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A Theory of Disasters and Long-run Growth

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Abstract

We examine the long-term consequences to economic growth of disasters using a discrete-time endogenous growth model. We consider two types of hypothetical disasters: historical disasters, which follow a Bernoulli process, and periodic disasters, which are taken as a regular event by assuming that one period is a sufficient time period. We show that the effects of historical disasters on the steady state growth rate depend on the intertemporal elasticity of substitution for consumption. Specifically, when it is less than one, more destructive disasters or more frequent occurrence of historical disasters foster investment in human capital, which results in a higher economic growth rate. This conditionally supports the empirical finding: disasters may positively affect long-run economic growth. We also show the effects of historical and periodic disasters on resource allocation and industrial composition at the steady state and on the convergence speed.

Keywords: Long-term growth; Disasters; Optimal growth model
JEL classification: O44, C61, D90

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1 Introduction

Recent empirical research has revealed an interesting relationship between disasters and long-run economic growth. The seminal paper by Skidmore and Toya (2002), for instance, conducted a cross-country empirical analysis of the long-run relationship between natural disasters, capital accumulation, total factor productivity (TFP), and economic growth. Based on data for 89 countries from 1960 to 1990, they found that higher frequencies of climatic disasters are associated with higher rates of human capital accumulation, increases in TFP, and higher economic growth in the long run, whereas geologic disasters have no significant effect on TFP growth. Blomberg et al. (2004), on the other hand, investigated the economic consequences of terrorism, an example of man-made disasters. Using data for 177 countries from 1968 to 2000, they found that the incidence of terrorism has an economically significant negative effect on growth although its impact is considerably smaller and less persistent than those associated with external wars or internal conflicts. More recently, Sawada et al. (2011) studied the long-term welfare impact of natural and man-made disasters based on data for 189 countries from 1968 to 2001. They found that while the incidence of natural or man-made disasters has a negative effect on per capita consumption in the short run, its impact is positive in the long run. In particular, natural disasters and wars have positive effects on per capita consumption in all of their 15-, 20-, and 25-year lag models.

These findings imply that disasters have positive or negative long-run economic consequences, depending on their types and characteristics. It might sound counterintuitive that disasters may have a positive impact on long-run economic growth. To rationalize this result, Skidmore and Toya (2002) argued that while the risk of disasters may reduce physical capital investment, a disaster can present an opportunity to update capital stocks, facilitating the early adoption of new technologies. This might boost economic growth when there exists some constraint on technology adoption.[1] Further-

[1] Motivated by this interpretation, the empirical research by Crespo Cuaresma et al. (2008) investigated the rela-

more, they suggested that the higher frequency of disasters may provide an incentive to reallocate resources from physical to human capital. In the standard endogenous growth framework, increased emphasis on human capital investment may have a positive effect on growth. These arguments indicate that we may have a paradoxical proposition, akin to the "resource curse" (van der Ploeg, 2011), in which a region with no or small risk of disasters misses opportunities to update capital stocks, invests less in human capital, and as a result ends up with a lower economic growth rate in the long run. While their argument is intuitively appealing, it is primarily based on empirical analysis, but not theoretically well-founded.

In this paper, we present a theoretical framework with which the long-run relationship between disasters and their economic consequences can be analyzed. Based upon an endogenous growth model, we investigate the interplay between disasters, consumption, and investment in the long run and show in particular that the risk of disasters can have positive or negative economic consequences, depending on their types as well as preferences and technology.

In a disaster database such as EM-DAT,[2] the destructive magnitude of a disaster is measured by the number of casualties and the size of the economic loss. These may be related to the losses of human resources and physical resources, respectively. The relative magnitudes of these two indices vary across individual disasters as well as the types of disasters. From the annual review based on EM-DAT (Guha-Sapir et al., 2011), we have the damage ratio, (billions of 2011 US$)/victim (millions), during the 2000s by natural disaster category: 0.12 for climatological disasters, 2.70 for geophysical disasters, 0.20 for hydrological disasters, and 1.40 for meteorological disasters.

Another important aspect of disasters is their frequency. The 2011 Tohoku earthquake and

tionship between foreign technology absorption and catastrophic events, concentrating on developing countries. They found no positive partial correlation between the frequency of natural disasters and the R&D content of imports in the period 1976–1990, except for in the countries with relatively high levels of GDP per capita.

[2] EM-DAT is an international disaster database maintained by the Center for Research on the Epidemiology of Disasters at the Université catholique de Louvain in Brussels. For details, see the website (http://www.emdat.be/).

Table 1: Natural disasters by type and regions in 1975--2007

	Total number	Regional Share (%)				
		Africa	Americas	Asia	Europe	Oceania
Drought	629	53.1	15.9	22.1	4.9	4.0
Earthquake	874	6.3	18.4	47.1	18.2	10.0
Epidemic	935	61.5	7.8	26.7	3.2	0.7
Exteme Temperature	360	3.1	20.0	30.8	45.0	1.1
Famine (Natural)	48	70.8	4.2	20.8	4.2	0.0
Flood	2982	19.1	22.9	39.9	12.9	5.1
Insect infestation	82	82.9	3.7	11.0	1.2	1.2
Slide	451	5.5	25.3	54.5	10.6	4.0
Volcano	161	8.7	34.2	38.5	9.9	8.7
Wild fire	308	5.8	34.1	19.8	29.2	11.0
Wild Stom	2339	6.4	30.7	39.5	13.7	9.6
Wave/ Surge	38	13.2	13.2	63.2	2.6	7.9

Source: ARDC-Natural Disasters Data Book-2007

Note: The original figures were compiled by CRED-EMDAT where an event is counted as a disaster if it falls under any of the following: a) 10 or more people reported killed, b) 100 or more people reported affected, c) declaration of a state of emergency, or d) call for international assistance.

following tsunami in Japan were very rare events[3]; however, this was not the first time such events had occurred as the Tohoku area was hit by similar large earthquakes in 390 BC, 430, 869 and 1500. The frequencies of disasters vary across their types and regions as indicated in Table 1.

We model these disaster characteristics by considering two types of disasters: *historical* disasters and *periodic* disasters. The first is a very rare event with a huge destructive magnitude such as the Tohoku earthquake and tsunami. This type of disaster seriously damages both physical and

[3] The Tohoku earthquake was the one of the five strongest earthquakes recorded since modern record keeping began. The earthquake and tsunami caused nearly 19,850 deaths and cost US$ 210 billion (Guha-Sapir et al., 2011). These figures respectively correspond to 64.5% of global disaster mortality and 57.4% of global reported damage in 2011.

human resources and the occurrence is probabilistic. We assume that these disasters occur following a Bernoulli process. A periodic disaster is a more common disaster. Although any disaster is a stochastic event, if we consider a time period of several decades, many disasters in a disaster database as in Table 1 should look like regular events. For periodic disasters, we distinguish between the damage to human resources and physical resources.

The damage from a disaster can be mitigated by our efforts. As an economy develops, we can take more precautions against disasters. Empirically, Kahn (2005) found that although richer nations do not experience fewer natural disasters than poorer nations, richer nations suffer fewer deaths from disasters. Similar results are obtained by Burton et al. (1993), Tol and Leek (1999), and Toya and Skidmore (2009). Although how the damage differs depending on the stage of economic development is also an important research topic, we refrain from considering this relationship in this paper, to avoid the model becoming too complicated. For the same reason, we ignore the possibility that the level of economic activity increases the risk of disasters such as for the relationship between global warming and climatic disasters. This relationship is analyzed by Ikefuji and Horii (2012) in the context of sustainable growth and a pollution tax.

The rest of the paper is organized as follows. Section 2 describes the model. In Section 3, we first show the existence of a unique balanced growth path. Then, we examine the effects of the destructive magnitude of disasters and other factors on the growth rate. We also analyze the comparative statics of resource allocation and industrial composition at the steady state. Section 4 examines the stability and the properties of the transition dynamics. Section 5 concludes.

2 The model

We use a discrete-time model. Time is expressed by a nonnegative integer $t \in \mathbb{Z}_+ := \{0, 1, ...\}$. The tth period is the time interval from $t-1$ to t. There are two types of disasters: historical and periodic. We assume that the time interval is long enough, e.g., a couple of decades, so that the frequency of periodic disasters in each period is equalized.

2.1 Historical disasters

A historical disaster destroys all types of capital in the same proportion. Once such a disaster occurs, the capital stock decreases from x to Δx, where $\Delta \in (0,1)$ and $x \in \mathbb{R}^n_+$ is the vector of capital stocks. The occurrence of historical disasters follows a Bernoulli process. Denote by $\lambda \in (0,1)$ the probability that a historical disaster occurs in the present period. The probability that a historical disaster first occurs in the Tth period is given by $(1-\lambda)^{T-1}\lambda$. We assume that a historical disaster occurs at the beginning of a period and the economy reacts to the disaster during the period. The economy is described in an abstract fashion with a one-period return function $R(x,y)$, where x is the capital stocks at the beginning of the period and y is those at the end of the period. The feasible transition is given by a correspondence $y \in \Gamma(x)$. The problem is given by:

$$V(x) := \sup_{\{x_t\}_{t=1}^{\infty}} \mathbb{E}\left[\sum_{t=1}^{\infty} \beta^{t-1} R(D_t x_{t-1}, x_t) \,\middle|\, x_t \in \Gamma(D_t x_{t-1}),\ D_t = \begin{cases} 1 & \text{for } t \in \mathbb{Z}_{++} \backslash T^D \\ \Delta & \text{for } t \in T^D \end{cases},\ x_0 = x \in X \right], \tag{2.1}$$

where $\mathbb{E}$ is the expectation operator, $\mathbb{Z}_{++}$ is the set of positive integers, $\beta \in (0,1)$ is the discount factor, $X \subseteq \mathbb{R}^n_+$ is a cone and $T^D \subset \mathbb{Z}_{++}$ is the set of time points at which historical disasters occur. We make the following assumptions.

