforce model in the new trade theory literature. Technology also has a similar impact on the price-cost margins and welfare in our Ricardian model. We show that, although a country with more advanced technology entails the higher relative wage, technology nonetheless reduces the price-cost margins in this country relative to another country and raises welfare there.

This equilibrium property of our model helps understand the role of country size and technology in the gravity equation. A recent body of empirical evidence using the firm-level dataset has highlighted the importance of decomposing aggregate trade flows into the extensive margin and the intensive margin, where the former refers to the number of exporting firms and the latter refers to the average export sales per firm (Bernard et al., 2007a; Helpman et al., 2008). If the gravity equation employs C.E.S. preferences and monopolistic competition, the model predicts that country size and technology affect only the number of export variety (extensive margin), leaving the average export sales (intensive margin) independent of these integrants. However, empirical evidence suggests that country size and technology have crucial impacts not only on the extensive margin but also on the intensive margin in estimating trade flows. For example, Bernard et al. (2007a) show that GDP (a proxy of country size) impacts positively on the extensive margin, whereas it impacts negatively on the intensive margin in aggregate U.S. exports in 2000. Our model offers a possible explanation for this empirical pattern by allowing country size to affect the firm-level variables.

Our model's prediction for the impact of country size is similar to Melitz and Ottaviano (2008) with quasi-linear-quadratic preferences: a larger country exhibits higher aggregate productivity and lower price-cost margins. It is important to note, however, that competition by country size operates through the different channels. In their paper, increased goods market competition shifts up residual demand price elasticity but factor market competition has no impact due to an outside good that equalizes wage rates across countries. In this paper, increased factor market competition reduces the relative wage (and hence the marginal cost) but goods market competition has no impact due to C.E.S. preferences. This difference is worth emphasizing since it gives rise to the different impact of country size on a trading partner. In contrast to Melitz and Ottaviano (2008) in which country size does not affect the productivity cutoffs and welfare of a trading partner, country size does affect the productivity cutoffs and welfare of a trading partner in this paper, through a change in the relative wage and the relative competitiveness across countries.

Clearly, the equilibrium property of our model arises only when the relative wage is endogenous. If the relative wage is exogenously fixed, country size impacts only on the number of variety without affecting the firm-level variables. In this respect, the current paper is closely related to Bernard et al. (2007b) who allow for asymmetry in factor proportions and thereby factor price equalization (FPE) does not necessarily hold. They find in the environment that trade-induced resource reallocations are more significant in comparative advantage sectors than comparative disadvantage sectors, creating a new welfare gain from trade. Although this finding is similar to that in our Ricardian model, one of key differences is that they have to resort to simulations for the outside FPE region (at least they do not examine the impact of country endowments on the firm-level variables in the analytical sections). In contrast, we are able to analytically examine the impact of country size and technology on the firm-level variables even with C.E.S. preferences, which proves useful for analyzing its consequence on welfare and the extensive and intensive margins in the gravity equation as noted above. This feature of the Ricardian model is not to imply however that the Heckscher-Ohlin model is less important. Indeed it is well-known that the two-factor Heckscher-Ohlin model has the advantage of being able to provide a rich framework for analyzing distributional consequences from trade, a feature that is missing in the one-factor Ricardian model. Our emphasis is instead that the Ricardian model can provide a different lens through which to understand the real world especially when large countries with different technology engage in international trade by exploiting wage differentials. As observed by Krugman (2008), we believe this is one of most striking aspects of recent trade flows.

Another novelty of this paper is in examining log-supermodularity studied by Costinot (2009) to explore implications of the Ricardian model with monopolistic competition and heterogeneous firms (Costinot (2009) only considers the Ricardian model with perfect competition). We show that if labor productivity is log-supermodular, not only is aggregate output but the other endogenous aggregate variables — including sector labor supply and the number of firms — are also log-supermodular. As a result, international trade allows laborers to be allocated relatively more to the sectors in which each country is relatively more productive, leading to the greater number of firms that operate in these sectors. This finding represents a sharp departure from that in perfect competition in which international trade simply allows all laborers to be allocated to the comparative advantage sectors and the number of firms is indeterminate.

A number of papers have employed the DFS model to capture bilateral trade volumes between dissimilar countries (see Helpman (2014) for a recent literature review). Eaton and Kortum (2002), while keeping perfect competition, extend the DFS model by allowing for an arbitrary number of countries to quantify the effect of country characteristics and geographic barriers on bilateral trade flows. In contrast, while keeping a two-country model, we extend the DFS model by allowing for monopolistic competition and heterogeneous firms to examine the effect of country characteristics and geographic barriers on the selection into export markets. In terms of the methodology and objective, the current paper is particularly close to Okubo (2009) in that the DFS model is integrated into a multi-sector version of the Melitz model. The most crucial difference is that, whereas Okubo (2009) restricts the Pareto distribution to obtain closed-form solutions, we develop a more general model without imposing any specific parameterizations to the firm distribution and show that most results of the Okubo model hold in such a setting. Furthermore, imposing the Pareto distribution, we derive the gravity equation in the Ricardian model that uncovers a new insight into the impact of country size and technology on the intensive margin as well as the extensive margin. Fan et al. (2013) and Huang et al. (2017) also adapt the DFS model to the environment of monopolistic competition and heterogeneous firms; however, they confine the analysis to the Pareto distribution without deriving the gravity equation. Although Huang et al. (2017) break away from FPE, they have to resort to simulations for the outside FPE region, just as in Bernard et al. (2007b).

The influence of the relative wage on firm selection under the assumptions of C.E.S. preferences and monopolistic competition is similar to that in Demidova and Rodríguez-Clare (2013). They show that endogenous wage considerations in the standard heterogeneous-firm model alter drastically the impact of asymmetric trade liberalization on welfare for a liberalizing country in a small economy. Our approach differs from theirs because we study Ricardian comparative advantage and the relative wage in a large economy. While we focus primarily on the impact of country size and technology, our model is tractable enough to examine the impact of asymmetric trade liberalization on the firm-level variables and welfare, yielding the similar result with theirs.

2 Setup

Consider a world composing of two large countries indexed by $i, j = 1, 2$. Throughout this paper, country subscripts are attached to all relevant variables.

2.1 Demand

Country i is populated by the mass of identical consumers $\overline{L}_i$ who devote their income into differentiated goods produced in a continuum of sectors over a unit interval $[0, 1]$. The preferences of a representative consumer are Cobb-Douglas across sectors and Dixit-Stiglitz within sectors:

$$U_i = \int_0^1 b_i(z) \ln Q_i(z) dz,$$

where $b_i(z)$ denotes a *constant* share of expenditure spent on sector z, which is *identical* between the two countries, and

$$Q_i(z) = \left[\sum_{h=i,j} \int_{v \in V_i(z)} q_{hi}(v, z)^{\frac{\sigma-1}{\sigma}} dv \right]^{\frac{\sigma}{\sigma-1}},$$

is the set of varieties consumed as an aggregate good in sector z. $V_i(z)$ is the set of available goods in the sector, and $\sigma(> 1)$ is a constant elasticity of substitution between varieties, which is the same across sectors. Given this aggregate good $Q_i(z)$, its dual aggregate price is given by

$$P_i(z) = \left[\sum_{h=i,j} \int_{v \in V_i(z)} p_{hi}(v, z)^{1-\sigma} dv \right]^{\frac{1}{1-\sigma}}.$$

Let $R_i(z) = P_i(z)Q_i(z)$ and $Y_i = w_i\overline{L}_i$ denote aggregate expenditure in sector z and aggregate labor income in the economy, where w_i is a wage rate. Then, the expenditure share $b_i(z)$ is defined as

$$b_i(z) = \frac{P_i(z)Q_i(z)}{Y_i} = \frac{R_i(z)}{w_i\overline{L}_i}, \qquad \int_0^1 b_i(z)dz = 1, \qquad b_i(z) = b_j(z).$$

Thus, the sum of aggregate sector expenditure equals aggregate labor income ($\int_0^1 R_i(z)dz = w_i\overline{L}_i$). As is well-known, the preferences yield the following demand functions for variety v:

$$q_{ji}(v, z) = R_i(z)P_i(z)^{\sigma-1}p_{ji}(v, z)^{-\sigma}.$$

In the analysis below, we focus on a particular variety and drop variety script v from relevant variables for notational simplicity.

It is important to note that demand structure is almost the same as the DFS model with perfect competition, except that all goods are differentiated in this model with monopolistic competition. In particular, the upper-tier Cobb-Douglas preferences imply that differentiated goods in sector z are associated with constant expenditure shares $b_i(z), b_j(z)$, which are exogenous preference parameters.

Further the sub-utility function is C.E.S. in all sectors and a freely traded outside good is excluded, implying that wage rates w_i, w_j cannot be normalized between the two countries. As stressed in the Introduction, this assumption is made to examine the role of *endogenous* factoral terms of trade in the DFS model with monopolistic competition and heterogeneous firms.

2.2 Production

There is a continuum of firms that produce a different variety in each sector. Labor is the only factor of production and firms in country i face a perfectly elastic supply of labor at country size $\overline{L}_i$. Since labor is completely mobile across sectors but immobile across countries, the wage rate w_i is the same across sectors but is different across countries.

To enter a sector in country i, potential entrants bear a fixed entry cost f_i^e, measured in country i's labor units. Upon paying this entry cost, these entrants draw their productivity level φ from a fixed distribution $G(\varphi)$, and each entrant decides whether to exit or not. If an entrant from country i chooses to serve a market in country j, it pays a variable trade cost $\tau_{ij}(\geq 1)$ and a fixed trade cost $f_{ij}(> 0)$, measured in country i's labor units. We assume that these costs satisfy $\tau_{ii} = \tau_{jj} = 1$ and $\tau_{ij}^{\sigma-1} f_{ij}/f_{ii} = \tau_{ji}^{\sigma-1} f_{ji}/f_{jj} > 1$. Following the literature (e.g., Huang et al., 2017), we further assume that these costs are the same across sectors. Labor used by a firm of productivity φ in sector z from country i to country j is a linear cost function of output for domestic production and exporting:

$$\begin{cases} l_{ii}(\varphi, z) = f_{ii} + \frac{q_{ii}(\varphi,z)}{\theta(\varphi,z,\mu)} = f_{ii} + \frac{q_{ii}(\varphi,z)}{\varphi\mu_i(z)} & \text{for domestic production,} \\ l_{ij}(\varphi, z) = f_{ij} + \frac{\tau_{ij}q_{ij}(\varphi,z)}{\theta(\varphi,z,\mu)} = f_{ij} + \frac{\tau_{ij}q_{ij}(\varphi,z)}{\varphi\mu_i(z)} & \text{for exporting,} \end{cases}$$

where $q_{ij}(\varphi, z)$ is output shipped by a firm of productivity φ in sector z from country i to country j, and $\theta(\varphi, z, \mu)$ is labor productivity.

A few points are in order for this specification. First, labor productivity $\theta(\varphi, z, \mu)$ depends on the three characteristics: (i) firm-specific φ; (ii) sector-specific z; and (iii) country-specific μ. As noted above, each firm has a different productivity level indexed by φ, which is drawn idiosyncratically from a fixed distribution $G(\varphi)$. This distribution is assumed the same across countries and sectors, with support in $[\varphi_{\min}, \infty)$. Moreover, each country also has a different productivity level indexed by $\mu_i(z)$. This productivity denotes country i's ability to produce in sector z, which varies systematically with country characteristics across a continuum of sectors. It follows immediately from this cost function that φ and $\mu_i(z)$ affect the variable cost only (leaving the fixed cost identical) and the variable cost is lower if φ and $\mu_i(z)$ are greater.

Second, we employ a reduced form of labor productivity $\theta(\varphi, z, \mu) = \varphi\mu_i(z)$ which can be justified by Costinot's (2009) log-supermodular argument. In our model, $\mu_i(z)$ is defined as the inverse of the unit labor requirement indexed by $a_i(z)$: $\mu_i(z) = 1/a_i(z)$. Let $\mu(z) \equiv \mu_1(z)/\mu_2(z)(= a_2(z)/a_1(z))$ denote the relative labor productivity (or labor requirement) in country 1. Without loss of generality, we assume that country 1 (country 2) has a relatively bigger cost advantage in high-z (low-z) sectors, which holds under the following assumption:

Assumption 1 The relative labor productivity $\mu(z) \equiv \mu_1(z)/\mu_2(z)$ is log-supermodular. Formally, for $z < z'$,

$$\frac{\mu_1(z)}{\mu_2(z)} \leq \frac{\mu_1(z')}{\mu_2(z')}.$$

Assumption 1 means that the relative labor productivity $\mu_i(z)/\mu_j(z)$ is increasing in the strength of country i's comparative advantage. By assumption of a continuum of sectors, Assumption 1 also means that $\mu(z)$ is increasing in z.

Following the literature, we say that country i has a comparative advantage in producing goods in sector z if country i's unit labor costs are less than or equal to country j's unit labor costs:

$$w_i a_i(z) \leq \tau_{ji} w_j a_j(z) \quad \Longleftrightarrow \quad \frac{w_i}{\tau_{ji} w_j} \leq \frac{\mu_i(z)}{\mu_j(z)}. \tag{1}$$

Let $\omega \equiv w_1/w_2$ denote the relative wage in country 1. Then (1) immediately reveals that country 1 has a comparative advantage in high-z sectors $\bar{z}_1 \leq z \leq 1$, where

$$\bar{z}_1 \equiv \mu^{-1}\left(\frac{\omega}{\tau_{21}}\right). \tag{2}$$

Similarly, country 2 has a comparative advantage in low-z sectors $0 \leq z \leq \bar{z}_2$, where

$$\bar{z}_2 \equiv \mu^{-1}(\tau_{12}\omega). \tag{3}$$

Note that, as long as $\tau_{ji} \geq 1$, these cutoff sectors satisfy $\bar{z}_1 \leq \bar{z}_2$.

Having defined Ricardian comparative advantage, we next turn to firm behavior. Consider sector z in country i where domestic firms in i and foreign firms from j monopolistically compete and choose its price to maximize the profit. Letting $p_{ii}(\varphi, z)$ and $p_{ji}(\varphi, z)$ denote the prices set by domestic firms in i and foreign firms from j, profit maximization yields the following pricing rules:

$$p_{ii}(\varphi, z) = \frac{\sigma}{\sigma - 1} \frac{w_i}{\varphi \mu_i(z)}, \qquad p_{ji}(\varphi, z) = \frac{\sigma}{\sigma - 1} \frac{\tau_{ji} w_j}{\varphi \mu_j(z)}.$$

With these pricing rules, the revenues of domestic firms and foreign firms are respectively given by

$$r_{ii}(\varphi, z) = \sigma B_i(z) \left(\frac{\mu_i(z)}{w_i}\right)^{\sigma-1} \varphi^{\sigma-1}, \qquad r_{ji}(\varphi, z) = \sigma B_i(z) \left(\frac{\mu_j(z)}{\tau_{ji} w_j}\right)^{\sigma-1} \varphi^{\sigma-1},$$

where

$$B_i(z) = \frac{(\sigma - 1)^{\sigma-1}}{\sigma^\sigma} R_i(z) P_i(z)^{\sigma-1}$$

is the index of aggregate market demand. In the revenues, aggregate market demand $B_i(z)$ is same since both domestic firms and foreign firms sell their goods to consumers in country i. In contrast, country-specific productivity levels $\mu_i(z), \mu_j(z)$ and wage rates w_i, w_j are different since foreign firms make use of foreign technology and labor in j. From these revenues, the operating profits of domestic

