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**A Model of Multi-Dimensional Human Capital Investment:
Specific vs. general investments under uncertainty***

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Abstract

Specialization and the division of labor are the sources of high productivity in modern society. When worker skills are multi-dimensional, workers may face a choice between general versus specific human capital investment. Given that individual agents face uncertainty in the relative output price, what are the optimal strategies for heterogeneous individual agents in human capital investment? In the absence of insurance markets, general investment gives an option value for changes in the environment. We analyze a model in which workers are born heterogeneous and are endowed with two-dimensional skills in different sectors, to determine if incentives exist for workers to invest in skills in which they had originally excelled or struggled. We find that some workers choose to invest in their weaker skill, via specific human capital investment, provided that the scale of the risk is big and that the parameter of relative risk aversion is greater than one. We find that there exist agents whose optimal human capital investment decisions reverse their ex ante comparative advantages ex post.

Keywords: a self-selection model of occupational choice with multidimensional heterogeneity, individual comparative advantage, choice under uncertainty, intersectoral labor mobility, and specific and general human capital investments.

JEL classification: D81; F16; J23; J24; J62

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1 Introduction

This paper analyzes the theoretical implications of multi-dimensional heterogeneity of agent-skills and of interactions between the innate and acquired comparative advantages of individual agents. While many existing models deal with one-dimensional human capital skills when they analyze investment decisions, the model in this paper looks at multi-dimensional skill investments. Several studies have looked at the comparative advantages of individual agents that come from multi-dimensional heterogeneity, but none examined the interaction between ex ante and ex post differences in agent's comparative advantages. This paper introduces the innate heterogeneity of agents with multi-dimensional skills and considers the directional incentives for individual agents to invest in their human capital skills: general skill investments versus specific skill investments.

The model looks at the distinction between general and specific human capital investment as in Becker (1993), although not in exactly the similar manner. While general skills are useful for many other employers, both inside and outside their current industry, specific skills in this paper are not only useful skills for the current employer but also for employers in the same sector. Therefore, in this paper, specific skills denote sector-specific skills.

This paper adopts multi-dimensional heterogeneity and follows the regular job-assignment models based on individual comparative advantages. The model in this paper starts from the Roy (1951) model from labor economics (as with Ohnsorge and Trefler 2007 and Costinot and Vogel 2010), but asks a different question: What are the directional incentives of human capital investment for individual agents given the terms of trade uncertainty?

When the heterogeneity of human capital skills is multi-dimensional, the investment decision involves not only how much to invest but also which skill (direction) to enhance. For example, if each agent is endowed with two types of skills such as mathematics and music, then the question is which school (say, MIT to enhance math skills or Juilliard to enhance musical skills) the agent should enter in addition to the regular human-capital-investment question of how long he or she should attend the chosen school. If an agent chooses between somewhat specialized schools, then he or she is deciding to make a specialized human capital investment. Instead, an agent might go to a school (say, Harvard to enhance both math and music skills) where he or she can do a double major in music and math. In this case, an agent conducts a generalized human capital investment. This paper asks what type of agents will choose specialized investment or generalized investment and explores the relationship between the innate (ex ante) comparative advantages and the

acquired or post-education (post-investment) comparative advantages.¹

The analysis will be conducted under uncertainty because, without it, all agents will make specific investments based on his or her innate comparative advantage. With uncertainty, many agents will enhance their comparative advantage skills regardless of their degree of risk aversion. This occurs if the agents have very strong comparative advantage in one type of skill. However, some risk averse agents may make general skill investments if their comparative advantage is not particularly strong in either type. Under certain conditions, we find that some agents make specific skill investments in the direction of their innately weak comparative advantages. In particular, we can find some agents whose optimal human capital investment decisions reverse their ex ante comparative advantages ex post. The acquired comparative advantage may well be different from the innate comparative advantage.

To introduce uncertainty in a simple manner, I adopt the assumption of small open economy where output prices are exogenous in the model. Several papers have looked at the problem of human capital investment with international trade, including Findlay and Kierzkowski (1983) and Falvey, Greenaway, and Silva (2010).² However, these papers dealt with one-dimensional skill. So there was no question about specialization versus generalization in human capital skill investments. In this paper, by working with multi-dimensional skill model, I will try to work on the dichotomy of specialized skill investments versus general skill investments within the set up of open economy. In particular, this paper offers the first model to link innate multi-dimensional heterogeneity of individuals with the directional (specific versus general) incentive to invest in their human capital skills.³

This paper offers an analysis of directional incentives of human capital investment by individual agents. When worker skills are multi-dimensional, the direction of investment becomes a choice problem. Each individual may specialize in one type of skill or may generalize in multiple skills. Both the specialization and generalization of skills have costs and benefits. Several previous studies have looked at these issues, but all of them (with which the

¹This paper does not address the question of how long agents choose to go to school. Please refer to Becker (1993) for this choice problem.

