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Abstract

This paper shows that new economic geography models are capable of simulating the real-world tendency for urban agglomeration to the primate city. It is often observed that while regional populations were dispersed in early times, they have been increasingly concentrated into one capital region over recent years. The present paper thus demonstrates that multi-region, new economic geography models are able to simulate the real-world population distribution trends witnessed over the past few centuries.

Keywords: Agglomeration, New economic geography, Historical population distribution
JEL classification: R12

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1 Introduction

Since the pioneering work of Krugman (1991), new economic geography (NEG) has been developed and sophisticated in several directions in order to show how the spatial distribution of economic activities is evolving in the real world. Specifically, the alternative modeling strategies proposed by Ottaviano, Tabuchi and Thisse (2002), Forslid and Ottaviano (2003), and Pflüger (2004), among others, have improved analytical tractability, which has enabled researchers to gain further insights into the space economy and its transition. Furthermore, NEG has been enriched by introducing important ingredients such as intermediate goods (Krugman and Venables, 1995), land for housing (Helpman, 1998; Tabuchi, 1998), urban costs (Tabuchi and Thisse, 2006), agricultural transport costs (Picard and Zeng, 2005), firm heterogeneity (Melitz, 2003), and economic growth (Martin and Ottaviano, 1999).

The scopes of most theoretical studies published thus far have been limited to two regions in order for researchers to reach meaningful analytical results. Their main result of the two-region NEG models is that the spatial distribution is dispersed in the early period (high trade costs or low manufacturing share) and agglomerated in one of the two regions in the late period (low trade costs or high manufacturing share).

However, it is no doubt that the two-region NEG models are too simple to describe the spatial distribution of economic activities in the real-world economy. Since there are only two regions, locations of them are necessarily symmetric, and hence, spatial distributions cannot be diverse. In order to consider asymmetric locations of regions, the number of regions should be more than two.

There have been a few attempts to extend the two-region to multi-region NEG models (notably Krugman, 1993; Venables and Limão, 2002; Picard and Tabuchi, 2010). Although there are attempts to simulate the spatial distribution of economic activities in the real world (notably Bosker *et al.*, 2010), previous studies have not yet succeeded in obtaining practical analytical results for the long-term transition of the spatial distribution of economic activities in the multi-region economy. The present paper thus fills this gap in the current body of knowledge on this topic by describing the transition of the spatial distribution of economic activities over the past few centuries despite considering NEG using an arbitrary number of regions in order to maintain analytical tractability.

The spatial distribution of economic activities has certain characteristics. The first is Zipf's law or the rank-size rule of city size distribution. The microfoundations of this law have been explained by Rossi-Hansberg and Wright (2007), among others. However, this is beyond the scope of the present paper, which focuses on explaining *long-term changes in the size distribution of regions rather than of cities*, and thus uses NEG models.

The second characteristic is the robustness of long-run regional population distribution trends to large temporary shocks, such as the bombing of Japan during World War II (Davis and Weinstein, 2002). However, although the regional population distribution is robust and stable during an intermediate period of time, it has been shown to be gradually changing over very long periods of time, as described in the next section. In particular, it is often observed that the capital region has experienced distinct growth patterns over centuries especially after the Industrial Revolution.

Therefore, this paper simply plots the long-term regional population distributions for several countries in order to assess how the aftermath of the Industrial Revolution and the recent IT revolution have influenced decreases in shipping and communications costs and thereby affected urbanization and agglomeration to core regions (Bairoch, 1988). It then presents NEG models that are capable of explaining the described changes in the spatial distributions of economic activities.

The remainder of this paper is organized as follows. The regional population distribution trends for the postwar period and for a few centuries in several countries are investigated in the next section. In order to explain these trends, a multi-region extension of Krugman's (1991) model is presented in section 3. The existence and stability of the spatial equilibrium are examined and interpreted in relation to the actual multi-region economy in section 4. Welfare analysis for different types of workers is conducted in section 5. Section 6 concludes.

2 Long-term trends in population distribution

2.1 Postwar trends

I first considered the rates of population growth and decline of the largest city in each sample country. Since the largest cities in these countries often spread out beyond municipal boundaries, I chose metropolitan areas rather than municipal city areas as the unit of analysis. Although the definitions of metropolitan areas differ by country, the United Nations database of urban agglomerations includes both central cities and suburbs, and provides a universal definition of metropolitan areas.[1] The data sources are listed in Appendix 1.

From the UN database, I chose the top 30 countries according to GDP in 2010 and then selected the largest metropolitan area (= agglomeration) in each country.[2] I collected these data on every fifth year between 1950 and 2010, resulting in 13 years. Even though the national populations in each sample country increased during the study period, the population shares of the largest metropolitan areas also increased owing to interregional migration. In order to confirm this trend, I calculated the correlation coefficients between the population share of the largest metropolitan area and sample years of $t = 1950$, $1955, \ldots, 2010$. It was found that these correlation coefficients were significantly positive in 24 countries, significantly negative in 4 countries, and insignificant in 2 countries out of 30 countries at the 5 percent level. This implied that *the population shares in most of*

[1] According to World Urbanization Prospects (http://esa.un.org/unup/index.asp?panel=6), the term "urban agglomeration" refers to the de facto population contained within the contours of a contiguous territory inhabited at urban density levels without regard to administrative boundaries. It usually incorporates the population in a city or town plus that in the suburban areas lying outside of but adjacent to the city boundaries.

[2] The largest cities in the top 30 countries by national GDP in 2010 are New York in USA, Shanghai in China, Tokyo in Japan, Mumbai in India, Berlin in Germany, Moscow in Russia, London in UK, São Paulo in Brazil, Paris in France, Rome in Italy, Mexico City in Mexico, Seoul in South Korea, Madrid in Spain, Toronto in Canada, Jakarta in Indonesia, Istanbul in Turkey, Teheran in Iran, Sydney in Australia, Warsaw in Poland, Amsterdam in Netherlands, Buenos Aires in Argentina, Ar-Riyadh in Saudi Arabia, Bangkok in Thailand, Johannesburg in South Africa, Cairo in Egypt, Karachi in Pakistan, Bogota in Colombia, Kuala Lumpur in Malaysia, Brussels in Belgium, and Lagos in Nigeria.

the largest cities in the top 30 countries by GDP have grown since World War II.

Figure 1a displays the 25 countries in which the population share of the largest metropolitan area is increasing over time, while Figure 1b shows the 5 countries in which the population share of the largest metropolitan area is decreasing over time (one of which is statistically insignificant). The marked growth in Tokyo, Ar-Riyadh, and Seoul, which have increased by about 15 percentage points over the 60-year study period, is notable.

