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Low economic growth tends to be seen a decade after financial crises. To explain this fact, we construct general equilibrium models based on a simplified version of Jermann and Quadrini (2012), in which exogenous shocks cause a substantial number of firms to default on their debts. Lenders cannot pre-commit to debt forgiveness, forcing them to allow "debt-ridden" firms, which are defined as firms whose lenders have a unilateral right to liquidate them, to continue. Although debt-ridden firms are under the control of their lenders, their borrowing constraints are tighter than those of normal firms. This implies that the emergence of debt-ridden borrowers may be a cause of the "financial shocks" seen in the recent macroeconomic literature.

Tightened borrowing constraints due to the emergence of debt-ridden firms lower aggregate productivity and may worsen the labor wedge.

Keywords: Borrowing constraint, Working capital, Labor wedge, Productivity

JEL classification: E30, G01

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1 Introduction

The decade after a financial crisis tends to be associated with low economic growth (Reinhart and Rogoff 2009, Reinhart and Reinhart 2010). An example of a decade-long recession after a financial crisis is the 1990s in Japan. The growth rates of the gross domestic product (GDP) and the total factor productivity (TFP) in the 1990s are both lower than in the 1980s. Figure 1 shows the GDP along with the potential capacity, which has an apparent kink at the beginning of the 1990s. Table 1 shows various estimates of TFP growth rate in Japan. Hayashi and Prescott (2002) emphasize that the growth of TFP slowed down in the 1990s in Japan.[1]

In this paper we propose a theoretical model in which the emergence of debt-ridden borrowers lowers the aggregate productivity persistently through tightening of the financial constraints. We construct a general equilibrium model, based on the models by Jermann and Quadrini (2006, 2011), in which an exogenous shock makes substantial number of firms default on their debt. There are two novel features in our model.

The first point is analysis of the decision making over continuation of firms' operation after they default on their debt. We show that the lack of commitment concerning debt forgiveness generates significant inefficiency when a mass default occurs. A lender has three options when the borrower defaults on the debt: to liquidate the firm, to forgive a part of the debt, or to allow the firm continue operation as a "debt-ridden" firm, where we define a debt-ridden firm as a firm whose lender has a unilateral discretion to liquidate it. We show that if the lender cannot precommit to debt forgiveness the bargaining outcome is that the lender accepts the firm to continue as a debt-ridden firm. This outcome is a coordination failure, because the optimal choice for both the lender and the firm would be debt forgiveness, by which the firm owner regains the full control over the firm by paying the lender the maximized value of the firm, which is less than the original debt. But the lack of commitment to debt forgiveness makes this transaction impossible and the lender continues to retain the right to liquidate the firm. The retention of the right to liquidate the firm causes inefficiency and lowers the aggregate productivity.

This is the second point that we show in this paper: We analyze the borrowing constraint for the debt-ridden firms and show that they face tighter borrowing constraint when they borrow working capital loans than normal firms do. This result seems counterintuitive since debt-ridden firms are under the control of their lenders, while the normal firms are not. There are two reasons that we have this counterintuitive result: First, a normal firm loses more when it defaults on its debt than a debt-ridden firm. This fact

[1] There are substantial debate on whether the TFP slowdown in Japan is truly a slowdown of technical progress or just a measurement error (see Kawamoto 2005, Fukao and Miyagawa 2008). Tentative conclusion on this issue in the literature is that there was a slowdown in technical progress in Japan, though it may not be so severe as Hayashi and Prescott originally measure.

allows a normal firm to borrow more than a debt-ridden firm can. The second reason is that the lender's bargaining position is weaker when the borrower is debt-ridden than when it is a normal firm. Note that the retained right to liquidate a debt-ridden firm is valuable for the lender because it enables her to exploit a positive amount of the surplus that the debt-ridden firm generates. When the lender liquidate the debt-ridden firm, she can obtain its liquidation value but she loses the value of the retained right of liquidation. So the lender's threat point is smaller when she bargains over repayment with the debt-ridden firm, while it is larger when she bargains with a normal firm because she can just obtain the liquidation value if the bargaining breaks down. Anticipating her weaker position in the bargaining over repayment, the lender limits the working capital loan to the debt-ridden firm to a smaller amount than the loan to a normal firm. These two effects combine to make the borrowing constraint tighter for debt-ridden firms than for normal firms.

The tighter borrowing constraint for working capital loans of debt-ridden firms makes their production activity inefficient. If substantial number of firms become debt-ridden, both the aggregate borrowing capacity and the aggregate productivity decline. This implies that emergence of debt-ridden borrowers may be a cause of the "financial shocks" in the recent macroeconomic literature. After the great recession in 2007–2009, many researchers argue that a shock in the financial sector can cause a severe recession (e.g., a risk shock in Christiano, Motto, and Rostagno 2009, and a financial shock in Jermann and Quadrini 2011). In our model, emergence of debt-ridden firms manifest itself as a tightening of aggregate borrowing constraint, which can be interpreted as a financial shock.

We show that tightened borrowing constraints due to emergence of debt-ridden firms may also increase the labor wedge, which is the gap between the marginal product of labor and the marginal rate of substitution between consumption and leisure (Chari, Kehoe and McGrattan 2007, Shimer 2009). The increase in the labor wedge is typically observed in persistent recessions after financial crises, in particular the US Great Depression (Mulligan 2002, Chari, Kehoe and McGrattan 2007) and Japan's lost decade (Kobayashi and Inaba 2006).

