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Abstract

In this paper, we study the social interactions between two populations of individuals living in a city. Agents consume land and benefit from intra and intergroup social interactions. We show that segregation arises in equilibrium: populations become separated in distinct spatial neighborhoods. Two- and three-district urban structures are characterized. For high population ratios or strong intergroup interactions, only three-district cities exist. In other cases, multiplicity of equilibria arises. Moreover, for sufficiently low population ratios or very weak intergroup interactions, all individuals agree on the optimal spatial equilibrium.

Keywords: Social interaction, Segregation, Multiple centers, Urban districts
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1 Introduction

In many cities population groups are sorted into distinct spatial neighborhoods. Many US metropolitan areas have a China town, a little Italy, or other ethnic enclaves which host significantly higher concentrations of ethnic or cultural groups. Such enclaves may range from a single block to areas of a few square miles. For most observers, the main reason for such spatial segregation lies in the economic ties and social interactions that people have with individuals of their reference group. The prevalence of such a segregation is exacerbated by poverty as poor people are more likely to see their economic prospects and social relationships improved within their own ethnic group. Spatial concentration also affects business and professional activities. The financial, retailing and manufacturing sectors often locate their economic activity in separate industrial areas. For instance, in Los Angeles, distinct neighborhoods host the movie, finance, fashion, and art industries. For many urban economists, such industrial concentration is partly explained by the stronger spillover that firms benefit from other firms active in the same industry.

The present paper aims at improving our understanding of spatial segregation and concentration as resulting from the emergence of endogenous urban districts. We study a one-dimensional city where agents engage in intra- and inter-group social interactions, choosing their land consumption, as well as their location. Agents are heterogeneous in the sense that each of them belongs to one of two distinct populations. Intra-group interactions are more frequent than inter-group interactions: agents interact more often with agents of their own population than with agents of the other group. Such preferential interactions reflect stronger relationships between individuals sharing a common culture, language, or ethnicity. They may also reflect professional relationships between group members sharing the same economic activity (e.g., bankers, lawyers, or designers) or economic status (e.g. employed or unemployed workers). We assume that populations are symmetric in terms of their benefit from intra- and inter-group interactions: the intensity of intra-group and inter-group social interactions is the same for both populations. Our model does not rely on the exogenous existence of a city center (Alonso, 1964). Instead, as in Beckmann (1976), each agent visits other agents so as to benefit from a face-to-face contacts, each trip involving a cost which is proportional to distance. In equilibrium, the benefit from social interactions balances the residence and the access cost. The main issue in this paper is about whether social interactions lead to spatial segregation and how these interactions structure urban neighborhoods. Though segregation is a common result in land market models in the urban literature, our paper constitutes the first attempt to deal with the issue without assuming the exogenous existence of a central place.

Our results are the following. First, we show that integration never is a spatial equilibrium. Both populations never cluster in an integrated city. This result is due to the agents' higher return from interacting with individuals of their own group. Given this, agents have an incentive to relocate close to agents of their own population, so as to meet more frequently, and therefore to save on trip costs.

Second, we analyze spatial structures involving segregation in two or three urban districts. In a two-district city, populations separate in two neighboring districts. In a three-district city, one population locates in the city center while the other one resides in the two city edges. We show that both cases are possible.

The two-district city is a spatial equilibrium if populations have similar sizes or if inter-group interactions are weak. In this case, the large population occupies a large central urban district repelling the small population towards a city edge. The small population accommodates such a situation as its interactions with the large group are weak, making the incentive to relocate close to the large population´s district not sufficiently high. In contrast, when the small population has stronger interactions with the large group, some of its individuals have an incentive to relocate to the opposite city edge, where the large population resides, so as to benefit from cheaper land rents and from a better accessibility to the large group. The two-district then ceases to be a spatial equilibrium. Interestingly, the two-district city can display one or two subcenters. When inter-group interactions are weak, two subcenters arise. As intra-group interactions dominate, a separate basin of attraction arises for each population. On the other hand, when one population is much larger than the other one, the city has a single subcenter as the large population's district also constitutes a basin of attraction for the small group.

The three-district city is always a spatial equilibrium when the large population locates in the central district. In this case, the large population occupies the city area where it benefits most from closer interactions though facing high land rents. The small population benefits from lower land prices at the expense of a lower accessibility to agents of its own group. The city is shown to exhibit a single subcenter occupied by the large population, which constitutes a large basin of attraction for both groups. In contrast, when the small population locates in the central district, we show that the three-district city may not always be sustained as a spatial equilibrium. This is the case only if population sizes are sufficiently similar or if inter-group interactions are weak. The existence of this equilibrium configuration stems from a coordination problem. Although the large population as a whole would benefit from occupying the city center, no individual agent has an incentive to relocate in the central district as he would face an excessive residence cost and lose access to individuals of his own group which are located in the city edges.

Third, we show that multiple equilibria may arise. Depending on the model parameters, various urban structures can coexist. The economy exhibits one, two, or three spatial equilibria. The more similar the population sizes, and the weaker the inter-group interactions, the more likely several equilibria to emerge. For high population ratios or strong inter-group interactions, only the three-district city with the large group occupying the central district exists. In other cases, multiplicity of equilibria arises. When several spatial configurations are possible, spatial equilibria can be ranked in terms of the utility derived by each population. A welfare analysis shows that for sufficiently low population ratios or very weak inter-group interactions, all individuals agree on which spatial equilibrium is best.

Our model suggests urban patterns that can be found for various language, racial or ethnic groups. For instance, the Island of Montreal in Canada presents a East-West division of the French- and the English-speaking communities. A similar North-South divide can be observed in the city of Brussels in Belgium between the Dutch- and the French-speaking communities. While the city of Paris accommodates most of the foreign ethnic groups away from the city center, US cities like New-York or Detroit host several small ethnic and racial groups around the city center. Spatial segregation can also have religious grounds. In the city of Belfast, the West and the East sides of the city are mostly inhabited by Catholics and Protestants respectively. In general, the spatial clustering of people in cities may be influenced by several other personal attributes, ranging from professional activities to sexual habits. The paper focuses on the spatial sorting of two communities differing in one such characteristic.

Our model helps understanding how city growth may affect urban structures. Given the multiplicity of equilibria, city growth may induce spontaneous transitions from one urban structure to another. So, the spatial structure of cities depends on history. Over time, old cities like Paris have undergone several transitions and are now locked in a configuration with the large (native) population in the city center. In contrast, younger US cities like New York or Detroit may not have undergone such transitions and display an urban configuration with the minority group in the city center.

Our model also sheds some light on the impact of social integration programs on urban structures. Interpreting the frequency of inter-group interactions as an indicator of social integration, our model provides some interesting insights. Spatial integration should not be considered as an indicator of efficiency of social integration programs. In many instances, social integration programs may be ineffective in reshaping the urban landscape. Moreover, social integration may even fragment spatially the minority group.

The paper relates to the literature in several respects. Beckmann (1976), Fujita and Thisse (2004), and Mossay and Picard (2011) have studied how non-market interactions can shape urban structures by studying social interactions in a land market model without assuming the pre-existence of an urban center to which residents commute. Fujita and Ogawa (1982) and Lucas and Rossi-Hansberg (2002) have analyzed how market interactions can shape the city structure by including firms and workers. Here, multiple equilibria along the line segment are due to the introduction of inter-group interactions as opposed to the corresponding single group model. Simple closed-form solutions are obtained for two- and three-district urban structures. This allows us to characterize and compare the equilibrium configurations, as well as to provide a rationale for these results. Our analysis therefore provides additional theoretical insights regarding the formation of multicentered cities as surveyed in Anas *et al.* (1998).

As in Schelling (1971), individual location choices lead to spatial segregation patterns. While the dynamics in Schelling´s model is driven by individual preferences for the neighborhood composition, spatial segregation in our model arises although no individual agent prefers or promotes this spatial outcome. Here, segregation hinges on the discrepancies of social interactions that individuals have in the whole city and on competition in the land market. So, our model reconciles Beckmann's urban land market with Schelling's idea of segregation. As in our paper, Kanemoto (1980, Chapter 6) studies the selection of spatial neighborhoods by two groups of households. He considers a monocentric city where the poor group imposes a negative externality on the rich group. In contrast to this, we do not assume the pre-existence of the city center and analyse the case of reciprocal segregation, where each population is affected by the location decisions of individuals of the other group. Moreover, our model may display three-district configurations, which do not arise in Kanemoto's work.

Glaeser *et al.* (2000) have stressed the important role of social interactions in the formation of cities. While we address social interactions in an explicit spatial framework, de Marti and Zenou (2012) study similar segregation issues arising in social networks. As in their paper, we focus on intra- and inter-group social interactions between two groups of individuals. The intra- and inter-group interactions can be interpreted as strong and weak ties in the sense of Granovetter (1973). While de Marti and Zenou address various social aspects (e.g. assimilation or oppositional identities), we study a land market model with spatial interactions. In our model, the access cost is assumed to be small enough so that each agent has an incentive to interact with all other agents distributed along the line segment. This means that according to the terminology of de Marti and Zenou, our economy always displays complete integration (e.g. each group is fully intra-connected and both groups are fully inter-connected). However, here, in contrast to de Marti and Zenou, the issue is not about whether an individual will maintain a social link with other individuals, nor it is about the impact of the geometry of the social network. Rather, we are interested in how location choices of individuals affect the structure of spatial neighborhoods.[1]

This paper is organized as follows. Section 2 describes the model. Section 3 studies the urban structure with integrated populations. Section 4 analyzes spatial segregation in two and three districts. Section 5 concludes.

2 The Model

We assume a linear city with a unit width which spreads over the interval $\mathcal{B} \equiv [-b, b]$ and hosts two populations of agents $P_1 > P_2$. The density of agents of population i residing at location x is denoted by the function $\lambda_i(x) : \mathcal{B} \to \mathbb{R}^+$, $i = 1, 2$. Each individual enjoys the same unitary benefit when interacting socially with another agent while incurring an access cost τ associated with the return trip to visit that agent. However, because of cultural differences or language barriers, social interactions are more frequent among individuals of a same group. While agents meet each agent of their own population with a frequency normalized to one, they meet each agent of the other group with a lower frequency $0 < \alpha < 1$. So the social utility derived by an agent of population i can be written as

$$S_i(x) = \int_{\mathcal{B}} (1 - \tau |x - y|) \lambda_i(y) dy + \alpha \int_{\mathcal{B}} (1 - \tau |x - y|) \lambda_j(y) dy \qquad , \ i \neq j$$

where the first term (resp. the second term) reflects the net benefit from intra-group interactions (resp. inter-group interactions) with $|x - y|$ denoting the Euclidian distance

[1] Note that the model by Hesley and Zenou (2012) addresses both location choices and endogenous network formation. However, it does not focus on segregation issues.

between locations x and y. The surplus $S(x)$ can also be interpreted in a context of uncertainty. In that case, it corresponds to the expected utility of an agent who plans to interact with a subset of agents whom location and identity are not known at the time of the residence choice. Such an interpretation applies to individuals moving to an urban area with no *a priori* acquaintances. This could also apply to the case of shopkeepers, sellers, as well as workers who expect to hold several jobs at different locations during their lifetime, or employers who do not have a precise idea about future workers' residences.