Most of the largest agglomerations experienced gradual postwar growth with a few exceptions such as New York. The regional evolution in the United States may be explained by immigration from Europe and settlement to the West for more than four centuries. Such an exogenous increase in population was modeled under the NEG framework by Fujita, Krugman and Mori (1999). Rather than focusing on these few exceptions, the present paper pays attention to the gradual increases in the largest agglomerations that started agrarian societies and then became industrialized societies.

2.2 Historical trends

In assessing the longer-term changes in the spatial distribution of economic activities, it is necessary to consider a spatial unit that does not change over time, and hence, metropolitan areas such as Metropolitan Statistical Areas are inappropriate.[3] It is also necessary that a spatial unit is large enough to include metropolitan areas which contain a central city and suburbs. In fact, according to the population density plots in Tokyo and New York metropolitan areas plotted by Nakamura (http://blogs.yahoo.co.jp/shohei_tokyo _1980/32233480.html), they are continuously and smoothly decreasing from each city center for a hundred kilometer radius, implying that each metropolitan area may spread over an area with a radius of more than a hundred kilometer radius. It must be that London region or Greater London is too small to cover London metropolitan area and Tokyo prefecture is too small to cover Tokyo metropolitan area.

Based on these observations, I collected historical data on the regional population and aggregated some of them for the following six sample countries: Brazil (sample period 1872-2010), France (1851-2009), Great Britain (1701-2010), Italy (1881-2001), Japan

[3] How varying definitions yield different population totals, see Forstall *et al.* (2009).

(1721-2010), and Spain (1900-2010). The data sources are again listed in Appendix 1. The historical population shares in each region are plotted in Figures 2a-2f, respectively.

The visual inspection of Figures 2a-2e reveals the striking feature that the region containing the largest metropolitan area (Sao Paulo in Brazil, Île-de France in France, London+East+South East in Great Britain, Lazio in Italy, and Minami Kanto in Japan) has experienced significant population growth in recent years in comparison with that in the rest of the country.[4] In particular, the region containing the largest metropolitan area in Brazil (Figure 2a), France (Figure 2b), and Japan (Figure 2e) showed remarkable growth relative to the rest of the country. Indeed, the share of the region tripled in France and was 2.5 times larger in Brazil and Japan during their respective study periods. This growth pattern was similar in Spain (Figure 2e). However, it displays the rapid population growth in the two central regions, Madrid and Barcelona, possibly due to their different cultures although Barcelona has lost share since the 1980s. Therefore, the same basic forces seem to be at work in these countries. I may thus conclude that *the region containing the largest metropolitan area has significantly increased in population share at the expense of the rest of the country in the long-term in these six countries.*

The observations in Figures 2a-2f are consistent with those in Figures 1a-1b with the exception of Great Britain. The share of the capital region in Great Britain increased after 1800 (Figure 2b), whereas the largest metropolitan area of London has lost share since World War II (Figure 1b). Such a discrepancy occurs because of the definition of the regional boundary: the latter region is not large enough to cover the whole area in which daily commuting to the center of capital is possible.

The foregoing raises the question of why the population share in the regions containing the largest metropolitan areas significantly increased. The level of urban concentration to capital regions may be partly determined by political factors (Ades and Glaeser, 1995), while the growth in urban concentration to these regions may be determined by the extent-

[4] Not all countries display the same regional growth patterns. I also collected historical data on population shares by state in the United States and Austria. Contrary to Figures 2a-2f, the population share in New York State (which contains the largest metropolitan area has been declining since early in the 19th century because of the settlement to the West mentioned previously. The population shares in Vienna and Lower Austria have also been decreasing.

of-the-market and aggregate demand (Ades and Glaeser, 1999). I attempt to explain and simulate these historical trends using NEG models in the next section.

3 Using NEG models to explain historical trends

The space economy consists of an arbitrary number of regions $n(\geq 2)$. Following the established tradition of NEG, each region is divided into rural and urban areas (or traditional and modern sectors). Agricultural production is carried out by farmers (or unskilled workers) in rural areas, whereas manufacturing production is carried out by workers (or skilled workers) in urban areas. The size of the workforce is fixed and split according to given shares between farmers and workers. Each person supplies one unit of labor inelastically. The given masses of farmers and workers are equal to $1-\mu>0$ and $\mu>0$, respectively. Farmers are immobile and evenly distributed, i.e., the number of farmers is $(1-\mu)/n$ in each region, while workers are assumed to be spatially mobile.

The agricultural good is homogeneous and is supplied under constant returns and perfect competition. In order to produce one unit of the homogenous good, one unit of farming labor is required. This good can be traded freely between regions and thus its price is identical across regions. Hence, this good may be chosen as the numéraire. As a result, farmers' wages are equal to one in each region.

The manufacturing good is horizontally differentiated and is supplied under increasing returns and monopolistic competition. Manufacturing technology is identical for all varieties and in all regions and involves a fixed input requirement F and marginal input requirement c. Thus, the production of a quantity q requires labor input l, given by

$$l = cq + F.$$

There is a continuum of potential firms, implying that the impact of each firm on market outcomes is negligible. Because of increasing returns to scale in production, each variety is produced by a single firm in only one region. Since firms are symmetric, each firm's output is equalized in equilibrium. Therefore, the total number of firms and varieties in region $r(=1,2,\ldots,n)$ are given by $m_r=\mu\lambda_r/l$, where the percentage distribution of workers is denoted by

$$\Lambda \equiv (\lambda_1, \lambda_2, \ldots, \lambda_n), \qquad \lambda_r \in [0,1], \ \sum_{r=1}^{n} \lambda_r = 1.$$

Interregional shipments of any variety are subject to iceberg trade costs: $\tau > 1$ units have to be shipped for one unit to reach its destination. That is,

$$\tau_{rs} = \begin{cases} \tau & \text{if } r \neq s, \\ 1 & \text{if } r = s. \end{cases}$$

Although trade costs differ according to the distance between regions, I assume common τ for each pair of regions: the transport cost of one unit of good is the same regardless of the origin and destination. This assumption may be justified by the fact that distance-related shipping costs are low, whereas distance-unrelated costs such as insurance, loading and unloading are relatively high (Boyer, 1997). Therefore, I focus on an arbitrary number of locationally symmetric regions and assess *what happens when fully symmetric equilibrium breaks.* In the general case of asymmetric transport costs, one can hardly obtain meaningful analytical results. However, it is partially analytically tractable in a race-track economy, where regions are symmetrically located on the circumference of a circle (Krugman, 1993; Picard and Tabuchi, 2010). It is also partially analytically tractable in a linear economy, where regions are located equidistant on a line (Venables and Limão, 2002; Ago, Isono and Tabuchi, 2006). It is worth of noting that the analytical results and numerical results in these literatures are similar to those obtained in the next sections.

