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KOBAYASHI Keiichiro
RIETI

Research Institute of Economy, Trade & Industry, IAA

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Banking in the Lagos-Wright Monetary Economy

KOBAYASHI Keiichiro

Institute of Economic Research,

Hitotsubashi University and RIETI

We introduce banks in a monetary economy and analyze the effect of monetary friction on the banking sector. The basic model is a cash-in-advance economy which is a simplified version of Lagos and Wright's (2005) model. We introduce the banks using Diamond and Rajan (2001) in this economy: Bankers can produce goods more efficiently than depositors but cannot pre-commit to the use of human capital on behalf of the latter. Demand deposit contracts work as a commitment device for bankers, while leaving banks susceptible to bank runs. We show that as the inflation rate increases, the size of the banking sector expands, and the probability of bank runs occurring rises.

Friedman's rule is not necessarily optimal.

Keywords: Money, Lagos-Wright model, Bank runs, Commitment, Diamond-Rajan model

JEL classification: E42, G01, G21

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Keiichiro Kobayashi (Institute of Economic Research, Hitotsubashi University; and Research Institute of Economy, Trade and Industry (RIETI) E-mail: kcrkbys@ier.hit-u.ac.jp.)

1 Introduction

We introduce the Diamond-Rajan banks into a simplified version of the Lagos-Wright monetary economy.

The motivation is that we want to analyze the collective action problem (i.e., the bank runs) in a business cycle model in which agents live forever.[1] To analyze the global financial crisis of 2007–2009, the business cycle models with financial frictions have been intensively studied (see Christiano, Motto, Rostagno 2009, Gertler and Karadi 2009, Gertler and Kiyotaki 2010). These models consider borrowing constraints due to costly state verification à la Carlstrom and Fuerst (1997) and Bernanke, Gertler and Gilchrist (1999). These models are intended to be a basis for the monetary policy analysis, while the collective action problem is not analyzed in these models. In the present paper, we explicitly consider the collective action problem in a business cycle model that can be used for the monetary policy analysis.[2]

The second motivation is to explicitly analyze the relationship between the monetary frictions and the bank runs. The Lagos-Wright model is a very tractable framework to analyze the frictions due to the role of money as a medium of exchange. Using this framework, we can analyze how the inflation rate affects the occurrence of bank runs.

2 The Model

The model is a simplified version of the monetary model following the framework of Lagos and Wright (2005).

[1]In the existing literature, the bank runs are usually analyzed in a two-period or three-period models (see, for example, Diamond and Dybvig 1983, Allen and Gale 1998). These models in the banking literature is not easily compatible with the standard business cycle models. There are the overlapping-generations models with infinite horizon, in which money and banks are incorporated. See, for example, Smith (2002), Schreft and Smith (1996), Paal and Smith (2000), Cooper and Ejarque (1995), and Cooper and Corbae (2002).

[2]The Lagos-Wright framework can be easily integrated with the new Keynesian models and can be used for monetary policy analysis. See Aruoba and Schorfheide (2009).

2.1 The Environment

The economy is a closed economy, with discrete time that continues forever: $t = 0, 1, 2, \ldots$. A unit mass of the infinitely-lived households are inhabited in this economy. The intertemporal discount factor for the utility flow is β for all households, where $0 < \beta < 1$. In each period t, there are perfectly competitive markets that open sequentially: the day market (DM) and the night market (NM). At the beginning of each period t, all households are identical and after the DM opens, the households receive an idiosyncratic technology shock $\delta_t \in \{0, 1\}$. A household receives $\delta_t = 1$ with probability ζ and $\delta_t = 0$ with probability $1 - \zeta$, where $0 < \zeta \ll 1$. We call the households who received $\delta_t = 0$ "depositors," and the households who received $\delta_t = 1$ "bankers." The depositors can produce the intermediate goods q in the DM, expending the utility cost $c(q)$, where $c'(q) > 0$, $c''(q) > 0$, and $c(0) = 0$. A depositor can transform q units of the intermediate goods that are produced by the other depositors to q units of the consumption goods. The bankers cannot produce the intermediate goods, but they have an access to a stochastic production technology by which a banker can transform Q units of the intermediate goods to $A_t Q$ units of the consumption goods, where $A_t = \theta + a_t$, where $0 < \theta < 1$ and $a_t \in (0, +\infty)$ is a random variable that follows the log-normal distribution with mean μ and variance σ^2. The random variable a_t is i.i.d. for each banker. The expected value of the productivity of the banker's technology is higher than that of the depositor's technology:

$$E[A_t] > 1.$$

The production of $A_t Q$ causes the utility cost $C(Q)$ to the bankers in the DM. The random variable A_t is revealed in the middle of the DM after the banker acquired Q units of the intermediate goods and he lost $C(Q)$. The NM opens after the production of the consumption goods is finished in the DM. The consumption takes place only in the NM and all households obtains $U(c)$ units of the utility from consuming c. In the NM, all households have access to a general technology by which they can transform h units of labor into κh units of the consumption goods, while the labor supply h causes h units of the additional utility cost to the households in the NM. Thus the utility flow of agents in period t is $U(c_t) - h_t$. Lagos and Wright (2005) utilizes this quasi-linearity of the utility à la Hansen (1985) and Rogerson (1988) to degenerate the distribution of money holdings to make the analysis tractable.

Cash-in-advance constraint for depositors: We assume that a transaction of the intermediate goods in the DM is *not verifiable* in the court. Therefore if a depositor buys the intermediate goods with credit, he can abscond without paying for them. There is no penalty for absconding because the existence of the transaction of the intermediate goods is not verifiable. Thus purchase of the intermediate goods in the DM should be paid with cash.

Cash-in-advance constraint for bankers: Our assumption that the goods transaction in the DM is not verifiable implies that the bankers should buy the intermediate goods by paying cash: If the banker buys the intermediate goods with credit, the banker can repudiate the debt without any penalty because the existence of the transaction of the intermediate goods is not verifiable.