Agents maximize the utility they derive from consumption and social interactions

$$U_i\left(s, z; x\right) = S_i(x) - \frac{\beta}{2s} + z$$

subject to their budget constraint

$$z + R(x)s = Y$$

where s and z are the consumption of land and of the composite good, $R(x)$ the land rent at location x, Y the agent's income[2], and β the preference parameter for land consumption. For the sake of simplicity, we assume that land has no alternate use, so that $R(x) = 0$ in uninhabited locations. In the above functional form of utility, we consider an hyperbolic preference for land instead of the logarithmic preference used in Beckmann (1976) and Fujita and Thisse (2002, Chapter 6). The present hyperbolic preference represents an intermediate case between Beckmann's demand and the inelastic demand for space that is regularly used in standard urban economics.[3]

Since Alonso (1964) and Fujita (1989), the urban economic literature has regularly relied on the bid rent approach to determine the spatial equilibrium. Because agents are free to relocate anywhere along the geographical space, the absence of locational arbitrage requires that the utility level of agents of a same population remains constant across all

[2] Y can also be interpreted as the valuation of the endowment in the composite good.

[3] The hyperbolic and logarithmic preferences for residential space are two particular instances of the same class of preferences $s^{1-\rho}/\left(1-\rho\right)$ where $\rho = 2$ and $\rho \rightarrow 1$ respectively, which yield iso-elastic demands for residential space with price elasticities equal to 1/2 and 1 respectively.

locations inhabited by this population. The agent's bid rent ψ_i in location x is defined as the maximum rent that an agent is willing to pay for residing in x

$$\psi_i(x) = \max_s \frac{Y - z}{s} \quad \text{s.t.} \quad U_i\,(s, z; x) \geq u_i \qquad i = 1, 2$$

for some given utility level u_i.

Let $\widehat{z}_i(x, u_i)$ and $\widehat{s}_i(x, u_i)$ denote the bid-maximizing consumption of land and of the composite good for an individual of population i residing in x. By using the agent's budget constraint, the bid rent $\psi_i(x)$ can be written as

$$\psi_i(x) = \max_s \frac{Y - u_i + S_i - \beta/(2s)}{s} = \max_s \left(\frac{Y - u_i + S_i}{s} - \frac{\beta}{2s^2} \right)$$

The optimal consumption of space corresponds to

$$\widehat{s}_i(x, u_i) = \frac{\beta}{Y - u_i + S_i}$$

which yields the following bid rent:

$$\psi_i(x) = \frac{(Y - u_i + S_i)^2}{2\beta} = \frac{\beta}{2\widehat{s}_i^{\,2}} \tag{1}$$

A competitive spatial equilibrium is then defined by spatial distributions of consumption $\{z_i(x),\ s_i(x)\}$, land rent $R(x)$, agents $\lambda_i(x)$, and utility levels u_i which

(i) maximize each population's bid rent ($z_i(x) = \widehat{z}_i(x, u_i)$ and $s_i(x) = \widehat{s}_i(x, u_i)$),

(ii) allocate land to the highest bid ($R(x) = \max_i[\psi_i(x), 0]$ so that $R(x) = \psi_i(x)$ if $\lambda_i(x) > 0$, and $R(x) = 0$ if $\lambda_i(x) = 0, \forall i$),

(iii) satisfy the land market equilibrium, $\sum_i \lambda_i(x) s_i(x) = 1$, and

(iv) meet the total population constraint $\int_{-b}^{b} \lambda_i(x) dx = P_i$.

3 Integrated populations

In this section, we investigate the existence of integrated districts where both population groups live together. Integrated urban structures are often advocated in the urban planning literature; they are also shown to result from the balance between dispersion and agglomeration forces in the urban economic literature (see e.g. Fujita (1989), Fujita and Ogawa (1982), or Lucas and Rossi-Hansberg (2002)). It turns out that our model of social interactions does not support spatial equilibria with integrated populations. We show below that this is because agents always have an incentive to relocate close to other agents of their own group so as to meet more frequently.

Suppose that the two populations are integrated in some interval so that $\lambda_1(x) > 0$ and $\lambda_2(x) > 0$ for all x in that interval. For this configuration to constitute an equilibrium, land should be allocated to both populations. Hence, by equilibrium condition (ii), the bid rents of both populations must be equal: $\psi_1(x) = \psi_2(x)$. By expression (1), this implies that land consumption should also be equal

$$s_1(x) = s_2(x) \equiv s(x)$$

Agents have an identical use of space because their benefit from social interactions and their preference for space are the same across both populations. As the land market equilibrium (iii) implies that $s(x) = [\lambda_1(x) + \lambda_2(x)]^{-1}$, the agent's utility becomes $U_i = S_i(x) + Y - \beta\,[\lambda_1(x) + \lambda_2(x)]$. The spatial gradient of utility is then given by

$$U_i'(x) = \tau[P_i^+(x) - P_i^-(x)] + \alpha\tau[P_j^+(x) - P_j^-(x)] - \beta\left(\lambda_i'(x) + \lambda_j'(x)\right), \quad i \neq j = 1,2 \quad (2)$$

where

$$P_i^+(x) = \int_x^b \lambda_i(y)dy \quad \text{and} \quad P_i^-(x) = \int_{-b}^x \lambda_i(y)dy = P_i - P_i^+(x)$$

denote the agents of population i to the right and to the left of location x. Clearly, $P_i^+(x)$ (resp. $P_i^-(x)$) is a decreasing (resp. increasing) continuous function. A necessary condition for spatial equilibrium is that the utility of agents remains constant across inhabited areas ($U_1'(x) = U_2'(x) = 0$). By using expressions (2) and solving for $(P_i^+ - P_i^-)(x)$, we get

$$P_1^+(x) - P_1^-(x) = P_2^+(x) - P_2^-(x) = \frac{\beta}{\tau} \frac{\lambda_1'(x) + \lambda_2'(x)}{1+\alpha} \tag{3}$$

In equilibrium, both types of agents should have the same access to agents of their own group. Any difference in population access reflects a change of benefit from social interactions which translates itself into a change of bid rent. If the bid rent gradients were to differ, then one population would be able to overbid the other one. A direct implication of this reasoning is that population densities should be identical in an integrated area, so that $\lambda_1(x) = \lambda_2(x)$. This result has two following consequences.

First, at equilibrium, an integrated district must be interior to the city support. In other words, no city border can be part of an integrated district. This is because at the city border $x = b$, the first equality in condition (3) simplifies to $-P_1 = -P_2$, which is impossible given that population sizes are different. Intuitively, as an integrated district located at the city edge should host the same share of each population, the rest of the city should consequently host a larger share of the large population. This will inevitably induce relocations of individuals of the large group living at city edges to locations where the access cost to the large group is lower.

Second, at equilibrium, only a single integrated district can exist. Stated differently, the city cannot include two integrated districts that are separated by an area hosting a single population or no population at all. On the one hand, interactions are more valuable within a district hosting a single population because agents benefit there from more intra-group interactions. This attracts other agents of the same population living in neighboring integrated districts. On the other hand, the presence of an uninhabited area within the city increases the access cost to agents, and as land is priced at its zero opportunity cost, agents have an incentive to relocate to a border of that inhabited area. This provides them with cheap land and agood access to their own population. The formal argument is provided in the proof of the following Lemma.

Lemma 1 *Population integration can only arise in a single interior district.*

Proof. (i) Consider two integrated areas which are separated by a segregated district $[x_1, x_2]$ hosting a mass $m > 0$ of agents of population 1. We have $P_1^+(x_1) - P_1^+(x_2) = m$ and $P_1^-(x_1) - P_1^-(x_2) = -m$ while $P_2^+(x_1) - P_2^+(x_2) = P_2^-(x_1) - P_2^-(x_2) = 0$. Then, taking the difference between conditions (3) evaluated at $x = x_1$ and $x = x_2$ yields $2m = 0$, which is a contradiction. Of course, the argument also holds for a segregated area hosting population 2.

(ii) Consider an uninhabited area (x_1, x_2) and two adjacent districts with integrated populations. Given that land has a zero opportunity cost outside inhabited areas, we have that $\psi_i(x_1) = \psi_i(x_2) = 0$. At the borders of the empty district, population densities are equal to zero and population imbalances are identical so that $\lambda_1(x_1) + \lambda_2(x_1) = \lambda_1(x_2) + \lambda_2(x_2) = 0$ and $P_i^+(x_1) - P_i^-(x_1) = P_i^+(x_2) - P_i^-(x_2)$, $\forall i$. The latter condition and condition (3) imply that the gradients of population densities are identical at the borders: $\lambda_1'(x_1) + \lambda_2'(x_1) = \lambda_1'(x_2) + \lambda_2'(x_2)$. By differentiating condition (3) with respect to x, the gradient $\lambda_1'(x + \lambda_2'(x)$ can be shown to be a decreasing function. As the density $\lambda_1(x) + \lambda_2(x)$ cannot simultaneously fall to zero at x_1 and rise from zero at x_2, this proves the absence of empty hinterlands in equilibrium. ■

Lemma 1 implies that integrated populations can only reside in a district surrounded by segregated districts. However, this turns out to be impossible for the following reason. When populations differ in size, imbalances in population access provide the individuals of some population with an incentive to relocate away from the integrated district towards the segregated one. So as to establish this result, we first present general properties about utility levels when both populations segregate in distinct districts. These results will also be of use in Section 4.

Suppose some segregated district $[x_1, x_2]$ where $\lambda_i(x) > 0$ and $\lambda_j(x) = 0$, $i \neq j = 1, 2$. We determine the utility and the density of the population residing within this district, as well as the utility level that other agents would obtain by relocating into this district.