The preferences of a typical resident of region s are represented by

$$U_s = \mu^{-\mu} (1-\mu)^{\mu-1} \left[\sum_{r=1}^{n} \int_0^{m_r} q_{rs}(v)^{\frac{\sigma-1}{\sigma}} dv \right]^{\frac{\mu\sigma}{\sigma-1}} A^{1-\mu}, \tag{1}$$

where $q_{rs}(v)$ is the consumption of variety v in region s that is produced in region r, $\sigma > 1$ measures the elasticity of substitution between any two varieties, and A is the consumption of the homogeneous good. The budget constraint of a consumer earning a wage w_s in region s is as follows:

$$\sum_r \int_0^{m_r} p_{rs}(v) q_{rs}(v) \mathrm{d}v + A = w_s, \tag{2}$$

where $p_{rs}(v)$ is the delivered price of variety v in region s that is produced in region r. Given these assumptions, firms differ only by their locations in equilibrium. Accordingly, I drop the variety label v hereafter.

The maximization of (1) subject to the budget constraint (2) yields the following demand for a variety produced in region r:

$$q_{rs} = \frac{p_{rs}^{-\sigma}}{P_s^{1-\sigma}}\mu Y_s, \tag{3}$$

where $Y_s = \mu\lambda_s w_s + \frac{1-\mu}{n}$ is the total income in region s and P_s is the price index in region s, given by

$$P_s = \left(\sum_t m_t p_{ts}^{1-\sigma}\right)^{1/(1-\sigma)}. \tag{4}$$

Accordingly, the indirect utility of a consumer residing in region s is

$$V_s = \frac{w_s}{P_s^{\mu}}. \tag{5}$$

Because of the iceberg assumption, a typical firm established in country r has to produce $x_{rs} = \tau_{rs}q_{rs}$ units to satisfy final demand q_{rs} in country s. The firm in region r takes (3) into account when maximizing its profit given by

$$\pi_r = \sum_s (p_{rs}q_{rs} - w_r c x_{rs}) - w_r F = \sum_s (p_{rs} - w_r c\tau_{rs}) \frac{p_{rs}^{-\sigma}}{P_s^{1-\sigma}}\mu w_s - w_r F. \tag{6}$$

The maximization of (6) yields the equilibrium price:

$$p_{rs} = \frac{\sigma}{\sigma-1}c\tau_{rs}w_r = \tau_{rs}w_r. \tag{7}$$

where I normalize the marginal labor requirement $c = 1 - 1/\sigma$. Assuming the free entry of firms, $\pi^* = 0$ holds, which leads to the equilibrium output:

$$\sum_s x_{rs} \leq F\sigma = l. \tag{8}$$

Let $\phi_{rs} \equiv \tau_{rs}^{1-\sigma}$ be the freeness of trade. Substituting (3) and (4) into (8), multiplying both sides by $p_{rr} > 0$, and using (7), I derive

$$\sum_s \frac{w_r^{-\sigma}\phi_{rs}\left(\lambda_s w_s + \frac{1-\mu}{n\mu}\right)}{\sum_t \lambda_t w_t^{1-\sigma}\phi_{ts}} \leq 1 \tag{9}$$

with equality if $\lambda_r > 0$. Hence, there are n wage equations (9) that determine the equilibrium wages w_r. By Walras law, one of these conditions is redundant, and thus manufacturing labor in one country can serve as the numéraire.

4 Equilibrium and stability

Workers must reach the same utility level in the long-run equilibrium. A spatial equilibrium is such that there exists a constant $\bar{V}$ for which

$$V_r \leq \bar{V} \qquad \text{and} \qquad \left(V_r - \bar{V}\right)\lambda_r = 0 \qquad \forall r = 1, \ldots, n. \tag{10}$$

Because V_r is continuous with respect to λ_s ($s = 1, \ldots, n$), it follows from Ginsburgh *et al.* (1985) that a spatial equilibrium exists for all parameter values.

For the stability of equilibrium, I choose the replicator dynamics of $n-1$ equations and $n-1$ variables:

$$\overset{\bullet}{\lambda}_r = \lambda_r \left(V_r - \sum_s \lambda_s V_s\right) \equiv J_r \qquad \text{for } r = 1, \ldots, n-1, \tag{11}$$

where $\bullet$ denotes the time derivative of λ_r and the equation for the nth region is out of consideration because of the identity $\lambda_n = 1 - \sum_{s=1}^{n-1} \lambda_s$. Since unstable equilibria are hardly observed in the real world, the stability is used to refine the equilibria.

4.1 Agglomeration at the break point

As a thought experiment, I consider trade costs to steadily fall, i.e., the freeness of trade gradually increases from 0 (autarky) to 1 (free trade).[5] First of all, the symmetric configuration defined by $\Lambda_{\text{sym}} \equiv (1/n, \ldots, 1/n)$ is obviously a spatial equilibrium for any values of the freeness of trade. The first task is thus to study the conditions under which this symmetric configuration is a stable equilibrium as follows.

Because $w_1, \ldots, w_n$ are determined by the wage equations (9), their marginal changes can be computed by the implicit function theorem as follows:

$$\left.\begin{pmatrix} \frac{\partial w_1}{\partial \lambda_s} \\ \vdots \\ \frac{\partial w_n}{\partial \lambda_s} \end{pmatrix}\right|_{\Lambda=\Lambda_{\text{sym}}} = \begin{pmatrix} \frac{\partial J_1}{\partial w_1} & \cdots & \frac{\partial J_1}{\partial w_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial J_n}{\partial w_1} & \cdots & \frac{\partial J_n}{\partial w_n} \end{pmatrix}^{-1} \left.\begin{pmatrix} \frac{\partial J_1}{\partial \lambda_s} \\ \vdots \\ \frac{\partial J_n}{\partial \lambda_s} \end{pmatrix}\right|_{\Lambda=\Lambda_{\text{sym}}} \qquad \forall s = 1, \ldots, n-1.$$

[5] The thought experiment may be conducted with respect to the manufacturing share μ rather than to trade costs τ. in order to describe the structural change in industrial composition or rural-to-urban migration. The main conclusions of this paper remain the same.