Demandable deposit in the DM: We assume that the lending and borrowing of cash between agents in the DM is verifiable as in Berentsen, Camera, and Waller (2007). We assume that just after the shock δ_t is revealed and before the intermediate goods are produced, there is a chance that the depositors and the bankers can lend cash with each other. Since $E[A_t] > 1$, in order to maximize the social welfare of the agents, it is desirable to have the bankers produce the consumption goods using a large amount of the intermediate goods. Since the purchase of the intermediate goods is subject to the cash-in-advance (CIA) constraint, it is desirable for the depositors and the bankers to agree that the depositors lend cash to the bankers and the bankers buy the intermediate goods using the borrowed cash. A banker borrows cash from $\frac{1-\zeta}{\zeta}$ depositors. We assume the following for the technology of financial contract:

Assumption 1 *The financial contract cannot be contingent on the realization of A_t, which is observable for all agents but not verifiable in the court. The banker can walk away* *in the midst of production of the consumption goods, leaving θQ_t units of consumption goods to the depositors. When A_t is realized but before the production of $A_t Q_t$ is finished, a banker has a chance to initiate a renegotiation with the depositors, threatening them that he will walk away unless the depositors accept to renew the terms and condition of their financial contract.*

This assumption is in line with Diamond and Rajan (2001). As Diamond and Rajan show in Section III in their paper, the demand deposit contract with many depositors is an optimal device to prevent the banker from initiating the renegotiation.

Definition 1 *Demand deposit contract in period t specifies the gross rate of return on deposit, R_t, and gives the depositor who deposited d_t units of cash the right to withdraw $R_t d_t$ units of cash at anytime he/she wants after A_t is revealed during period t.*

Each banker borrows cash from $\frac{1-\zeta}{\zeta}$ depositors, where $\frac{1-\zeta}{\zeta}$ is a large number. If a banker tries to initiate the renegotiation with depositors to decrease R_t, the depositors must simultaneously choose whether to accept he banker's offer to renegotiate or to withdraw. It is easily shown under Assumption 2 that the dominant strategy for the depositors is to withdraw (the proof is almost identical to the arguments in Sections III C and D of Diamond and Rajan 2001). When all depositors run on the banker, the banker has to pay cash $R_t d_t$ to each depositor who deposited d_t by selling the bank assets. We assume the following for the fire sale of bank assets:

Assumption 2 *If a banker is forced to sell his/her assets before finishing the production of $A_t Q_t$ units of the final good, the banker can transform Q_t units of the intermediate goods into θQ_t units of final good and can sell only θQ_t in the market (to the depositors), where $0 < \theta < 1$.*

Therefore, if a banker offer to renegotiate, the depositors' run on the banker occurs and the banker is forced to stop producing $A_t Q_t$ and sell θQ to repay cash to the depositors. Since θQ is smaller than total liability of the banker, all surplus that the banker could produce is destroyed by the bank run. Anticipating this result, the banker never try to initiate renegotiation with the depositors to decrease R_t. Note that we assume that the banker, who are run by the depositors, can sell the consumption goods, θQ, to the households (i.e., depositors) in the midst of the DM, *after* the household sells the intermediate goods q to the bankers. This assumption is big difference from the models following the Lagos-Wright framework, which assume that the agents can implement a transaction of the goods only once in the DM. On the other hand, in the present paper, we assume that an agent (a depositor) can participate in two transactions during the DM: he/she sells the intermediate goods to a banker, and he/she can buy the consumption goods from the banker who are subject to a bank run.

Money supply: We assume that there exists the central bank that can freely implement a lump-sum transfer to/from each agents in the NM. Therefore the central bank can set the growth rate of money:

$$\pi_t = \frac{M_{t+1}}{M_t},$$

which equals the inflation rate in the steady-state equilibrium.

Sequence of events: The sequence of events during a representative period t is as follows. Identical households with cash holdings m enter the DM. They receive the idiosyncratic shock δ_t. $1-\zeta$ agents who receive $\delta_t = 1$ become depositors and ζ agents who receive $\delta_t = 0$ become bankers. The depositors deposit (a part of) their cash with the bank in the form of demand deposit. Then the depositors produce the intermediate goods and sell them to the bankers and other depositors. The intermediate goods purchased by the depositors are transformed into the consumption goods at one-for-one basis. After the bankers start producing the consumption goods from the intermediate goods Q_t, the aggregate productivity A_t is revealed. Then the bankers have a chance to make an offer to renegotiate with the depositors and the depositors may run on the bankers to withdraw their deposits immediately. If the bank runs occur, the bankers sell the consumption goods, the amount of which is $\theta Q_t < A_t Q_t$, in the market (to depositors) and they repay the deposits to the depositors. If the bank runs do not occur, the bankers finish production of $A_t Q_t$ units of the consumption good. Then the NM opens. Unless the bank runs occurred in the DM, the bankers payout the deposits $R_t d_t$ to each depositor and all households (bankers and depositors) choose the consumption (c), the labor in the NM (h), and the amount of cash (m_{+1}) to bring into the next period $t+1$.

2.2 Optimization Problems

Since all agents are identical when they enter a new period t, the value function for an agent entering period t holding cash m can be written as

$$V(m) = \zeta V^B(m) + (1-\zeta)V^D(m), \tag{1}$$

where $V^B(m)$ is the value function for a banker and $V^D(m)$ is the value function for a depositor. The depositor's optimization is written as the following Bellman equation:

$$V^D(m) = \max_{d,q,q'} E_A[\max_{\tilde{h},\tilde{c},\tilde{m}_{+1}} U(\tilde{c}) - \tilde{h} + \beta V(\tilde{m}_{+1})] - c(q), \tag{2}$$

subject to

$$\tilde{c} + \phi\tilde{m}_{+1} = \phi\tilde{R}d + \kappa\tilde{h} + \phi pq + q' + \tau, \tag{3}$$