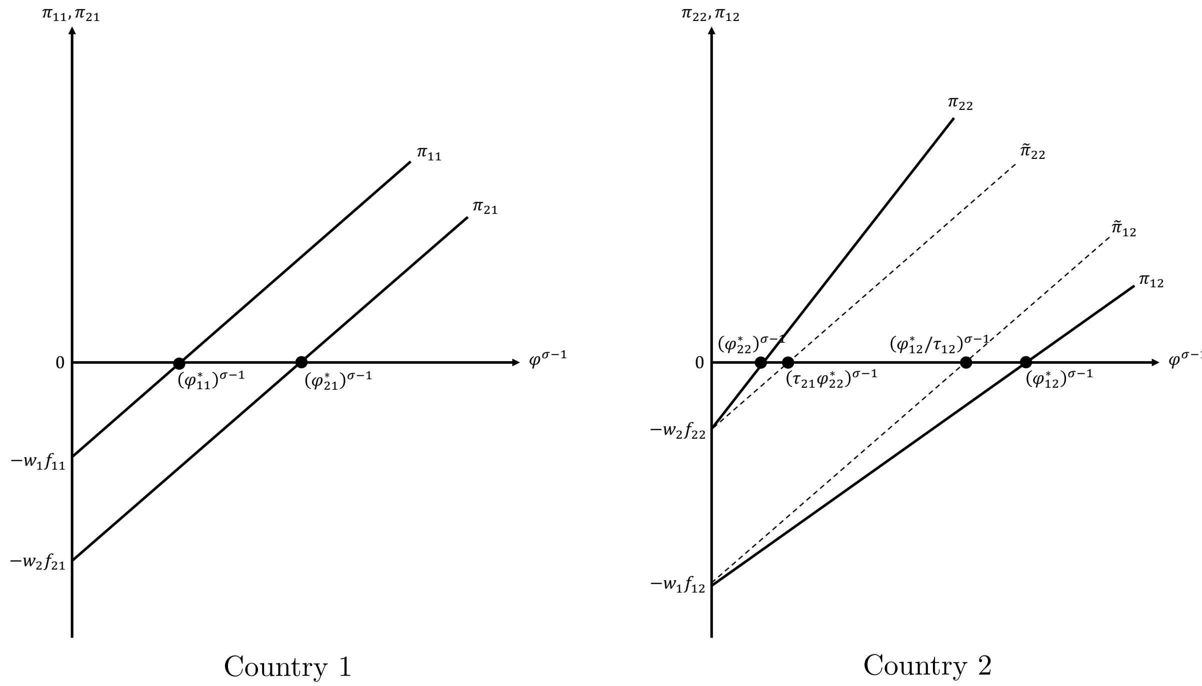

FIGURE 1 – Profits from domestic and export markets

firms are

$$\pi_{ii}(\varphi, z) = \frac{r_{ii}(\varphi, z)}{\sigma} - w_i f_{ii} = B_i(z) \left(\frac{\mu_i(z)}{w_i} \right)^{\sigma-1} \varphi^{\sigma-1} - w_i f_{ii},$$

and those of foreign firms are

$$\pi_{ji}(\varphi, z) = \frac{r_{ji}(\varphi, z)}{\sigma} - w_j f_{ji} = B_i(z) \left(\frac{\mu_j(z)}{\tau_{ji} w_j} \right)^{\sigma-1} \varphi^{\sigma-1} - w_j f_{ji}.$$

These profits can be drawn in $(\varphi^{\sigma-1}, \pi_{ji})$ space, with slope $B_i(z) \left(\frac{\mu_j(z)}{\tau_{ji} w_j} \right)^{\sigma-1}$ and intercept $-w_j f_{ji}$. Figure 1 depicts $\pi_{ii}(\varphi, z)$ and $\pi_{ji}(\varphi, z)$ for country 1's market ($i = 1$) and country 2's market ($i = 2$) in the cutoff sector $\bar{z}_1$. Note $p_{ii}(\varphi, z) \leq p_{ji}(\varphi, z)$ and $r_{ii}(\varphi, z) \geq r_{ji}(\varphi, z)$ if and only if (1) holds (for given φ). Thus firms in comparative advantage sectors set lower price and earn higher revenue than firms in comparative disadvantage sectors. Reflecting this fact, $\pi_{11}(\varphi, z)$ and $\pi_{21}(\varphi, z)$ are parallel for country 1's market, whereas $\pi_{22}(\varphi, z)$ is steeper than $\pi_{12}(\varphi, z)$ for country 2's market in the cutoff sector $\bar{z}_1$. The converse is true in another cutoff sector $\bar{z}_2$.

3 General Equilibrium

This section examines the interplay among the key endogenous variables of the model and addresses comparative static questions in general equilibrium.

3.1 Equilibrium Conditions

In this subsection, we outline several equilibrium conditions that play a central role in characterizing the endogenous variables in general equilibrium. In the subsequent subsections, we solve this general-equilibrium model with some restrictions on the exogenous variables.

Firstly, a zero profit condition holds for *all* sectors $z \in [0, 1]$ of the domestic and export markets. The productivity cutoffs that satisfy $\pi_{ii}(\varphi_{ii}^*, z) = 0$ and $\pi_{ij}(\varphi_{ij}^*, z) = 0$ are respectively given by

$$B_i(z) \left(\frac{\mu_i(z)}{w_i} \right)^{\sigma-1} (\varphi_{ii}^*(z))^{\sigma-1} = w_i f_{ii}, \tag{4}$$

$$B_j(z) \left(\frac{\mu_i(z)}{\tau_{ij} w_i} \right)^{\sigma-1} (\varphi_{ij}^*(z))^{\sigma-1} = w_i f_{ij}. \tag{5}$$

Since (4) and (5) respectively apply to domestic firms in i and exporting firms from i to j, aggregate market demands $B_i(z), B_j(z)$ are different between (4) and (5), but the country-specific productivity level $\mu_i(z)$ and wage rate w_i are the same.

Secondly, a free entry condition holds for *all* sectors:

$$\int_{\varphi_{ii}^*(z)}^{\infty} \pi_{ii}(\varphi, z) dG(\varphi) + \int_{\varphi_{ij}^*(z)}^{\infty} \pi_{ij}(\varphi, z) dG(\varphi) = w_i f_i^e, \tag{6}$$

where the first and second terms in the left-hand side respectively denote the expected operating profits in the domestic and export markets by potential entrants. The sum of these expected profits should be equal to the fixed entry cost $w_i f_i^e$. Note that (6) holds so long as there is a positive mass of potential entrants denoted by $M_i^e(z)$. In this paper, we focus on the case where $M_i^e(z) > 0$ in all sectors and international trade leads both countries to incomplete specialization.[1]

Finally, a labor market clearing condition must be taken into account:

$$\int_0^1 M_i^e(z) \int_{\varphi_{ii}^*(z)}^{\infty} l_{ii}(\varphi, z) dG(\varphi) dz + \int_0^1 M_i^e(z) \int_{\varphi_{ij}^*(z)}^{\infty} l_{ij}(\varphi, z) dG(\varphi) dz + \int_0^1 M_i^e(z) f_i^e dz = \overline{L}_i, \tag{7}$$

where the first and second terms in the left-hand side are respectively the expected amounts of labor for domestic production and exporting by potential entrants, and the third is the expected amounts of labor for investment by these entrants. The sum of these expected amounts of labor should be equal to the fixed aggregate labor supply $\overline{L}_i$.

Now, it is possible to endogenize the important variables in general equilibrium. Since there are the eight equations ((4), (5), (6) and (7) that hold in countries 1 and 2), these conditions provide implicit solutions for the following eight unknowns:

$$\varphi_{11}^*(z),\ \varphi_{22}^*(z),\ \varphi_{12}^*(z),\ \varphi_{21}^*(z),\ B_1(z),\ B_2(z),\ w_1,\ w_2,$$

[1] If the expected profits are smaller than the fixed entry cost, $M_i^e(z) = 0$ and county j specializes in this sector. For the sake of parsimony, we rule out this case by imposing some restrictions on the exogenous variables. (The condition of incomplete specialization is given later).

where (7) for $i = 2$ can be omitted by Walras's law, thereby normalizing $w_2 = 1$ as a numeraire of the model. The mass of potential entrants $M_i^e(z)$ is written as a function of these eight unknowns as shown later.

As is evident from the dependence of z among the eight unknowns, the productivity cutoffs and the aggregate market demands are allowed to vary across sectors; in contrast, the wage rates are the same across sectors due to perfect inter-sectoral mobility of labor. This means that (4), (5) and (6) for $i = 1, 2$ are the six equations that characterize $\{\varphi_{11}^*(z), \varphi_{22}^*(z), \varphi_{12}^*(z), \varphi_{21}^*(z), B_1(z), B_2(z)\}$ in each sector, whereas (7) for $i = 1, 2$ are the two additional equations that characterize $\{w_1, w_2\}$, aggregating the use of labor across all sectors in each country. From this reason, the next subsections first characterize the sectoral equilibrium by focusing on (4), (5) and (6), and then explore the full general equilibrium by integrating (7) into the model.

3.2 Sectoral Equilibrium

This subsection sets forth characterizations of the eight unknowns derived from the eight equilibrium conditions. It is however difficult to solve the general equilibrium with asymmetric countries in this general setting; in particular, closed-form solutions of these unknowns cannot be obtained without specifying a functional form of the distribution. To avoid this difficulty, the main analysis is devoted to characterizing the *relative* terms of these unknowns, instead of the absolute terms of them.

In what follows, we derive the sectoral equilibrium which is characterized in terms of the relative aggregate market demand and the relative productivity cutoffs. First, dividing (4) by (5), the relative aggregate market demand is given by

$$\frac{B_i(z)}{B_j(z)} = \left(\frac{1}{\tau_{ij}} \frac{\varphi_{ij}^*(z)}{\varphi_{ii}^*(z)}\right)^{\sigma-1} \frac{f_{ii}}{f_{ij}}. \tag{8}$$

Solving the system of equations (4), (5) and (6) simultaneously yields the following lemma regarding the relative market demand $B(z) \equiv B_1(z)/B_2(z)$.

Lemma 1 *The relative market demand $B(z) \equiv B_1(z)/B_2(z)$ is log-submodular. For $z < z'$,*

$$\frac{B_1(z)}{B_2(z)} \geq \frac{B_1(z')}{B_2(z')}.$$ [2]

Lemma 1 means that the relative aggregate market demand $B_i(z)/B_j$ is decreasing in the strength of i's comparative advantage. The intuition stems from the fact that $B_i(z)/B_j(z)$ is proportional to the relative price index $P_i(z)/P_j(z)$. By definition, the stronger is country i's comparative advantage,

[2] To be more precise, this lemma holds under the following restriction:

$$\frac{J'(\varphi_{ii}^*(z))J'(\varphi_{jj}^*(z))}{J'(\varphi_{ij}^*(z))J'(\varphi_{ji}^*(z))} > \tau_{ij}\tau_{ji}\left(\frac{f_{ij}f_{ji}}{f_{ii}f_{jj}}\right)^{\frac{\sigma}{\sigma-1}},$$

where $J(\varphi_{ji}^*(z)) = \int_{\varphi_{ji}^*(z)}^{\infty}[(\varphi/\varphi_{ji}^*(z))^{\sigma-1} - 1]dG(\varphi)$ is decreasing in $\varphi_{ji}^*(z)$.

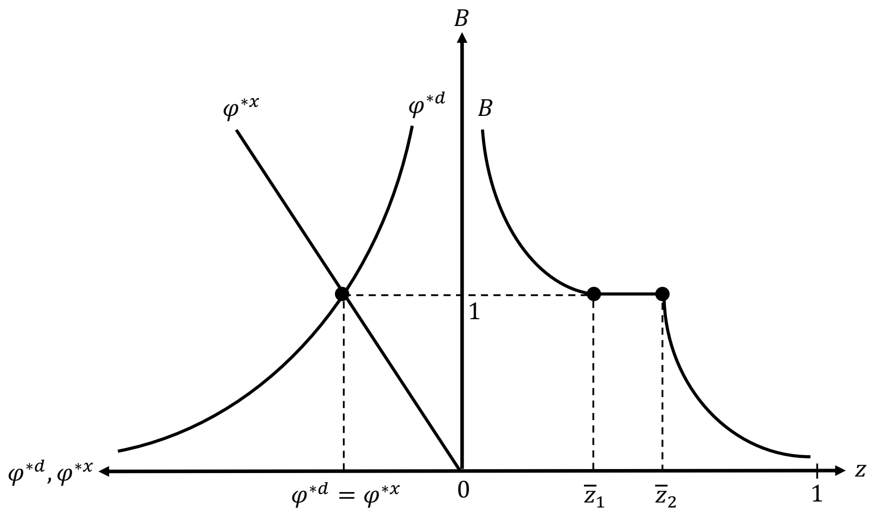

FIGURE 2 – Market demand and productivity cutoffs

the more productive is country i relative to country j, and the lower is $P_i(z)$ relative to $P_j(z)$. As a result, $P_i(z)/P_j(z)$ (and $B_i(z)/B_j(z)$) is decreasing in the strength of i's comparative advantage. Since country 1 (country 2) has a comparative advantage in high-z (low-z) sectors under Assumption 1, Lemma 1 alternatively means that, by assumption of a continuum of sectors, $B(z) \equiv B_1(z)/B_2(z)$ is decreasing in z. The first quadrant of Figure 2 depicts this relationship in (z, B) space.

Next, dividing (4) of i by (4) of j, the relative domestic productivity cutoff is given by

$$\frac{\varphi_{ii}^*(z)}{\varphi_{jj}^*(z)} = \frac{w_i}{w_j}\frac{\mu_j(z)}{\mu_i(z)}\left(\frac{w_i}{w_j}\frac{f_{ii}}{f_{jj}}\frac{B_j(z)}{B_i(z)}\right)^{\frac{1}{\sigma-1}}. \tag{9}$$

Similarly, dividing (5) of i by (5) of j, the relative export productivity cutoff is given by

$$\frac{\varphi_{ij}^*(z)}{\varphi_{ji}^*(z)} = \frac{\tau_{ij}w_i}{\tau_{ji}w_j}\frac{\mu_j(z)}{\mu_i(z)}\left(\frac{w_i}{w_j}\frac{f_{ij}}{f_{ji}}\frac{B_i(z)}{B_j(z)}\right)^{\frac{1}{\sigma-1}}. \tag{10}$$

Since B's are only different endogenous variables between (9) and (10), we have the following lemma.

Lemma 2

(i) The relative domestic productivity cutoff $\varphi^{*d}(z) \equiv \varphi_{11}^*(z)/\varphi_{22}^*(z)$ is log-supermodular. For $z < z'$,

$$\frac{\varphi_{11}^*(z)}{\varphi_{22}^*(z)} \leq \frac{\varphi_{11}^*(z')}{\varphi_{22}^*(z')}.$$

(ii) The relative export productivity cutoff $\varphi^{*x}(z) \equiv \varphi_{12}^*(z)/\varphi_{21}^*(z)$ is log-submodular. For $z < z'$,

$$\frac{\varphi_{12}^*(z)}{\varphi_{21}^*(z)} \geq \frac{\varphi_{12}^*(z')}{\varphi_{21}^*(z')}.$$

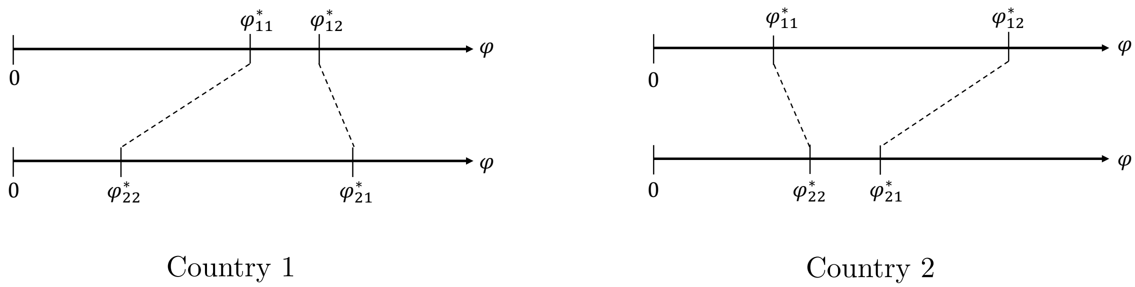

FIGURE 3 – Relationship among productivity cutoffs

Lemma 2 means that the relative domestic (export) productivity cutoff is increasing (decreasing) in the strength of i's comparative advantage. In fact, solving for the system of equations (4), (5) and (6) immediately reveals that the productivity cutoffs satisfy

$$\varphi^{*'}_{11}(z) \geq 0,\ \varphi^{*'}_{22}(z) \leq 0,\ \varphi^{*'}_{12}(z) \leq 0,\ \varphi^{*'}_{21}(z) \geq 0. \tag{11}$$

Thus, the stronger is each country's comparative advantage, the more intense is firm selection in the domestic market ($\varphi^{*'}_{11}(z) \geq 0$, $\varphi^{*'}_{22}(z) \leq 0$), but the less intense is firm selection in the export market ($\varphi^{*'}_{12}(z) \leq 0$, $\varphi^{*'}_{21}(z) \geq 0$). Note that under $\tau_{ij}^{\sigma-1} f_{ij}/f_{ii} = \tau_{ji}^{\sigma-1} f_{ji}/f_{jj}$,

$$\varphi^{*d}(z) \gtreqless \varphi^{*x}(z) \quad \Longleftrightarrow \quad B(z) \lesseqgtr 1.$$

Further $\varphi^{*d}(z) = \varphi^{*x}(z)$ and $B(z) = 1$ if $z = \bar{z}_1, \bar{z}_2$ (see the Appendix). It then follows from Lemma 1 that $B(z)$ is *weakly* decreasing in z where $B(z) = 1$ for $z \in [\bar{z}_1, \bar{z}_2]$. The second quadrant of Figure 2 depicts this relationship in (B, φ^*) space.