Assumption 1: $R : X \times X \mapsto \mathbb{R}$ is strictly quasi-concave and homogeneous of degree $\theta \in (-\infty, 0) \cup$

$(0,1)$. $R(x,y)$ is increasing in x and decreasing in y.

Assumption 2: $\Gamma : \mathbb{R}^n_+ \mapsto \mathbb{R}^n_+$ satisfies (a) $\Gamma(0) = \{0\}$ and (b) for any (x,y) such that $y \in \Gamma(x)$ and for any $k > 0$, $ky \in \Gamma(kx)$, i.e., the graph of Γ is a positive cone.

Lemma 2.1 *The value function V in (2.1) is the same as that in the following deterministic model:*

$$V(x) = \sup \left\{ \sum_{t=1}^{\infty} \left[\beta(\lambda\Delta^\theta + 1 - \lambda)\right]^{t-1} \{(\lambda\Delta^\theta + 1 - \lambda) R\left(x_{t-1}, x_t\right)\} \;\middle|\; x_t \in \Gamma\left(x_{t-1}\right), x_0 = x \in X \right\}. \tag{2.2}$$

Proof. Let $D^t := (D_1, D_2, \ldots, D_t) \in \{1, \Delta\}^t$ be a realization of historical disasters up until period $t \in \mathbb{Z}_{++}$ and $\pi(D^t) > 0$ be the probability of D^t being realized. Then

$$\sum_{D^t \in \{1,\Delta\}^t} \prod_{\tau=1}^{t} D_\tau^\theta \pi(D^t) = (\lambda\Delta^\theta + 1 - \lambda) \sum_{D^{t-1} \in \{1,\Delta\}^{t-1}} \prod_{\tau=1}^{t-1} D_\tau^\theta \pi(D^{t-1}) = (\lambda\Delta^\theta + 1 - \lambda)^t, \tag{2.3}$$

and thus

$$\begin{aligned}
V(x) &= \sup \left\{ \sum_{t=1}^{\infty} \beta^{t-1} \sum_{D^t \in \{1,\Delta\}^t} R[D_t x_{t-1}(D^{t-1}), x_t(D^t)] \pi(D^t) \;\middle|\; x_t(D^t) \in \Gamma[D_t x_{t-1}(D^{t-1})], x_0 = x \right\} \\
&= \sup \left\{ \sum_{t=1}^{\infty} \beta^{t-1} \sum_{D^t \in \{1,\Delta\}^t} \prod_{\tau=1}^{t} D_\tau^\theta R \left[\left(\prod_{\tau=1}^{t-1} D_\tau \right)^{-1} x_{t-1}(D^{t-1}), \left(\prod_{\tau=1}^{t} D_\tau \right)^{-1} x_t(D^t) \right] \pi(D^t) \right. \\
&\qquad \left. \;\middle|\; \left(\prod_{\tau=1}^{t} D_\tau \right)^{-1} x_t(D^t) \in \Gamma \left[\left(\prod_{\tau=1}^{t-1} D_\tau \right)^{-1} x_{t-1}(D^{t-1}) \right], x_0 = x \right\} \\
&= \sup \left\{ \sum_{t=1}^{\infty} \beta^{t-1} \sum_{D^t \in \{1,\Delta\}^t} \prod_{\tau=1}^{t} D_\tau^\theta \pi(D^t) R\left(x_{t-1}, x_t\right) \;\middle|\; x_t \in \Gamma\left(x_{t-1}\right), x_0 = x \right\} \\
&= \sup \left\{ \sum_{t=1}^{\infty} \beta^{t-1} (\lambda\Delta^\theta + 1 - \lambda)^t R\left(x_{t-1}, x_t\right) \;\middle|\; x_t \in \Gamma\left(x_{t-1}\right), x_0 = x \right\}.
\end{aligned} \tag{2.4}$$

■

Alvarez and Stokey (1998) established the conditions under which a solution to the deterministic problem (2.2) exists. We call their conditions the regularity conditions for a dynamic programming problem with homogeneous functions. We assume the following.

Assumption 3: The regularity conditions for a dynamic programming problem with homogeneous functions are satisfied.

Let $g : X \mapsto X$ be the optimal policy function for the deterministic problem (2.2).

Lemma 2.2 *(a) Under Assumptions 1–3, there is a solution to the deterministic problem (2.2). (b) V is homogeneous of degree θ and g is homogeneous of degree one: For any $x \in X$ and any $k > 0$,*

$$V(kx) = k^{\theta} V(k), \tag{2.5}$$

$$g(kx) = kg(x). \tag{2.6}$$

Proof. (a) See Alvarez and Stokey (1998). (b) They follow from the assumptions that R is homogeneous of degree θ and Γ is a cone. ■

Now we can show the following.

Proposition 2.1 *Under Assumptions 1–3, there is a solution to the problem (2.1). The optimal policy is given by g.*

Proof. Consider the transition policy

$$x_t(D^t) = g(D_t x_{t-1}(D^{t-1})). \tag{2.7}$$

As g is homogeneous of degree one (Lemma 2.2), it holds that:

$$x_t(D^t) = D_t g(x_{t-1}(D^{t-1})) = D_t g^2(D_{t-1}x_{t-2}(D^{t-2})) = \dots = \left(\prod_{\tau=1}^{t} D_\tau\right) g^t(x_0), \tag{2.8}$$

where $g^t(x) = g(g^{t-1}(x))$ and $g^0(x) = x$. Then, the total utility obtained from this transition policy is given by:

$$\begin{aligned}
&\sum_{t=1}^{\infty} \beta^{t-1} \sum_{D^t \in \{1,\Delta\}^t} R[D_t x_{t-1}(D^{t-1}), x_t(D^t)]\pi(D^t) \\
&= \sum_{t=1}^{\infty} \beta^{t-1} \sum_{D^t \in \{1,\Delta\}^t} R\left[\left(\prod_{\tau=1}^{t} D_\tau\right) g^{t-1}(x_0), \left(\prod_{\tau=1}^{t} D_\tau\right) g^t(x_0)\right] \pi(D^t) \\
&= \sum_{t=1}^{\infty} \beta^{t-1} \sum_{D^t \in \{1,\Delta\}^t} R\left[g^{t-1}(x_0), g^t(x_0)\right] \left(\prod_{\tau=1}^{t} D_\tau\right) \pi(D^t) \\
&= \sum_{t=1}^{\infty} \beta^{t-1} (\lambda\Delta^\theta + 1 - \lambda)^t R\left[g^{t-1}(x_0), g^t(x_0)\right] \\
&= V(x_0).
\end{aligned} \tag{2.9}$$

Then, the claim follows from Lemma 2.1. ■

In subsequent sections, we shall use, instead of (2.2), the following deterministic problem:

$$\max_{\{x_t\}_{t=1}^{\infty}} \left[\sum_{t=1}^{\infty} \tilde{\beta}^{t-1} R(x_{t-1}, x_t) | x_0 = x \in X,\ x_t \in \Gamma(x_{t-1}) \text{ for all } t \geq 1\right], \tag{2.10}$$

where

$$\tilde{\beta} = \Big(\lambda\Delta^\theta + (1-\lambda)\Big)\beta. \tag{2.11}$$

Obviously, g is the optimal policy function of this problem. Furthermore, note that in the case of $\theta < 0$, we may have $\tilde{\beta} > 1$. Even in this case, the solution of (2.10) exists as long as Assumption 3 holds. More specifically, the condition is that there is homogeneous of degree one selection γ from Γ and a positive number ν such that $\tilde{\beta}\nu^{\theta} < 1$ and $\|\gamma(x)\| \geq \nu \|x\|$ for all $x \in X$ (Alvarez and Stokey, 1998, Assumption 2c.)

2.2 Periodic disasters

The periodic disaster destroys each production factor in a different manner. To capture this feature, we distinguish physical capital, human capital and raw labor. Let $F(k,l)$ be the production function for the final good where k is the physical capital stock and l is the effective labor. The effective labor is given by the product of the human capital stock B and raw labor input $1-n$, i.e., $l = B(1-n)$, where we assume that the total labor input is constant over time and normalized to one. The accumulation of the physical capital is given by:

$$K_t = F\left[\delta K_{t-1}, \zeta\left(1-n_t\right) B_{t-1}\right] - C_t, \tag{2.12}$$

where C_t is the consumption at the tth period, $\delta \in (0,1)$ is the measure of the destructive magnitude of periodic disasters on the physical capital in a period and $\zeta \in (0,1)$ is the measure on the raw labor input.[4] Note that the depreciation rate of physical capital is 100 percent because we assume that the time interval of one period is adequately long. We assume the following.

Assumption 4: $F : \mathbb{R}_+^2 \mapsto \mathbb{R}_+$ is smooth, concave and homogeneous of degree one with:

$$F(0,0) = 0,\ \ F_k > 0, F_l > 0, F_{kk} < 0, F_{ll} < 0, \tag{2.13}$$

$$\lim_{z \to 0} F_k(z,l) = \infty,\ \ \lim_{z\to\infty} F_k(z,l) = 0, \tag{2.14}$$

[4] We assume that the human damage by a periodic disaster is recovered in a period and thus the raw labor supply after the disaster equals the constant ζ over all periods.

for all $k > 0$ and $l > 0$.

The production function of the human capital sector is $G(l) = \eta l$ where $l = Bn$ is the effective labor and $\eta > 0$ is the productivity coefficient. As with the physical capital, we assume that the depreciation rate of human capital is 100 percent.[5] Thus, the accumulation of the human capital is given by:

$$B_t = \eta\zeta n_t B_{t-1}. \tag{2.15}$$

One-period utility is given by:

$$u(C) = \frac{C^\theta}{\theta}, \quad \theta \neq 0, \theta < 1, \tag{2.16}$$

where θ is the elasticity of the marginal utility. Note that when $\theta = 0$, (2.16) is not well defined. Furthermore, note that $u(C) = \ln C$ is not a homogeneous function. For these reasons, we exclude the case of $\theta = 0$.