$$pq' + d \leq m, \tag{4}$$

where the variables with tilde ($\tilde{h}, \tilde{c}, \tilde{m}_{+1}$) are chosen in the NM and those without tilde (d, q, q') are chosen in the DM; $E_A[\,\cdot\,]$ is the expectation over A taken at the beginning of the DM of period t; ϕ is the value of cash in terms of the consumption good; $\tilde{R}$ is the return on the bank deposit, which varies contingent on the realization of A, because $\tilde{R}$ is a constant R as long as the bank run does not occur and $\theta Q/(\phi D)$ if the bank run occurs; p is the price of the intermediate good in terms of money; q is the amount of the intermediate good that the depositor produces in the DM; d is the nominal amount of bank deposit that the depositor lends to the banker in the DM; and q' is the amount of the intermediate good that the depositor purchases from other depositors. Condition (4) is the CIA constraint for the depositor. Note that when the bank run occurs, the budget constraint (3) can be written as

$$\tilde{c} + \phi\tilde{m}_{+1} = \frac{\theta Q}{\phi D}\phi d - \frac{\theta Q}{\phi D}\phi d + \frac{\theta Q}{\phi D}\phi d + \kappa\tilde{h} + \phi pq + q' + \tau,$$

where the first term in the right-hand side is the return from the bank deposit, the second term is the payment of cash (in terms of the consumption good) to the banker at the fire sale of the consumption good, and the third term is the amount of the consumption good that the depositor purchases from the banker at the fire sale. The reduced form of the above problem is

$$V^D(m) = \max_{q,q'} E_A \left[\max_{\tilde{c},\tilde{m}_{+1}} U(\tilde{c}) - \frac{1}{\kappa}\tilde{c} - \frac{\phi}{\kappa}\tilde{m}_{+1} + \beta V(\tilde{m}_{+1}) \right] + \frac{\phi}{\kappa} E_A[\tilde{R}]m - \frac{\phi p E_A[\tilde{R}] - 1}{\kappa} q' + \frac{\phi p}{\kappa} q - c(q), \tag{5}$$

subject to

$$0 \leq q' \leq \frac{m}{p}. \tag{6}$$

The first-order conditions (FOCs) imply that $\tilde{c}$ and $\tilde{m}_{+1}$ are independent from m and are determined by

$$U'(\tilde{c}) = \frac{1}{\kappa}, \tag{7}$$

$$\frac{\phi}{\kappa} = \beta V'(\tilde{m}_{+1}). \tag{8}$$

The supply of the intermediate goods is determined by

$$c'(q) = \frac{\phi p}{\kappa}. \tag{9}$$

The FOC with respect to q' implies that if $\phi p E_A[\tilde{R}] > 1$, then $q' = 0$, if $\phi p E_A[\tilde{R}] < 1$, then $q' = m/p$, and if $\phi p E_A[\tilde{R}] = 1$, then $q' \in [0, m/p]$ is indeterminate.

The banker's optimization is written as the following Bellman equation:

$$V^B(m) = \max_{D,Q,R} E_A[\max_{\tilde{h},\tilde{c},\tilde{m}_{+1}} U(\tilde{c}) - \tilde{h} + \beta V(\tilde{m}_{+1})] - C(Q), \tag{10}$$

subject to

$$\tilde{c} + \phi\tilde{m}_{+1} = \max\{A_t Q - \phi R D, 0\} + \kappa\tilde{h} + \tau, \tag{11}$$

$$pQ \leq D + m, \tag{12}$$

$$E_A[\tilde{R}] \equiv R \int_{\frac{\phi RD}{Q}}^{\infty} f(A)dA + \frac{\theta Q}{\phi D} \int_0^{\frac{\phi RD}{Q}} f(A)dA \geq \frac{B}{\phi p}, \tag{13}$$

$$D \leq \frac{1-\zeta}{\zeta}\overline{m}, \tag{14}$$

where $B/(\phi p)$ is the outside rate of returns that the depositors can get from depositing their money with other banks, which is determined as an equilibrium outcome; Q is the amount of the intermediate good that the banker purchases in the DM; D is the nominal amount of the bank deposits that the banker accepts from the depositors; R is the deposit rate that the banker promises to the depositors; and $\overline{m}$ is the per capita cash holdings in the economy.

The condition for $d > 0$ in the depositor's problem is that $\phi p E_A[\tilde{R}] \geq 1$. This condition implies that the equilibrium condition for the existence of the equilibrium where $D > 0$ is

$$B \geq 1. \tag{15}$$

Define the variable y by

$$y = \phi p R\left(1 - \frac{m}{pQ}\right).$$

The reduced form of the banker's problem is

$$V^B(m) = \max_{y,Q} \frac{1}{\kappa} Q \int_y^\infty (A-y) f(A) dA - C(Q) + E_A\left[\max_{\tilde{c},\tilde{m}_{+1}} U(\tilde{c}) - \frac{\tilde{c}}{\kappa} - \frac{\phi}{\kappa}\tilde{m}_{+1} + \beta V(\tilde{m}_{+1})\right], \tag{16}$$

subject to

$$y\int_y^\infty f(A)dA + \theta\int_0^y f(A)dA \geq B\left(1 - \frac{m}{pQ}\right), \tag{17}$$

$$pQ - m \leq \frac{1-\zeta}{\zeta}\overline{m}, \tag{18}$$

The FOCs are $U'(\tilde{c}) = 1/\kappa$, $\beta V'(m_{+1}) = \phi/\kappa$, and

$$\frac{1}{\kappa}\int_y^\infty (A-y)f(A)dA - C'(Q) - p\mu = \frac{m}{pQ^2}B\lambda, \tag{19}$$

$$\lambda\left\{\int_y^\infty f(A)dA - (y-\theta)f(y)\right\} = \frac{1}{\kappa}Q\int_y^\infty f(A)dA, \tag{20}$$

where λ and μ are the Lagrange multipliers for (17) and (18). These FOCs imply that

$$C'(Q) = \frac{1}{\kappa}\left[\int_y^\infty (A-y)f(A)dA - \frac{mB}{pQ}\frac{\int_y^\infty f(A)dA}{\int_y^\infty f(A)dA - (y-\theta)f(y)}\right] - p\mu. \tag{21}$$