²In a recent paper, Bougheas and Riezman (2007) looked at the determinants of trade in relation to the distribution of human capital. There, even if the countries are the same in the aggregate endowments of human capital, the difference in the properties of distribution of human capital explains the different patterns of trade.

³Note that, in traditional labor literature, specific human capital investment usually means firm-specific skills rather than industry- or sector-specific. In this paper, it is the latter.

author is familiar) started from identical agents who conduct either specific or general investments. No previous study looked at the case of the innate heterogeneity of individual agents with multi-dimensional skills.

When there are increasing returns on human capital investment, Sherwin Rosen (1983) showed that specialized investments tend to prevail in a model without uncertainty. The importance of specialization has also long been widely recognized. (See, for example, the very first chapter in *Wealth of Nations* by Adam Smith 1776.) The division of labor and specialization are the key sources of higher productivity in the modern world. The division of labor may be the result of endowed differences in individual comparative advantages as in Rosen (1978). Specialization, however, may also be a result of specialized human capital investment as in Rosen (1983). It is now widely recognized that individual agents have an incentive to specialize when there is no uncertainty about which sector individuals are employed. [Kevin M. Murphy (1986), p. 37.]

When the world is uncertain, however, ex-ante specific human capital investment may be a risky strategy because specialization makes individual agents inflexible as factors of production ex post. To borrow the expressions from Gene M. Grossman and Carl Shapiro (1982), an investment in specialization is like reducing “the degree of intersectoral mobility” in a subsequent period. A general (as opposed to specific) human capital investment is a form of *self-insurance* if insurance markets are absent. (See, for example, Ehrlich and Becker 1972.) A general investment brings the benefit of greater flexibility in responding to economic change. Murphy (1986, section 5.2) also looked at a case where there is uncertainty about which sector will become favorable. He also concluded that less specialization exposes workers to less risk. Both Grossman and Shapiro (1982) and Murphy (1986) examined the individuals’ incentives to make general versus specific human capital investment when every agent is *identical* before conducting an investment in human capital skill(s). So, what happens to these results if we start from a situation in which agents are heterogeneous in their innate comparative advantage before making the human capital investment? In this paper, I aim to analyze the incentives of individual agents to invest in human capital skills when the role of endowed differences is important.

By looking at the directional incentives to invest in human capital skills for the self-selection model of occupational choice, I am able to address the following (new) set of research questions: What is the role of endowed differences of individuals in human capital investment? Do people with heterogeneous skills specialize or generalize their innate skills when the world is uncertain? What kind of people do invest in their innately strong (compara-

tive advantage) skills? Do people specialize in what they were relatively good at when they were born? If so, under what conditions? Do agents invest their time (and money) to enhance their skills in a socially inefficient way? Do they go to schools in which they will learn something they are innately poor at? Will there be the same number of job-switching individuals if we allow the dynamic development of human capital? All of these questions were unanswered by previous work in this field since they assumed identical agents before the decision of human capital investment. Although there is a large volume of literature on human capital investment, to my knowledge there is no previous work that starts from multi-dimensional heterogeneity. I introduce such a model in the next section.

I find that, in general, many workers tend to specialize in their strengths even under uncertainty. This is true especially for the workers who have a very strong innate comparative advantage in one sector. For example, if you are born a good singer and you did not do well in high school, then you will choose to go to a school of music to enhance your strong innate skills in terms of comparative advantage.

For workers who are born almost equally good in both sectors, the problem becomes very difficult. Some workers try to invest in general skills so that they can choose their jobs in a flexible manner once the uncertainty resolves. Others may specialize in their innately strong or weak skills depending on their attitude toward risk.

If workers are very risk averse, we find that some workers decide to make specialized investments in their weak skills (in the direction of their comparative disadvantages) rather than in their strong ones (in the direction of their comparative advantages). This happens to those who have skills whose levels are similar enough so that their degree of comparative advantage is not very strong. For example, consider a person A who has high talent in music but also has a high IQ. This person might choose to give up music to go to MIT even if his talent in music is actually the stronger of his skills. On the other hand, suppose there is another person, B, who has similar skills as person A. Suppose also that B's music skills are not as good as A's and that B's IQ is slightly higher than A's. Therefore, if there is no uncertainty, A chooses to be a musician and B gives up the idea of being a musician to focus on academic study. However, given the setup of the model with uncertainty, A goes to MIT by giving up becoming a musician and B goes to Juilliard to be a musician! So the reversal of training occurs for those who are in certain classes of skills in this model. In the end, we may see some cases where a person's comparative advantage reverse from *ex ante* to *ex post*. Person A (respectively, B) has a comparative advantage in music (respec-

tively, academic skills) ex ante, but after the training A (respectively, B) has a comparative advantage in academic skills (respectively, music). This reversal result is quite new as far as we know.