On the one hand, the agents residing in this district have a utility level given by

$$U_i = S_i(x) + Y - \frac{\beta}{s_i} = S_i(x) + Y - \beta\lambda_i$$

where the last term includes the density of population i only. The utility gradient is given by

$$U_i'(x) = \tau[P_i^+(x) - P_i^-(x)] + \alpha\tau[P_j^+(x) - P_j^-(x)] - \beta\lambda_i'(x) = 0 \tag{4}$$

so that the population gradient can be written as

$$\lambda_i'(x) = \frac{\tau}{\beta}\left\{[P_i^+(x) + \alpha P_j^+(x)] - [P_i^-(x) + \alpha P_j^-(x)]\right\} \tag{5}$$

Population densities and land rents fall when less population can be accessed to. In equilibrium, the marginal residence cost $\beta\lambda_i'$ equates the sum of the marginal access costs to individuals of her own group $\tau(P_i^+ - P_i^-)$ and to individuals of the other group $\alpha\tau(P_j^+ - P_j^-)$. The frequency of interaction α discounts inter-group interactions as they are less frequent than intra-group ones.

On the other hand, consider an agent of population j who does not reside in the segregated district $[x_1, x_2]$. When considering relocating to location $x \in [x_1, x_2]$, she will maximize her utility $U_j = S_j(x) - \beta/(2s_j) + z_j$ subject to her budget constraint $z_j + R(x)s_j = Y$, where the equilibrium land rent, $R(x)$, is equal to the highest bid made by population i, $\psi_i(x) = \beta/\left[2s_i(x)^2\right]$. Given that $\lambda_i(x) = 1/s_i(x)$ and $s_j(x) = s_i(x)$, agent j's utility becomes

$$U_j(x) = S_j(x) - \beta\lambda_i(x) + Y, \qquad x \in [x_1, x_2]$$

This expression reflects her social interactions (first term) and her use of space that diminishes with the density of population i (second term). Differentiating this expression and using the expression of the population gradient (5) gives

$$U_j'(x) = \tau(1-\alpha)\left\{[P_j^+(x) - P_j^-(x)] - [P_i^+(x) - P_i^-(x)]\right\} \tag{6}$$

In the above expression, population imbalances reflects the trade-off between population access and land prices. Agent j's utility increases as she relocates to the right in the segregated district ($U_j'(x) > 0$) because she gets a better access to her own population ($P_j^+(x) - P_j^-(x) > 0$). Her utility also increases when the access to the other population i worsens ($P_i^+(x) - P_i^-(x) < 0$) as this reduces the equilibrium land rent.

We now show that an integrated district surrounded by segregated districts cannot be sustained in equilibrium. Consider some integrated district $[x_1, x_2]$ hosting both populations as well as two neighboring segregated districts. Consider some location $x > x_2$ in the segregated right-district. Let denote the mass of population 1 between locations x_2 and x by $n(x) \equiv \int_{x_2}^{x} \lambda_1(x) > 0$. For this configuration to constitute an equilibrium, it is necessary that $U_2(x_2) - U_2(x) > 0$. By using the utility gradient (6), we successively get

$$\begin{aligned} U_2'(x) &= \tau\,(1-\alpha)\left\{[P_2^+(x) - P_2^-(x)] - [P_1^+(x) - P_1^-(x)]\right\} \\ &= \tau\,(1-\alpha)\left\{[P_2^+(x_2) - P_2^-(x_2)] - \left[\left(P_1^+(x_2) - n(x)\right) - \left(P_1^-(x_2) + n(x)\right)\right]\right\} \end{aligned}$$

Given condition (3) in the integrated area (x_1, x_2), the above expression simplifies to $U_2'(x) = 2\tau\,(1-\alpha)\,n(x) > 0$. As a result, $U_2(x) - U_2(x_2) = \int_{x_2}^{x} U_2'(z)dz = 2\tau\,(1-\alpha)$ $\int_{x_2}^{x} n(z)dz > 0$ meaning that this city configuration cannot be sustained in equilibrium. An analogous argument applies when population 2 lives in the segregated district. This reasoning leads to the following Proposition.

Proposition 2 *No city includes an integrated district.*

Proposition 2 results from the fact intra-group interactions are more frequent than inter-group ones. At any equilibrium land price, agents have an incentive to relocate close to agents of their own population so as to lower their access cost. Our analysis shows that integrated residential patterns are inherently unstable as individuals have an incentive to relocate closer to their own group. In contrast to Schelling´s paradigm, spatial segregation does not stem from explicit preferences for the neighborhood composition. Spatial segregation hinges on the discrepancies in social interactions and competition in the land market. This sets the stage for the next questions: how do segregation patterns arise and how do spatial neighborhoods form? We now turn to the analysis of cities with segregated populations.

4 Segregated populations

According to the analysis of the previous section, cities must structure into a set of segregated districts, each of them hosting a single population. In each district, individuals favor intra- over inter-group interactions as they have a closer access to agents of their own group while having a more remote access to agents of the other group. As we will show below in this section, a population may well spread into several districts, in which case, agents lose access even to agents of their own group.

We first establish the functional form of the population density within a segregated district. In a district hosting population i only, land market clears so that $\lambda_i(x)s_i(x) = 1$ for all locations x where $\lambda_i(x) > 0$ (condition (iii)). Given this, the utility and the population gradients are given by expressions (4) and (5) respectively. Differentiating once more the utility expression yields

$$U_i'' = 0 \iff -2\tau\lambda_i - \beta\lambda_i'' = 0$$

This second order ordinary differential equation accepts the following class of solutions

$$\lambda_i(x) = C_i \cos\delta\,(x - \phi_i) \tag{7}$$

where $\delta^2 = 2\tau/\beta$ and the coefficients C_i and ϕ_i are constants to be determined. We define a subcenter as a location where the density $\lambda_i(x)$, and therefore, the land rent $R(x)$, are maximal. As function (7) has a maximum at $x = \phi_i$, the coefficient ϕ_i determines the location of a subcenter (if it exists), and the coefficient C_i measures the amplitude of population i's density and therefore the population density, which corresponds to the population density at its subcenter.

We now analyze the structure of cities with two and three segregated districts.

4.1 Two districts

Let the districts $[0, b_1]$ and $[-b_2, 0]$ host populations 1 and 2 respectively. Such an urban structure is best illustrated by the Island of Montreal or the city of Belfast where individuals segregate in two distinct areas based on language or religious grounds. Population densities are described by $\lambda_1(x) \geq 0$ for $x \in [0, b_1]$ and by $\lambda_2(x) \geq 0$ for $x \in [-b_2, 0]$. A spatial equilibrium is then defined by a set of scalars and functions (b_i, λ_i), $i = 1, 2$, which satisfy the following conditions:

(a) the no-relocation arbitrage conditions within a district: $U_1'(x) = 0$, $\forall x \in [0, b_1]$ and $U_2'(x) = 0$, $\forall x \in [-b_2, 0]$

(b) the no-relocation arbitrage conditions between districts: $U_1(x) \leq U_1(0)$, $\forall x \in [-b_2, 0]$ and $U_2(x) \leq U_2(0)$, $\forall x \in [0, b_1]$

(c) the continuity of bid rents at the borders of each district: $\psi_2(0) = \psi_1(0)$ and $\psi_1(b_1) = \psi_2(-b_2) = 0$, and

(d) the total population constraint: $P_1 = \int_0^{b_1} \lambda_1(x)dx$ and $P_2 = \int_{-b_2}^0 \lambda_2(x)dx$

Conditions (a) and (b) ensure that agents have no incentive to relocate to another location regardless of which population inhabits it. Conditions (c) ensure that land is allocated to the highest bidder at the district border and that land is prices at its opportunity cost at the city edge. Conditions (d) guarantees that each districts is occupied by its corresponding population. Note that the bid rent conditions (c) imply that $\lambda_2(0) = \lambda_1(0)$, $\lambda_1(b_1) = 0$, and $\lambda_2(-b_2) = 0$. This is because the bid rent $\psi_i(x)$ is inversely related to the use of space, $s_i(x)$, which is itself inversely related to the population density, $\lambda_i(x)$.

Using conditions (a), (c) and (d), we compute the spatial distributions (7) as (see details provided in Appendix A)

$$\lambda_1 = C_1 \cos[\delta(x - \phi_1)] \quad \text{and} \quad \lambda_2 = C_2 \cos[\delta(x + \phi_2)] \tag{8}$$

where

$$C_1 = \frac{\delta}{2}(P_1 + \alpha P_2) \quad \text{and} \quad C_2 = \frac{\delta}{2}(\alpha P_1 + P_2) \tag{9}$$

$$\sin(\delta\phi_1) = \frac{P_1 - \alpha P_2}{P_1 + \alpha P_2} \quad \text{and} \quad \sin(\delta\phi_2) = \frac{P_2 - \alpha P_1}{P_2 + \alpha P_1} \tag{10}$$

City borders are given by $b_i = \phi_i + \pi/(2\delta)$, $i = 1, 2$.

Because $P_1 > P_2$, we have $C_1 > C_2$ so that *population* 1 *reaches higher densities than population* 2. From expression (10), it is readily checked that $\phi_1 > 0$, $\phi_1 > \phi_2$, and $b_1 > b_2$. The maximum density of population 1 is C_1 whereas that of population 2 may be less than C_2. Thus population 1 is more concentrated and benefits from a better access to agents of its own group. Figure 1 depicts the urban structure with two spatial districts.

INSERT FIGURE 1 HERE

We still have to check whether this urban structure satisfies the no-relocation arbitrage conditions (b). The first condition states that agents of population 1 have no incentive to relocate in population 2's district. This can be checked by using relation (6),

$$U_1'(x) = \tau (1 - \alpha) \left\{P_1 - [P_2 - 2P_2^-(x)]\right\} > 0$$

which means that $U_1(x) \leq U_1(0)$, $x \in [-b_2, 0]$. This is because P_1 is larger than P_2 and $P_2^-(x)$ increases from 0 to P_2 in the interval $[-b_2, 0]$. No individual of the large population has an incentive to relocate to the small population area. In contrast, the second condition (b) does not always hold. By expression (6), we have

$$U_2'(x) = \tau (1 - \alpha) \left\{-P_2 - [P_1 - 2P_1^-(x)]\right\}$$

where the curly bracket increases from $-(P_1 + P_2) < 0$ to $(P_1 - P_2) > 0$ in the interval $[0, b_1]$. Hence, the utility differential $U_2(x) - U_2(0) = \int_0^x U_2'(z)dz$ is a convex function that first falls under zero and then eventually increases above zero. Clearly, $U_2(0) \geq U_2(x)$, $\forall x \in [0, b_1]$, if and only if $U_2(0) \geq U_2(b_1)$. Given that $U_2(0) = U_2(-b_2) = S_2(-b_2) + Y$ and $U_2(b_1) = S_2(b_1) + Y$, the no-relocation arbitrage condition (b) can be rewritten as $S_2(-b_2) \geq S_2(b_1)$. An individual of the small population area may well gain from moving to the large population area so as to benefit from a better access to the large group. In Appendix A, we show that the latter condition can be rewritten as

$$2\sqrt{\alpha P_1/P_2} < (1 + \alpha) \left[\pi - \arccos\left(\frac{P_1/P_2 - \alpha}{P_1/P_2 + \alpha}\right)\right] \tag{11}$$

This argument yields the following Proposition.