Now, we have specified the growth model (2.1) with specific technology and preferences given by (2.12), (2.15), (2.16) and the discount factor β. The problem (2.1) is now written as

$$V(x) := \max_{\{C_t\}_{t=1}^{\infty}} \mathbb{E}\left[\sum_{t=1}^{\infty} \beta^{t-1} u(C_t)\right]$$

$$\text{subject to } K_t = F(D_t\delta K_{t-1}, D_t\zeta B_{t-1}(1-n_t)) - C_t,$$

$$B_t = D_t\left[\eta\zeta\left(n_t B_{t-1}\right)\right], \; D_t = \begin{cases} 1 & \text{for } t \in \mathbb{Z}_{++}\backslash T^D \\ \Delta & \text{for } t \in T^D \end{cases},$$

$$(K_0, B_0) \in X \text{ given.}$$

[5] Note that $n = 0$ implies no investment in education over decades. It will deprive people of the ability to learn from past knowledge. This is the meaning of a 100 percent depreciation rate for human capital.

Note that this two-sector endogenous growth model satisfies the three assumptions in the previous subsection. Then, by Proposition 2.1, the optimal policy function of the problem coincides with the following deterministic growth model with the effective discount factor $\tilde{\beta}$:

$$\max_{\{C_t\}_{t=1}^{\infty}} \sum_{t=1}^{\infty} \tilde{\beta}^{t-1} u(C_t) \text{ subject to (2.12), (2.15), } C_t \geq 0, \ K_0 \text{ and } B_0 \text{ are given.} \tag{2.17}$$

It is convenient to define the reduced form return function associated with (2.17) as follows:

$$R(K_{t-1}, B_{t-1}, K_t, B_t) = \frac{\left[F\left(\delta K_{t-1}, \zeta B_{t-1} - \eta^{-1} B_t\right) - K_t\right]^{\theta}}{\theta}. \tag{2.18}$$

The transition possibility set is given by $(K_t, B_t) \in \Gamma(K_{t-1}, B_{t-1})$ where

$$\Gamma(K, B) = \left\{(k, b) \in \mathbb{R}_+^2 \left| F\left(\delta K, \zeta B - \eta^{-1} b\right) - k \geq 0 \right.\right\}. \tag{2.19}$$

The problem (2.17) is expressed as:

$$\max_{\{(K_t,B_t)\}_{t=1}^{\infty}} \sum_{t=1}^{\infty} \tilde{\beta}^{t-1} R(K_{t-1}, B_{t-1}, K_t, B_t) \text{ subject to } (K_t, B_t) \in \Gamma(K_{t-1}, B_{t-1}), \ K_0 \text{ and } B_0 \text{ given.} \tag{2.20}$$

3 Balanced growth path

3.1 Existence and comparative statics of the growth path

This section studies the balanced growth path. We shall demonstrate its existence and investigate its properties. The Euler equations associated with (2.20) are given by:

$$-(C_t)^{\theta-1} + \tilde{\beta}(C_{t+1})^{\theta-1}\delta F_k\left(\delta K_t, \zeta B_t - \eta^{-1}B_{t+1}\right) = 0, \tag{3.1a}$$

$$-\eta^{-1}(C_t)^{\theta-1}F_l\left(\delta K_{t-1}, \zeta B_{t-1} - \eta^{-1}B_t\right) + \tilde{\beta}(C_{t+1})^{\theta-1}\zeta F_l\left(\delta K_t, \zeta B_t - \eta^{-1}B_{t+1}\right) = 0. \tag{3.1b}$$

Let

$$\hat{k}_t := \frac{\eta\delta K_t}{\eta\zeta B_t - B_{t+1}} \text{ and } \hat{x}_t := \frac{C_{t+1}}{C_t}. \tag{3.2}$$

As F_k and F_l are homogeneous of degree zero, the Euler equations are written as:

$$\tilde{\beta}(\hat{x}_t)^{\theta-1}\delta F_k\left(\hat{k}_t, 1\right) = 1, \tag{3.3a}$$

$$\tilde{\beta}(\hat{x}_t)^{\theta-1}\eta\zeta F_l\left(\hat{k}_t, 1\right) = F_l\left(\hat{k}_{t-1}, 1\right). \tag{3.3b}$$

Then, we have the following.

$$F_k\left(\hat{k}_t, 1\right) = \frac{\eta\zeta}{\delta}\frac{F_l\left(\hat{k}_t, 1\right)}{F_l\left(\hat{k}_{t-1}, 1\right)}, \tag{3.4a}$$

$$\hat{x}_t = \left[\tilde{\beta}\delta F_k\left(\hat{k}_t, 1\right)\right]^{\frac{1}{1-\theta}}. \tag{3.4b}$$

Denote by $\hat{k}^{ss}$ the interior steady state of the dynamic system (3.4a). $\hat{k}^{ss}$ is implicitly defined as:

$$F_k\left(\hat{k}^{ss}, 1\right) = \frac{\eta\zeta}{\delta}. \tag{3.5}$$

By Assumption 4, $F_k(k,1)$ is strictly decreasing in k and $\lim_{k\to 0} F_k(k,1) = \infty$ and $\lim_{k\to\infty} F_k(k,1) = 0$ hold. Thus, there is a unique steady state $\hat{k}^{ss}$ that satisfies (3.5). From (3.4b) and (3.5), the associated steady state consumption growth rate $\hat{x}^{ss}$ is given by:

$$\hat{x}^{ss} = (\tilde{\beta}\eta\zeta)^{\frac{1}{1-\theta}}. \tag{3.6}$$

Then, we have:

Proposition 3.1 *There is a unique steady state that is a balanced growth path, i.e., the products, consumptions, physical capital stocks and human capital stocks have the same growth rate:*

$$\hat{x}^{ss} = \left\{\beta\left[\lambda\Delta^{\theta} + (1-\lambda)\right]\eta\zeta\right\}^{\frac{1}{1-\theta}}. \tag{3.7}$$

Proof. The existence and uniqueness of $\hat{x}^{ss}$ are equivalent to those of $\hat{k}^{ss}$. The existence of $\hat{k}^{ss}$ follows from Assumption 4 and (3.5), as mentioned above. Denote by Y_t^{ss}, C_t^{ss}, K_t^{ss} and B_t^{ss}, the products, consumptions, physical capital stocks and human capital stocks along the steady state, respectively. They satisfy:

$$K_t^{ss} = Y_t^{ss} - C_t^{ss} = Y_t^{ss} - (\hat{x}^{ss})^{t-1} C_1^{ss}, \tag{3.8a}$$

$$K_t^{ss} = \delta^{-1}\left[\zeta B_t^{ss} - \eta^{-1} B_{t+1}^{ss}\right]\hat{k}^{ss}, \tag{3.8b}$$

where (3.8a) follows from (2.17) and the definition of $\hat{x}^{ss}$, and (3.8b) follows from the definition of $\hat{k}^{ss}$. (3.8a) implies that the growth rates of K_t^{ss} and Y_t^{ss} are equal to $\hat{x}^{ss}$. Then, (3.8b) implies that the growth rate of B_t^{ss} is also $\hat{x}^{ss}$. Finally, (3.7) is obtained by substituting (2.11) into (3.6). ■

Now consider the effect of θ on $\hat{x}^{ss}$. As the intertemporal elasticity of substitution for consumption is given by $1/(1-\theta)$, it is usual in a growth model that the gross growth rate $\hat{x}^{ss}$ approaches one as θ decreases. This law is modified in this model, because θ affects not only the curvature of the one-period utility function but also the effective discount factor after adjusting the risk of a historical disaster, $\tilde{\beta}$. The derivative of the steady state growth rate with respect to θ is given by:

$$\frac{\partial \hat{x}^{ss}}{\partial \theta} = \hat{x}^{ss} \frac{\partial \ln \hat{x}^{ss}}{\partial \theta} = \hat{x}^{ss} \left[\frac{\ln \hat{x}^{ss}}{1-\theta} + \frac{1}{1-\theta} \frac{\lambda \Delta^{\theta} \ln \Delta}{\lambda \Delta^{\theta} + (1-\lambda)} \right]. \tag{3.9}$$

The first term in brackets is the usual "egalitarian" effect. That is, $\ln \hat{x}^{ss} / (1-\theta) \gtreqless 0$ if $\hat{x}^{ss} \gtreqless 1$. The second term is added because of the risk of a disaster. This risk effect always increases the steady state growth rate as θ decreases. Furthermore, note that:

$$\lim_{\theta \to -\infty} \ln \hat{x}^{ss} = \lim_{\theta \to -\infty} \frac{\ln \beta \eta \zeta + \ln \left[\lambda \Delta^{\theta} + (1-\lambda) \right]}{1-\theta} = \lim_{\theta \to -\infty} \frac{-\ln \Delta}{1 + (1-\lambda)\lambda \Delta^{-\theta}} = -\ln \Delta. \tag{3.10}$$

Therefore, at the limit of $\theta \to -\infty$, the steady state growth rate is equal to $1/\Delta$, which is greater than one.

Using Proposition 3.1, we have the effects of several types of disasters on the long-run growth rate.

