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Abstract

Total factor productivity (TFP) is considered as a good measure of productivity. However, empirical TFP, often calculated from ordinary least squares (OLS) residuals from production function estimates, normally includes demand shocks as well as productivity shocks. The appropriate policy differs depending on which factor is the main cause. Konishi and Nishiyama (2013, KN hereafter) attempt to provide a method in this direction to decompose the TFP shock into demand and supply shocks using the Current Survey of Production and Census of Manufacture. They do not consider a demand side model and implicitly assume that the difference in the production capacity and the realized production identifies the demand shock. This note extends their approach to model explicitly the demand side structure and its shock. Assuming a log-linear demand function, we allow a demand shock of a constant shift as well as a slope change. We show that different quantities identify the demand shock in the two cases. The KN method works in the case of additive demand shocks, but not for slope changes under perfect competition. We further discuss the case of monopolistic competition and find a qualitatively similar result.

Keywords: TFP; Decomposition; Supply shock; Demand shock; Production capacity
JEL classification: C14, D21, D24

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1 Introduction

Estimation of production function is one of an important issues in empirical economics. Least squares method is a simple and common way to estimate production function. Cobb-Douglas production technology $Y = AK^{\beta_k}L^{\beta_l}$ is often used as a parametric model since Cobb and Douglas (1928), where Y, K, and L are the output level, labor input, and capital input, respectively. β_k, β_l, and A, are unknown constants. We transform it to log-linear form, add an error component u and apply OLS to

$$y = \alpha + \beta_k k + \beta_l l + u$$

given observations of (Y, K, L). Here $y = \log Y$, $k = \log K$, $l = \log L$, $\alpha = \log A$. Translog function is also often used as a generalized version of Cobb-Douglas technology. Since Solow (1957), a common method is to estimate this equation by OLS and compute TFP. As pointed out by Marschak and Andrews (1944) and subsequent papers, there can exist an endogeneity problem in the least squares estimation. Each firm can observe its own u at least in part because it is likely to include firm's technological shock. It should maximize the profit using the observation of a part of u and thus the optimal inputs must depend on u. Therefore u and the optimal inputs K, L are correlated. This endogeneity causes bias in OLS estimates. Ackerberg, Caves and Frazer (2006), Ichimura, Konishi, and Nishiyama (2011, IKN hereafter), Levinsohn and Petrin (1999, 2003), Olley and Pakes (1996), Wooldridge (2009), and some other papers propose several methods to estimate the parameters consistently using panel data. These papers consider a model splitting error term u into two components as follows :

$$y_{it} = \beta_0 + \beta_l l_{it} + \beta_k k_{it} + \omega_{it} + \epsilon_{it}.$$

Here, i indicates a firm or a plant and t is time. ω_{it} is considered as the firm-specific productivity or technological shock, which firms can, but econometricians cannot, observe. This causes a bias in OLS regression. ϵ_{it} denotes the ordinary error term independent of the system. We refer to Doraszelski, U. and J. Jaumandreu (2007), Fukao et.al. (2006, 2007), Kim (2008) and many others for empirical research in this line.

After obtaining consistent estimates of production function from one of the above methods, we can calculate TFP of each firm and many empirical studies attempt to decompose the

TFP into entry/exit/change of share/technological shock factors. IKN focuses on another decomposition for TFP, because they are more interested in abstracting the technological productivity from TFP, they encounter a difficulty in decomposing ω_{it} and ϵ_{it}. They make use of the fact that ω_{it} is correlated with the inputs to obtain the estimate of ω_{it}, but this captures only some part of ω_{it}. Konishi and Nishiyama (2013, KN hereafter) proposes a method of decomposing supply and demand shocks in TFP. They employ IKN method to estimate production function. Furthermore, they use an additional dataset of Current Survey of Production that reports the production capacity and the operational rate of labor in order to decompose the shock. They construct a model to describe the capacity and realized production under a Cobb-Douglas technology and suppose the difference comes from demand shocks. Using such a model, they attempt to compute the two shocks separately. Log-production capacity $\bar{y}_{it}$ is determined using technology and present log-inputs $\bar{k}_{it}$, $\bar{l}_{it}$ as follows:

$$\bar{y}_{it} = \alpha + \beta_k \bar{k}_{it} + \beta_l \bar{l}_{it} + \omega_{it}. \tag{1}$$

Applying IKN to (1) and inserting the estimates of α, β_k, β_l in it, it is convenient, unlike in IKN, in that we can estimate ω_{it} directly by

$$\hat{\omega}_{it} = \bar{y}_{it} - \hat{\alpha} - \hat{\beta}_k \bar{k}_{it} - \hat{\beta}_l \bar{l}_{it}$$

because (1) does not include ϵ_{it}.

