Competitive Search with Moving Costs

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Abstract

We developed a competitive search model involving multiple regions, geographically mobile workers, and moving costs. Equilibrium mobility patterns were analyzed and characterized, and the results indicate that shocks to a particular region, such as a productivity shock, can propagate to other regions through workers' mobility. Moreover, equilibrium mobility patterns are not efficient because of the existence of moving costs, implying that they affect social welfare because not only are they costs but also they distort equilibrium allocation. By calibrating our framework to Japanese regional data, we demonstrate the extent to which changes in moving costs affect unemployment and social welfare.

Keywords: Geographical mobility of workers, Competitive job search, Moving costs, Efficiency

JEL classification: J61, J64, R13, R23

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1 Introduction

This study analyzes the possible impacts of inter-regional moving costs on local and national labor markets as well as social welfare. As observed in many countries, considerable labor mobility exists within a country, and such migration has been shown to be sensitive to local labor market conditions.\(^1\) We then naturally expect that migration should eventually eliminate regional differences in labor market conditions, such as those in wages and unemployment rates. However, contrary to this expectation, we observe persistent and significant differences in such labor market outcomes. For instance, Lkhagvasuren \(^8\) showed that the magnitude of cross-state unemployment differences is approximately identical with the cyclical variation occurring in the national unemployment rate.\(^2\)

Migration sensitivity to labor market conditions and persistent regional differences in labor market outcomes imply that regional labor markets are only imperfectly integrated. This attribute can be primarily ascribed to the existence of moving costs in general. Such moving costs include those of moving, selling, and finding houses, which may depend on transportation and communication technologies, and those of adjusting to a new environment and re-constructing social networks, and those related to job turnover, which depends on institution and regulations affecting labor markets, such as mutual recognition of professional degrees among different regions and occupational licenser requirements. Thus, these costs can constitute a substantial barrier to labor mobility. This paper aims to qualify and quantify the effects of moving costs on local and national labor markets.\(^3\)

We develop a competitive search model involving multiple regions and moving costs. As modeled in Acemoglu and Shimer \(^1\) \(^2\) and Moen \(^14\), firms post wages when opening their vacancies, and job searches are directed.\(^4\) Search is off-the-job and only unemployed workers can move between regions. Although job seekers can search for jobs (i.e., can access information on vacancies) both within and outside their places of residence, a new job in a region different from where they live will require moving, which incurs costs.

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\(^1\) For earlier contributions on this issue, see Blanchard and Katz \(^4\), Borjas et al \(^5\), and Topel \(^24\) among others. Recent contributions include Hatton and Tani \(^7\), Kennan and Walker \(^9\), and Rabe and Taylor \(^21\).

\(^2\) The same holds true for Japanese prefectures. A population census of Japan reports prefectural unemployment rates every five years. The coefficients of variation for cross-prefecture unemployment in 1985, 1995, and 2005 are approximately 0.35, 0.31, and 0.23, respectively, while that of time-series unemployment from 1985 to 2005 is 0.27.

\(^3\) In the international context, the degree of labor market integration also depends on the formation of political and economic unions such as the European Union. Although our arguments in this study base on migration within a nation, our framework is applicable to such unions as well.

\(^4\) See, among others, Rogerson et al \(^22\) for recent developments in the literature on job search models that include a competitive search model.
from the initial places of residence incurs moving costs.

Our analysis first examines the qualitative effects of moving costs on migration patterns. Intriguingly, we find that a change in moving costs results in spillover effects through migration responses, resulting in a counter-intuitive outcome: Better access from one region to another, which is characterized as having better economic conditions such as higher productivity, may negatively affect the source region’s unemployment rate. It increases job settlements from the source region to the better region, but it decreases job settlements to other regions besides the better one, which may result in a higher unemployment rate in the source region. Hence, improved access between two regions may widen the difference in labor market conditions between the two regions.

Second, equilibrium of the model is shown to be inefficient: A migration flow is inefficiently small when the destination (resp. source) region offers a relatively high (resp. low) asset value of an unemployed worker. A high asset value of an unemployed worker in the destination region implies that in-migration of job seekers to the region is socially beneficial. However, firms in the destination region ignore such migration benefits when opening their vacancies, which result in insufficient job settlements and migration. When the asset value of an unemployed worker in the source region is low, out-migration of job seekers from the region is socially beneficial. Again, firms in the destination region ignore such benefits when opening vacancies, resulting in insufficient migration. Thus, migration costs reduces social welfare not only because they decrease social surplus when migration occurs but also because they distorts the equilibrium allocation.

Furthermore, we demonstrate a method to quantify losses from moving costs. We calibrate our framework to Japanese prefectural data and then consider counterfactual experiments in which moving costs decrease. From the counterfactual analyses, we show that (i) a 1% decrease in moving costs decreases the national unemployment rate by 0.5 – 2% and increases the social welfare by 0.1 – 0.2%, and (ii) removal of moving costs has a significant impact on unemployment and welfare, which are comparable to those caused by a 30% productivity increase.

Several previous studies have investigated the role of migration in determining labor market outcomes. Lkhagvasuren [8] extended the island model of Lucas and Prescott [10] by introducing job search frictions in each island as modeled in the Mortensen-Pissarides model. In Lkhagvasuren’s model, a worker’s productivity is subject to a shock specific to the worker-location match. Therefore, a job seeker hit by a negative productivity shock may have incentive to move to other islands even if her/his current location offers a high probability of finding a job, leading

\footnote{For details on the Mortensen-Pissarides model, see, among others, Mortensen and Pissarides [16] and Pissarides [20].}
to the possibility of simultaneous in- and out-migration. Using this framework, he showed that regional differences in the unemployment rate may persist, regardless of high labor mobility between regions, and that labor mobility is procyclical. Although our model is similar to that developed in Lkhagvasuren [8] in the sense that both exhibit labor mobility and regional unemployment differences simultaneously, they are different in focus: We uncover the possible role of moving costs in determining migration patterns, whereas Lkhagvasuren [8] examined the role of productivity shocks.

In the immigration literature, Ortega [18] developed a two-country job search model in which workers could decide where to search for jobs. Workers need to incur moving costs if they search for jobs abroad. Differences in the job separation rate may incentivize workers in the high job separation country to migrate to the low job separation country. Because wages are determined by Nash bargaining, firms expect to make low wage payments to immigrants who have high search costs, thereby incentivizing them to increase vacancies. Thus, workers’ incentives to migrate and firms’ incentives to increase vacancies reinforce each other, resulting in Pareto-ranked multiple equilibria. In contrast, we employ a competitive search model in which wages are posted and searches are directed. This modeling strategy results in a unique equilibrium, enabling us to focus on the analysis on geographical mobility patterns.

The following studies highlight the positive effects of decreases in moving costs between regions on human capital accumulation and specialization. Miyagiwa [12], in the context of immigration between countries, showed that if economies of scale exist in education, skilled worker migration benefits the host region by increasing the skilled labor ratio, whereas it negatively influences the source region by discouraging skill formation. In such an environment, lower moving costs induce people in the host region to invest more in human capital whereas it discourages people in the source region from investing in it. Wildasin [25] presented a multi-region model in which human capital investment increases specialization but exposes skilled workers to region specific earnings risk. Wildasin [25] then showed that the skilled workers’ mobility across regions mitigates such risk and improves efficiency, and examined how the ways of financing investments, such as local taxes, affect efficiency. However, the simple treatment of migration decisions in these studies fail to provide a substantive and detailed analysis of migration patterns and their efficiency properties, which forms the focus of this paper.

Our quantitative analysis is also related to recent studies such as Bayer and Juessen [3], Coen-Pirani [6], and Kennan and Walker [9]. Bayer and Juessen [3] and Kennan and Walker [9] estimated partial equilibrium models in which worker’s moving decisions are motivated by idiosyncratic and location-specific factors. Bayer and Juessen [3], in particular, share common
aspects with our quantitative analysis: They obtained a moving cost estimate, which is approximately two-thirds the average annual household income, and considered a counterfactual experiment in which moving costs are set to zero. They focus on the effects on moving flows: Moving cost elimination increases the U.S. interstate migration rate from 3.7% to 12.6% in the baseline case. In contrast, we focus on the general equilibrium effects of moving costs, which is in common with Coen-Pirani [6]. Coen-Pirani [6] developed a general equilibrium model of migration based on the island model of Lucas and Prescott [10] to show that the model can replicate several stylized facts regarding moving patterns in the United States. In contrast, we investigate the quantitative impacts of moving costs on unemployment and welfare.

The remainder of this paper is organized as follows. Section 2 presents the basic setups. Section 3 analyzes the equilibrium geographical mobility patterns. Section 4 presents the efficiency property of equilibrium. Section 5 quantifies the effects of moving costs. Section 6 concludes.