The remaining part of the paper is organized as follows: the next section develops the basic model of human capital investment with two dimensional heterogeneity of innate skills. The final section summarizes the results and suggests some possible extensions.

2 The Model

We consider a simple two-period and two-sector model of a small open economy that faces exogenously given international output prices. In period 0, no production takes place, agents are endowed with multi-dimensional heterogeneous skills, and they invest in their human capital to enhance their innate skills. Uncertainty about the terms of trade will be realized in period 1 and agents choose their occupations and engage in production. Multi-dimensional skills are embodied in an individual agent and can only be sold as a package. Therefore, we assume that an individual agent can take only one job at a time.

Output markets for sectors X and Y are assumed to be competitive, both internationally and domestically. In making the investment decision in period 0, each agent is assumed to have rational expectations concerning the prices that will prevail in period 1.

The economy consists of a continuum of self-employed agents $j \in J$, each of whom is endowed with an individual-specific occupational skill vector (θ^j, τ^j) jointly distributed over a unit square $[0, 1] \times [0, 1] \subset \mathbb{R}^2$ in period 0.⁴ Let Θ_t and $F_t(\theta, \tau)$ denote the space and the joint distribution function of human capital skills for each period $t = 0, 1$. Thus, we know that $\Theta_0 = [0, 1] \times [0, 1]$. Let $f_t(\theta, \tau) > 0$ denote the joint density function for $F_t(\theta, \tau)$, and assume that f_t is integrable over any partition of the human capital skill space Θ_t for $t = 0, 1$. Agents are price takers in the output markets. Each component of the skill vector (θ^j, τ^j) represents an sector-specific human capital skill; their magnitudes measure the capabilities of the agent j in the production of X and Y in efficiency units.

Once the terms of trade become known in period 1, each agent decides either to produce X using θ , or Y using τ . Each individual undertakes only one occupation at a time because human capital skills are assumed to

⁴Type space does not have to be a unit square. Here for simplicity, a unit square type space is assumed.

be embodied in human beings. [Murphy (1986, Sec.II)] Each component of the skill vector (θ^j, τ^j) is indivisible and non-transferable. The size of the components of individual skill vectors in each period $(\theta^j, \tau^j) \in \Theta_t$ is private information for j , but its aggregate distribution F_t is publicly known.

To summarize, the timing of the model is as follows:

1. In period 0, the type of each individual $(\theta^j, \tau^j) \in [0, 1] \times [0, 1] = \Theta_0$ is given.
2. An individual choose to invest in human capital skills for either sector X or Y , or both. This individual decision will create the new skill space Θ_1 and the new distribution of skills $F_1(\theta, \tau)$ for period 1.
3. In period 1, a new relative price $P_1 = (P_X, P_Y)$ is realized.
4. An individual chooses occupation and produce.

Let us now state the assumptions about human capital investment and uncertainty.

2.1 Key Assumptions of the Model

Let us now state the assumptions about how individual agents can invest in their skills in period 0. There are two kinds of human capital investment: (1) General Human Capital Investment and (2) Sector Specific Human Capital Investment. Because there are two directions in sector specific investments, we can categorize possible investments into three investment types.

A1: [Human Capital Investment] An individual with the skill vector (θ, τ) can choose among the three types of human capital investment: $I^{HC} \in \{S_X, G, S_Y\}$ which are listed as:

1. $S_X =$ Specific Investment in Sector X: $(\theta, \tau) \implies (\beta\theta, \tau)$
2. $G =$ General Investment in both sectors: $(\theta, \tau) \implies (\alpha\theta, \alpha\tau)$
3. $S_Y =$ Specific Investment in Sector Y: $(\theta, \tau) \implies (\theta, \beta\tau)$

where α and β represent parameters such that

$$\beta > \alpha > 1. \tag{1}$$

The fact that both parameters are larger than 1 implies that human capital investment, regardless of specificity or generality, is effective. The size of

investment efficiency for specific investment β is assumed to be larger than that for general investment α . Otherwise, everyone will invest in general skills alone because specific skill investment will be dominated by general investment in either direction.

It is assumed that the specific investments in two sectors share the same parameter β . This means that the effect of the specific investment is symmetric between sector X and Y. It is also assumed that only one type of general investment can occur. General investment enhances skills in both sectors in a symmetric manner. Both components will be multiplied by the same α . These symmetry assumptions are made to simplify the analysis.

Another peculiarity about investment technology is that the effect of investments is proportional to the individual's original strength (innate capabilities) in each sector. If your innate skill θ in sector X production is very large, then your post investment skill in the same sector $\beta\theta$ will be proportionally large. This assumption is appropriate if all agents in the economy are considered to be young. For example, young Michael Jordan can be trained to be a superstar because of his exceptional innate talent as a basketball player. The effect of training Jordan is much larger than the effect of training a mediocre player. The assumption of proportional effectiveness of human capital skill investment would create a theoretical problem if we were to start thinking about an economy with matured (and old) agents who already have invested in their skills and have little margin for additional skill development. However, we focus on this particular case of proportionality in this paper.