2.3 Equilibrium

Following the literature on the Lagos-Wright monetary models, we focus on the steady-state equilibrium where the labor productivity in the NM is unity: $\kappa = 1$ and the inflation rate is constant: $\phi/\phi_{+1} = \pi$. Since all agents hold the same amount of cash at the beginning of each period,

$$\overline{m} = m. \tag{22}$$

We define the size of the banking sector x by

$$x = \frac{D}{m} = \frac{pQ}{m} - 1. \tag{23}$$

Since $1 - \frac{m}{pQ} = \frac{x}{1+x}$, the participation condition for the depositor (17) becomes as follows when it is binding:

$$F(y) = \frac{Bx}{1+x} - \theta, \tag{24}$$

where

$$F(y) \equiv (y - \theta) \int_y^{\infty} f(A) dA. \tag{25}$$

There are at most two solutions to (24) as shown in *** Figure 1 ***. Since the profit maximizing banker chooses the smallest value of y that satisfies (24), the value of $y = y(x, B)$ is uniquely determined as the smallest solution to (24). It is obvious from Figure 1 that there exists an upper bound for B that allows the existence of $y(x, B)$. Together with (15), in equilibrium, B must satisfy

$$1 \leq B \leq \overline{B}, \tag{26}$$

where $\overline{B}$ is defined by

$$(1 - \zeta)\overline{B} = \theta + \sup_y F(y). \tag{27}$$

First, we specify the banker's demand for the intermediate goods, Q, as a function of x and B under the assumption that constraint (18) is nonbinding, i.e., $\mu = 0$, in the equilibrium. As we show later, there exists an equilibrium where $\mu > 0$ if the inflation rate is sufficiently high. We will show that the amount of the intermediate goods is solely determined by the supply decision of the depositors in this case and the value of μ (> 0) is determined such that the demand, Q, is consistent with the supply.

Under the assumption that $\mu = 0$, the condition (21) is rewritten as

$$C'(Q) = \Gamma(x, B), \tag{28}$$

where

$$\Gamma(x, B) = \int_{y(x,B)}^{\infty} \{A - y(x, B)\} f(A) dA - \frac{B}{1+x} \left[\frac{\int_{y(x,B)}^{\infty} f(A) dA}{\int_{y(x,B)}^{\infty} f(A) dA - \{y(x, B) - \theta\} f(y(x, B))} \right]. \tag{29}$$

We define $q^b \equiv \frac{\zeta}{1-\zeta} Q$, the amount of the intermediate goods per one depositor that the bankers demand.

$$q^b = \hat{\Gamma}(x, B), \tag{30}$$

where $\hat{\Gamma}(x, B) = \frac{\zeta}{1-\zeta} C'^{-1}(\Gamma(x, B))$, where $C'^{-1}(\ \cdot\ )$ is the inverse of $C'(\ \cdot\ )$, which is well defined because $C'(\ \cdot\ )$ is a monotonically increasing function. Since $f(A) = 0$ for $0 \leq A \leq \theta$ and $\int_y^{\infty} f(A) dA = 1$ for $0 \leq y \leq \theta$, (24) implies that $y = Bx/(1 + x)$ for $0 \leq x \leq \theta/(B - \theta)$. It is shown that $\Gamma(x, B) = E(A) - B = \theta + \ln \mu - B$ for $0 \leq x \leq \theta/(B - \theta)$. Therefore, $q^b = \hat{\Gamma}(x, B)$ is invariant with respect to x for $0 \leq x \leq \theta/(B - \theta)$. Since $\lim_{x\to\infty} y(x, B) = \overline{y}(B)$, where $\overline{y}(B)$ is the solution to $F(y) = B - \theta$, it is shown that $\Gamma_{\infty}(B) \equiv \lim_{x\to\infty} \Gamma(x, B) = \int_{\overline{y}(B)}^{\infty} \{A - \overline{y}(B)\} f(A) dA$. $\Gamma(0, B)$ is larger than $\Gamma_{\infty}(B)$, because $\Gamma(0, B) - \Gamma_{\infty}(B) = \int_0^{\overline{y}(B)} (A - \theta) f(A) dA > 0$. Note that $\Gamma(0, B) - \Gamma_{\infty}(B)$ is increasing in B, because $\overline{y}(B)$ is increasing in B, as long as $\overline{y}(B)$ exists. $\hat{\Gamma}(x, B)$ also has the same features. *** See Figures 1 and 2 for illustration.***

The FOCs with respect to m_{+1} and the envelope condition for $V^B(m)$ and $V^D(m)$ imply that

$$\frac{\phi}{\beta} = V'(m_{+1}) = (1 - \zeta) E_A[\phi_{+1} \tilde{R}] + \zeta E_A[\phi_{+1} \hat{R}], \tag{31}$$

where $\hat{R} = R$ if $A \geq y(x, B)$ and $\hat{R} = 0$ if $A < y(x, B)$. Since $y = \phi p R \frac{x}{1+x}$, this condition and $c'(q) = \phi p$ imply that

$$c'(q) = \frac{\beta}{\pi} \Lambda(x, B) \tag{32}$$

where

$$\Lambda(x, B) = \begin{cases} B & \text{if } 0 \leq x < \frac{\theta}{B-\theta}, \\ B - \zeta\theta \frac{1+x}{x} \int_0^{y(x,B)} f(A) dA, & \text{if } x \geq \frac{\theta}{B-\theta}. \end{cases} \tag{33}$$