Our theory is apparently related to the literature on debt overhang, pioneered by Myers 1977 in the corporate finance literature and applied to macroeconomics by Lamont (1995). See also Krugman (1988) and Philippon (2010), for example. Debt overhang problem is typically that a firm cannot borrow new money if it has too large amount of existing debt. This inefficiency arises when the existing debt holder is different from the potential lender who would lend new money. In this paper we take a small step forward by proposing a new theory that an inefficiency can arise even if the lender of new money *is* the existing debt holder.

This paper is organized as follows. Section 2 proposes a basic model in which there is no collateral asset. Section 3 concludes.

2 The Model

In this section we consider a model in which firms are monopolistic competitors and they produce goods from labor input. In our model, when a firm defaults on the debt, the lender can choose whether to liquidate the firm or to allow it to continue operation as a "debt-ridden firm." In this paper a debt-ridden firm is a firm whose lender has a unilateral discretion to liquidate the firm. Later we clarify the difference between normal firms and debt-ridden firms by formally defining optimization problems they solve, which are (2) and (7), respectively.

In Section 2.1 we describe the basic model, which is based on a simplified version of Jermann and Quadrini (2006). In Section 2.2, we consider emergence of debt-ridden firms in the basic model. In Section 2.3, we characterize the equilibrium and inefficiency due to emergence of debt-ridden firms.

2.1 Basic setup without debt-ridden borrowers

The economy is a closed economy with discrete time, in which a representative household and monopolistic firms live. There is an unit mass of monopolistic firms indexed by $i \in [0,1]$, which produce the intermediate goods. A representative household owns these firms and solve the following program:

$$\begin{aligned} \max \quad & E_0 \left[\sum_{t=0}^{\infty} \beta^t \left\{ \ln C_t + \gamma \ln(1-L_t) \right\} \right], \\ \text{subject to} \quad & C_t + \frac{b_{t+1}}{1+r_t} \leq w_t L_t + \int_0^1 \pi_{it} di + b_t, \end{aligned}$$

where β is the subjective discount factor, C_t is consumption, L_t is labor, w_t is wage rate, r_t is the market interest rate, b_t is bond issued by the firms, and π_{it} is the profit from firm i, where $i \in [0,1]$. The bond is risk free.[2] The first-order conditions (FOCs) imply that

$$\begin{aligned} w_t &= \frac{\gamma C_t}{1-L_t}, \\ \frac{1}{1+r_t} &= \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \right], \end{aligned}$$

where $\lambda_t = C_t^{-1}$ is the Lagrange multiplier for the budget constraint.

[2] In Section 2.3 we consider the case that substantial fraction of monopolistic firms default on bonds. We can formulate the bond b_t as a risky debt with positive default probability but it makes the analysis unnecessarily complicated for exposition of our point. In this paper we simply assume that the default is a measure-zero event. See Section 2.3.

The final good, Y_t, is produced competitively from the intermediate goods x_{it}, $i \in [0,1]$, by the following production function:

$$Y_t = \left(\int_0^1 x_{it}^{\eta} di \right)^{\frac{1}{\eta}},$$

where $0 < \eta < 1$. The final good producer maximizes $Y_t - \int_0^1 p_{it} x_{it} di$, where p_{it} is the real price of the intermediate good i. Perfect competition in the final good market implies that

$$p_{it} = p(x_{it}) = Y_t^{1-\eta} x_{it}^{\eta-1}.$$

Firm i produces the intermediate good i in the monopolistically competitive market by the following production function:

$$x_{it} = A_{it} l_{it},$$

where A_{it} is the productivity parameter and l_{it} is the labor input for firm i. We assume

$$A_{it} = A, \text{ for all } i \text{ and } t.$$

Firm i needs to pay wages $w_t l_{it}$ before production and, therefore, firm i needs to borrow working capital $w_t l_{it}$ from the representative household. Firm i borrows both intertemporal debt b_{it} and intratemporal debt (working capital). Timing of actions of firm i is as follows. At the end of period $t-1$, firm i borrows $\frac{b_{it}}{1+r_{t-1}}$. At the beginning of period t a shock to b_{it} is revealed, if any, and then the firm borrows working capital $w_t l_{it}$, pays wage to the worker, and produce x_{it}. After selling x_{it} it repays debt $w_t l_{it} + b_{it}$. Note that there is no stochastic shock during the short span when the firm borrows the working capital. So the interest rate for the intraperiod working capital is zero.

Now we specify the borrowing constraint for firm i. In what follows we omit the subscript i unless there is a risk of confusion. The firm's debt is subject to the borrowing constraint:

$$w_t l_t + b_t \leq \phi p(x_t) x_t + V_{nt} - V_{zt},$$

where V_{nt} is the value of a normal firm and V_{zt} is the value of a debt-ridden firm, while later we will specify V_{nt} by (2) and V_{zt} by (7). The reason why the debt is subject to this constraint is clarified in what follows. We assume the following assumption on the commitment ability of the lender:

Assumption 1. In period t, firm i can default on the repayment of $w_t l_t + b_t$ after the firm obtains the proceeds $p(x_t)x_t$. If firm i defaults then the firm and the lender can renegotiate on the amount of repayment f_t.

