An Eaton-Kortum Model of Trade and Growth (Revised)

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Abstract

We combine a multi-country, continuum-good Ricardian model of Eaton and Kortum (2002) with a multi-country AK model of Acemoglu and Ventura (2002) to examine how trade liberalization affects countries’ growth rates and extensive margins of trade over time. Focusing on the three-country case, we obtain three main results. First, a permanent fall in any trade cost raises the balanced growth rate. Second, trade liberalization increases the liberalizing countries’ long-run fractions of exported varieties to all destinations. Third, the long-run effects of trade liberalization are different from its short-run effects, which can reverse the welfare implications of the static Eaton-Kortum model.

Keywords: Eaton-Kortum model; Trade and growth; Trade liberalization; Extensive margins of trade; Preferential trade agreement

JEL classification: F13; F43

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1 Introduction

We are now witnessing a revival of the Ricardian model of international trade.¹ This revival is led by Eaton and Kortum (2002), who extend a two-country, continuum-good Ricardian model of Dornbusch et al. (1977) to an arbitrary number of countries by assuming that each country’s productivity of each variety in the unit interval is randomly drawn from a country-specific probability distribution. One of the remarkable features of the Eaton-Kortum model is that it enables us to analyze the effects of trade liberalization on the extensive margins of trade (i.e., the numbers or fractions of varieties each country imports from and exports to other countries) under high levels of asymmetry across countries.² However, their static formulation overlooks the fact that countries tend to export more and more varieties as they grow faster than the rest of the world (e.g., Hummels and Klenow, 2005; Kehoe and Ruhl, 2013). Allowing for different growth paths across countries may help explain the evolution of the extensive margins that the static framework cannot describe.³ The purpose of this paper is to develop an Eaton-Kortum model of trade and growth to examine how trade liberalization affects countries’ growth rates and extensive margins of trade over time.

To extend Eaton and Kortum (2002) dynamically, we use a multi-country AK model of Acemoglu and Ventura (2002). In the Acemoglu-Ventura model, countries trade only (exogenously or endogenously) differentiated intermediate goods, which in turn are produced from domestic capital under constant returns to scale. When a country grows faster than the rest of the world, its increased relative supply of capital lowers its rental rate of capital and hence its terms of trade. Since this pulls down the country’s rate of return to capital, its growth rate tends to fall back. Thanks to such stabilization forces, the countries growing at different rates during the transition converge to a common growth rate on a balanced growth path (BGP). Naito (2012) introduces international competition à la Dornbusch et al. (1977) in place of product differentiation in the two-country version of Acemoglu and Ventura (2002), and investigates the effects of unilateral trade liberalization on countries’ growth rates, extensive margins, and welfare. While obtaining clear-cut results, the two-country model cannot be applied to problems involving more than two countries. For example, considering a preferential trade agreement between two countries requires at least one outsider country.⁴ By combining Eaton and Kortum (2002) with Acemoglu and Ventura (2002), we formulate a model of endogenous growth and extensive margins which is applicable to multi-country problems.

After providing a general multi-country model, we focus on the three-country case to understand the mechanics of our model more deeply. We obtain the following main results. First, a permanent fall in any trade cost raises the balanced growth rate. For example, a fall in country 1’s import trade cost from country 2 increases its growth potential by allowing it to import varieties from country 2 more cheaply. Since this raises the relative rental rates and hence the terms of trade of countries 2 and 3 against country 1 through the above-mentioned stabilization mechanism, all countries grow at a higher balanced growth rate in the long run. This result is consistent with the recent sophisticated empirical research finding the

¹See Matsuyama (2008) and Eaton and Kortum (2012) for reviews of recent developments in the Ricardian trade theory.
²Although Melitz (2003) studies the same topic in a multi-country monopolistic competition model with firm heterogeneity, his analysis is based on the assumption of symmetric countries having a common productivity distribution. Baldwin and Robert-Nicoud (2008) embed the Melitz framework into an R&D-based endogenous growth model with expanding product variety, but their analysis is also subject to the symmetry assumption.
³Eaton and Kortum (2001) extend Eaton and Kortum (2002) to formulate an R&D-based growth. Assuming an exogenous growth rate of population which is common to all countries, they show that the long-run growth rate of technology in each country is equal to the population growth rate and is independent of trade costs due to their semi-endogenous growth specification. However, they do not examine how countries’ growth rates evolve depending on the trade costs during the transition.
⁴Dinopoulos and Syropoulos (1997) construct a three-country R&D-based endogenous growth model to see the long-run growth and welfare effects of unilateral, bilateral, and multilateral trade liberalization. Due to the presence of imperfect competition and nontraded goods, reductions in trade costs do not always raise countries’ long-run growth. However, they do not address the evolution of trade patterns.
positive relationship between trade liberalization and economic growth (e.g., Wacziarg and Welch, 2008; Estevadeordal and Taylor, 2013).

Second, trade liberalization increases the liberalizing countries’ long-run fractions of exported varieties to all destinations. Country 1’s unilateral trade liberalization for imports from country 2 shifts its extensive margins of imports away from country 3 to country 2. Not only that, it also increases country 1’s extensive margins of exports to all destinations because of its decreased long-run rental rates relative to countries 2 and 3. The result also applies to preferential trade liberalization between countries 1 and 2, with their long-run bilateral terms of trade unchanged: it increases their extensive margins of exports to both inside and outside destinations through their decreased long-run rental rates relative to country 3. Moreover, the fact that their fractions of domestic varieties and those of imported varieties from country 3 decrease means that both trade creation and trade diversion at the extensive margins occur as a result of such preferential trade liberalization. This result provides one theoretical explanation for why faster-growing countries export increasingly more varieties (e.g., Hummels and Klenow, 2005; Kehoe and Ruhl, 2013).

Third, the long-run effects of trade liberalization are different from its short-run effects, which can reverse the welfare implications of the static Eaton-Kortum model. In particular, starting from near the symmetric BGP, both trade liberalization schemes mentioned above raise the growth rates in countries 1 and 2 but lower the growth rate in country 3 in the initial period. The last result implies that country 3’s rate of return to capital and hence its welfare fall in the static Eaton-Kortum model. In our dynamic model, however, even country 3’s overall welfare can rise through accelerated long-run growth. This is indeed true in our numerical experiments based on the calibrated trade costs and technological parameters. The present result is remarkable in the context of regional trade agreements. The two-country model of Naito (2012).

The rest of this paper is organized as follows. Section 2 first sets up the multi-country model, and then concentrate on the three-country case. Section 3 examines analytically the long-run effects of unilateral and bilateral trade liberalization, and compare them with the short-run effects. Section 4 conducts some numerical experiments to gain additional insights. Section 5 concludes.

2 The model

Consider a world with \( N \geq 2 \) countries. In each country \( j (= 1, ..., N) \), there is only one nontradable final good, which can either be consumed or invested for capital accumulation. The final good is produced from a continuum of tradable intermediate goods indexed by \( i \in [0, 1] \) under constant returns to scale and perfect competition. Each variety \( i \) in turn can be produced using nontradable capital under constant returns to scale and perfect competition. Capital is the only primary factor.
2.1 Households

The representative household in country $j$ maximizes its utility $U_j = \int_0^\infty \ln C_{jt} \exp(-\rho_j t) dt$, subject to the budget constraint:

$$p_j^Y (C_{jt} + \dot{K}_{jt} + \delta_j K_{jt}) = r_{jt} K_{jt},$$  \hspace{1cm} (1)

with $\{p_j^Y, r_{jt}\}_{t=0}^\infty$ and $K_{j0}$ given, where $C_j$ is consumption; $\rho_j$ is the subjective discount rate; $p_j^Y$ is the price of the final good; $K_j$ is the stock of capital; $\delta_j$ is the depreciation rate of capital; and $r_j$ is the rental rate of capital; and a dot over a variable represents differentiation with respect to time $t$ (e.g., $\dot{K}_{jt} \equiv dK_{jt}/dt$).

We omit the time subscripts whenever there is no confusion. Under the logarithmic instantaneous utility function, it is optimal for the representative household to keep the consumption/capital ratio constant at $\rho_j$ for all periods, which immediately implies that capital always grows at the same (but not necessarily constant) rate as consumption given by the Euler equation:

$$\dot{K}_{jt}/K_{jt} = C_{jt}/C_j = r_{jt}/p_j^Y - \delta_j - \rho_j \forall t \in [0, \infty).$$  \hspace{1cm} (2)

We call this common growth rate "the growth rate in country $j".  

2.2 Final good firms

The representative final good firm in country $j$ maximizes its profit $\Pi_j^Y = p_j^Y Y_j - \int_0^1 p_j(i)x_j(i)di$, subject to the production function $Y_j = B_j(\int_0^1 x_j(i)(\sigma_j^{-1})/(\sigma_i^{-1})di)^{\sigma_j/(\sigma_i-1)}; \sigma_j > 1$, with $p_j^Y$ and $\{p_j(i)\}_{i=0}^1$ given, where $Y_j$ is the supply of the final good; $p_j(i)$ is the demand price of variety $i$; $x_j(i)$ is the demand for variety $i$; $B_j$ is the productivity in the final good sector; and $\sigma_j$ is the elasticity of substitution between any two varieties. The minimized cost is written as:

$$\int_0^1 p_j(i)x_j(i)di = q_j Y_j; q_j(\{p_j(i)\}_{i=0}^1) \equiv B_j^{-1}(\int_0^1 p_j(i)^{1-\sigma_j}di)^{1/(1-\sigma_j)}; \hspace{1cm} (3)$$

where $q_j(\cdot)$ is the unit cost function, working as the price index of intermediate goods. With Eq. (3), the first-order condition for profit maximization, also implying zero profit, is given by:

$$p_j^Y = q_j.$$  \hspace{1cm} (4)

2.3 Intermediate good firms

Our formulation of the intermediate good sector is based on Eaton and Kortum (2002). The representative intermediate good firm producing variety $i$ in country $j$ maximizes its profit $\Pi_j^x(i_j) = p(i_j)x(i_j) - r_j K^x(i_j)$, subject to the production function $x(i_j) = K^x(i_j)/a_j(i_j)$, with $p(i_j)$ and $r_j$ given, where $p(i_j)$ is the supply price of variety $i$; $x(i_j)$ is the supply of variety $i$; $K^x(i_j)$ is the demand for capital in producing variety $i$; and $a_j(i_j)$ is the unit capital requirement of variety $i$; and a subscript after $i$ indicates the country producing variety $i$. If variety $i$ is actually produced in country $j$, then its supply price should be equal to its unit cost, which results in zero profit:

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5We reach the former statement by integrating the budget constraint (1) from $s = t$ to infinity, and using the Euler equation and the transversality condition.
\[
x(i_j) > 0 \Rightarrow p(i_j) = r_j a_j(i_j), i_j \in I_j \subset [0, 1],
\]

where \( I_j \) is the set of varieties produced in country \( j \).

We consider iceberg trade costs: one has to ship \( \tau_{nj}(\geq 1) \) units of each variety in country \( j \) to deliver one unit of that variety to country \( n \). It is assumed that \( \tau_{jj} = 1 \forall j, \tau_{nj} > 1 \forall j \neq n, \tau_{nj} \leq \tau_{nk} \forall j, k, n \). If \( j \neq n \), then we call \( \tau_{nj} \) country \( n \)'s import trade cost from country \( j \), or country \( j \)'s export trade cost to country \( n \).