We have $\Lambda_\infty(B) \equiv \lim_{x \to \infty} \Lambda(x, B) = B - \zeta\theta \int_0^{\overline{y}(B)} f(A) dA$. Therefore, $\Lambda(0, B) - \Lambda_\infty(B) = \zeta\theta \int_0^{\overline{y}(B)} f(A) dA > 0$, which is increasing in B. Since $c'(\,\cdot\,)$ is a monotonically increasing function, the supply of the intermediate goods per a depositor, $q = \hat{\Lambda}(\pi, x, B)$, where $\hat{\Lambda}(\pi, x, B) \equiv c'^{-1}\left(\frac{\beta}{\pi}\Lambda(x, B)\right)$, has the same features as $\frac{\beta}{\pi}\Lambda(x, B)$. *** See Figure 3 for illustration. *** The conditions $\frac{pQ}{m} = 1 + x$ and $Q = \frac{1-\zeta}{\zeta} q^b$ imply that

$$\frac{m}{p} = \frac{1-\zeta}{(1+x)\zeta} q^b. \tag{34}$$

The condition (4) with $q' = q - q^b$ and $d = \frac{\zeta}{1-\zeta} D = \frac{\zeta}{1-\zeta} xm$ implies that

$$q - q^b = \left(1 - \frac{\zeta x}{1-\zeta}\right) \frac{m}{p} \tag{35}$$

From these equations we have

$$q = \frac{1}{(1+x)\zeta} q^b. \tag{36}$$

The equilibrium conditions are

$$q \geq q^b, \tag{37}$$

$$x \leq \frac{1-\zeta}{\zeta}. \tag{38}$$

There are two kinds of steady-state equilibria: one with the condition (18) is nonbinding, i.e., $\mu = 0$, and one with (18) is binding, i.e., $\mu > 0$. When (18) is binding, the bankers cannot obtain sufficient amount of the bank deposits and are subject to rationing. Therefore, we call the steady-state equilibrium with $\mu = 0$ the equilibrium with rationing and the one without $\mu > 0$ the equilibrium without rationing.

Definition 2 *Given the inflation rate π, the steady-state equilibrium without rationing is the set of variables (x, B, q, q^b) that satisfy (26), (30), (32), (36), (37), and (38).*

The following lemma is useful to characterize the equilibrium.

Lemma 1 *In the equilibrium, either $x < \frac{1-\zeta}{\zeta}$ and $B = 1$ or $x = \frac{1-\zeta}{\zeta}$ and $B > 1$.*

(Proof) Suppose that in the equilibrium $x < \frac{1-\zeta}{\zeta}$. Since $x = \frac{D}{m}$ and $D = \frac{1-\zeta}{\zeta}d$, the CIA constraint for depositors (4) implies that $q' > 0$. If $B > 1$, the FOC for the depositor's problem implies that $q' = 0$. Therefore, if $x < \frac{1-\zeta}{\zeta}$, then B cannot exceed 1. (End of Proof)

This lemma with the equations (30), (32), and (36) implies that if the following equation:

$$\hat{\Lambda}(\pi, x, 1) = \frac{1}{(1+x)\zeta}\hat{\Gamma}(x, 1), \tag{39}$$

where $\hat{\Lambda}(\pi, x, 1) \equiv c'^{-1}\left(\frac{\beta}{\pi}\Lambda(x, 1)\right)$, has the solution in the region $0 \leq x \leq \frac{1-\zeta}{\zeta}$, then the equilibrium value of x is the solution to (39). We assume that the functions $C(\cdot)$ and $c(\cdot)$ are convex enough to make $\hat{\Gamma}(x, 1)$ and $\hat{\Lambda}(x, 1)$ are both monotonically decreasing functions. As *** Figure 4 *** shows, the right-hand side of (39) is steeply decreasing and the solution to (39) increases as the inflation rate, π, increases. The total supply of the intermediate goods, $q = \hat{\Lambda}(\pi, x, 1)$, and the amount of the intermediate goods that are purchased by the bankers, $Q = \frac{1-\zeta}{\zeta}q^b = \frac{1-\zeta}{\zeta}\hat{\Gamma}(x, 1)$, are both decreasing in π because $\hat{\Gamma}(x, 1)$ and $\hat{\Lambda}(\pi, x, 1)$ are both decreasing in x. Since $\frac{m}{pQ} = \frac{1}{1+x}$, the real balance $\phi m = z$ is time-invariant and determined by $z = (\zeta^{-1} - 1)\frac{c'(q)q^b}{1+x}$. The probability that the bank runs occur is $P(x, B) \equiv \int_0^{y(x,B)} f(A)dA$ in this economy and $y(x, 1)$ is increasing in x. Therefore, $P(x, 1)$ increases as the inflation rate π increases. Since the graph of $q = \hat{\Lambda}(\pi, x, 1)$ shifts downward as π increases in Figure 4, there exist $\underline{\pi}$ and $\overline{\pi}$ such that

$$\hat{\Lambda}(\underline{\pi}, 0, 1) = \frac{1}{\zeta}\hat{\Gamma}(0, 1),$$
$$\hat{\Lambda}(\overline{\pi}, \zeta^{-1} - 1, 1) = \hat{\Gamma}(\zeta^{-1} - 1, 1).$$

The above equilibrium with $B = 1$ exists if $\underline{\pi} < \pi \leq \overline{\pi}$. (See Appendix A for the details about the equilibrium for $\pi \leq \underline{\pi}$.) If π exceeds $\overline{\pi}$, there is no equilibrium with $B = 1$.

For $\pi > \overline{\pi}$, the equilibrium value of x is fixed at $x = \frac{1-\zeta}{\zeta}$ and B is determined by

$$\hat{\Lambda}(\pi, \zeta^{-1} - 1, B) = \hat{\Gamma}(\zeta^{-1} - 1, B). \tag{40}$$

In this case, $q = q^b = \hat{\Lambda}(\pi, \zeta^{-1} - 1, B)$. The real balance $\phi m = z$ is time-invariant and $z = (1 - \zeta)c'(q)q$. It is easily shown that B increases and q decreases as π increases. There exists $\hat{\pi}$ such that the solution to (40) at $\pi = \hat{\pi}$ equals $B = \overline{B}$. If π exceeds $\hat{\pi}$, the steady-state equilibrium without rationing does not exist. In this case, (18) is binding and the bankers cannot obtain the sufficient amount of deposits although they offer $\overline{B}/(\phi p)$ as the expected return on the bank deposits. The Lagrange multiplier for (18), μ, is determined such that q^b equals $q = \hat{\Lambda}(\pi, \zeta^{-1} - 1, \overline{B})$ in the equilibrium. Thus the equilibrium with rationing is given as follows.