Finally, combining the first and second quadrants of Figure 2, we obtain the sectoral equilibrium characterized by the relative aggregate market demand and the relative productivity cutoffs:

$$\begin{aligned} 0 \leq z \leq \bar{z}_2 \quad &\Longleftrightarrow \quad B(z) \geq 1 \quad \Longleftrightarrow \quad \varphi^{*x}(z) \geq \varphi^{*d}(z), \\ \bar{z}_1 \leq z \leq 1 \quad &\Longleftrightarrow \quad B(z) \leq 1 \quad \Longleftrightarrow \quad \varphi^{*x}(z) \leq \varphi^{*d}(z). \end{aligned} \tag{12}$$

Figure 3 depicts the relationship among the productivity cutoffs in the comparative advantage sectors of country 1 ($\bar{z}_1 \leq z \leq 1$) and country 2 ($0 \leq z \leq \bar{z}_2$). From (11), the gap between $\varphi^*_{ij}(z)$ and $\varphi^*_{ii}(z)$ is decreasing in the strength of country i's comparative advantage, and from (12), this gap is relatively narrower than the gap between $\varphi^*_{ji}(z)$ and $\varphi^*_{jj}(z)$ in country i's comparative advantage sectors. These findings can be seen more formally in terms of their relative gap:

$$\frac{\varphi^*_{ij}(z)}{\varphi^*_{ii}(z)} = \tau_{ij} \left(\frac{B_i(z)}{B_j(z)} \frac{f_{ij}}{f_{ii}} \right)^{\frac{1}{\sigma-1}}. \tag{13}$$

From Lemma 1, $\varphi^*_{ij}(z)/\varphi^*_{ii}(z)$ is decreasing in the strength of i's comparative advantage. In addition, from (12), $\varphi^*_{ij}(z)/\varphi^*_{ii}(z)$ is smaller than $\varphi^*_{ji}(z)/\varphi^*_{jj}(z)$ in i's comparative advantage sectors.

(13) shows that the selection into export markets occurs in the comparative *disadvantage* sectors of both countries.[3] In the comparative *advantage* sectors, the selection occurs in both countries if

$$\varphi_{12}^*(z) > \varphi_{11}^*(z),\ \varphi_{21}^*(z) > \varphi_{22}^*(z) \iff \frac{1}{\tau_{12}^{\sigma-1}}\frac{f_{11}}{f_{12}} < B(z) < \tau_{21}^{\sigma-1}\frac{f_{21}}{f_{22}}, \tag{14}$$

whereas this does not hold in country i if

$$\varphi_{ii}^*(z) \geq \varphi_{ij}^*(z) \iff \begin{cases} B(z) \leq \frac{1}{\tau_{12}^{\sigma-1}}\frac{f_{11}}{f_{12}} & \text{for } \ i = 1, \\ B(z) \geq \tau_{21}^{\sigma-1}\frac{f_{21}}{f_{22}} & \text{for } \ i = 2. \end{cases}$$

Clearly, the selection might not occur in the strong comparative *advantage* sectors. If $\varphi_{ii}^*(z) \geq \varphi_{ij}^*(z)$, however, all surviving firms in i could export to j, which is not supported by empirical evidence (see Bernard et al., 2007a). Thus, we hereafter assume that (14) is satisfied across countries and sectors in the following analysis.

Recall that we have assumed that (6) holds for all sectors and no country fully specializes in any sector, i.e., $M_e^i(z) > 0$ for all z. To derive $M_i^e(z)$, rewrite the price index $P_i(z)$ as

$$(P_i(z))^{1-\sigma} = M_i^e(z)\left(\frac{\sigma}{\sigma-1}\frac{w_i}{\mu_i(z)}\right)^{1-\sigma} V(\varphi_{ii}^*(z)) + M_j^e(z)\left(\frac{\sigma}{\sigma-1}\frac{\tau_{ji}w_j}{\mu_j(z)}\right)^{1-\sigma} V(\varphi_{ji}^*(z)),$$

where $V(\varphi_{ji}^*(z)) \equiv \int_{\varphi_{ji}^*(z)}^{\infty} \varphi^{\sigma-1} dG(\varphi)$ is decreasing in $\varphi_{ji}^*(z)$. Solving $P_i(z)$ and $P_j(z)$ for $M_i^e(z)$ and $M_j^e(z)$ and using $B_i(z)$, we obtain the mass of potential entrants:

$$M_i^e(z) = \frac{1}{\sigma}\left(\frac{w_i}{\mu_i(z)}\right)^{1-\sigma} \frac{V(\varphi_{jj}^*(z))\frac{R_i(z)}{B_i(z)} - \tau_{ji}^{1-\sigma}V(\varphi_{ji}^*(z))\frac{R_j(z)}{B_j(z)}}{\Delta(z)},$$

where

$$\Delta(z) \equiv V(\varphi_{ii}^*(z))V(\varphi_{jj}^*(z)) - (\tau_{ij}\tau_{ji})^{(1-\sigma)}V(\varphi_{ij}^*(z))V(\varphi_{ji}^*(z)).$$

Note that $\Delta(z)$ is positive since $\varphi_{ij}^*(z) > \varphi_{ii}^*(z)$ from (14). Then, there is a positive mass of potential entrants in all sectors of both countries if

$$M_1^e(z) > 0,\ M_2^e(z) > 0 \iff \frac{1}{\tau_{12}^{\sigma-1}}\frac{V(\varphi_{12}^*(z))}{V(\varphi_{11}^*(z))}\frac{R_1(z)}{R_2(z)} < B(z) < \tau_{21}^{\sigma-1}\frac{V(\varphi_{22}^*(z))}{V(\varphi_{21}^*(z))}\frac{R_1(z)}{R_2(z)}, \tag{15}$$

whereas this does not hold in country i if

$$M_i^e(z) \leq 0 \iff \begin{cases} B(z) \geq \tau_{21}^{\sigma-1}\frac{V(\varphi_{22}^*(z))}{V(\varphi_{21}^*(z))}\frac{R_1(z)}{R_2(z)} & \text{for } \ i = 1, \\ B(z) \leq \frac{1}{\tau_{12}^{\sigma-1}}\frac{V(\varphi_{12}^*(z))}{V(\varphi_{11}^*(z))}\frac{R_1(z)}{R_2(z)} & \text{for } \ i = 2. \end{cases}$$

[3] Under the condition $\tau_{ij}^{\sigma-1}f_{ij}/f_{ii} > 1$, the comparative disadvantage sectors of country 1, for example, must satisfy

$$0 \leq z < \bar{z}_1 \iff B(z) > 1 \implies \varphi_{12}^*(z) > \varphi_{11}^*(z).$$

From Lemma 1 and (11), there might not be a positive mass of entrants in the strong comparative *disadvantage* sectors (see also Huang et al., 2017). For the sake of parsimony, we restrict attention to the situation in which not only is (14) but (15) is also satisfied, so that incomplete specialization occurs in all sectors of both countries. Conditions (14) and (15) require country size not too different between the two countries, because $B(z)$ is proportional to relative country size $\overline{L}_1/\overline{L}_2$.[4]

Proposition 1

(*i*) *The domestic (export) productivity cutoff* $\varphi^*_{ii}(z)$ ($\varphi^*_{ij}(z)$) *is increasing (decreasing) in the strength of country i's comparative advantage.*

(*ii*) *The productivity cutoff ratio* $\varphi^*_{ij}(z)/\varphi^*_{ii}(z)$ *is smaller than* $\varphi^*_{ji}(z)/\varphi^*_{jj}(z)$ *in country i's comparative advantage sectors.*

Proposition 1 shows that aggregate productivity premium of exporting firms relative to domestic firms is smaller, the stronger is each country's comparative advantage in the Ricardian model with monopolistic competition and heterogeneous firms. Note importantly that the findings in Proposition 1 and Figure 3 are very similar to those in Bernard et al. (2007b) who develop the Heckscher-Ohlin model with monopolistic competition and heterogeneous firms. Our contribution is in demonstrating that the relationship between firm selection and comparative advantage rests only on comparative cost advantage, but not on whether the cost advantage stems from factor proportions or technology. Despite this similarity, the difference emerges in the situation in which country endowments impact endogenously on the factor prices across the two countries.

Before proceeding further, it is worth emphasizing that the other aggregate variables in the model exhibit log-supermodularity and log-submodularity. Let $R_{ii}(z)$ and $R_{ij}(z)$ denote aggregate domestic sales and aggregate export sales in sector z from country i to country j:

$$R_{ii}(z) = M^e_i(z)\int_{\varphi^*_{ii}(z)}^{\infty} r_{ii}(\varphi,z)dG(\varphi), \quad R_{ij}(z) = M^e_i(z)\int_{\varphi^*_{ij}(z)}^{\infty} r_{ij}(\varphi,z)dG(\varphi).$$

Similarly, let $L_i(z)$ denote aggregate labor supply in sector z of country i:

$$L_i(z) = M^e_i(z)\int_{\varphi^*_{ii}(z)}^{\infty} l_{ii}(\varphi,z)dG(\varphi) + M^e_i(z)\int_{\varphi^*_{ij}(z)}^{\infty} l_{ij}(\varphi,z)dG(\varphi) + M^e_i(z)f^e_i,$$

where

$$l_{ii}(\varphi,z) = f_{ii} + \frac{\sigma-1}{\sigma}\frac{r_{ii}(\varphi,z)}{w_i}, \quad l_{ij}(\varphi,z) = f_{ij} + \frac{\sigma-1}{\sigma}\frac{r_{ij}(\varphi,z)}{w_i}.$$

Noting that these aggregate variables are functions of the endogenous variables in Lemmas 1 and 2, the following lemma is obtained from the characterization of sectoral equilibrium above.

[4]This comes from that $\frac{B_i(z)}{B_j(z)} = \frac{R_i(z)}{R_j(z)}\left(\frac{P_i(z)}{P_j(z)}\right)^{\sigma-1}$ and $R_i(z) = b_i(z)w_i\overline{L}_i$ where $b_i(z) = b_j(z)$. While the condition that requires firm selection is usually imposed in the literature (e.g., Bernard et al., 2007b), the condition that rules out the possibility of complete specialization is also often imposed in the literature (e.g., Melitz and Ottaviano, 2008).

Lemma 3

(i) The relative output $Q(z) \equiv Q_1(z)/Q_2(z)$ is log-supermodular, whereas the relative price $P(z) \equiv P_1(z)/P_2(z)$ is log-submodular. For $z < z'$,

$$\frac{Q_1(z)}{Q_2(z)} \leq \frac{Q_1(z')}{Q_2(z')}, \quad \frac{P_1(z)}{P_2(z)} \geq \frac{P_1(z')}{P_2(z')}.$$

(ii) The relative sales in the domestic market $R^d(z) \equiv R_{11}(z)/R_{22}(z)$ and those in the export market $R^x(z) \equiv R_{12}(z)/R_{21}(z)$ are log-supermodular. For $z < z'$,

$$\frac{R_{11}(z)}{R_{22}(z)} \leq \frac{R_{11}(z')}{R_{22}(z')}, \quad \frac{R_{12}(z)}{R_{21}(z)} \leq \frac{R_{12}(z')}{R_{21}(z')}.$$

(iii) The relative labor supply $L(z) \equiv L_1(z)/L_2(z)$ and the relative mass of potential entrants $M^e(z) \equiv M_1^e(z)/M_2^e(z)$ are log-supermodular. For $z < z'$,

$$\frac{L_1(z)}{L_2(z)} \leq \frac{L_1(z')}{L_2(z')}, \quad \frac{M_1^e(z)}{M_2^e(z)} \leq \frac{M_1^e(z')}{M_2^e(z')}.$$

In Lemma 3, the ranking of the relative output and its relative price suggests that each country produces relatively more associated with relatively lower price indices, the stronger is its comparative advantage. The relative sales in the domestic and export markets also belong to the ranking, since each country sells in these markets relatively more, the stronger is its comparative advantage. Finally, the relative labor supply and relative mass of entrants also belong to the ranking, since labor resources are relatively more allocated to sectors in which outputs and sales are greater. Note that since the equality holds for $z \in [\bar{z}_1, \bar{z}_2]$ in Lemmas 1 and 2, the equality also holds for the interval sectors in Lemma 3.

3.3 Full General Equilibrium

The last subsection characterized the equilibrium vector $\{\varphi_{11}^*(z), \varphi_{22}^*(z), \varphi_{12}^*(z), \varphi_{21}^*(z), B_1(z), B_2(z)\}$ for *given* wage rates. Now that the sectoral equilibrium is characterized by these six unknowns, this subsection embeds the sectoral equilibrium into general equilibrium.

To close the model in general equilibrium, we explicitly take account of the labor market clearing condition (7) below. Substituting the amount of labor required by individual firms $l_{ii}(\varphi, z), l_{ij}(\varphi, z)$ into (7) and using (6), equation (7) is simplified as

$$\frac{\int_0^1 R_i(z)dz}{w_i} = \overline{L}_i, \tag{16}$$

where $\int_0^1 R_i(z)dz = \int_0^1 P_i(z)Q_i(z)dz$ is aggregate expenditure in country i. Thus, country i's wage w_i is determined by the equality between aggregate expenditure $\int_0^1 R_i(z)dz$ and aggregate payments to labor $w_i\overline{L}_i$ as in usual general-equilibrium trade models without an outside good.

To derive the relative wage, we first show that (16) is equivalent with the balance-of-payments condition. Since $B_i(z), B_j(z)$ are finite in all sectors under (14), $\varphi^*_{ij}(z), \varphi^*_{ji}(z)$ are finite in all sectors. Further, $M^e_i(z), M^e_j(z)$ are positive in all sectors under (15). From the distribution with unbounded upper support, it follows that bilateral trade occurs in all sectors:

$$\int_0^1 R_{ij}(z)dz = \int_0^1 R_{ji}(z)dz. \tag{17}$$

Note that aggregate expenditure in i consists of expenditure spent on domestic goods in i and foreign goods from j, $\int_0^1 R_i(z)dz = \int_0^1 (R_{ii}(z) + R_{ji}(z))dz$. On the other hand, aggregate labor income in i consists of revenues earned by domestic firms and exporting firms of i, $w_i\overline{L}_i = \int_0^1 (R_{ii}(z) + R_{ij}(z))\,dz$. As a result, (17) is equivalent with (16) in the sense that both (16) and (17) induce the same equality: $\int_0^1 R_i(z)dz = w_i\overline{L}_i$.

From Lemma 3, aggregate export sales are increasing in the strength of comparative advantage ($\frac{R_{12}(z)}{R_{21}(z)} \leq \frac{R_{12}(z')}{R_{21}(z')}$ for $z < z'$ where equality holds for $z \in [\bar{z}_1, \bar{z}_2]$). Let $\bar{z}_i$ denote the *hypothetical* sector in which net exports are zero in two-way trade. Since net aggregate export sales are the differences between aggregate labor income and aggregate expenditure, (17) is expressed as

$$\int_{\bar{z}_1}^1 (w_1 L_1(z) - R_1(z))\,dz = \int_0^{\bar{z}_2} (w_2 L_2(z) - R_2(z))\,dz,$$

which simply indicates that each country runs trade surplus in the comparative advantage sectors, and trade deficit in the comparative disadvantage sectors. This equation is further rewritten as

$$(\kappa_1(\bar{z}_1) - \lambda_1(\bar{z}_1))w_1\overline{L}_1 = (\kappa_2(\bar{z}_2) - \lambda_2(\bar{z}_2))w_2\overline{L}_2,$$

where $\kappa_i(\bar{z}_i)$ and $\lambda_i(\bar{z}_i)$ respectively denote the labor share and the expenditure share devoted in i's comparative advantage sectors:

$$\kappa_1(\bar{z}_1) \equiv \int_{\bar{z}_1}^1 \frac{L_1(z)}{\overline{L}_1}dz, \quad \kappa_2(\bar{z}_2) \equiv \int_0^{\bar{z}_2} \frac{L_2(z)}{\overline{L}_2}dz,$$
$$\lambda_1(\bar{z}_1) \equiv \int_{\bar{z}_1}^1 b_1(z)dz, \quad \lambda_2(\bar{z}_2) \equiv \int_0^{\bar{z}_2} b_2(z)dz.$$

Then ω can be explicitly solved as

$$\omega = \frac{\kappa_2(\bar{z}_2) - \lambda_2(\bar{z}_2)}{\kappa_1(\bar{z}_1) - \lambda_1(\bar{z}_1)}\left(\frac{\overline{L}_2}{\overline{L}_1}\right). \tag{18}$$

The other conditions that pin down the relative wage ω come from the cutoff conditions (2) and (3): $\omega = \tau_{21}\mu(\bar{z}_1)$ and $\omega = \mu(\bar{z}_2)/\tau_{12}$, where the relative labor productivity $\mu(z)$ is increasing in z. These three conditions (2), (3) and (18) provide implicit solutions for the following three unknowns:

$$\bar{z}_1,\ \bar{z}_2,\ \omega.$$

Substituting (2) and (3) into (18) reveals that the right-hand side of (18) is decreasing in ω, which guarantees a unique equilibrium relative wage. The relative wage in (18), together with (2) and (3), determines the pattern of comparative advantage of country 1 and country 2. Given the relative wage, the system of equations (4), (5) and (6) in turn leads to $\{\varphi^*_{11}(z), \varphi^*_{22}(z), \varphi^*_{12}(z), \varphi^*_{21}(z), B_1(z), B_2(z)\}$. This completes the characterization of the eight unknowns in general equilibrium.

Proposition 2

(*i*) *There exist the two cutoff sectors $\bar{z}_1, \bar{z}_2$ that pin down comparative advantage of country 1 and country 2 in bilateral trade.*

(*ii*) *The equilibrium relative wage ω is unique.*

While the results in Proposition 2 are similar with the results in DFS, it should be noted that the variable trade costs τ_{12}, τ_{21} do not allow for nontraded goods in the interval sectors $z \in [\bar{z}_1, \bar{z}_2]$ here: each country does trade differentiated goods in $z \in [\bar{z}_1, \bar{z}_2]$, but *net* exports are zero in these sectors. More important differences, however, are comparative static questions for the firm-level variables.