Compared to more general investment frontier depicted in Murphy (1986) and Grossman and Shapiro (1982), this paper's specification of human capital investment given by the above assumption is somewhat restrictive by not allowing a range of intermediate cases (such as 40% on X and 60% on Y, etc.) in general investment. This restrictive assumption is made for simplicity. The fact that β is larger than α means an implicit assumption about some form of increasing returns from specialization in human capital investment. Because β is larger than α , we can conclude that the type space in period 1 should be $\Theta_1 \subset [0, \beta] \times [0, \beta]$.

Now let us assume that all consumers have identical and homothetic preferences that can be represented by a utility function $u(x, y)$ for positive consumption amounts for each product: $x > 0$ and $y > 0$. When we denote the income of the individuals as $M > 0$ and the vector of output prices as $P \equiv (P_X, P_Y)$, then the indirect utility function for homothetic preferences

can be written as the following separable function:

$$V(P, M) = C(P) \cdot v(M) \quad (2)$$

We know, by Deaton and Muellbauer (1980), that the price-index function $C(P)$ is linear homogeneous and concave in P . The part $v(M)$ is an increasing function of income M of an individual. If the original utility function $u(x, y)$ is homogeneous of degree one, then $v(M) = M$ should hold. Therefore, if the agents are risk neutral, the indirect utility can be written as

$$V(P, M) = C(P) \cdot M \quad (3)$$

and if the agents are risk averse, then $v(M)$ in the equation (2) is strictly concave in M . [Jack Hirshleifer and John G. Riley, 1992, p. 32.] Thus, we could write $v' > 0$ and $v'' < 0$. For now, let us leave the second one as $v'' \leq 0$, in order to include both cases of risk neutrality and risk aversion.

Now let us introduce uncertainty in period 1. Since we assume that we live in a small open economy, every individual agent takes terms of trade (output prices) as given. Suppose there are two states of nature, as follows:

A2. [Uncertainty] Uncertainty about the terms of trade in period 1 takes the following symmetric form:

$$P_1 = \begin{cases} (p, 1) & \text{with probability } \frac{1}{2} \\ (1, p) & \text{with probability } \frac{1}{2} \end{cases} \quad (4)$$

where p is a positive parameter larger than 1.

We concentrate on a case with the same probability for the two states of nature. When the state of nature is such that $P_1 = (p, 1)$, producers in sector X will benefit in period 1 because P_X will be more expensive relative to P_Y . When $P_1 = (1, p)$ occurs, producers in sector Y will benefit because P_Y will be more expensive relative to P_X . As we have seen in the regular self-selection models based on individual comparative advantage, the realization of particular terms of trade may induce some workers to take different jobs once the uncertainty is resolved. (See, for example, Sattinger 1975.) Thus, some agents will choose to work in a favorable sector while others may stay in the sector at which they were innately good.

Let us further assume that the demand condition is symmetric in sectors X and Y . In the equation of indirect utility (2), $C(P)$ represents a consumer price index (CPI).

A3. [Equal CPI] The consumer price index (CPI) in period 1 is the same for the different states of the world, namely

$$C(p, 1) = C(1, p) \tag{5}$$

holds in the neighborhood.

This assumption will enable us to compare welfare by directly looking at $v(M)$ in (2), the utility part only from income in period 1 without worrying about substitution effects in consumption. (See Appendix for an explanation.) This way, irrespective of the outcome of uncertain terms of trade, we can compare the economic welfare of individual agents only by looking at $v(M)$.

Assume also that no income-insurance market exists. (See Grossman and Shapiro [1982] for a comparison between self-insurance versus insurance markets.)

A4. [No Insurance] The market for income insurance is absent.

This may be because there is no market for skills per se. Or it may be because information asymmetry⁵ prevents insurance firms from operating profitably. According to an article in *The Economist* (March 26, 1994), the capital markets for human capital investment may be imperfect. The article reads as follows: “For instance, borrowing to finance an investment in human capital may be difficult because would-be trainees lack collateral, or because the costs of administration and collection make such loans unattractive to private lenders.” In any case, we focus on the case without insurance.

Given the investment made in period 0 and given the resolved terms of trade, individual agents choose their occupations and start producing either X or Y in period 1. How do individual agents decide in which sector to work? To determine this, let’s first introduce a constant-returns-to-scale production function for each sector:

$$\begin{cases} x &= N_X \\ y &= N_Y \end{cases} \tag{6}$$

where N_X and N_Y are the total sum of (effective) skill levels employed in each sector.⁶ Given the realization of terms of trade, P , the skill space in

⁵Skill levels are private information about the individual agents and the insurance company cannot know them.