The unit cost of producing variety \( i \) in country \( j \) and delivering it to country \( n \) is expressed as:

\[
p_{nj}(i) = \tau_{nj} r_j a_j(i), i \in [0, 1], j, n = 1, ..., N.
\]

Since the representative final good firm in country \( n \) buys variety \( i \) from the cheapest source, its demand price is given by:

\[
p_n(i) = \min\{p_{nj}(i)\}_{j=1}^N.
\]

Let \( A_j \) denote an independent and identically distributed random variable for \( a_j(i) \). Following Eaton and Kortum (2002), we impose a Fréchet distribution on country \( j \)'s productivity of capital in each variety \( 1/A_j \):

\[
F_j(z) \equiv \Pr\left(\frac{1}{A_j} \leq z\right) \equiv \exp(-b_j z^{-\theta}); b_j > 0, \theta > 1.
\]

The parameter \( b_j \) captures country \( j \)'s overall state of intermediate good technology: the higher \( b_j \) is, the higher \( 1/A_j \) tends to be. On the other hand, the parameter \( \theta \) indicates (the inverse of) variability of the productivity distribution. The fact that \( \theta \) is common to all countries will give us a useful representation of the intermediate good price index function (3). With \( F_j(z) \), the distributions of \( P_{nj} = \tau_{nj} r_j A_j \) and \( P_n = \min\{P_{nj}\}_{j=1}^N \), the i.i.d. random variables for \( p_{nj}(i) \) and \( p_n(i) \), respectively, are expressed in the following simple forms:

\[
G_{nj}(p) \equiv \Pr(P_{nj} \leq p) = 1 - \exp(-p^\theta b_j (\tau_{nj} r_j)^{-\theta}),
\]

\[
G_n(p) \equiv \Pr(P_n \leq p) = 1 - \exp(-p^\theta \Phi_n); \Phi_n \equiv \sum_{j=1}^N b_j (\tau_{nj} r_j)^{-\theta}.
\]

Moreover, these price distributions provide us with three well-known properties:

**Lemma 1 (Eaton and Kortum (2002)) .**

1. The probability that country \( n \) buys a variety from country \( j \) is:

\[
\pi_{nj}\{\tau_{nk} r_k\}_{k=1}^N \equiv b_j (\tau_{nj} r_j)^{-\theta}/\Phi_n = b_j (\tau_{nj} r_j)^{-\theta}/[\sum_{k=1}^N b_k (\tau_{nk} r_k)^{-\theta}];
\]

\[
\sum_{j=1}^N \pi_{nj}(\cdot) = 1 \forall n.
\]

2. The conditional distribution of \( P_{nj} \), given that country \( n \) buys a variety from country \( j \), is the same as \( G_n(p) \) for all \( j \).

3. The intermediate good price index function (3) for country \( n \) is rewritten as:
where $\Gamma(1 + (1 - \sigma_n)/\theta)$ is the Gamma function.

Lemma 1 has some implications. First, $\pi_{nj}$ also shows the fraction of varieties country $n$ buys from country $j$. This is because the probability $\pi_{nj}$ applies to a large number of varieties in the unit interval. If $j \neq n$, then $\pi_{nj}$ is called country $n$’s extensive margin of imports from country $j$, or country $j$’s extensive margin of exports to country $n$. Second, $\pi_{nj}$ is homogeneous of degree zero, whereas $Q_n$ is homogeneous of degree one, in the source countries’ rental rates multiplied by country $n$’s trade costs. The assumption of common $\theta$ is responsible for the convenient result.

2.4 Markets

If variety $i$ is actually produced in country $j$ and delivered to country $n$, then its demand price is expressed as:

$$p_n(i_j) = \tau_{nj} p(i_j) = \tau_{nj} r_j a_j(i_j), i_j \in I_{nj} \subset I_j,$$

(8)

where $I_{nj}$ is the set of varieties country $n$ buys from country $j$. In each country, the market-clearing conditions for the final good, capital, and the intermediate goods are given by:

$$Y_j = C_j + \dot{K}_j + \delta_j K_j,$$

(9)

$$K_j = \int_{I_j} K^x(i_j) di_j,$$

(10)

$$x(i_j) = \sum_{n=1}^{N} \tau_{nj} x_n(i_j), i_j \in I_j.$$

(11)

Finally, summing up Eqs. (1), (3), (4), and (5) for all countries, and considering Eq. (8), we obtain Walras’ law: the sum of the values of excess demands for the three types of markets is identically zero.

2.5 Dynamic system

Before deriving the dynamic system, it is worth mentioning two key relationships. First, from Eqs. (4) and (7), the rate of return to capital gross of depreciation in the Euler equation (2) for country $n$ is rewritten as:

$$r_n/p_n^V = r_n/Q_n(\{\tau_{nj} r_j\}_{j=1}^{N}) = 1/Q_n(\{\tau_{nj} r_j/r_n\}_{j=1}^{N}),$$

(12)

where linear homogeneity of $Q_n(\{\tau_{nj} r_j\}_{j=1}^{N})$ is used. This means that the growth rate in country $n$ is decreasing in $\tau_{nj} r_j/r_n$. This intuitively makes sense: a fall in $\tau_{nj}$ or a rise in $r_n/r_j$, with the latter indicating an improvement in country $n$’s terms of trade $p(i_n)/p(i_j) = (r_n/r_j) a_n(i_n)/a_j(i_j)$, raises its growth rate. Second, the cost share of varieties country $n$ buys from country $j$ is equal to the fraction of varieties country $n$ buys from country $j$ (6):
\[
\int_{I_{nj}} p_n(ij)x_n(ij)\,dj/(Q_nY_n) = \pi_{nj}\left(\{\tau_{nk}r_k\}_{k=1}^N\right) = \pi_{nj}\left(\{\tau_{nk}r_k/r_n\}_{k=1}^N\right).
\]  

(13)

The first equality holds because, from property 2 of Lemma 1, the conditional expectation of the expenditure for a variety, given that country \(n\) buys it from country \(j\), is the same as \(Q_nY_n\) for all varieties.\(^6\) The second equality follows from the fact that \(\pi_{nj}\left(\{\tau_{nk}r_k\}_{k=1}^N\right)\) is homogeneous of degree zero. Eq. (13) implies that all adjustments in the cost shares occur at the extensive margins. Note that country \(n\)'s cost shares depend on the same ratios \(\tau_{nk}r_k/r_n\) as its rate of return to capital.

Let us choose capital in the last country \(N\) as the numeraire: \(r_N \equiv 1\). And, let \(\kappa_j \equiv K_j/K_N\) denote the relative supply of capital in country \(j\) to country \(N\). Then our model is dramatically reduced to just two types of equations:\(^7\)

\[
\begin{align*}
\dot{\kappa}_j &= \kappa_j(\gamma_j(\{\tau_{jn}r_n/r_j\}_{n=1}^N) - \gamma_N(\{\tau_{Nn}r_n\}_{n=1}^N)), j = 1, \ldots, N - 1; \\
\gamma_j(\{\tau_{jn}r_n/r_j\}_{n=1}^N) &\equiv \bar{C}_j/C_j = 1/Q_j(\{\tau_{jn}r_n/r_j\}_{n=1}^N) - \delta_j - \rho_j, \\
\kappa_j &= \sum_{n=1}^N \pi_{nj}\left(\{\tau_{nk}r_k/r_n\}_{k=1}^N\right)\kappa_n/(r_j/r_n), j = 1, \ldots, N - 1,
\end{align*}
\]

(14)

(15)

where \(\gamma_j\) is the growth rate in country \(j\). Eq. (14) states that \(\kappa_j\) evolves according to the difference between the growth rates in countries \(j\) and \(N\). On the other hand, Eq. (15) can be interpreted as the capital market-clearing condition in country \(j\) relative to country \(N\).\(^8\) This corresponds to the labor market-clearing condition (21) of Eaton and Kortum (2002). Our model is different from Eaton and Kortum (2002) in that factor supplies and hence factor prices are endogenously changing over time. For all \(t \in [0, \infty)\), with \(\{\tau_{jn}\}_{j,n=1}^N\) exogenous and with \(\{\kappa_j\}_{j=1}^N\) predetermined, Eq. (15) determines \(\{r_{j}\}_{j=1}^{N-1}\), and then Eq. (14) determines \(\{\kappa_j\}_{j=1}^{N-1}\).

Before proceeding, we calculate changes in \(Q_n, \pi_{nj}, \) and \(\gamma_j\) contained in our dynamic system. First, from Eq. (7), we obtain:

\[
dQ_n/Q_n = \sum_{j=1}^N \pi_{nj}(d\tau_{nj}/\tau_{nj} + dr_j/r_j).
\]

(16)

Interestingly, the partial elasticity of \(Q_n\) with respect to \(\tau_{nj}r_j\) is just \(\pi_{nj}\). Second, substituting \(\Phi_n = (Q_n/c_n)^{-\theta}\) from Eq. (7) into Eq. (6), and using Eq. (16), \(d\pi_{nj}/\pi_{nj}\) is calculated as:

\[
d\pi_{nj}/\pi_{nj} = -\theta\sum_{k \neq j} \pi_{nk}(d\tau_{nj}/\tau_{nj} + dr_j/r_j - d\tau_{nk}/\tau_{nk} - dr_k/r_k).
\]

(17)

The usual substitution effects work here: the higher \(\tau_{nj}r_j\) is and/or the lower \(\tau_{nk}r_k\) is for \(k \neq j\), the less likely country \(n\) is to buy varieties from country \(j\). Third, totally differentiating \(\gamma_j = r_j/p_j^Y - \delta_j - \rho_j\) from

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\(^6\) Deriving country \(n\)'s demand for variety \(i\) from Eq. (3), and multiplying it by its demand price, the expenditure for variety \(i\) is expressed as \(p_n(i)x_n(i) = B_n\pi_n^{-1}q_n^i Y_n p_n(i)^{1-\sigma_n}\). On the other hand, the conditional expectation of \(I_{nj}\), given that country \(n\) buys a variety from country \(j\), is calculated as \(\int_{I_{nj}} p^{1-\sigma_n}(dG_n(p)/dp)\,dp = (B_nQ_n)^{1-\sigma_n}\). Combining these results, the conditional expectation of the expenditure for variety \(i_j\), given that country \(n\) buys it from country \(j\), is \(B_n\pi_n^{-1}Q_n^i Y_n (B_nQ_n)^{1-\sigma_n} = Q_n Y_n r_{ij} \in I_{nj}\).

\(^7\) We obtain Eq. (14) from Eqs. (2), (12), and the definition of \(\kappa_j\). Eq. (15) is obtained by rewriting Eq. (10) using Eqs. (1), (4), (8), (9), (11), and (13).

\(^8\) Eq. (15) is equivalent to \(r_j K_j = \sum_{n=1}^N \pi_{nj}r_n K_n\). This equation implies two further results. First, it is rewritten as \(\sum_{n \neq j} \pi_{jn}r_j K_j = \sum_{n \neq j} \pi_{nj}r_n K_n\). The latter equation shows country \(j\)'s balanced trade condition (i.e., value of imports = value of exports). Second, summing up the former equation for \(j = 1, \ldots, N - 1\), we obtain the same equation for \(j = N\). This means that country \(N\)'s capital market-clearing condition is indeed redundant.
Eq. (2), and using Eqs. (4) and (16), we obtain:

\[
d\gamma_j = -\Gamma_j \sum_{n \neq j} \pi_{jn} (d\tau_{jn}/\tau_{jn} + dr_n/r_n - dr_j/r_j); \Gamma_j \equiv \gamma_j + \delta_j + \rho_j. \tag{18}
\]

This is totally consistent with the discussion right after Eq. (12), with \( n \) and \( j \) interchanged.