Proposition 3.2 *The steady state growth rate $\hat{x}^{ss}$ satisfies:*

$$\frac{\partial \hat{x}^{ss}}{\partial \Delta} \gtreqless 0 \textit{ if } \theta \gtreqless 0, \quad \frac{\partial \hat{x}^{ss}}{\partial \lambda} \lesseqgtr 0 \textit{ if } \theta \gtreqless 0, \quad \frac{\partial \hat{x}^{ss}}{\partial \delta} = 0, \quad \frac{\partial \hat{x}^{ss}}{\partial \zeta} > 0. \tag{3.11}$$

Proof. From (3.7) in Proposition 3.1, $\partial \hat{x}^{ss}/\partial \delta = 0$ is obvious. The other results follow from simple calculations:

$$\frac{\partial \hat{x}^{ss}}{\partial \Delta} = \frac{1}{1-\theta} \frac{\hat{x}^{ss}\left(\lambda \theta \Delta^{\theta-1}\right)}{\left[\lambda \Delta^{\theta} + (1-\lambda)\right]} \gtrless 0 \text{ if } \theta \gtrless 0, \tag{3.12a}$$

$$\frac{\partial \hat{x}^{ss}}{\partial \lambda} = \frac{1}{1-\theta} \frac{\hat{x}^{ss}\left(\Delta^{\theta} - 1\right)}{\left[\lambda \Delta^{\theta} + (1-\lambda)\right]} \lessgtr 0 \text{ if } \theta \gtrless 0, \tag{3.12b}$$

$$\frac{\partial \hat{x}^{ss}}{\partial \zeta} = \frac{1}{1-\theta} \frac{\hat{x}^{ss}}{\zeta} > 0. \tag{3.12c}$$

■

There are both intuitive and less intuitive results. An intuitive result is $\partial \hat{x}^{ss}/\partial \zeta > 0$: the long-run growth rate decreases as the destructive magnitude of the periodic disasters on labor $(1-\zeta)$ increases. As ζ decreases, the effective productivity in the growth engine sector (the human capital sector) $\zeta\eta$ decreases and the steady state growth rate decreases.

One of the less intuitive results is $\partial \hat{x}^{ss}/\partial \delta = 0$: the destructive magnitude of the periodic disasters on physical capital $(1-\delta)$ does *not* affect the long-run growth rate. The reason is that the productivity of the growth engine sector is not affected by the destructive magnitude of periodic disaster on physical capital. The implication of these results is that saving people's lives against a periodic disaster is more important than saving physical capital not only from a humanitarian viewpoint, but also to sustain a higher growth rate.

Another less intuitive result is that the effects of the historical disaster depend on the sign of the elasticity of marginal utility. An undesirable change, a decrease in Δ or an increase in λ, causes a reduction in the growth rate $\hat{x}^{ss}$ if $\theta > 0$. However, if $\theta < 0$, then we have the opposite result, i.e., when these parameters change in an undesirable direction, the steady state growth rate increases. This is the paradoxical case that corresponds to the empirical finding by Skidmore and Toya (2002).

To see why the result depends on the sign of θ, let:

$$\varphi(\Delta, \lambda; \theta) = \lambda \Delta^{\theta} + (1 - \lambda). \tag{3.13}$$

We first consider the case of $\theta > 0$. In this case,

$$\frac{\partial \varphi}{\partial \Delta} = \theta \lambda \Delta^{\theta - 1} > 0, \quad \frac{\partial \varphi}{\partial \lambda} = \Delta^{\theta} - 1 < 0. \tag{3.14}$$

This shows that an undesirable change (a decrease in Δ or an increase in λ) decreases the risk-adjusted discount factor $\tilde{\beta} = \varphi\beta$, because the change lowers the present value of future utility. As a result, the steady state growth rate decreases. In the case of $\theta < 0$, we have opposite signs for (3.14), i.e., an undesirable change increases the risk-adjusted discount factor. Note that this also makes the present value of future utility lower, because the utility takes a negative value. As a result, the steady state growth rate increases in this case. The following alternative explanation in the case of $\theta < 0$ might make more sense. Rewrite the objective function of the problem (2.20) as:

$$\sum_{t=1}^{\infty} \beta^{t-1} \left[\varphi^{t-1} u(C_t) \right], \quad u(C_t) := (C_t)^{\theta}/\theta, \tag{3.15}$$

where $\varphi^{t-1} u(C_t)$ is the expected (or risk-adjusted) current-value one-period utility. The derivatives of marginal expected utility with respect to Δ and λ are given by:

$$\frac{\partial \varphi^{t-1} u'(C_t)}{\partial \Delta} = \theta (t-1) \varphi^{t-2} \lambda \Delta^{\theta-1} (C_t)^{\theta-1} < 0, \tag{3.16a}$$

$$\text{and } \frac{\partial \varphi^{t-1} u'(C_t)}{\partial \lambda} = (t-1) \varphi^{t-2} \left(\Delta^{\theta} - 1 \right) (C_t)^{\theta-1} > 0. \tag{3.16b}$$

Their time derivatives are:

$$\frac{\partial \varphi(\Delta,\lambda;\theta)^{t-1}u'(C_t)}{\partial t \partial \Delta} = \theta\lambda\Delta^{\theta-1}(C_t)^{\theta-1}\left[\varphi^{t-2}+(t-1)\varphi^{t-2}\ln\varphi\right] < 0, \tag{3.17a}$$

$$\text{and } \frac{\partial \varphi(\Delta,\lambda;\theta)^{t-1}u'(C_t)}{\partial t \partial \lambda} = \left(\Delta^{\theta}-1\right)(C_t)^{\theta-1}\left[\varphi^{t-2}+(t-1)\varphi^{t-2}\ln\varphi\right] > 0. \tag{3.17b}$$

(3.16) implies that an undesirable change increases the marginal expected utility, and (3.17) implies that the gain is greater in a later period. Then, optimality requires greater consumption in the future, and the steady state growth rate is adjusted upward.

3.2 Resource allocation and industrial composition at the steady state

We then investigate the effects of disasters on resource allocation at the steady state. Regarding the final goods sector, we have the following.

Proposition 3.3 *The effective capital/labor ratio $\hat{k}^{ss}$ in the final goods sector satisfies:*

$$\frac{\partial \hat{k}^{ss}}{\partial \Delta}=0, \quad \frac{\partial \hat{k}^{ss}}{\partial \lambda}=0, \quad \frac{\partial \hat{k}^{ss}}{\partial \delta}>0, \quad \frac{\partial \hat{k}^{ss}}{\partial \zeta}<0. \tag{3.18}$$

The raw labor input share n^{ss} to the human capital sector at the steady state satisfies:

$$\frac{\partial n^{ss}}{\partial \Delta} \gtreqless 0 \textit{ if } \theta \gtreqless 0, \quad \frac{\partial n^{ss}}{\partial \lambda} \lesseqgtr 0 \textit{ if } \theta \gtreqless 0, \quad \frac{\partial n^{ss}}{\partial \delta}=0, \quad \frac{\partial n^{ss}}{\partial \zeta} \gtreqless 0 \textit{ if } \theta \gtreqless 0. \tag{3.19}$$

Proof. From (3.5), we have:

$$F_{kk}\left(\hat{k}^{ss},1\right)d\hat{k}^{ss}+F_k\left(\hat{k}^{ss},1\right)\frac{d\delta}{\delta}-F_k\left(\hat{k}^{ss},1\right)\frac{d\zeta}{\zeta}=0. \tag{3.20}$$

Then, by Assumption 4, we have (3.18). Proposition 3.1 and (2.15) imply $\hat{x}^{ss} = B_t^{ss}/B_{t-1}^{ss} = \eta\zeta n^{ss}$. Then, we have:

$$\frac{\partial n^{ss}}{\partial \Delta} = \frac{1}{\eta\zeta}\frac{\partial \hat{x}^{ss}}{\partial \Delta}, \quad \frac{\partial n^{ss}}{\partial \lambda} = \frac{1}{\eta\zeta}\frac{\partial \hat{x}^{ss}}{\partial \lambda}, \quad \frac{\partial n^{ss}}{\partial \delta} = \frac{1}{\eta\zeta}\frac{\partial \hat{x}^{ss}}{\partial \delta}, \text{ and } \frac{\partial n^{ss}/n^{ss}}{\partial \zeta/\zeta} = \frac{\partial \hat{x}^{ss}/\hat{x}^{ss}}{\partial \zeta/\zeta} - 1 = \frac{\theta}{1-\theta}, \tag{3.21}$$

where we use (3.12c) for the derivation of $\partial n^{ss}/\partial \zeta$. Then, by using (3.11), we have (3.19). ■

The results for $\hat{k}^{ss}$ are intuitive. As historical disasters destroy all capital types in the same proportion, it does not affect the relative prices of the capital stocks, which results in $\partial \hat{k}^{ss}/\partial \Delta = 0$ and $\partial \hat{k}^{ss}/\partial \lambda = 0$. The sign of $\partial \hat{k}^{ss}/\partial \alpha$ $(\alpha = \delta, \zeta)$ indicates a normal substitution effect.

Some results for n^{ss} are the same as those for $\hat{x}^{ss}$. This is reasonable because the steady state growth rate $\hat{x}^{ss}$ is linear in ζn^{ss} and the sign of $\partial \zeta n^{ss}/\partial \alpha$ is the same as that for $\partial n^{ss}/\partial \alpha$ for parameters, $\alpha = \Delta$, λ, and δ. The sign of $\partial n^{ss}/\partial \zeta$ depends on θ. As is seen in (3.21), if the ζ elasticity of $\hat{x}^{ss}$ is greater than one, then it is positive and vice versa. As θ decreases, the egalitarian effect strengthens and the ζ elasticity of $\hat{x}^{ss}$ becomes small, accompanying the reduction in labor input in the human capital sector.