⁶We could add coefficients in the production functions such as $x = \gamma N_X$, but we omit them for the sake of simplicity.

period 1, Θ_1 , can be partitioned into two: (1) $\Theta_X(P)$ \dots group of producers of X and (2) $\Theta_Y(P)$ \dots group of producers of Y. Then, N_X and N_Y can be written as follows:

$$\begin{cases} N_X &= \int_{\Theta_X(P)} \theta dF_1(\theta, \tau) \\ N_Y &= \int_{\Theta_Y(P)} \tau dF_1(\theta, \tau) \end{cases} \quad (7)$$

All individual agents earn wages that are equal to the value of their marginal product. Therefore, when the terms of trade is given as $P = (P_X, P_Y)$, if an individual with a vector of skills (θ, τ) who engages in production in sector X, then that individual earns a wage of $P_X\theta$, while the same individual earns $P_Y\tau$ if he or she engages in production in sector Y. So whether an individual (θ, τ) works in sector X or Y depends on the direction of inequality in

$$P_X\theta \geq P_Y\tau. \quad (8)$$

Thus, an individual who had made human capital investment $I^{HC} \in \{S_X, G, S_Y\}$ in period 0 and faces terms of trade $P \equiv (P_X, P_Y) \in \{(p, 1), (1, p)\}$ in period 1 will earn the income $M(I^{HC}, P)$. The possible combinations can be written as follows:

$$\begin{cases} M(S_X, (p, 1)) &= \max [p\beta\theta, \tau] \\ M(G, (p, 1)) &= \max [p\alpha\theta, \alpha\tau] \\ M(S_Y, (p, 1)) &= \max [p\theta, \beta\tau] \\ M(S_X, (1, p)) &= \max [\beta\theta, p\tau] \\ M(G, (1, p)) &= \max [\alpha\theta, p\alpha\tau] \\ M(S_Y, (1, p)) &= \max [\theta, p\beta\tau] \end{cases} \quad (9)$$

Given this information about possible income in period 1, we can now consider the expected utility of an individual (θ, τ) who conducted human capital investment $I^{HC} \in \{S_X, G, S_Y\}$ in period 0. Since the lottery about terms of trade in period 1 is given by (4), expected utility $U(I^{HC})$ for each investment pattern is written as

$$\begin{cases} U(S_X) &= \left\{ \frac{1}{2}v[M(S_X, (p, 1))] + \frac{1}{2}v[M(S_X, (1, p))] \right\} \cdot C(P_1) \\ U(G) &= \left\{ \frac{1}{2}v[M(G, (p, 1))] + \frac{1}{2}v[M(G, (1, p))] \right\} \cdot C(P_1) \\ U(S_Y) &= \left\{ \frac{1}{2}v[M(S_Y, (p, 1))] + \frac{1}{2}v[M(S_Y, (1, p))] \right\} \cdot C(P_1) \end{cases} \quad (10)$$

where $C(P_1) \equiv C(p, 1) = C(1, p)$ is defined as such from (5). Thus, to compare the sizes among (10), we only need to compare the inside of the curly brackets $\{\}$ because we can think of $C(P_1)$ as an exogenous parameter. We will come back to this point later.

Now, to leave space for analysis, I would like to make additional innocuous assumption about the size of risk. Let us consider the case with large risk in this paper.

A5. [Risk Size] We focus on our analysis for the case of $p > \beta$. The size of the price parameter is larger than the size of a parameter for specific human capital investment.

The reason for this assumption is a matter of simplicity. By assuming otherwise, the analysis is very similar except for the combination of parameters and cases. For those who are interested in the other case of $p < \beta$, please consult my Ph.D. Dissertation, Ichida (2004, Ch. 4), for the case of risk-neutral agents.

2.2 Incentives for Each Individual Agent

With the assumptions given by A1.-A5. above, let us now consider the incentives for each individual agent. What type of human capital investment do agents choose? What jobs do agents take once the uncertainty is resolved?

If all agents were identical before the investment decision as in Grossman and Shapiro (1982) and in Murphy (1986), then in equilibrium all investment choices must give the same expected utility. Otherwise, every agent will pick the best alternative that gives the highest expected utility level. The income of all agents in period 1 must also be equalized across sectors for identical-agents-case. That is how the allocation of jobs would be done. However, this shall not be the case for the model in this paper because agents are born to be different. In fact, agents are heterogeneous in both absolute and relative skill levels in both periods. Depending on the relative sizes of skills in the vector (θ, τ) , the largest income that is chosen among those in (9) may differ.