2.6 Three-country model

In the rest of this paper, we focus on the case where \( N = 3 \). This is the minimum number of countries that allows us to consider regional trade agreements. For example, a preferential trade agreement between countries 1 and 2 can be expressed by reducing \( \tau_{12} \) and \( \tau_{21} \) but keeping \( \tau_{13} \) and \( \tau_{23} \) (as well as \( \tau_{31} \) and \( \tau_{32} \)) unchanged. For \( N = 3 \), Eqs. (14) and (15) are restated as follows:

\[
\begin{align*}
\dot{k}_1 &= \kappa_1(\gamma_1(1, \tau_{12} r_2^*/r_1^*, \tau_{13}/r_1^*) - \gamma_3(\tau_{31} r_1^*, \tau_{32} r_2^*, 1)), \\
\dot{k}_2 &= \kappa_2(\gamma_2(\tau_{21} r_1^*/r_2^*, 1, \tau_{23}/r_2^*) - \gamma_3(\tau_{31} r_1^*, \tau_{32} r_2^*, 1)), \\
\kappa_1 &= \pi_{11}(1, \tau_{12} r_2^*/r_1^*, \tau_{13}/r_1^*) + \pi_{21}(\tau_{21} r_1^*/r_2^*, 1, \tau_{23}/r_2^*) \kappa_2/(r_1^*/r_2^*) \\
&+ \pi_{31}(\tau_{31} r_1^*, \tau_{32} r_2^*, 1)/r_1^*, \\
\kappa_2 &= \pi_{12}(1, \tau_{12} r_2^*/r_1^*, \tau_{13}/r_1^*) + \pi_{22}(\tau_{21} r_1^*/r_2^*, 1, \tau_{23}/r_2^*) \kappa_2/(r_1^*/r_2^*) \\
&+ \pi_{32}(\tau_{31} r_1^*, \tau_{32} r_2^*, 1)/r_2^*. 
\end{align*}
\]

We define a balanced growth path (BGP) as a path along which all variables grow at constant rates. From Eqs. (19) and (20), for both \( \dot{k}_1/\kappa_1 \) and \( \dot{k}_2/\kappa_2 \) to be constant, \( r_1 \) and \( r_2 \) should be constant. Then Eqs. (21) and (22) imply that \( \kappa_1 \) and \( \kappa_2 \) should be constant. With \( \dot{k}_1 = \dot{k}_2 = 0 \), a BGP is implicitly determined by:

\[
\begin{align*}
0 &= \gamma_1(1, \tau_{12} r_2^*/r_1^*, \tau_{13}/r_1^*) - \gamma_3(\tau_{31} r_1^*, \tau_{32} r_2^*, 1), \\
0 &= \gamma_2(\tau_{21} r_1^*/r_2^*, 1, \tau_{23}/r_2^*) - \gamma_3(\tau_{31} r_1^*, \tau_{32} r_2^*, 1), \\
\kappa_1^* &= \pi_{11}(1, \tau_{12} r_2^*/r_1^*, \tau_{13}/r_1^*) \kappa_1^* + \pi_{21}(\tau_{21} r_1^*/r_2^*, 1, \tau_{23}/r_2^*) \kappa_2^*/(r_1^*/r_2^*) \\
&+ \pi_{31}(\tau_{31} r_1^*, \tau_{32} r_2^*, 1)/r_1^*, \\
\kappa_2^* &= \pi_{12}(1, \tau_{12} r_2^*/r_1^*, \tau_{13}/r_1^*) \kappa_1^*/(r_2^*/r_1^*) + \pi_{22}(\tau_{21} r_1^*/r_2^*, 1, \tau_{23}/r_2^*) \kappa_2^* \\
&+ \pi_{32}(\tau_{31} r_1^*, \tau_{32} r_2^*, 1)/r_2^*. 
\end{align*}
\]

where an asterisk over a variable represents a BGP. Eqs. (23) and (24) determine \( r_1^* \) and \( r_2^* \), and then Eqs. (25) and (26) determine \( \kappa_1^* \) and \( \kappa_2^* \). Let us call the common growth rate satisfying Eqs. (23) and (24) "the balanced growth rate". Note that the long-run rental rates are not determined by the capital market-clearing conditions, but by the balanced growth conditions. This will make the long-run effects of trade liberalization different from the short-run effects corresponding to the static Eaton-Kortum model.

The existence, uniqueness, and stability of a BGP is discussed in Appendix C. Simply stated, there exists a unique BGP that is locally stable unless the three countries are extremely different. Here we just assume that a unique and stable BGP exists.
3 Long-run effects of trade liberalization

3.1 Balanced growth rate

In this section, we examine the long-run impacts of trade liberalization analytically. Substituting Eq. (18) into the totally differentiated forms of Eqs. (23) and (24), we have:

\[
\begin{align*}
    a_{11}^*dr_1^*/r_1^* + a_{12}^*dr_2^*/r_2^* &= \Gamma_1^*(\pi_{12}d\tau_{12}/\tau_{12} + \pi_{13}d\tau_{13}/\tau_{13}) - \Gamma_3^*(\pi_{31}d\tau_{31}/\tau_{31} + \pi_{32}d\tau_{32}/\tau_{32}), \\
    a_{21}^*dr_1^*/r_1^* + a_{22}^*dr_2^*/r_2^* &= \Gamma_2^*(\pi_{21}d\tau_{21}/\tau_{21} + \pi_{23}d\tau_{23}/\tau_{23}) - \Gamma_3^*(\pi_{31}d\tau_{31}/\tau_{31} + \pi_{32}d\tau_{32}/\tau_{32});
\end{align*}
\]

The coefficients in the left-hand sides of these equations represent how \( \gamma_1 - \gamma_3 \) and \( \gamma_2 - \gamma_3 \) respond to the rental rates, respectively. For example, \( a_{11}^* \) is always positive because a rise in \( r_1 \) raises \( \gamma_1 \) but lowers \( \gamma_3 \). On the other hand, since a rise in \( r_2 \) lowers both \( \gamma_1 \) and \( \gamma_3 \), the sign of \( a_{12}^* \) is ambiguous, depending on the openness of countries 1 and 3 against country 2. Consider what happens when \( \tau_{12} \) falls. Since this raises country 1’s growth potential, \( r_1 \) tends to fall for \( \gamma_1 - \gamma_3 \) to go back to zero. Moreover, the fall in \( r_1 \) affects \( \gamma_2 - \gamma_3 \), which in turn causes \( r_2 \) to change. Solving the above equations for \( dr_1^*/r_1^* \) and \( dr_2^*/r_2^* \), we obtain:

\[
\begin{align*}
    dr_1^*/r_1^* &= \left(1/a^*\right)[a_{12}^*\Gamma_1^*(\pi_{12}d\tau_{12}/\tau_{12} + \pi_{13}d\tau_{13}/\tau_{13}) \\
    &\quad - (a_{22}^* - a_{12}^*)\Gamma_3^*(\pi_{31}d\tau_{31}/\tau_{31} + \pi_{32}d\tau_{32}/\tau_{32})], \tag{27}
\end{align*}
\]

\[
\begin{align*}
    dr_2^*/r_2^* &= \left(1/a^*\right)[a_{11}^*\Gamma_1^*(\pi_{12}d\tau_{12}/\tau_{12} + \pi_{13}d\tau_{13}/\tau_{13}) \\
    &\quad + a_{12}^*\Gamma_2^*(\pi_{21}d\tau_{21}/\tau_{21} + \pi_{23}d\tau_{23}/\tau_{23}) \\
    &\quad - (a_{11}^* - a_{21}^*)\Gamma_3^*(\pi_{31}d\tau_{31}/\tau_{31} + \pi_{32}d\tau_{32}/\tau_{32})]. \tag{28}
\end{align*}
\]

As expected, a fall in either \( \tau_{12} \) or \( \tau_{13} \) lowers \( r_1^* \), whereas its effect on \( r_2^* \) is ambiguous. Similarly, a fall in either \( \tau_{21} \) or \( \tau_{23} \) lowers \( r_2^* \), whereas its effect on \( r_1^* \) is ambiguous. Finally, since \( a_{22}^* - a_{12}^* = \Gamma_2^*(\pi_{21}^* + \pi_{23}) + \Gamma_1^*\tau_{12} > 0 \) and \( a_{11}^* - a_{21}^* = \Gamma_1^*(\pi_{12}^* + \pi_{13}) + \Gamma_2^*\pi_{21}^* > 0 \), a fall in either \( \tau_{31} \) or \( \tau_{32} \) raises both \( r_1^* \) and \( r_2^* \).

Substituting Eqs. (27) and (28) back into Eq. (18) for country 3, the change in the balanced growth rate is generally expressed as:
\[ d\gamma_1^* = d\gamma_2^* = d\gamma_3^* = - (\Gamma_1^* r_{12}^* / a^*) \]
\[ \times \left\{ [\pi_{23} \pi_{31} + \pi_{21} (\pi_{12}^* \pi_{31}^*)] (\pi_{12}^* dr_{12}^* / \tau_{12} + \pi_{13}^* d\tau_{12} / \tau_{13}) \right. \]
\[ + [\pi_{31} \pi_{12}^* + \pi_{32} (\pi_{13}^* + \pi_{12}^*)] (\pi_{21}^* dr_{21}^* / \tau_{21} + \pi_{23}^* d\tau_{23} / \tau_{23}) \]
\[ + [\pi_{12} \pi_{23} + \pi_{13} (\pi_{21}^* + \pi_{23}^*)] (\pi_{31}^* d\tau_{31} / \tau_{31} + \pi_{32}^* d\tau_{32} / \tau_{32}) \} \].

**Proposition 1** For all \( j, n = 1, 2, 3, n \neq j \), a permanent fall in \( \tau_{jn} \) raises the balanced growth rate.

This is a very powerful result: regardless of the signs and sizes of \( a_{12}^* \) and \( a_{21}^* \), trade liberalization anywhere does raise the balanced growth rate. This is because trade liberalization in one country not only raises its own growth potential, but also raises the terms of trade of the other countries against the liberalizing country. For example, a fall in \( \tau_{12} \) lowers \( r_1^* \), which raises the terms of trade of country 3 against country 1. Also, since \( (dr_1^*/r_1^* - dr_2^*/r_2^*)/(dr_{12}/\tau_{12}) = (1/a^*) (a_{12}^* + a_{21}^*) \Gamma_1^* \pi_{12}^* > 0 \) from Eqs. (27) and (28), it raises the terms of trade of country 2 against country 1. These terms of trade improvements for the non-liberalizing countries contribute to the higher balanced growth rate. This proposition implies that the positive long-run growth effect of unilateral trade liberalization in Naito (2012) still holds in a three-country case, although we will see in section 3.3 that it might not be true for some country in the short run. On the other hand, our robust result is different from the three-country R&D-based endogenous growth model of Dinopoulos and Syropoulos (1997), where unilateral trade liberalization can either raise or lower the long-run growth rate of the liberalizing country depending on whether the growth intensity (summarizing the productivity in production and R&D and expenditure share) of its export sector is larger or smaller than its nontraded sector.