Then, we show the effects of disasters on the industrial composition at the steady state, which we represent by $\delta K_t^{ss}/B_t^{ss}$. Note that from the definition of $\hat{k}^{ss}$ in (3.2) and Proposition 3.1, we have:

$$(K/B)^{ss} := \frac{\delta K_t^{ss}}{B_t^{ss}} = \frac{\eta\zeta - \hat{x}^{ss}}{\eta}\hat{k}^{ss}. \tag{3.22}$$

Denote by ε^{ss} the elasticity of the marginal productivity of the physical capital at the steady state:

$$\varepsilon^{ss} = -\frac{\hat{k}^{ss} F_{kk}\left(\hat{k}^{ss}, 1\right)}{F_k\left(\hat{k}^{ss}, 1\right)}. \tag{3.23}$$

Proposition 3.4 *For the effective physical/human capital ratio at the steady state, it holds that:*

$$\frac{\partial\left(K/B\right)^{ss}}{\partial\Delta}\lesseqgtr 0 \;\; if \; \theta\gtreqless 0, \quad \frac{\partial\left(K/B\right)^{ss}}{\partial\lambda}\gtreqless 0 \;\; if \; \theta\gtreqless 0, \quad \frac{\partial\left(K/B\right)^{ss}}{\partial\delta}>0, \tag{3.24}$$

and

$$\frac{\partial\left(K/B\right)^{ss}}{\partial\zeta}\left\{\begin{array}{c} > \\ \\ < \end{array}\right\}0 \;\; if \left\{\begin{array}{l} \frac{1}{1-\theta}<\frac{1}{\varepsilon^{ss}}<1 \\ \\ 1<\frac{1}{\varepsilon^{ss}}<\frac{1}{1-\theta} \end{array}\right. . \tag{3.25}$$

Proof. A simple calculation yields:

$$\frac{\partial\left(K/B\right)^{ss}}{\partial\Delta}=\frac{-\hat{k}^{ss}}{\eta}\frac{\partial\hat{x}^{ss}}{\partial\Delta}, \quad \frac{\partial\left(K/B\right)^{ss}}{\partial\lambda}=\frac{-\hat{k}^{ss}}{\eta}\frac{\partial\hat{x}^{ss}}{\partial\lambda}, \quad \frac{\partial\left(K/B\right)^{ss}}{\partial\delta}=\frac{\eta\zeta-\hat{x}^{ss}}{\eta}\frac{\partial\hat{k}^{ss}}{\partial\delta}. \tag{3.26}$$

Then the results in (3.24) follow from (3.11) in Proposition 3.2 and (3.18) in Proposition 3.3. As for ζ, (3.25) follows from:

$$\begin{aligned}\frac{\partial\left(K/B\right)^{ss}}{\partial\zeta}&=\hat{k}^{ss}\left(1-\eta^{-1}\frac{\partial\hat{x}^{ss}}{\partial\zeta}\right)+\frac{\eta\zeta-\hat{x}^{ss}}{\eta}\frac{\partial\hat{k}^{ss}}{\partial\zeta}\\&=\hat{k}^{ss}\left[\left(1-\frac{1}{\varepsilon^{ss}}\right)-\frac{\hat{x}^{ss}}{\eta\zeta}\left(\frac{1}{1-\theta}-\frac{1}{\varepsilon^{ss}}\right)\right].\end{aligned}$$

■

The effect of historical disasters on the industrial composition depends on the sign of θ. This result is consistent with the effect on the steady state growth rate in Proposition 3.2. That is, when θ is positive, the growth rate decreases responding to the decrease in Δ or the increase in λ. This change accompanies the change in capital allocation from human capital to physical capital. This causes final goods production to increase and capital accumulation in the growth engine sector to slow. When θ is negative, the opposite adjustment occurs.

As for periodic disasters, the result for δ, $\partial\left(K/B\right)^{ss}/\partial\delta > 0$, is clear and intuitive. In contrast, the result for ζ is ambiguous, except for the two cases in (3.25). The reason is that the destruction of labor affects both the human capital sector and the final goods sector.

Finally, we examine the effects of disasters on the saving rate at the steady state:

$$\hat{s}^{ss} := 1 - \frac{C_t^{ss}}{Y_t^{ss}} = \frac{K_t^{ss}}{Y_t^{ss}}. \tag{3.27}$$

Proposition 3.5

$$\frac{\partial \hat{s}^{ss}}{\partial \Delta} \gtreqless 0 \ \textit{if}\ \theta \gtreqless 0, \quad \frac{\partial \hat{s}^{ss}}{\partial \lambda} \lesseqgtr 0 \ \textit{if}\ \theta \gtreqless 0, \quad \frac{\partial \hat{s}^{ss}}{\partial \delta} < 0 \ \textit{if}\ \varepsilon^{ss} \geq 1, \quad \frac{\partial \hat{s}^{ss}}{\partial \zeta} > 0 \ \textit{if}\ \varepsilon^{ss} \geq 1-\theta. \tag{3.28}$$

Proof. Note first that:

$$\frac{K_t^{ss}}{Y_t^{ss}} = \frac{\hat{x}^{ss}K_{t-1}^{ss}}{F\left(\delta K_{t-1}^{ss}, \zeta B_{t-1}^{ss} - \eta^{-1}B_t^{ss}\right)} = \frac{\hat{x}^{ss}\hat{k}^{ss}/\delta}{F\left(\hat{k}^{ss}, 1\right)}. \tag{3.29}$$

Therefore:

$$\ln \hat{s}^{ss} = \ln \hat{x}^{ss}(\Delta, \lambda, \zeta) + \ln \hat{k}^{ss}(\delta, \zeta) - \ln F\left(\hat{k}^{ss}(\delta,\zeta), 1\right) - \ln \delta. \tag{3.30}$$

Here, we explicitly express the dependence of $\hat{x}^{ss}$ and $\hat{k}^{ss}$ on the disaster-related parameters. From (3.30):

$$\frac{\partial \ln \hat{s}^{ss}}{\partial \Delta} = \frac{1}{\hat{x}^{ss}} \frac{\partial \hat{x}^{ss}}{\partial \Delta}, \tag{3.31}$$

$$\frac{\partial \ln \hat{s}^{ss}}{\partial \lambda} = \frac{1}{\hat{x}^{ss}} \frac{\partial \hat{x}^{ss}}{\partial \lambda}, \tag{3.32}$$

$$\begin{aligned}
\frac{\partial \ln \hat{s}^{ss}}{\partial \delta} &= \left[\frac{1}{\hat{k}^{ss}} - \frac{F_k\left(\hat{k}^{ss}, 1\right)}{F\left(\hat{k}^{ss}, 1\right)} \right] \frac{\partial \hat{k}^{ss}}{\partial \delta} - \frac{1}{\delta} \\
&= \frac{1}{\delta} \left[\frac{\left(\varepsilon^{ss}\right)^{-1} F_l\left(\hat{k}^{ss}, 1\right)}{F\left(\hat{k}^{ss}, 1\right)} - 1 \right] \text{ (by (3.20))} \\
&< \frac{1}{\delta} \left[\frac{1}{\varepsilon^{ss}} - 1 \right],
\end{aligned} \tag{3.33}$$

$$\begin{aligned}
\frac{\partial \ln \hat{s}^{ss}}{\partial \zeta} &= \frac{1}{\hat{x}^{ss}} \frac{\partial \hat{x}^{ss}}{\partial \zeta} + \left[\frac{1}{\hat{k}^{ss}} - \frac{F_k\left(\hat{k}^{ss}, 1\right)}{F\left(\hat{k}^{ss}, 1\right)} \right] \frac{\partial \hat{k}^{ss}}{\partial \zeta} \\
&= \frac{1}{(1-\theta)\zeta} + \frac{1}{\hat{k}^{ss}} \frac{\partial \hat{k}^{ss}}{\partial \zeta} \frac{F_l\left(\hat{k}^{ss}, 1\right)}{F\left(\hat{k}^{ss}, 1\right)} \text{ (by (3.12c))} \\
&= \frac{1}{\zeta} \left[\frac{1}{1-\theta} - \frac{\left(\varepsilon^{ss}\right)^{-1} F_l\left(\hat{k}^{ss}, 1\right)}{F\left(\hat{k}^{ss}, 1\right)} \right] \text{ (by (3.20))} \\
&> \frac{1}{\zeta} \left[\frac{1}{1-\theta} - \frac{1}{\varepsilon^{ss}} \right].
\end{aligned} \tag{3.34}$$

These results prove the proposition. ■

The effects of the historical disasters on the saving rate $\hat{s}^{ss}$ depend on the elasticity of the marginal utility and are consistent with the effects on the steady state growth rate. The effects of periodic disasters are ambiguous, except for the cases in (3.28). As seen in (3.29), the parameters affect the saving rate through three terms, $\hat{x}^{ss}$, $\hat{k}^{ss}$ and δ. Regarding the destructive magnitude on the physical capital, the direct effect of δ is negative, while the indirect effect through $\hat{k}^{ss}$ is positive. Note that $\partial \hat{k}^{ss}/\partial \delta > 0$ and the increment rate of $\hat{k}^{ss}$ in the numerator of (3.29) is greater than $F\left(\hat{k}^{ss}, 1\right)$ in the denominator. Regarding the effect of the destructive magnitude on the human resource, the effect of ζ through $\hat{x}^{ss}$ is positive because of $\partial\hat{x}^{ss}/\partial\zeta > 0$, while the effect through $\hat{k}^{ss}$ is positive because of $\partial\hat{k}^{ss}/\partial\zeta < 0$.

The analysis in this section shows how the economy is influenced by disasters in the long run and in particular reveals how the paradoxical case emerges. It should be worth emphasizing that despite the seemingly paradoxical relationship between disasters and the long-term growth rate, its welfare implication is always unambiguous. In the case of $\theta < 0$, a higher risk of historical disasters facilitates more savings, shifts emphasis from physical to human capital, and as a result yields a higher growth rate in the long run. This is only possible, however, at the cost of suppressed consumption by the current generation, suggesting that a higher long-term growth rate does not necessarily mean improved welfare. In fact, decreases in Δ or increases in λ decrease welfare even though the corresponding long-term rate of economic growth becomes higher. Hence, disaster-prevention efforts improve welfare. In the case of $\theta > 0$, on the other hand, the same risk of historical disasters rather drives higher consumption by the current generation in anticipation of sudden drops of productivity in the future, ending up with a lower long-term growth rate. In this case, preventing disasters either by increasing Δ or by decreasing λ improves social welfare as well as the long-term growth rate.