Proposition 3 *The spatial configuration with two segregated districts is a spatial equilibrium if $S_2(-b_2) \geq S_2(b_1)$; that is if condition (11) holds.*

When $P_1/P_2 \to \infty$, the inequality (11) is never satisfied. When $P_1/P_2 \to 1$, the condition becomes $2\sqrt{\alpha} < (1+\alpha)\left\{\pi - \arccos\left[(1-\alpha)/(\alpha+1)\right]\right\}$, which can be shown to be always satisfied. More generally, it can be shown that there exists a unique threshold for P_1/P_2 below which this condition is satisfied. So, the two-district configuration is an equilibrium if populations are of similar sizes or if inter-group interactions are weak. In equilibrium, the large population occupies a larger share of the urban area repelling the small population towards the other city edge. The small population accommodates this situation because its interactions with the large population are weak and because land rents are too high in the other district.

We also investigate whether the spatial distribution of agents exhibit one or two subcenters, where a subcenter is defined as a location where the density $\lambda_i(x)$, and therefore, the land rent $R(x)$, are maximal. It readily comes from (10) that the city exhibits one center at $x > 0$ if $P_1/P_2 > 1/\alpha$ (i.e. $\phi_1 > 0, \phi_2 < 0$), while it exhibits two of them - one on each side of $x = 0$ - if $1 < P_1/P_2 < 1/\alpha$ (i.e. $\phi_1 > 0$, $\phi_2 > 0$).

Corollary 4 *The two-district city exhibits a single subcenter if $\alpha > (P_1/P_2)^{-1}$ and two subcenters otherwise.*

Both urban structures are depicted in the two panels of Figure 1. Of course, when intra- and inter-group social interactions become equally frequent ($\alpha \to 1$), the location choices made by agents lead to the emergence of a single subcenter. On the other hand, when intra-group social interactions dominate inter-group ones ($\alpha \to 0$), each population group locates around its own subcenter while still benefiting from inter-group interactions as both groups live in the same city. Relative population sizes also matter. When these are similar, each population also forms its own subcenter. This is because strong intra-group interactions create a separate basin of attraction for each population. In contrast, when one population is much larger than the other one, its density becomes so high that it also becomes a basin of attraction for the small population, which ceases to have its own subcenter.

4.2 Three districts

We consider urban structures with three districts. In these structures, the large population may locate at either the center or the edge of the city.

Large population in the central district We consider a symmetric spatial configuration in which the large population 1 resides within the central district $[-b_1, b_1]$ and the small population 2 within the two edge districts $[-b_2, -b_1]$ and $[b_1, b_2]$.[4] Such an urban structure is reminescent of some European cities like Paris where the native population concentrates around the city center and ethnic populations reside in the suburbs.

A spatial equilibrium is a set of non negative scalars b_i, $i = 1, 2$ and two even functions $\lambda_1 : [-b_1, b_1] \to \mathbb{R}^+$ and $\lambda_2 : [-b_2, -b_1] \cup [b_1, b_2] \to \mathbb{R}^+$ which satisfy:

(a) the no-relocation arbitrage conditions within each district: $U_1'(x) = 0$, $\forall x \in [0, b_1]$ and $U_2'(x) = 0$, $\forall x \in [b_1, b_2]$

(b) the no-relocation arbitrage conditions between districts: $U_2(x) \leq U_2(b_1)$, $\forall x \in [0, b_1]$ and $U_1(x) \leq U_1(0)$, $\forall x \in [b_1, b_2]$

(c) the land rent continuity at district borders $\psi_2(b_1^-) = \psi_1(b_1^+)$ and $\psi_2(b_2) = 0$

(d) the total population constraint $P_1 = \int_{-b_1}^{b_1} \lambda_1(x)dx$ and $P_2 = 2\int_{b_1}^{b_2} \lambda_2(x)dx$

These conditions have an interpretation similar to that provided in the previous section. Conditions (a), (c) and (d) allows us to determine the spatial distributions as (see

[4] It can be shown that no asymmetric configuration with three districts can be a spatial equilibrium (see Appendix C).

details provided in Appendix B)

$$\lambda_1(x) = \lambda_1(-x) = C_1 \cos(\delta x) \text{ if } x \in [0, b_1]$$

$$\lambda_2(x) = \lambda_2(-x) = C_2 \cos[\delta(x - \phi_2)] \text{ if } x \in [b_1, b_2] \tag{12}$$

where

$$C_1 = \frac{\delta}{2}\sqrt{P_1^2 + P_2^2 + 2\alpha P_1 P_2} \quad \text{and} \quad C_2 = \frac{\delta}{2}(P_2 + \alpha P_1) \tag{13}$$

while $\phi_2 = b_2 - \pi/(2\delta)$ and the district borders b_1 and b_2 solve

$$\sin \delta b_1 = \frac{P_1}{\sqrt{P_1^2 + P_2^2 + 2\alpha P_1 P_2}} \tag{14}$$

$$\cos \delta (b_2 - b_1) = \frac{\alpha P_1}{\alpha P_1 + P_2} \tag{15}$$

The corresponding urban structure is illustrated in the left panel of Figure 2.

INSERT FIGURE 2 HERE

In the above urban structure, the no-relocation arbitrage conditions (b) are always satisfied. This means that no individual has an incentive to relocate in the district hosting the other population. On the one hand, because of its size, the large population benefits from more numerous social interactions. It is better off locating around the city center where it gets a closer access to agents of its own group. For $x \in [b_1, b_2]$, condition (6) leads to the utility gradient $U_1'(x) = \tau(1-\alpha)\left[-P_1 - (P_2^+(x) - P_2^-(x))\right] < 0$ as $P_2 < P_1$. Hence $U_1(x) \leq U_1(0)$ for $x \in [b_1, b_2]$. On the other hand, the small population has no incentive to relocate to the center. To show this, observe that by condition (6), for $x \in [0, b_1]$, we get $P_2^+(x) = P_2^-(x) = P_2/2$ so that the utility gradient $U_2'(x) = \tau(1-\alpha)\left[-(P_1^+(x) - P_1^-(x))\right]$ increases from zero to $\tau(1-\alpha)P_1 > 0$ when x rises from 0 to b_1. This means that $U_2'(x) \geq 0$ and thus $U_2(x) \leq U_2(b_1)$, $\forall x \in [0, b_1]$. Intuitively, the higher density of the large population in the city center also benefits the small population. Although it interacts less frequently with the large group, it gets a close access to a large number of agents of the other group.

These arguments yield the following Proposition.

Proposition 5 *The urban structure with three segregated districts and the large population in the central district is always a spatial equilibrium.*

In this three-district city, the large population occupies the city area where it benefits from a closer access to both populations at a high residence cost while the small population benefits from lower land rents in the city edges at the expense of a higher access cost. Moreover, the city exhibits a single subcenter in $x = 0$. This is because the location $x = \phi_2$ cannot be a subcenter for population 2. If it were so, one should have $\phi_2 > b_1$, which contradicts the condition $\phi_2 = b_2 - \pi/(2\delta)$ and $\delta\,(b_2 - b_1) < \pi/2$ imposed by (15). Hence, in this urban structure, the large population constitutes a basin of attraction that is large enough to impede the creation of subcenters within the small population' districts.

Large population in the edge district We now consider a spatial configuration where the small population 2 resides within the central district $[-b_2, b_2]$ and the large population 1 within the two edge districts $[-b_1, -b_2]$ and $[b_2, b_1]$. This structure is reminescent of some US cities like Detroit where the White population resides away from the city center while the Black (ethnic) population resides within the center.

The equilibrium analysis performed in the previous subsection applies here by simply swapping subscripts 1 and 2. The corresponding urban structure is depicted in the right panel of Figure 2. Yet, an important change concerns the no-relocation arbitrage condition (b) as the small population may here have an incentive to relocate in a peripheral district. We have that, for $x \in [b_2, b_1]$, $U_2'(x) = \tau\,(1-\alpha)\left[-P_2 - (P_1^+(x) - P_1^-(x))\right]$ which rises from $-\tau\,(1-\alpha)\,P_2 < 0$ to $\tau\,(1-\alpha)\,(P_1 - P_2) > 0$. Hence, $U_2(x)$ is a convex function on the interval $[b_2, b_1]$. Therefore, because $U_2(x)$ is constant for all $x \in [b_2, b_1]$, the condition $U_2(0) \geq U_2(x)$ is equivalent to $U_2(0) \geq U_2(b_1)$. The utility differential $U_2(0) - U_2(b_1) \geq 0$ can be written as (see details provided in Appendix B)

$$\pi(1+\alpha) - 2\sqrt{(2\alpha + P_1/P_2)\,P_1/P_2} \geq 2(1+\alpha)\arcsin\left(\frac{\alpha}{\alpha + P_1/P_2}\right) \tag{16}$$

As in the analysis of two-district cities, only individuals of the small population may benefit from relocating to the large population area.

Proposition 6 *The urban structure with three segregated districts and the small population in the central district is a spatial equilibrium if $U_2(0) \geq U_2(b_1)$; that is, if condition (16) holds.*

A numerical analysis of condition (16) shows that the above three-district city is a spatial equilibrium if population sizes are sufficiently similar or if inter-group interactions are weak enough. The explanation for this is as follows. Consider the case where the small population in the city center shrinks and the large population in the city edges grows. The growth of the large population increases the benefits of intra- and inter-group interactions while the decline of the small population diminishes these benefits. At city edges, the stronger intra-group interactions entice the large population to increase their bid for land use. This pressure on land rents in city edges transmits to the city center. At the same time, individuals of the small population benefits more from inter-group interactions than from intra-group ones, and have less incentive to stay close to each other. At some point, when the small population is small enough, its agents find the city edges more attractive and start relocating there. The three-district urban structure with the small population at the central district can then no longer be a spatial equilibrium. Note that the existence of such an equilibrium configuration stems from a coordination problem. Although the large population would benefit from locating around the city center, no individual agent has an incentive to do so as she would face an excessive land rent and lose access to her own group which is located in the city edges.