3.4 Comparative Statics

Building on the equilibrium characterization, this subsection addresses comparative static questions with respect to relative country size $\overline{L} \equiv \overline{L}_1/\overline{L}_2$ and relative labor productivity $\mu(z) \equiv \mu_1(z)/\mu_2(z)$. Regarding $\overline{L}$, we focus on the analysis within the ranges of (14) and (15). Regarding $\mu(z)$, we are concerned with the effect of uniform changes across sectors.

The comparative static results are facilitated by the recursive structure of the equilibrium: any change in $\overline{L}$ or $\mu(z)$ first has an impact on $\{\bar{z}_1, \bar{z}_2, \omega\}$ from (2), (3) and (18); and the impact of ω on $\{\varphi^*_{11}(z), \varphi^*_{22}(z), \varphi^*_{12}(z), \varphi^*_{21}(z), B_1(z), B_2(z)\}$ is then obtained from (4), (5) and (6) for $i = 1, 2$. Let the latter set of the sectoral equilibrium variables express in the relative terms $\{\varphi^{*d}(z), \varphi^{*x}(z), B(z)\}$. The main results are provided in the next proposition.

Proposition 3

(*i*) *The equilibrium vector $\{\bar{z}_1, \bar{z}_2, \omega\}$ characterized by (2), (3) and (18) satisfies*

$$\frac{\partial \bar{z}_1}{\partial \overline{L}} \leq 0, \ \frac{\partial \bar{z}_2}{\partial \overline{L}} \leq 0, \ \frac{\partial \omega}{\partial \overline{L}} \leq 0,$$
$$\frac{\partial \bar{z}_1}{\partial \mu(z)} \leq 0, \ \frac{\partial \bar{z}_2}{\partial \mu(z)} \leq 0, \ \frac{\partial \omega}{\partial \mu(z)} \geq 0.$$

(*ii*) *The equilibrium vector $\{\varphi^{*d}(z), \varphi^{*x}(z), B(z)\}$ characterized by (4), (5) and (6) satisfies*

$$\frac{\partial \varphi^{*d}(z)}{\partial \overline{L}} \geq 0, \ \frac{\partial \varphi^{*x}(z)}{\partial \overline{L}} \leq 0, \ \frac{\partial B(z)}{\partial \overline{L}} \leq 0,$$
$$\frac{\partial \varphi^{*d}(z)}{\partial \mu(z)} \geq 0, \ \frac{\partial \varphi^{*x}(z)}{\partial \mu(z)} \leq 0, \ \frac{\partial B(z)}{\partial \mu(z)} \leq 0.$$

The first part of this proposition is exactly the same as that in DFS. The second part says that an increase in relative country size, for example, makes the relative selection into the domestic (export) market more (less) intense. In fact, solving for the system of equations (4), (5) and (6) reveals that

$$\frac{\partial \varphi_{11}^*(z)}{\partial \overline{L}} \geq 0, \ \frac{\partial \varphi_{22}^*(z)}{\partial \overline{L}} \leq 0, \ \frac{\partial \varphi_{12}^*(z)}{\partial \overline{L}} \leq 0, \ \frac{\partial \varphi_{21}^*(z)}{\partial \overline{L}} \geq 0.$$

Intuitively, a country with larger size entails the lower relative wage and lower price-cost margins, which makes competition more intense and raises the productivity cutoffs of domestic and exporting firms operating in that country. This impact of country size on firm selection is similar to that in Melitz and Ottaviano (2008).[5] In our Ricardian model, an increase in relative labor productivity has the similar impact on firm selection: although a country with higher labor productivity entails the higher relative wage, it still entails the lower price-cost margins because the relative wage increases proportionally short of an increase in relative labor productivity.

One of key insights from the comparative statics is that, even with C.E.S. preferences, country size does affect firm selection $\varphi_{ii}^*(z), \varphi_{ij}^*(z)$ through the relative wage in the Ricardian model with heterogeneous firms. (If the relative wage is exogenously fixed, country size impacts only on the mass of potential entrants $M_i^e(z)$ without affecting the firm-level variables.) This finding is in line with recent theoretical work, although the mechanism differs. For example, Bertoletti and Etro (2017) show that national income does affect firm selection through the variable markups when consumers' preferences are represented by additively separable indirect utilities. In our Ricardian model, even though consumers' preferences are represented by C.E.S. (and hence the markups are constant), the price-cost margins are no longer constant because per-capita income w_i varies with country size. See also Demidova and Rodríguez-Clare (2013) for the influence of the relative wage on firm selection.

3.5 Welfare

Let us next consider welfare in the Ricardian model with monopolistic competition and heterogeneous firms. As shown in the Appendix, the real wage is given by

$$\frac{w_i}{P_i(z)} = \frac{\sigma - 1}{\sigma} \left(\frac{b_i(z)\overline{L}_i}{\sigma f_{ii}} \right)^{\frac{1}{\sigma-1}} \mu_i(z)\varphi_{ii}^*(z). \tag{19}$$

In this economy with a continuum of sectors, welfare per worker in country i is defined as

$$W_i = \int_0^1 b_i(z) \ln \left(\frac{w_i}{P_i(z)} \right) dz.$$

This welfare expression means that the productivity cutoff of domestic production $\varphi_{ii}^*(z)$ is a sufficient statistic for welfare (because $\overline{L}_i$ is exogenous and $b_i(z)$ is constant). Applying the comparative static results in Proposition 3 to (19), we have the following proposition within the ranges of (14) and (15).

[5] From the marginal cost $\frac{w_i}{\varphi \mu_i(z)}$ for a domestic firm in i, the price-cost margins are $\frac{1}{\sigma-1}\frac{w_i}{\varphi \mu_i(z)}$, which is relatively lower in a larger country. In contrast to Melitz and Ottaviano (2008), country size of i affects the productivity cutoffs of j in the current model, because country size affects the relative wage and the relative competitiveness across countries.

Proposition 4

(i) A rise in relative country size raises welfare in country 1, but reduces welfare in country 2.

$$\frac{\partial W_1}{\partial \overline{L}} \geq 0, \quad \frac{\partial W_2}{\partial \overline{L}} \leq 0.$$

(ii) A rise in relative labor productivity raises welfare in country 1, but reduces welfare in country 2.

$$\frac{\partial W_1}{\partial \mu(z)} \geq 0, \quad \frac{\partial W_2}{\partial \mu(z)} \leq 0.$$

This result is obtained by noting that the productivity cutoff of domestic production in country 1 (country 2) is increasing (decreasing) in relative country size or relative labor productivity. Note that the welfare implication stands in sharp contrast to the Ricardian model with perfect competition. Regarding the impact of relative country size, for example, a growing country experiences a welfare loss by worsening the terms of trade as in the Ricardian model with perfect competition. At the same time, the lower relative wage reduces the price-cost margins and makes the country's competition more intense. The fact that the productivity cutoff of domestic production rises with country size implies that the welfare loss from the terms of trade is dominated by the welfare gain from increased competition and aggregate productivity in our Ricardian model.[6]

3.6 Margins of Specialization and Trade

We have analyzed the equilibrium characterization and comparative statics. This subsection explores the impacts on the extensive and intensive margins and derives the gravity equation in the Ricardian model with monopolistic competition and heterogeneous firms. To obtain closed-form solutions of these two margins, we hereafter assume that firm productivity φ is drawn from a Pareto distribution:

$$G(\varphi) = 1 - \left(\frac{\varphi_{\min}}{\varphi}\right)^k, \quad \varphi \geq \varphi_{\min} > 0,$$

where $k > \sigma - 1$. It is useful to decompose aggregate export sales $R_{ij}(z)$ into

$$\begin{aligned} R_{ij}(z) &= M_i^e(z) \int_{\varphi_{ij}^*(z)}^{\infty} r_{ij}(\varphi, z) dG(\varphi) \\ &= [1 - G(\varphi_{ij}^*(z))] M_i^e(z) \times \frac{1}{[1 - G(\varphi_{ij}^*(z))]} \int_{\varphi_{ij}^*(z)}^{\infty} r_{ij}(\varphi, z) dG(\varphi) \\ &= M_{ij}(z) \times \bar{r}_{ij}(z), \end{aligned}$$

where $M_{ij}(z)$ is the mass of exporting firms (extensive margin), and $\bar{r}_{ij}(z)$ is average sales per firm (intensive margin). Similarly, aggregate domestic sales are decomposed into $R_{ii}(z) = M_{ii}(z) \times \bar{r}_{ii}(z)$. Then, the following lemma records the impact of comparative advantage on the two margins.

[6] The impact of relative labor productivity also has the different welfare implications, because an increase in relative labor productivity leads to a welfare gain for both countries in the Ricardian model with perfect competition.

Lemma 4

(*i*) *The relative extensive margin of domestic firms* $M^d(z) \equiv M_{11}(z)/M_{22}(z)$ *and that of exporting firms* $M^x(z) \equiv M_{12}(z)/M_{21}(z)$ *are log-supermodular. For* $z < z'$,

$$\frac{M_{11}(z)}{M_{22}(z)} \leq \frac{M_{11}(z')}{M_{22}(z')}, \quad \frac{M_{12}(z)}{M_{21}(z)} \leq \frac{M_{12}(z')}{M_{21}(z')}.$$

(*ii*) *The relative intensive margin of domestic firms* $\bar{r}^d(z) \equiv \bar{r}_{11}(z)/\bar{r}_{22}(z)$ *and that of exporting firms* $\bar{r}^x(z) \equiv \bar{r}_{12}(z)/\bar{r}_{21}(z)$ *are neither log-supermodular nor log-submodular. For* $z < z'$,

$$\frac{\bar{r}_{11}(z)}{\bar{r}_{22}(z)} = \frac{\bar{r}_{11}(z')}{\bar{r}_{22}(z')}, \quad \frac{\bar{r}_{12}(z)}{\bar{r}_{21}(z)} = \frac{\bar{r}_{12}(z')}{\bar{r}_{21}(z')}.$$

Lemma 4 means that the relative mass of domestic firms $M_{ii}(z)/M_{jj}(z)$ and that of exporting firms $M_{ij}(z)/M_{ji}(z)$ are increasing in the strength of i's comparative advantage, whereas the relative average sales of these firms are the same across sectors. To establish this lemma, let us first derive the mass of potential entrants under the Pareto distribution. Applying this specific parameterization to (6) and (7) and rearranging, we have that

$$M_i^e(z) = \frac{\sigma - 1}{k\sigma} \frac{L_i(z)}{f_i^e}. \tag{20}$$

Although $M_i^e(z)$ is a function of the eight unknowns in general (as shown in Lemma 3), it depends only on aggregate labor supply $L_i(z)$ under the Pareto distribution. Using (20), the extensive and intensive margins are expressed as

$$\begin{aligned} M_{ii}(z) &= \left(\frac{\varphi_{\min}}{\varphi_{ii}^*(z)}\right)^k \frac{\sigma - 1}{k\sigma} \frac{L_i(z)}{f_i^e}, \qquad \bar{r}_{ii}(z) = \frac{k\sigma}{k - (\sigma - 1)} w_i f_{ii}, \\ M_{ij}(z) &= \left(\frac{\varphi_{\min}}{\varphi_{ij}^*(z)}\right)^k \frac{\sigma - 1}{k\sigma} \frac{L_i(z)}{f_i^e}, \qquad \bar{r}_{ij}(z) = \frac{k\sigma}{k - (\sigma - 1)} w_i f_{ij}. \end{aligned} \tag{21}$$

Lemma 4 follows immediately from noting Lemma 3 and (21).

The decomposition into the extensive and intensive margins allows us to express aggregate export sales as the gravity equation in the Ricardian model with monopolistic competition and heterogeneous firms. Substituting $\varphi_{ij}^*(z)$ from (5) into $M_{ij}(z)$ given in (21), we obtain the following proposition.

Proposition 5 *Aggregate export sales* $R_{ij}(z)$ *in sector* z *from country* i *to country* j *are given by*

$$R_{ij}(z) = \psi_i L_i(z) B_j(z)^{\frac{k}{\sigma-1}} \left(\frac{\mu_i(z)}{\tau_{ij} w_i}\right)^k (w_i f_{ij})^{1 - \frac{k}{\sigma - 1}}, \tag{22}$$

where $\psi_i \equiv \frac{\sigma-1}{k-(\sigma-1)} \frac{(\varphi_{\min})^k}{f_i^e}$. *An increase in* $R_{ij}(z)$ *due to country* i*'s comparative advantage is mainly accounted for by an increase in the extensive margin.*

The functional form in (22) is similar to that in Chaney (2008) for the elasticities of trade flows with respect to the variable and fixed trade costs, though he does not impose free entry to simplify the analysis. More importantly, he does not investigate the impact of comparative advantage on the gravity equation. To see this impact in the current model, let us express (22) in the relative term:

$$\frac{R_{ij}(z)}{R_{ji}(z)} = \underbrace{\frac{f_j^e}{f_i^e}\frac{L_i(z)}{L_j(z)}\left(\frac{B_j(z)}{B_i(z)}\right)^{\frac{k}{\sigma-1}}\left(\frac{\tau_{ji}w_j}{\tau_{ij}w_i}\frac{\mu_i(z)}{\mu_j(z)}\right)^k}_{\frac{M_{ij}(z)}{M_{ji}(z)}}\underbrace{\left(\frac{w_i}{w_j}\frac{f_{ij}}{f_{ji}}\right)^{1-\frac{k}{\sigma-1}}}_{\frac{\bar{r}_{ij}(z)}{\bar{r}_{ji}(z)}}.$$

From Assumption 1, Lemmas 1 and 3, country i's comparative advantage increases aggregate export sales $R_{ij}(z)$ (relative to $R_{ji}(z)$) by increasing labor allocation in exporting country i ($L_i(z)$), market demand in importing country j ($B_j(z)$), and labor productivity in exporting country i ($\mu_i(z)$), which all contribute to an increase in the extensive margin of trade.

Having described the impact of comparative advantage on the extensive and intensive margins, let us turn to examining the impact of country size and technology on these two margins. From (21), the relative extensive margins, $M^d(z) \equiv M_{11}(z)/M_{22}(z), M^x(z) \equiv M_{12}(z)/M_{21}(z)$, are given by

$$M^d(z) = \frac{L(z)}{(\varphi^{*d}(z))^k}\frac{f_2^e}{f_1^e}, \quad M^x(z) = \frac{L(z)}{(\varphi^{*x}(z))^k}\frac{f_2^e}{f_1^e}. \tag{23}$$

Similarly, the relative intensive margins, $\bar{r}^d(z) \equiv \bar{r}_{11}(z)/\bar{r}_{22}(z), \bar{r}^x(z) \equiv \bar{r}_{12}(z)/\bar{r}_{21}(z)$, are given by

$$\bar{r}^d(z) = \omega\frac{f_{11}}{f_{22}}, \quad \bar{r}^x(z) = \omega\frac{f_{12}}{f_{21}}. \tag{24}$$

The following proposition is obtained by applying the comparative static results in Proposition 3 to (23) and (24).

Proposition 6 *The relative extensive and intensive margins in (23) and (24) satisfy*

$$\begin{aligned}
&\frac{L}{M^d(z)}\frac{\partial M^d(z)}{\partial \overline{L}} \leq \frac{L}{M^x(z)}\frac{\partial M^x(z)}{\partial \overline{L}}, \quad \frac{\partial \bar{r}^d(z)}{\partial \overline{L}} \leq 0, \quad \frac{\partial \bar{r}^x(z)}{\partial \overline{L}} \leq 0, \\
&\frac{\mu(z)}{M^d(z)}\frac{\partial M^d(z)}{\partial \mu(z)} \leq \frac{\mu(z)}{M^x(z)}\frac{\partial M^x(z)}{\partial \mu(z)}, \quad \frac{\partial \bar{r}^d(z)}{\partial \mu(z)} \geq 0, \quad \frac{\partial \bar{r}^x(z)}{\partial \mu(z)} \geq 0.
\end{aligned}$$

This proposition means that country size, for example, impacts positively on the extensive margin, whereas it impacts negatively on the intensive margin in the gravity equation, which accords well with recent empirical evidence (e.g., Bernard et al., 2007a). The comparative static results for the intensive margin are obtained immediately from Proposition 3 and (24). As for the extensive margin, (20) shows that the mass of potential entrants is proportional to aggregate labor supply and there is no home market effect for entry:

$$\frac{M_1^e(z)}{M_2^e(z)} = \frac{L_1(z)}{L_2(z)}\frac{f_2^e}{f_1^e}. \tag{25}$$

In contrast, (23) shows that the masses of domestic firms and exporting firms are not proportional to entry since the productivity cutoffs vary with country size in the Ricardian model with monopolistic competition and heterogeneous firms. Noting that $M^e(z) \equiv M_1^e(z)/M_2^e(z)$ and $L(z) \equiv L_1(z)/L_2(z)$, $M^e(z) = L(z)/(f_1^e/f_2^e)$ from (25) and $M^d(z) = M^e(z)/(\varphi^{*d}(z))^k, M^x(z) = M^e(z)/(\varphi^{*x}(z))^k$ from (23). Further since $\varphi^{*d}(z)$ ($\varphi^{*x}(z)$) is increasing (decreasing) in $\overline{L}$ from Proposition 3, the relative extensive margins in (23) must satisfy

$$\frac{L}{M^d(z)}\frac{\partial M^d(z)}{\partial \overline{L}} \leq \frac{L}{M^e(z)}\frac{\partial M^e(z)}{\partial \overline{L}} \leq \frac{L}{M^x(z)}\frac{\partial M^x(z)}{\partial \overline{L}}.$$

Thus, the mass of exporting firms (domestic firms) increases more (less) than proportionally to entry. This reasoning also explains why relative labor productivity raises both the extensive and intensive margins through the impact on the productivity cutoffs and relative wage. The comparative statics suggest that any change in country size or technology should have an impact not only on the structure of comparative advantage (*inter*-sectoral adjustment) characterized by (2), (3) and (18), but also on the extensive and intensive margins (*intra*-sectoral adjustment) characterized by (4), (5) and (6) in the Ricardian model with monopolistic competition and heterogeneous firms.