Next, they consider how much firms produce in fact, given this production capacity. Firms decide their production amounts (say, monthly) looking at the inventory and the economic situation or demand. Suppose firms use only $100\Delta_{it}\% \in (0, 100)$ of labor input and $100\Delta_{it}^{\nu}\%$ of capital input in general; namely, $(\Delta_{it}^{\nu} K_{it}, \Delta_{it} L_{it})$. Here, ν is introduced to allow for different rates of operation for capital and labor. The realized production level is represented as follows:

$$\begin{aligned} y_{it} &= \alpha + \beta_k(\nu \log \Delta_{it} + \bar{k}_{it}) + \beta_l(\log \Delta_{it} + \bar{l}_{it}) + \omega_{it} + \epsilon_{it} & (2) \\ &= \alpha + (\nu\beta_k + \beta_l)\delta_{it} + \beta_k \bar{k}_{it} + \beta_l \bar{l}_{it} + \omega_{it} + \epsilon_{it}, & (3) \end{aligned}$$

where $\delta_{it} = \log \Delta_{it}$ and ϵ_{it} are idiosyncratic errors independent of the inputs and ω_{it}. Given observations $(\bar{y}_{it}, y_{it}, \delta_{it}, \bar{k}_{it}, \bar{l}_{it})$, we can estimate both equations (1) and (2).

KN uses (2) in the next step to identify and estimate the idiosyncratic shock. Subtracting

(2) from (1), they have

$$y_{it} - \bar{y}_{it} = (\nu\beta_k + \beta_l)\delta_{it} + \epsilon_{it}.$$

They have the estimates of β_k, β_l and the data on δ_{it}, the operation rate of labor input. Then, we can estimate ν simply using OLS without constant because ϵ_{it} is the idiosyncratic error from the regression:

$$y_{it} - \bar{y}_{it} - \hat{\beta}_l\delta_{it} = \nu(\hat{\beta}_k\delta_{it}) + \epsilon_{it}. \tag{4}$$

Given these estimates, we can estimate ϵ_{it} by the residual:

$$\hat{\epsilon}_{it} = y_{it} - \bar{y}_{it} - \hat{\beta}_l\delta_{it} - \hat{\nu}\hat{\beta}_k\delta_{it}.$$

Therefore, we can compute all the shocks from this model using the parameter estimates.

Finally, given all these estimates, we can compute the demand shock as

$$\hat{\xi}_{it} = (\hat{\nu}\hat{\beta}_k + \hat{\beta}_l)\delta_{it}.$$

In the empirical study, KN apply this method to two industries, machinery (2110) and die-cast (2560). They show the decomposition results as in Figure 1 and Figure 2. They found no negative productivity shocks but severe demand shocks during the financial crisis of 2007-2008.

<Insert Figures 1 and 2 here>

KN assumed that the difference of production capacity and realized production is caused by the demand shock. This is intuitively appealing, however, demand shocks should be formally defined as shocks in demand function parameters. If we would like to quantify the demand shocks, their approach does not suffice. In order for this purpose, this paper specifies demand function in a simple log-linear form and explicitly introduce demand shocks in either the intercept or the slope. This enables us to carefully investigate what is identified by KN, or quantify the demand shocks.

Section 2 explains how we can identify shocks under perfect competition, while the case of monopolistic competition is discussed in Section 3. Section 4 concludes and addresses the possible future research.

2 Economic Models under perfect competition

2.1 Static model without shocks

We first present a simple static model of producers' and consumers' behavior under perfect competition, that one can find in any microeconomics textbook. We later allow shocks in both demand function and production function. Suppose a number of firms produce an identical good and a number of consumers purchase it. Let P be the price of the product and $p = \log P$. A common demand specification is log-linear,

$$p = \alpha_0 - \alpha_1 y \tag{5}$$

where $\alpha_1 > 0$, and y is the log-quantity. This demand function is obtained under Cobb-Douglas utility function where α_0 depends on the income and the parameters of the utility function, and $\alpha_1 = 1$ in theory. In empirical framework, we do not stick to $\alpha_1 = 1$ and allow for any a negative price elasticity.

Firms determine the inputs and output given a production function. We assume a Cobb-Douglas technology,

$$Y = f(K, L) = AK^{\beta_k} L^{\beta_l}, \;\; \beta_k + \beta_l \leq 1$$

where K and L are capital and labor inputs respectively, or

$$y = \log A + \beta_k k + \beta_l l$$

in log-form. Here k and l are the natural logarithm of K and L. Profit maximizing input levels are characterized by,

$$\max_{K,L} PY - rK - wL \;\; s.t. \;\; Y = AK^{\beta_k} L^{\beta_l}$$

where w and r are prices of labor and capital. Under the above parameter restriction, this yields,

$$K = \left(\frac{w}{PA\beta_l}\right)^{\frac{1}{\beta_k+\beta_l-1}} \left(\frac{w\beta_k}{r\beta_l}\right)^{\frac{\beta_l-1}{\beta_k+\beta_l-1}}, \tag{6}$$

$$L = \left(\frac{r}{PA\beta_k}\right)^{\frac{1}{\beta_k+\beta_l-1}} \left(\frac{r\beta_l}{w\beta_k}\right)^{\frac{\beta_k-1}{\beta_k+\beta_l-1}}, \tag{7}$$

and the optimal output is, as a result,

$$
\begin{aligned}
Y &= A(\frac{w}{PA\beta_l})^{\frac{\beta_k}{\beta_k+\beta_l-1}}(\frac{w\beta_k}{r\beta_l})^{\frac{\beta_k(\beta_l-1)}{\beta_k+\beta_l-1}}(\frac{r}{PA\beta_k})^{\frac{\beta_l}{\beta_k+\beta_l-1}}(\frac{r\beta_l}{w\beta_k})^{\frac{\beta_l(\beta_k-1)}{\beta_k+\beta_l-1}} \\
&= \{A(\frac{\beta_l}{w})^{\beta_l}(\frac{\beta_k}{r})^{\beta_k}\}^{\frac{1}{1-\beta_k-\beta_l}} P^{\frac{\beta_k+\beta_l}{1-\beta_k-\beta_l}}. \qquad (8)
\end{aligned}
$$