Lemma 2 *Suppose that $\pi > \hat{\pi}$. In this case, there exists the steady-state equilibrium with deposit rationing, which is the set of variables (x, B, q, z) that satisfy $x = \zeta^{-1} - 1$, $B = \overline{B}$, $q = \hat{\Lambda}(\pi, \zeta^{-1} - 1, \overline{B})$, and $z = (1 - \zeta)c'(q)q$.*

In summary, if $\underline{\pi} < \pi \leq \overline{\pi}$, the equilibrium is without rationing, where $0 < x < \zeta^{-1} - 1$ and $B = 1$; if $\overline{\pi} < \pi \leq \hat{\pi}$, the equilibrium is without rationing, where $x = \zeta^{-1} - 1$ and $1 < B < \overline{B}$; and if $\pi > \hat{\pi}$, the equilibrium is with rationing, where $x = \zeta^{-1} - 1$ and $B = \overline{B}$.

Social welfare: The supply of the intermediate goods, q, and the amount of the intermediate goods purchased by the bankers, Q, decrease as π increases. It is not clear whether or not the Friedman rule, i.e., setting $\pi = \beta$, maximizes the social welfare in this economy. Note that $\underline{\pi}$ may be way below β so that the steady-state equilibrium at $\pi = \beta$ may be the one without rationing, where $0 < x < \zeta^{-1} - 1$ and $B = 1$. As we show in Appendix B, given that $E[A] > 1$, the social optimal is attained at $q = q^*$, where q^* is determined by

$$c'(q^*) + C'((\zeta^{-1} - 1)q^*) = E[A], \tag{41}$$

because all the intermediate goods should be purchased by the bankers in the first-best allocation. The following lemma holds:

Lemma 3 *If q in the steady-state equilibrium at $\pi = \beta$ is less than q^*, then the social welfare in the steady-state equilibrium at $\pi = \beta$ is higher than at $\forall \pi > \beta$. If q in the steady-state equilibrium at $\pi = \beta$ is larger than q^*, there may exist $\pi^* > \beta$ such that the social welfare of the steady-state equilibrium without rationing is maximized at $\pi = \pi^*$.*

(Proof) Since q and Q are both decreasing in π, the lemma is obvious.

This lemma implies that the social welfare may not be maximized by the Friedman rule. This is because the social welfare may be higher at $\pi > \beta$ than at $\pi = \beta$.

3 Conclusion

In this paper, we construct a monetary model with infinitely lived agents, where the bank runs occur at the low realizations of the aggregate productivity in each period. The probability of the occurrence of bank runs is endogenously determined, which increases as the inflation rate becomes higher. The production of the goods involving the banking sector decreases as the inflation rate increases. The size of the banking sector, which is measured by the size of deposits, x, increases as the inflation rate increases. The high inflation is welfare reducing and the low inflation may be welfare enhancing depending on the functional forms and parameter values. The Friedman rule may not be optimal in this economy.[3]

Appendix A

In this appendix, we specify the steady-state equilibrium at $\pi < \underline{\pi}$. If $\pi < \underline{\pi}$, the steady-state equilibrium without rationing does not exist. Since $x = 0$ at $\pi = \underline{\pi}$, the depositors

[3] Note that the effects of the inflation rate on the banking sector and the production of the goods are based on one crucial assumption, that is, the bank deposit can be made only by depositing cash and it cannot be made by depositing the goods (i.e., the intermediate goods). It is easily confirmed that if the depositors can deposit the intermediate goods they produce with the bankers in our model, then the probability of the bank runs, the amount of the intermediate goods produced, and the size of the banking sector become all irrelevant to the inflation rate. Therefore, in the economy where depositing the real goods with the banks is possible, the bank runs are not a monetary but real problem.

do not deposit their cash with the bankers when $\pi < \underline{\pi}$. Therefore, the bankers no longer constrained by (17) and the banker's problem in this case becomes

$$V(m) = \max U(\tilde{c}) - \tilde{h} + \beta V(\tilde{m}_{+1}) - C(Q), \tag{42}$$

subject to

$$\tilde{c} + \phi \tilde{m}_{+1} = \tilde{A}Q + \tilde{h}, \tag{43}$$

$$pQ \leq m. \tag{44}$$

The reduced form of this problem is

$$V(m) = \max_{\tilde{c}, \tilde{m}_{+1}} \{U(\tilde{c}) - \tilde{c} - \phi \tilde{m}_{+1} + \beta V(\tilde{m}_{+1})\} + \tilde{A}\frac{m}{p} - C(\frac{m}{p}). \tag{45}$$

Since $d = 0$ and $q' = m/p$ in the depositor's problem, the FOC with respect to $\tilde{m}_{+1}$ implies that

$$\frac{\pi}{\beta} = \frac{1 - \zeta + \zeta\{E[A] - C'(m_{+1}/p_{+1})\}}{c'(q)}.$$

In the equilibrium, $q' = Q = m/p$ and $(1-\zeta)q' + \zeta Q = (1-\zeta)q$. Thus $q' = Q = m/p = (1-\zeta)q$. The equilibrium amount of the intermediate good q is determined by

$$\frac{\pi}{\beta} = \frac{1 - \zeta + \zeta\{E[A] - C'((1-\zeta)q)\}}{c'(q)}.$$

This condition determines the value of q, which increases as π decreases. The real balance $z = \phi m$ is determined by $z = \phi p Q = c'(q)(1-\zeta)q$.

Appendix B

The first-best allocation is given as the solution to the following social planner's problem:

$$\max_{c_d, h_d, c_b, h_b, q, q^b} E_A[(1-\zeta)\{U(c_d) - h_d - c(q)\} + \zeta\{U(c_b) - h_b - C(Q)\}]$$

subject to

$$(1-\zeta)c_d + \zeta c_b = \zeta \tilde{A}\frac{1-\zeta}{\zeta}q^b + (1-\zeta)(q - q^b) + \{(1-\zeta)h_b + \zeta h_d\},$$

$$q^b \leq q,$$

where q and q^b are chosen before the realization of $\tilde{A}$, and c_d, c_b, h_d, and h_b are chosen after the realization of $\tilde{A}$. Given that $E[A] > 1$, the constraint $q^b \leq q$ is binding in the first-best allocation. Thus the optimal production of the intermediate good $q = q^b = q^*$ is given by (41).