4 Stability of the steady state and transition phase

4.1 Translation of the results of the deterministic model into the stochastic model

In this section, we examine the stability of the steady state, the property of the transition path, and the speed of convergence to the steady state. We shall continue to study the deterministic model (2.20). However, because the original model is stochastic, the results obtained from the deterministic model should be carefully addressed. We first discuss how the results from the deterministic model are interpreted within the original stochastic model (2.1).

Let $\{C_t^o\}_{t=1}^{\infty}$ be the optimal consumption path induced by the deterministic model. Furthermore, let

$$\hat{x}_t^o = C_{t+1}^o / C_t^o \tag{4.1}$$

be the associated growth rate. In the original stochastic model, this path is realized when a historical disaster never occurs. Although the probability that we have this path is zero, it is useful as a reference.

Now move to the original stochastic model and consider what happens to the economy when a historical disaster hits at t. As the production technology is a cone and a historical disaster destroys all the capital stocks at the same rate $1-\Delta$, there is no change to the physical/human capital ratio K_t/B_t. As a result, although the levels of these capital stocks are reduced by historical disasters, the optimal path of ratio $\{K_t/B_t\}_{t=1}^{\infty}$ does not change. Note that the optimal consumption policy, say $C = C(K, B)$, satisfies $C(\Delta K, \Delta B) = \Delta C(K, B)$. A sample path of consumptions $\{C_t\}_{t=1}^{\infty}$ is expressed as $C_t = \Pi_{\tau=1}^{t} D_\tau C_t^o$, where D_τ is defined in (2.1). The associated growth rate path is given by:

$$\hat{x}_t = \Pi_{\tau=1}^{t+1} D_\tau C_{t+1}^o / \Pi_{\tau=1}^{t} D_\tau C_t^o = D_{t+1} \hat{x}_t^o.$$

Therefore, the growth rate is the same as the reference growth rate $\hat{x}_t^o$, unless a historical disaster occurs in $t+1$. The results obtained from the deterministic models (2.20) are true in the stochastic model except for these periods. Let $\# : \mathbb{Z}_{++} \to \mathbb{Z}_{+}$ be the number of historical disasters that occur during period t. The average number of occurrences of historical disasters is given by:

$$\lim_{T\to\infty} \frac{\mathbb{E}[\#(T)]}{T} = \lim_{T\to\infty} \frac{1}{T} \sum_{s=0}^{T} \begin{pmatrix} T \\ s \end{pmatrix} s \lambda^s (1-\lambda)^{T-s} = \lambda.$$

As long as λ is very small (because the historical disaster is a very rare event), disturbances by historical disasters have little effect on the dynamics of the growth rate path $\{x_t\}_{t=1}^{\infty}$.

4.2 Stability and properties of the transition paths

Now we proceed to the analysis. We specify the production function to be CES.

Assumption 5:

$$F(k,l) = \left[\gamma k^{\frac{\sigma-1}{\sigma}} + (1-\gamma) l^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \quad \gamma \in (0,1), \sigma > 0, \sigma \neq 1. \tag{4.2}$$

As is well known, a CES function does not satisfy the Inada conditions (2.14) in Assumption 4, except for the case of $\sigma = 1$, i.e., the Cobb–Douglas production function. More precisely, when $\sigma > 1$, the Inada condition $\lim_{k\to\infty} F_k(k,l) = 0$ does not hold, whereas when $\sigma < 1$, another Inada condition $\lim_{k\to 0} F_k(k,l) = \infty$ does not hold. These Inada conditions are used only to ensure the existence of the steady state. We substitute these Inada conditions with some parameter restrictions.[6]

Assumption 6:

$$\lim_{k\to\infty} F_k(k,1) = \gamma^{\frac{\sigma}{\sigma-1}} < \frac{\eta\zeta}{\delta} \text{ if } \sigma > 1, \tag{4.3a}$$

$$\lim_{k\to 0} F_k(k,1) = \gamma^{\frac{\sigma}{\sigma-1}} > \frac{\eta\zeta}{\delta} \text{ if } \sigma < 1. \tag{4.3b}$$

In this section, we maintain Assumptions 5 and 6. Note that the CES function satisfies (2.13) of Assumption 4.

[6] This follows from (3.5).

The first derivatives of the production function (4.2) become:

$$F_k(k,l) = \gamma \left[\gamma k^{\frac{\sigma-1}{\sigma}} + (1-\gamma) l^{\frac{\sigma-1}{\sigma}}\right]^{\frac{1}{\sigma-1}} k^{-\frac{1}{\sigma}}, \tag{4.4a}$$

$$F_l(k,l) = (1-\gamma) \left[\gamma k^{\frac{\sigma-1}{\sigma}} + (1-\gamma) l^{\frac{\sigma-1}{\sigma}}\right]^{\frac{1}{\sigma-1}} l^{-\frac{1}{\sigma}}. \tag{4.4b}$$

Then, (3.4a) is expressed as:

$$\begin{aligned} 0 &= \frac{\delta}{\eta\zeta} F_l\left(\hat{k}_{t-1}, 1\right) F_k\left(\hat{k}_t, 1\right) - F_l\left(\hat{k}_t, 1\right) \\ &= \frac{\delta(1-\gamma)\gamma}{\eta\zeta}\left[\gamma \hat{k}_{t-1}^{\frac{\sigma-1}{\sigma}} + (1-\gamma)\right]^{\frac{1}{\sigma-1}} \left[\gamma \hat{k}_t^{\frac{\sigma-1}{\sigma}} + (1-\gamma)\right]^{\frac{1}{\sigma-1}} \hat{k}_t^{-\frac{1}{\sigma}} - (1-\gamma)\left[\gamma \hat{k}_t^{\frac{\sigma-1}{\sigma}} + (1-\gamma)\right]^{\frac{1}{\sigma-1}}. \end{aligned} \tag{4.5}$$

Thus, we have:

$$\hat{k}_t = \left(\frac{\delta\gamma}{\eta\zeta}\right)^{\sigma} \left[\gamma \hat{k}_{t-1}^{\frac{\sigma-1}{\sigma}} + (1-\gamma)\right]^{\frac{\sigma}{\sigma-1}} = \left(\frac{\delta\gamma}{\eta\zeta}\right)^{\sigma} F(\hat{k}_{t-1}, 1). \tag{4.6}$$

Since $F(0,1) \geq 0$, $F_k(\hat{k},1) > 0$ and $F_{kk}(\hat{k},1) < 0$,

$$\left(\frac{\delta\gamma}{\eta\zeta}\right)^{\sigma} F_k(\hat{k}_{ss}, 1) < 1 \tag{4.7}$$

is the necessary and sufficient condition for the steady state to be globally asymptotically stable. We can show that this inequality holds and thus the steady state is stable.

Proposition 4.1 *Suppose that the production function is of the CES type (4.2). Then, the steady state in Proposition 3.2 is globally asymptotically stable.*

Proof. The elasticity of the marginal productivity of physical capital at $(k,1)$ is given by:

$$\varepsilon := \frac{-kF_{kk}}{F_k} = \frac{k}{F_k}\frac{F_k\left(F - F_k k\right)}{\sigma k F} = \frac{F/k - F_k}{\sigma F/k} = \frac{1 - F_k/\left(\gamma^{-1}F_k\right)^{\sigma}}{\sigma} = \frac{1-\gamma^{\sigma}\left(F_k\right)^{1-\sigma}}{\sigma}, \tag{4.8}$$

where the second equality follows from (A.3a) and the last one from (A.2a). At the steady state, by substituting (3.5), we have:

$$\varepsilon^{ss} = \frac{1-\gamma^{\sigma}\left(F_k(\hat{k}_{ss},1)\right)^{1-\sigma}}{\sigma} = \frac{1}{\sigma}\left[1-\left(\frac{\gamma\delta}{\eta\zeta}\right)^{\sigma} F_k(\hat{k}_{ss},1)\right] > 0, \tag{4.9}$$

which implies (4.7), the necessary and sufficient condition for the steady state to be globally asymptotically stable. ■

Let us turn to the transition dynamics.

Proposition 4.2 *Suppose that the production function is of the CES type (4.2). Then, the consumption growth rate $\hat{x}_t$ monotonically converges to the steady state growth rate $\hat{x}^{ss}$.*

Proof. As shown above, $\hat{k}_t$ monotonically converges to the steady state $\hat{k}^{ss}$. This monotonicity property is shared with $\hat{x}_t$, because:

$$\frac{\partial \hat{x}_t}{\partial \hat{k}_t} = \frac{1}{1-\theta}\frac{\hat{x}_t F_{kk}\left(\hat{k}_t,1\right)}{F_k\left(\hat{k}_t,1\right)} < 0, \tag{4.10}$$

which is obtained from (3.4b). ■

Finally, we show the relation between the destructive magnitude of periodic disasters and the speed of convergence. Consider two parameter sets $\Phi^i = \left(\delta^i,\zeta^i;\lambda,\Delta,\theta,\beta,\eta,\gamma,\sigma\right)$, $i = A, B$. We compare two paths of the consumption growth rate, $\{\hat{x}^i_t\}_{t=1}^{\infty}$, $i = A, B$, which are generated with the corresponding parameter sets. We assume that two paths are comparable if the initial states are the same in the relative distance to their steady states and both of them are either greater or less than their steady states. Therefore, we consider two paths whose initial states $\hat{x}^A_1$ and $\hat{x}^B_1$ satisfy $\hat{x}^A_1/\hat{x}^{ss}_A = \hat{x}^B_1/\hat{x}^{ss}_B$, where $\hat{x}^{ss}_i = \lim_{t\to\infty}\hat{x}^i_t$. We say that path $\{\hat{x}^A_t\}$ converges faster than path $\{\hat{x}^B_t\}$

if for any period $t \geq 2$, the relative distances to the steady states satisfy:

$$|\hat{x}_t^A/\hat{x}_A^{ss} - 1| < |\hat{x}_t^B/\hat{x}_B^{ss} - 1|. \tag{4.11}$$

Define:

$$\hat{y}_t = \left(\frac{\hat{x}_t}{\hat{x}^{ss}}\right)^{\sigma(1-\theta)}. \tag{4.12}$$

Then, path $\{\hat{x}_t^A\}_{t=1}^{\infty}$ converges faster than path $\{\hat{x}_t^B\}_{t=1}^{\infty}$ if and only if

$$|\hat{y}_t^A - 1| < |\hat{y}_t^B - 1| \tag{4.13}$$

for all $t \geq 2$. Furthermore, note that $\lim_{t\to\infty} \hat{y}_t = 1$ and

$$\hat{y}_1 > 1 \iff \hat{y}_t > 1 \ \forall t \in \mathbb{Z}_{++}. \tag{4.14}$$

Let

$$\xi = \frac{\delta}{\eta\zeta}. \tag{4.15}$$

We prepare two lemmas for the proposition on the convergence speed.