We conclude this subsection by examining the the impact on the fraction of firms that export. From (4), (5) and (21), this fraction is given by

$$\frac{M_{ij}(z)}{M_{ii}(z)} = \begin{cases} \left(\frac{1}{B(z)}\frac{1}{\tau_{12}^{\sigma-1}}\frac{f_{11}}{f_{12}}\right)^{\frac{k}{\sigma-1}} & \text{for} \quad i=1, \\ \left(B(z)\frac{1}{\tau_{21}^{\sigma-1}}\frac{f_{22}}{f_{21}}\right)^{\frac{k}{\sigma-1}} & \text{for} \quad i=2, \end{cases} \tag{26}$$

which is between zero and unity under (14). Since $B(z)$ is decreasing in z (Lemma 1), (26) shows that $M_{ij}(z)/M_{ii}(z)$ is increasing in the strength of i's comparative advantage and thus log-supermodular ($\frac{M_{12}(z)/M_{11}(z)}{M_{21}(z)/M_{22}(z)} \leq \frac{M_{12}(z')/M_{11}(z')}{M_{21}(z')/M_{22}(z')}$ for $z < z'$). It also follows from (12) that $M_{ij}(z)/M_{ii}(z)$ is greater than $M_{ji}(z)/M_{jj}(z)$ in i's comparative advantage sectors. Further, from Proposition 3, $M_{ij}(z)/M_{ii}(z)$ is increasing (decreasing) in relative country size $\overline{L}$ for country 1 (country 2). The mechanism of the last result stems from the above comparative statics: an increase in relative country size makes firm selection into the domestic (export) market more (less) intense, which increases the mass of domestic firms (exporting firms) less (more) than proportionally to entry, and consequently raises the fraction of firms that export. It is important to stress that the analysis applies only for large countries where any exogenous shocks in a country have an influence on another country. Though rigorous empirical work examining this channel is yet to come, if we treat the U.S. and China as representatives of such large countries, our theoretical prediction is consistent with the existing evidence: 18% of U.S. firms export in 2002 (Bernard et al., 2007a), while 25% of Chinese firms export in either 1999 or 2007 (Huang et al., 2017).[7] Clearly, $M_{ij}(z)/M_{ii}(z)$ is increasing (decreasing) in relative labor productivity $\mu(z)$ for country 1 (country 2).

[7] As in Bernard et al. (2007a), Huang et al. (2017) focus on manufacturing firms, using the Chinese Annual Industrial Survey that covers all State Owned Enterprises (SOEs) and non-SOEs with annual sales higher than 5 million yuan.

4 Discussions

This section first discusses the impact of the variable trade cost, and then relates the DFS model with perfect competition and the DFS model with monopolistic competition and heterogeneous firms.

4.1 Variable Trade Cost

A simple inspection of (2), (3) and (18) reveals that while a symmetric reduction in the variable trade cost narrows the the interval sectors $z \in [\bar{z}_1, \bar{z}_2]$, it can shift the relative wage ω in either direction. Because of this, it also can shift the equilibrium vector $\{\varphi_{11}^*(z), \varphi_{22}^*(z), \varphi_{12}^*(z), \varphi_{21}^*(z), B_1(z), B_2(z)\}$ in either direction too (though it necessarily shifts the productivity cutoffs $\varphi_{ii}^*(z), \varphi_{ij}^*(z)$ in opposite directions). We can consider the impact of zero gravity ($\tau_{ij} = \tau_{ji} = 1$) on the equilibrium outcomes. In this setting, country i's unit labor costs are less than or equal to country j's unit labor costs if

$$w_i a_i(z) \leq w_j a_j(z) \quad \Longleftrightarrow \quad \frac{w_i}{w_j} \leq \frac{\mu_i(z)}{\mu_j(z)},$$

which means that the cutoff sector is unique and country 1 (country 2) has a comparative advantage in high-z (low-z) sectors $\bar{z} \leq z \leq 1$ ($0 \leq z \leq \bar{z}$), where

$$\bar{z} \equiv \mu^{-1}(\omega). \tag{27}$$

In the zero gravity world, we can easily show that (18) is given by

$$\omega = \frac{\kappa_2(\bar{z}) - \lambda_2(\bar{z})}{\kappa_1(\bar{z}) - \lambda_1(\bar{z})} \left(\frac{\overline{L}_2}{\overline{L}_1} \right). \tag{28}$$

As in the main analysis, (27) and (28) provide implicit solutions for $\{\bar{z}, \omega\}$; and then (4), (5) and (6) with $\tau_{ij} = 1$ for $i = 1, 2$ provide implicit solutions for $\{\varphi_{11}^*(z), \varphi_{22}^*(z), \varphi_{12}^*(z), \varphi_{21}^*(z), B_1(z), B_2(z)\}$. While the equilibrium characterization is similar as before, a key difference arises in the presence of zero gravity: $B(z)$ is *strictly* decreasing in z, while $\varphi^{*d}(z)$ ($\varphi^{*x}(z)$) is *strictly* increasing (decreasing) in z, because the interval sectors disappear. As a result, all the endogenous variables in Lemmas 1–4 are *strictly* log-supermodular or log-submodular. (See the Appendix for details.)

While the impact of a symmetric reduction in the variable trade cost is ambiguous in this model, the impact of an asymmetric reduction is unambiguous. Recalling that τ_{ij} is the variable trade cost from country i to country j, let $\tau \equiv \tau_{21}/\tau_{12}$ denote the relative variable trade cost in country 1. Clearly, this τ declines when country 1 unilaterally reduces its variable trade cost of importing τ_{21}. From (2), (3) and (18), a reduction in τ narrows the the interval sectors $z \in [\bar{z}_1, \bar{z}_2]$ as above, but it always reduces the relative wage in this case. Given this impact on the relative wage, solving for the system of equations (4), (5) and (6) by keeping $\tau_{ij}^{\sigma-1} f_{ij}/f_{ii} = \tau_{ji}^{\sigma-1} f_{ji}/f_{jj}$ satisfied, a reduction in τ has the following impact on the productivity cutoffs of the liberalizing country:

$$\frac{\partial \varphi_{11}^*(z)}{\partial \tau} \leq 0, \ \frac{\partial \varphi_{12}^*(z)}{\partial \tau} \geq 0. \tag{29}$$

Since the productivity cutoff of domestic production is a sufficient statistic for welfare in this model, (29) implies that the liberalizing country gains from a reduction in τ. Intuitively, while liberalization in country 1 leads to a decline in its relative wage, this is smaller than the decline in the price index and hence raises welfare there. (In contrast, the liberalized country might gain or lose from such a reduction in our setting.) This impact on the liberalizing country, which depends crucially on the endogenous relative wage, is the same as Demidova and Rodríguez-Clare (2013) but it is opposite to Demidova (2008) and Melitz and Ottaviano (2008) due to the presence of an outside good.[8]

4.2 Relationship to DFS

It is important to stress that, in the DFS model with monopolistic competition and heterogeneous firms, the finding of the DFS model with perfect competition arises as a special case in which product differentiation and firm heterogeneity are absent. If the current model assumes perfect competition, international trade will lead to complete specialization for traded goods in each country, allowing all laborers to be allocated to the comparative advantage sectors:

$$\int_{\bar{z}_1}^{1} \frac{L_1(z)}{\overline{L}_1} dz = \int_{0}^{\bar{z}_2} \frac{L_2(z)}{\overline{L}_2} dz = 1,$$

or $\kappa_1(\bar{z}_1) = \kappa_2(\bar{z}_2) = 1$. Substituting this equality into (18), we obtain

$$\omega = \frac{1 - \lambda_2(\bar{z}_2)}{1 - \lambda_1(\bar{z}_1)} \left(\frac{\overline{L}_2}{\overline{L}_1} \right). \tag{30}$$

The equilibrium characterization determined by (2), (3) and (30) is exactly the same as that of DFS, while making (4), (5) and (6) irrelevant for the analysis of perfect competition.

The above labor reallocation does not occur in the current model since international trade leads to incomplete specialization, allowing laborers to be allocated relatively more to the sectors where each country's comparative advantage is relatively stronger. This implies that the DFS model with perfect competition can be understood as a special case of the DFS model with monopolistic competition and heterogeneous firms in that the equilibrium characterization and comparative statics give rise to exactly the same outcomes as those of the DFS model with perfect competition once we abstract from product differentiation and firm heterogeneity. Thus, the DFS model with monopolistic competition and heterogeneous firms can generate richer predictions through the intra-sectoral adjustment in the firm-level variables that are absent in the DFS model with perfect competition.

Finally, we mention a welfare comparison between the two models. While the real wage in the DFS model with perfect competition is given by $w_i/P_i(z) = \mu_i(z)$, we cannot say for sure whether this real wage is necessarily greater or smaller than that in the DFS model with monopolistic competition and heterogeneous firms, which is given by (19). This makes it difficult to compare welfare between the two models.

[8] Since asymmetric trade liberalization reduces the relative wage, it does not always improve welfare of the liberalizing country in the Ricardian model with perfect competition.

5 Conclusions

This paper presented a general-equilibrium Ricardian model with heterogeneous firms to explore the impact of country size and technology on the firm-level variables. We demonstrated that a country with larger size and higher technology exhibits higher productivity and lower price-cost margins even under the assumptions of C.E.S. preferences and monopolistic competition by changing the relative wage. Welfare is higher in this country, not only due to increased product variety but also due to increased competition in a domestic market. We also showed that the equilibrium property of our model helps understand the role of country size and technology in the gravity equation. In particular, our model predicts that country size impacts positively on the extensive margin, whereas it impacts negatively on the intensive margin, which accords well with recent empirical evidence using the firm-level dataset. Our model offers a possible explanation for this empirical pattern by allowing country size to affect the firm-level variables, while preserving the usefulness of the workforce model in the new trade theory literature.

To make the analysis simple, we have restricted our attention to an open economy and abstracted from comparing welfare in autarky and costly trade, but it is straightforward to extend our setup to explore the impact of trade on inter-/intra-sectoral resource allocations and welfare gains from trade. From the impact of asymmetric trade liberalization on the firm-level variables, we expect that trade liberalization would allocate labor resources relatively more to more productive firms within sectors, whereas these trade-induced reallocations would be more significant in comparative advantage sectors than comparative disadvantage sectors, thereby creating additional welfare gains from trade. The rationale in our analysis suggests that this welfare consequence of trade should be similar between the Ricardian model and the Heckscher-Ohlin model. Since the movement from autarky to costly trade would not lead to the same factor prices between two countries in the Ricardian model, however, this difference in the factor prices would lead to different implications for the role of country endowments in the firm-level variables.

Appendix A: Proofs

A.1 Proofs of Lemmas 1 and 2

We first prove Lemmas 1 and 2. Taking the log and differentiating (4) and (5) with respect to z,

$$\frac{B_1'(z)}{B_1(z)} - (\sigma-1)\frac{\mu_1'(z)}{\mu_1(z)} + (\sigma-1)\frac{\varphi_{11}^{*'}(z)}{\varphi_{11}^*(z)} = 0, \tag{A.1}$$

$$\frac{B_2'(z)}{B_2(z)} - (\sigma-1)\frac{\mu_2'(z)}{\mu_2(z)} + (\sigma-1)\frac{\varphi_{22}^{*'}(z)}{\varphi_{22}^*(z)} = 0, \tag{A.2}$$

$$\frac{B_2'(z)}{B_2(z)} - (\sigma-1)\frac{\mu_1'(z)}{\mu_1(z)} + (\sigma-1)\frac{\varphi_{12}^{*'}(z)}{\varphi_{12}^*(z)} = 0, \tag{A.3}$$

$$\frac{B_1'(z)}{B_1(z)} - (\sigma-1)\frac{\mu_2'(z)}{\mu_2(z)} + (\sigma-1)\frac{\varphi_{21}^{*'}(z)}{\varphi_{21}^*(z)} = 0. \tag{A.4}$$

Further, using (4) and (5), rewrite (6) as

$$f_{ii}J(\varphi_{ii}^*(z)) + f_{ij}J(\varphi_{ij}^*(z)) = f_i^e,$$

where $J(\varphi_{ij}^*(z)) = \int_{\varphi_{ij}^*(z)}^{\infty}[(\varphi/\varphi_{ij}^*(z))^{\sigma-1}-1]dG(\varphi)$ is decreasing in $\varphi_{ij}^*(z)$, with $\lim_{\varphi_{ij}^*(z)\to 0} J(\varphi_{ij}^*(z)) = \infty$ and $\lim_{\varphi_{ij}^*(z)\to\infty} J(\varphi_{ij}^*(z)) = 0$. Differentiating this equality with respect to z and rearranging,

$$\varphi_{12}^{*'}(z) = -C_1(z)\varphi_{11}^{*'}(z), \tag{A.5}$$

$$\varphi_{21}^{*'}(z) = -C_2(z)\varphi_{22}^{*'}(z), \tag{A.6}$$

where $C_i(z) \equiv \frac{f_{ii}J'(\varphi_{ii}^*(z))}{f_{ij}J'(\varphi_{ij}^*(z))} > 0$. Note that (A.1) – (A.6) are six equations which have six unknowns $(\varphi_{11}^{*'}(z), \varphi_{22}^{*'}(z), \varphi_{12}^{*'}(z), \varphi_{21}^{*'}(z), B_1'(z), B_2'(z))$. Substituting (A.5) and (A.6) respectively into (A.3) and (A.4), and subtracting (A.2) and (A.1) respectively from these yields

$$\frac{C_1(z)\varphi_{11}^{*'}(z)}{\varphi_{12}^*(z)} + \frac{\varphi_{22}^{*'}(z)}{\varphi_{22}^*(z)} = \frac{\mu'(z)}{\mu(z)}, \quad \frac{C_2(z)\varphi_{22}^{*'}(z)}{\varphi_{21}^*(z)} + \frac{\varphi_{11}^{*'}(z)}{\varphi_{11}^*(z)} = -\frac{\mu'(z)}{\mu(z)},$$

where $\mu'(z) \geq 0$. These are two equations with two unknowns $(\varphi_{11}^{*'}(z), \varphi_{22}^{*'}(z))$, which are solved for

$$\varphi_{11}^{*'}(z) = \frac{\frac{\mu'(z)}{\mu(z)}\left(\frac{1}{\varphi_{22}^*(z)} + \frac{C_2(z)}{\varphi_{21}^*(z)}\right)}{\Xi(z)}, \quad \varphi_{22}^{*'}(z) = -\frac{\frac{\mu'(z)}{\mu(z)}\left(\frac{1}{\varphi_{11}^*(z)} + \frac{C_1(z)}{\varphi_{12}^*(z)}\right)}{\Xi(z)},$$

where

$$\Xi(z) \equiv \frac{1}{\varphi_{11}^*(z)\varphi_{22}^*(z)}\left(\frac{\varphi_{11}^*(z)\varphi_{22}^*(z)}{\varphi_{12}^*(z)\varphi_{21}^*(z)}C_1(z)C_2(z) - 1\right).$$

From (4), (5) and $C_i(z)$ defined above, $\Xi(z)$ is positive if

$$\frac{J'(\varphi_{11}^*(z))J'(\varphi_{22}^*(z))}{J'(\varphi_{12}^*(z))J'(\varphi_{21}^*(z))} > \tau_{12}\tau_{21}\left(\frac{f_{12}f_{21}}{f_{11}f_{22}}\right)^{\frac{\sigma}{\sigma-1}}. \tag{A.7}$$

(A.7) holds for the Pareto distribution since the left-hand side is $(\tau_{12}\tau_{21})^{k+1}(f_{12}f_{21}/f_{11}f_{22})^{(k+1)/(\sigma-1)}$. Under condition (A.7), $\varphi_{11}^{*'}(z) \geq 0, \varphi_{22}^{*'}(z) \leq 0$, and from (A.5) and (A.6), $\varphi_{12}^{*'}(z) \leq 0, \varphi_{21}^{*'}(z) \geq 0$, and hence $\varphi^{*d'}(z) \geq 0,\ \varphi^{*x'}(z) \leq 0$. From (8), these imply that $B'(z) \leq 0$.