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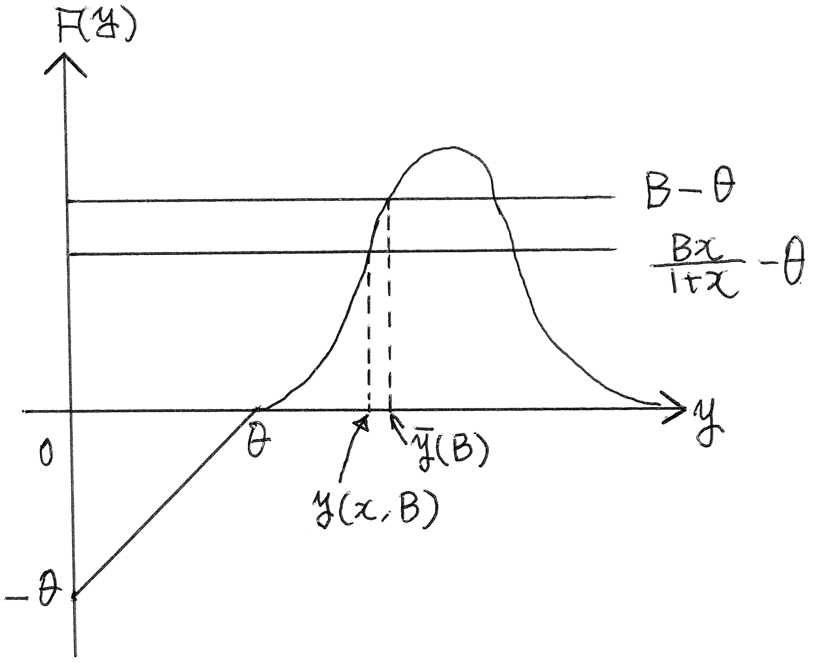
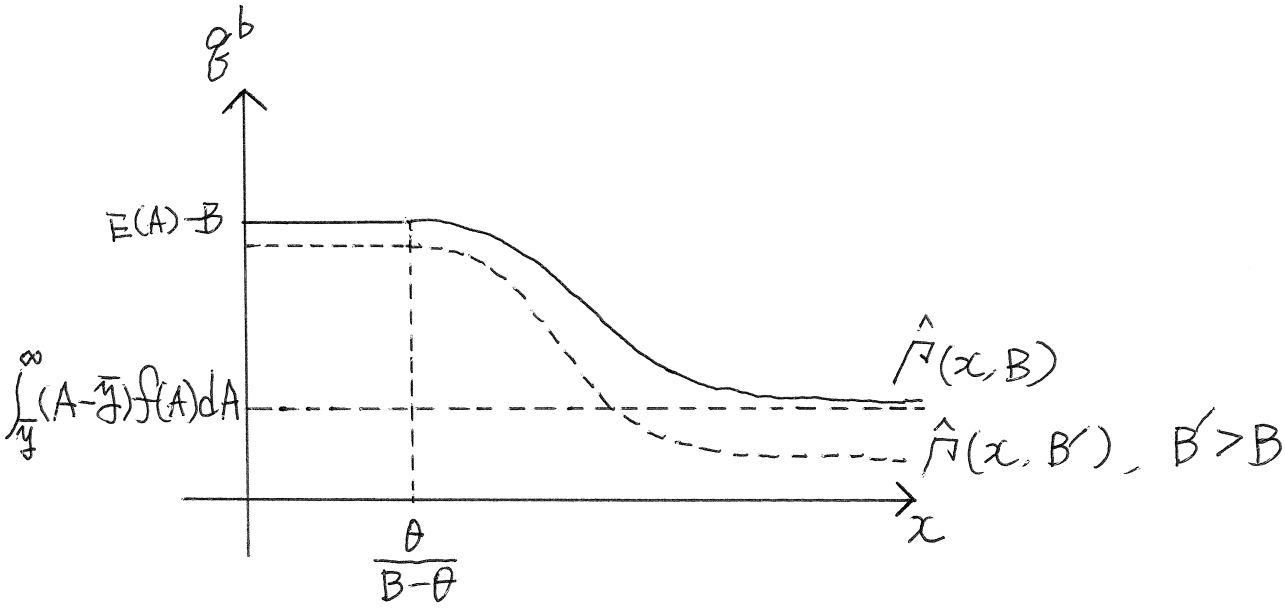
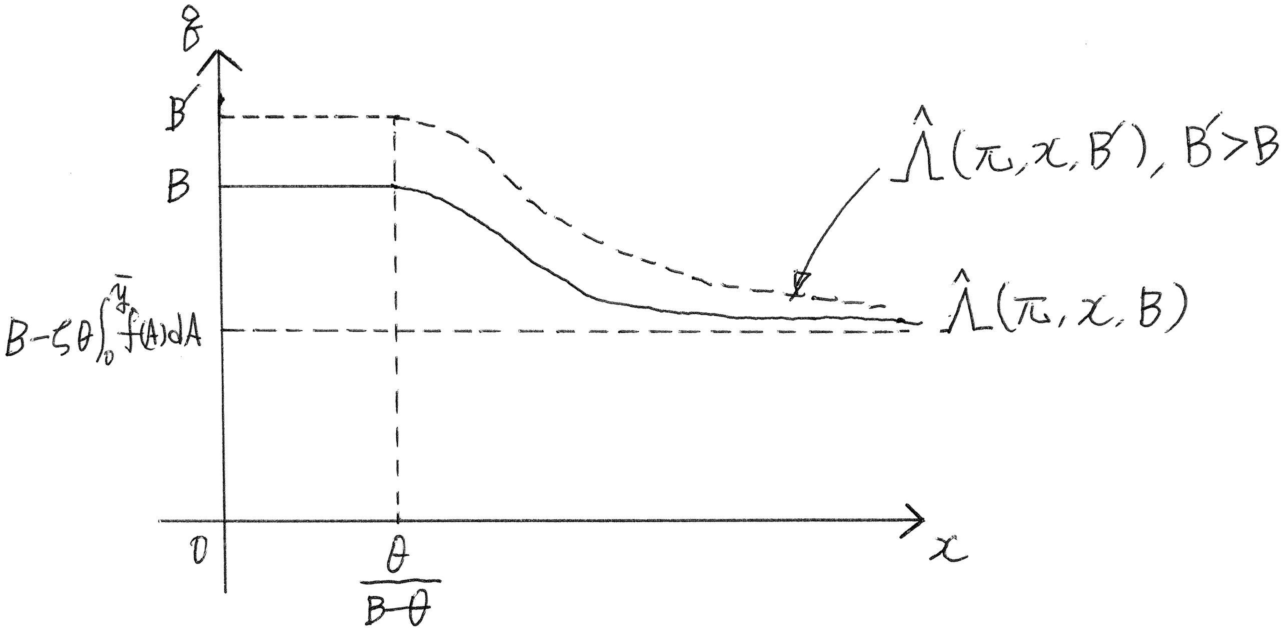