Lemma 4.1 *There is a strictly increasing function $h : \mathbb{R}_+ \mapsto \mathbb{R}_+$ such that the law of motion of $\hat{y}_t$ is given by $\hat{y}_{t+1} = h(\hat{y}_t)$.*

Proof. From (3.4a), (3.4b), and (4.12), we have:

$$\frac{d\hat{k}_{t+1}}{d\hat{k}_t} = \frac{-\xi F_{lk}(\hat{k}_t, 1) F_k(\hat{k}_{t+1}, 1)}{\xi F_l(\hat{k}_t, 1) F_{kk}(\hat{k}_{t+1}, 1) - F_{lk}(\hat{k}_{t+1}, 1)} > 0,$$

$$\frac{d\hat{x}_{t+1}}{d\hat{k}_{t+1}} = \frac{1}{1-\theta}\left(\tilde{\beta}\delta\right)^{\frac{1}{1-\theta}} F_k\left(\hat{k}_{t+1},1\right)^{\frac{\theta}{1-\theta}} F_{kk}\left(\hat{k}_{t+1},1\right) < 0,$$

and

$$\frac{d\hat{y}_{t+1}}{d\hat{x}_{t+1}} = \frac{\sigma(1-\theta)}{\hat{x}_{t+1}}\left(\frac{\hat{x}_{t+1}}{\hat{x}^{ss}}\right)^{\sigma(1-\theta)} > 0.$$

Combining these, we have:

$$\frac{d\hat{y}_{t+1}}{d\hat{y}_t} = \frac{d\hat{y}_{t+1}}{d\hat{x}_{t+1}}\frac{d\hat{x}_{t+1}}{d\hat{k}_{t+1}}\frac{d\hat{k}_{t+1}}{d\hat{k}_t}\frac{d\hat{k}_t}{d\hat{x}_t}\frac{d\hat{x}_t}{d\hat{y}_t} > 0.$$

Thus, the function h is strictly increasing. ■

Lemma 4.2 *Let $\hat{y}_1^A = \hat{y}_1^B$. Then,*

$$\xi^A < \xi^B \iff |\hat{y}_2^A - 1| < |\hat{y}_2^B - 1| \text{ if } \sigma > 1,$$

and

$$\xi^A < \xi^B \iff |\hat{y}_2^A - 1| > |\hat{y}_2^B - 1| \text{ if } \sigma < 1.$$

Proof. $\{\hat{y}_t\}_{t=1}^{\infty}$ satisfies:

$$\hat{y}_t = \left[\xi F_k\left(\hat{k}_t,1\right)\right]^{\sigma} \tag{4.16a}$$

$$= (\xi\gamma)^{\sigma} F\left(\hat{k}_t,1\right)/\hat{k}_t, \tag{4.16b}$$

where we use (3.4) and (3.6) to obtain the first equality and use (A.2a) to obtain the second equality. Using $\hat{y}_1^A = \hat{y}_1^B$ and (4.16a), we have:

$$\hat{y}_1^A = \xi^A F_k\left(\hat{k}_1^A,1\right) = \xi^B F_k\left(\hat{k}_1^B,1\right) = \hat{y}_1^B. \tag{4.17}$$

From (4.16b) and (4.6), we have:

$$\hat{y}_1^A = \hat{k}_2^A / \hat{k}_1^A = \hat{k}_2^B / \hat{k}_1^B = \hat{y}_1^B. \tag{4.18}$$

(4.17) and (4.18) imply that:

$$\xi^A < \xi^B \iff \hat{k}_1^A < \hat{k}_1^B \iff \hat{k}_2^A < \hat{k}_2^B. \tag{4.19}$$

Then, using (4.16b) and (4.18), we have:

$$\begin{aligned}
\frac{\hat{y}_2^B}{\hat{y}_2^A} &= \frac{\left(\xi^B \gamma\right)^\sigma F\left(\hat{k}_2^B, 1\right)}{\left(\xi^A \gamma\right)^\sigma F\left(\hat{k}_2^A, 1\right)} \frac{\hat{k}_2^A}{\hat{k}_2^B} = \frac{\hat{y}_1^B}{\hat{y}_1^A} \frac{F\left(\hat{k}_1^A, 1\right) F\left(\hat{k}_2^B, 1\right)}{F\left(\hat{k}_1^B, 1\right) F\left(\hat{k}_2^A, 1\right)} \frac{\hat{k}_2^A / \hat{k}_1^A}{\hat{k}_2^B / \hat{k}_1^B} \\
&= \frac{F\left(\hat{k}_2^B, 1\right) / F\left(\hat{k}_1^B, 1\right)}{F\left(\hat{k}_2^A, 1\right) / F\left(\hat{k}_1^A, 1\right)} = \frac{F\left(\hat{k}_1^B \hat{y}_1^B, 1\right) / F\left(\hat{k}_1^B, 1\right)}{F\left(\hat{k}_1^A \hat{y}_1^A, 1\right) / F\left(\hat{k}_1^A, 1\right)} \\
&= \frac{F\left(\hat{k}_1^B, 1/\hat{y}_0\right) / F\left(\hat{k}_1^B, 1\right)}{F\left(\hat{k}_1^A, 1/\hat{y}_0\right) / F\left(\hat{k}_1^A, 1\right)},
\end{aligned} \tag{4.20}$$

where $\hat{y}_1 := \hat{y}_1^A = \hat{y}_1^B$. As (A.2a)

$$\frac{\partial F\left(k, 1/\hat{y}_1\right) / F\left(k, 1\right)}{\partial k} = \frac{\gamma F\left(k, 1/\hat{y}_1\right) F\left(k, 1\right)}{k F\left(k, 1\right)^2} \left[F\left(k, 1/\hat{y}_1\right)^{\frac{1-\sigma}{\sigma}} - F\left(k, 1\right)^{\frac{1-\sigma}{\sigma}}\right], \tag{4.21}$$

we have:

$$\frac{\partial F\left(k, 1/\hat{y}_1\right) / F\left(k, 1\right)}{\partial k} > 0 \iff \left[\frac{F\left(k, 1/\hat{y}_1\right)}{F\left(k, 1\right)}\right]^{1-\sigma} > 1. \tag{4.22}$$

As $F_{kl} > 0$ (see (A.3c)), it follows from (4.14), (4.19), (4.20), (4.22) that if $\sigma > 1$,

$$\begin{aligned}
\xi^A < \xi^B &\iff \hat{k}_1^A < \hat{k}_1^B \\
&\iff \hat{y}_1 > \hat{y}_2^B > \hat{y}_2^A > 1 \text{ or } \hat{y}_1 < \hat{y}_2^B < \hat{y}_2^A < 1 \\
&\iff |\hat{y}_2^A - 1| < |\hat{y}_2^B - 1|,
\end{aligned}$$

and if $\sigma < 1$,

$$\begin{aligned}
\xi^A < \xi^B &\iff \hat{k}_1^A < \hat{k}_1^B \\
&\iff \hat{y}_1 > \hat{y}_2^A > \hat{y}_2^B > 1 \text{ or } \hat{y}_1 < \hat{y}_2^A < \hat{y}_2^B < 1 \\
&\iff |\hat{y}_2^A - 1| > |\hat{y}_2^B - 1|.
\end{aligned}$$

■

Proposition 4.3 *Assume that two parameter sets Φ^A and Φ^B satisfy*

$$\frac{\delta^A}{\eta\zeta^A} < \frac{\delta^B}{\eta\zeta^B}.$$

Let $\{\hat{x}_t^A\}_{t=1}^\infty$ and $\{\hat{x}_t^B\}_{t=1}^\infty$ be the growth rate paths associated with these parameter sets and starting from $\hat{x}_1^A/\hat{x}_A^{ss} = \hat{x}_1^B/\hat{x}_B^{ss}$. If $\sigma > 1$, then path $\{\hat{x}_t^A\}$ converges faster than path $\{\hat{x}_t^B\}$ and if $\sigma < 1$, then path $\{\hat{x}_t^B\}$ converges faster than path $\{\hat{x}_t^A\}$.