Thus, the log-supply function is,

$$
\begin{aligned}
p &= \frac{1-\beta_k-\beta_l}{\beta_k+\beta_l}y - \frac{\log\{A(\frac{\beta_l}{w})^{\beta_l}(\frac{\beta_k}{r})^{\beta_k}\}}{\beta_k+\beta_l} \qquad (9) \\
&= \kappa_0 + \kappa_1 y \qquad (10)
\end{aligned}
$$

where

$$
\begin{aligned}
\kappa_0 &= -\frac{\log\{A(\frac{\beta_l}{w})^{\beta_l}(\frac{\beta_k}{r})^{\beta_k}\}}{\beta_k+\beta_l} \\
\kappa_1 &= \frac{1}{\beta_k+\beta_l} - 1.
\end{aligned}
$$

2.2 Demand and production functions with shocks.

Total factor productivity (TFP) is often regarded as a productivity measure, where large TFP indicates high productivity. It may be a simple and straightforward way of studying the productivity or technology. However, based on the model above, we can look at the shocks more carefully and identify the demand shocks and technological (or supply) shocks.

A common way to introduce shocks is to include additive components like regression errors. Namely, we consider a shock ξ in demand function as,

$$p = \alpha_0 - \alpha_1 y + \xi, \qquad (11)$$

and a shock ω in production function

$$y = \log A + \beta_k k + \beta_l l + \omega. \qquad (12)$$

The demand shock ξ indicates a shock in the intercept of the demand function, and ω is a shock on TFP. (12) is the common model assumed in Ichimura, Konishi and Nishiyama (2012, IKN hereafter), Konishi and Nishiyama (2013), Levinsohn and Petrin (1999, 2003, LP hereafter), Olley and Pakes (1996, OP hereafter) and many others. (11) is also a typical setting, that is implicitly assumed by Konishi and Nishiyama (2012) as shown later. The model of (11) and (12) is certainly one reasonable setting to describe demand and supply structure with shocks. Demand shock, however, could be a slope change, not intercept shift. Also, technological shock may not be on the TFP, or A, but a change in parameters β_k, β_l. In such a case, it may be better to specify the model as, for example,

$$
\begin{aligned}
p &= \alpha_0 - (\alpha_1 + \xi)y, \\
y &= \log A + (\beta_k + \omega)k + \beta_l l,
\end{aligned}
$$

instead of (11) and (12). The specification of the second equation above is an interesting topic which we should examine, but our main interest is how to identify demand shocks under different demand specifications in this paper. We do not consider the case of slope shocks in supply side any further in this paper.

2.3 Firms' decision in the case of additive demand shock

We first consider the simplest additive shock model of (11) and (12). We suppose firms observe a part of shocks in demand and production, and make an input decision as the following order. It is natural to consider that firms cannot change the capital and labor inputs in short term, so that we also suppose that there is a time lag between the input decision and production.

1. Firms input optimal levels of K and L based on the present information and prediction. This input levels determine the production capacity. We call shocks observed up to or at this stage as "long-run" shocks.

2. Firms observe further shocks and decide an optimal amount of products applying a suitable operation rate. Shocks here are called "short-run" shocks.

3. Final production level is realized where an idiosyncratic shock is added.

Figure 3 shows this flow of shock observation and the firm's decision.

We can now formally describe the profit maximization of firm i as follows. Denoting ω_i^L be the long-run technological shock, firm i observes its log-production function

$$
y_i = \beta_0 + \beta_k k_i + \beta_l l_i + \omega_i^L
$$

and maximizes its profit given (P, w, r). To compute the optimal level of inputs, we only need to replace A by $\exp(\beta_0 + \omega_i^L)$ in (6) and (7), that yields

$$
\begin{aligned}
K_i &= \{\frac{w}{P\exp(\beta_0 + \omega_i^L)\beta_l}\}^{\frac{1}{\beta_k+\beta_l-1}}(\frac{w\beta_k}{r\beta_l})^{\frac{\beta_l-1}{\beta_k+\beta_l-1}}, \\
L_i &= \{\frac{r}{P\exp(\beta_0 + \omega_i^L)\beta_k}\}^{\frac{1}{\beta_k+\beta_l-1}}(\frac{r\beta_l}{w\beta_k})^{\frac{\beta_k-1}{\beta_k+\beta_l-1}},
\end{aligned}
$$

and the resulting output is, from (8),

$$
Y_i = \{\exp(\beta_0 + \omega_i^L)(\frac{\beta_l}{w})^{\beta_l}(\frac{\beta_k}{r})^{\beta_k}\}^{\frac{1}{1-\beta_k-\beta_l}} P^{\frac{\beta_k+\beta_l}{1-\beta_k-\beta_l}}
$$

or the log-output

$$
y_i = \frac{\beta_0 + \omega_i^L + \log\{(\frac{\beta_l}{w})^{\beta_l}(\frac{\beta_k}{r})^{\beta_k}\}}{1-\beta_k-\beta_l} + \frac{\beta_k+\beta_l}{1-\beta_k-\beta_l}p
$$