Figure 1.



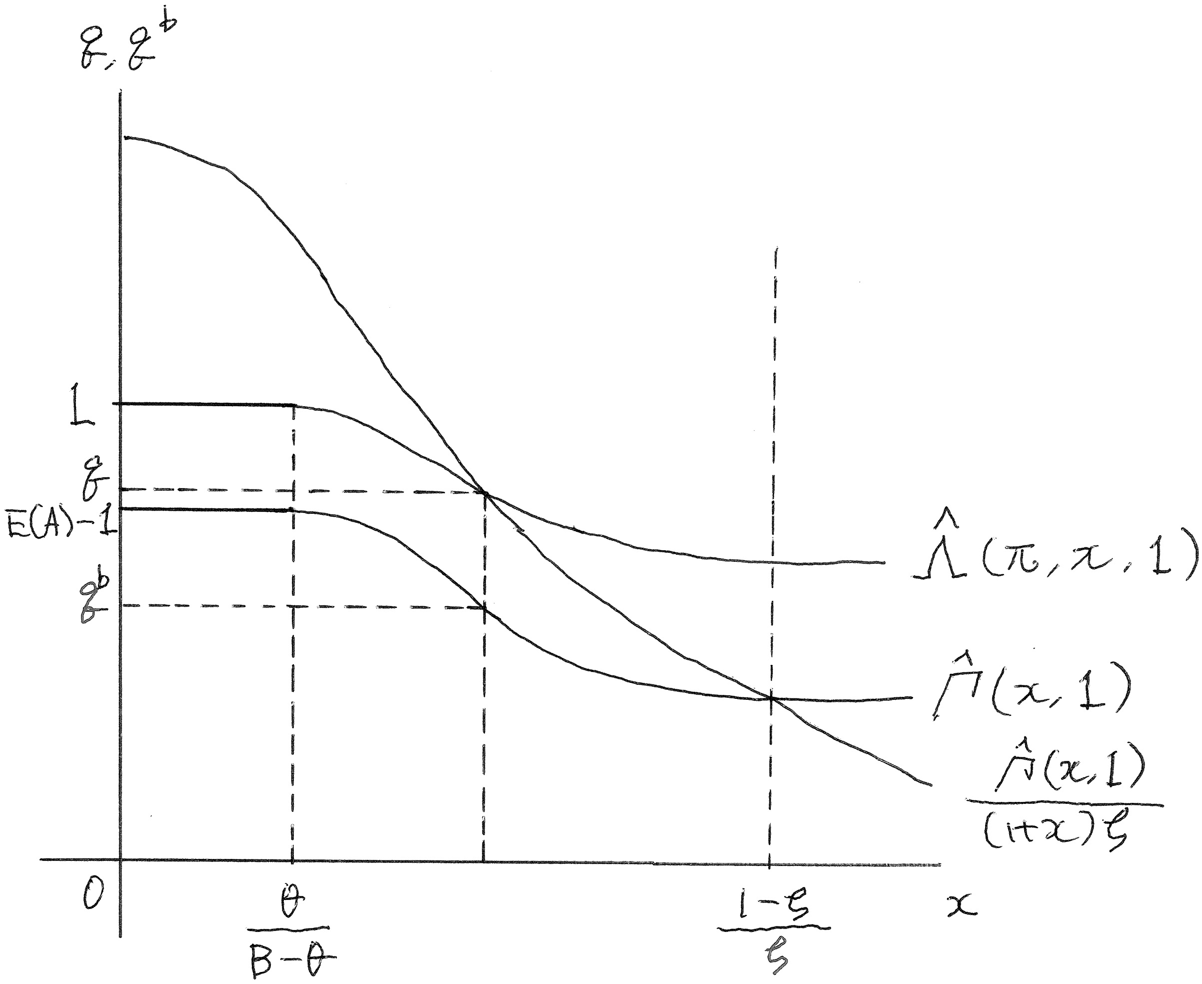
q^b
$E(A)-B$
$\int_{\bar{y}}^{\infty}(A-\bar{y})f(A)dA$
$\hat{r}(x,B)$
$\hat{r}(x,B'),\ B'>B$
x
$\frac{\theta}{B-\theta}$

Figure 2.



q
B'
B
$B-\zeta\theta\int_0^{\bar{y}} f(A)dA$
$\hat{\Lambda}(\pi, x, B'),\ B'>B$
$\hat{\Lambda}(\pi, x, B)$
0
$\frac{\theta}{B-\theta}$
x

Figure 3.



q, q^b
1
q
E(A)-1
q^b
0
$\frac{\theta}{B-\theta}$
$\frac{1-\xi}{\xi}$
x
$\hat{\Lambda}(\pi, x, 1)$
$\hat{\Pi}(x, 1)$
$\frac{\hat{\Pi}(x,1)}{(1+x)\xi}$

Figure 4.