2 General settings

Consider $H$ regions (region 1, 2, ..., $H$) in which there is a continuum of risk-neutral workers of size $N$. Workers are either employed or unemployed. While employed, workers can not move between regions. In contrast, unemployed workers can move but must bear moving costs $t_{ij}$. They can seek employment opportunities both beyond and within their region of residence, however, they incur moving costs $t_{ij}$ in case they become employed outside their region of residence.\footnote{We later show that an unemployed worker may move only when she/he gets employed. While being unemployed, she/he has no incentive to move.} We employ the following standard assumptions regarding moving costs: (i) Finding a job in the current region of residence incurs no moving cost $t_{ii} = 0$, (ii) moving costs are symmetric, $t_{ij} = t_{ji}$, and (iii) moving costs satisfy the triangle inequality, $t_{ij} \leq t_{ih} + t_{hj}$. Such moving costs include the costs of selling and buying/renting a house and any psychological costs incurred in renewing social networks. This study primarily analyzes the impacts of the existence of and changes in such moving costs on labor market outcomes and welfare.\footnote{Alternatively, we can assume that workers can only search for local employment opportunities, referred to as the "move then search" regime. In our framework, workers can move between regions while searching for jobs, so this regime does apply. In addition, workers can search for jobs outside of their region of residence, implying that the "search then move" regime is also possible. However, as shown later, only the "search then move" regime emerges in equilibrium. See Molho [15] for a comparison of equilibrium unemployment rates between the "move then search" regime and the "search then move" regime.}

\footnote{One may suspect that migration costs are different across people. Under a competitive search framework, such heterogeneity does not alter our results qualitatively because of the block recursivity that we will refer to in the next section.}
We assume that only unemployed workers seek employment opportunities. Once a worker is employed by a firm, the firm-worker pair in region $i$ produces output $y_i$, where without loss of generality, we assume that a region with a larger number $i$ is associated with higher productivity, $y_{i+1} \geq y_i$. A worker exits the economy according to a Poisson process with rate $\delta (> 0)$, who is replaced by a new worker thereby keeping the total population size, $N$, constant. A new worker enters the economy as an unemployed worker in the same region as her/his parent. The following figure summarizes the model’s structure:

[Figure 1 around here]

2.1 Matching framework

Because arguments are based on a competitive search model, the overall job search market is divided into sub-markets, each of which is characterized by a wage rate, and hence, by a geographical mobility pattern, known as the "block recursivity" (Menzio and Shi [13]; Shi [23]). Job matches accompanied by migration from region $i$ to region $j$ are generated by a Poisson process with rate $M_{ij} = \mu_j m(u_{ij}, v_{ij})$, where $u_{ij}$ and $v_{ij}$ are the number of unemployed workers who seek employment in region $j$ while living in region $i$, and the number of vacancies directed at such job searchers, respectively. This sub-market is called "sub-market $ij". \mu_j$ represents location-specific matching efficiency. $\mu_j m(\cdot, \cdot)$ is the matching function defined on $\mathbb{R}_+ \times \mathbb{R}_+$, and assumed to be strictly increasing in both arguments, twice differentiable, strictly concave, and homogeneous of degree one. Moreover, we assume that $\mu_j m(\cdot, \cdot)$ satisfies $0 \leq M_{ij} \leq \min[u_{ij}, v_{ij}]$, $\mu_j m(u_{ij}, 0) = \mu_j m(0, v_{ij}) = 0$ and the Inada condition for both arguments.

In each sub-market, worker-job matching occurs at the rate of $p_{ij} = p(\theta_{ij}) = M_{ij}/u_{ij} = \mu_j m(1, \theta_{ij})$ for a job seeker, and $q_{ij} = q(\theta_{ij}) = M_{ij}/v_{ij} = \mu_j m(1/\theta_{ij}, 1)$ for a firm seeking to fill a vacancy. $\theta_{ij}$ is the measure of labor market tightness in sub-market $ij$ defined as $\theta_{ij} = v_{ij}/u_{ij}$. From the assumptions regarding $\mu_j m(\cdot, \cdot)$, we obtain that $p_{ij} u_{ij} = q_{ij} v_{ij}$, $dp_{ij}/d\theta_{ij} > 0$ and $dq_{ij}/d\theta_{ij} < 0$ for any $\theta_{ij} \in (0, +\infty)$. We can also see that $\lim_{\theta_{ij} \to 0} p_{ij} = 0$, $\lim_{\theta_{ij} \to \infty} p_{ij} = \infty$, $\lim_{\theta_{ij} \to 0} q_{ij} = \infty$, and $\lim_{\theta_{ij} \to \infty} q_{ij} = 0$. Moreover, we assume that the elasticity of the firm’s contact rate with respect to market tightness, $\eta_{ij} \equiv -\theta_{ij}/q_{ij} dp_{ij}/d\theta_{ij} = 1 - (\theta_{ij}/p_{ij}) dp_{ij}/d\theta_{ij}$, is constant and common across all submarkets ($\eta_{ij} = \eta, \forall i, j$).9

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9This assumption leads to a set of functions that include the Cobb-Douglass function, which is standard in the literature on theoretical and empirical search models (See Petrongolo and Pissarides [19]).
2.2 Asset value functions

Let $\rho > 0$ denote the discount rate and define $r = \delta + \rho$. When locating region $i$, the asset value functions for an employed worker, $W_i(w)$, an unemployed worker, $U_i$, a firm with a filled position, $J_i(w)$, a firm with a vacancy, $V_{ji}$, are given by (1)-(4), respectively.

\begin{align*}
    rW_i(w) &= w, \quad (1) \\
    rJ_i(w) &= y_i - w, \quad (2) \\
    rU_i &= b + \sum_{h=1}^{H} p_{ih} (W_h(w_{ih}) - U_i - t_{ih}), \quad (3) \\
    rV_{ji} &= -k + q_{ji} (J_i(w_{ji}) - V_{ji}), \quad (4)
\end{align*}

where $b$ and $k$ represent the flow utility of an unemployed worker, including the value of leisure and unemployment benefits, and the cost of posting a vacancy, respectively. We assume that $y_i > b, \forall i$. Moreover, subscript $i$ represents the region where agents (i.e., workers and firms) are located, and subscripts $h$ and $j$ represent the region in which unemployed workers seek employment and firms post vacancies, respectively. Note that the block recursivity divides the labor market into sub-markets, and each sub-market $ij$ is characterized by the combination of the place of residence, $i$, and the place of job search, $j$. Wage rate may differ between sub-markets within a region and hence the asset values $W_i(w)$ and $J_i(w)$ may also differ: We may observe that $w_{ji} \neq w_{j'i}$, $W_i(w_{ji}) \neq W_i(w_{j'i})$, and $J_i(w_{ji}) \neq J_i(w_{j'i})$ ($j \neq j'$). In (3), the second term represents the sum of expected gains in the asset values from finding jobs net of moving costs. Thus, moving costs are described as reductions in asset values.\(^{10}\) In (4), $V_{ji}$ depends on the firm’s location, $i$, and the location of posting a vacancy, $j$.

2.3 Equilibrium

Because this is a competitive search model, that is, firms post wages and searches are directed, the job search market in each region is divided into sub-markets according to the region’s individual migration pattern. An unemployed worker in region $i$ chooses sub-markets to search for jobs in order to maximize her/his asset value. In doing so, she/he can search for jobs in multiple sub-markets.\(^{11}\) In equilibrium, the asset value in region $i$ takes the same value $U_i$ regardless of the submarkets that she/he choose.

\(^{10}\) Alternatively, we can assume that a mover need to pay the flow costs of moving until she/he exits the economy. This alteration does not change any of our results.

\(^{11}\) From the assumption of the Poisson process, the probability that an unemployed worker obtains multiple offers at one time is zero.
A firm providing a vacancy determines the wage to post while anticipating the market response: It regards $U_i$ as given and thus takes the relationship between $w_{ij}$ and $\theta_{ij}$, which is determined by (3) into consideration. The firm’s decision is described as follows:

$$\max_{w_{ij}, \theta_{ij}} V_{ij} \quad \text{s.t. (3), where } U_i \text{ is treated as given.}$$

Using (1), (2), and (4), this optimization is written as

$$\max_{w_{ij}, \theta_{ij}} -k + q_{ij} \left( \frac{y_j - w_{ij}}{r} - V_{ij} \right) \quad \text{s.t. } rU_i = b + \sum_{h=1}^{H} p_{ih} \left( \frac{w_{ih}}{r} - U_i - t_{ih} \right), \text{ where } U_i \text{ is treated as given.}$$

The related first-order conditions are given by

$$0 = -q_{ij} - \lambda p_{ij},$$

$$0 = \frac{d q_{ij}}{d \theta_{ij}} \left( \frac{y_j - w_{ij}}{r} - V_{ij} \right) - \lambda \frac{d p_{ij}}{d \theta_{ij}} \left( \frac{w_{ij}}{r} - U_i - t_{ij} \right).$$

We assume free entry and exit of firms, which drives the asset value of posting a vacancy to zero: $V_{ij} = 0$.

The first-order conditions then yield the wage rate posted by a firm in sub-market $ij$:

$$w_{ij} = \eta y_j + (1 - \eta) r (U_i + t_{ij}). \quad (6)$$

Thus, for a given market tightness, the wage rate rises as the productivity, $y_j$, asset value of an unemployed worker, $U_i$, and moving cost, $t_{ij}$, increase. A higher $y_j$ enables a firm to offer a higher wage rate whereas a higher $U_i$ or $t_{ij}$ requires a firm to pay higher compensation in order to attract job applicants. Plugging (6) into the zero-profit condition, $V_{ij} = 0$, we obtain

$$r k = q_{ij} (1 - \eta) (y_j - r U_i - r t_{ij}). \quad (7)$$

Of course, there may be some region $j$ where $y_j - r t_{ij} - r U_i \leq 0$. In such a case, no vacancy is posted in sub-market $ij$ and $p_{ij} = 0$.

We focus on the steady state. Although total population remains constant, the population in each region may change over time. Here, the steady state requires that the unemployment rate in each region, $u_n_i$, is constant. The dynamics of the unemployment rate are given by $d u_n_i / d \tau = \delta - u_n_i \tau (\delta + \sum_{h=1}^{H} p_{ih}\tau)$, where $\tau$ represents time. This yields the steady state level of unemployment rate as

$$u_n_i = \frac{\delta}{\delta + \sum_{h=1}^{H} p_{ih}}. \quad (8)$$

Once the asset value of an unemployed worker, $U_i$, is given, other endogenous variables are well determined: Equation (7) uniquely determines the market tightness, $\theta_{ij}$. Then, (6) and (8)
yield the wage and unemployment rates, $w_{ij}$ and $u_{ni}$, respectively. Asset values other than $U_{i}$ are determined accordingly.

The wage equation (6) is rewritten as

$$(1 - \eta)(y_{j} - rt_{ij} - rU_{i}) = y_{j} - w_{ij}. \tag{9}$$

Using this, we can rearrange the zero-profit condition (7) as

$$rk = q_{ij}(y_{j} - w_{ij}) \tag{10}$$

Substituting (1), (10), and $q_{ij} = p_{ij}/\theta_{ij}$ into (3), we can rewrite the asset value of an unemployed worker, (3), as

$$rU_{i} = b + \sum_{h=1}^{H} \left[ p_{ih} \left( \frac{y_{h}}{r} - U_{i} - t_{ih} \right) - k\theta_{ih} \right]. \tag{11}$$

Equations (9) and (10) imply that $\theta_{ij}$ is a function of $U_{i}$ for all $j$. Thus, (11) implicitly determines $U_{i}$. The following proposition establishes the existence and uniqueness of the solution:

**Proposition 1** The steady state equilibrium exists and is unique.

**Proof.** See Appendix A. ■

### 3 Equilibrium properties

Here, we summarize several equilibrium properties that are worth specifying, thereby focusing on interior solutions though we can obtain qualitatively the same results as those obtained below even if we allow corner solutions.