### 3.2 Fractions of domestic and traded varieties

#### 3.2.1 Unilateral trade liberalization

Since it is difficult to draw general conclusions about changes in the long-run fractions of domestic and traded varieties by simply substituting Eqs. (27) and (28) into Eq. (17), we first consider a fall in one trade cost \( \tau_{12} \) at a time. In view of Eq. (17), we need information about changes in \( r_1^*, r_2^*, r_1^*/r_2^*, \tau_{12} r_2^*, \) and \( \tau_{12} r_2^*/r_1^* \). We already know from section 3.1 that a fall in \( \tau_{12} \) lowers \( r_1^* \) and \( r_1^*/r_2^* \), but its effect on \( r_2^* \) is ambiguous depending on the sign of \( a_{21}^* \). For changes in \( \tau_{12} r_2^* \) and \( \tau_{12} r_2^*/r_1^* \), we obtain:

\[ \]
Setting the left-hand side of the above equation equal to zero, we obtain:

\[
\frac{d\tau_{12}}{\tau_{12}} + \frac{dr_1^*}{r_1^*} = 0
\]

\[
(1/a^*)\{\Gamma_1^2\pi_{12}^* (\pi_{13}^* + \pi_{23}^*) + \Gamma_1^2\Gamma_2^3[\pi_{23}\pi_{31}^* + \pi_{21}\pi_{32}^* + \pi_{31}^*]
+ \Gamma_2^2\pi_{32}^*(\pi_{13}^* + \pi_{12}^*)\}
> 0,
\]

\[
(\frac{d\tau_{13}}{\tau_{13}} + \frac{dr_1^*}{r_1^*} - \frac{dr_2^*}{r_2^*} + \frac{dr_1}{r_1} + \frac{dr_2}{r_2})/(\frac{d\tau_{12}}{\tau_{12}})
= (1/a^*)\{\Gamma_1^2\pi_{13}^* (\pi_{21}^* + \pi_{23}^*) + \Gamma_2^2\Gamma_3^3[\pi_{24}\pi_{31}^* + \pi_{21}\pi_{32}^* + \pi_{31}^*]
+ \Gamma_2^2\pi_{32}^*(\pi_{13}^* + \pi_{12}^*)\}
> 0.
\]

Hence, a fall in \(\tau_{12}\) lowers \(\tau_{12}r_1^*\) and \(\tau_{12}r_2^*/r_1^*\) as well as \(r_1^*/r_2^*\). Combining this with Eq. (17), we reach the following proposition:

**Proposition 2** A permanent fall in \(\tau_{12}\) increases \(\pi_{12}^*, \pi_{21}^*,\) and \(\pi_{31}^*\), whereas it decreases \(\pi_{13}^*\).

Although one might be disappointed that we find the directions of changes in only four out of nine fractions of varieties, this proposition actually has rich implications. First, in country 1, some varieties imported from country 3 are replaced by those imported from country 2. This is because the fall in \(\tau_{12}\) makes the latter varieties cheaper than the former for the liberalizing country. Second, and more interestingly, country 1’s fractions of exported varieties to all destinations increase. For either country 2 or 3, the falls in \(r_1^*\) and \(r_1^*/r_2^*\) caused by the fall in \(\tau_{12}\) make it cheaper to import varieties from country 1 than the other sources. In this sense, import promotion also works as export promotion at the extensive margins.

### 3.2.2 Bilateral trade liberalization

We next see the case where \(\tau_{12}\) and \(\tau_{21}\) are reduced at the same time. Taking the difference between Eqs. (27) and (28), with \(d\tau_{13} = d\tau_{23} = d\tau_{31} = d\tau_{32} = 0\), we have:

\[
a^*(dr_1^*/r_1^* - dr_2^*/r_2^*) = (a_2^* + a_2^*)\Gamma_1^2\pi_{12}^* d\tau_{12}/\tau_{12} - (a_1^* + a_1^*)\Gamma_2^2\pi_{21}^* d\tau_{21}/\tau_{21} - (a_2^* + a_2^*)\Gamma_1^2\pi_{31}^* d\tau_{31}/\tau_{31} - (a_1^* + a_1^*)\Gamma_2^2\pi_{32}^* d\tau_{32}/\tau_{32},
\]

where \(a_2^* + a_2^* > 0\) and \(a_1^* + a_1^* > 0\). We consider a particular type of bilateral trade liberalization such that \(r_1^*/r_2^*\) and hence the long-run bilateral terms of trade between countries 1 and 2 should be unchanged. Setting the left-hand side of the above equation equal to zero, we obtain:

\[
(\frac{d\tau_{21}}{\tau_{21}})/(\frac{d\tau_{12}}{\tau_{12}})|_{dr_1^*/r_1^*=dr_2^*/r_2^*} = [(a_2^* + a_2^*)\Gamma_1^2\pi_{12}^*]/[(a_1^* + a_1^*)\Gamma_2^2\pi_{21}^*].
\]  

(29)

Under the liberalization rule (29), the rate of change in \(r_1^*\) (and also \(r_2^*\)) is calculated as:

\[
(\frac{dr_1^*}{r_1^*})/(\frac{d\tau_{12}}{\tau_{12}})|_{dr_1^*/r_1^*=dr_2^*/r_2^*} = \Gamma_1^2\pi_{12}^*/(a_1^* + a_1^*) > 0.
\]

This means that \(r_1^*\) and \(r_2^*\) always fall at the same rate. Observing Eq. (17), we can easily find that \(\pi_{12}^*, \pi_{21}^*, \pi_{31}^*\), and \(\pi_{32}^*\) increase, whereas \(\pi_{13}^*, \pi_{23}^*,\) and \(\pi_{33}^*\) decrease. For \(\pi_{11}^*\) and \(\pi_{22}^*\), Eq. (17) gives:
countries. Fortunately, the short- and long-run effects of unilateral and bilateral trade liberalization on the falling, the directions of changes in the other six fractions of varieties remain ambiguous.

\[
\begin{align*}
\frac{d\pi_{11}}{\pi_{11}} &= -\theta(-\pi_{12}^*d\tau_{12}/\tau_{12} + \pi_{13}^*dr_{1}^*/r_{1}^*), \\
\frac{d\pi_{22}}{\pi_{22}} &= -\theta(-\pi_{21}^*d\tau_{21}/\tau_{21} + \pi_{23}^*dr_{2}^*/r_{2}^*).
\end{align*}
\]

The terms in the parentheses are calculated as:

\[
\begin{align*}
-\pi_{12}^* + \pi_{13}^*(dr_{1}^*/r_{1}^*)/(d\tau_{12}/\tau_{12})|_{dr_{1}^*/r_{1}^*=dr_{2}^*/r_{2}^*} &= -\Gamma_3^*(\pi_{31} + \pi_{32})\pi_{12}^*/(\kappa_{11} + \kappa_{12}) < 0, \\
-\pi_{21}^* + \pi_{23}^*(dr_{2}^*/r_{2}^*)/(d\tau_{21}/\tau_{21})|_{dr_{1}^*/r_{1}^*=dr_{2}^*/r_{2}^*} &= -\Gamma_3^*(\pi_{32} + \pi_{31})\pi_{21}^*/(\kappa_{22} + \kappa_{21}) < 0.
\end{align*}
\]

Therefore, \(\pi_{11}^*\) and \(\pi_{22}^*\) decrease. The following proposition summarizes the results:

**Proposition 3** Permanenent falls in \(\tau_{12}\) and \(\tau_{21}\), with \(r_{1}^*/r_{2}^*\) unchanged, increase \(\pi_{12}^*, \pi_{21}^*, \pi_{31}^*, \text{ and } \pi_{32}^*\), whereas they decrease \(\pi_{11}^*, \pi_{13}^*, \pi_{22}^*, \pi_{23}^*, \text{ and } \pi_{33}^*\).

This proposition is stronger than the previous one in that the directions of changes in all of the nine fractions of varieties are unambiguously determined under bilateral trade liberalization with the long-run bilateral terms of trade unchanged. In fact, Proposition 3 can be seen as just a composite of Proposition 2 applied to \(\tau_{12}\) and \(\tau_{21}\): a preferential trade agreement between countries 1 and 2 shifts their import demands away from the outsider to each insider, and also increases their fractions of exported varieties to both inside and outside destinations. Moreover, the fractions of domestic varieties decrease in all of the three countries. This indicates that the preferential trade agreement brings about trade creation (i.e., \(\pi_{11}^*\) and \(\pi_{22}^*\) are replaced by \(\pi_{12}^*\) and \(\pi_{21}^*\), respectively) as well as trade diversion (i.e., \(\pi_{13}^*\) and \(\pi_{23}^*\) are replaced by \(\pi_{12}^*\) and \(\pi_{21}^*\), respectively). Finally, the fact that country 3’s fractions of exported varieties decrease does not mean that the non-member country loses from the preferential trade agreement. It actually enjoys the higher balanced growth rate along with the improved terms of trade against both countries 1 and 2.

### 3.3 Comparison with short-run effects

To compare the long-run effects of unilateral trade liberalization with its short-run effects, we investigate analytically the impacts of a change in \(\tau_{12}\) on main endogenous variables in the initial period in Appendix A. A fall in \(\tau_{12}\) makes country 1 substitute country 2’s varieties for its own, thereby raising \(r_{2}\). Although it is unclear if \(\tau_{1}\) falls or not, \(\tau_{1}/\tau_{2}, \tau_{1}\tau_{2}/\tau_{1}\), and \(\tau_{1}\tau_{2}/\tau_{1}\) necessarily fall. Since country 2’s terms of trade against the other two countries improve, its growth rate \(\gamma_{2}\) necessarily rises. The directions of changes in \(\gamma_{1}\) and \(\gamma_{3}\) are generally ambiguous due to the ambiguous change in \(\tau_{1}\). Finally, although \(\pi_{12}\) rises whereas \(\pi_{22}\) and \(\pi_{32}\) fall, the directions of changes in the other six fractions of varieties remain ambiguous.

To find clearer results analytically, we focus on a special case where the old BGP is symmetric across countries. Fortunately, the short- and long-run effects of unilateral and bilateral trade liberalization on the growth rates and fractions of varieties are all identified in Appendix B and summarized in Table 1.

In the short run, unilateral trade liberalization lowers \(\tau_{1}\). Although this tends to lower \(\gamma_{1}\) but raise \(\gamma_{3}\), these effects are outweighed by the fall in \(\tau_{1}\tau_{2}/\tau_{1}\) and the rise in \(\tau_{2}\), thereby raising \(\gamma_{1}\) but lowering \(\gamma_{3}\), respectively. Due to the fall in \(\tau_{1}\tau_{2}/\tau_{1}\), country 1 buys more of country 2’s varieties but less of the others. On the other hand, the price changes induce both countries 2 and 3 to buy less of country 2’s varieties but more of the others. Since countries 1 and 2 start to grow faster than country 3, both \(\kappa_{1}\) and \(\kappa_{2}\) increase,
which in turn lowers both $r_1$ and $r_2$. In the long run, $r_2$ goes back to its old BGP value. The balanced growth rate necessarily rises. The directions of changes in the fractions of varieties in the long run are the same as in the short run, except $\pi_{23}$ and $\pi_{31}$: they decrease from their old BGP values because country 3’s cost advantage over country 2 disappears in the long run.

Unlike unilateral trade liberalization, bilateral trade liberalization with the long-run bilateral terms of trade unchanged raises not only $r_2$ but also $r_1$ in the short run. This is because, for both countries 1 and 2, the decreased demands for their domestic varieties are more than compensated by the increased demands for each other’s varieties. Since country 3’s terms of trade against all other countries deteriorate, country 3 starts to grow more slowly, whereas countries 1 and 2 start to grow faster, than the old BGP just like unilateral trade liberalization. Due to the strong direct effects of the falls in $\tau_{12}$ and $\tau_{21}$, both trade creation and diversion occur even in the short run. On the other hand, reflecting the price changes, country 3 imports less from both countries 1 and 2. The gaps in the short-run growth rates pull both $r_1$ and $r_2$ down below their old BGP values, which in turn raises the balanced growth rate. Since country 3’s varieties become relatively more expensive, country 3 imports more from both countries 1 and 2 in the long run.

There are many variables whose long-run responses are opposite to their short-run responses representing the static model of Eaton and Kortum (2002): $\gamma_3, \pi_{23}$, and $\pi_{33}$ for unilateral trade liberalization; or $r_1, r_2, \gamma_3, \pi_{31}, \pi_{32}$, and $\pi_{33}$ for bilateral trade liberalization. The most remarkable result is stated in the following proposition:

**Proposition 4** Starting from the symmetric BGP, both trade liberalization schemes specified in Propositions 2 and 3 raise $\gamma_1$ and $\gamma_2$ but lower $\gamma_3$ in the initial period.