Summing up with respect to all the n firms, we obtain the total output in the market as

$$
\sum_{i=1}^{n} Y_i = \{\exp(\beta_0)(\frac{\beta_l}{w})^{\beta_l}(\frac{\beta_k}{r})^{\beta_k}\}^{\frac{1}{1-\beta_k-\beta_l}} P^{\frac{\beta_k+\beta_l}{1-\beta_k-\beta_l}} \sum_{i=1}^{n} \exp(\frac{\omega_i^L}{1-\beta_k-\beta_l})
$$

and its logarithm

$$
y = \frac{\beta_k+\beta_l}{1-\beta_k-\beta_l}p + \frac{\beta_0 + \log\{(\frac{\beta_l}{w})^{\beta_l}(\frac{\beta_k}{r})^{\beta_k}\}}{1-\beta_k-\beta_l} + \log\{\frac{1}{n}\sum_{i=1}^{n}\exp(\frac{\omega_i^L}{1-\beta_k-\beta_l})\} + \log n.
$$

Then the total log-supply function is

$$
p = \frac{1-\beta_k-\beta_l}{\beta_k+\beta_l}y - \frac{\beta_0 + \log\{(\frac{\beta_l}{w})^{\beta_l}(\frac{\beta_k}{r})^{\beta_k}\}}{\beta_k+\beta_l} - \frac{1-\beta_k-\beta_l}{\beta_k+\beta_l}\left[\log\{\frac{1}{n}\sum_{i=1}^{n}\exp(\frac{\omega_i^L}{1-\beta_k-\beta_l})\} + \log n\right].
$$

Assuming ω_i^L are i.i.d., the term in the square bracket is approximately equal to

$$
\theta = \log\{E\exp(\frac{\omega_i^L}{1-\beta_k-\beta_l}) + \log n\}
$$

when n is large, which is a constant depending on β_k, β_l, n. Rewriting the total log-supply function as,

$$
p = \kappa_0' + \kappa_1 y \tag{13}
$$

where $\kappa_0' = -\frac{\beta_0+\log\{(\frac{\beta_l}{w})^{\beta_l}(\frac{\beta_k}{r})^{\beta_k}\}}{\beta_k+\beta_l} - \frac{1-\beta_k-\beta_l}{\beta_k+\beta_l}\theta$.

Given an additive long-term demand shock ξ^L, the predicted (or ex-ante) demand function is

$$p = \alpha_0 - \alpha_1 y + \xi^L.$$

The ex-ante equilibrium price that clears the market is,

$$p = \frac{\kappa_1\alpha_0 + \kappa_0'\alpha_1}{\kappa_1 + \alpha_1} + \frac{\kappa_1}{\kappa_1+\alpha_1}\xi^L.$$

Given this price, firm i plans to produce

$$\bar{y}_i = \frac{\beta_0 + \omega_i^L + \log\{(\frac{\beta_l}{w})^{\beta_l}(\frac{\beta_k}{r})^{\beta_k}\}}{1-\beta_k-\beta_l} + \frac{\beta_k+\beta_l}{1-\beta_k-\beta_l}(\frac{\kappa_1\alpha_0+\kappa_0'\alpha_1}{\kappa_1+\alpha_1} + \frac{\kappa_1}{\kappa_1+\alpha_1}\xi^L)$$

This $\bar{y}_i$ determines the production capacity of firm i, which satisfies

$$\bar{y}_i = \beta_0 + \beta_k\bar{k}_i + \beta_l\bar{l}_i + \omega_i^L \tag{14}$$

where the optimal log-inputs are,

$$\begin{aligned} \bar{k}_i &= \frac{\kappa_1\xi^L}{(1-\beta_k-\beta_l)(\kappa_1+\alpha_1)} + \frac{\omega_i^L}{1-\beta_k-\beta_l} + c_k \\ \bar{l}_i &= \frac{\kappa_1\xi^L}{(1-\beta_k-\beta_l)(\kappa_1+\alpha_1)} + \frac{\omega_i^L}{1-\beta_k-\beta_l} + c_l, \end{aligned} \tag{15}$$

and c_k and c_l are constants. After this input decision, firms observe the short-run demand shock ξ^S and know that the ex-post optimal input level is,

$$\begin{aligned} k_i^* &= \frac{\kappa_1(\xi^L+\xi^S)}{(1-\beta_k-\beta_l)(\kappa_1+\alpha_1)} + \frac{\omega_i^L}{1-\beta_k-\beta_l} + c_k \\ l_i^* &= \frac{\kappa_1(\xi^L+\xi^S)}{(1-\beta_k-\beta_l)(\kappa_1+\alpha_1)} + \frac{\omega_i^L}{1-\beta_k-\beta_l} + c_l. \end{aligned} \tag{16}$$

Because firms cannot change the capital and labor inputs in short term, the firm adjusts the short-term shock by changing the log-operation rate by

$$\delta = \frac{\kappa_1}{(1-\beta_k-\beta_l)(\kappa_1+\alpha_1)}\xi^S \tag{17}$$

or

$$(\beta_k + \beta_l)\delta = \frac{\xi^S}{\kappa_1 + \alpha_1}.$$

We easily see that δ is the log-operation rate as follows. Letting Δ be the operation rate, we have, for a capital level K, $\log(\Delta K) = \log(\Delta) + \log(K)$. From (15), (16), (17), $k^* = \delta + \bar{k}$ holds. Therefore, δ is the log-operation rate. As a result, the realized production level turns out

$$y_i = \beta_0 + \beta_k k_i^* + \beta_l l_i^* + \omega_i^L + \epsilon_i \tag{18}$$

where ϵ_i is an idiosyncratic error.