Substituting the above into

$$\left.\frac{dJ_r}{d\lambda_s}\right|_{\Lambda=\Lambda_{\mathrm{sym}}} = \left.\frac{\partial J_r}{\partial \lambda_s} + \sum_t \frac{\partial J_r}{\partial w_t}\frac{\partial w_t}{\partial \lambda_s}\right|_{\Lambda=\Lambda_{\mathrm{sym}}} \quad \forall r, s = 1, \ldots, n-1$$

and evaluating it yields

$$\begin{aligned}
\left.\frac{dJ_r}{d\lambda_s}\right|_{\Lambda=\Lambda_{\mathrm{sym}}} &= \begin{cases} B\left(1-\phi\right)\left(\phi-\phi_B\right) & \text{if } s=r \\ 0 & \text{if } s\neq r \end{cases} \qquad \forall r, s = 1, \ldots, n-1 \\
\phi_B &\equiv \frac{\left(1-\mu\right)\left(\sigma-1-\mu\sigma\right)}{\left(1-\mu\right)\left(\sigma-1-\mu\sigma\right)+n\mu\left(2\sigma-1\right)},
\end{aligned}$$

where B is a positive constant. Here, ϕ_B is the symmetry break point. Solving $\phi_B > 0$ yields the no-black-hole condition as given by

$$\mu < \mu_{\max} \equiv \frac{\sigma-1}{\sigma}. \tag{12}$$

Thus, this procedure has extended the well-known NEG result on dispersion and agglomeration by Krugman (1991) to an arbitrary number of regions. This leads us to the following proposition:

Proposition 1 *Assume that the no-black-hole condition (12) holds. The symmetric equilibrium is stable only if* $0 \leq \phi < \phi_B$.

Next, we pay attention to the full agglomeration equilibrium $\Lambda_{\mathrm{agg}} \equiv (0, \ldots, 0, 1)$. Plugging $\Lambda = \Lambda_{\mathrm{agg}}$ into the nth wage equation (9), I get $w_n = 1$. Substituting $\Lambda = \Lambda_{\mathrm{agg}}$ and $w_n = 1$ with $w_r = w$ for all $r = 1, \ldots, n-1$ into the other wage equations yields

$$w = \left[\frac{1+\left(n-2\right)\phi+\phi^2-\mu\left(1-\phi\right)\left(1+n\phi\right)}{n\phi}\right]^{1/\sigma}.$$

Therefore, the agglomeration at the symmetry break point $\phi = \phi_B$ is sustained if

$$V_n - V_1\big|_{\Lambda=\Lambda_{\mathrm{agg}},\, \phi=\phi_B,\, w_1=\cdots=w_{n-1}=w,\, w_n=1} = 1 - g^{1/\sigma} > 0, \tag{13}$$

where

$$g \equiv \frac{\phi^{\frac{\mu\sigma}{\sigma-1}-1}}{n}\left[1+\left(n-2\right)\phi+\phi^2-\mu\left(1-\phi\right)\left(1+\left(n-1\right)\phi\right)\right].$$

The inequality (13) can be rewritten as $\phi > \phi_S$, where ϕ_S is defined by the unique solution of $g = 1$. This leads us to the following proposition:

Proposition 2 *The agglomerated equilibrium is stable if $\phi > \phi_S$ or if the no-black-hole condition (12) is violated.*

Although the research interest of this paper is assessing what happens when the symmetry breaks. Unfortunately, it is far from possible to find all equilibria when the symmetry breaks in the case of an arbitrary number of regions. Therefore, this paper proceeds by investigating the full agglomeration Λ_{agg} only. In other words, the question is whether the full agglomeration is a stable equilibrium when the symmetry breaks at $\phi = \phi_B$. This can be answered by finding whether the following inequality holds:

$$g|_{\phi=\phi_B} < 1. \tag{14}$$

Note that the LHS is a function of the three parameters: n, μ and σ. The full agglomeration is a stable equilibrium if the inequality (14) holds. Similar to Robert-Nicoud (2005), this statement can be proven in the following manner. First, $h \equiv \log g|_{\phi=\phi_B}$ is shown to be decreasing in n by differentiation. Then, letting $\widetilde{h} \equiv h|_{n=2}$, it can be readily verified that $\partial^2\widetilde{h}/\partial\mu^2 < 0$ for all $0 < \mu < \mu_{\max}$ and $\partial\widetilde{h}/\partial\mu\Big|_{\mu=0} = \widetilde{h}\Big|_{\mu=0} = 0$. Thus, I get $\widetilde{h} < 0$ and $h < 0$ for all $0 < \mu < \mu_{\max}$. This leads us to the following proposition:

Proposition 3 *When trade costs steadily fall, the symmetric equilibrium breaks and full agglomeration to one city emerges for any parameter value of n, μ and σ in Krugman's (1991) model with an arbitrary number of regions.*[6]

An identical result can be shown to hold in the case of Forslid and Ottaviano (2003) with an arbitrary number of regions. A similar result also holds in the case of Pflüger (2004) with an arbitrary number of regions (see Appendix 2). In the latter case, when the symmetry breaks, either partial agglomeration or full agglomeration emerges depending on the parameter values of n, μ and σ. Partial agglomeration is expressed as $\Lambda_{\text{par}} = \left(\frac{1-\lambda}{n-1}, \ldots, \frac{1-\lambda}{n-1}, \lambda\right)$ with $\frac{1-\lambda}{n-1} < \lambda$, implying that the number of workers in one region is larger than that in the remainder of each region. The following theorem is thus implied from these three major NEG models:

[6] For three regions, this proposition is the same as Proposition 3 in Castro, Correia-da-Silva and Mossay (2012).

Theorem 1 *When trade costs steadily fall, the symmetric equilibrium breaks and partial or full agglomeration to one city emerges for any parameter value of n, μ and σ in the models of Krugman (1991), Forslid and Ottaviano (2003), and Pflüger (2004) with an arbitrary number of regions.*

Because regional populations consist of farmers and workers, the population distributions of the symmetric and agglomerated configurations are given by $\left(\frac{1}{n}, \ldots, \frac{1}{n}\right)$ and $\left(\frac{1-\mu}{n}, \ldots, \frac{1-\mu}{n}, \frac{1+(n-1)\mu}{n}\right)$, respectively. Theorem 1 approximates the early and late periods of the historical trends experienced in Brazil, France, Great Britain, Italy, Japan, and Spain (see Figures 2a-2f). Thus, I conclude that these NEG models are capable of replicating the real-world tendency for urban agglomeration to the primate city.