As long as we are around the symmetric BGP, trade liberalization, whether unilateral or bilateral, must lower the growth rate in a non-liberalizing country (i.e., country 3) in the short run. In the two-country model of Naito (2012), a fall in $\tau_{12}$ improves the terms of trade of country 2 against country 1, which in turn raises the growth rate in country 2 in the initial period. It is true that a fall in $\tau_{12}$ still improves country 2’s terms of trade against all other countries even in our three-country model, thereby pushing up country 2’s growth rate in the short run. However, this fact implies that country 3’s terms of trade against country 2 must deteriorate, which pulls down country 3’s growth rate in the short run. It is the presence of a third country that is responsible for the new result.

Propositions 1 and 4 have welfare implications that differentiate our model from Eaton and Kortum (2002). If $\dot{K}_j + \delta_j K_j = 0$ in Eq. (1), then our model reduces to the static Eaton-Kortum model, where country j’s welfare is given by $\ln C_j = \ln((r_j/p_j^Y)K_j)$, and the equilibrium rental rates are determined by Eqs. (21) and (22). The fact that the initial growth rate in country 3 falls from its old BGP value in our model means that $r_3/p_3^Y$ and hence country 3’s welfare must fall in the static model. In contrast, the liberalization-induced rise in the balanced growth rate in our model can raise country 3’s overall utility, reversing the pessimistic view from the static Eaton-Kortum model.

## 4 Numerical analysis

In this section, we run some numerical experiments on unilateral and bilateral trade liberalization based on the calibrated trade costs $\tau_{jn}$ and the overall states of intermediate good technologies $b_j$ in a hypothetical world consisting of three regions, Asia ($j = 1$), North America ($j = 2$), and Europe ($j = 3$), according to
the WTO statistics database. There are two purposes for doing this. One is to see how well our analytical results obtained around the symmetric BGP hold in a more realistic situation. The other is to compare the welfare effects in our model with the static Eaton-Kortum model. Parameters other than $\tau_{jn}$ and $b_j$ are set as follows: $\rho_j = 0.02$ and $\delta_j = 0.05$ from Barro and Sala-i-Martin (2004); $K_{30} = 100$ for welfare to take positive values; and $B_j = 0.1$ to obtain reasonable values of the growth rates. For $\theta$, the estimates of Eaton and Kortum (2002) are 2.84 to 3.60 (Table V), 8.28 (Section 3), and 12.86 (Section 5.3). We choose $\theta = 3$ from their lowest estimate because $\theta = 8$ or higher yields almost 0% trade shares in our model with a small number of countries. Following this, we set $\sigma_j = 3$ so that the restriction $1 + (1 - \sigma_j)/\theta > 0$ in Eq. (7) should hold. The following calculations are conducted using Mathematica 9.

First, we simply solve nine equations characterizing the BGP, that is, Eqs. (23), (24), $\gamma_3(\cdot) = \gamma_3^\ast$, and $\pi_{jn}(\cdot) = \pi_{jn}^\ast$ for $n \neq j$, for nine parameters, $\tau_{jn}$ for $n \neq j$ and $b_j$. This requires data on $r_1^\ast$, $r_2^\ast$, $\gamma_3^\ast$, and $\pi_{jn}^\ast$ for $n \neq j$. To express the long-run situation, all of our processed data are averaged annually over the period 2001-2010. Our primary data source is the World Bank’s World Development Indicators. To obtain $r_j^\ast$, we add the common depreciation rate of 0.05 to the real interest rates of Japan, United States, and Italy, and divide each of the first two by the third. As for $\gamma_3^\ast$, we just use the annual average GDP growth rate of the world. To obtain $\pi_{jn}^\ast$ for $n \neq j$, we first divide region $j$’s share of merchandise trade (the sum of exports and imports) in GDP in half to obtain region $j$’s import/GDP ratio under the balanced trade, and then multiply the result by the ratio of region $j$’s value of imports from region $n$ to region $j$’s value of imports from the world. The resulting values are reported in the first column of Table 2. For example, Asia’s relative rental rate to Europe is 0.964, the balanced growth rate is 2.560%, Asia’s import share from North America is 7.464%, and so on. For our model to reproduce these data, the nine parameters are calibrated as follows: $\tau_{12} = 2.33905, \tau_{13} = 2.02213, \tau_{21} = 2.4053, \tau_{23} = 2.66304, \tau_{31} = 1.95887, \tau_{32} = 2.56534, b_1 = 0.166886, b_2 = 0.180617, b_3 = 0.161854$.

Next, we explain our simulation method. Unilateral trade liberalization is expressed as an exogenous and permanent fall in a net trade cost $\tau_{12} - 1$ by half (i.e., $\tau_{12}' = (\tau_{12} - 1)/2 + 1$) from $t = 0$ on. On the other hand, bilateral trade liberalization combines unilateral trade liberalization with a permanent fall in $\tau_{21}$ according to Eq. (29) evaluated at the old BGP. However, this way of calculating the change in $\tau_{21}$ contains an error arising from linearization, which could be large for a large-scale change in $\tau_{12}$. To obtain the exact value of $\tau_{21}'$ corresponding to $\tau_{21}'$, we instead solve Eqs. (23), (24), and $r_1' \rho_2' \gamma_3' = r_2' \rho_2' \gamma_3'$ numerically for $r_1', r_2', \gamma_3'$, and $\tau_{21}'$. Before trade liberalization, we first compute the old BGP values of $r_1, r_2, \kappa_1, \kappa_2$ from Eqs. (23), (24), (25), and (26). Once they are calculated, the other endogenous variables at the old BGP are also calculated immediately. Of course, they should replicate data on $r_1', r_2', \gamma_3'$, and $\pi_{jn}^\ast$ for $n \neq j$. Next, for each liberalization regime, we apply the NDSolve command to Eqs. (19), (20), (21), and (22) with the initial conditions $\kappa_{10}' = \kappa_1', \kappa_{20}' = \kappa_2'$ to obtain the equilibrium paths of $r_{1t}', r_{2t}', \kappa_{1t}', \kappa_{2t}'$ from $t = 0$ to $t = 1,000,000$. Finally, we use $U_j = \int_0^\infty \ln C_j \exp(-\rho_j t) dt = (1/\rho_j) \ln C_{j0}' + \int_0^\infty \gamma_j \exp(-\rho_j t) dt$.

9We assume that they correspond to East Asia & Pacific (all income levels), North America, and European Union, respectively, according to the World Development Indicators. They account for 25.5%, 25.8%, and 25.4% of the world GDP in 2010, respectively.

10A country’s real interest rate is defined as its lending interest rate adjusted for inflation as measured by the GDP deflator. Italy is the largest country in the Euro Area which reports the real interest rate for all years during 2001-2010.

11Region $j$’s values of imports from region $n$ and the world are constructed from the WTO statistics database.

12Since each region in fact imports goods from regions other than the three, our calculation of region $j$’s total import share $\sum_{n \neq j} \pi_{jn}^\ast$ underestimates its actual import/GDP ratio.

13In fact, in Table 2, all figures except welfare can be calculated without solving for the full paths of $r_{1t}', r_{2t}', \kappa_{1t}', \kappa_{2t}'$. We can use Eqs. (21) and (22) for $t = 0$ to obtain $r_{10}'$ and $r_{20}'$, and Eqs. (23) and (24) to compute $r_{1t}'$ and $r_{2t}'$, for each set of trade costs. However, since $U_j$ is a function of {$\gamma_{jt}$}_{t=0}^{\infty}, exact computation of the former requires calculating the full paths of $r_{1t}'$ and $r_{2t}'$. 

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with \( C_j^0 = \rho_j K_j^0, \kappa'_{10} = \kappa_1^*, \kappa'_{20} = \kappa_2^*, \) and \( K_{30}' = 100 \) to compute welfare under each regime.

Our quantitative results are summarized in Table 2. The balanced growth rate rises by 0.113 percentage points to 2.673% under unilateral trade liberalization, and also rises by 0.309 percentage points to 2.869% under bilateral trade liberalization. Quite surprisingly, the short- and long-run effects of unilateral and bilateral trade liberalization on the growth rates and fractions of varieties are qualitatively almost the same as our analytical results obtained around the symmetric BGP and reported in Table 1. The only exception is that \( \pi_{23} \) is now larger than the old BGP in the long run under unilateral trade liberalization, because the long-run value of \( r_2 \) is still higher than the old BGP. Finally, for all regions, welfare is higher under unilateral trade liberalization than the old BGP, and even higher under bilateral trade liberalization. As mentioned in the last paragraph of section 3.3, the fact that \( \gamma_3 \) falls in the initial period under unilateral and bilateral trade liberalization means that region 3’s welfare falls in the static Eaton-Kortum model. On the contrary, our dynamic model reveals that region 3’s welfare does rise. Our dynamic extension of the Eaton-Kortum model indeed overturns its welfare implications: considering the long-run growth effect, trade liberalization by Asia and North America will benefit Europe.

Fig. 1 and Fig. 2 illustrate the paths of main endogenous variables under unilateral and bilateral trade liberalization experiments we have done, respectively. In all panels except the top right corner of both figures, the blue curve represents the path of the variable on the vertical axis, which eventually merges with the yellow horizontal line indicating the new BGP value. The blue curve should be compared with the red horizontal line showing the old BGP value. In the top right panel, the blue, red, and yellow curves represent the paths of \( \gamma_1, \gamma_2, \) and \( \gamma_3, \) respectively, compared with the green horizontal line indicating the old balanced growth rate. These figures provide some additional information. First, in both experiments, all variables converge to their new BGP values until around \( t = 1,500. \) Second, although \( \gamma_3 \) falls from its old BGP value in the initial period, the former overtakes the latter in a very short period of time: \( t = 2.1763 \) for unilateral trade liberalization; and \( t = 28.9468 \) for bilateral trade liberalization. This implies that the path of consumption in country 3 also exceeds that at the old BGP very soon, contributing to its higher welfare.

As a simple robustness check, we raise the subjective discount rates gradually by 0.01. Since this shifts down \( \gamma_j \) functions equally, only the implied \( b_j \)’s increase, but the implied trade costs are unchanged, to recover the data. The simulated values of the rental rates and fractions of varieties in the short- and long-run as a result of unilateral and bilateral trade liberalization are unchanged as well. Welfare of all regions continue to rise under unilateral and bilateral trade liberalization up to \( \rho_j = 0.04. \) When \( \rho_j = 0.05, \) region 3’s welfare falls for the first time under bilateral trade liberalization, whereas it still rises under unilateral trade liberalization. Therefore, our welfare effects are robust for a sufficiently wide range of the subjective discount rates.

5 Concluding remarks

Our three-country, continuum-good Ricardian model of trade and growth with endogenous extensive margins has some policy implications. First, trade liberalization, be it unilateral, bilateral, or multilateral, raises global growth. This is because it raises the growth potential of the liberalizing countries, which in turn raises the relative rental rates and hence the terms of trade of the non-liberalizing countries against the liberalizing ones. This supports the recent empirical research such as Wacziarg and Welch (2008) and Estevadeordal and Taylor (2013) reporting the positive relationship between trade liberalization and economic growth. Second, import promotion acts as export promotion at the extensive margins. The falling long-run rental rates of the
liberalizing countries relative to the non-liberalizing ones arising from their faster growth make it cheaper to buy varieties from the liberalizing countries. This explains the mechanism underlying the empirical evidence on economic growth and extensive margins of exports found by Hummels and Klenow (2005) and Kehoe and Ruhl (2013). Third, the difference between the short- and long-run effects of trade liberalization opens up the possibility that its welfare effects in the static Eaton-Kortum model can be reversed in a positive direction. This contributes to the literature on regional trade agreements: unlike Bagwell and Staiger (1999) and Ornelas (2005), where a free trade agreement between two countries benefits a third country only if each member country voluntarily lowers its optimal external tariff, welfare of the third country can rise without adjustments in their external trade costs in our model.