We do not consider short-run technological shocks ω^S. What we want to capture is the technological improvement that affects the optimal input levels. Because firms cannot adjust capital and labor inputs in short term by assumption, even if there exist short-run technological shocks, firms cannot change their technology. They will only adjust it in the future, and thus present production should not be affected by ω^S. If ever ω^S is a shock that affects the present output level, we can absorb it in ϵ at present because it cannot change the input levels and thus does not cause endogeneity. This technological improvement will be used for the future production.

2.4 Identification of shocks in the case of additive demand shock

We carefully examine how we can identify shocks ω^L and ξ^L, ξ^S in the additive case. We have observations of $(y, \bar{y}, \bar{k}, \bar{l}, \delta, m, e)$ from Current Survey of Production and Census of Manufacture. In view of (14), ω_i^L is identified as,

$$\omega_i^L = \bar{y}_i - \beta_0 - \beta_k \bar{k}_i - \beta_l \bar{l}_i$$

given $\beta_0, \beta_k, \beta_l$. Obviously, $\bar{k}, \bar{l}$ are correlated with ω and ξ^L as computed in the above section. KN propose to estimate (14) by IKN estimation in the first stage. It is still a valid method because we only use the moment condition $E(\eta | m_{-1}, e_{-1}) = 0$ that holds even if the inputs depend on both ω_i^L and ξ^L. Then we obtain the estimates of $\beta_0, \beta_k, \beta_l$ and the residuals must be reasonable estimates of ω_i^L.

Because of a favorable relationship (17), we can identify ξ^S up to scale using the operation rate data.

$$\begin{aligned} \xi^S &= (\kappa_1 + \alpha_1)(\beta_k + \beta_l)\delta \\ &= \{1 - (\beta_k + \beta_l)(1 - \alpha_1)\}\delta. \end{aligned} \tag{19}$$

We now show that this model is consistent with an implicit assumption of KN that $y_i - \bar{y}_i$ consists of the demand shock and the idiosyncratic shock in fact as follows. From (14) and (18), we have

$$y_i - \bar{y}_i = \frac{\beta_k \kappa_1 \xi^S}{(1 - \beta_k - \beta_l)(\kappa_1 + \alpha_1)} + \frac{\beta_l \kappa_1 \xi^S}{(1 - \beta_k - \beta_l)(\kappa_1 + \alpha_1)} + \epsilon_i \tag{20}$$

$$= \frac{(\beta_k + \beta_l)\kappa_1 \xi^S}{(1 - \beta_k - \beta_l)(\kappa_1 + \alpha_1)} + \epsilon_i$$

$$= \frac{\xi^S}{\kappa_1 + \alpha_1} + \epsilon_i \tag{21}$$

Here different firms may employ different operation rates δ_i. Also operation rate of capital and labor could be different in practice. Thus, substituting (19) into (21) and making these modifications, we obtain an estimation model

$$y_i - \bar{y}_i = (\nu\beta_k + \beta_l)\delta_i + \epsilon_i.$$

This model is equivalent to KN. Therefore, we may understand that KN implicitly assumes additive shocks under perfect equilibrium. Once this model is estimated, we can identify the idiosyncratic shocks as

$$\epsilon_i = y_i - \bar{y}_i - (\nu\beta_k + \beta_l)\delta_i.$$

Therefore, $\omega^L, \xi^S, \epsilon$ are identified.

2.5 Shock in the slope of demand function and its identification

Now we consider the case when the shock appears in the slope of the demand function. The demand function is specified as,

$$p = \alpha_0 - (\alpha_1 + \xi^L)y. \tag{22}$$

We suppose the production shock is additive as in the previous section here again, then the total log-supply function remains the same as (13). (13) and (22) yield the ex-ante log-price,

$$p = \kappa_0' + \frac{\kappa_1(\alpha_0 - \kappa_0')}{\alpha_1 + \kappa_1 + \xi^L}$$

so that the production capacity of firm i becomes

$$\begin{aligned}\bar{y}_i &= \frac{\beta_0 + \omega_i^L + \log\{(\frac{\beta_l}{w})^{\beta_l}(\frac{\beta_k}{r})^{\beta_k}\}}{1-\beta_k-\beta_l} + \frac{\beta_k+\beta_l}{1-\beta_k-\beta_l}\{\kappa_0' + \frac{\kappa_1(\alpha_0-\kappa_0')}{\alpha_1+\kappa_1+\xi^L}\} \\ &= \frac{\beta_0 + \omega_i^L + \log\{(\frac{\beta_l}{w})^{\beta_l}(\frac{\beta_k}{r})^{\beta_k}\}}{1-\beta_k-\beta_l} + \frac{\kappa_0'}{\kappa_1} + \frac{\alpha_0-\kappa_0'}{\alpha_1+\kappa_1+\xi^L}.\end{aligned}$$