#### 3.1 Migration patterns

In equilibrium, we can confirm that unemployed workers, while searching for a job, do not have incentive to migrate:

**Proposition 2** The difference between the asset value of an unemployed worker in region $i$ and that in region $j$ is smaller than the costs of moving between the two regions:

$$t_{ij} \geq |U_{i} - U_{j}|, \quad \forall i, j \in H,$$

where the equality holds true if and only if $t_{ij} = 0.$
**Proof.** See Appendix B.

Thus, we know that migration occurs only when unemployed workers find jobs. Moreover, this proposition implies that if there are no moving costs \((t_{ij} = 0, \forall i, j)\), the asset value of an unemployed worker is the same across regions. From (9) and (10), \(\theta_{ij}\) and \(p_{ij}\) are also the same across regions, which, combined with (8), results in equalization of regional unemployment rates.

The probability of such migration depends on the difference between the social gains from making a match, \(y_j - rt_{ij} - rU_i\), which is the output of a match minus the value of an unemployed worker and the related moving costs, as well as the matching efficiency of the destination region, \(\mu_j\):

**Proposition 3** The job finding rate associated with migration from region \(i\) to region \(j\) increases as the social gains from a match and the location specific matching efficiency increase:

\[
p_{ij} > p_{i'j'} \quad \text{if } y_j - rt_{ij} - rU_i > y_{j'} - rt_{i'j'} - rU_{i'} \quad \text{and } \mu_j \geq \mu_{j'},
\]

**Proof.** See Appendix C.

This proposition has several implications. First, a particular destination attracts more people from a region with low moving costs and a low asset value of an unemployed worker (i.e., in destination \(j\), the job finding rate from region \(i\), \(p_{ij}\), is higher than that from region \(i'\), \(p_{i'j}\), if \(t_{i'j} + U_{i'} > t_{ij} + U_i\)). Second, a destination with low moving costs, high productivity and a high matching efficiency attracts more employed workers from a particular region (i.e., for a job seeker in region \(i\), the job finding rate in region \(j\), \(p_{ij}\), is higher than that in region \(j'\), \(p_{ij'}\), if \(y_j - rt_{ij} > y_{j'} - rt_{ij'}\) and \(\mu_j > \mu_{j'}\)). Finally, the net migration from region \(i\) to \(j\) is positive when the productivity, asset value of the unemployed worker, and matching efficiency are higher in region \(j\) than in region \(i\) (i.e., \(p_{ij} > p_{ji}\) if \(y_j + rU_j > y_i + rU_i\) and \(\mu_j > \mu_i\)).

### 3.2 Spillover effects of shocks through migration

Next, we examine the effects of various shocks on local labor markets and show that a shock to a particular region spills over to other regions through migration. We start by considering a decrease in moving costs \(t_{ij}\).

**Proposition 4** A decrease in moving costs from region \(i\) to region \(j\), \(t_{ij}\), (i) increases the asset value of an unemployed worker in region \(i\), \(U_i\), (ii) increases the job finding rate from region \(i\) to region \(j\), \(p_{ij}\), but decreases that from region \(i\) to region \(j' \neq j\), \(p_{ij'}\) \((j' \neq j)\), (iii) decreases

\(^{12}\text{This automatically implies that } t_{ji} \text{ also decreases. Hence, such a change affects region } j \text{ and related sub-markets in a similar fashion to those explained in Proposition 4. However, by block recursivity of the competitive search model, it does not affect other sub-markets.}\)}}
the wage rate when an unemployed worker in region $i$ finds a job in region $j$, $w_{ij}$, but increases the wage rate when an unemployed worker in region $i$ finds a job in other regions, $w_{ij'}$, and (iv) has ambiguous effects on the unemployment rate in region $i$, $u_i$.

**Proof.** See Appendix D. ■

A decrease in moving costs $t_{ij}$ increases job searchers’ gains in region $i$ from a job match in region $j$, increasing their asset value, $U_i$. From (7), we can see that a decrease in $t_{ij}$ directly increases $\theta_{ij}$ (a direct effect) and influences $\theta_{ij}$ through changes in $U_i$ (an indirect effect). Although the direct effect positively influences $\theta_{ij}$ and increases $p_{ij}$ and the indirect effect has the opposite impact, the direct effect dominates the indirect effect in region $j$. In other regions, we observe no direct effect, implying that $p_{ij'}$ ($j' \neq j$) unambiguously declines. The wage rate $w_{ij}$ is lower for a lower $t_{ij}$ because firms are able to pay lower compensation in order to attract job seekers from region $i$ to region $j$, which in turn, implies that firms in other regions need to pay higher wages in order to attract workers from region $i$. Although a lower moving costs, $t_{ij}$, implies a higher job finding rate for unemployed workers in region $i$ who search for jobs in region $j$, $p_{ij}$, it leads to lower job finding rates for unemployed workers in region $i$ who search for jobs in other regions, $p_{ij'}$ ($j' \neq j$), through changes in $U_i$. The former effect lowers the unemployment rate in region $i$, $u_i$, whereas the latter effect raises it. When $y_j - rt_{ij}$ is sufficiently large, a change in $t_{ij}$ significantly affects $U_i$ and hence it becomes possible that the latter effect dominates the former. Put differently, improved access from region $i$ to a region with good job opportunities, i.e., a region with high $y_j$, may reduce job placement flows to other regions and increase the unemployment rate in region $i$. This is counter-intuitive since we normally expect that such a better access would lower the unemployment rate in the source region. The spillover effects on the job finding rate in other regions give rise to this intriguing result.

Moreover, due to the responses of migration flows, a productivity shock in a particular region spills over to other regions.

**Proposition 5** An increase in productivity in region $j$, $y_j$, (i) increases the asset value of an unemployed worker in region $i$ ($i \neq j$), $U_i$, (ii) increases the job finding rate for unemployed workers in region $i$ searching for jobs in region $j$, $p_{ij}$, but decreases that of unemployed workers in region $i$ searching for jobs in other regions, $p_{ij'}$ ($j' \neq j$), (iii) increases not only the wage rate when an unemployed worker in region $i$ finds a job in region $j$, $w_{ij}$, but also the wage rate when an unemployed worker in region $i$ finds a job in other regions, $w_{ij'}$, and (iv) has ambiguous effects on the unemployment rate in region $i$, $u_i$.

**Proof.** See Appendix D. ■
Productivity improvement in region $j$ increases the employment flows from all regions into region $j$, $p_{ij}, \forall i$, which increases the asset values of an unemployed worker in these worker-exporting regions, $U_i$. However, it decreases the employment flows to other regions, i.e., region $j'$, $(i \neq j, j' \neq j)$, $p_{ij'}$. In contrast, it increases the wage rate in all regions while such an effect is most prominent in the region where the productivity shock arises. With higher productivity in region $j$, firms can afford to post higher wages, forcing firms in other regions to pay higher wages in order to attract workers. The effect on the unemployment rate, $u_{ni}$, is again ambiguous because of the opposing effects of changes in $p_{ij}$ and $p_{ij'}$ on $u_{ni}$. This finding is in contrast to the results of standard job search models with no moving costs, wherein a positive productivity shock always lowers the unemployment rate (see Rogerson et al [22], for instance).

### 4 Inefficiencies arising from moving costs

Now, we characterize the efficiency of equilibrium. We use the social surplus, $S$, as the efficiency criterion, which is standard in job search models (See Pissarides [20]). $S$ is the sum of total output and flow utility of unemployed workers minus the costs of posting vacancies and migration:

$$S \equiv \int_{0}^{\infty} \sum_{i=1}^{H} \left[ y_i (N_{it} - u_{it}) + b u_{it} - u_{it} \sum_{h=1}^{H} (k \theta_{ih} + p_{ih} t_{ih}) \right] e^{-\rho \tau} d\tau \quad (12)$$

We start by describing the social planner’s problem. The social planner maximizes the social surplus subject to the laws of motion of regional population and unemployment:

$$\max_{\theta_{ij}, N_{it}, u_{it}} S \quad (13)$$

s.t. $\frac{dN_{it}}{d\tau} = \sum_{h=1}^{H} u_{h\tau} p_{ih\tau} - u_{it} \sum_{h=1}^{H} p_{ih\tau}$

and $\frac{du_{it}}{d\tau} = \delta N_{it} - u_{it} \left( \sum_{h=1}^{H} p_{ih\tau} + \delta \right)$

where $\tau$ represents time. Changes in regional population arise from social changes (differences between in-migration $\sum u_{h\tau} p_{ih\tau}$ and out-migration $u_{it} \sum p_{ih\tau}$). Inflows to the unemployment pool are newcomers to the economy and outflows are those who become employed. We relegate the derivation of the optimal conditions to Appendix E. After evaluating the first-order conditions for the social planner’s maximization at the steady state, we obtain the following proposition:
Proposition 6 Define \( D_{ij} \) as

\[
D_{ij} = \frac{(1 - \eta) q_{ij}}{r + \eta \sum_{h=1}^{H} \sum_{j=1}^{H} p_{ih}} \left[ r (U_i - U_j) + \eta \sum_{h=1}^{H} p_{ih} (U_{ih} - U_{ij}) \right].
\] (14)

Equilibrium market tightness \( \theta_{ij} \) is socially optimal if and only if \( D_{ij} = 0 \) in equilibrium. If and only if \( D_{ij} > 0 \), \( \theta_{ij} \) is greater than the optimal tightness. The opposite holds true if and only if \( D_{ij} < 0 \).