There are some directions for future research. First, replacing iceberg trade costs with import tariffs will somewhat complicate the effect of trade liberalization on welfare of the liberalizing countries. By the standard optimal tariff argument, a fall in a country’s import tariff will partly lower its welfare in the short run through decreased tariff revenue associated with deteriorated terms of trade. However, since the tariff reduction raises the balanced growth rate just like a fall in the corresponding iceberg trade cost, the liberalizing country’s welfare partly rises in the long run. As long as countries’ subjective discount rates are sufficiently low that the long-run welfare gains from faster growth outweigh the possible short-run welfare losses from decreased tariff revenue, not only the non-liberalizing but also the liberalizing countries will still gain from tariff reductions. Second, it will be interesting to perform counterfactual experiments based on structurally estimated parameters. Since our model works for an arbitrary number of countries, we could do this in a world with much more than three countries as Eaton and Kortum (2002) do. Even then, the qualitative results obtained in our dynamic three-country model provide a benchmark against which quantitative results will be evaluated.

Appendix A. Short-run effects of a change in $\tau_{12}$

Substituting Eq. (17) into the logarithmically differentiated forms of Eqs. (21) and (22) with $d\kappa_j/\kappa_j = 0 \forall j$, we obtain:

\[
c_{11}dr_1/r_1 + c_{12}dr_2/r_2 = -\theta \pi_{11}r_1r_1\pi_{12}d\tau_{12}/\tau_{12} - \theta \pi_{11}r_1\kappa_{13}d\tau_{13}/\tau_{13} \\
+ \theta \pi_{21}r_2\kappa_2(1 - \pi_{21})d\tau_{21}/\tau_{21} - \theta \pi_{21}r_2\kappa_2\pi_{23}d\tau_{23}/\tau_{23} \\
+ \theta \pi_{31}(1 - \pi_{31})d\tau_{31}/\tau_{31} - \theta \pi_{31}\pi_{32}d\tau_{32}/\tau_{32},
\]

\[
c_{21}dr_1/r_1 + c_{22}dr_2/r_2 = \theta \pi_{12}r_1\kappa_1(1 - \pi_{12})d\tau_{12}/\tau_{12} - \theta \pi_{12}r_1\kappa_{13}d\tau_{13}/\tau_{13} \\
- \theta \pi_{22}r_2\kappa_2\pi_{21}d\tau_{21}/\tau_{21} - \theta \pi_{22}r_2\kappa_2\pi_{23}d\tau_{23}/\tau_{23} \\
- \theta \pi_{32}\pi_{31}d\tau_{31}/\tau_{31} + \theta \pi_{32}(1 - \pi_{32})d\tau_{32}/\tau_{32};
\]
\[ c_{11} \equiv -\theta [\pi_{11} r_1 \kappa_1 (1 - \pi_{11}) + \pi_{21} r_2 \kappa_2 (1 - \pi_{21}) + \pi_{31} (1 - \pi_{31})] + (1 - \pi_{11}) r_1 \kappa_1 < 0, \]
\[ c_{12} \equiv \theta (\pi_{11} r_1 \kappa_1 \pi_{12} + \pi_{21} r_2 \kappa_2 \pi_{22} + \pi_{31} \pi_{32}) + \pi_{21} r_2 \kappa_2 > 0, \]
\[ c_{21} \equiv \theta (\pi_{22} r_2 \kappa_2 \pi_{21} + \pi_{12} r_1 \kappa_1 \pi_{11} + \pi_{32} \pi_{31}) + \pi_{12} r_1 \kappa_1 > 0, \]
\[ c_{22} \equiv -\theta [\pi_{22} r_2 \kappa_2 (1 - \pi_{22}) + \pi_{12} r_1 \kappa_1 (1 - \pi_{12}) + \pi_{32} (1 - \pi_{32})] + (1 - \pi_{22}) r_2 \kappa_2 < 0, \]
\[ c \equiv c_{11} c_{22} - c_{12} c_{21} > 0. \]

Focusing on a change in \( \tau_{12} \), its short-run effects on \( r_1 \) and \( r_2 \) are given by:

\[
\frac{(dr_1/r_1)}{(d\tau_{12}/\tau_{12})} = (\theta \pi_{12} r_1 \kappa_1/c) \times [\theta [\pi_{22} r_2 \kappa_2 (\pi_{11} \pi_{23} - \pi_{21} \pi_{13}) + \pi_{32} (\pi_{11} \pi_{33} - \pi_{31} \pi_{13}) + r_2 \kappa_2 (\pi_{11} \pi_{23} - \pi_{21} \pi_{13})],
\]
\[
\frac{(dr_2/r_2)}{(d\tau_{12}/\tau_{12})} = (\theta \pi_{12} r_1 \kappa_1/c) \times [-\theta (\pi_{11} r_1 \kappa_1 \pi_{13} + \pi_{21} r_2 \kappa_2 \pi_{23} + \pi_{31} \pi_{33}) + \pi_{13} r_1 \kappa_1 \pi_{11} + c_{11} \pi_{13}] < 0.
\]

The rates of changes in \( r_1/r_2, \tau_{12} r_2 \), and \( \tau_{12} r_2/r_1 \) are then calculated as:

\[
\frac{(dr_1/r_1 - dr_2/r_2)}{(d\tau_{12}/\tau_{12})} = (\theta \pi_{12} r_1 \kappa_1/c) \times [\pi_{11} [\theta (\pi_{22} r_2 \kappa_2 \pi_{23} + \pi_{12} r_1 \kappa_1 \pi_{13} + \pi_{32} \pi_{33}) + \pi_{32}]],
\]
\[
+ (1 - \pi_{12}) \theta (\pi_{11} r_1 \kappa_1 \pi_{13} + \pi_{21} r_2 \kappa_2 \pi_{23} + \pi_{31} \pi_{33}) + \pi_{32}] \}
\]
\[
> 0,
\]
\[
\frac{(d\tau_{12}/\tau_{12} + dr_2/r_2)}{(d\tau_{12}/\tau_{12})} = (1/c) \times [\theta (\pi_{11} r_1 \kappa_1 \pi_{13} + \pi_{21} r_2 \kappa_2 \pi_{23} + \pi_{31} \pi_{33}) + \pi_{13} r_1 \kappa_1]
\]
\[
\times [\theta (\pi_{22} r_2 \kappa_2 \pi_{21} + \pi_{32} \pi_{31}) + \pi_{21} r_2 \kappa_2] - c_{11} \theta (\pi_{22} r_2 \kappa_2 \pi_{23} + \pi_{32} \pi_{33}) + \pi_{23} r_2 \kappa_2] \}
\]
\[
> 0,
\]
\[
\frac{(d\tau_{12}/\tau_{12} + dr_2/r_2 - dr_1/r_1)}{(d\tau_{12}/\tau_{12})} = (1/c) \times [\theta (\pi_{11} r_1 \kappa_1 \pi_{13} + \pi_{21} r_2 \kappa_2 \pi_{23} + \pi_{31} \pi_{33}) + \pi_{32}] \times \theta (\pi_{22} r_2 \kappa_2 \pi_{21} + \pi_{32} \pi_{31}) + \pi_{21} r_2 \kappa_2]
\]
\[
+ [\theta (\pi_{11} r_1 \kappa_1 \pi_{13} + \pi_{21} r_2 \kappa_2 \pi_{23} + \pi_{31} \pi_{33}) + \pi_{31}] \}
\]
\[
> 0.
\]

From these results and Eq. (18), the short-run effects of a change in \( \tau_{12} \) on the growth rates are obtained as:

17
Suppose that all parameters are symmetric across countries: \( \rho_j = \rho, \delta_j = \delta, B_j = B, \sigma_j = \sigma, b_j = b \forall j, \tau_{nj} = \tau > 1 \forall n, j, j \neq n \). Then we can easily see from Eq. (7) that \( r_1^* = r_2^* = 1 \) solve Eqs. (23) and (24). Since the balanced growth rates as well as the depreciation and subjective discount rates are equal across countries, we have \( \Gamma_j^* = \Gamma^* \forall j \). Moreover, Eq. (6) implies that:

\[
\pi_{nj}^* = \tau^{-\theta}/(1 + 2\tau^{-\theta}) \equiv \pi^* < 1/3 \forall n, j, j \neq n,
\]

\[
\pi_{nn}^* = 1 - 2\pi^* > 1/3 \forall n.
\]

Finally, Eqs. (25) and (26) are solved for \( \kappa_1^* = \kappa_2^* = 1 \). The following comparative dynamics are evaluated at this symmetric BGP.

**Appendix B. Comparative dynamics around the symmetric BGP**

Finally, in view of Eq. (17), only three out of nine fractions of varieties have definite signs:

\[
(d\pi_{12}/\pi_{12})/(d\tau_{12}/\tau_{12}) < 0, (d\pi_{22}/\pi_{22})/(d\tau_{12}/\tau_{12}) > 0, (d\pi_{32}/\pi_{32})/(d\tau_{12}/\tau_{12}) > 0.
\]

Unilateral trade liberalization

We first examine the directions of changes in the long run fractions of varieties which are not covered in Proposition 2. From Eqs. (27) and (28), we have:

\[
a^* = a_{11}^*a_{22}^* - a_{12}^*a_{21}^* = (3\Gamma^*\pi^*)^2 - 0^2 = 9\Gamma^{*2}\pi^{*2},
\]

\[
(d\pi_{12}/\pi_{12})/(d\tau_{12}/\tau_{12}) = (1/a^*)(a_{22}^*\Gamma_1^*\pi_{12}^*) = 3\Gamma^{*2}\pi^{*2}/a^* = 1/3,
\]

\[
(d\pi_{22}/\pi_{22})/(d\tau_{12}/\tau_{12}) = (1/a^*)(a_{21}^*\Gamma_1^*\pi_{12}^*) = 0.
\]

Then Eq. (17) implies that

\[
(d\pi_{22}/\pi_{22})/(d\tau_{12}/\tau_{12}) > 0, (d\pi_{23}/\pi_{23})/(d\tau_{12}/\tau_{12}) > 0, (d\pi_{32}/\pi_{32})/(d\tau_{12}/\tau_{12}) > 0,
\]

\[
(d\pi_{33}/\pi_{33})/(d\tau_{12}/\tau_{12}) > 0, (d\pi_{11}/\pi_{11})/(d\tau_{12}/\tau_{12}) = \theta\pi^*[(d\tau_{12}/\tau_{12} + dr_{12}^*/r_{12}^*) - dr_{12}^*/r_{12}^* - (d\tau_{12}/\tau_{12} - dr_{12}^*/r_{12}^*)/(d\tau_{12}/\tau_{12})]>0.
\]

However, since

\[
(d\tau_{12}/\tau_{12} + dr_{12}^*/r_{12}^*)/(d\tau_{12}/\tau_{12}) = 1 - (dr_{12}^*/r_{12}^*)/(d\tau_{12}/\tau_{12}) = 2/3 > (dr_{12}^*/r_{12}^*)/(d\tau_{12}/\tau_{12}),
\]

we obtain

\[
(d\pi_{11}/\pi_{11})/(d\tau_{12}/\tau_{12}) > 0.
\]