5 Welfare considerations

Finally, it is of interest of explore welfare of manufacturing workers and farmers. Fortunately, Charlot *et al.* (2006) have already obtained a set of welfare results for two regions. This can be extended to an arbitrary number of regions because the nominal wages of workers and farmers are readily shown to be equal to one for the symmetric and agglomerated configurations in this model. The utility differentials are determined by the differences in the price indices, which are decreasing in the market access. Hence, those located in the agglomeration obviously enjoy better market access.

Furthermore, the same results are shown to hold in the cases of Forslid and Ottaviano (2003) and Pflüger (2004) with an arbitrary number of regions, although the nominal wages of workers are different. In sum, we establish the following robust results.

Proposition 4 *Workers prefer agglomeration to symmetric dispersion in the models of Krugman (1991), Forslid and Ottaviano (2003), and Pflüger (2004) with an arbitrary number of regions.*

Proposition 5 *Farmers in the core prefer agglomeration to dispersion whereas farmers in the periphery prefer dispersion to agglomeration in the models of Krugman (1991), Forslid and Ottaviano (2003), and Pflüger (2004) with an arbitrary number of regions.*

These two propositions correspond to Propositions 3 and 4 in Charlot *et al.* (2006). It follows from Propositions 4 and 5 that neither the agglomerated configuration nor the symmetric configuration is Pareto dominant. However, extending Proposition 5 in Charlot *et al.* (2006) to an arbitrary number of regions, one can say that the agglomerated configuration is socially preferable to the symmetric configuration in the sense of both Kaldor's and Hicks' criteria when the trade costs are sufficiently low.

6 Conclusion

There is consensus that while regional populations were dispersed in early times, they have been increasingly concentrated into one capital region over recent years. Although two-region NEG models can depict such an agglomeration tendency, the two-region economy itself is not realistic. This paper thus extended the two-region NEG models to multi-region ones, which can reasonably simulate these historical trends of urban agglomeration to the primate city. I have also shown that the multi-region models of NEG are similar to the two-region ones in terms of the market outcomes and social welfare.

Appendix 1: Data Sources

Figure 1a-1b: United Nations

World Urbanization Prospects, the 2011 Revision, Data on Cities and Urban Agglomerations (http://esa.un.org/unup/index.asp?panel=2).

Figure 2a: Brazil

Instituto Brasileiro de Geografia e Estatística (http://seriesestatisticas.ibge.gov.br/).

Figure 2b: France

Population of metropolitan France at the census, by region, Institut National de la Statistique et des Études Économiques

(http://www.bdm.insee.fr/bdm2/choixCriteres.action?request_locale=en&codeGroupe=27).

Figure 2c: Great Britain

For 1701-1951. Table 7.1 in Lee (1986).

For 1971-2010. Tables 1.2-1.3 in Table Vital Statistics: Population and Health Reference Tables, Office for National Statistics (www.ons.gov.uk/ons/rel/vsob1/vital.../rft-august-2011.xls).

Figure 2d: Italy

For 1881-1981. Table A.2 in Zamagni (1987).

For 1991-2001. Table 2, Italian National Institute of Statistics,

(http://dawinci.istat.it/daWinci/jsp/dawinci.jsp?q=pl02000100032301).

Figure 2e: Japan

For 1721-1846. Table 1 in Kito (1996).

For 1873-1890. Tables 20-21 in Umemura (1983).

For 1920-2010. Population Census of Japan, Statistical Bureau.

Figure 2f: Spain

Instituto Nacional de Estadistica (http://www.ine.es/en/welcome_en.htm).

Appendix 2: Pflüger's (2004) model with multiple regions

Extending Pflüger's (2004) model to an arbitrary number of regions, I get the indirect utility of a consumer living in region r:

$$V_r = \frac{1}{\sigma-1}\log\left(\sum_s \mu\lambda_s\phi_{sr}\right) + \frac{1}{\sigma}\sum_s \frac{\lambda_s + \frac{1-\mu}{\mu n}}{\sum_t \phi_{ts}\lambda_t}\phi_{sr}.$$

The symmetry break point is computed as

$$\widehat{\phi}_B \equiv \frac{\sigma-1-\mu\left(2\sigma-1\right)}{\sigma-1-\mu\left(2\sigma-1\right)\left(n-1\right)} \in (0,1)$$

and the no-black-hole condition is

$$\mu < \widehat{\mu}_{\max} \equiv \frac{\sigma-1}{2\sigma-1}. \tag{15}$$

Let $\widehat{\mu}_{\min}$ be a unique solution of $V_n - V_1|_{\Lambda=\Lambda_{\text{agg}},\, \phi=\widehat{\phi}_B} = 0$. These lead us to the following lemmas:

Lemma 1 *The interior equilibrium configuration with* $n-1$ *small cities and* 1 *large city is stable for all* $\mu_{\min} < \mu < \mu_{\max}$.

Lemma 2 *The fully agglomerated configuration is a stable equilibrium for all* $0 < \mu \leq \mu_{\min}$.

Summing up the foregoing results, I establish the following proposition.

Proposition 6 *When the trade costs steadily fall, the symmetric equilibrium breaks and partial or full agglomeration in one city appears for any parameter values of* n*,* μ *and* σ *in Pflüger's (2004) model with an arbitrary number of regions. More precisely, at* $\phi = \widehat{\phi}_B$*,*

(i) if $\mu \geq \widehat{\mu}_{\max}$*, the no-black-hole condition (15) is violated and there exists a stable equilibrium with full agglomeration in one city;*

(ii) if $\widehat{\mu}_{\max} > \mu > \widehat{\mu}_{\min}$*, there exists a stable equilibrium with one big city and* $n-1$ *small cities of equal size;*

(iii) if $0 < \mu \leq \widehat{\mu}_{\min}$*, there exists a stable equilibrium with full agglomeration in one city at* $\phi = \widehat{\phi}_B$.

Proofs of these lemmas and proposition are available upon request to the author.