By swapping subscripts 1 and 2 in expressions (13)-(14), we get the coefficients C_i and the district borders b_i

$$C_1 = \frac{\delta}{2}\left(P_1 + \alpha P_2\right) \quad \text{and} \quad C_2 = \frac{\delta}{2}\sqrt{P_1^2 + P_2^2 + 2\alpha P_1 P_2}$$

where $\phi_1 = b_1 - \pi/(2\delta)$ and b_1 and b_2 solve

$$\sin \delta b_2 = \frac{P_2}{\sqrt{P_1^2 + P_2^2 + 2\alpha P_1 P_2}}$$

$$\cos \delta \left(b_1 - b_2\right) = \frac{\alpha P_2}{\alpha P_2 + P_1}$$

The city exhibits a single subcenter at $x = 0$ as $b_2 - \phi_1 > 0$. Indeed, the last condition implies that $b_1 - b_2 < \pi/(2\delta)$, which yields $\phi_1 - b_2 < 0$ because $\phi_1 = b_1 - \pi/(2\delta)$. This result is similar to that found in the previous case, where the large population locates in the central district.

Corollary 7 *Regardless of which population locates in the central district, the three-district city exhibits a single subcenter.*

In a three-district city, any population located around the city center creates a strong basin of attraction for both populations and impedes the creation of subcenters in the periphery, see Figure 2.

5 Discussion

In this section, we study the properties of the equilibrium structures obtained in Section 4, discuss the multiplicity of equilibria, and compare the utilities derived by each population group.

5.1 Comparative statics

Table 1 summarizes the comparative statics analysis of the two- and three-district configurations denoted respectively by (21), (212) and (121) indicating which population occupies the central district (see Appendices A and B for mathematical expressions). So as to ease the comparison, we denote the district area by $B_i = b_i$ for the two-district city and the central district of a three-district city, and by $B_i = 2(b_i - b_j)$ for an edge district hosting population $i \neq j$ in a three-district city. Many results are identical to all city structures. For instance, more frequent inter-group interactions (higher α), weaker preferences for space or larger access costs (higher $\delta^2 = 2\tau/\beta$) induce spatial concentration: agents locate closer to other individuals of their own group and reside in districts with smaller areas B_i and larger densities C_i; see columns 1, 2, 5 and 6. This is intuitive: agents substitute the use of space for social interactions so as to benefit from more frequent interactions with individuals of their own groups.

City structure	$\frac{dB_i}{d\alpha}$	$\frac{dB_i}{d\delta}$	$\frac{dB_1}{d(P_1/P_2)}$	$\frac{dB_2}{d(P_1/P_2)}$	$\frac{dC_i}{d\alpha}$	$\frac{dC_i}{d\delta}$	$\frac{dC_i}{d(P_1/P_2)}$	subcenter(s)
12	-	-	+	-	+	+	+	1 or 2
212	-	-	-	-	+	+	+	1
121	-	-	+	-	+	+	+	1

Table 1: Comparative statics summary

Other results may differ across city structures. On the one hand, the emergence of a second subcenter may arise in a two-district city only, see column 8. This point has been commented in Section 4 already. On the other hand, the impact of a rise in the population ratio P_1/P_2 on the area of districts depends on the city structure. For any urban structure, a rise in the ratio P_1/P_2 induces the small population 2 to live in a district with a smaller area B_2 simply because fewer agents demand land space (see column 4). This also increases the density amplitude C_i for both populations (see column 7). This is because the increased number of individuals of population 1 raises their incentive to locate closer to each others. The pressure on land rents exerted by population 1 increases, which transmits to the district hosting population 2 that is then enticed to use less space.

By contrast, the effect of rise in the ratio P_1/P_2 on the district hosting the large population 1 depends on whether the large population locates in the city edge. When this is the case (i.e. configurations 21 and 121), population 1 can expand *horizontally* through an increase of the district area (that is a larger B_1 as reflected in see rows 1 and 3 in column 3). This horizontal expansion is due to the availability of cheap land at city edges. On the other hand, when the large population 1 occupies the central district (i.e. configuration 212), land rents at the edge of the central district are so high that any horizontal expansion is refrained. Instead, population 1 concentrates around the district center. Moreover, the rising share of population 1 increases the benefit from intra-group social interactions, which induces population 1 to concentrate even further. As a consequence, the area of the central district B_1 shrinks (see row 2 in column 3).

5.2 Multiplicity of equilibria

Here we analyze the conditions under which the urban structures analyzed in Section 4 exist. In particular, we highlight the possibility of multiple spatial equilibria. Figure 3 depicts the equilibrium urban structures with two or three segregated districts in terms of the population ratio P_1/P_2 and the intensity of inter-group interactions α. A population residing in a district exhibiting a subcenter (resp. no subcenter) is indicated by a bold number (resp. regular number). Note that the curves displayed in Figure 3 are independent of δ, and therefore of the preference for land β and the access cost τ. This means that Figure 3 accounts for all the parameters of the model (α and P_1/P_2). The two-district city is an equilibrium provided that P_1/P_2 and α are not too large (see areas **1**2 and **12**). The three-district city with the large population 1 living in the central district is always an equilibrium regardless of parameter values (see area 2**1**2). In contrast, the three-district city with the small population 2 living in the central district is an equilibrium only for a low population ratio P_1/P_2 and a low α (see area 1**2**1). Figure 3 illustrates the existence of multiple equilibria. Depending on parameter values (P_1/P_2 and α), the economy exhibits one, two, or three spatial equilibria. The more similar the population sizes P_1 and P_2, and the weaker the inter-group interactions α, the more likely several equilibria to emerge.

INSERT FIGURE 3 HERE

When several spatial configurations are possible, spatial equilibria can be ranked in terms of the utility derived by each population. Though this ranking could not be established analytically, it does not depend on the preference for land β nor on the access cost τ. Figure 4 displays each population's preferred urban configuration in terms of the population ratio (P_1/P_2) and the intensity α of inter-group interactions.[5]

INSERT FIGURE 4 HERE

For high population ratios and strong inter-group interactions, the urban structure (212) is the unique spatial equilibrium and leaves no other choice to individuals. However, multiple equilibria exist for low population ratios or weak inter-group interactions. Figure 4 shows that population 1 prefers the three-district structure (P_1:212) where it resides in the central district while population 2 prefers the two-district configuration (P_2:21). In this case, both populations therefore disagree about the urban structure to adopt. Finally, for sufficiently low population ratios, both populations prefer the two-district urban configuration (P_1:21 and P_2:21). In this latter case, urban configurations are Pareto ranked and a common agreement can be reached so as to which spatial equilibrium is best.

A somewhat surprising result of our analysis is that the small population 2 always gets a higher utility in the two-district configuration. This configuration is preferred over the three-district urban configuration even if population 2 resides in the central district. This is because in the three-district urban configuration (121), the pressure on land prices exerted by population 1 in city edges transmits to the central district and outweights the benefits of intra-group interactions of population 2. Figure 4 also shows that population 1 displays a similar preference for the two-district configuration when populations have similar sizes. In this case, population 1 is worse off in configuration (212) as it faces too high land prices in the central district which outweights the benefit of intra-group interactions. It is the lack of access to city edges where land is cheaper that makes population 1 prefer configuration (21).

[5]Figure 4 summarizes the information contained in Tables 2 and 3 provided in Appendix D.

5.3 City growth and social integration

Our model helps understanding how city growth may affect urban structures. When city growth is not anticipated by agents, the urban structure depends only on population sizes. However, if both populations grow at the same rate, the urban structure remains unchanged. If some population grows at a faster rate than the other one, the city may incur a spontaneous restructuring process. To illustrate such a transition, suppose that the small population remains constant and has initially a size similar to that of the large group. In this case, it is possible that it resides in the central district surrounded by two edge districts hosting the large population (configuration 121). As the large population grows in size, it exerts a high pressure on land rents, which transmits from the city edges into the city center through the land market. At some point, the small population moves to a city edge, replacing the former population which was living there. This corresponds to a transition from configuration 121 to configuration 21, see transition a in Figure 3. The intuition is as follows. Rents have become too high in the central district so that some individuals of the small population have an incentive to move to the city edge so as to benefit from lower rents even though their intra-group interactions become more costly. As more of these individuals move to the edge district, their intra-group interactions become less costly, which makes the city edge more attractive. The restructuring process ends when all individuals of the small population 2 have relocated to the city edge. Of course, in our model this transition is instantaneous. Moreover, consequences of city growth do not end up here.

As the large population grows further in size, the city district hosting it expands horizontally, repelling the small population further away. At some point, the small population relocates in the two city edges (configuration 212). This corresponds to a transition from configuration 21 to configuration 212, see transition b in Figure 3. When the population ratio P_1/P_2 increases, configuration 21 has to restructure as it ceases to be an equilibrium. Intuitively, as the large population derives larger benefits from its intra-group social interactions, it can bid more for land. This pressure on land rents transmits in the small population's district through the land market so that this latter population spread across locations to face lowee land rents. By relocating to both city edges, the small population ends up splitting into two sub-groups. By doing so, it compensates the loss in inter-group interactions by larger land plots.

Because of multiple equilibria, city growth may induce spontaneous transition from one urban structure to another. So, urban structures depend on history. Whereas the growth of the large population can reshape the urban structure, a decline of this population has no effect on it. This is because any urban structure, which is an equilibrium for some initial population levels, remains so as the large population falls in size, see Figure 3. Interestingly, this suggests that over time, old cities like Paris have undergone several transitions and are now locked in configuration 212 with the large (native) population in the city center. In contrast, younger US cities like New York or Detroit may not have undergone such transitions and display an urban configuration with the minority group in the city center (configuration 121). Our model implies that over the long run, the minority group will be repelled to the city edge if the native population grows a faster rate than the minority group.

Our model also sheds some light on the impact of social integration programs on urban structures. Schooling and social programs aim at fostering social integration of immigrants with the native population. Urban planners and labor and urban economists often advocate a better social integration for efficiency and equity reasons. For instance, in Benabou (1993), under-investment in education is due to market imperfections arising from local human capital spillovers. Also, de Marti and Zenou (2012) point out that only substantial (versus partial) lower inter-community socialization costs can improve efficiency. Interpreting the frequency α of inter-group interactions as an indicator of social integration, our model provides some interesting insights, see Figure 3. First, segregation prevails as long as $\alpha < 1$. Therefore, the level of social integration should be very high (actually $\alpha = 1$) to eliminate spatial segregation and yield spatial integration. So, the lack of spatial integration should not be considered as an indicator of inefficiency of social integration programs. Second, when both populations have similar sizes, social integration programs that promote higher frequencies of inter-group interactions have no effect on the city structure. The population residing in the city center and in the city edges does not relocate as the frequency α of inter-group interactions decreases. This means that social integration programs may be ineffective in reshaping the urban landscape. Third, when one population is significantly larger than the other one, social integration may even fragment spatially the minority group and split into subgroups (see transition c in Figure 3).