Just before production, firms observe a short-run demand shock ξ^S in the slope, namely the demand function of

$$p = \alpha_0 - (\alpha_1 + \xi^L + \xi^S)y.$$

They adjust it through the operation rate. The ex-post or realized equilibrium price is

$$p = \kappa_0' + \frac{\kappa_1(\alpha_0 - \kappa_0')}{\alpha_1 + \kappa_1 + \xi^L + \xi^S}$$

and the corresponding output level of firm i turns out

$$y_i = \frac{\beta_0 + \omega_i^L + \log\{(\frac{\beta_l}{w})^{\beta_l}(\frac{\beta_k}{r})^{\beta_k}\}}{1-\beta_k-\beta_l} + \frac{\kappa_0'}{\kappa_1} + \frac{\alpha_0-\kappa_0'}{\alpha_1+\kappa_1+\xi^L+\xi^S} + \epsilon_i.$$

If we mechanically apply KN method to compute $y_i - \bar{y}_i$, we obtain

$$y_i - \bar{y}_i = \frac{\alpha_0-\kappa_0'}{\alpha_1+\kappa_1+\xi^L+\xi^S} + \epsilon_i - \frac{\alpha_0-\kappa_0'}{\alpha_1+\kappa_1+\xi^L}$$

which does not identify ξ^S. If we would like to extract ξ^S, one possible way is to compute

$$\frac{1}{y_i - v_i - \epsilon_i} - \frac{1}{\bar{y}_i - v_i} = \frac{\xi^S}{\alpha_0 - \kappa_0'}$$

where $v_i = \frac{\beta_0+\omega_i^L+\log\{(\frac{\beta_l}{w})^{\beta_l}(\frac{\beta_k}{r})^{\beta_k}\}}{1-\beta_k-\beta_l} + \frac{\kappa_0'}{\kappa_1}$. Construction of its empirical counterpart may need some more considerations and/or assumptions. This expression should be convenient because it does not involve ξ^L which does not look easy to identify. Also, ϵ_i may be a nuisance component, then we may first take an expectation conditional on ξ^S to remove it, or

$$\frac{1}{E(y_i - v_i|\xi^S)} - \frac{1}{E(\bar{y}_i - v_i|\xi^S)} = \frac{\xi^S}{\alpha_0 - \kappa_0'}.$$

This identifies ξ^S up to scale.

3 The case of monopolistic competition

We consider a simple model of monopolistic competition that may be more realistic than perfect competition in some industries. Suppose there are n firms producing similar but not identical goods. Also assume that entry is not free, thus $n < \infty$ is fixed. A representative consumer has CES utility function

$$U(Y_1, \cdots, Y_n) = \left(\sum_{i=1}^{n} Y_i^{\rho} \right)^{1/\rho}, \; 0 < \rho < 1.$$

The elasticity of substitution between two goods is the same for any pairs, and it is $\sigma = \frac{1}{1-\rho} > 1$. S/he maximizes the utility function under the budget constraint of $\sum_{i=1}^{n} P_i Y_i \leq I$ where P_i are the output prices and I is the income. Then we obtain the demand function for good i as,

$$Y_i = \frac{I}{P_i^{\sigma} \bar{P}^{1-\sigma}}$$

where $\bar{P} = (\sum_{i=1}^{n} P_i^{1-\sigma})^{\frac{1}{1-\sigma}}$. The log-demand function is

$$\begin{aligned} y_i &= \log I - (1-\sigma)\log \bar{P} - \sigma p_i \\ &= \delta_0 - \delta_1 p_i, \end{aligned}$$

where $\delta_0 = \log I - (1-\sigma)\log \bar{P}$, $\delta_1 = \sigma$.

Supposing the cost function is

$$C_i = cY_i + F,$$

the profit for firm i is

$$\pi_i = P_i Y_i - (cY_i + F) = (P_i - c)Y_i - F.$$

Each firm behaves like a monopoly firm, namely they decide the price and output level such that the profit is maximized, or

$$\frac{d\pi_i}{dP_i} = \frac{d}{dP_i}\{(P_i - c)\frac{I}{P_i^{\sigma}\bar{P}^{1-\sigma}} - F\} = 0. \tag{23}$$

When n is sufficiently large and $\bar{P}$ does not move for a small change in P_i, the optimal price is approximately

$$P_i \approx \frac{c\sigma}{\sigma - 1}$$

and the corresponding output level is

$$Y_i \approx \frac{I}{(\frac{c\sigma}{\sigma-1})^{\sigma}\bar{P}^{1-\sigma}} = \frac{\exp(\delta_0)}{(\frac{c\delta_1}{\delta_1-1})^{\delta_1}}. \tag{24}$$

This is the ex-ante output level, or the production capacity and its logarithm is

$$\bar{y}_i = \delta_0 - \delta_1\{\log c + \log\delta_1 - \log(\delta_1 - 1)\}.$$