Proof. See Appendix E. □

The equilibrium market tightness \( \theta_{ij} \) and the job finding rate \( p_{ij} \) are insufficient when the destination region has a relatively high asset value of an unemployed worker, \( U_j \), or when the source region has a relatively low \( U_i \). This can be explained as follows. Because of the existence of moving costs, job searchers’ arbitrage works only imperfectly and regional differences exist in the asset value of an unemployed worker. In such a scenario, movements of job searchers from a region with a low \( U_i \) to a region with a high \( U_j \) are socially beneficial. Job creation in region \( j \) induces job searchers to move to region \( j \), and hence, social welfare is increased when region \( j \) has a high \( U_j \). However, firms ignore such benefits of improving the distribution of job searchers when opening their vacancies. Put differently, a job creation accompanies a positive externality, resulting in insufficient market tightness. For this reason, equilibrium in the presence of of moving costs fails to attain the socially optimum outcome.

In case of identical moving costs for all migration patterns \((t_{ij} = t, i \neq j, \forall i, j)\), migration from any region \( i \) to region \( H \), where productivity is the highest, is always insufficient and that to region 1, where productivity is the lowest, is always excessive, and a threshold region \( \hat{j}(i) \) exists for which flows to region \( j > \hat{j}(i) \) are too small and those to region \( j \leq \hat{j}(i) \) are too large.\(^{13} \)

Moreover, Proposition 2 implies that the absence of migration costs \((t_{ij} = 0, \forall i, j)\) implies that \( U_i = U_j \forall i, j \) and hence \( D_{ij} = 0 \):

Corollary 7 If migration costs do not exist, i.e., \( t_{ij} = 0, \forall i, j \), equilibrium is socially optimal.

\(^{13}\)We can prove the result as follows. We readily know that \( U_i = U_j \) if \( y_i = y_j \). Moreover, (16) proves that

\[
\frac{dU_i}{dy_j} = \frac{dU_j}{dy_j} = \frac{p_{ij}}{r + \sum_{h=1}^{H} p_{ih}} - \frac{p_{ij}}{r + \sum_{h=1}^{H} p_{ih}}.
\]

Proposition 3 implies that \( p_{ii} = p_{ij} > p_{ij} = p_{ji} \) and \( p_{ij} = p_{ji} \) if \( y_i = y_j \), which lead to

\[
\frac{dU_i}{dy_j} - \frac{dU_j}{dy_j} \bigg|_{y_j = y_i} > 0.
\]

Hence, the continuity of \( U_i \) with respect to \( y_j, \forall i, j \), proves that \( U_i > U_j \) if \( y_i > y_j \). From the assumption that \( y_i > \cdots > y_{i+1} > y_i > \cdots > y_1 \), we know that \( U_H > \cdots > U_{i+1} > U_i > \cdots > U_1 \). From (14), we readily know that \( D_{ij} < 0 \) and \( D_{ij} > 0 \) for all \( i \), and there exists a threshold region \( \hat{j}(i) \) for which \( D_{ij} < 0 \) for \( j < \hat{j}(i) \) and \( D_{ij} > 0 \) for \( j \leq \hat{j}(i) \).
In the absence of moving costs, our framework becomes a standard competitive search model, of which equilibrium is socially optimal (see Moen [14] and Rogerson et al [22], among others). Thus, moving costs reduce the social surplus not only because they reduce the movers’ asset values but also because they distort the equilibrium.

5 Quantitative analysis

In this section, we demonstrate how our framework can be used to quantify overall losses incurred from moving costs. This exercise also serves to reveal the impacts of regional integration on the economy. Here, we calibrate our model to Japanese prefectural data, and provide counterfactual analysis regarding changes in moving costs.

We use data on Japanese prefectures for 2000-2009. In calibrating our model, we focus on the long-run characteristics of regional labor markets in Japan to ensure that the calibration is consistent with the steady state analysis given in the previous section. More concretely, we focus on the level and regional variation of the unemployment rate averaged over these periods, which are represented in the following figure.

[Figure 2 around here]

The overall unemployment rate of these 46 prefectures averaged over 2000-2009, $un_N$, is 0.0455, and the unemployment rate of each prefecture ranges from 0.0305 (Fukui prefecture) to 0.064 (Osaka prefecture) (Population Census, Ministry of Internal Affairs and Communications). The degree of dispersion can be measured by the coefficient of variation: $CV = (1/\overline{un})\sqrt{(1/46)\sum_{i=1}^{46}(un_i - \overline{un})^2}$ where $\overline{un}$ is the average of regional unemployment rates. The $CV$ for the regional unemployment rate averaged over 2000-2009 is 0.182, which is somewhat lower than that in the United States. We will examine the extent to which moving costs affect the overall unemployment rate, the dispersion of regional unemployment rates, and welfare.

---

14 We excluded Okinawa prefecture and used data covering the remaining 46 prefectures because Okinawa prefecture comprises islands and is located extremely far from other prefectures, making it an outlier. In fact, its distance from its closest neighboring prefecture is around 650km whereas in most cases, the distance between two neighboring prefectures is less than 100km. Note here that the distance between prefectures is measured by the distance between the locations of prefectural governments. This elimination reduces the coefficient of variation regarding regional unemployment. For instance, the figure for the year 2000 decreases from 0.232 to 0.172.

15 Lkhagyasuren [8] reported that between January 1976 and May 2011, the coefficient of variation of cross-state unemployment rates in the United States ranges from 0.175 to 0.346 with an average of 0.237.
5.1 Calibration

In the following analysis, we normalize the total population, \( N \), to one. The values of the job separation rate, \( \delta \), the regional output per capita, \( y_i \), and the distance between regions, \( z_{ij} \), are taken from the Japanese data: \( \delta \) is set to 0.16, which is the annual job separation rate in Japan averaged over the years 2000-2009 (Survey on Employment Trends, Ministry of Health, Labour and Welfare). We employ the per capita gross prefectural domestic product (in million yen, Prefectural Accounts, Department of National Accounts, Cabinet Office) as \( y_i \). We measure \( z_{ij} \) as the distance (in 100km) between prefectural governments (which was taken on February 20, 2013 from http://www.gsi.go.jp/KOKUJOYOHO/kenchokan.html, Geographical Information Authority of Japan). We normalize the flow utility of an unemployed worker, \( b \), too on one.

We set the value of the discount rate, \( \rho \), to 0.0151, which comes from the average annual interest rate of Japanese 10-year national bond during 2000-2009 (which was taken on February 20, 2013 from http://www.mof.go.jp/jgbs/reference/interest_rate/data/jgbcm_2000-2009.csv, Ministry of Finance). In existing studies such as Coen-Pirani [6], Lkhagvasuren [8], and Kennan and Walker [9], this value is set to 0.04−0.05. We will verify the robustness of our results against a higher value of \( \rho \) (\( \rho = 0.05 \)).

We specify moving costs, \( t_{ij} \), as a linear function of the distance between prefectures \( i \) and \( j \), that is, \( t_{ij} = tz_{ij} \), where \( z_{ij} \) is the distance between regions and \( t \) is a positive constant. We will verify the robustness of our results against a different functional form for moving cost.

In the following quantitative analysis, we employ a Cobb-Douglas form of the matching function, given by

\[
\mu_j m(u_{ij}, v_{ij}) = \mu_j u_{ij}^{\eta} v_{ij}^{1-\eta},
\]

where \( \mu_j \) and \( \eta \) are constants satisfying that \( \mu_j > 0 \) and \( 0 < \eta < 1 \). As surveyed by Petrongolo and Pissarides [19], the Cobb-Douglas matching function is very standard in the literature of theoretical and empirical search models. We rearrange the matching function as

\[
\ln[\mu_j m(u_{ij}, v_{ij})/u_{ij}] = \ln[\mu_j] + (1-\eta) \ln[v_{ij}/u_{ij}] = \ln[\mu_j] + (1-\eta) \ln[\theta_{ij}],
\]

and estimate it by using the data on job applicants, job openings, and job placements (Monthly Report of Public Employment Security Statistics, Ministry of Health, Labour and Welfare). Note here that the job seekers’ job finding rate \( \mu_j m(u_{ij}, v_{ij})/u_{ij} \) is given by the number of job placements per job applicant, and the market tightness \( \theta_{ij} \) is given by the number of job openings per job applicant. Our spatial units are Japanese prefectures.16 The Monthly Report of Public Employment Security Statistics reports the number of active job applicants, active job openings, and job placements in every month. To eliminate seasonal volatility, we aggregate monthly data into annual data by taking averages. Because figures for job placements within prefectures are available, we can estimate the matching function

16Here, again, we eliminated Okinawa prefecture from our sample.
\[
\ln[\mu_j m(u_{ij}, v_{ij})/u_{jj}] = \ln[\mu_j] + (1-\eta) \ln[\theta_{jj}]
\]
to obtain \( \eta = 0.512 \) and \( \mu_j \). Details of the estimation are provided in Appendix F. In the benchmark case, we estimate the matching function using the fixed effects (FE) model. We will check for the possible bias arising from the endogeneity of \( \theta_{jj} \).

The remaining two parameters, the moving cost parameter, \( t \), and the cost of providing a vacancy, \( k \), are chosen by targeting the coefficient of variation of the unemployment rate and the overall unemployment rate, which results in \( t = 5.348 \) and \( k = 0.0196 \) in the benchmark case. Tables 1 and 2 summarize the parameter values and calibration results, respectively.

In Table 2, we also report the value of social surplus given by (12). Using this calibrated model, we will execute a counterfactual analysis regarding moving costs.

5.2 Counterfactual analysis

In order to uncover the quantitative impacts of moving costs, we consider the following two counterfactual analyses. First, we assume a 1% decrease in moving costs. Second, we assume that space does not matter at all, i.e., there is no moving cost (\( t = 0 \)). In both counterfactual analyses, we change the value of \( t \) while keeping other parameters fixed as described in Table 1, and run counterfactual simulations. We compare the resulting unemployment rate and welfare with the calibrated values shown in the previous sub-section. The results of our analyses show that decreasing moving costs has the following two effects. First, it directly results in decreases in losses from moving and increases the social surplus. Second, as shown in Proposition 6 and Corollary 7, it improves the efficiency of equilibrium by increasing inter-regional mobility, thereby improving the labor force’s distribution to enhance job creation efficiency and increase the social surplus.