We next show that $B(\bar{z}_1) = B(\bar{z}_2) = 1$. To prove this, it is useful to apply geometry to Figure 1. For this purpose, let us suppose the following two conditions:

- $\pi_{ii}(\varphi, z)$ and $\pi_{ji}(\varphi, z)$ are parallel; $\tilde{\pi}_{jj}(\varphi, z)$ and $\tilde{\pi}_{ij}(\varphi, z)$ are parallel. $\tilde{\pi}_{jj}(\varphi, z)$ and $\tilde{\pi}_{ij}(\varphi, z)$ are the operating profits that adjust the variable trade cost (see the dotted lines in Figure 1).

$$\tilde{\pi}_{jj}(\varphi, z) = B_j(z)\left(\frac{\mu_j(z)}{\tau_{ji}w_j}\right)^{\sigma-1}\varphi^{\sigma-1} - w_j f_{jj}, \quad \tilde{\pi}_{ij}(\varphi, z) = B_j(z)\left(\frac{\mu_i(z)}{w_i}\right)^{\sigma-1}\varphi^{\sigma-1} - w_i f_{ij}.$$

- $\pi_{ii}(z)$ and $\tilde{\pi}_{ij}(z)$ are parallel; $\tilde{\pi}_{jj}(z)$ and $\pi_{ji}(z)$ are parallel.

From (2) and (3), the first condition is satisfied in the cutoff sectors $\bar{z}_1, \bar{z}_2$. In addition, since the slopes of $\pi_{ii}(\varphi, z)$ and $\tilde{\pi}_{ij}(\varphi, z)$ are $B_i(z)\left(\frac{\mu_i(z)}{w_i}\right)^{\sigma-1}$ and $B_j(z)\left(\frac{\mu_i(z)}{w_i}\right)^{\sigma-1}$ respectively, the second condition is satisfied if and only if $B_i(z) = B_j(z)$. Thus, these profit functions are parallel if and only if $B_i(\bar{z}_i) = B_j(\bar{z}_i)$, or equivalently $B(\bar{z}_i) \equiv \frac{B_1(\bar{z}_i)}{B_2(\bar{z}_i)} = 1$. Then, from simple geometry in Figure 1, we have the following relationships in the cutoff sector $\bar{z}_1$:

$$(\varphi_{11}^*(\bar{z}_1))^{\sigma-1} : (\tau_{21}\varphi_{22}^*(\bar{z}_1))^{\sigma-1} = w_1 f_{11} : w_2 f_{22} \quad \Leftrightarrow \quad \varphi^{*d}(\bar{z}_1) \equiv \frac{\varphi_{11}^*(\bar{z}_1)}{\varphi_{22}^*(\bar{z}_1)} = \tau_{21}\left(\frac{w_1 f_{11}}{w_2 f_{22}}\right)^{\frac{1}{\sigma-1}},$$

$$\left(\frac{\varphi_{12}^*(\bar{z}_1)}{\tau_{12}}\right)^{\sigma-1} : (\varphi_{21}^*(\bar{z}_1))^{\sigma-1} = w_1 f_{12} : w_2 f_{21} \quad \Leftrightarrow \quad \varphi^{*x}(\bar{z}_1) \equiv \frac{\varphi_{12}^*(\bar{z}_1)}{\varphi_{21}^*(\bar{z}_1)} = \tau_{12}\left(\frac{w_1 f_{12}}{w_2 f_{21}}\right)^{\frac{1}{\sigma-1}}.$$

Thus, under $\tau_{ij}^{\sigma-1} f_{ij}/f_{ii} = \tau_{ji}^{\sigma-1} f_{ji}/f_{jj}$, $\varphi^{*d}(\bar{z}_1) = \varphi^{*x}(\bar{z}_1)$ and $B(\bar{z}_1) = 1$ in the cutoff sector $\bar{z}_1$. The similar proof also shows that $\varphi^{*d}(\bar{z}_2) = \varphi^{*x}(\bar{z}_2)$ and $B(\bar{z}_2) = 1$ in another cutoff sector $\bar{z}_2$.

A.2 Proof of Lemma 3

We first show that $Q(z) \equiv \frac{Q_1(z)}{Q_2(z)}$ is increasing in z whereas $P(z) \equiv \frac{P_1(z)}{P_2(z)}$ is decreasing in z. Using $R_i(z) = b_i(z)w_i\overline{L}_i$ and $b_i(z) = b_j(z)$, we have

$$R(z) \equiv \frac{R_1(z)}{R_2(z)} = \omega\overline{L}. \tag{A.8}$$

Using this, it follows from $B_i(z) = \frac{(\sigma-1)^{\sigma-1}}{\sigma^\sigma}R_i(z)(P_i(z))^{\sigma-1}$ that

$$B(z) \equiv \frac{B_1(z)}{B_2(z)} = \omega\overline{L}P(z)^{\sigma-1}.$$

Differentiating $B(z)$ with respect to z, $\frac{B'(z)}{B(z)} = (\sigma-1)\frac{P'(z)}{P(z)}$. Since $B'(z) \leq 0$ and $\sigma > 1$, we have that $P'(z) \leq 0$. Moreover, using $R_i(z) = P_i(z)Q_i(z)$, (A.8) is alternatively expressed as

$$R(z) = P(z)Q(z).$$

Noting that the right-hand side of (A.8) is independent of z, differentiating this with respect to z yields $\frac{R'(z)}{R(z)} = \frac{P'(z)}{P(z)} + \frac{Q'(z)}{Q(z)} = 0$. Since $P'(z) \leq 0$, we have that $Q'(z) \geq 0$. This proves that $\frac{Q_1(z)}{Q_2(z)} \leq \frac{Q_1(z')}{Q_2(z')}$ and $\frac{P_1(z)}{P_2(z)} \geq \frac{P_1(z')}{P_2(z')}$ for $z < z'$.

We next show that $R^d(z) \equiv \frac{R_{11}(z)}{R_{22}(z)}$ and $R^x(z) \equiv \frac{R_{12}(z)}{R_{21}(z)}$ are increasing in z, where $R_{ii}(z)$ and $R_{ij}(z)$ are expressed as

$$R_{ii}(z) = M_i^e(z) \int_{\varphi_{ii}^*(z)}^{\infty} r_{ii}(\varphi, z) dG(\varphi), \quad R_{ij}(z) = M_i^e(z) \int_{\varphi_{ij}^*(z)}^{\infty} r_{ij}(\varphi, z) dG(\varphi).$$

Using $V(\varphi_{ii}^*) \equiv \int_{\varphi_{ii}^*}^{\infty} \varphi^{\sigma-1} dG(\varphi)$ where $V'(\varphi_{ii}^*) < 0$, rewrite these revenues as

$$\begin{aligned} R_{ii}(z) &= M_i^e(z) \sigma B_i(z) \left(\frac{\mu_i(z)}{w_i}\right)^{\sigma-1} V(\varphi_{ii}^*(z)), \\ R_{ij}(z) &= M_i^e(z) \sigma B_j(z) \left(\frac{\mu_i(z)}{\tau_{ij} w_i}\right)^{\sigma-1} V(\varphi_{ij}^*(z)), \end{aligned}$$

and its ratio is given by

$$\frac{R_{ii}(z)}{R_{ij}(z)} = \tau_{ij}^{\sigma-1} \frac{B_i(z)}{B_j(z)} \frac{V(\varphi_{ii}^*(z))}{V(\varphi_{ij}^*(z))}.$$

Taking the log and differentiating $\frac{R_{ii}(z)}{R_{ij}(z)}$ with respect to z, we have that

$$\frac{R'_{11}(z)}{R_{11}(z)} \leq \frac{R'_{12}(z)}{R_{12}(z)}, \quad \frac{R'_{21}(z)}{R_{21}(z)} \leq \frac{R'_{22}(z)}{R_{22}(z)}, \tag{A.9}$$

where the inequalities come from the results in Lemmas 1 and 2. Further, noting that $b_i(z) = b_i(z')$ for $z \neq z'$ and $R_i(z) = R_{ii}(z) + R_{ji}(z) = b_i(z) w_i \overline{L}_i$, we have that $R'_{ii}(z) = -R'_{ji}(z)$. Substituting this into (A.9) and rearranging, we have

$$\begin{aligned} \frac{R_{21}(z)}{R_{22}(z)} R'_{12}(z) \leq R'_{11}(z) \leq \frac{R_{11}(z)}{R_{12}(z)} R'_{12}(z), \\ \frac{R_{22}(z)}{R_{21}(z)} R'_{21}(z) \leq R'_{22}(z) \leq \frac{R_{12}(z)}{R_{11}(z)} R'_{21}(z). \end{aligned} \tag{A.10}$$

Note that $\frac{R_{21}(z)}{R_{22}(z)} < \frac{R_{11}(z)}{R_{12}(z)}$ if and only if $\frac{V(\varphi_{21}^*(z))}{V(\varphi_{22}^*(z))} < \frac{V(\varphi_{11}^*(z))}{V(\varphi_{12}^*(z))}$, which holds true because $\varphi_{ij}^*(z) > \varphi_{ii}^*(z)$ under (14). Then, (A.10) implies that $R'_{11}(z) \geq 0$, $R'_{22}(z) \leq 0$, $R'_{12}(z) \geq 0$, $R'_{21}(z) \leq 0$, which in turn implies that $R^{d'}(z) \geq 0$ and $R^{x'}(z) \geq 0$. This proves that $\frac{R_{11}(z)}{R_{22}(z)} \leq \frac{R_{11}(z')}{R_{22}(z')}$ and $\frac{R_{12}(z)}{R_{21}(z)} \leq \frac{R_{12}(z')}{R_{21}(z')}$ for $z < z'$.

Finally, we show that $L(z) \equiv \frac{L_1(z)}{L_2(z)}$ and $M^e(z) \equiv \frac{M_1^e(z)}{M_2^e(z)}$ are increasing in z. Regarding $L(z)$, we will show in Appendix A.3 that $L_i(z)$ is written as

$$L_i(z) = \frac{R_{ii}(z) + R_{ij}(z)}{w_i}. \tag{A.11}$$

From (A.11), its ratio is given by

$$L(z) \equiv \frac{L_1(z)}{L_2(z)} = \frac{1}{\omega}\frac{R_{11}(z)+R_{12}(z)}{R_{22}(z)+R_{21}(z)}.$$

Since $R_{11}'(z) \geq 0$, $R_{22}'(z) \leq 0$, $R_{12}'(z) \geq 0$, $R_{21}'(z) \leq 0$, (A.11) implies that $L_2'(z) \leq 0 \leq L_1'(z)$, and hence $L'(z) \geq 0$. As for $M_i^e(z)$, from the expression of $M_i^e(z)$ in the main text, it follows that its ratio is given by

$$M^e(z) \equiv \frac{M_1^e(z)}{M_2^e(z)} = \left(\frac{\mu(z)}{\omega}\right)^{\sigma-1} \frac{V(\varphi_{22}^*(z))(P(z))^{1-\sigma} - \tau_{21}^{1-\sigma}V(\varphi_{21}^*(z))}{V(\varphi_{11}^*(z)) - \tau_{12}^{1-\sigma}V(\varphi_{12}^*)(P(z))^{1-\sigma}}.$$

Since $\mu'(z) \geq 0, \varphi_{11}^{*\prime}(z) \geq 0, \varphi_{22}^{*\prime}(z) \leq 0, \varphi_{12}^{*\prime}(z) \leq 0, \varphi_{21}^{*\prime}(z) \geq 0, P'(z) \leq 0$, we have that $M^{e\prime}(z) \geq 0$. This proves that $\frac{L_1(z)}{L_2(z)} \leq \frac{L_1(z')}{L_2(z')}$ and $\frac{M_1^e(z)}{M_2^e(z)} \leq \frac{M_1^e(z')}{M_2^e(z')}$ for $z < z'$.

A.3 Proofs of Proposition 2

A.3.1 Proof of Equation (16)

We show that equation (7) is written as equation (16). Aggregate labor supply in sector z of country i is given by

$$L_i(z) = M_i^e(z)\int_{\varphi_{ii}^*(z)}^{\infty} l_{ii}(\varphi,z)dG(\varphi) + M_i^e(z)\int_{\varphi_{ij}^*(z)}^{\infty} l_{ij}(\varphi,z)dG(\varphi) + M_i^e(z)f_i^e. \tag{A.12}$$

Substituting $l_{ii}(\varphi,z)$ and $l_{ij}(\varphi,z)$, the first two terms in the right-hand side are

$$\begin{aligned}
&M_i^e(z)\int_{\varphi_{ii}^*(z)}^{\infty} l_{ii}(\varphi,z)dG(\varphi) + M_i^e(z)\int_{\varphi_{ij}^*(z)}^{\infty} l_{ij}(\varphi,z)dG(\varphi) \\
&= \frac{M_i^e}{w_i}\left\{[1-G(\varphi_{ii}^*(z))]\,w_i f_{ii} + \frac{\sigma-1}{\sigma}\int_{\varphi_{ii}^*(z)}^{\infty} r_{ii}(\varphi,z)dG(\varphi) + \left[1-G(\varphi_{ij}^*(z))\right]w_i f_{ij} + \frac{\sigma-1}{\sigma}\int_{\varphi_{ij}^*(z)}^{\infty} r_{ij}(\varphi,z)dG(\varphi)\right\},
\end{aligned}$$

On the other hand, the last term in the right-hand side is

$$M_i^e(z)f_i^e = \frac{M_i^e(z)}{w_i}\left\{\frac{1}{\sigma}\int_{\varphi_{ii}^*(z)}^{\infty} r_{ii}(\varphi,z)dG(\varphi) - [1-G(\varphi_{ii}^*(z))]\,w_i f_{ii} + \frac{1}{\sigma}\int_{\varphi_{ij}^*(z)}^{\infty} r_{ij}(\varphi,z)dG(\varphi) - \left[1-G(\varphi_{ij}^*(z))\right]w_i f_{ij}\right\},$$

which is derived from (6):

$$\begin{aligned}
f_i^e &= \int_{\varphi_{ii}^*(z)}^{\infty} \frac{\pi_{ii}(\varphi,z)}{w_i}dG(\varphi) + \int_{\varphi_{ij}^*(z)}^{\infty} \frac{\pi_{ij}(\varphi,z)}{w_i}dG(\varphi) \\
&= \frac{1}{w_i}\left\{\int_{\varphi_{ii}^*(z)}^{\infty} \frac{r_{ii}(\varphi,z)}{\sigma}dG(\varphi) - [1-G(\varphi_{ii}^*(z))]\,w_i f_{ii} + \int_{\varphi_{ij}^*(z)}^{\infty} \frac{r_{ij}(\varphi,z)}{\sigma}dG(\varphi) - \left[1-G(\varphi_{ij}^*)\right]w_i f_{ij}\right\}.
\end{aligned}$$

Summing up these terms, (A.12) is equivalent with (A.11):

$$\begin{aligned} L_i(z) &= \frac{M_i^e(z)}{w_i}\left\{\int_{\varphi_{ii}^*(z)}^{\infty} r_{ii}(\varphi,z)dG(\varphi) + \int_{\varphi_{ij}^*(z)}^{\infty} r_{ij}(\varphi,z)dG(\varphi)\right\} \\ &= \frac{R_{ii}(z)+R_{ij}(z)}{w_i}. \end{aligned}$$

Integrating the above aggregate labor supply over the interval [0,1] and noting $\int_0^1 L_i(z)dz = \overline{L}_i$,

$$\begin{aligned} \overline{L}_i &= \frac{\int_0^1 R_{ii}(z)dz + \int_0^1 R_{ij}(z)dz}{w_i} \\ &= \frac{\int_0^1 R_{ii}(z)dz + \int_0^1 R_{ji}(z)dz}{w_i} \\ &= \frac{\int_0^1 R_i(z)dz}{w_i}, \end{aligned}$$

where the second equality comes from (17), and the third equality comes from $R_i(z) = R_{ii}(z)+R_{ji}(z)$.

A.3.2 Proof of Equation (18)

We first show that (17) is expressed as

$$\int_{\bar{z}_1}^{1} (w_1L_1(z) - R_1(z))\, dz = \int_0^{\bar{z}_2} (w_2L_2(z) - R_2(z))\, dz. \tag{A.13}$$

Since net exports are zero in the interval sectors $z \in [\bar{z}_1, \bar{z}_2]$, we have that $\int_{\bar{z}_1}^{\bar{z}_2} R_{12}(z)dz = \int_{\bar{z}_1}^{\bar{z}_2} R_{21}(z)dz$. Noting this relationship, rewrite (17) as

$$\int_{\bar{z}_1}^{1} (R_{12}(z) - R_{21}(z))\, dz = \int_0^{\bar{z}_2} (R_{21}(z) - R_{12}(z))\, dz.$$

From $w_iL_i(z) = R_{ii}(z)+R_{ij}(z)$ and $R_i(z) = R_{ii}(z)+R_{ji}(z)$, we have $w_iL_i(z)-R_i(z) = R_{ij}(z)-R_{ji}(z)$. Substituting this into the above equality gives us equation (A.13).