Under the symmetric equilibrium of $P = P_1 = P_2 = \cdots = P_n$, $Y = Y_1 = Y_2 = \cdots = Y_n$, we obtain from (23),

$$P = \frac{c\{(n-1)\sigma + 1\}}{(n-1)(\sigma - 1)}.$$

Because the demand function is $Y = \frac{I}{P^{\sigma}\sum_{j=1}^{n} P^{1-\sigma}}$, the production capacity is

$$Y = \frac{I}{nP} = \frac{I(n-1)(\sigma-1)}{nc\{(n-1)\sigma+1\}}. \tag{25}$$

We are now ready to examine the effect of ξ^S, demand shock observed after the input or capacity decision. As in the competitive case we consider the cases of intercept shock and slope shock in the demand function. We do not include ξ^L explicitly here because it does not play an essential role as in the case of perfect competition. Also note that we put subscript i here in ξ_i^S because each market can have its own demand shock. The demand function of the intercept shock case is

$$y_i = \delta_0 - \delta_1 p_i + \xi_i^S.$$

Replacing δ_0 by $\delta_0 + \xi_i^S$ in (24), we easily see that the realized output becomes

$$y_i = \delta_0 + \xi_i^S - \delta_1\{\log c + \log\delta_1 - \log(\delta_1 - 1)\} + \epsilon_i$$

and thus, we have,

$$y_i - \bar{y}_i = \xi_i^S + \epsilon_i$$

Interestingly, approach of KN perfectly suits in this case, even better than the case of perfect competition in the sense that (20) determine ξ^S only up to the scale.

On the other hand, in the case of slope shock, the realized demand function is

$$y_i = \delta_0 - (\delta_1 + \xi_i^S)p_i.$$

Then, the realized output is

$$y_i = \delta_0 - (\delta_1 + \xi_i^S)\{\log c + \log(\delta_1 + \xi_i^S) - \log(\delta_1 + \xi_i^S - 1)\} + \epsilon_i.$$

It is not easy to nicely identify the short-term demand shock in this case because y_i is highly nonlinear in ξ_i^S. When ξ_i^S is small, we could approximate the simple output difference in KN as

$$y_i - \bar{y}_i \approx \xi_i^S\{\frac{1}{\delta_1 - 1} - 1 - \log c - \log(\delta_1) + \log(\delta_1 - 1)\} + \epsilon_i.$$

Then we may regard that the difference approximately identifies ξ_i^S up to scale. However, this may be of very limited use because there is no reason why we believe ξ_i^S is small.

Under the symmetric equilibrium, the demand function is

$$y = \log(\frac{I}{n}) - p = \delta_0 - p$$

so that we cannot consider the slope shock in theory because the price elasticity of demand always equals to unity. When there exists intercept shock, which can be regarded as the income shock or shock in the number of firms, the realized demand function is

$$y = \delta_0 + \xi^S - p$$

and the ex-ante optimal output is

$$Y = \frac{I\exp(\xi^S)(n-1)(\sigma-1)}{nc\{(n-1)\sigma+1\}}.$$

Therefore, the log-difference is

$$y_i - \bar{y}_i = \xi^S + \epsilon_i.$$

4 Concluding remarks and future research

Total factor productivity (TFP) has been widely adopted as a measure of productivity in both economic theory and empirical analysis. After examing the TFP, when policy makers observe a low TFP or low TFP growth, they often claim the productivity is low so that we should implement policy which promotes firms' R&D investment, for example. However, empirical TFP, often calculated from OLS residuals from production function estimates, normally includes not only the productivity shocks but also demand shocks. Appropriate policy will be different depending on which factor is the main cause. Therefore, it is of some help if we know which causes the low TFP, the demand side factor or supply side factor, in determining economic policy.

IKN propose an estimation method of production function under endogeneity and KN uses it to decompose technological shocks and demand shocks using the data of realized production and production capacity. KN consider that the difference of the two comes from demand shocks and use this information to estimate demand shocks. It may be one reasonable approach, but demand shocks occur in various ways. We specify demand function as a log-linear function and consider two types of demand shock. One is intercept shock and the other is slope shock. We examine how we can identify such demand shock under two market clearing mechanism of perfect competition and monopolistic competition.

We found that the difference $y_i - \bar{y}_i$ considered in KN can identify the intercept demand shock up to scale under the perfect competition. Interestingly, $y_i - \bar{y}_i$ identifies intercept demand shock itself, not up to scale, under monopolistic competition. In the case of slope demand shock, the structure becomes nonlinear and the difference $y_i - \bar{y}_i$ does not work well for the identification both perfect and monopolistic competition. Under perfect competition, we found a reciprocal difference could provide us good estimates. On the other hand, no simple identification seems possible in the case of monopolistic competition with slope demand shocks due to the high non-linearity. We only provide a simple case when a linear approximation is applicable, though it may not be useful in practice. Once such quantities are estimated, they provide a reasonable foundation to quantify a suitable level of policy variables to stimulate demand side.

There are some topics that should be considered in the future. We only consider how we can identify the shocks in this paper. Needless to say, unidentified quantities are impossible to estimate in principle. Identifiability, however, does not guarantee the estimability. In order for the practical use, we must consider feasible ways of estimation under suitable econometric models. Also, we would like to apply such methods to real data. One possible difficulty is that the above models explicitly consider the demand function, that involves additional parameters to IKN or KN estimation models. If we attempt to estimate such demand side parameters, we will require further instruments which is orthogonal to the explanatory variables in the demand function.