The results of the counterfactual analyses are reported in Table 3.

Let us begin by examining the effects of a 1% decrease in moving costs. As shown in Table 3, in the benchmark case, the overall unemployment rate \( u_{N} \), drops by 0.0002 points from 0.0455 to 0.0453, which corresponds to a 0.43% decrease. Moreover, the coefficient of variation for the regional unemployment rate, \( CV \), decreases by 0.004 points from 0.182 to 0.178, which corresponds to a 2.19% decrease. The social surplus, \( S \), increases by 0.6 points from 362.0 to 362.6, which corresponds to a 0.16% increase. In order to gauge the magnitude of such impacts,
we consider an additional counterfactual in which productivity increases in all regions by 1%, and compare the changes in the two counterfactuals. Such a productivity change results in a 1.31% decrease in $un_N$, a 1.64% decrease in $CV$, and a 1.10% increase in $S$. This result shows that the effects of these changes on $CV$ are comparable. It can, therefore, be concluded that the effects of a 1% change in moving costs on $un_N$ and $S$ are quantitatively non-negligible although the effects of a 1% change in productivity.

In the second counterfactual analysis where local labor markets are spatially integrated and there are no moving costs ($t = 0$), the overall unemployment rate, $un_N$, drops by 0.0199 points from 0.0455 to 0.0256, which corresponds to a 43.7% decrease. As shown above, when $t = 0$, the unemployment rate is the same across regions, and hence, the coefficient of variation for the regional unemployment rate, $CV$, becomes zero. The social surplus, $S$, increases by 104.5 points from 362.0 to 466.5, which corresponds to a 28.8% increase. Such large welfare gains arise from the two effects explained above. In order to gauge the magnitude of such impacts, we consider an additional counterfactual in which productivity increases in all regions. By comparing these two counterfactuals, we can see that the effects of labor market integration are comparable to those of a 30% productivity increase. In Table 3, we provide the results of the counterfactual analysis, where output per capita in each region increases by 30%. Such productivity changes results in a 27.9% decrease in $un_N$, 30.7% decrease in $CV$, and a 33.6% increase in $S$. This result shows that losses from moving costs can be highly significant in a quantitative sense.

5.3 Robustness check

In this section, we discuss the robustness of our results against possible alternative settings.

Endogeneity bias in estimating the matching function

First, as is well known, market tightness, $\theta_{jj}$, i.e., the independent variable in estimating the matching function, $\ln[m(u_{jj}, v_{jj})/u_{jj}] = \ln[\mu_j] + (1 - \eta) \ln[\theta_{jj}]$, is also an endogenous variable in search models. Such endogeneity may bias the estimated coefficient obtained by the standard fixed effects (FE) model. In order to verify the robustness against endogeneity, we conducted a fixed effects instrumental variable (FEIV) estimation. We follow several recent studies that estimated the matching function in using lags of market tightness as instruments (see e.g., Yashiv [26]). As explained in Appendix F, we used the two-period and three-period lags of market tightness as instruments, and obtained 0.575 as the estimated value of $\eta$. Table 4 reports the parameter values in the robustness check.
In Table 4, the column labeled Robustness check (1) presents parameter values in the case where the matching function is estimated by the FEIV method. Because we obtained 0.512 in the benchmark case (i.e., under FE estimation), FEIV estimation yields a slightly higher value. Still, the main results are highly similar to those of the benchmark case. The results of calibration and counterfactual analysis are provided in the column labeled Robustness check (1) in Tables 2 and 3.

Here, a 1% decrease in moving costs lowers $u_n$ by 0.87% and $CV$ by 7.69%, and raises $S$ by 0.16% whereas a 1% increase in productivity lowers $u_n$ by 1.53% and $CV$ by 7.69%, and raises $S$ by 1.10%. In the absence of moving costs, the unemployment rate, $u_n$, would be lower by 42.8%, and social surplus, $S$, would be higher by 29.3%. These figures are again comparable to the effects of a 30% productivity increase, which has effects of lowering $u_n$ by 26.8% and the coefficient of variation of the regional unemployment rates, $CV$, by 31.3%, and of raising $S$ by 33.6%. These results confirm the findings of the benchmark case.

**Concave moving costs**

Second, we need to examine the degree to which our results depends on the specification of moving costs. In the benchmark case, we specified the moving costs as a linear function of the distance between regions, i.e., $t_{ij} = tz_{ij}$. However, the marginal moving costs may decline with distance because the cost difference between moving versus not moving 10km would be significant whereas that between moving 100km and moving 110km may not be substantial. In order to represent this possibility, we assume a concave function of the distance between regions as the moving costs. More specifically, we use a logarithmic function, i.e., $t_{ij} = t \ln[z_{ij}]$. Parameter values in this case are shown in the column of Robustness check (2) in Table 4. The calibration results and counterfactual analysis are presented in the column labeled Robustness check (2) in Tables 2 and 3.

In this case, a 1% decrease in moving costs lowers $u_n$ by 2.41% and $CV$ by 2.74%, and raises $S$ by 0.14% whereas a 1% increase in productivity lowers $u_n$ by 3.29% and $CV$ by 2.19%, and raises $S$ by 1.06%. If there were no moving cost, $u_n$ would be lower by 72.0%, and $S$ would be higher by 41.2%. In contrast, a 30% productivity increase lowers $u_n$ by 53.6% and $CV$ by 75.2%, and increases $S$ by 25.1%. Thus, we observe that moving costs exert an even more significant effect in this case than in the benchmark case.
5.3.1 Distance-neutral moving costs

Third, we consider existence of distance-neutral moving costs, i.e., those that do not change with the distance between regions. In order to avoid increasing the number of parameters, we assume that moving costs take the form as \( t_{ij} = t(z + z_{ij}) \), where \( z \) is the average distance between regions and is common for all \( i-j \) combinations. Parameter values in this case are shown in the column labeled Robustness check (3) in Table 4. The calibration results and counterfactual analysis are presented in the column labeled Robustness check (3) in Tables 2 and 3.

These tables indicate that a 1% decrease in moving costs lowers \( uN \) by 1.53% and \( CV \) by 8.24%, and raises \( S \) by 0.02% whereas a 1% increase in productivity lowers \( uN \) by 2.41% and \( CV \) by 8.79%, and raises \( S \) by 0.92%. If there were no moving cost, \( uN \) would be lower by 63.9%, and \( S \) would be higher by 40.9%. In contrast, a 30% productivity increase lowers \( uN \) by 38.0% and \( CV \) by 41.2%, and increases \( S \) by 32.5%. The results shown in this and previous subsections imply that our baseline results are robust against choices regarding moving costs.

Higher discount rate

Fourth, the value of discount rate, \( \rho \), that we use (\( \rho = 0.0151 \)) is lower than that used in existing studies such as Coen-Pirani [6], Lkhagvasuren [8], and Kennan and Walker [9] (\( \rho = 0.04 \) or 0.05). This is because the Japanese interest rate was at a unprecedentedly low level in the 2000s. In order to confirm that our results are not attributable to this low discount rate, we run a counterfactual simulation in which the discount rate is higher (\( \rho = 0.05 \)). Parameter values in this case are shown in the column labeled Robustness check (4) in Table 4. The results of calibration and counterfactual analysis are given in the column labeled Robustness check (4) in Tables 2 and 3.

In this case, a 1% decrease in moving costs lowers \( uN \) by 0.43% and \( CV \) by 1.64%, and raises \( S \) by 0.08% whereas a 1% increase in productivity lowers \( uN \) by 1.31% and \( CV \) by 1.09%, and raises \( S \) by 1.05%. If there were no moving cost, \( uN \) would be lower by 44.1%, and \( S \) would be higher by 23.3%. In contrast, a 30% productivity increase lowers \( uN \) by 27.9% and \( CV \) by 30.7%, and increases \( S \) by 33.2%. Again, these checks confirm the robustness of our results.

Difference in periods

Finally, we check whether our results change with the period of analysis. Accordingly, we divide the sample into two periods (2000-2004 and 2005-2009). As explained in Appendix F, we obtained \( \eta = 0.456 \) for 2000-2004 and \( \eta = 0.608 \) for 2005-2009. Parameter values for 2000-2004 and for 2005-2009 are shown in the columns labeled Robustness check (5) and (6) in Table 4,
respectively. The calibration results and counterfactual analysis for 2000-2004 and those for 2005-2009 are presented in the columns of Robustness check (5) and (6) of Tables 2 and 3, respectively.

For 2000-2004, a 1% decrease in moving costs lowers $u_{nN}$ by 0.60% and CV by 3.20%, and raises $S$ by 0.14% whereas a 1% increase in productivity lowers $u_{nN}$ by 1.42% and CV by 4.27%, and raises $S$ by 1.07%. Eliminating moving costs lowers $u_{nN}$ by 49.5% and increases $S$ by 31.7% whereas a 30% productivity increase lowers $u_{nN}$ by 28.0% and CV by 4.27%, and increases $S$ by 33.3%. For 2005-2009, a 1% decrease in moving costs lowers $u_{nN}$ by 0.71% and CV by 2.12%, and raises $S$ by 0.14% whereas a 1% increase in productivity lowers $u_{nN}$ by 1.43% and CV by 1.59%, and raises $S$ by 1.03%. Elimination of moving costs lowers $u_{nN}$ by 46.4% and increases $S$ by 31.8% whereas a 30% productivity increase lowers $u_{nN}$ by 25.5% and CV by 25.0%, and increases $S$ by 33.6%. Thus, the effects of moving costs are very similar over these periods and comparable to the effects of a 30% productivity increase. The only difference between these periods occurs in the effect of productivity improvements on the unemployment differential, which is smaller for the early 2000s than for the late 2000s.