1. **The lack of commitment to debt forgiveness:** Once firm i defaults, the lender obtains the unilateral discretion to liquidate it. The legal institution in this economy is such that as long as the repayment f_t is strictly less than the original debt $(w_t l_t + b_t)$ the lender can hold the right to liquidate firm i. And the lender can hold this right without any penalty even after she receives f_t $(< w_t l_t + b_t)$ from firm i, implying that the lender cannot credibly commit to sell the right to liquidate firm i at any price cheaper than the original debt.

2. If the lender liquidates the firm the lender obtains a part of the proceeds of sales $\phi p(x_t) x_t$ and a part of the firm value ψV_{nt}, where $0 < \phi < 1$ and $0 < \psi < 1$.[3] When firm i is liquidated, the intermediate good i continues to be produced by a new firm from the next period on with probability ψ and the good i disappears with probability $1 - \psi$.

3. If the lender and the firm agree on f_t, which is equal to or greater than the original debt, then the firm redeems the right to choose whether or not to continue its own business. Both the lender and the firm can verify that the lender loses the right to liquidate the firm at the moment she receives f_t.

4. If the lender and the firm agree on f_t, which is strictly less than the original debt, then the lender retains the right to liquidate the firm. In this case at the end of period t, the lender and the firm negotiate over the amount d_{t+1}, which the firm must repay in period $t + 1$ (in addition to the working capital $w_{t+1} l_{t+1}$).

   (a) If they agree on d_{t+1} the lender allows the firm to continue operating in the next period.

   (b) If they do not agree on d_{t+1}, the lender liquidates the firm. Since the bargaining over d_{t+1} takes place at the end of period t when all the output in period t has been consumed, the lender can confiscate only ψV_{nt} by liquidating the firm at this stage.

   (c) Suppose that they agree on d_{t+1} and firm i continues operating from period $t + 1$ on. Unpaid debt of firm i evolves to $B_{t+j+1} = (1 + r_{t+j})\{w_{t+j} l_{t+j} + B_{t+j} - f_{t+j}\}$ in period $t + j + 1$ for $j \geq 0$ with $B_t = b_t$. Suppose that the firm borrows the working capital $w_{t+j} l_{t+j}$ and pays f_{t+j} in period $t + j$. As long as $B_{t+j} + w_{t+j} l_{t+j} > f_{t+j}$ the lender retains the right to liquidate firm i. And the lender can hold this right without any penalty even after she receives f_{t+j} $(< w_{t+j} l_{t+j} + B_{t+j})$ from firm i, implying that the lender cannot credibly

[3] We can interpret that the lender confiscates the blueprint of the intermediate good i from firm i and can sell the blueprint to a new firm with probability ψ at the price of V_{nt}.

commit to sell the right to liquidate firm i at any price cheaper than the unpaid debt.

This assumption implies that if firm i defaults on $w_t l_t + b_t$, the lender and the firm renegotiate over the repayment f_t, and the firm is liquidated if they do not reach an agreement. If firm i is liquidated it receives $(1-\phi)p(x_t)x_t$ and the lender receives $\phi p(x_t)x_t + \psi V_{nt}$. If they agree on f_t $(< w_t l_t + b_t)$, firm i continues operation while the lender retains the right to liquidate it. We call firm i under this situation a "debt-ridden firm." If firm i becomes a debt-ridden firm by paying f_t, it obtains $p(x_t)x_t - f_t + V_{zt}$ and the lender obtains $f_t + D_t$, where D_t is the present value of the expected cashflow which the lender can receive from a debt-ridden firm from period $t+1$ on, and V_{zt} and D_t are specified in Section 2.2. The bargaining over f_t after firm i defaults on $w_t l_t + b_t$ is described as a Nash bargaining:

$$[\phi p(x_t)x_t + V_{zt} - f_t]^{\sigma}[f_t + D_t - \phi p(x_t)x_t - \psi V_{nt}]^{1-\sigma}.$$

Throughout this paper, we assume that the firm has all the bargaining power for simplicity of analysis, i.e., $\sigma = 1$. Thus the bargaining outcome is $f_t = \phi p(x_t)x_t + \psi V_{nt} - D_t$. In this section we just assume that $D_t = \psi V_{nt}$ and $V_{zt} > 0$, which will be verified in Section 2.2. Therefore, if firm i defaults on $w_t l_t + b_t$, the lender and the firm will agree on the payment $f_t = \phi p(x_t)x_t$ and firm i continues as a debt-ridden firm as a result of the bargaining.

Now we specify the condition for firm i not to default on $w_t l_t + b_t$. After receiving the proceeds, $p(x_t)x_t$, if firm i does not default, it obtains $p(x_t)x_t - w_t l_t - b_t + V_{nt}$. On the other hand, if it defaults, the firm and the lender bargains over repayment f_t, which leads to the agreement $f_t = \phi p(x_t)x_t$, and firm i obtains $(1-\phi)p(x_t)x_t + V_{zt}$. (Note that the firm's value is V_{zt} because the lender allows it to continue as a debt-ridden firm if they reach an agreement in the bargaining.) The no default condition for firm i is $p(x_t)x_t - w_t l_t - b_t + V_{nt} \geq (1-\phi)p(x_t)x_t + V_{zt}$, which can be rewritten as

$$w_t l_t + b_t \leq \phi p(x_t)x_t + V_{nt} - V_{zt}.$$

To exclude the equilibrium in which all firms intentionally defaults after borrowing too much b_t, we assume that the values of the parameters are chosen such that

$$\forall t, \quad V_{nt} - V_{zt} \geq \psi V_{nt}, \tag{1}$$

in the equilibrium path.[4] Given the borrowing constraint above, firm i maximizes its own