Then, by manipulating (A.13),

$$\begin{aligned} &\int_{\bar{z}_1}^{1} (w_1L_1(z) - R_1(z))\, dz = \int_0^{\bar{z}_2} (w_2L_2(z) - R_2(z))\, dz \\ \iff &\int_{\bar{z}_1}^{1} \left(\frac{L_1(z)}{\overline{L}_1} - \frac{R_1(z)}{w_1\overline{L}_1}\right) dz = \int_0^{\bar{z}_2} \left(\frac{L_2(z)}{\overline{L}_2}\frac{w_2\overline{L}_2}{w_1\overline{L}_1} - \frac{R_2(z)}{w_2\overline{L}_2}\frac{w_2\overline{L}_2}{w_1\overline{L}_1}\right) dz \\ \iff &\int_{\bar{z}_1}^{1} \left(\frac{L_1(z)}{\overline{L}_1} - b_1(z)\right) dz = \frac{1}{\omega\overline{L}}\int_0^{\bar{z}_2} \left(\frac{L_2(z)}{\overline{L}_2} - b_2(z)\right) dz, \end{aligned}$$

where the second equation comes from dividing both sides of the first equation by $w_1\overline{L}_1$, and the third equation comes from the definition of $b_i(z)$. Solving the third equation for ω gives us (18).

We next show that (18) is decreasing in ω. To prove this, it suffices to show that (18) is decreasing in $\bar{z}_1, \bar{z}_2$ because $\bar{z}_1$ and $\bar{z}_2$ are increasing in ω (see (2) and (3)). Let

$$\xi_1(\bar{z}_1) \equiv \kappa_1(\bar{z}_1) - \lambda_1(\bar{z}_1), \quad \xi_2(\bar{z}_2) \equiv \kappa_2(\bar{z}_2) - \lambda_2(\bar{z}_2)$$

respectively denote the denominator and numerator of (18), which are positive for any $\bar{z}_1$ and $\bar{z}_2$. Differentiating these with respect to $\bar{z}_1$ and $\bar{z}_2$ respectively yields

$$\begin{aligned}
\frac{d\xi_1(\bar{z}_1)}{d\bar{z}_1} &= -\frac{L_1(\bar{z}_1)}{\overline{L}_1} + \int_{\bar{z}_1}^{1} \frac{L_1'(\bar{z}_1)}{\overline{L}_1} dz + \frac{R_1(\bar{z}_1)}{w_1 \overline{L}_1} = \int_{\bar{z}_1}^{1} \frac{L_1'(\bar{z}_1)}{\overline{L}_1} dz, \\
\frac{d\xi_2(\bar{z}_2)}{d\bar{z}_2} &= \frac{L_2(\bar{z}_2)}{\overline{L}_2} + \int_{0}^{\bar{z}_2} \frac{L_2'(\bar{z}_2)}{\overline{L}_2} dz - \frac{R_2(\bar{z}_2)}{w_2 \overline{L}_2} = \int_{0}^{\bar{z}_2} \frac{L_2'(\bar{z}_2)}{\overline{L}_2} dz,
\end{aligned}$$

where the first equality comes from $b_i(z) = \frac{R_i(z)}{w_i \overline{L}_i}$ and the second one comes from $w_i L_i(\bar{z}_i) = R_i(\bar{z}_i)$. Since $L_2'(z) \leq 0 \leq L_1'(z)$ (see Appendix A.2) and this property of $L_i(z)$ must hold for $z = \bar{z}_1, \bar{z}_2$, we have that $\xi_1'(\bar{z}_1) \geq 0$ and $\xi_2'(\bar{z}_2) \leq 0$.

A.4 Proof of Proposition 3

We first show the comparative statics for $\{\bar{z}_1, \bar{z}_2, \omega\}$ characterized by (2), (3) and (18). Regarding comparative statics with respect to $\overline{L}$, since the right-hand side of (18) is decreasing in ω, a rise in $\overline{L} \equiv \overline{L}_1/\overline{L}_2$ must decrease ω. Further, since the right-hand sides of (2) and (3) are increasing in ω, a rise in $\overline{L}$ must decrease $\bar{z}_1$ and $\bar{z}_2$. Regarding comparative statics with respect to $\mu(z)$, it follows from (2) and (3) that a proportional rise in $\mu(z)$ must increase ω. Further, since the right-hand sides of $\omega = \tau_{21}\mu(\bar{z}_1)$ and $\omega = \mu(\bar{z}_2)/\tau_{12}$ are increasing in $\mu(z)$, a proportional rise in $\mu(z)$ must decrease $\bar{z}_1$ and $\bar{z}_2$. This proves that $\frac{\partial \bar{z}_1}{\partial \overline{L}} \leq 0, \frac{\partial \bar{z}_2}{\partial \overline{L}} \leq 0, \frac{\partial \omega}{\partial \overline{L}} \leq 0, \frac{\partial \bar{z}_1}{\partial \mu(z)} \leq 0, \frac{\partial \bar{z}_2}{\partial \mu(z)} \leq 0, \frac{\partial \omega}{\partial \mu(z)} \geq 0$.

We next show the comparative statics for $\{\varphi^{*d}(z), \varphi^{*x}(z), B(z)\}$ characterized by (4), (5) and (6). Regarding comparative statics with respect to $\overline{L}$, consider a rise in $\overline{L}_1$ (while keeping $\overline{L}_2$ constant) and normalize $w_2 = 1$ as a numeraire of the model. Differentiating (4), (5) and (6) with respect to $\overline{L}_1$ gives us the following six equations:

$$\frac{\dot{B}_1(z)}{B_1(z)} - (\sigma - 1)\frac{\dot{w}_1}{w_1} + (\sigma - 1)\frac{\dot{\varphi}_{11}^*(z)}{\varphi_{11}^*(z)} = \frac{\dot{w}_1}{w_1}, \tag{A.14}$$

$$\frac{\dot{B}_2(z)}{B_2(z)} + (\sigma - 1)\frac{\dot{\varphi}_{22}^*(z)}{\varphi_{22}^*(z)} = 0, \tag{A.15}$$

$$\frac{\dot{B}_2(z)}{B_2(z)} - (\sigma - 1)\frac{\dot{w}_1}{w_1} + (\sigma - 1)\frac{\dot{\varphi}_{12}^*(z)}{\varphi_{12}^*(z)} = \frac{\dot{w}_1}{w_1}, \tag{A.16}$$

$$\frac{\dot{B}_1(z)}{B_1(z)} + (\sigma - 1)\frac{\dot{\varphi}_{21}^*(z)}{\varphi_{21}^*(z)} = 0, \tag{A.17}$$

$$\dot{\varphi}_{12}^*(z) = -C_1(z)\dot{\varphi}_{11}^*(z), \tag{A.18}$$

$$\dot{\varphi}_{21}^*(z) = -C_2(z)\dot{\varphi}_{22}^*(z), \tag{A.19}$$

where a dot is used to represent the derivative with respect to $\overline{L}_1$ (e.g., $\dot{B}_1(z) \equiv \frac{\partial B_1(z)}{\partial \overline{L}_1}$). Note that (A.14) – (A.19) are six equations with six unknowns $(\dot{\varphi}^*_{11}(z), \dot{\varphi}^*_{22}(z), \dot{\varphi}^*_{12}(z), \dot{\varphi}^*_{21}(z), \dot{B}_1(z), \dot{B}_2(z))$. Following the same steps in Appendix A.1, we can solve for

$$\dot{\varphi}^*_{11}(z) = -\frac{\frac{\sigma}{\sigma-1}\frac{\dot{w}_1}{w_1}\left(\frac{1}{\varphi^*_{22}(z)} + \frac{C_2(z)}{\varphi^*_{21}(z)}\right)}{\Xi(z)}, \quad \dot{\varphi}^*_{22}(z) = \frac{\frac{\sigma}{\sigma-1}\frac{\dot{w}_1}{w_1}\left(\frac{1}{\varphi^*_{11}(z)} + \frac{C_1(z)}{\varphi^*_{12}(z)}\right)}{\Xi(z)}.$$

From $\dot{w}_1 \leq 0$, we have $\dot{\varphi}^*_{11}(z) \geq 0, \dot{\varphi}^*_{22}(z) \leq 0$; and from (A.18) and (A.19), $\dot{\varphi}^*_{12}(z) \leq 0, \dot{\varphi}^*_{21}(z) \geq 0$. Further, from (8), we have $\dot{B}(z) \leq 0$. This proves that $\frac{\partial \varphi^*_{11}(z)}{\partial \overline{L}} \geq 0$, $\frac{\partial \varphi^*_{22}(z)}{\partial \overline{L}} \leq 0$, $\frac{\partial \varphi^*_{12}(z)}{\partial \overline{L}} \leq 0$, $\frac{\partial \varphi^*_{21}(z)}{\partial \overline{L}} \geq 0$, $\frac{\partial B(z)}{\partial \overline{L}} \leq 0$ and hence $\frac{\partial \varphi^{*d}(z)}{\partial \overline{L}} \geq 0$, $\frac{\partial \varphi^{*x}(z)}{\partial \overline{L}} \leq 0$.

Regarding comparative statics with respect to $\mu(z)$, consider a proportional rise in $\mu_1(z)$ (while keeping $\mu_2(z)$ constant) and normalize $w_2 = 1$. Differentiating (4), (5) and (6) with respect to $\mu_1(z)$ gives us the following six equations:

$$\frac{\ddot{B}_1(z)}{B_1(z)} + (\sigma-1)\frac{1}{\mu_1(z)} - (\sigma-1)\frac{\ddot{w}_1}{w_1} + (\sigma-1)\frac{\ddot{\varphi}^*_{11}(z)}{\varphi^*_{11}(z)} = \frac{\ddot{w}_1}{w_1}, \tag{A.20}$$

$$\frac{\ddot{B}_2(z)}{B_2(z)} + (\sigma-1)\frac{\ddot{\varphi}^*_{22}(z)}{\varphi^*_{22}(z)} = 0, \tag{A.21}$$

$$\frac{\ddot{B}_2(z)}{B_2(z)} + (\sigma-1)\frac{1}{\mu_1(z)} - (\sigma-1)\frac{\ddot{w}_1}{w_1} + (\sigma-1)\frac{\ddot{\varphi}^*_{12}(z)}{\varphi^*_{12}(z)} = \frac{\ddot{w}_1}{w_1}, \tag{A.22}$$

$$\frac{\ddot{B}_1(z)}{B_1(z)} + (\sigma-1)\frac{\ddot{\varphi}^*_{21}(z)}{\varphi^*_{21}(z)} = 0, \tag{A.23}$$

$$\ddot{\varphi}^*_{12}(z) = -C_1(z)\ddot{\varphi}^*_{11}(z), \tag{A.24}$$

$$\ddot{\varphi}^*_{21}(z) = -C_2(z)\ddot{\varphi}^*_{22}(z), \tag{A.25}$$

where a double dot is used to represent the derivative with respect to $\mu_1(z)$ (e.g., $\ddot{B}_1(z) \equiv \frac{\partial B_1(z)}{\partial \mu_1(z)}$). Solving (A.20) – (A.25), we have

$$\ddot{\varphi}^*_{11}(z) = \frac{\left(\frac{1}{\mu_1} - \frac{\sigma}{\sigma-1}\frac{\ddot{w}_1}{w_1}\right)\left(\frac{1}{\varphi^*_{22}(z)} + \frac{C_2(z)}{\varphi^*_{21}(z)}\right)}{\Xi(z)}, \quad \ddot{\varphi}^*_{22}(z) = -\frac{\left(\frac{1}{\mu_1(z)} - \frac{\sigma}{\sigma-1}\frac{\ddot{w}_1}{w_1}\right)\left(\frac{1}{\varphi^*_{11}(z)} + \frac{C_1(z)}{\varphi^*_{12}(z)}\right)}{\Xi(z)}.$$

From $\ddot{w}_1 \geq 0$, we have $\ddot{\varphi}^*_{11}(z) \geq 0, \ddot{\varphi}^*_{22}(z) \leq 0$ if $\frac{1}{\mu_1(z)} - \frac{\sigma}{\sigma-1}\frac{\ddot{w}_1}{w_1} \geq 0$, or equivalently

$$\frac{\mu(z)}{\omega}\frac{\partial \omega}{\partial \mu(z)} \leq \frac{\sigma-1}{\sigma}. \tag{A.26}$$

Note that not only is the right-hand side of (A.26) but the left-hand side of (A.26) is less than one, because a proportional rise in $\mu_1(z)$ (or $\mu(z)$) means that the relative wage increases proportionally short of an increase in relative labor productivity ($\frac{\partial \omega}{\omega} \leq \frac{\partial \mu(z)}{\mu(z)}$). Under (A.26), $\ddot{\varphi}^*_{11}(z) \geq 0, \ddot{\varphi}^*_{22}(z) \leq 0$, and from (A.24), (A.25) and (8), we have $\ddot{\varphi}^*_{12}(z) \leq 0, \ddot{\varphi}^*_{21}(z) \geq 0$, and $\ddot{B}(z) \leq 0$. This proves that $\frac{\partial \varphi^*_{11}(z)}{\partial \mu(z)} \geq 0$, $\frac{\partial \varphi^*_{22}(z)}{\partial \mu(z)} \leq 0$, $\frac{\partial \varphi^*_{12}(z)}{\partial \mu(z)} \leq 0$, $\frac{\partial \varphi^*_{21}(z)}{\partial \mu(z)} \geq 0$, $\frac{\partial B(z)}{\partial \mu(z)} \leq 0$ and hence $\frac{\partial \varphi^{*d}(z)}{\partial \mu(z)} \geq 0$, $\frac{\partial \varphi^{*x}(z)}{\partial \mu(z)} \leq 0$.

A.5 Proof of Proposition 4

We first show the derivation of (19). From $b_i(z) = \frac{R_i(z)}{w_i \overline{L}_i}$, aggregate market demand $B_i(z)$ is given by

$$B_i(z) = \frac{(\sigma-1)^{\sigma-1}}{\sigma^\sigma} b_i(z) w_i \overline{L}_i P_i(z)^{\sigma-1}.$$

Substituting this $B_i(z)$ into (4) and rearranging,

$$\left(\frac{\sigma-1}{\sigma} \frac{P_i(z)}{w_i} \mu_i(z) \varphi_{ii}^*(z) \right)^{\sigma-1} = \frac{\sigma f_{ii}}{b_i(z) \overline{L}_i}.$$

Solving this equality for $w_i / P_i(z)$ establishes the result.

We next show that the impact of $\overline{L}$ on each country's welfare. To show this, note first that our Cobb-Douglas demand assumption makes the expenditure share $b_i(z)$ *constant* and thus any change in $\overline{L}$ does not affect $b_i(z)$. Then, applying $\frac{\partial \varphi_{11}^*(z)}{\partial \overline{L}} \geq 0$ and $\frac{\partial \varphi_{22}^*(z)}{\partial \overline{L}} \leq 0$ (see Appendix A.4) to (19) and combining this with the welfare expression establishes the result. The similar proof also applies for the impact of $\mu(z)$ on each country's welfare.

A.6 Proofs of Lemma 4

A.6.1 Proof of Equation (20)

We show the derivation of (20). Applying the Pareto distribution to (A.11) yields

$$L_i(z) = M_i^e(z) \left[\left(\frac{\varphi_{\min}}{\varphi_{ii}^*(z)} \right)^k \left(\frac{k\sigma}{k-(\sigma-1)} \right) f_{ii} + \left(\frac{\varphi_{\min}}{\varphi_{ij}^*(z)} \right)^k \left(\frac{k\sigma}{k-(\sigma-1)} \right) f_{ij} \right]. \tag{A.27}$$

Further, applying the Pareto distribution to (6) yields

$$\left(\frac{\varphi_{\min}}{\varphi_{ii}^*(z)} \right)^k \left(\frac{\sigma-1}{k-(\sigma-1)} \right) f_{ii} + \left(\frac{\varphi_{\min}}{\varphi_{ij}^*(z)} \right)^k \left(\frac{\sigma-1}{k-(\sigma-1)} \right) f_{ij} = f_i^e. \tag{A.28}$$

Substituting (A.28) into (A.27) and rearranging gives us equation (20).