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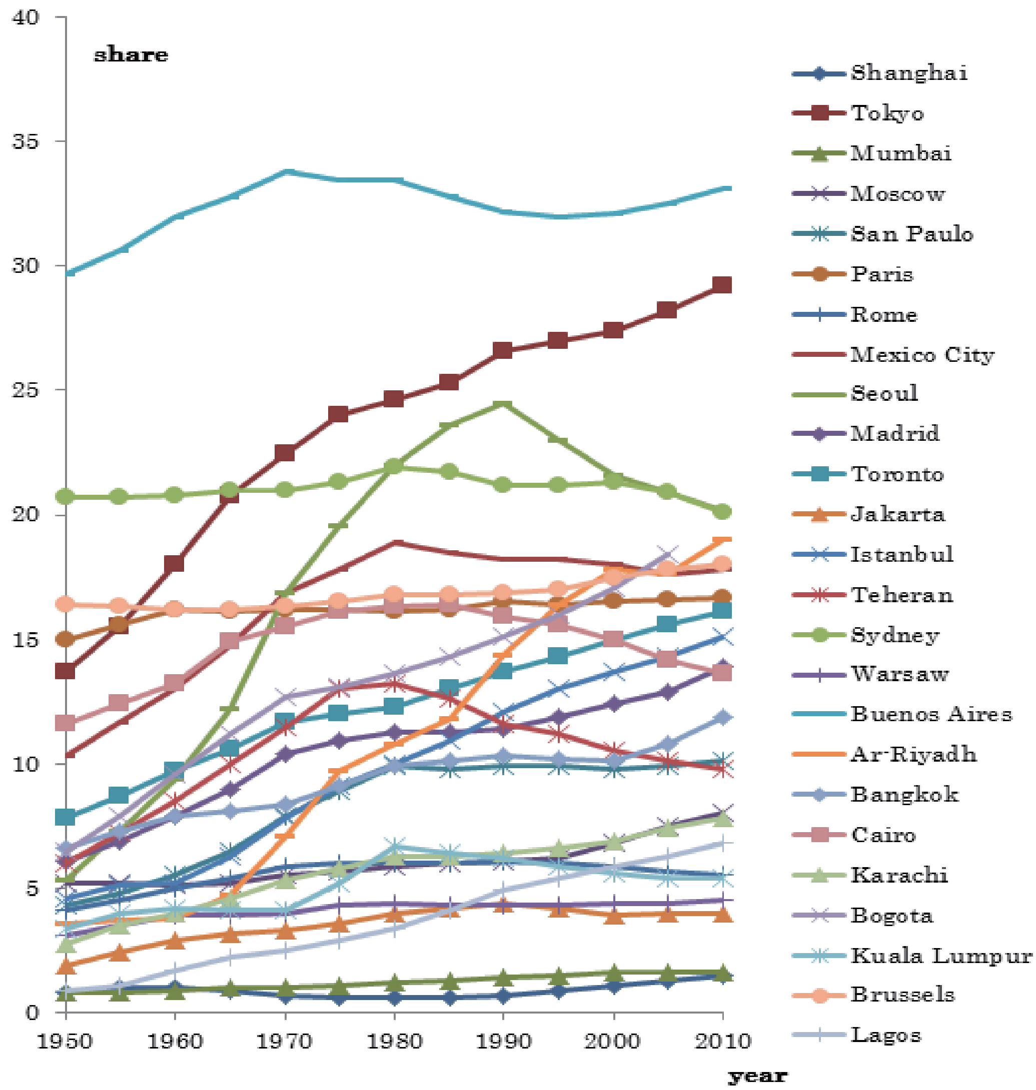