[4] Note that the lender is willing to lend as long as $w_t l_t + b_t \leq \phi p(x_t)x_t + \psi V_{nt}$, because she can obtain $\phi p(x_t)x_t + \psi V_{nt}$ by liquidating the firm if it defaults on the debt. So if $V_{nt} - V_{zt} < \psi V_{nt}$, all firms set the highest possible b_t at the end of period $t-1$ such that they choose optimal l_t and $\phi p(x_t)x_t + V_{nt} - V_{zt} < w_t l_t + b_t \leq \phi p(x_t)x_t + \psi V_{nt}$, and all of them intentionally default in period t. In this case, all firms become debt-ridden firms from period t on. This equilibrium path is self-consistent but does not seem to be relevant to the reality. Condition (1) excludes the possibility of emergence of this "all debt-ridden" equilibrium.

value V_{nt}, which is defined by the following Bellman equation:

$$V_{nt} = \max_b \frac{b}{1+r_t} + E_t \left[\max_l \beta \frac{\lambda_{t+1}}{\lambda_t} \{p(x)x - w_{t+1}l - b + V_{nt+1}\} \right], \tag{2}$$

$$\text{subject to} \quad x = A_{t+1}l, \tag{3}$$

$$w_{t+1}l \leq \max\{0, \quad \phi p(x)x + V_{nt+1} - V_{zt+1} - b\}, \tag{4}$$

where we assume that $\max\{0, \quad \phi p(x)x + V_{nt+1} - V_{zt+1} - b\} = \phi p(x)x + V_{nt+1} - V_{zt+1} - b$ with probability one. The resource constraints are

$$C_t = Y_t,$$
$$L_t = \int_0^1 l_{it} di.$$

We assume that in equilibrium path the borrowing constraint (4) is not binding, i.e., $w_{t+1}l^*_{t+1} \leq \phi Y^{1-\eta}_{t+1}(A_{t+1}l^*_{t+1})^\eta + V_{nt+1} - V_{zt+1}$, where $l^*_{t+1} = \arg\max_l Y^{1-\eta}_{t+1}(A_{t+1}l)^\eta - w_{t+1}l$. In this case, b_{t+1} is indeterminate. But we assume that there is infinitesimally small tax benefit for issuing intertemporal debt such that firms are willing to borrow intertemporal debt up to the borrowing limit. In the case of a deterministic equilibrium, the amount of b_{t+1} is determined by $b_{t+1} = \phi Y^{1-\eta}_{t+1}(A_{t+1}l^*_{t+1})^\eta - w_{t+1}l^*_{t+1} + V_{nt+1} - V_{zt+1}$. Note that V_{nt} does not depend on b_{t+1}, because the tax benefit is infinitesimal, and the loan rate and the market rate are equal and satisfy

$$\frac{b_{t+1}}{1+r_t} = \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \right] b_{t+1}.$$

2.2 Emergence of debt-ridden borrowers

Now we consider a measure-zero event: Some idiosyncratic shock on firm i makes its intertemporal debt b_t so large that

$$b_t > \max_l \{\phi p(A_t l)A_t l - w_t l\} + V_{nt} - V_{zt}$$

in period t.

If it occurs in period t, the firm cannot obtain working capital and thus cannot produce anything. This is because $w_t l_t$ is constrained by $\max\{0, \ \phi p(x_t)x_t - b_t + V_{nt} - V_{zt}\}$. Thus $w_t l_t = 0$ if $b_t > \max_l\{\phi p(A_t l)A_t l - w_t l\} + V_{nt} - V_{zt}$. Then firm i chooses whether to default on b_t at the end of period t. If it pays b_t, it obtains $V_{nt} - b_t$, while it obtains V_{zt} if it defaults on b_t. Since $b_t > V_{nt} - V_{zt}$, the firm chooses to default and become a debt-ridden firm.

Note that Assumption 1-1 (The lack of commitment on debt relief) is crucial in the above argument. If the lender can commit to debt forgiveness, she can sell the right to liquidate firm i to the firm at the price of $V_{nt} - V_{zt}$ and forgive the remaining debt

$b_t - V_{nt} + V_{zt}$, while she can get only D_t if she makes firm i a debt-ridden firm. Since $V_{nt} - V_{zt} \geq \psi V_{nt}$ and $D_t = \psi V_{nt}$ as we show later, the optimal choice for the lender would be to release firm i in exchange for $V_{nt} - V_{zt}$. But this optimal action is not feasible because of Assumption 1-1.

Assumption 1-1 implies that the lender cannot offer any debt forgiveness to firm i because she cannot credibly commit to debt forgiveness. Thus firm i cannot redeem the discretion to liquidate its own business by paying any amount less than b_t. Since the amount that the firm is willing to pay is $V_{nt} - V_{zt}$ ($< b_t$), the lender has no choice but to make firm i a debt-ridden firm.

Now, we consider how D_t is determined. When the lender decides whether to liquidate the defaulted borrower or to permit it to continue operation, the lender and firm i negotiate over d_{t+1}, the amount to be repaid at the end of period $t+1$ in addition to the working capital. If they agree on d_{t+1}, firm i continues operation in period $t+1$ and otherwise the lender liquidates firm i. We assume the following concerning commitment ability of the lender:

Assumption 2. If the debt-ridden firm purchases risk-free bonds s_{t+1} as its financial asset in period $t+1$, the lender confiscates s_{t+1} at the end of period $t+1$. Namely, the lender cannot commit not to confiscate the financial asset of the firm.