In the short run, the coefficients \( c_{11}, c_{12}, c_{21}, c_{22} \), and \( c = c_{11}c_{22} - c_{12}c_{21} \) at the symmetric BGP are calculated as:

\[
dr_{12}^*/(d\tau_{12}/\tau_{12})
\]

\[
dr_{12}^*/(d\tau_{12}/\tau_{12})
\]

\[
dr_{12}^*/(d\tau_{12}/\tau_{12})
\]

\[
dr_{12}^*/(d\tau_{12}/\tau_{12})
\]
\[ c_{11} = c_{22} = -\{\theta[(1 - 2\pi^*)2\pi^* + \pi^*(1 - \pi^*) + \pi^*(1 - \pi^*)]| + 2\pi^*\} = -2\pi^*[\theta(2 - 3\pi^*) + 1], \]
\[ c_{12} = c_{21} = \theta[(1 - 2\pi^*)\pi^* + \pi^*(1 - 2\pi^*) + \pi^*^2] + \pi^* = \pi^*[\theta(2 - 3\pi^*) + 1], \]
\[ c^* = c_{11}c_{22} - c_{12}c_{21} = (-2c_{12}^2 - c_{12}^3) = 3c_{12}^3. \]

Then \((dr_1/r_1)/(dr_{12}/\tau_{12})\) and \((dr_2/r_2)/(dr_{12}/\tau_{12})\) are respectively given by:

\[ (dr_1/r_1)/(dr_{12}/\tau_{12}) = (\theta\pi*c_{12}^*/c^*)(1 - 3\pi^*) > 0, \]
\[ (dr_2/r_2)/(dr_{12}/\tau_{12}) = -\theta\pi*c_{12}^*/c^*. \]

From Appendix A, the signs of \(d\tau_{1}/(dr_{12}/\tau_{12})\) and \(d\gamma_3/(dr_{12}/\tau_{12})\) at the symmetric BGP are entirely determined by the signs of \((d\tau_{12}/\tau_{12} + dr_2/r_2 - dr_1/r_1)/(dr_{12}/\tau_{12})\) and \((dr_1/r_1)/(dr_{12}/\tau_{12}) + (dr_2/r_2)/(dr_{12}/\tau_{12})\), respectively. Since they are calculated as \((d\tau_{12}/\tau_{12} + dr_2/r_2 - dr_1/r_1)/(dr_{12}/\tau_{12}) - (dr_1/r_1)/(dr_{12}/\tau_{12}) = (3c_{12}^3\pi^*/c^*)[\theta(1 - \pi^*) + 1] > 0\) and \((dr_1/r_1)/(dr_{12}/\tau_{12}) + (dr_2/r_2)/(dr_{12}/\tau_{12}) = -3\theta\pi^*^2c_{12}^*/c^* < 0\), we have \(d\tau_{1}/(dr_{12}/\tau_{12}) < 0\) and \(d\gamma_3/(dr_{12}/\tau_{12}) > 0\). The last inequality means that, around the symmetric BGP, a fall in \(\tau_{12}\) indeed lowers \(\gamma_3\) in the short run.

For the fractions of varieties, we immediately know from Eq. (17) that

\[ (d\pi_{12}/\pi_{12})/(dr_{12}/\tau_{12}) > 0, \text{ and } (d\pi_{31}/\pi_{31})/(dr_{12}/\tau_{12}) < 0. \]

Moreover, \((dr_1/r_1)/(dr_{12}/\tau_{12}) + dr_2/r_2 - dr_1/r_1)/(dr_{12}/\tau_{12}) - (dr_1/r_1)/(dr_{12}/\tau_{12}) + (dr_2/r_2)/(dr_{12}/\tau_{12}) < 0\) implies that \((d\pi_{11}/\pi_{11})/(dr_{12}/\tau_{12}) > 0\) and \((d\pi_{33}/\pi_{33})/(dr_{12}/\tau_{12}) < 0\), respectively. Finally, we obtain \((d\pi_{23}/\pi_{23})/(dr_{12}/\tau_{12}) < 0\) from \(\pi^*(dr_1/r_1)/(dr_{12}/\tau_{12}) + (1 - 2\pi^*)(dr_2/r_2)/(dr_{12}/\tau_{12}) = (\theta\pi*c_{12}^*/c^*)[3\pi^*(1 - \pi^*) - 1] < 0\).

**Bilateral trade liberalization**

For bilateral trade liberalization, we only have to examine its short-run effects around the symmetric BGP.

First of all, Eq. (29) reduces to \((d\tau_{21}/\tau_{21})/(dr_{12}/\tau_{12})\mid_{dr_{1}r_{1}=dr_{2}r_{2}} = [(3\pi^* + 0)\Gamma^*\pi^*]/[(3\pi^* + 0)\Gamma^*\pi^*] = 1\), meaning that \(\tau_{21}\) is reduced by the same rate as \(\tau_{12}\) in order to keep \(r_1^*/r_2^*\) constant. Then, noting that

\[ -\theta\pi_{11}^r\pi_{12}^r\pi_{12}^r\tau_{12}\tau_{12} + \theta\pi_{22}^r\pi_{22}^r\pi_{22}^r(1 - \pi_{21}^r)dt_{12}/\tau_{21} = [\theta(1 - 2\pi^*)\pi^* + \theta\pi^*(1 - \pi^*)]dt_{12}/\tau_{12} = \theta\pi^*^2dr_{12}/\tau_{12}\]

and \(\theta\pi_{12}^r\pi_{12}^r(1 - \pi_{12}^r)dt_{12}/\tau_{12} - \theta\pi_{22}^r\pi_{22}^r\pi_{22}^r\pi_{22}^r dt_{12}/\tau_{21} = \theta\pi^*^2dr_{12}/\tau_{12}\), the rates of changes in \(r_1\) and \(r_2\) are respectively given by:

\[ (dr_1/r_1)/(dr_{12}/\tau_{12})\mid_{dr_{1}r_{1}=dr_{2}r_{2}} = -3\theta\pi^*^2c_{12}^*/c^* < 0, \]
\[ (dr_2/r_2)/(dr_{12}/\tau_{12})\mid_{dr_{1}r_{1}=dr_{2}r_{2}} = -3\theta\pi^*^2c_{12}^*/c^* = (dr_1/r_1)/(dr_{12}/\tau_{12})\mid_{dr_{1}r_{1}=dr_{2}r_{2}} < 0. \]

In contrast to its long-run effects, bilateral trade liberalization raises both \(r_1\) and \(r_2\), with \(r_1/r_2\) unchanged even in the short run. Therefore, \(\gamma_1(1, \tau_{12}r_2/r_1, \tau_{12}r_1/r_1)\) and \(\gamma_2(\tau_{21}r_1/r_2, 1, \tau_{23}r_2/r_2)\) rise whereas \(\gamma_3(\tau_{31}r_1, \tau_{32}r_2, 1)\) falls in the short run.

For the fractions of varieties, we immediately know from Eq. (17) that

\[ (d\pi_{11}/\pi_{11})/(dr_{12}/\tau_{12}) > 0, (d\pi_{22}/\pi_{22})/(dr_{12}/\tau_{12}) > 0, (d\pi_{31}/\pi_{31})/(dr_{12}/\tau_{12}) > 0, (d\pi_{32}/\pi_{32})/(dr_{12}/\tau_{12}) > 0, \text{ and } (d\pi_{33}/\pi_{33})/(dr_{12}/\tau_{12}) < 0, \]

where the condition that \(dr_{1}r_{1} = dr_{2}r_{2}\) after each vertical line is omitted.
to save space. Moreover, since \((d\tau_{12}/\tau_{12} + dr_2/r_2)/(d\tau_{12}/\tau_{12}) = 1 + (dr_2/r_2)/(d\tau_{12}/\tau_{12}) = (3c_{12}^*\pi^*/c^*)[2\theta(1 - 2\pi^*) + 1] > 0\) and \((d\tau_{23}/\tau_{23} + dr_1/r_1)/(d\tau_{12}/\tau_{12}) = (d\tau_{12}/\tau_{12} + dr_2/r_2)/(d\tau_{12}/\tau_{12}) > 0\), Eq. (17) implies that \((d\tau_{12}/\tau_{12})(d\tau_{12}/\tau_{12}) < 0\) and \((d\tau_{21}/\tau_{21})(d\tau_{12}/\tau_{12}) < 0\). Finally, we obtain \((d\tau_{13}/\tau_{13})(d\tau_{12}/\tau_{12}) > 0\) and \((d\tau_{23}/\tau_{23})(d\tau_{12}/\tau_{12}) > 0\) from \((1 - 2\pi^*)(dr_1/r_1)/(d\tau_{12}/\tau_{12}) + \pi^*(d\tau_{12}/\tau_{12} + dr_2/r_2)/(d\tau_{12}/\tau_{12}) = (3\pi^2c_{12}^*/c^*)[\theta(1 - 2\pi^*) + 1] > 0\) and \(\pi^*(d\tau_{21}/\tau_{21} + dr_1/r_1)/(d\tau_{12}/\tau_{12}) + (1 - 2\pi^*)(dr_2/r_2)/(d\tau_{12}/\tau_{12}) = (1 - 2\pi^*)(dr_1/r_1)/(d\tau_{12}/\tau_{12}) + \pi^*(d\tau_{12}/\tau_{12} + dr_2/r_2)/(d\tau_{12}/\tau_{12}) > 0\).

**Appendix C. The existence, uniqueness, and stability of a BGP**

Consider a three-country dynamic system consisting of Eqs. (19)-(22). Its BGP is defined by Eqs. (23)-(26).

We first show that there exists a BGP, where \(r^*_j \in (0, \infty)\forall j = 1, 2\). (If it is true, then Eqs. (25) and (26) are uniquely solved for positive and finite \(\kappa_{13}^*\) and \(\kappa_{23}^*\).) In view of Eq. (7), as \(r_1\) approaches zero with \(r_2 \in (0, \infty)\) given, \(Q_2(\tau_{21}r_1/r_2, 1, \tau_{23}/r_2)\) and \(Q_3(\tau_{31}r_1, \tau_{32}r_2, 1)\) approach zero whereas \(Q_1(1, \tau_{12}r_2/r_1, \tau_{13}/r_1)\) remains positive. This implies that \(\lim_{r_1 \to 0}(\gamma_1(1, \tau_{12}r_2/r_1, \tau_{13}/r_1) - \gamma_3(\tau_{31}r_1, \tau_{32}r_2, 1)) = -\infty < 0\). On the other hand, since \(Q_1(1, \tau_{12}r_2/r_1, \tau_{13}/r_1)\) approaches zero whereas \(Q_2(\tau_{21}r_1/r_2, 1, \tau_{23}/r_2)\) and \(Q_3(3\tau_{31}r_1, \tau_{32}r_2, 1)\) are positive as \(r_1\) approaches infinity, we have \(\lim_{r_1 \to \infty}(\gamma_1(1, \tau_{12}r_2/r_1, \tau_{13}/r_1) - \gamma_3(\tau_{31}r_1, \tau_{32}r_2, 1)) = \infty > 0\). Thus, from the intermediate value theorem, there exists \(r_1 = R_1(r_2) \in (0, \infty)\) such that \(\gamma_1(1, \tau_{12}r_2/r_1, \tau_{13}/r_1) - \gamma_3(\tau_{31}r_1, \tau_{32}r_2, 1) = 0\). Similarly, with \(r_1 \in (0, \infty)\) given, there exists \(r_2 = R_2(r_1) \in (0, \infty)\) such that \(\gamma_2(\tau_{21}r_1/r_2, 1, \tau_{23}/r_2) - \gamma_3(\tau_{21}r_1, \tau_{32}r_2, 1) = 0\). Solving \(r_1 = R_1(r_2)\) and \(r_2 = R_2(r_1)\), we obtain \(r_1^*\) and \(r_2^*\), which are positive and finite.