6 Concluding remarks

In this study, we developed a multi-region job search model and analyzed the impacts of moving costs both qualitatively and quantitatively. In our qualitative analysis, we showed that shocks to a particular region, such as a productivity shock or improvement in access to another region, cause spillover effects to other regions through migration responses. We proved that equilibrium is inefficient in the presence of moving costs. Thus, moving costs reduce the social welfare not only because they decrease the social surplus when migration occurs but also because they cause distortions. Furthermore, we calibrated our framework to Japanese prefectural data and demonstrated by a counterfactual simulation that the impacts of reduced moving costs on the economy would be quantitatively significant.

We will now briefly mention the limitations and possible extensions of our model. First, in order to concentrate on analyzing migration patterns, we ignored one important dimensions related to migration and labor market integration. As shown in Miyagiwa [12] and Wildasin [25], labor market integration enhances human capital accumulation and specialization. Moreover, it may affect firms’ investment decisions. Although incorporating these investment decisions into our framework would not change the efficiency results because investment decisions are know to be efficient in a competitive search model (e.g., Acemoglu and Shimer [2]; Masters [11]), it would amplify the effects of migration: A region receiving many migrants or having better access
from other regions enjoys the benefits of larger investments whereas such benefits are absent in a region experiencing out-migration or suffering poor access from other regions. Agglomeration economies and diseconomies would also be relevant in evaluating the migration patterns from the welfare point of view. Reductions in moving costs induce people to concentrate in a region with high productivity. If we introduce agglomeration economies and diseconomies, productivity can be affected by changes in population distribution. It would be worth examining the properties of such interactions.

Second, we represented moving costs as a function of distance between regions in the quantitative analysis. However, this is evidently a coarse approximation: A region having better transportation infrastructure such as a hub airport may be easier to move both into and out from as compared with a region without it, for example. Indeed, Nakajima and Tabuchi [17] mentioned a case in which one should exclude distances when estimating moving costs (a case in which there is no employment uncertainty and migration takes place based on utility differentials). Fortunately, our framework does not correspond to such a case. Still, it would be worth exploring a more detailed description of moving costs than we were able to utilize in our model.

Third, related to the second point, we may be able to endogenize moving costs. One possible way is to introduce housing loans. Suppose that people buy houses using mortgage loans. If negative productivity shocks hit a region, its income level and housing price would decline. As a result, people may want to move to another region. This would require people to repay the mortgage loans. However, if decreases in income level and housing price are sufficiently large, people can not do so because selling houses at sufficiently high prices becomes difficult. Thus, mortgage loans may act as moving costs in the face of economic fluctuations.

Finally, our framework can be extended to represent the relationships between countries. For instance, we can consider an expansion of the European Union (EU). We can then examine the possible impacts of accession by a new member country on each member country’s labor market and the overall EU labor market. All these are important topics for future research.

Appendices: For online publication

Appendix A: Proof of Proposition 1.

Define \( \Gamma_i \) as

\[
\Gamma_i(U_i) \equiv rU_i - b - \sum_{h=1}^{H} \left[ p_{ih} \left( \frac{y_h}{r} - U_i - t_{ih} \right) - k\theta_{ih} \right].
\]

If equation \( \Gamma_i(U_i) = 0 \) has a unique solution for all \( i \), we know that there exists a unique steady state equilibrium. Equation (7) is rearranged as

\[
k = \frac{dp_{ij}}{d\theta_{ij}} \left( \frac{y_j}{r} - U_i - t_{ij} \right),
\]
which, combined with the Inada condition of the matching function, implies that $\theta_{ij}$ and $p_{ij}$ are positive when $U_i$ is equal to zero and that $\theta_{ij}$ and $p_{ij}$ converge to zero as $U_i$ goes to $y_j/r - t_{ij}$.

Hence, letting $\overline{U}_i$ denote $\max[y_i/r, \max_j[y_j/r - t_{ij}]]$, we readily know that

$$
\Gamma_i(0) < 0,
\Gamma_i(\overline{U}_i) = r\overline{U}_i - b \geq y_i - b > 0.
$$

Note that even though $\Gamma_i(U_i)$ may be kinked at $U_i = y_j/r - t_{ij}$, it is continuous at $U_i \in [0, \overline{U}_i]$. Thus, $\Gamma_i(U_i) = 0$ has at least one solution in $[0, \overline{U}_i]$, which shows the existence of equilibrium.

$\Gamma_i(U_i)$ may not be differentiable at $U_i = y_j/r - t_{ij}$. However, except for these points, it is differentiable, and by differentiating $\Gamma_i(U_i)$ with respect to $U_i$, we obtain

$$
d\Gamma_i(U_i) = \frac{d\Gamma_i(U_i)}{dU_i} = r + \sum_h p_{ih} - \sum_h \frac{\partial [p_{ih}(y_h/r - U_i - t_{ih}) - k\theta_{ih}]}{\partial \theta_{ih}} \frac{\partial \theta_{ih}}{\partial U_i} = r + \sum_h p_{ih} > 0,
$$

where the second equality comes from (15). Combined with the continuity of $\Gamma_i(U_i)$, this proves that the solution of $\Gamma_i(U_i) = 0$ is unique.

**Appendix B: Proof of Proposition 2.**

From (1) and (3), we have

$$
rU_i = b + \sum_{h=1}^H \left[ p_{ih} \left( \frac{y_h}{r} - U_i - t_{ih} \right) - k\theta_{ih} \right],
$$

which yields

$$
U_i = \frac{b + \sum_h [p_{ih}(y_h/r - t_{ih}) - k\theta_{ih}]}{r + \sum_h p_{ih}}.
$$

From (7), we know that $\theta_{ij} = \arg\max U_i, \forall i, j \in H$. Hence, we readily know that

$$
U_j = \frac{b + \sum_h [p_{jh}(y_h/r - t_{jh}) - k\theta_{jh}]}{r + \sum_h p_{jh}} \geq \frac{b + \sum_h [p_{jh}(y_h/r - t_{jh}) - k\theta_{jh}]}{r + \sum_h p_{jh}}.
$$

This implies that

$$
U_i - U_j \leq \frac{b + \sum_h [p_{ih}(y_h/r - t_{ih}) - k\theta_{ih}]}{r + \sum_h p_{ih}} - \frac{b + \sum_h [p_{jh}(y_h/r - t_{jh}) - k\theta_{jh}]}{r + \sum_h p_{jh}} = \frac{\sum_h p_{ih}(t_{jh} - t_{ih})}{r + \sum_h p_{ih}} \leq \frac{\sum_h p_{ih}t_{ij}}{r + \sum_h p_{ih}} \leq t_{ij},
$$

21
where the second inequality comes from the triangle inequality $t_{jh} \leq t_{ji} + t_{ih} = t_{ij} + t_{ih}$. Similar arguments show that $U_j - U_i \leq t_{ij}$.

Appendix C: Proof of Proposition 3.

Suppose temporarily that $U_i$ is fixed. Differentiation of (7) with respect to $y_j - rU_i - rt_{ij}$ yields

$$0 = \frac{dq_{ij}}{d\theta_{ij}} (y_j - rU_i - rt_{ij}) \frac{\partial \theta_{ij}}{\partial (y_j - rU_i - rt_{ij})} + q_{ij}. $$

Plugging $q_{ij} = p_{ij}/\theta_{ij}$ and (7) into this, we obtain

$$0 = \frac{r k \theta_{ij} dq_{ij}/d\theta_{ij}}{q_{ij} (1 - \eta)} \frac{\partial \theta_{ij}}{\partial (y_j - rU_i - rt_{ij})} + p_{ij} = -\frac{r k \eta}{1 - \eta} \frac{\partial \theta_{ij}}{\partial (y_j - rU_i - rt_{ij})} + p_{ij}, $$

which implies that

$$\frac{\partial \theta_{ij}}{\partial (y_j - rU_i - rt_{ij})} = \frac{1 - \eta p_{ij}}{\eta r k} > 0. $$

Also, differentiation of (7) with respect to $\mu_j$ gives

$$\frac{\partial \theta_{ij}}{\partial \mu_j} = -m \left(1, \theta_{ij}^{-1}\right) \frac{dq_{ij}/d\theta_{ij}}{\partial (y_j - rU_i - rt_{ij})} > 0. $$

Because $dp_{ij}/d\theta_{ij} > 0$, these inequalities imply that $p_{ij} > p_{ij'}$ if $y_j - rU_i - rt_{ij} > y_{ij'} - rU_{ij'} - rt_{ij'}$ and $\mu_j > \mu_{ij'}$.

Appendix D: Proof of Propositions 4 and 5.

We start by deriving the effect on the asset value of an unemployed worker, $U_i$. $y_j$ and $t_{ij}$ affect $U_i$ only through changes in $y_j - rt_{ij}$. Differentiating (11) with respect to $y_j - rt_{ij}$ and using (15), we obtain

$$\frac{\partial U_i}{\partial (y_j - rt_{ij})} = \frac{p_{ij}}{r + \sum_{h=1}^{H} p_{ih}} > 0. $$

We readily see that $\partial U_i/\partial y_j = \partial U_i/\partial (y_j - rt_{ij}) > 0$ and $\partial U_i/\partial t_{ij} = -r \partial U_i/\partial (y_j - rt_{ij}) < 0$.