Figure 1a:　Increasing population share of the largest metropolitan area

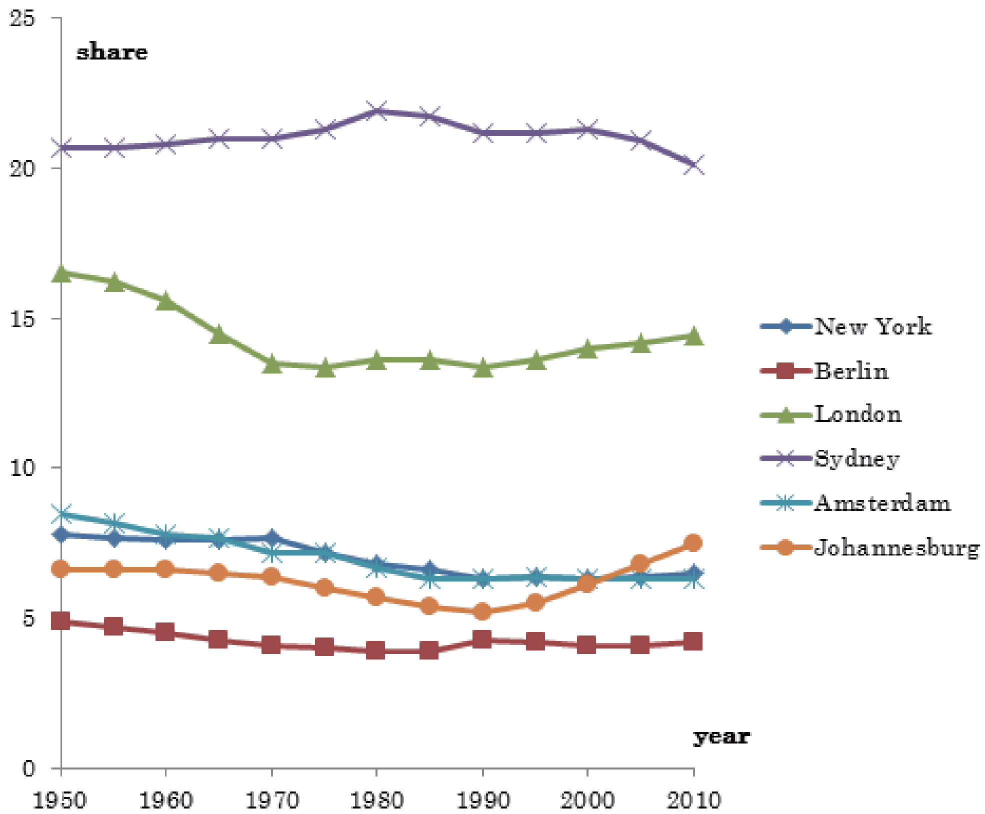

Figure 1b:　Decreasing population share of the largest metropolitan area

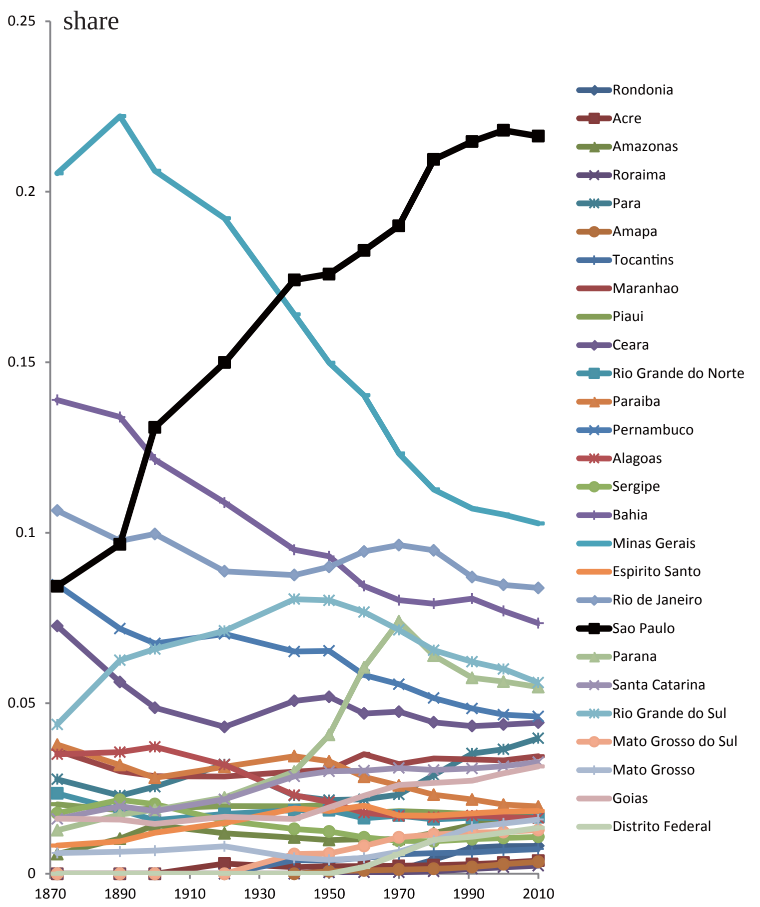

Figure 2a: Regional population share in Brazil

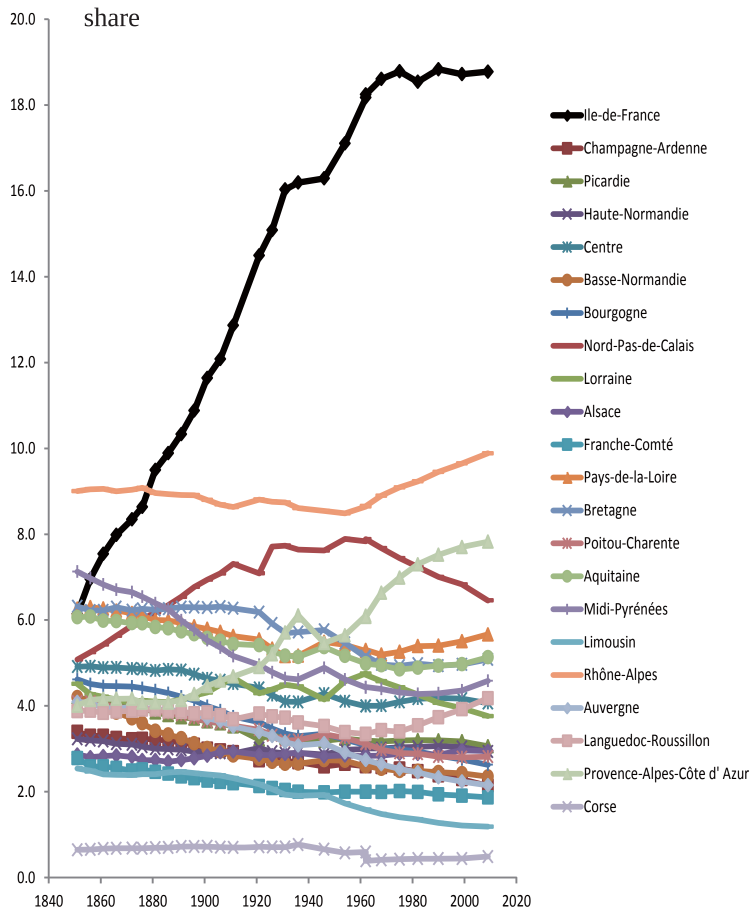

Figure 2b: Regional population share in France