Therefore, let us break down the initial type space $\Theta_0 = [0, 1]^2$ (in period 0) into the following eight partitions by possible occupational choice with

different investment choices and with different risk outcomes.

$$\left\{ \begin{array}{l} K_1 : (\theta, \tau) \in [0, 1]^2 \cap \left\{ 0 < \tau < \frac{1}{p\beta}\theta \right\} \\ K_2 : (\theta, \tau) \in [0, 1]^2 \cap \left\{ \frac{1}{p\beta}\theta < \tau < \frac{1}{p}\theta \right\} \\ K_3 : (\theta, \tau) \in [0, 1]^2 \cap \left\{ \frac{1}{p}\theta < \tau < \frac{\beta}{p}\theta \right\} \\ K_4 : (\theta, \tau) \in [0, 1]^2 \cap \left\{ \frac{\beta}{p}\theta < \tau < \theta \right\} \\ K_5 : (\theta, \tau) \in [0, 1]^2 \cap \left\{ \theta < \tau < \frac{p}{\beta}\theta \right\} \\ K_6 : (\theta, \tau) \in [0, 1]^2 \cap \left\{ \frac{p}{\beta}\theta < \tau < p\theta \right\} \\ K_7 : (\theta, \tau) \in [0, 1]^2 \cap \left\{ p\theta < \tau < p\beta\theta \right\} \\ K_8 : (\theta, \tau) \in [0, 1]^2 \cap \left\{ p\beta\theta < \tau < \infty \right\} \end{array} \right. \quad (11)$$

I will explain the meaning of these partitions in words.

1. Think about the case where an individual conducted S_X (specific investment in sector X skill) in period 0. We think of the case of $M(S_X, (p, 1))$ or $M(S_X, (1, p))$. If the terms of trade turn out to be favorable to sector X, i.e. $(p, 1)$ is realized,⁷ then agents in $\bigcup_{i=1}^7 K_i$ will produce in sector X while those in K_8 will produce in sector Y. If the terms of trade turn out to be favorable to sector Y, i.e. $(1, p)$ is realized, then agents in $\bigcup_{i=1}^3 K_i$ will produce in sector X while those in $\bigcup_{i=4}^8 K_i$ will produce in sector Y.
2. Think about the case where an individual conducted G (general investment) in period 0. We think of the case of $M(G, (p, 1))$ or $M(G, (1, p))$. If the terms of trade turn out to be favorable to sector X, i.e. $(p, 1)$ is realized, then agents in $\bigcup_{i=1}^6 K_i$ will produce in sector X while those in $\bigcup_{i=7}^8 K_i$ will produce in sector Y. If the terms of trade turn out to be favorable to sector Y, i.e. $(1, p)$ is realized, then agents in $\bigcup_{i=1}^2 K_i$ will produce in sector X while those in $\bigcup_{i=3}^8 K_i$ will produce in sector Y.
3. Think about a case in which an individual conducted S_Y (a specific investment in sector Y skill) in period 0. We think of the case of $M(S_Y, (p, 1))$ or $M(S_Y, (1, p))$. If the terms of trade turn out to be favorable to sector X, i.e. $(p, 1)$ is realized,⁸ then agents in $\bigcup_{i=1}^5 K_i$ will produce in sector X while those in $\bigcup_{i=6}^8 K_i$ will produce in sector Y. If the terms of trade turn out to be favorable to sector Y, i.e. $(1, p)$ is realized, then agents in K_1 will produce in sector X while those in $\bigcup_{i=2}^8 K_i$ will produce in sector Y.

⁷This also means that the individual guessed correctly.

⁸This also means that the individual guessed wrong.

Given the above examination of individual decisions about which sector agents work in period 1, we want to think about the investment incentives of individuals in each partition by using backward induction. However, the kind of investment each individual chooses depends on the agent's attitude toward risk.

2.3 Expected Utility and Attitude toward Risk

Now, because the expected utility function can represent the same preference up to the monotonic linear transformation,⁹ we can define the expected utility for the agents in partition K_i who conducted investment I^{HC} as follows: for all $i \in \{1, 2, \dots, 8\}$, a new von-Neumann Morgenstern expected utility function $V(K_i, I^{HC})$ is defined by

$$V(K_i, I^{HC}) \equiv \frac{U(I^{HC})}{C(P_1)} \Big|_{(\theta, \tau) \in K_i}. \quad (12)$$

This can be done because $C(P_1) \equiv C(p, 1) = C(1, p)$ is an exogenous parameter, as we noted before. For the agents in different partitions in (11), we are able to calculate the von-Neumann Morgenstern expected utility function $V(K_i, I^{HC})$ by combining the equations of income (9) and the equations of the expected utility (10). The $v(M)$ part of the equations (10) can be thought of as the Bernoulli utility function. When $v'' = 0$, we can say that agents are risk neutral and when $v'' < 0$, we can say that agents are strictly risk averse.