The effects on the job finding rate, $p_{ij}$, also appears through changes in $y_j - rt_{ij}$. Differentiation of (7) with respect to $y_j - rt_{ij}$, combined with (16), yields

$$\frac{\partial p_{ij}}{\partial (y_j - rt_{ij})} = (1 - \eta)^2 \frac{p_{ij} q_{ij}}{\eta k} \left(1 - \frac{p_{ij}}{r + \sum_{h=1}^{H} p_{ih}}\right) > 0, $$

$$\frac{\partial p_{ij'}}{\partial (y_j - t_{ij})} = -\frac{(1 - \eta)^2 p_{ij'q_{ij'}}}{\eta k} \frac{p_{ij'}}{r + \sum_{h=1}^{H} p_{ih}} < 0, $$

22
which lead to \( \partial p_{ij} / \partial y_j > 0 \), \( \partial p_{ij} / \partial t_{ij} < 0 \), \( \partial p_{ij'} / \partial y_j < 0 \), and \( \partial p_{ij'} / \partial t_{ij} > 0 \). From (6), and by using (16), we obtain the effects on the wage rate:

\[
\begin{align*}
\frac{\partial w_{ij}}{\partial y_j} &= \eta + (1 - \eta) \frac{p_{ij}}{r + \sum_{h=1}^{H} p_{ih}} > 0, \\
\frac{\partial w_{ij}}{\partial y_j} &= (1 - \eta) \frac{p_{ij}}{r + \sum_{h=1}^{H} p_{ih}} > 0, \\
\frac{\partial w_{ij}}{\partial t_{ij}} &= (1 - \eta) r \left( 1 - \frac{p_{ij}}{r + \sum_{h=1}^{H} p_{ih}} \right) > 0, \\
\frac{\partial w_{ij}}{\partial t_{ij}} &= - (1 - \eta) r \frac{p_{ij}}{r + \sum_{h=1}^{H} p_{ih}} < 0.
\end{align*}
\]

Finally, from (17), we can see that

\[
\sum_{h=1}^{H} \frac{\partial p_{ih}}{\partial (y_j - rt_{ij})} = \frac{(1 - \eta)^2 p_{ij} q_{ij}}{\eta k} - \sum_{h=1}^{H} \frac{(1 - \eta)^2 p_{ih} q_{ih}}{\eta k} \frac{p_{ij}}{r + \sum_{h=1}^{H} p_{ih}} = \frac{(1 - \eta)^2 p_{ij} q_{ij}}{\eta k} \left[ \frac{(r + \sum_{h} p_{ih}) q_{ij} - \sum_{h} p_{ih} q_{ih}}{r + \sum_{h} p_{ih}} \right].
\]

When \( y_j - rt_{ij} \) is sufficiently large, market tightness \( \theta_{ij} \) is also large and \( q_{ij} \) is small, under which \( \sum_{h} \partial p_{ih} / \partial (y_j - rt_{ij}) \) is likely to be negative. Because the unemployment rate, \( u_{it} \), is given by (8), this raises \( u_{it} \).

**Appendix E: Proof of Proposition 6.**

The present-value Hamiltonian for the welfare maximization (13) is defined as

\[
H_r = \sum_{i=1}^{H} \left[ y_i (N_{it} - u_{ir}) + bu_{it} - u_{ir} \sum_{h=1}^{H} (k \theta_{ihr} + p_{ihr} t_{ih}) \right] e^{-\rho r t}
+ \sum_{h=1}^{H} \lambda_{it}^N \left( \sum_{h=1}^{H} p_{ihr} u_{ihr} - u_{ir} \sum_{h=1}^{H} p_{ihr} \right) + \sum_{i} \lambda_{it}^u \left( \delta N_{it} - u_{ir} \sum_{h=1}^{H} p_{ihr} - \delta u_{ir} \right).
\]

Note here that the control variables are \( \theta_{ijr} \), and the state variables are \( N_{it} \) and \( u_{it} \), \( \lambda_{it}^N \) and \( \lambda_{it}^u \) are the co-state variables. The first-order conditions are given by

\[
ke^{-\rho r t} = \frac{\partial p_{ijr}}{\partial \theta_{ijr}} \left( \lambda_{jr}^N - \lambda_{it}^N - \lambda_{it}^u - t_{ij} e^{-\rho r t} \right) = (1 - \eta) q_{ijr} \left( \lambda_{jr}^N - \lambda_{it}^N - \lambda_{it}^u - t_{ij} e^{-\rho r t} \right) \tag{18}
\]

\[
\lambda_{it}^N = \frac{y_i e^{-\rho r t} + \delta \lambda_{it}^u}{r - \delta} \tag{19}
\]

\[
0 = - \left[ y_i - b + \sum_{h=1}^{H} (k \theta_{ihr} + p_{ihr} t_{ih}) \right] e^{-\rho r t} + \sum_{h=1}^{H} \lambda_{ihr}^N p_{ihr} - \lambda_{it}^N \sum_{h=1}^{H} p_{ihr} - \lambda_{it}^u \left( \sum_{h=1}^{H} p_{ihr} + r \right) \tag{20}
\]
where (18) determines the optimal \( \theta_{ijr} \), and (19) and (20) can be solved to yield \( \lambda_i^N \) and \( \lambda_i^u \). We evaluate these values at the steady state. Hence, we do not need \( \tau \) in the following equations and \( N_i \) and \( u_i \) are determined by \( dN_i/\tau = 0 \) and \( du_i/\tau = 0 \).

Equations (18) and (20) yield

\[
\lambda_i^u = -\left( y_i - b + \eta \sum_h p_{ih} t_{ih} \right) e^{-\rho \tau} + \eta \sum_h p_{ih} \left( \lambda_i^N - \lambda_h^N \right) \frac{r + \eta \sum_h p_{ih}}{r + \eta \sum_h p_{ih}} \tag{21}
\]

Moreover, (19) is rearranged as

\[
\lambda_i^N - \lambda_j^N = \frac{(y_i - y_j) e^{-\rho \tau} + \delta \left( \lambda_i^u - \lambda_j^u \right)}{r - \delta} \tag{22}
\]

Plugging (19), (20) and (22) into (18), we obtain

\[
k = (1 - \eta) q_{ij} \left\{ \frac{(y_i - b + \eta \sum_h p_{ih} t_{ih}) + \eta \sum_h p_{ih} \left[ (y_i - y_h) + \delta \left( \lambda_i^u - \lambda_h^u \right) e^{\rho \tau} \right]}{r + \eta \sum_h p_{ih}} - \frac{\lambda_i^N - \lambda_j^N}{r - \delta} \right\} - \frac{t_{ij}}{r - \delta}
\]

\[
= \pi_{ij} - \frac{\delta}{r} D_{ij},
\]

where \( \pi_{ij} \) and \( D_{ij} \) are defined as

\[
\pi_{ij} \equiv (1 - \eta) q_{ij} \left[ \frac{y_j}{r} - t_{ij} - \frac{b + \eta \sum_h p_{ih} (y_h/r - t_{ih})}{r + \eta \sum_h p_{ih}} \right], \tag{23}
\]

\[
D_{ij} \equiv (1 - \eta) q_{ij} \frac{[y_i - b - (r + \eta \sum_h p_{ih}) t_{ij}] - r \left( \lambda_i^u - \lambda_j^u \right) e^{\rho \tau} - \eta \sum_h p_{ih} \left( \lambda_j^N - \lambda_i^N - t_{ih} e^{-\rho \tau} \right) e^{\rho \tau}}{r + \eta \sum_h p_{ih}} - \frac{k}{r - \delta}.
\]

In equilibrium, because \( p_{ij} = \theta_{ij} q_{ij} \), (7) is rewritten

\[
r k \theta_{ij} = (1 - \eta) p_{ij} (y_j - rU_i - rt_{ij}) .
\]

Summing up the both sides of it for \( j = 1...H \), we obtain

\[
r k \sum_{j=1}^H \theta_{ij} = (1 - \eta) \sum_{j=1}^H p_{ij} (y_j - rU_i - rt_{ij}) ,
\]

which is rearranged as

\[
\eta \sum_{j=1}^H p_{ij} \left( \frac{y_j - rU_i - rt_{ij}}{r} \right) = \frac{\eta}{1 - \eta} k \sum_{j=1}^H \theta_{ij} .
\]

Plugging (1), (6) and the above equation into (3), the asset value of an unemployed worker in
equilibrium can be rewritten as
\[ r U_i = b + \sum_{j=1}^{H} p_{ij} \left[ \eta y_j + (1 - \eta) \frac{r (t_{ij} + U_i)}{r} - U_i - t_{ij} \right] \tag{24} \]
\[ = b + \eta \sum_{j=1}^{H} \frac{y_j - r U_i - rt_{ij}}{r} \]
\[ = b + \frac{\eta}{1 - \eta} k \sum_{j=1}^{H} \theta_{ij}. \]

The second equality implies that
\[ U_i = \frac{b + \eta \sum_h p_{ih} (y_h/r - t_{ih})}{r + \eta \sum_h p_{ih}}. \tag{25} \]

Using this, we can rewrite the zero-profit condition (7) as
\[ k = (1 - \eta) q_{ij} \left( \frac{y_j}{r} - t_{ij} - \frac{b + \eta \sum_h p_{ih} (y_h/r - t_{ih})}{r + \eta \sum_h p_{ih}} \right). \tag{26} \]

Plugging (25) into \( \pi_{ij} \) of (23), we can see that in equilibrium,
\[ \pi_{ij} = (1 - \eta) q_{ij} \left( \frac{y_j}{r} - U_i - t_{ij} \right), \]
which, combined with (7), implies that \( \pi_{ij} = k \) holds true in equilibrium. From this, we know that the equilibrium market tightness is optimal if and only if \( D_{ij} \) evaluated at the equilibrium is zero. Moreover, from the second-order condition of firm’s optimization (5), the equilibrium market tightness is larger than the social optimum if and only if \( D_{ij} \) evaluated at the equilibrium is positive, and the opposite holds true if and only if it is negative.

From (18), we obtain
\[ \sum_{h} p_{ih} (\lambda_{h}^N - \lambda_{i}^N) = \frac{k}{1 - \eta} \sum_{h} \theta_{ih} e^{-\rho \tau} + \sum_{h} p_{ih} (\mu_{i}^u + t_{ih} e^{-\rho \tau}) . \]
Substituting this and (24) into (20), we know that in equilibrium,
\[ \lambda_{i}^u = - \left( \frac{y_i - b + \eta \sum_h p_{ih} t_{ih}}{r} \right) e^{-\rho \tau} - \left[ \eta (1 - \eta) \right] k \sum_{h} \theta_{ih} e^{-\rho \tau} - \eta \sum_{h} p_{ih} (\mu_{i}^u + t_{ih} e^{-\rho \tau}) \]
\[ = - \frac{y_i - r U_i}{r} e^{-\rho \tau} \]

Using this and (24), we can write \( D_{ij} \) of (23) evaluated at the equilibrium as
\[ D_{ij} = \frac{(1 - \eta) q_{ij}}{r + \eta \sum_h p_{ih}} \left\{ -b + \left( \frac{r + \eta \sum_h p_{ih}}{r} \right) \left( \frac{y_j}{r} - t_{ij} \right) + \frac{\eta}{1 - \eta} k \left( \sum_{h} \theta_{ih} - \sum_{h} \theta_{jh} \right) \right\} - k. \]
From (26), this can be further rewritten as

\[ D_{ij} = \frac{\delta (1 - \eta) q_{ij}}{r + \eta \sum_{h=1}^{H} p_{ih}} \frac{\eta}{1 - \eta} \left[ k \sum_{h} \theta_{ih} - k \sum_{h} \theta_{jh} - \frac{\eta}{r} \sum_{h} p_{ih} \left( k \sum_{h} \theta_{jh'} - k \sum_{h} \theta_{hh'} \right) \right]. \]

Finally, from (24), we obtain \( D_{ij} \) evaluated at the equilibrium as

\[ D_{ij} = \frac{(1 - \eta) q_{ij}}{r + \eta \sum_{h} p_{ih}} \left[ r (U_i - U_j) + \eta \sum_{h} p_{ih} (U_h - U_j) \right]. \]

Appendix F: Estimation of the matching function.