Proof. Let $\{y_t^i\}_{t=1}^\infty, i = A, B$ be the sequences associated with $\{\hat{x}_t^i\}_{t=1}^\infty, i = A, B$. Furthermore, let $h : \mathbb{R}_+ \mapsto \mathbb{R}_+$ in Lemma 4.1 describe the evolution of $\{y_t^B\}$, i.e., $y_{t+1}^B = h(y_t^B)$. Suppose that $\sigma > 1$ and $\hat{x}_1^A/\hat{x}_A^{ss} = \hat{x}_1^B/\hat{x}_B^{ss} > 1$. In this case, by Lemma 4.2, $1 < \hat{y}_2^A < \hat{y}_2^B$. Then, we have $1 < \hat{y}_3^A = h(\hat{y}_2^A) < h(\hat{y}_2^B) = \hat{y}_3^B$ by Lemma 4.1. This argument is also applicable to any

$t \geq 3$. Therefore, $1 < \hat{y}_t^A < \hat{y}_t^B$ for all $t \geq 2$, which is equivalent to $|\hat{x}_t^A/\hat{x}_A^{ss} - 1| < |\hat{x}_t^B/\hat{x}_B^{ss} - 1|$. Thus, we have proved that path $\{\hat{x}_t^A\}$ converges faster than path $\{\hat{x}_t^B\}$ in the case of $\sigma > 1$ and $\hat{x}_1^A/\hat{x}_A^{ss} = \hat{x}_1^B/\hat{x}_B^{ss} > 1$. It is obvious that the same argument can be applied to other cases for the parameter σ and the initial values $\hat{x}_1^A$, $\hat{x}_1^B$. ■

The speed of convergence is influenced by the relative magnitude of the effects of periodic disasters on physical capital and their influence is qualitatively different depending on the elasticity of substitution σ between physical capital and effective labor in the production function. When the two inputs are substitutes (i.e., when $\sigma > 1$), the economy converges more rapidly to its steady state if the relative magnitude of the effects of periodic disaster on human resource is small. To interpret this result, consider first the case where $\hat{k}_t > \hat{k}_{ss}$. In this case, human capital needs to be accumulated more rapidly than physical capital along the transition path to the steady state, which is obviously easier when the effects of periodic disasters on human resource are not large. In the case with $\hat{k}_t < \hat{k}_{ss}$, it is not necessary to use much effective labor for human capital investment in the transition path. This implies that a relatively large amount of effective labor is available for the consumption good sector, in particular when only small quantities of human resource are destroyed by periodic disasters. The abundant effective labor then increases output in the consumption good sector as long as the two inputs are good substitutes, which in turn makes it easier to increase physical capital keeping consumption growth intact.

If the two inputs are complements (i.e., if $\sigma < 1$), however, the economy converges rather slowly to its steady state if the relative magnitude of the effect of periodic disasters on physical capital is large. When $\hat{k}_t < \hat{k}_{ss}$, physical capital needs to be increased more rapidly than human capital along the transition path to the steady state, which should be more difficult when periodic disasters are more destructive to physical capital. When $\hat{k}_t > \hat{k}_{ss}$, human capital needs to be accumulated. But because physical capital and effective labor are complements, reallocation of labor into human capital investment is difficult without sacrificing consumption growth, in particular when the consumption good sector already suffers from low productivity because of the effect of destructive periodic disasters on physical capital.

Therefore, following occasional disturbances by historical disasters, the economy may or may not come back to its steady state quickly, depending on both the elasticity of substitution and the relative magnitude of the effect of the disasters on each type of capital.

5 Concluding remarks

A growing number of empirical studies have investigated the long-run economic consequences of disasters. Based on empirical evidence, it has been argued that, under some circumstances, disasters of a particular kind have positive impacts on the economy in the long run. While reasonable explanations for this result have been provided, little is known about the formal mechanism lying behind those empirical observations. As a first attempt to formalize the argument, this paper has presented an endogenous growth model where disasters of different kinds are taken into account, and has examined the long-run relationship between disasters and economic growth. Although the model presented in this paper is not entirely realistic in some respects, it captures well the essence of the interplay between disasters and the economy. Our analysis of the unique steady state of the model, together with its comparative statics, has in fact provided some novel insights.

First, the magnitude or frequency of periodic disasters matters for the long-run economic growth rate, but only when human resources are affected by the disasters. In a region where physically destructive disasters are frequent, the economy in effect suffers from the lower productivity of physical capital. As far as the long-run consequence is concerned, this apparently negative impact does not affect the economic growth rate, possibly because it is counteracted by the positive impact of resource reallocation from physical to human capital investment. When periodic disasters are destructive in terms of human resources, however, the long-run growth rate decreases. This implies that a lower return to human capital investment because of human-targeted disasters cannot be compensated by reallocating resources to physical capital investment. This might not be so surprising because the engine of economic growth is in the process of human capital accumulation.

What is less obvious is the finding that some of the economic consequences of disasters crucially depend on the preferences and technology parameters. In particular, the elasticity of intertemporal substitution is critical in the long-run relationship between historical disasters and economic growth. Our analysis showed that the greater magnitude of or the higher frequency of historical disasters can imply a higher long-run growth rate by shifting emphasis from physical to human capital investment. This result, however, is contingent upon the relatively low elasticity of intertemporal substitution. If the preferences of people in disaster-prone regions are highly elastic in terms of intertemporal substitution, the long-run growth rate in the regions would be smaller. The best available evidence suggests that both cases are possible (Hall, 1988; Vissing-Jorgensen, 2002). Furthermore, it has been shown that the elasticity of substitution between physical capital and labor in final good production plays a key role in the speed of convergence. This implies that the recovery time from historical disasters in a region is dependent upon the substitutability between the two inputs in the region. For further empirical research of the long-run economic consequences of disasters, regional differences in terms of preferences and technology should therefore be considered carefully.

We draw some policy implications from our analysis. First, human protection against disasters is important, not only from a humanism aspect, but also for long-run economic growth. Second, however, it is not always true that a higher economic growth rate implies improved social welfare. In the case of high elasticity preferences, a high disaster risk foments people's worries about the future and encourages them to save current consumption for the uncertain future, which results in a high economic growth rate. In this case, although disaster prevention measures should lower economic growth, this does not mean a decrease, but rather an increase in social welfare, because those measures enable them to increase current consumption free from anxiety. Contrarily, in the case of low elasticity preferences, a disaster risk goads people into increasing current consumption. Then, disaster prevention measures should contribute to both improved social welfare and a higher economic growth rate. Last, the convergence speed to the steady state is interpreted as a measure of economic resilience. Seeking a policy measure to increase resilience against disasters should be an important goal. Our analysis indicates that the appropriate measure depends on the production technology, and that there is no general rule.

A Appendix: Properties of CES function

This Appendix states some properties of the CES function:

$$F(k,l) = \left[\gamma k^{\frac{\sigma-1}{\sigma}} + (1-\gamma) l^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \quad \gamma \in (0,1),\ \sigma > 0,\ \sigma \neq 1. \tag{A.1}$$

The first-order derivatives of the CES function are:

$$F_k(k,l) = \gamma \left[\gamma k^{\frac{\sigma-1}{\sigma}} + (1-\gamma) l^{\frac{\sigma-1}{\sigma}}\right]^{\frac{1}{\sigma-1}} k^{-\frac{1}{\sigma}} = \gamma \left(F/k\right)^{\frac{1}{\sigma}} > 0, \tag{A.2a}$$

$$F_l(k,l) = (1-\gamma) \left[\gamma k^{\frac{\sigma-1}{\sigma}} + (1-\gamma) l^{\frac{\sigma-1}{\sigma}}\right]^{\frac{1}{\sigma-1}} l^{-\frac{1}{\sigma}} = (1-\gamma) \left(F/l\right)^{\frac{1}{\sigma}} > 0. \tag{A.2b}$$

They verify the strict increasing property in (2.13). The second-order derivatives are calculated as:

$$F_{kk} = F_k \frac{\partial \ln F_k}{\partial k} = \frac{F_k}{\sigma} \frac{\partial (\ln F - \ln k)}{\partial k} = \frac{F_k}{\sigma k F} (F_k k - F) < 0, \tag{A.3a}$$

$$F_{ll} = F_l \frac{\partial \ln F_l}{\partial l} = \frac{F_l}{\sigma l F} (F_l l - F) < 0, \tag{A.3b}$$

$$F_{kl} = \frac{1}{\sigma k} \gamma (F/k)^{\frac{1}{\sigma}-1} (F_l) = \frac{1}{\sigma k} (F/k)^{-1} (F_k F_l) = \frac{F_k F_l}{\sigma F}. \tag{A.3c}$$

From these:

$$\begin{aligned}
F_{kk}F_{ll} - (F)^2 &= \left(\frac{1}{\sigma F}\right)^2 F_k F_l \left(F_k - \frac{F}{k}\right)\left(F_l - \frac{F}{l}\right) - \left(\frac{F_k F_l}{\sigma F}\right)^2 \\
&= \frac{F_k F_l}{(\sigma F)^2} \left[\left(F_k - \frac{F}{k}\right)\left(F_l - \frac{F}{l}\right) - F_k F_l\right] \\
&= \frac{F_k F_l}{(\sigma F)^2} \left(-\frac{F}{k} F_l - \frac{F}{l} F_k + \frac{F}{k}\frac{F}{l}\right) \\
&= \frac{F F_k F_l}{kl (\sigma F)^2} (F - F_l l - F_k k) \\
&= 0,
\end{aligned} \tag{A.4}$$

where the last equality follows from Euler's theorem about a homogeneous function of degree one. Then, the concavity follows from (A.3a), (A.3b) and (A.4). Finally, let us check the Inada conditions (2.14). For the physical capital, we have:

$$\lim_{z \to 0} F_k(z,l) = \lim_{z \to 0} \gamma \left[\gamma + (1-\gamma)(l/z)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{1}{\sigma-1}} = \begin{cases} \infty & \text{if } \sigma < 1 \\ \gamma^{\frac{\sigma}{\sigma-1}} & \text{if } \sigma > 1 \end{cases},$$

$$\lim_{z \to \infty} F_k(z,l) = \lim_{z \to \infty} \gamma \left[\gamma + (1-\gamma)(l/z)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{1}{\sigma-1}} = \begin{cases} \gamma^{\frac{\sigma}{\sigma-1}} & \text{if } \sigma < 1 \\ 0 & \text{if } \sigma > 1 \end{cases}.$$

Similarly, for the human capital, we have:

$$\lim_{z\to\infty} F_l(k,z) = \begin{cases} \infty & \text{if } \sigma < 1 \\ (1-\gamma)^{\frac{\sigma}{\sigma-1}} & \text{if } \sigma > 1 \end{cases},$$

$$\lim_{z\to\infty} F_l(k,z) = \begin{cases} \gamma^{\frac{\sigma}{\sigma-1}} & \text{if } \sigma < 1 \\ 0 & \text{if } \sigma > 1 \end{cases}.$$

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