6 Conclusion

In this paper, we have studied how segregated districts emerge endogenously in a city and how multiple spatial equilibria arise. Our analysis embeds Shelling's (1971) spatial segregation framework and Beckmann's (1976) urban land market. We have discussed the implications of our model regarding city growth and social integration programs. The paper also sets the stage for future research. Dynamic considerations, which are absent from our model, may be useful in understanding the evolution of spatial neighborhoods and how history may select spatial equilibria. The spatial segregation of several groups into several urban districts is another issue to be examined. It would also be interesting to compare the equilibrium outcome with the socially optimal allocation of resources, as well as with the outcome of spatial/social integration programs that involve some specific social mix within urban districts. Finally, the present analysis might be usefully exploited to discuss issues related to urban labor markets, school segregation and social capital. Indeed, part of the benefits of social interactions is the access to information about jobs (see Granovetter, 1973; Zenou, 2012). School pupils' composition may shape the long run frequencies of inter-group social interactions and therefore affect urban segregation. Also, social interactions and the spatial distribution of agents contribute to the social capital that agents have in urban areas.

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Appendix A: two-district urban structure

By expression (7), we can set $\lambda_1 = C_1 \cos[\delta(x - \phi_1)]$ and $\lambda_2 = C_2 \cos[\delta(x + \phi_2)]$. First, $\psi_1(b_1) = \psi_2(-b_2) = 0$ implies that

$$\lambda_1(b_1) = C_1 \cos[\delta(b_1 - \phi_1)] = C_2 \cos[\delta(-b_2 + \phi_2)] = \lambda_2(-b_2) = 0$$

so that

$$b_1 = \phi_1 + \frac{\pi}{2\delta} \; ; \; b_2 = \phi_2 + \frac{\pi}{2\delta}$$

Second, $U_1'(b_1) = U_2'(-b_2) = 0$ implies that

$$-\tau P_2 - \alpha\tau P_1 + \beta C_2 \delta \sin[\delta(-b_2 + \phi_2)] = 0$$
$$-\tau P_1 - \alpha\tau P_2 + \beta C_1 \delta \sin[\delta(b_1 - \phi_1)] = 0$$

so that

$$C_1 = \frac{\delta(P_1 + \alpha P_2)}{2} \; ; \; C_2 = \frac{\delta(\alpha P_1 + P_2)}{2}$$

Therefore,

$$\frac{C_1}{C_2} = \frac{P_1 + \alpha P_2}{\alpha P_1 + P_2}$$

Third population constraints $\int_0^{b_1} \lambda_1(x)dx = P_1$ and $\int_{-b_2}^{0} \lambda_2(x)dx = P_2$ imply

$$\frac{C_1}{\delta}(1 + \sin(\delta\phi_1)) = P_1$$
$$\frac{C_2}{\delta}(\sin(\delta\phi_2) + 1) = P_2$$

which yields

$$\phi_1 : \sin(\delta\phi_1) = \frac{P_1 - \alpha P_2}{P_1 + \alpha P_2} = -\cos(\delta b_1)$$
$$\phi_2 : \sin(\delta\phi_2) = \frac{P_2 - \alpha P_1}{P_2 + \alpha P_1} = -\cos(\delta b_2)$$

Surplus differential: We now compute the equilibrium condition $S_2(-b_2) \geq S_2(b_1)$. For any $x \in [0, b_1]$, we can write

$$
\begin{aligned}
S_2(x) &= \int (1-\tau\,|x-y|)\,\lambda_2(y)dy + \alpha \int (1-\tau\,|x-y|)\,\lambda_1(y)dy \\
&= \int_{-b_2}^{0} (1-\tau\,|x-y|)\,\lambda_2(y)dy + \alpha \int_{0}^{b_1} (1-\tau\,|x-y|)\,\lambda_1(y)dy
\end{aligned}
$$

Applying this expression to the locations $x=-b_2$ and $x=b_1$ we get

$$
S_2(-b_2) - S_2(b_1) = -\tau \int_{-b_2}^{0} (2y-b_1+b_2)\,\lambda_2(y)dy - \alpha\tau \int_{0}^{b_1} (2y-b_1+b_2)\,\lambda_1(y)dy
$$

One computes

$$
\begin{aligned}
\int_{-b_2}^{0} (2y-b_1+b_2)\,\lambda_2(y)dy &= (-b_1+b_2)\,P_2 + \int_{-b_2}^{0} (2y)\,\lambda_2(y)dy \\
&= (-b_1+b_2)\,P_2 + C_2 \int_{-b_2}^{0} (2y)\cos[\delta(y+\phi_2)]dy \\
&= (-b_1+b_2)\,P_2 + C_2 \frac{2}{\delta^2}\,(\cos\delta\phi_2 - \delta b_2)
\end{aligned}
$$

where the last line obtains because

$$
\begin{aligned}
\int_{-b_2}^{0} (2y)\cos(\delta\,(y+\phi_2))\,dy &= \frac{2}{\delta^2} \int_{-\delta b_2}^{0} \delta y \cos(\delta y + \delta\phi_2)\;d\delta y \\
&= \frac{2}{\delta^2} \int_{-\delta b_2}^{0} z\cos(z+\delta\phi_2)\,dz \\
&= \frac{2}{\delta^2}\,(\cos\delta\phi_2 - \cos\delta\,(\phi_2-b_2) + \delta b_2 \sin\delta\,(\phi_2-b_2)) \\
&= \frac{2}{\delta^2}\,(\cos\delta\phi_2 - \delta b_2)
\end{aligned}
$$

Also, one computes

$$
\begin{aligned}
\int_{0}^{b_1} (2y-b_1+b_2)\,\lambda_1(y)dy &= (-b_1+b_2)\,P_1 + \int_{0}^{b_1} (2y)\,\lambda_1(y)dy \\
&= (-b_1+b_2)\,P_1 + 2C_1 \int_{0}^{b_1} y\cos[\delta(y-\phi_1)]dy \\
&= (-b_1+b_2)\,P_1 + 2C_1 \frac{1}{\delta^2}\,(-\cos\delta\phi_1 + \delta b_1)
\end{aligned}
$$

where the last line obtains because

$$\begin{aligned}
\int_0^{b_1} y\cos[\delta(y-\phi_1)]dy &= \frac{1}{\delta^2}\int_0^{\delta b_1} \delta y\cos\left(\delta y-\delta\phi_1\right)d\delta y \\
&= \frac{1}{\delta^2}\int_0^{\delta b_1} z\cos\left(z-\delta\phi_1\right)dz \\
&= \frac{1}{\delta^2}\left(-\cos\delta\phi_1+\delta b_1\right)
\end{aligned}$$

Therefore, the surplus differential is positive iff $S_2(-b_2)-S_2(b_1)\geq 0$; that is, if

$$C_2\left(\cos\delta\phi_2-\delta b_2\right)+\alpha C_1\left(-\cos\delta\phi_1+\delta b_1\right)\leq\frac{\delta^2}{2}\left(b_1-b_2\right)\left(P_2+\alpha P_1\right)$$

or equivalently

$$-\delta b_1 P_2(-1+\alpha^2)+\alpha(P_1+\alpha P_2)\cos\delta\phi_1-(P_2+\alpha P_1)\cos\delta\phi_2>0 \tag{17}$$

Because

$$\begin{aligned}
\cos^2\delta\phi_1 &= 1-\sin^2\delta\phi_1 = 1-\left(\frac{P_1-\alpha P_2}{P_1+\alpha P_2}\right)^2 = 4\alpha\frac{P_1P_2}{\left(P_1+\alpha P_2\right)^2} \\
\cos^2\delta\phi_2 &= 1-\sin^2\delta\phi_2 = 1-\left(\frac{P_2-\alpha P_1}{P_2+\alpha P_1}\right)^2 = 4\alpha\frac{P_1P_2}{\left(P_2+\alpha P_1\right)^2}
\end{aligned}$$

we have that

$$\left(\alpha P_1+P_2\right)\cos\delta\phi_2-\alpha\left(P_1+\alpha P_2\right)\cos\delta\phi_1=2\left(1-\alpha\right)\sqrt{\alpha P_1P_2}$$

So, the condition (17) becomes

$$2\sqrt{\alpha P_1/P_2}<(1+\alpha)\left[\pi-\arccos\left(\frac{P_1/P_2-\alpha}{P_1/P_2+\alpha}\right)\right]$$

Comparative statics Here is a comparative statics analysis of the city equilibrium. First, population densities increase and district borders shrink as the access cost increases and the preference for space falls (a higher $\delta^2 = 2\tau/\beta$ raises C_i and reduces b_i), see relations (9) and (10). The population density increases as population sizes grow in equal proportions (keeping P_1/P_2 constant, higher values of P_1 and P_2 raise C_i only). A larger share of population 1 (P_1/P_2) leads district 1 to expand and district 2 to shrink. The city expands (larger b_i) if the frequency of inter-group interaction (α) falls, see relation (10). So, more frequent inter-group interactions concentrate populations further as they are able to bid more for land.

Utilities Utilities can be computed as

$$\begin{aligned} U_1 &= P_1\left(1-\alpha^2\right) + \frac{\tau}{2} b_2 \alpha \left(P_2 + \alpha P_1\right) - \frac{\tau}{2} b_1 \left(P_1 + \alpha P_2\right) + \frac{\tau}{\delta}\left(\alpha - 1\right)\sqrt{\alpha P_1 P_2} + Y \\ U_2 &= \frac{\tau}{2} b_2 (\alpha P_1 + P_2) - \alpha \frac{\tau}{2} b_1 (P_1 + \alpha P_2) - \frac{\tau}{\delta}\left(1-\alpha\right)\sqrt{\alpha P_1 P_2} + Y \end{aligned}$$

Their difference is given by

$$\Delta U = P_1\left(1-\alpha^2\right) - \frac{1}{2}(1-\alpha)\left[b_1 \tau (P_1 + \alpha P_2) - b_2 \tau (\alpha P_1 + P_2)\right]$$

Appendix B: three-district urban structure

Here, we focus on the case when the large population 1 is in the city center. The converse configuration can be obtained by swapping subscripts 1 and 2. By using the expressions $\lambda_1 = C_1 \cos(\delta x)$ if $x \in [0, b_1]$ and $\lambda_2 = C_2 \cos[\delta(x - \phi_2)]$ if $x \in [b_1, b_2]$, the conditions for population mass conservation and land rent arbitrage at district borders become

$$\begin{cases} P_1 = \int_{-b_1}^{b_1} \cos(\delta x)\, dx = 2C_1 \delta^{-1} \sin \delta b_1 \\ P_2 = 2C_2 \delta^{-1}\left[\sin \delta(b_2 - \phi_2) - \sin \delta(b_1 - \phi_2)\right] \\ \psi_2(b_1) = \frac{1}{2\beta} C_2^2 \cos^2[\delta(b_1 - \phi_2)] = \frac{1}{2\beta} C_1^2 \cos^2(\delta b_1) = \psi_1(b_1) \\ \lambda_2(b_2) = C_2 \cos[\delta(b_2 - \phi_2)] = 0 \end{cases} \tag{18}$$

The last line of (18) implies that $b_2 - \phi_2 = \pi/2\delta$. In addition, note that $U_2'(b_2) = 0$ implies that by (5), $\lambda_2'(b_2) = -\frac{\tau}{\beta}\left[P_2 + \alpha P_1\right]$, which yields

$$C_2 = \frac{1}{2}\delta\left(P_2 + \alpha P_1\right)$$

because $\lambda_2'(b_2) = C_2 \sin \delta(b_1 - \phi_2) = C_2$.