Here I would like to introduce specific functional forms for the Bernoulli utility function. Let it be the Constant Relative Risk Aversion (CRRA), written as

$$v(M) = \begin{cases} \frac{M^{1-\rho}}{1-\rho} & \text{for } \rho \neq 1 \\ \ln M & \text{for } \rho = 1 \end{cases}$$

where the size of ρ represents the coefficient of relative risk aversion. To simplify the analysis, I focus on some specific cases in this paper.

Three cases of different attitude toward risk will be analyzed: (1) RN = risk neutral case, (2) RA1 = risk aversion with a coefficient of relative risk aversion is 1, and (3) RA2 = risk aversion with a coefficient of relative risk aversion is 2. Let us start with the risk neutral case.

RN When the agents are risk neutral, the Bernoulli utility function takes the linear form: $v(M) = M$.

⁹See Prop.6B1 in page 173 of Mas-Colell et al. (1995).

The case for risk neutrality (RN) is straightforward. We adopt simply the linear function without coefficients for the Bernoulli utility.

The case for risk averse is more complicated because there are many ways to be risk averse. To make it easier to obtain the analytical results, let us focus on the following two cases of risk aversion.

RA1 One type of risk averse agents has a Bernoulli utility function in the form of $v(M) = \ln M$. The parameter for relative risk aversion for this log form is constant at 1.

The first type of risk aversion RA1 utilizes log utility form. This utility has a constant elasticity for any income level.

RA2 Another type of risk averse agent has a Bernoulli utility function in the form of $v(M) = -\frac{1}{M}$. The parameter for relative risk aversion for this form is 2.

The second type of risk aversion RA2 utilizes the negative inverse form of utility. The parameter for relative risk aversion is greater than 1. Therefore, the degree of risk aversion is the strongest among these three cases.

2.4 The results for individual agents

Let us state the main result for the RN case as Theorem 1.

Theorem 1 *When the assumptions given by A1.-A5. hold in the model with the RN Bernoulli utility function $v(M) = M$, the following three situations will occur depending on the size of the parameter value of α .*

- (i) *All agents in the economy will make specific investments in the direction of their innate comparative advantage; agents $(\theta, \tau) \in \Theta_0$ with $\tau < \theta$ will make S_X (Specific investment in sector X) and agents $(\theta, \tau) \in \Theta_0$ with $\tau > \theta$ will invest S_Y (Specific investment in sector Y), if the parameter of general investment is smaller than a particular threshold value, or*

$$\alpha < \frac{\beta + 1}{2}.$$

- (ii) *There will be some agents who will make general investments G. In particular, agents $(\theta, \tau) \in \Theta_0$ with*

$$\frac{\beta - \alpha}{\alpha - 1} \cdot \theta < \tau < \frac{\alpha - 1}{\beta - \alpha} \cdot \theta$$

will invest G (General investment in both sectors), agents $(\theta, \tau) \in \Theta_0$ with

$$0 < \tau < \frac{\beta - \alpha}{\alpha - 1} \cdot \theta$$

will invest S_X , and agents $(\theta, \tau) \in \Theta_0$ with

$$\frac{\alpha - 1}{\beta - \alpha} \cdot \theta < \tau < \infty$$

will invest S_Y , if the parameter of general investment is within a particular threshold values, or

$$\frac{\beta + 1}{2} < \alpha < \frac{(p + 1)\beta}{p + \beta}.$$

(iii) There will be some agents who will make general investments G . In particular, agents $(\theta, \tau) \in \Theta_0$ with

$$\frac{p\beta + \beta - p\alpha}{p\alpha} \cdot \theta < \tau < \frac{p\alpha}{p\beta + \beta - p\alpha} \cdot \theta$$

will invest G (General investment in both sectors), agents $(\theta, \tau) \in \Theta_0$ with

$$0 < \tau < \frac{p\beta + \beta - p\alpha}{p\alpha} \cdot \theta$$

will invest S_X , and agents $(\theta, \tau) \in \Theta_0$ with

$$\frac{p\alpha}{p\beta + \beta - p\alpha} \cdot \theta < \tau < \infty$$

will invest S_Y , if the parameter of general investment is larger than a particular threshold value, or

$$\frac{(p + 1)\beta}{p + \beta} < \alpha.$$

The proof is in the later section. In the case of risk neutrality, specific investments are made only to strengthen the agent's innate comparative advantage. When the parameter of general investment is smaller than a particular threshold value, no general investment occurs. If the parameter is larger, then the general investment occurs for agents with an intermediate level of comparative advantage. Risk neutral agents will never make specific investments in the opposite direction, namely, the direction of the agent's innate comparative disadvantage.

Let us state the main result for the RA1 case with log utility.