This assumption implies that the debt-ridden firms cannot make savings. We simply assumed away the possibility of savings by debt-ridden firms in this assumption, while in Appendix A we consider the case where the lender can allow the debt-ridden firm to accumulate the financial asset s_t as savings. It is shown in Appendix A that in a deterministic equilibrium the savings s_t by the debt-ridden firm has no effect on the equilibrium path and the firm has no incentive to accumulate s_t.

Denote the value of the debt-ridden firm by V_{zt}. If firm i and the lender agree on d_{t+1}, firm i obtains V_{zt}, which depends on d_{t+1}, and the lender obtains $\beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \{d_{t+1} + D_{t+1}\} \right]$. Note that the lender and firm i take D_{t+1} as given at period t because D_{t+1} is determined by the bargaining at the end of period $t+1$ and they have no ability to precommit to the outcome of the future bargaining. If firm i and the lender does not agree on d_{t+1}, then firm i obtains zero and the lender obtains ψV_{nt} by liquidating firm i. This is due to Assumption 1-4 (b). The Nash bargaining between a debt-ridden firm and the lender is therefore,

$$\max_{d_{t+1}} [V_{zt}(d_{t+1})]^{\sigma} \left[\beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \{d_{t+1} + D_{t+1}\} \right] - \psi V_{nt} \right]^{1-\sigma},$$

with $\sigma = 1$, which implies that the bargaining outcome is

$$\beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \{d_{t+1} + D_{t+1}\} \right] = \psi V_{nt}. \tag{5}$$

Since the definition of D_{t+1} implies that $D_{t+1} = \beta E_{t+1}\left[\frac{\lambda_{t+2}}{\lambda_{t+1}}\{d_{t+2}+D_{t+2}\}\right] = \psi V_{nt+1}$. Assuming that d_{t+1} is repaid with probability 1,

$$d_{t+1} = \frac{\psi V_{nt}}{\beta E_t\left[\frac{\lambda_{t+1}}{\lambda_t}\right]} - \frac{E_t\left[\frac{\lambda_{t+1}}{\lambda_t}\psi V_{nt+1}\right]}{E_t\left[\frac{\lambda_{t+1}}{\lambda_t}\right]}. \tag{6}$$

After d_{t+1} is agreed at the end of period t, firm i (the debt-ridden firm) operates in period $t+1$. Actions of the debt-ridden firm are as follows. At the beginning of period $t+1$, it borrows working capital $w_{t+1}l_{zt+1}$, where l_{zt+1} is the labor input for the debt-ridden firm. It produces the intermediate good $A_{t+1}l_{zt+1}$ and sells it in the monopolistically competitive market. After it receives the proceeds of the sales, it repays the debt $w_{t+1}l_{zt+1}+d_{t+1}$. The debt-ridden firm cannot precommit to repayment of the debt $(w_{t+1}l_{zt+1}+d_{t+1})$ beforehand and can default on the debt after production.

When the firm defaults, the firm and the lender (household) renegotiate over repayment f. If the debt-ridden firm and the lender reach an agreement, the debt-ridden firm obtains $p(x_{t+1})x_{t+1} + V_{zt+1} - f$ and the lender obtains $f + D_{t+1}$. If there is no agreement, the debt-ridden firm obtains $(1-\phi)p(x_{t+1})x_{t+1}$ and exits the market, and the lender obtains $\phi p(x_{t+1})x_{t+1} + \psi V_{nt+1}$ by liquidating firm i. As the bargaining power of the debt-ridden firm is 1 and that of the lender is zero, we have

$$f = \phi p(x_{t+1})x_{t+1} + \psi V_{nt+1} - D_{t+1}.$$

The no renegotiation condition implies that $w_{t+1}l_{zt+1} + d_{t+1} \le f$. Therefore,

$$w_{t+1}l_{zt+1} + d_{t+1} + D_{t+1} \le \phi p(x_{t+1})x_{t+1} + \psi V_{nt+1}.$$

Since $D_{t+1} = \psi V_{nt+1}$, this condition is rewritten as

$$w_{t+1}l_{zt+1} + d_{t+1} \le \phi p(x_{t+1})x_{t+1}.$$

which is the borrowing constraint for the debt-ridden firm. The debt-ridden firm maximizes its own value V_{zt}, which is defined by the following Bellman equation.[5] Given d_{t+1}

[5]The Bellman equation (7) implies that given d_{t+1} the debt-ridden firm (firm i) can freely choose l_{it+1} and x_{it+1} under the borrowing constraint (9). There may be another way of modeling the relationship between a debt-ridden firm and the lender. For example, we can assume that there is no negotiation over d_{t+1} at the end of period t and the lender just allows firm i to continue, and the lender directly decides the amount of l_{it+1} and x_{it+1} by setting the amount of the working capital loan to the firm. In this setting, l_{t+1}, d_{t+1} and D_{t+1} are determind by

$$\begin{aligned}
l_{t+1} &= \arg\max_{l}\{\phi p(A_{t+1}l)A_{t+1}l - w_{t+1}l\},\\
d_{t+1} &= \phi p(A_{t+1}l_{t+1})A_{t+1}l_{t+1} - w_{t+1}l_{t+1} + \psi V_{nt+1} - D_{t+1},\\
D_{t+1} &= \beta E_{t+1}\left[\frac{\lambda_{t+2}}{\lambda_{t+1}}\{d_{t+2}+D_{t+2}\}\right].
\end{aligned}$$