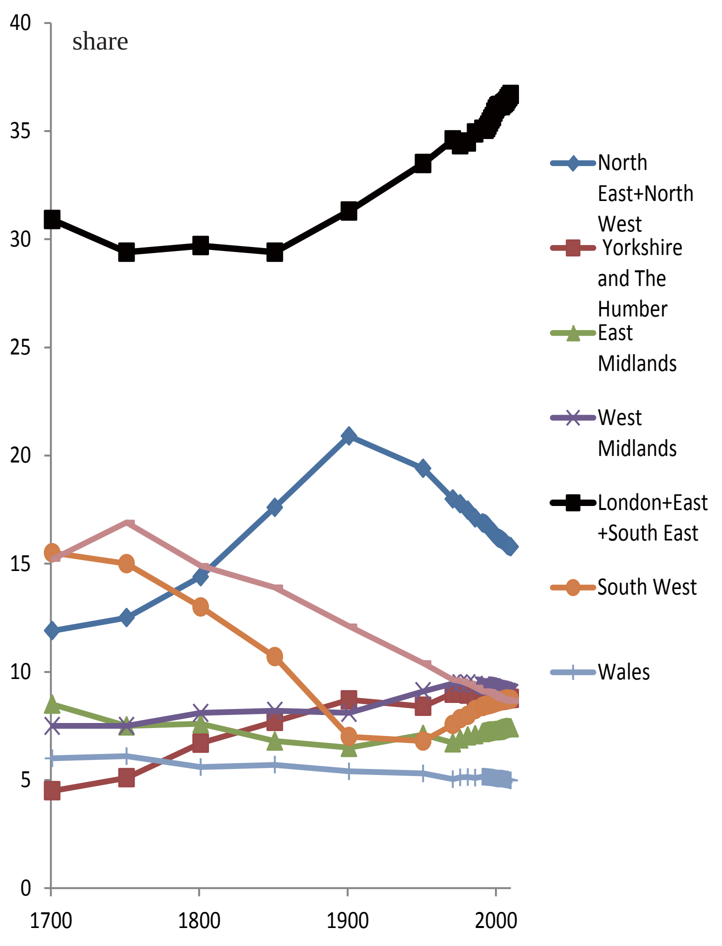

Figure 2c: Regional population share in Great Britain

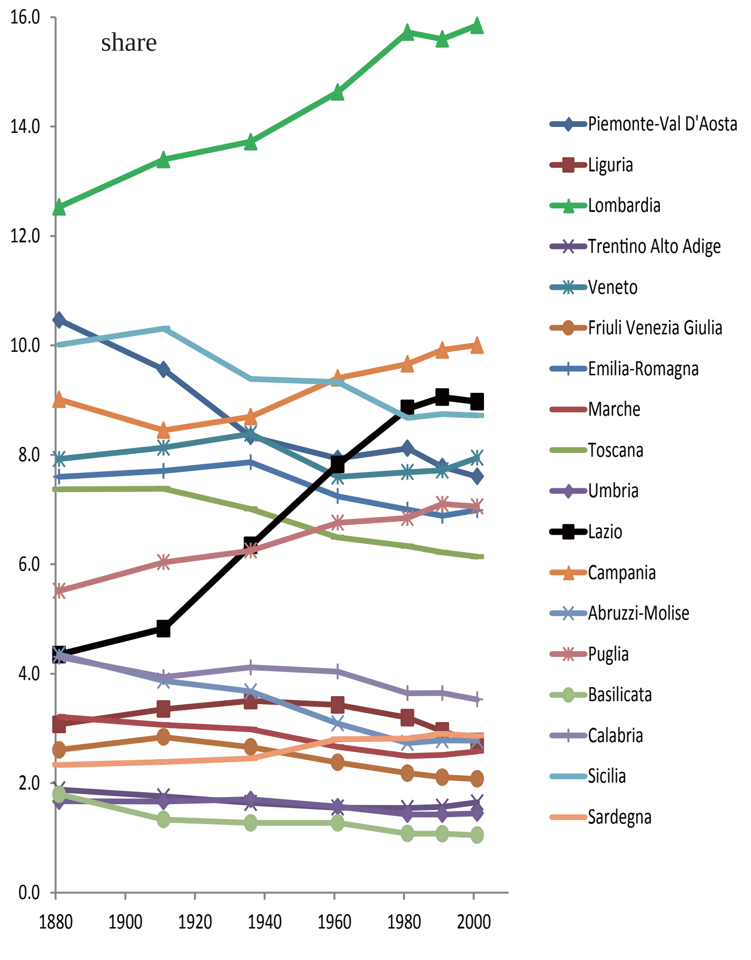

Figure 2d: Regional population share in Italy

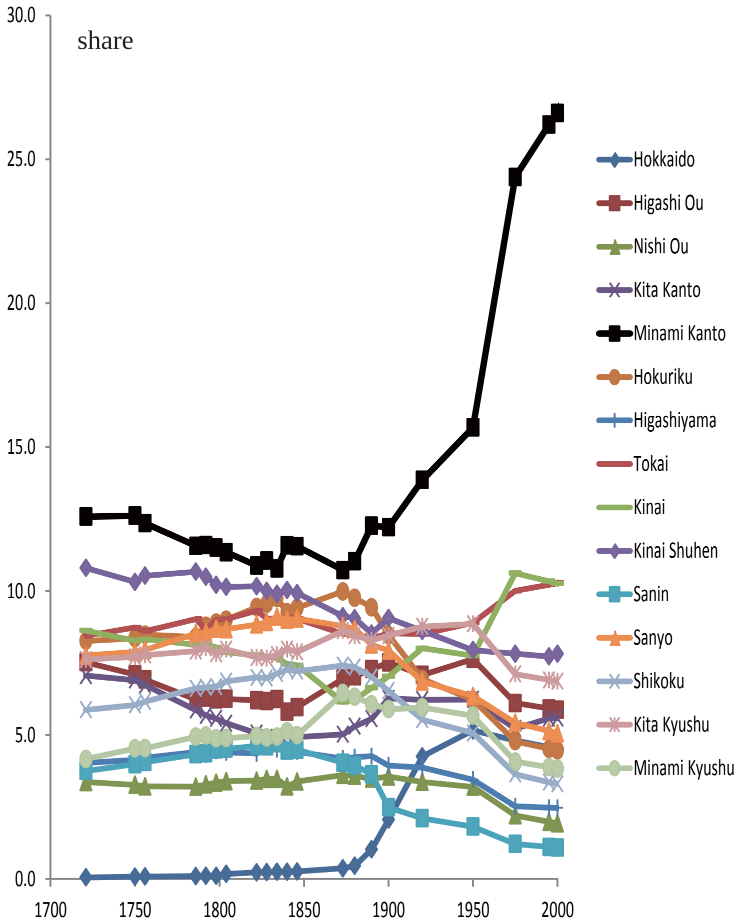

Figure 2e: Regional population share in Japan

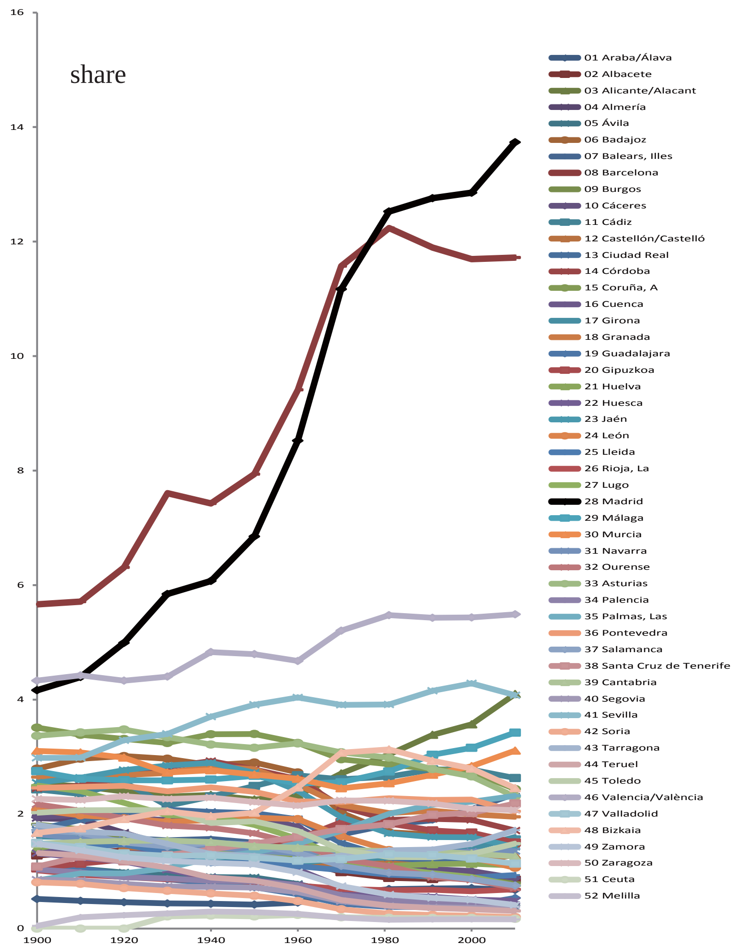

Figure 2f: Regional population share in Spain