We, as well as many articles in this line including OP, LP and many others, have considered technological shocks in only the TFP. However, technological shocks may occur in, say, capital productivity, namely in the parameter of β_k, as in the industrial revolution in 20th century. Such kind of technological shocks might not be able to capture correctly if we only look at the TFP. Research toward this direction must be necessary. This may not be easy in view of the highly nonlinear structure of the optimal inputs and output with respect to the parameters. Also, given production data, we should provide econometric tools to test or select a suitable model. Specifically, we need to know which type of shock occurs in fact, slope shocks or intercept shocks in both demand and supply sides. Also, as shown above, the identification strategy becomes different depending on how market clears.

An interesting topic in a slightly different line of research is the data envelope analysis (DEA) that is used to estimate the production frontier. In traditional DEA, researchers do not pay attention to the possible endogeneity problem. Recently, Cordero, Santin and Sicilia (2013) show that there can exist severe bias in the DEA estimates under the presence of endogeneity by Monte Carlo simulation. Our analysis of production capacity could be regarded as an estimate of production frontier under endogeneity. These two approaches could be complimentary and we can investigate how DEA should be modified to cope with the problem.

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Figure 1: Product 2110

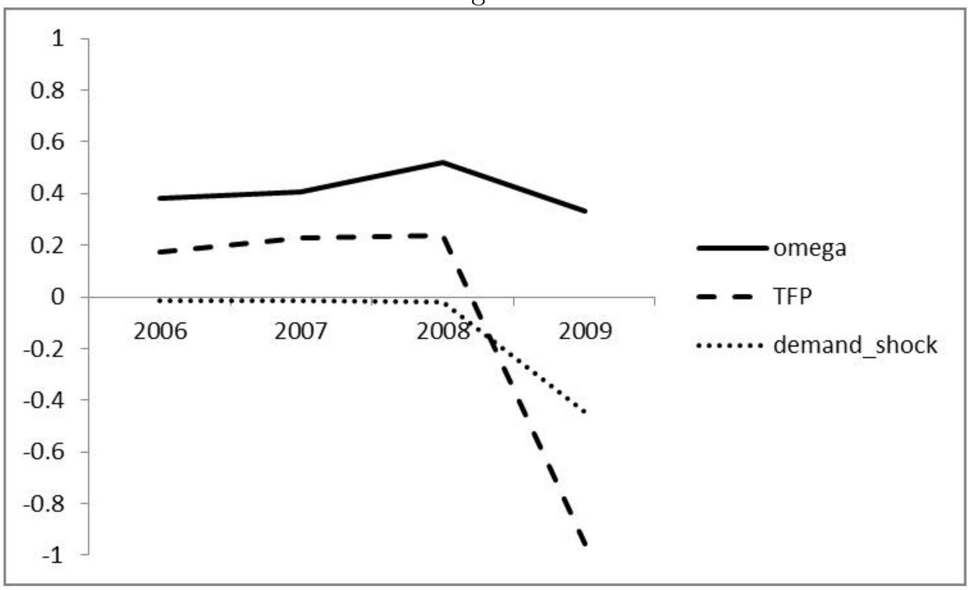

Figure 2: Product 2560

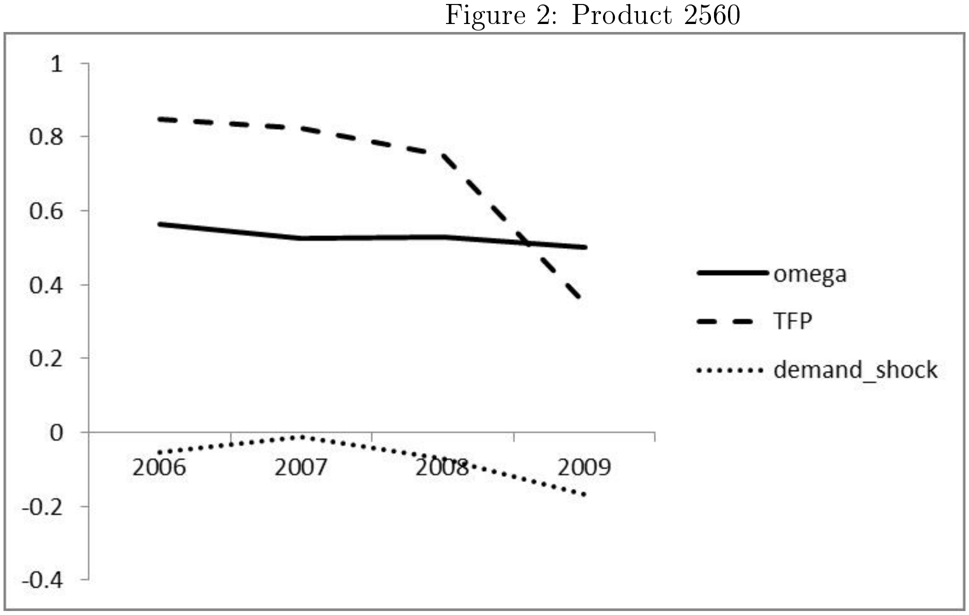

Figure 3: Shocks and firm's decision