and D_{t+1}, the debt-ridden firm solves

$$V_{zt} = E_t \left[\max_{l_{t+1}} \beta \frac{\lambda_{t+1}}{\lambda_t} \{p(x)x - w_{t+1}l_{t+1} - d_{t+1} + V_{zt+1}\} \right], \tag{7}$$

$$\text{subject to} \quad x = A_{t+1}l_{t+1}, \tag{8}$$

$$w_{t+1}l_{t+1} + d_{t+1} \le \phi p(x)x. \tag{9}$$

Note that Assumption 2 that prohibits savings by the debt-ridden firms seems to be crucial in deriving a persistent inefficiency due to the binding borrowing constraint (9). If the firm can accumulate financial assets, it could relaxed the borrowing constraint eventually. But we show in Appendix A that at least in the deterministic equilibrium the borrowing constraint is identical to (9) even if the debt-ridden firm can make savings in the form of risk-free bonds, under the assumption that the lender can confiscate the bonds when she liquidates the firm. Thus there is no incentive for the debt-ridden firm to make savings in the deterministic case.

2.3 Equilibrium with debt-ridden borrowers

We consider that at the beginning of period 0, firms indexed by $i \in [0, Z]$, where $0 < Z < 1$, default on their debt, while firms indexed by $i \in [Z, 1]$ do not default. The mass default is a measure-zero event. In this case the intertemporal borrowing b_{i0} of firm $i \in [0, Z]$ satisfies

$$b_{i0} > \max_l \{p(A_{i0}l)A_{i0}l - w_0 l\} + V_{n0} - V_{z0}. \tag{10}$$

Firm $i \in [0, Z]$ cannot obtain working capital and cannot produce anything in period 0. At the end of period 0, firms and their lenders negotiate on the amount of d_1. In the equilibrium all firm $i \in [0, Z]$ continue operation as debt-ridden firms. In this section we consider the deterministic equilibrium where $A_{it} = A$ for all $i \in [0, 1]$ and all $t \ge 1$. Since there is no state variable that changes over time, the equilibrium is a steady state.

We assume parameter values are such that the borrowing constraints for normal firms $i \in [Z, 1]$ does not bind and those for debt-ridden firms $i \in [0, Z]$ binds. Thus the labor input, l_n, for a normal firm is

$$l_n = \left(\frac{\eta A^\eta}{w} \right)^{\frac{1}{1-\eta}} Y. \tag{11}$$

The equilibrium outcome in the alternative setting is qualitatively the same as in the text: It is easily shown that the debt-ridden firms are inefficient and the aggregate productivity declines and the labor wedge worsens as the measure of debt-ridden firms increases. But we do not adopt this setting here because it is not realistic to assume that the lender directly sets the labor input of the borrowing firm. There are various information asymmetry and agency problems that prevent the lenders from directly setting the firms' labor input. If the lender could set the labor input directly, there would have been no reason that prevents the lenders from directly operating the firms rather than lending working capital to them.

The value of a normal firm is

$$V_n = (1-\eta)\frac{\beta}{1-\beta}\left(\frac{\eta A}{w}\right)^{\frac{\eta}{1-\eta}} Y. \tag{12}$$

Since $D = \beta\{d + D\} = \psi V_n$, the repayment d is

$$d = (1-\eta)\psi\left(\frac{\eta A}{w}\right)^{\frac{\eta}{1-\eta}} Y. \tag{13}$$

Since the borrowing constraint for the debt-ridden firms is binding, the labor input for the debt-ridden firms, l_z, satisfies

$$wl_z = \phi Y^{1-\eta} A^\eta l_z^\eta - d. \tag{14}$$

The production function of the final good implies

$$Y = A\{(1-Z)l_n^\eta + Zl_z^\eta\}^{\frac{1}{\eta}}. \tag{15}$$

The FOC for the household problem and $C = Y$ imply

$$w = \frac{\gamma Y}{1-(1-Z)l_n - Zl_z}. \tag{16}$$

Finally the value of the debt-ridden firm V_z must satisfy

$$V_z = (1-\phi)\frac{\beta}{1-\beta} Y^{1-\eta} A^\eta l_z^\eta. \tag{17}$$

The steady state equilibrium is pinned down by the system of equations (11)–(17). Condition (1), which must be satisfied in the equilibrium, can be written as $(1-\psi)V_n > V_z$ and we will check this condition numerically. Solution to (11)–(17) is given in the Appendix B. Figure 2 shows the variables as functions of Z. $A_z = \frac{Y}{(1-Z)l_n + Zl_z}$ is the aggregate productivity in this economy. The labor wedge, τ, is defined by $1 + \tau = A_z/w$. For moderate values of Z, the productivity declines and the labor wedge worsens as Z increases. This figure shows that an increase of the debt-ridden firms lowers the aggregate productivity and worsens the labor wedge.

These negative macroeconomic effects are apparently driven by tightening of the borrowing constraint for the debt-ridden firms. Note that these negative effect must be amplified by the aggregate demand externality, which is the fact that the aggregate demand Y_t directly enters the income of the monopoly firm: $p(x_t)x_t = Y_t^{1-\eta} x_t^\eta$.