Fig. 3 illustrates the determination of \(r_1^*\) and \(r_2^*\) in a particular case. In the \((r_1, r_2)\)-plane, both curves \(r_1 = R_1(r_2)\) and \(r_2 = R_2(r_1)\) are positively sloped, and the former is steeper than the latter. These graphs intersect only once at point A: \((r_1^*, r_2^*)\), which gives a unique BGP. To understand when this is the case, we turn to mathematics. Substituting Eq. (18) with \(dr_{jn}/\tau_{jn} = \theta_{ij}n\) into the totally differentiated forms of Eqs. (23) and (24), we obtain:

\[
\begin{align*}
0 &= d\gamma_1^* - d\gamma_3^* = a_{11}^*dr_1^*/r_1^* + a_{12}^*dr_2^*/r_2^*, \\
0 &= d\gamma_2^* - d\gamma_3^* = a_{21}^*dr_1^*/r_1^* + a_{22}^*dr_2^*/r_2^*,
\end{align*}
\]

where \(a_{11}^*, a_{12}^*, a_{21}^*,\) and \(a_{22}^*\) are defined in section 3.1. Since a rise in \(r_1\) raises \(\gamma_1\) but lowers \(\gamma_3\), \(a_{11}^*\) is always positive. This ensures that \(R_1(\cdot)\) is single-valued, and that \(\gamma_1 - \gamma_3 > 0\) if and only if \(r_1 > R_1(r_2)\). On the other hand, \(a_{12}^*\) is negative if and only if a rise in \(r_2\) lowers \(\gamma_1\) by more than the fall in \(\gamma_3\). This is likely to occur when \(\pi_{12}^*\) is larger than \(\pi_{13}^*\), that is, country 1 is more open to country 2 than country 3. By the same reasoning, \(a_{22}^*\) is always positive, but \(a_{21}^*\) can either be positive or negative. Eqs. (C.1) and (C.2) are rewritten as, respectively:

\[
\begin{align*}
\frac{dr_2^*}{dr_1^*}|_{r_1 = R_1(r_2)} &= -\left(r_2^*/r_1^*\right)a_{11}^*/a_{12}^*, \\
\frac{dr_2^*}{dr_1^*}|_{r_2 = R_2(r_1)} &= -\left(r_2^*/r_1^*\right)a_{21}^*/a_{22}^*.
\end{align*}
\]

These expressions give us some information about the shapes of curves \(r_1 = R_1(r_2)\) and \(r_2 = R_2(r_1)\). First, curve \(r_1 = R_1(r_2)\) is positively sloped if and only if \(a_{12}^*\) is negative. Similarly, curve \(r_2 = R_2(r_1)\) is
that the relative supply of capital in country 1 to country 3 positively sloped if and only if $a_{21}^*$ is negative. Fig. 3 corresponds to the case where both $a_{12}^*$ and $a_{21}^*$ are negative. In this case, a rise in $r_2$ from a point on curve $r_1 = R_1(r_2)$ pulls $\gamma_1 - \gamma_3$ down from zero. Then $r_1$ has to rise so that $\gamma_1 - \gamma_3$ should go back to zero. Second, curve $r_1 = R_1(r_2)$ becomes more vertical, the smaller is $a_{12}^*$ in absolute value. Similarly, curve $r_2 = R_2(r_1)$ becomes more horizontal, the smaller is $a_{21}^*$ in absolute value. This implies that, the more similar the three countries are, the steeper curve $r_2 = R_2(r_1)$ is whereas the flatter curve $r_2 = R_2(r_1)$ is, and the more likely a BGP is to be unique. Third, taking the difference between the slopes at an intersection, we have:

$$
\frac{dr_2^*/dr_1^*}{r_1=R_1(r_2)} - \frac{dr_2^*/dr_1^*}{r_2=R_2(r_1)} = -(r_2^*/r_1^*)a^*/(a_{12}^*a_{22}^*),
$$

where $a^* \equiv a_{11}a_{22} - a_{12}a_{21} > 0$. This means that curve $r_1 = R_1(r_2)$ crosses curve $r_2 = R_2(r_1)$ from below (i.e., $dr_2^*/dr_1^*|_{r_1=R_1(r_2)} > dr_2^*/dr_1^*|_{r_2=R_2(r_1)}$) if and only if the former is positively sloped (i.e., $dr_2^*/dr_1^*|_{r_1=R_1(r_2)} > 0$). Fig. 3 applies to this case.

To study transitional dynamics, we first see how $r_1$ and $r_2$ respond to $\kappa_1$ and $\kappa_2$ in each period. Logarithmically differentiating Eqs. (21) and (22), and using Eq. (17) with $dt_{jn}/\tau_{jn} = 0\forall j, n$, we obtain:

$$
c_{11}dr_1/r_1 + c_{12}dr_2/r_2 = (1 - \pi_{11})r_1\kappa_1 d\kappa_1/\kappa_1 - \pi_{21}r_2\kappa_2 d\kappa_2/\kappa_2,
$$

$$
c_{21}dr_1/r_1 + c_{22}dr_2/r_2 = -\pi_{12}r_1\kappa_1 d\kappa_1/\kappa_1 + (1 - \pi_{22})r_2\kappa_2 d\kappa_2/\kappa_2,
$$

where $c_{11}, c_{12}, c_{21}$, and $c_{22}$ are defined in Appendix A. The relative demand for capital in country 1 to country 3, the right-hand side of Eq. (21), is decreasing in $r_1$ but is increasing in $r_2$. Similarly, the relative demand for capital in country 2 to country 3 is decreasing in $r_2$ but is increasing in $r_1$. Suppose, for example, that the relative supply of capital in country 1 to country 3 $\kappa_1$ increases. This directly tends to lower $r_1$ but raise $r_2$ from Eqs. (21) and (22), respectively. Not only that, it indirectly tends to raise $r_1$ through the increase in its relative demand caused by the rise in $r_2$, and also tends to lower $r_2$ through a similar demand substitution. The total effects of changes in $\kappa_1$ and $\kappa_2$ on $r_1$ and $r_2$ are obtained by solving the above two equations for $dr_1/r_1$ and $dr_2/r_2$:

$$
\frac{dr_1}{r_1} = (r_1\kappa_1/c)e_{11}d\kappa_1/\kappa_1 + (r_2\kappa_2/c)e_{12}d\kappa_2/\kappa_2, \tag{C.3}
$$

$$
\frac{dr_2}{r_2} = (r_1\kappa_1/c)e_{21}d\kappa_1/\kappa_1 + (r_2\kappa_2/c)e_{22}d\kappa_2/\kappa_2; \tag{C.4}
$$
As a result, an increase in $\kappa_1$ always lowers $r_1$, but its total effect on $r_2$ is ambiguous. Similarly, an increase in $\kappa_2$ always lowers $r_2$, but its total effect on $r_1$ is ambiguous.

We turn to the dynamic system. Linearizing Eqs. (19) and (20) around the BGP, using Eqs. (23), (24), (C.1), (C.2), (C.3), and (C.4), and noting that $d\kappa_j/\kappa_j = \ln \kappa_j - \ln \kappa_j^*$ and $\dot{\kappa}_j/\kappa_j = d(\ln \kappa_j - \ln \kappa_j^*)/dt$, the linearized dynamic system is given by:

\[
\begin{bmatrix}
  d(\ln \kappa_1 - \ln \kappa_1^*)/dt \\
  d(\ln \kappa_2 - \ln \kappa_2^*)/dt
\end{bmatrix} = J^* \begin{bmatrix}
  \ln \kappa_1 - \ln \kappa_1^* \\
  \ln \kappa_2 - \ln \kappa_2^*
\end{bmatrix} ; J^* \equiv \begin{bmatrix}
  j_{11} & j_{12} \\
  j_{21} & j_{22}
\end{bmatrix},
\]

where all components of the Jacobian matrix $J^*$ are evaluated at the BGP. Since both $\kappa_1$ and $\kappa_2$ are state variables, local stability requires that both eigenvalues associated with $J^*$ should have negative real parts. In our two-dimensional system, the condition is equivalent to $\text{tr} J^* = j_{11}^* + j_{22}^* < 0$ and $\text{det} J^* = j_{11}^* j_{22}^* - j_{12}^* j_{21}^* > 0$. The trace and determinant of $J^*$ are calculated as:

\[
\text{tr} J^* = (1/c^*)[r_1^* \kappa_1^*(a_{11} e_{11} + a_{12} e_{21}) + r_2^* \kappa_2^*(a_{21} e_{12} + a_{22} e_{22})],
\]

\[
\text{det} J^* = (r_1^* \kappa_1^* r_2^* \kappa_2^*/c^2) a^*(e_{11}^* e_{22} - e_{12}^* e_{21}).
\]

Since it is easily verified that $e_{11} e_{22} - e_{12} e_{21} = [(1 - \pi_{11})(1 - \pi_{22}) - \pi_{12} \pi_{21}] c > 0$, we conclude that a balanced growth path is locally stable if and only if:

\[
r_1^* \kappa_1^*(a_{11} e_{11} + a_{12} e_{21}) + r_2^* \kappa_2^*(a_{21} e_{12} + a_{22} e_{22}) < 0.
\]

To interpret this condition, consider the case where $a_{12}, a_{21}, e_{12}^*$, and $e_{21}^*$ are close to zero in absolute values. Suppose, for example, that countries 1 and 2 are so small relative to country 3 in the initial period:
\( \kappa_{10} < \kappa^*_1 \) and \( \kappa_{20} < \kappa^*_2 \). Then, reflecting the scarcity of capital, \( r_{10} \) and \( r_{20} \) should be higher than \( r^*_1 \) and \( r^*_2 \), respectively (because \( e_{11} < 0 \) and \( e_{22} < 0 \) in Eqs. (C.3) and (C.4)). With the terms of trade of the first two countries being high, they start to grow faster than the last country (because \( a^*_{11} > 0 \) and \( a^*_{22} > 0 \) in Eqs. (C.1) and (C.2)). Since this increases \( \kappa_{1t} \) and \( \kappa_{2t} \) toward \( \kappa^*_1 \) and \( \kappa^*_2 \), respectively, \( r_{1t} \) and \( r_{2t} \) fall toward \( r^*_1 \) and \( r^*_2 \), respectively. Because of the diminishing terms of trade for countries 1 and 2, their growth advantages over country 3 are going to disappear in the long run.

More generally, an ambiguity arises due to the "cross effects" \( a^*_{12}, a^*_{21}, e^*_{12}, \) and \( e^*_{21} \). They result from the assumption of more than two countries: a similar two-country model of Naito (2012) does not have such cross effects because there is only one relative supply of capital. Acemoglu and Ventura (2002) stress the same stability logic as the above paragraph in their continuum-country model, seeming to ignore the cross effects. However, even if the cross effects are not negligible, our dynamic system is still stable as long as the "own effects" mentioned in the previous paragraph are dominant.
References


Fig. 1. Paths of main endogenous variables under unilateral trade liberalization in the calibrated case.
Fig. 2. Paths of main endogenous variables under bilateral trade liberalization in the calibrated case.
Fig. 3. Rental rates at the balanced growth path: $a_{12}^* < 0, a_{21}^* < 0$. 

$r_1 = R_1(r_2)$

$r_2 = R_2(r_1)$

$r_2^*$
(a) Unilateral trade liberalization

(b) Bilateral trade liberalization

Table 1: Effects of trade liberalization around the symmetric BGP
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Table 2: Effects of unilateral and bilateral trade liberalization in the calibrated case: $\tau_{12} = 2.33905, \tau_{13} = 2.02213, \tau_{21} = 2.4053, \tau_{23} = 2.66304, \tau_{31} = 1.95887, \tau_{32} = 2.56534, b_1 = 0.166886, b_2 = 0.180617, b_3 = 0.161854$