A.6.2 Proof of Equation (21)

We show the derivation of (21). It is straightforward to obtain the extensive margins $M_{ii}(z), M_{ij}(z)$ by applying the Pareto distribution and substituting (20) into

$$M_{ii}(z) = [1 - G(\varphi_{ii}^*(z))] M_i^e(z), \quad M_{ij}(z) = [1 - G(\varphi_{ij}^*(z))] M_i^e(z).$$

Regarding the intensive margins $\bar{r}_{ii}(z), \bar{r}_{ij}(z)$, let us consider first the intensive margin of exporting $\bar{r}_{ij}(z)$. By definition, this intensive margin is given by

$$\begin{aligned}
\bar{r}_{ij}(z) &= \frac{1}{1-G(\varphi_{ij}^*(z))}\int_{\varphi_{ij}^*(z)}^{\infty} r_{ij}(\varphi,z)dG(\varphi) \\
&= \frac{1}{1-G(\varphi_{ij}^*(z))}B_j(z)\sigma\left(\frac{\mu_i(z)}{\tau_{ij}w_i}\right)^{\sigma-1}V(\varphi_{ij}^*(z)) && \text{(using } V(\varphi)\text{)} \\
&= \left(\frac{\varphi_{ij}^*(z)}{\varphi_{\min}}\right)^{k}B_j(z)\sigma\left(\frac{\mu_i(z)}{\tau_{ij}w_i}\right)^{\sigma-1}\frac{k\varphi_{\min}^k}{k-(\sigma-1)}\frac{1}{(\varphi_{ij}^*(z))^{k-(\sigma-1)}} && \text{(using Pareto)} \\
&= \frac{k\sigma}{k-(\sigma-1)}B_j(z)\left(\frac{\mu_i(z)}{\tau_{ij}w_i}\right)^{\sigma-1}(\varphi_{ij}^*(z))^{\sigma-1} \\
&= \frac{k\sigma}{k-(\sigma-1)}B_j(z)\left(\frac{\mu_i(z)}{\tau_{ij}w_i}\right)^{\sigma-1}\frac{1}{B_j(z)}\left(\frac{\mu_i(z)}{\tau_{ij}w_i}\right)^{1-\sigma}w_i f_{ij} && \text{(using (5))} \\
&= \frac{k\sigma}{k-(\sigma-1)}w_i f_{ij}.
\end{aligned}$$

By following the similar steps, it is easily confirmed that the intensive margin of domestic production $\bar{r}_{ii}(z)$ is given by

$$\bar{r}_{ii}(z) = \frac{k\sigma}{k-(\sigma-1)}w_i f_{ii}.$$

This establishes the desired result.

A.7 Proof of Asymmetric Trade Liberalization

We show the derivation of (29). Note that when examining comparative statics with respect to τ_{21}, we need to change f_{12} or f_{11} proportionately to $\tau_{21}^{\sigma-1}$ since we assume that $\tau_{21}^{\sigma-1}f_{21}/f_{22} = \tau_{12}^{\sigma-1}f_{12}/f_{11}$. Consider a decline in $\tau_{21}^{\sigma-1}$ and a proportionate decline in f_{12} (while keeping the other costs constant) and normalize $w_2 = 1$. Substituting $f_{12} = (\tau_{21}/\tau_{12})^{\sigma-1}f_{11}f_{21}/f_{22}$ into (5) and (6), and differentiating (4), (5) and (6) with respect to τ_{21} gives us the following six equations:

$$\begin{aligned}
\frac{\dddot{B}_1(z)}{B_1(z)} - (\sigma-1)\frac{\dddot{w}_1}{w_1} + (\sigma-1)\frac{\dddot{\varphi}_{11}^*(z)}{\varphi_{11}^*(z)} &= \frac{\dddot{w}_1}{w_1}, \\
\frac{\dddot{B}_2(z)}{B_2(z)} + (\sigma-1)\frac{\dddot{\varphi}_{22}^*(z)}{\varphi_{22}^*(z)} &= 0, \\
\frac{\dddot{B}_2(z)}{B_2(z)} - (\sigma-1)\frac{\dddot{w}_1}{w_1} + (\sigma-1)\frac{\dddot{\varphi}_{12}^*(z)}{\varphi_{12}^*(z)} &= \frac{\dddot{w}_1}{w_1} + \frac{\sigma-1}{\tau_{21}}, \\
\frac{\dddot{B}_1(z)}{B_1(z)} - (\sigma-1)\frac{1}{\tau_{21}} + (\sigma-1)\frac{\dddot{\varphi}_{21}^*(z)}{\varphi_{21}^*(z)} &= 0,
\end{aligned}$$

$$\dddot{\varphi}_{12}^*(z) = -C_1(z)\dddot{\varphi}_{11}^*(z) - \frac{\sigma-1}{\tau_{21}}\frac{J(\varphi_{12}^*)}{J'(\varphi_{12}^*)}, \tag{A.29}$$

$$\dddot{\varphi}_{21}^*(z) = -C_2(z)\dddot{\varphi}_{22}^*(z), \tag{A.30}$$

where a triple dot is used to represent the derivative with respect to τ_{21} (e.g., $\dddot{B}_1(z) \equiv \frac{\partial B_1(z)}{\partial \tau_{21}}$). Solving these, we have

$$\dddot{\varphi}^*_{11}(z) = \frac{-\frac{\sigma}{\sigma-1}\frac{\dddot{w}_1}{w_1}\left(\frac{1}{\varphi^*_{22}(z)} + \frac{C_2(z)}{\varphi^*_{21}(z)}\right) - \frac{1}{\tau_{21}}\left((1-(\sigma-1)\rho(z))\frac{C_2(z)}{\varphi^*_{21}(z)} - \frac{1}{\varphi^*_{22}(z)}\right)}{\Xi(z)},$$

$$\dddot{\varphi}^*_{22}(z) = \frac{\frac{\sigma}{\sigma-1}\frac{\dddot{w}_1}{w_1}\left(\frac{1}{\varphi^*_{11}(z)} + \frac{C_1(z)}{\varphi^*_{12}(z)}\right) - \frac{1}{\tau_{21}}\left(\frac{C_1(z)}{\varphi^*_{12}(z)} - (1-(\sigma-1)\rho(z))\frac{1}{\varphi^*_{11}(z)}\right)}{\Xi(z)},$$

where $\rho(z) \equiv -\frac{J(\varphi^*_{12}(z))}{\varphi^*_{12}(z)J'(\varphi^*_{12}(z))} > 0$, satisfying $1-(\sigma-1)\rho(z) > 0$ (from $J'(\varphi^*_{ij}(z)) = -\frac{\sigma-1}{\varphi^*_{ij}(z)}[J(\varphi^*_{ij}(z)) + 1 - G(\varphi^*_{ij}(z))]$. Since $\dddot{w}_1 \geq 0$ and the second term in the numerator of $\dddot{\varphi}^*_{11}(z)$ and $\dddot{\varphi}^*_{22}(z)$ is negative (from $\Xi(z) > 0$), we have $\dddot{\varphi}^*_{11}(z) \leq 0$ and $\dddot{\varphi}^*_{12}(z) \geq 0$ (from (A.29)). In contrast, $\dddot{\varphi}^*_{22}(z)$ receives two opposing effects that are captured by the first and second terms in the numerator of $\dddot{\varphi}^*_{22}(z)$. First, a reduction in τ_{21} raises the relative wage and reduces the competitiveness of country 2, which decreases $\varphi^*_{22}(z)$. Second, a reduction in τ_{21} allows country 2 to have better access to country 1 and decreases $\varphi^*_{21}(z)$, which in turn increases $\varphi^*_{22}(z)$ (from (A.30)). As a result, the signs of $\dddot{\varphi}^*_{22}(z)$ and $\dddot{\varphi}^*_{21}(z)$ are generally ambiguous. This proves that $\frac{\partial \varphi^*_{11}(z)}{\partial \tau_{21}} \leq 0$, $\frac{\partial \varphi^*_{12}(z)}{\partial \tau_{21}} \geq 0$, whereas $\frac{\partial \varphi^*_{22}(z)}{\partial \tau_{21}} \gtreqless 0$, $\frac{\partial \varphi^*_{21}(z)}{\partial \tau_{21}} \gtreqless 0$.

Appendix B: Zero Gravity

In this Appendix, we provide the detailed analysis of zero gravity ($\tau_{ij} = \tau_{ji} = 1$) in Section 5.1, and relate it with the general case with the variable trade cost.

Following the literature, we say that country i has a comparative advantage in producing goods in sector z if country i's unit labor costs are less than or equal to country j's unit labor costs:

$$w_i a_i(z) \leq w_j a_j(z) \quad \Longleftrightarrow \quad \frac{w_i}{w_j} \leq \frac{\mu_i(z)}{\mu_j(z)}.$$

It follows immediately that country 1 (country 2) has a comparative advantage in high-z (low-z) sectors $\bar{z} \leq z \leq 1$ ($0 \leq z \leq \bar{z}$), where

$$\bar{z} \equiv \mu^{-1}(\omega).$$

As in the main text, the sectoral equilibrium is characterized by (4), (5) and (6) with $\tau_{ij} = \tau_{ji} = 1$. From the fact that the cutoff sector $\bar{z}$ is unique, the relative market demand $B(z) \equiv B_1(z)/B_2(z)$ is *strictly* log-submodular in Lemma 1. Formally, for $z < z'$,

$$\frac{B_1(z)}{B_2(z)} > \frac{B_1(z')}{B_2(z')}.$$

Since $B(z)$ is strictly log-submodular, $B(z)$ is strictly decreasing in z. The first quadrant of Figure B.1 depicts the relationship in (z, B) space.

Next, the relative domestic productivity cutoff $\varphi^{*d}(z) \equiv \varphi^*_{11}(z)/\varphi^*_{22}(z)$ is *strictly* log-supermodular, whereas the relative export productivity cutoff $\varphi^{*x}(z) \equiv \varphi^*_{12}(z)/\varphi^*_{21}(z)$ is *strictly* log-submodular in Lemma 2. For $z < z'$,

$$\frac{\varphi^*_{11}(z)}{\varphi^*_{22}(z)} < \frac{\varphi^*_{11}(z')}{\varphi^*_{22}(z')}, \quad \frac{\varphi^*_{12}(z)}{\varphi^*_{21}(z)} > \frac{\varphi^*_{12}(z')}{\varphi^*_{21}(z')}.$$

It can be shown that $\varphi^{*d}(z) = \varphi^{*x}(z)$ and $B(z) = 1$ if and only if $z = \bar{z}$ under $f_{ij}/f_{ii} = f_{ji}/f_{jj}$. The second quadrant of Figure B.1 depicts the relationship between (B, φ^*) space.

Finally, combining the first and second quadrants of Figure B.1, we obtain the sectoral equilibrium characterized in the relative terms of the productivity cutoffs and aggregate market demand:

$$\begin{aligned} 0 \leq z \leq \bar{z} \quad &\Longleftrightarrow \quad B(z) \geq 1 \quad \Longleftrightarrow \quad \varphi^{*x}(z) \geq \varphi^{*d}(z), \\ \bar{z} \leq z \leq 1 \quad &\Longleftrightarrow \quad B(z) \leq 1 \quad \Longleftrightarrow \quad \varphi^{*x}(z) \leq \varphi^{*d}(z). \end{aligned}$$

As in the case with $\tau_{ij} \neq 1, \tau_{ji} \neq 1$, the gap between $\varphi^*_{ij}(z)$ and $\varphi^*_{ii}(z)$ is relatively narrower than the gap between $\varphi^*_{ji}(z)$ and $\varphi^*_{jj}(z)$ in country i's comparative advantage sectors (see Figure 3).

It is important to stress that, if $\tau_{ij} = \tau_{ji} = 1$, not only is $B(z)$ but also $\varphi^{*d}(z)$ and $\varphi^{*x}(z)$ are *strictly* increasing or decreasing in z. Since the aggregate variables in Lemma 3 are written as a function of the key equilibrium variables $\{B(z), \varphi^{*d}(z), \varphi^{*x}(z)\}$, the aggregate variables in Lemma 3 are also *strictly* increasing or decreasing in z. As a result, $R_{12}(z)$ is *strictly* increasing in z and $R_{21}(z)$ is *strictly* decreasing in z.

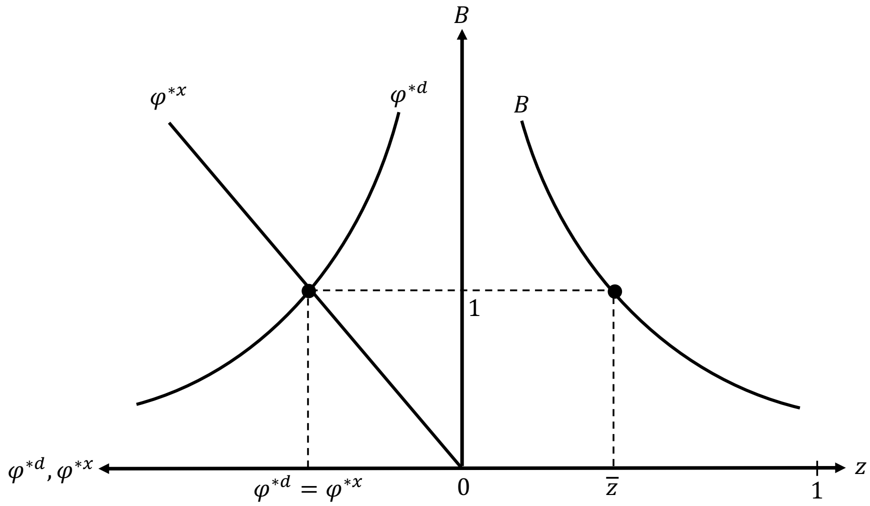

Figure B.1 – Market demand and productivity cutoffs

We next embed the sectoral equilibrium into general equilibrium. Let us define next exports from country 1 to country 2 as

$$NEXP(z) = R_{12}(z) - R_{21}(z).$$

If $\tau_{ij} = \tau_{ji} = 1$, $NEXP(z)$ is strictly increasing in z since $R_{12}(z)$ is strictly increasing in z and $R_{21}(z)$ is strictly decreasing in z. Let $\bar{z}$ denote the *hypothetical* cutoff sector in which $NEXP(\bar{z}) = 0$. Noting that $\bar{z}$ is the unique cutoff sector in which net exports are zero in intra-industry trade, country 1 runs trade surplus (deficit) in $z > (<)\bar{z}$. Then, using $R_{ij}(z) - R_{ji}(z) = w_i L_i(z) - R_i(z)$,

$$\int_{\bar{z}}^{1} (w_1 L_1(z) - R_1(z))\, dz = \int_{0}^{\bar{z}} (w_2 L_2(z) - R_2(z))\, dz,$$

which can be solved for

$$\omega = \frac{\kappa_2(\bar{z}) - \lambda_2(\bar{z})}{\kappa_1(\bar{z}) - \lambda_1(\bar{z})} \left(\frac{\overline{L}_2}{\overline{L}_1}\right). \tag{B.1}$$

Another condition that pins down the equilibrium is

$$\omega = \mu(\bar{z}). \tag{B.2}$$

Since (B.1) is strictly decreasing in $\bar{z}$ and (B.2) is strictly increasing in $\bar{z}$, these two conditions jointly determine the equilibrium variables $\{\bar{z}, \omega\}$, where $\bar{z}$ defines the equilibrium relative wage ω, and the cutoff sector $\bar{z}$ is special in that net exports are zero in intra-industry trade. Note that the logic is borrowed from DFS (1977, p.825-826); please refer to equations (10') and (11) in their paper. If we assume perfect competition, $\kappa_i(\bar{z}) = 1$ and the equilibrium characterized by (B.1) and (B.2) is exactly the same as DFS (1977).

If $\tau_{ij} \neq 1, \tau_{ji} \neq 1$, there are the two cutoff sectors $\bar{z}_1, \bar{z}_2$ and $B(\bar{z}_1) = B(\bar{z}_2) = 1$. Since $B(z)$ is decreasing in z (from Lemma 2), $B(z)$ must be *weakly* decreasing in z where $B(z) = 1$ for $z \in [\bar{z}_1, \bar{z}_2]$.

Further, not only $B(z)$ but also $\varphi^{*d}(z)$ and $\varphi^{*x}(z)$ are *weakly* increasing or decreasing in z where $B(z), \varphi^{*d}(z)$ and $\varphi^{*x}(z)$ are flat for $z \in [\bar{z}_1, \bar{z}_2]$ (see Figure 2). As a result, $R_{12}(z)$ is *weakly* increasing in z and $R_{21}(z)$ is *weakly* decreasing in z, and $NEXP(z) = R_{12}(z) - R_{21}(z)$ is *weakly* increasing in z where $NEXP(z)$ is flat for $z \in [\bar{z}_1, \bar{z}_2]$.

When embedding the sectoral equilibrium into general equilibrium, the similar argument with the zero-gravity case applies to the general case with the variable trade cost. In particular, as shown in Section 3.3, the corresponding equations to (B.1) and (B.2) are

$$\begin{aligned} \omega &= \frac{\kappa_2(\bar{z}_2) - \lambda_2(\bar{z}_2)}{\kappa_1(\bar{z}_1) - \lambda_1(\bar{z}_1)} \left(\frac{\overline{L}_2}{\overline{L}_1} \right), \\ \omega &= \tau_{21}\mu(\bar{z}_1), \\ \omega &= \mu(\bar{z}_2)/\tau_{12}. \end{aligned}$$

These three equations jointly determine the equilibrium variables $\{\bar{z}_1, \bar{z}_2, \omega\}$, where $\bar{z}_1, \bar{z}_2$ define the equilibrium relative wage ω, and the interval sectors $z \in [\bar{z}_1, \bar{z}_2]$ are special in that net exports are zero in intra-industry trade. As before, the logic is borrowed from DFS (1977, p.829-830); please refer to equations (19') and (21) in their paper. If we assume perfect competition, $\kappa_i(\bar{z}_i) = 1$ and the equilibrium characterized by the three equations is exactly the same as DFS (1977) as discussed in Section 5.2.

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