3 Conclusion

In this paper we show that if firms continue operation after defaulting on their debt they must be subject to tighter borrowing constraint for working capital finance. The tighter borrowing constraint makes the aggregate TFP lower when the mass default occurs as a result of a financial crisis. It is also shown that the deterioration of the labor wedge can be replicated in our model, while the deterioration of the labor wedge is widely observed in the financial crisis episodes.

Key assumption in our model is that the lender cannot credibly commit to debt forgiveness for the defaulted borrowers. Although this assumption may be too artificial for the US economy, it seems quite natural as a model of the Japanese institutional environment in the 1990s. I suspect that this assumption may be relevant to the European economy during and after the Great Recession, too.

We can derive simple policy implications from this model: Since the persistent productivity declines and the labor-wedge deterioration are due to continuation of debt-ridden firms after defaulting on their debt, the government can restore the normalcy of the economy by facilitating debt forgiveness. Institutional reforms that enable the lenders (i.e., financial institutions) to commit to debt forgiveness is an effective policy measure. For example,

- Debtor friendly reform of the bankruptcy procedures,

- Provision of a credible debt reduction procedures outside of the court,

- Implementation of a government-coordinated debt reduction program.

Subsidy to the debt-ridden firms that enables them to repay all debt arrears is also effective policy, if the government has sufficient fiscal resources. Forcing the lenders (i.e., financial institutions) to liquidate the debt-ridden firms may also restore the normalcy by inducing new R&D and entry of new firms. But liquidation is not socially optimal policy because the technology of existing firm will be lost and entry of new firms are socially costly.

Appendix A: Neutrality of corporate savings

In the text, firms cannot make savings (Assumption 2). Here we consider the case where the debt-ridden firms can accumulate financial assets. First, we eliminate Assumption 2, and we assume that the lender *can* commit not to confiscate the financial asset of the debt-ridden firm. Instead we assume that if the lender liquidates the firm at the end of period t, she obtains $\psi V_{nt} + s_t$, where s_t is the financial asset (risk-free bond), which the firm buys in period t. Suppose that the debt-ridden firm chooses a positive amount of savings, $s_t > 0$. The bargaining over d_{t+1} at the end of period t leads to $\beta E_t\left[\frac{\lambda_{t+1}}{\lambda_t}\{d_{t+1}+D_{t+1}\}\right] = \psi V_{nt} + s_t$. The risk-free rate r_t is defined by $(1+r_t)^{-1} = \beta E_t\left[\frac{\lambda_{t+1}}{\lambda_t}\right]$. Thus we have

$$d_{t+1} = (1+r_t)(\psi V_{nt} + s_t) - (1+r_t)\beta E_t\left[\frac{\lambda_{t+1}}{\lambda_t}D_{t+1}\right]. \tag{18}$$

At the beginning of period $t+1$, the firm obtains $(1+r_t)s_t$ from its asset and it borrows $w_{t+1}l_{t+1} - (1+r_t)s_t$ as the working capital. After production, the firm has to repay $w_{t+1}l_{t+1} + d_{t+1} - (1+r_t)s_t$. The firm and the lender renegotiate over the repayment f. Note that at this stage of bargaining the firm does not own financial asset s_{t+1} yet. It buys s_{t+1} after the repayment is done. Now we consider the bargaining. If they agree on f, the lender obtains $f + D_{t+1}$, while she obtains $\phi p(x_{t+1})x_{t+1} + \psi V_{nt+1}$ if they do not agree on f. Since the bargaining power of the firm is one, the bargaining outcome is $f = \phi p(x_{t+1})x_{t+1} + \psi V_{nt+1} - D_{t+1}$, which implies the borrowing constraint for the working capital:

$$w_{t+1}l_{t+1} + d_{t+1} - (1+r_t)s_t + D_{t+1} \leq \phi p(x_{t+1})x_{t+1} + \psi V_{nt+1}, \tag{19}$$

Equations (18) and (19) imply that

$$w_{t+1}l_{t+1} + (1+r_t)\psi V_{nt} - \psi V_{nt+1} + D_{t+1} - (1+r_t)\beta E_t\left[\frac{\lambda_{t+1}}{\lambda_t}D_{t+1}\right] \leq \phi p(x_{t+1})x_{t+1}. \tag{20}$$

This borrowing constraint is close to but not equal to (9) because D_{t+1} may depend on s_{t+1}. But in the deterministic case where $(1+r_t)^{-1} = \beta\frac{\lambda_{t+1}}{\lambda_t}$ and $(1+r_t)\beta E_t\left[\frac{\lambda_{t+1}}{\lambda_t}D_{t+1}\right] = D_{t+1}$, the borrowing constraint (20) reduces to (9), because d_{t+1} in (9) is rewritten as $(1+r_t)\psi V_{nt} - \psi V_{nt+1}$. Thus we have shown that in the deterministic equilibrium, the corporate savings s_t does not affect the equilibrium prices and allocations, and the debt-ridden firm has no incentive to accumulate s_t.

Appendix B : Steady State of the Model

We define $e = l_z/l_n$. Equations (11) and (17) implies

$$\frac{\gamma l_n}{1 - Zel_n - (1-Z)l_n} = \frac{\eta}{1 - Z + Ze^{\eta}}. \tag{21}$$

This equation and (14), along with $Y = Al_n\{1 - Z + Ze^{\eta}\}^{\frac{1}{\eta}}$ and $w = \frac{\gamma Y}{1-Zel_n-(1-Z)l_n}$, imply that

$$\eta e = \phi e^{\eta} - (1-\eta)\psi. \tag{22}$$

The value of e is specified by this equation.[6] Once e is pinned down, the labor l_n and $l_z = el_n$ are specified by :

$$l_n = \left[1 - (1-e)Z + \frac{\gamma}{\eta}\{1 - (1-e^{\eta})Z\}\right]^{-1}. \tag{23}$$

Other variables are also pinned down by equations (11) – (17).