The three first lines of (18) become

$$\begin{cases} \frac{\delta P_1}{2C_1} = \sin \delta b_1 \\ 1 - \frac{\delta P_2}{2C_2} = \sin \delta(b_1 - \phi_2) \\ \frac{\cos[\delta(b_1 - \phi_2)]}{\cos(\delta b_1)} = \frac{C_1}{C_2} \end{cases}$$

Squaring those expressions and using $\cos^2 x = 1 - \sin^2 x$ yields

$$C_1^2 = (\delta/2)^2 \left(P_1^2 + P_2^2 + 2\alpha P_1 P_2\right)$$

So,

$$\sin \delta(b_1 - \phi_2) = 1 - \frac{P_2}{(P_2 + \alpha P_1)} = \frac{\alpha P_1}{\alpha P_1 + P_2}$$
$$\sin^2 \delta b_1 = \frac{P_1^2}{P_1^2 + P_2^2 + 2\alpha P_1 P_2}$$

Utility differential: We now compute the equilibrium condition $U_2(0) - U_2(b_1)$. For any $x \in [0, b_1]$, we can write

$$\begin{aligned} U_2(0) &= \alpha \int_{-b_1}^{-b2} (1 + \tau y)\, \lambda_1(y) dy + 2 \int_0^{b_2} (1 - \tau y)\, \lambda_2(y) dy \\ &\quad + \alpha \int_{b_2}^{b_1} (1 - \tau y)\, \lambda_1(y) dy - \beta \lambda_2(0) \\ U_2(b_1) &= \alpha \int_{-b_1}^{-b2} (1 - \tau(b_1 - y)) \lambda_1(y) dy + \int_{-b_2}^{b_2} (1 - \tau(b_1 - y)) \lambda_2(y) dy \\ &\quad + \alpha \int_{b_2}^{b_1} (1 - \tau(b_1 - y)) \lambda_1(y) dy - \beta \lambda_1(b_1) \end{aligned}$$

By replacing the value of $(b_1, b_2, \phi_1, \phi_2, C_1, C_2)$, we get that the condition $U_2(0) - U_2(b_1) \geq 0$ is equivalent to

$$\pi(1 + \alpha) - 2\sqrt{P(P + 2\alpha)} \geq 2(1 + \alpha) \arcsin(\frac{\alpha}{P + \alpha})$$

where $P = P_1/P_2$.

Comparative statics when the large population is at the city center First, population densities increase and district borders shrink as the access cost increases and the preference for space falls (a higher $\delta^2 = 2\tau/\beta$ raises both C_1 and C_2 by relation (13), while it reduces $b_2 - b_1$ and b_1 by relations (15) and (14)). Second, population densities increase as population sizes grow in equal proportions: keeping P_1/P_2 constant, larger populations P_1 and P_2 raise C_1 and C_2 only. Third, the city expands (larger b_1 and $b_2 - b_1$) if the frequency of inter-group interactions α decreases. The lower returns from inter-group interactions induce lower bid rents, and thus the dispersion of agents. Fourth, a larger share of population 1 (P_1/P_2) leads the central district to shrink and the edge district to expand (a higher ratio P_1/P_2 raises $b_1 - b_2$ and decreases b_1 by relations (15) and (14)).

Comparative statics when the large population is at the city edge The comparative statics analysis is derived in a way similar to that used in the previous case. We simply need to swap subscripts 1 and 2. Hence, population densities increase and the district borders shrink as the access cost increases and the preference for space falls (a higher $\delta^2 = 2\tau/\beta$ raises C_1 and C_2 while it reduces $b_1 - b_2$ and b_2). Population densities increase as population sizes grow in equal proportions (keeping P_1/P_2 constant, higher values of P_1 and P_2 raise C_1 and C_2 only). The city expands (larger b_2 and $b_1 - b_2$) when the frequency of inter-group interaction (α) decreases. A larger share of population 1 (P_1/P_2) decreases the area of the central district hosting population 2 (smaller b_2) and increases that of the edge district hosting population 1 (larger $b_1 - b_2$).

Appendix C

We here show that no asymmetric configuration with three districts can be a spatial equilibrium. We consider the following equilibrium candidate

$$\lambda_1(x) = C_1 \cos(\delta x),\ y \in [-b_2, b_1]$$
$$\lambda_2(x) = \begin{cases} C_2 \cos(\delta(x-\phi_1))\ ,\ x \in [b_1, b_3] \\ C_3 \cos(\delta(x+\phi_2))\ ,\ x \in [-b_4, -b_2] \end{cases}$$

and we show that equilibrium conditions are not compatible with the above asymmetric solution.

The conditions $U_2'(b_3) = U_2'(-b_4) = 0$ lead to $-\tau P_2 - \alpha\tau P_1 + C_2 \sin(\delta(b_3 - \phi_1)) = \tau P_2 + \alpha\tau P_1 + C_3 \sin(\delta(-b_4 + \phi_2)) = 0$. By using conditions $\psi_1(b_1) = \psi_2(b_1)$, $\psi_1(b_1) = \psi_2(b_1)$, and $\psi_2(-b_2) = \psi_1(-b_2)$,we get $\delta(b_3 - \phi_1) = -\delta(-b_4 + \phi_2) = \pi/2$ and

$$C_1^2 = C_2^2 \cos^2(\delta(b_1 - \phi_1))/\cos^2(\delta b_1) = C_3^2 \cos^2(\delta(-b_2 + \phi_2))/\cos^2(\delta b_2)$$

The conditions $U_2'(b_3) = U_2'(-b_4) = U_1'(0) = 0$ lead respectively to $C_2 = C_3 = \tau(P_2 + \alpha P_1)/(\beta\delta)$ and

$$\sin(\delta b_2) - \sin(\delta b_1) - \alpha(\sin(\delta(-b_2 + \phi_2)) + \sin(\delta(b_1 - \phi_1))) = 0$$

The two above conditions can be written as

$$m_2 - m_1 - \alpha(m_4 + m_3) = 0$$
$$(1 - m_3^2)(1 - m_2^2) - (1 - m_4^2)(1 - m_1^2) = 0$$

where m_1, m_2, m_3, and m_4 denote $\sin(\delta b_1)$, $\sin(\delta b_2)$, $\sin(\delta(b_1 - \phi_1))$, and $\sin(\delta(-b_2 + \phi_2))$.

The solutions for m_2 and m_4 are

$(m_1, -m_3)$ and

$$\left(\frac{-m_1 + m_1^3 - 2\alpha m_3 + 2\alpha m_1^2 m_3 - \alpha^2 m_1 + \alpha^2 m_1 m_3^2}{-1 + m_1^2 + \alpha^2 - \alpha^2 m_3^2}, \frac{-m_3 + m_1^2 m_3 - 2\alpha m_1 + 2\alpha m_1 m_3^2 - \alpha^2 m_3 + \alpha^2 m_3^3}{-1 + m_1^2 + \alpha^2 - \alpha^2 m_3^2}\right)$$

The first solution corresponds to the symmetric equilibrium while the second one is our asymmetric candidate.

By plugging the second solution into the two constraints, $C_1/\delta(m_1 + m_2) = P_1$ and $C_2/\delta(m_4 - m_3 + 2) = P_2$, we get a system of equations for m_1 and m_3. The solutions are given by $(m_1, m_3) = (P_1(\alpha - 1)/(P_2 + \alpha P_1), P_1(\alpha - 1)/(P_2 + \alpha P_1))$ and $(m_1, m_3) = (-P_1(\alpha+1)/(P_2+P_1\alpha), P_1(1+\alpha)(P_2+P_1\alpha))$. Both solutions imply that $m_1 < 0$ meaning that b_1 would be negative.

Appendix D

Table 2 (resp. Table 3) provides the ranking of urban configurations (21, 212, and 121) for individuals of population 1 (resp. for individuals of population 2).

INSERT TABLES 2 AND 3 HERE

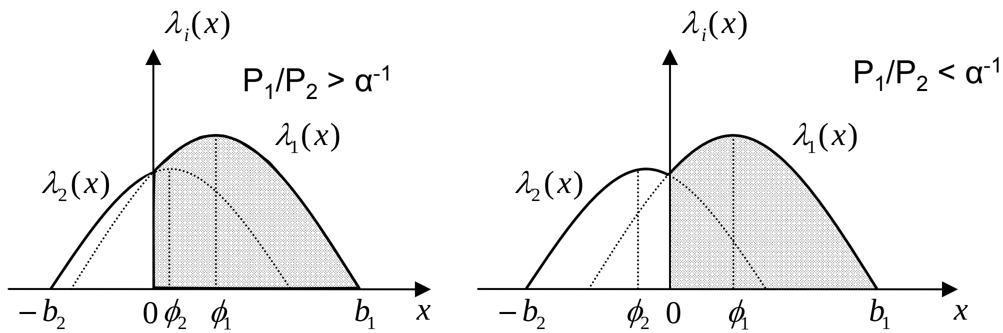

Figure 1: Two-district cities

The shaded area corresponds to the large population 1. In the left panel (resp. right panel), the urban structure displays a single subcenter (resp. two subcenters).