Data

Our spatial unit is the Japanese prefectures. For job status, we use the Monthly Report of Public Employment Security Statistics (Ministry of Health, Labour and Welfare). It contains numbers of active job applicants, active job openings, and job placements per month. Here, the number of job placements is available for within prefecture and outside of prefecture. We use the former in estimating the matching function. To eliminate seasonal volatility, we aggregate monthly data into annual data by taking average. In the analysis, Okinawa prefecture is excluded and hence we have 46 prefectures. We use data for 2000-2009, giving us a sample size of 460. Here, we do not take the average over years because the relationship represented in the matching function is not limited to the steady state. The following table provides the descriptive statistics.

[Table 5 around here]

Empirical strategy

As we explained in Section 5.1, we employ a Cobb-Douglas form of the matching function:

\[ \mu_j(t, v_{ij}) = \mu_j(t) u_{ij}^{\eta} v_{ij}^{1-\eta}, \]

where \( t \) represents time. From the assumption of the constant returns to scale, the matching function can be redefined in terms of a job seeker’s job finding rate:

\[ f_{ijt} = \mu_j(t) \theta_{ijt}^{1-\eta}, \]

where \( f_{ijt} = \mu_j(u_{ij}, v_{ij})/u_{ijt} \) is the job seeker’s job finding rate, and \( \theta_{ijt} = v_{ijt}/u_{ijt} \) is labor market tightness. In the estimation, \( f_{ijt} \) is given by the ratio of the number of job placements to the number of job applicants whereas \( \theta_{ijt} \) is given by the ratio of the number of job openings to the number of job applicants. By taking the natural logarithm, we can rewrite the matching function as

\[ \ln[f_{ijt}] = \ln[\mu_j(t)] + (1 - \eta) \ln[\theta_{ijt}]. \]
From this, we obtain an estimable equation as follows:

$$\ln[f_{ijt}] = \xi_j + (1 - \eta) \ln[\theta_{ijt}] + \varepsilon_{jt}. $$

We assume that the matching efficiency $\ln[\mu_{ijt}]$ can be decomposed into a time-invariant term $\xi_j$ and a time-variant term $\varepsilon_{jt}$. We assume that $\varepsilon_{jt}$ satisfies the assumption of the standard error term. Because our data cover job placements within prefectures, the equation to be estimated becomes

$$\ln[f_{jjt}] = \xi_j + (1 - \eta) \ln[\theta_{jjt}] + \varepsilon_{jt}. $$ (27)

In the benchmark case, we estimate (27) by the fixed effect (FE) model. This allows us to deal with concern that the matching efficiency may be correlated with labor market tightness. For example, existence of efficient matching intermediaries induces more job postings from local firms. If so, time-invariant match efficiency, $\xi_j$, may be correlated to labor market tightness, $\ln[\theta_{jjt}]$. The FE model can be used even in the presence of such correlation between $\xi_j$ and $\ln[\theta_{jjt}]$.

Furthermore, one may be concerned that the time-variant matching efficiency, $\varepsilon_{jt}$, might also be correlated with labor market tightness, $\ln[\theta_{jjt}]$. For example, firms may post their vacancies in response to changes in the labor market’s matching efficiency in the current period. If so, $\ln[\theta_{jjt}]$ correlates with $\varepsilon_{jt}$ and the standard FE model does not work. To respond to this concern, we use instrumental variables in estimating the fixed effect model, which we refer to as FEIV model. We follow several recent studies that estimated the matching function by using lags of market tightness as instruments (see e.g., Yashiv [26]): we use two periods and three periods lagged labor market tightness, $\ln[\theta_{jjt-2}]$ and $\ln[\theta_{jjt-3}]$, as instruments for labor market tightness, $\ln[\theta_{jjt}]$.\(^{17}\)

Moreover, because we examine the difference between the early and late 2000s, in addition to the baseline analysis that uses the full periods from 2000 to 2009, we separately estimate the matching function (by FE model) for 2000-2004 and 2005-2009.

**Estimation Results**

The estimation results are shown in Table 6.

\[\text{[Table 6 around here]}\]

\(^{17}\)One may be concerned that the time-variant matching efficiency may serially correlated across periods. In that case, system generalized method of moments (GMM) will work well. Our theoretical model, however, does not allow for the serial correlation of matching efficiency across periods. Because our purpose is conducting a counterfactual simulation by using a rigorously built theoretical model, we do not allow serial correlation in matching efficiency, and we do not use system GMM for parameter estimation.
Column (i) shows the result by the FE model. The point estimate of $\eta$ is 0.512 and is significantly different from zero. Column (ii) shows the result by the FEIV model. The point estimate of $\eta$ becomes slightly higher under the FEIV model than under the FE model. Columns (iii) and (iv) show the results for 2000-2004 and for 2005-2009, respectively. The estimated $\eta$ is larger for the late 2000s than for the early 2000s.

In the quantitative analysis, we also need matching efficiency, which is captured by the estimated prefectural fixed effects. Table 7 shows the descriptive statistics of the estimated fixed effects for each case.

On average, the estimated matching efficiency is stable across the estimation methods and periods.

References


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<th>Values</th>
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Table 1. Parameter values for the benchmark model.

Notes: The value of $\rho$ comes from Japanese long-term interest rates. The values of $\delta$, $y_i$, and $z_{ij}$ are taken from Japanese data. We estimated the Japanese matching function to obtain $\eta$ and $\mu_i$. We normalize the total population, $N$, and the flow utility of an unemployed worker, $b$, to one. The remaining two parameters, $t$ and $k$ are chosen by targeting the data listed in Table 2.
Table 2. Calibration results.

Notes: Data columns represent different time periods: (a) Years 2000-2009, (b) Years 2004-2009, (c) Years 2005-2009. Benchmark and Robustness check (1)-(4) calibrate Data (a). Robustness check (5) and (6) calibrate Data (b) and (c), respectively.
## Table 3. Counterfactual results.

Notes: Robustness check columns represent different cases: (1) FEIV estimation of the matching function, (2) Concave moving costs, (3) Distance-neutral moving costs, (4) Higher discount rate, (5) Years 2000-2004, (6) Years 2005-2009. Percentage changes are in parentheses.
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Table 3. Counterfactual results (continued).

Notes: Robustness check columns represent different cases: (1) FEIV estimation of the matching function, (2) Concave moving costs, (3) Distance-neutral moving costs, (4) Higher discount rate, (5) Years 2000-2004, (6) Years 2005-2009. Percentage changes are in parentheses.
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</table>

Table 4. Alternative parameter values.

Notes: Columns represent different cases: (1) FEIV estimation of the matching function, (2) Concave moving costs, (3) Distance-neutral moving costs, (4) Higher discount rate, (5) Years 2000-2004, (6) Years 2005-2009
<table>
<thead>
<tr>
<th>Variables</th>
<th>Observations</th>
<th>Mean</th>
<th>SD</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of active job openings</td>
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<td>311451</td>
<td>353851.6</td>
<td>53409</td>
<td>2715521</td>
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<tr>
<td>Number of active job applicants</td>
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<td>475205.2</td>
<td>99061</td>
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<tr>
<td>Number of job placements</td>
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<td>27502.22</td>
<td>25779.06</td>
<td>7273</td>
<td>194951</td>
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</table>

Table 5. Descriptive statistics of data used in estimating the matching function
Table 6. Estimation results of the matching function.

Notes: Standard errors are in parentheses. 

\*\*\*, \*\*\*, and \*\*\* represent \( p < 0.10 \), \( p < 0.05 \), and \( p < 0.01 \), respectively.

<table>
<thead>
<tr>
<th>Estimation procedures</th>
<th>(i)</th>
<th>(ii)</th>
<th>(iii)</th>
<th>(iv)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimated ( \eta )</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>FE</td>
<td>0.512***</td>
<td>0.574***</td>
<td>0.456***</td>
<td>0.608***</td>
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<td>(0.0140)</td>
<td>(0.0168)</td>
<td>(0.0195)</td>
<td>(0.0158)</td>
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<tr>
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<td>-2.579***</td>
<td>-2.568***</td>
<td>-2.582***</td>
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<td></td>
<td>(0.00780)</td>
<td>(0.00852)</td>
<td>(0.0136)</td>
<td>(0.00663)</td>
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<tr>
<td>Sample periods</td>
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<tr>
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<td>322</td>
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<tr>
<td>2005 – 2009</td>
<td></td>
<td></td>
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<tr>
<td>Observations</td>
<td></td>
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<tr>
<td>Adjusted ( R^2 )</td>
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<td>0.841</td>
<td>0.851</td>
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<tr>
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<td>years</td>
<td>Observations</td>
<td>Mean</td>
<td>SD</td>
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<tr>
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<td>0.2372374</td>
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</tbody>
</table>

Table 7. Descriptive statistics of estimated regional matching efficiency.
Figure 1. Description of the model
Figure 2: Prefectural output per employed worker and unemployment rates in Japan averaged over 2000-2009

Note: Dots represent prefectural unemployment rates and the thick line represents the overall unemployment rate.