[6]Equation (22) has at most two solutions. The value of e is the larger solution. It is shown by contradiction. Suppose that e is the smaller solution of (22). In this case, the debt-ridden firm can make l_z larger than el_n with the borrowing constraint nonbinding. This contradicts the premise that (22) is derived from binding borrowing constraint.

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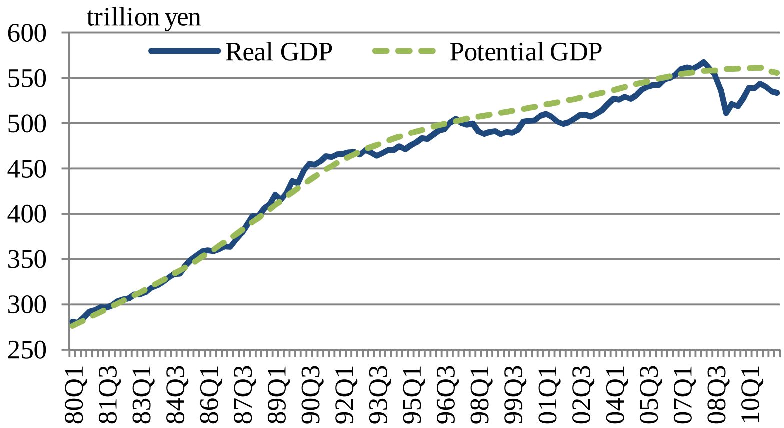


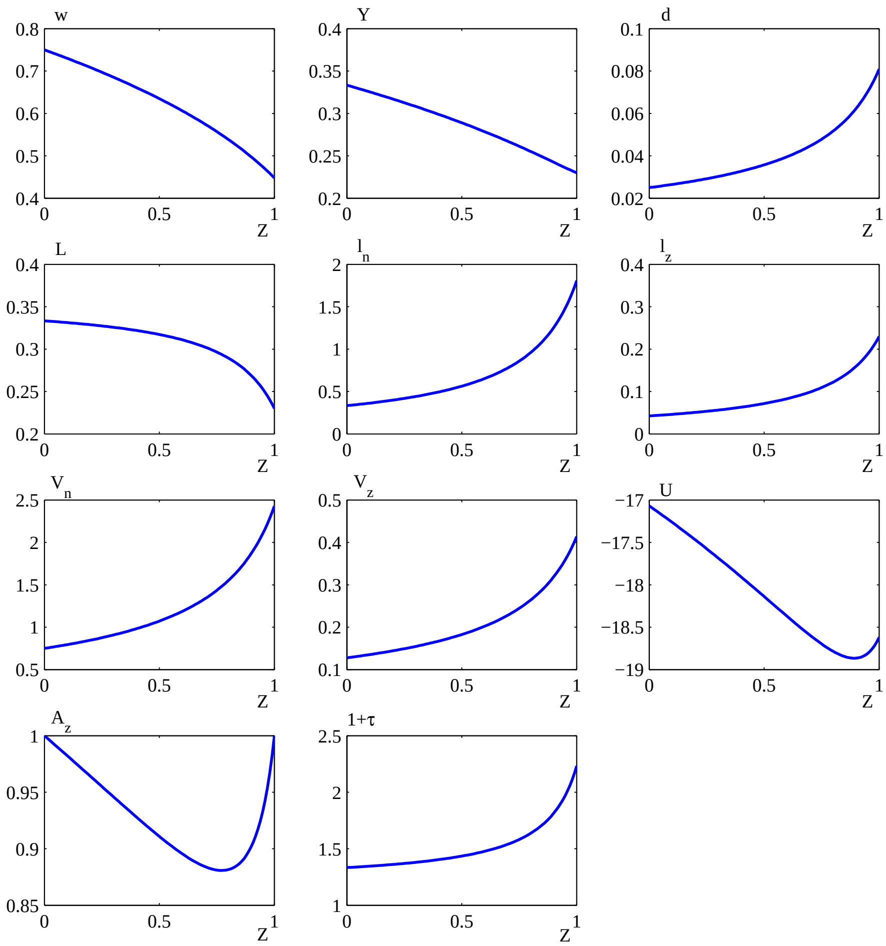
Source: Cabinet Office, Government of Japan, "Annual Report on National Accounts"

Figure 1: Real GDP and Potential GDP

	HP	KI	JIP2011
1971–80	1.33		1.68
1981–90	3.05	2.06	1.39
1991–2000	0.57	0.35	0.04
2001–2007		0.48	1.13

Note: HP, KI, JIP2011 are from updated versions of Hayashi and Prescott(2002), Kobayashi and Inaba (2006), and Fukao and Miyagawa (2008).

Table 1: TFP growth rate



Parameters: $\beta = 0.9, \quad \phi = 0.8, \quad \eta = 0.75, \quad \psi = 0.3, \quad \gamma = 1.5,\ A = 1, \quad K = 1$

Result: $e = 0.1272$

Figure 2: Equilibrium with debt-ridden firms in the Model