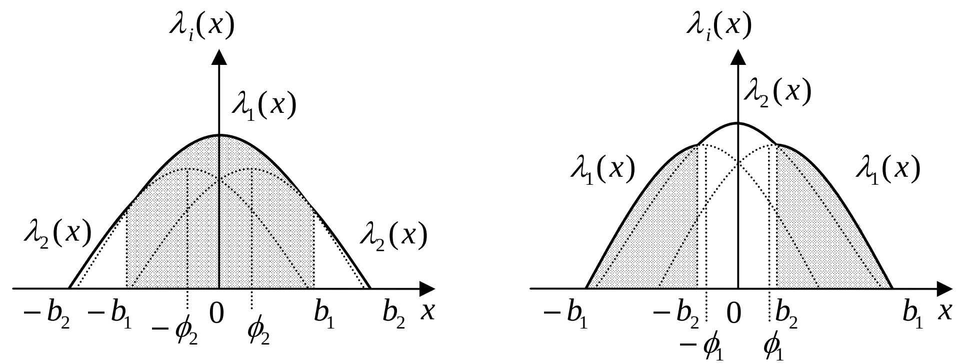

Figure 2: Three-district cities

The shaded area corresponds to the large population 1. In the left panel (resp. right panel), the large population 1 is hosted in the central district (resp. in the edge districts).

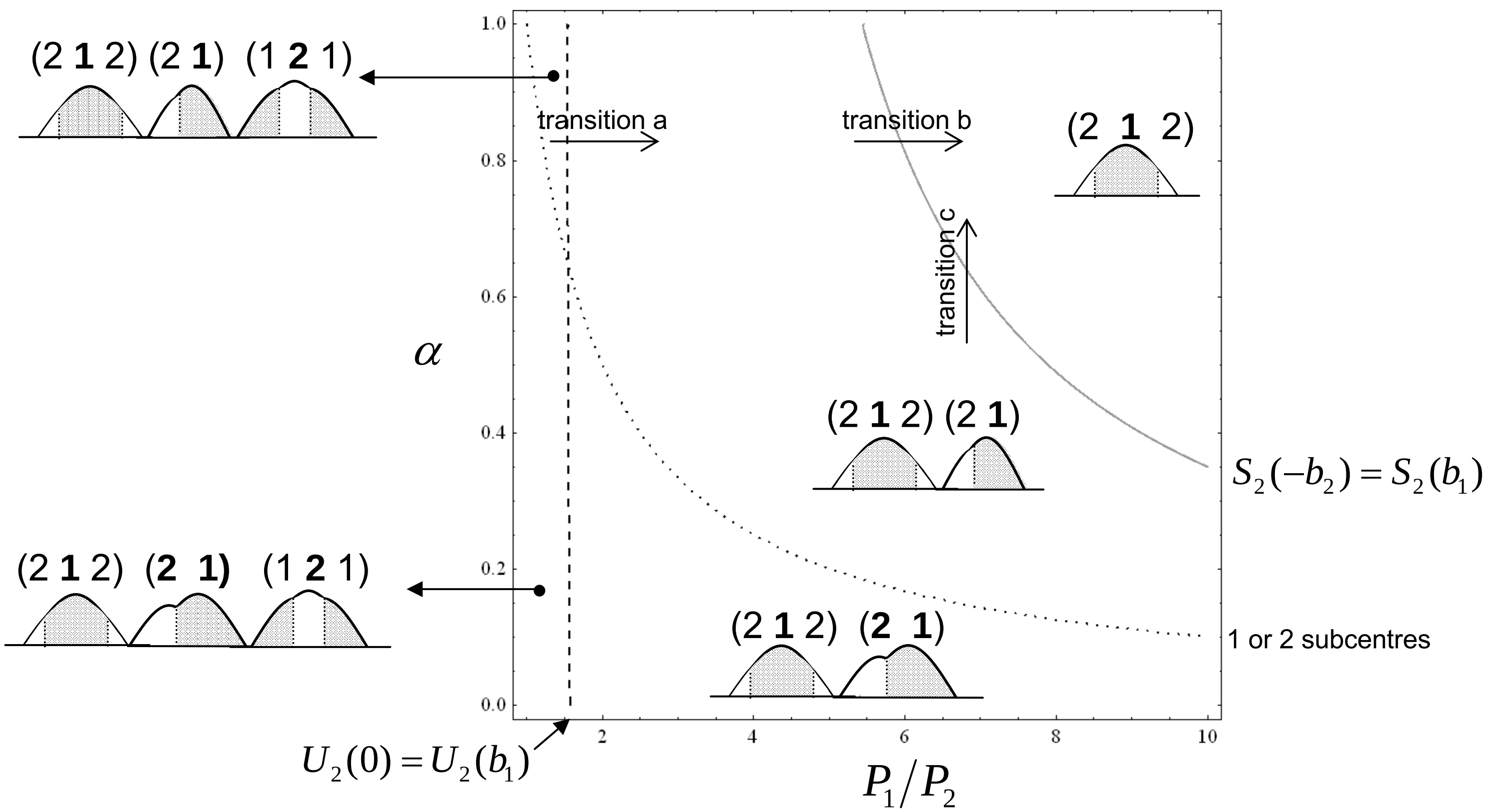

Figure 3: Urban structure equilibria

Two- and three-district equilibria in terms of the population ratio P_1/P_2 and the intensity of inter-group interactions α. The two-district city (21) (resp. the three-district city (121)) is a spatial equilibrium for parameter values at the left of the solid curve, representing condition (13) (resp. the dashed curve, representing condition (20)). Note that the three-district city (212) is a spatial equilibrium for all parameter values. The condition determining the number of subcentres in Corollary 4 is represented by the dotted curve so that a population residing in a district exhibiting a subcentre (resp. no subcentre) is indicated by a bold number (resp. a regular number).

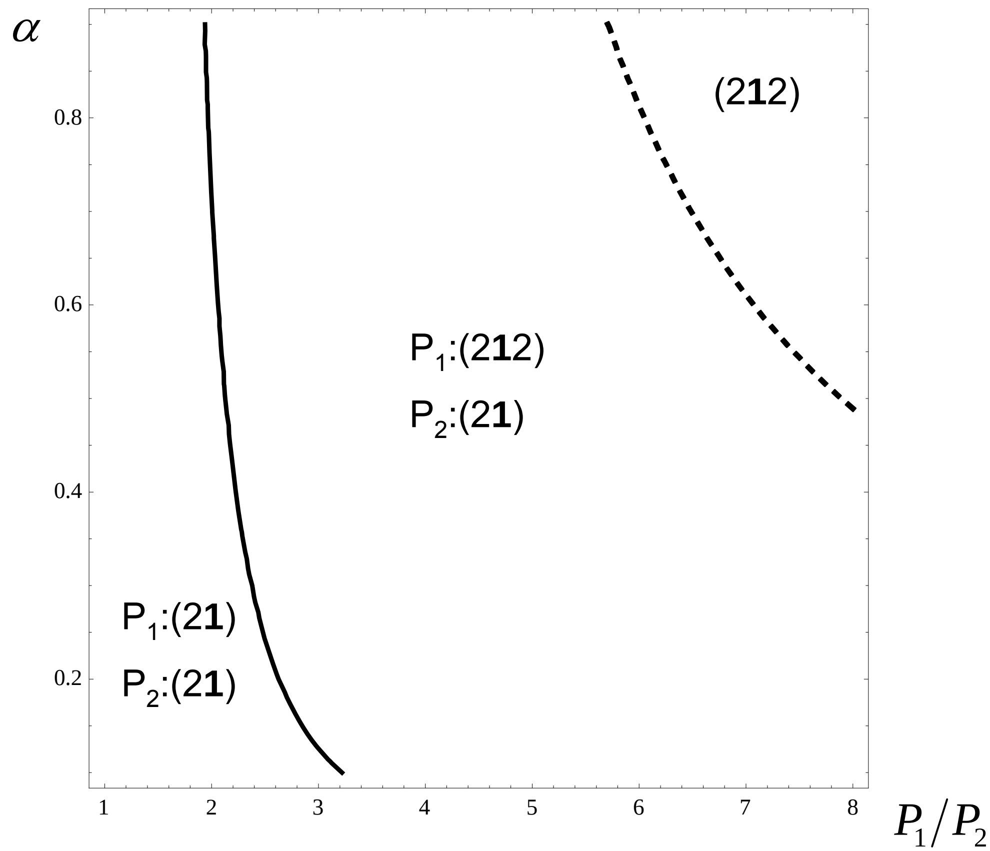

Figure 4: Preferred urban configuration by each population.

Above the dashed curve, the only equilibrium is (212). Below the solid curve, both populations prefer the two-district city (21). In between these two curves, population 1 prefers the three-district city (212) while population 2 prefers the two-district structure (21).

	P1/P2=1.5	2	2.5	3	3.5	4	4.5	5	5.5	6	6.5	7	7.5	8
α=0.9	21 212 121	212 21	212 21	212 21	212 21	212 21	212 21	212 21	212 21	212	212	212	212	212
0.8	21 212 121	212 21	212 21	212 21	212 21	212 21	212 21	212 21	212 21	212 21	212	212	212	212
0.7	21 212 121	21 212	212 21	212 21	212 21	212 21	212 21	212 21	212 21	212 21	212	212	212	212
0.6	21 212 121	21 212	212 21	212 21	212 21	212 21	212 21	212 21	212 21	212 21	212 21	212 21	212	212
0.5	21 212 121	21 212	212 21	212 21	212 21	212 21	212 21	212 21	212 21	212 21	212 21	212 21	212 21	212
0.4	21 212 121	21 212	212 21	212 21	212 21	212 21	212 21	212 21	212 21	212 21	212 21	212 21	212 21	212 21
0.3	21 212 121	21 212	212 21	212 21	212 21	212 21	212 21	212 21	212 21	212 21	212 21	212 21	212 21	212 21
0.2	21 212 121	21 212	21 212	212 21	212 21	212 21	212 21	212 21	212 21	212 21	212 21	212 21	212 21	212 21
0.1	21 212 121	21 212	21 212	21 212	212 21	212 21	212 21	212 21	212 21	212 21	212 21	212 21	212 21	212 21

Table 2: Urban configuration ranking for population 1

Each cell presents the ranking of urban configurations from most preferred to least preferred by population 1.

	P1/P2=1.5	2	2.5	3	3.5	4	4.5	5	5.5	6	6.5	7	7.5	8
α=0.9	21 121 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	212	212	212	212	212
0.8	21 121 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	212	212	212	212
0.7	21 121 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	212	212	212	212
0.6	21 121 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	212	212
0.5	21 121 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	212
0.4	21 121 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212
0.3	21 121 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212
0.2	21 121 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212
0.1	21 121 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212

Table 3: Urban configuration ranking for population 2

Each cell presents the ranking of urban configurations from most preferred to least preferred by population 2.