Theorem 2 *When the assumptions given by A1.-A5. hold in the model with the RA1 Bernoulli utility function $v(M) = \ln M$, the following two situations will occur depending on the size of the parameter value of α .*

- (i) *All agents in the economy will make specific investments, but the direction of investments vary depending on the strength of their innate comparative advantage; agents $(\theta, \tau) \in \Theta_0$ with*

$$\left\{ \begin{array}{ll} 0 < \tau < \frac{\beta^2}{p\alpha}\theta & \iff \text{invest } S_X \\ \frac{\beta^2}{p\alpha}\theta < \tau < \frac{p\alpha}{\beta^2}\theta & \iff \text{indifferent between } S_X \text{ and } S_Y \\ \frac{p\alpha}{\beta^2}\theta < \tau < \infty & \iff \text{invest } S_Y \end{array} \right.$$

if the parameter of general investment is smaller than a particular threshold value, or

$$\alpha < \sqrt{\beta}.$$

- (ii) *There will be some agents who will make general investments G . In particular, agents $(\theta, \tau) \in \Theta_0$ with*

$$\frac{\beta}{p}\theta < \tau < \frac{p}{\beta}\theta$$

will invest G (General investment in both sectors), agents $(\theta, \tau) \in \Theta_0$ with

$$0 < \tau < \frac{\beta}{p}\theta$$

will invest S_X , and agents $(\theta, \tau) \in \Theta_0$ with

$$\frac{p}{\beta}\theta < \tau < \infty$$

will invest S_Y , if the parameter of general investment is larger than a particular threshold value, or

$$\sqrt{\beta} < \alpha.$$

As with the risk neutral case, when the parameter of general investment is larger than a particular threshold value, then the general investment occurs for agents with an intermediate level of comparative advantage. When the parameter of general investment is smaller, however, the direction of specific investments is quite different. Those with a very strong innate comparative advantage make specific investments in their innately strong skills.

Those with an intermediate comparative advantage are indifferent when it comes to conducting specific investments in sector X and sector Y. This is a striking result. For those who are closer to the 45-degree line, the expected utility values from S_X and S_Y are the same. This is because of the flexibility of agents who are in the range of

$$\frac{\beta^2}{p\alpha}\theta < \tau < \frac{p\alpha}{\beta^2}\theta \quad (13)$$

about which sector they work in after the realization of the terms of trade. For those in partition (13), they work in sector X (respectively, Y) if the terms of trade are favorable for workers in X (respectively, Y).

Let us state the main result for the RA2 case as Theorem 3.

Theorem 3 *When the assumptions given by A1.-A5. hold in the model with the RA2 Bernoulli utility function $v(M) = -\frac{1}{M}$, the following two situations will occur depending on the size of the parameter value of α .*

(i) *All agents in the economy will make specific investments, but the direction of investments vary depending on the strength of their innate comparative advantage; agents $(\theta, \tau) \in \Theta_0$ with*

$$\left\{ \begin{array}{ll} 0 < \tau < \frac{1}{1+p-\beta}\theta & \iff \text{invest } S_X \\ \frac{1}{1+p-\beta}\theta < \tau < \theta & \iff \text{invest } S_Y \\ \theta < \tau < (1+p-\beta)\theta & \iff \text{invest } S_X \\ (1+p-\beta)\theta < \tau < \infty & \iff \text{invest } S_Y \end{array} \right.$$

if the parameter of general investment is smaller than a particular threshold value, or

$$\alpha < \frac{2\beta}{\beta+1}.$$

(ii) *There will be some agents who will make general investments G. In particular, agents $(\theta, \tau) \in \Theta_0$ with*

$$\left\{ \begin{array}{ll} 0 < \tau < \frac{1}{1+p-\beta}\theta & \iff \text{invest } S_X \\ \frac{1}{1+p-\beta}\theta < \tau < \frac{\beta-\alpha}{\beta(\alpha-1)}\theta & \iff \text{invest } S_Y \\ \frac{\beta-\alpha}{\beta(\alpha-1)}\theta < \tau < \frac{\beta(\alpha-1)}{\beta-\alpha}\theta & \iff \text{invest } G \\ \frac{\beta(\alpha-1)}{\beta-\alpha}\theta < \tau < (1+p-\beta)\theta & \iff \text{invest } S_X \\ (1+p-\beta)\theta < \tau < \infty & \iff \text{invest } S_Y \end{array} \right.$$

if the parameter of general investment is larger than a particular threshold value, or

$$\frac{2\beta}{\beta + 1} < \alpha.$$

When the degree of risk aversion is very strong for the agents, we observe a very interesting phenomenon. Some agents specialize in their innately weak skills. When agents are born with very strong comparative advantages, then they try to develop their strong skills regardless of the uncertainty about future terms of trade. As the relative ability of the agents in two sectors get closer, their individual incentive to invest in their human capital skills becomes something like personal insurance against a possible misfortune affecting their innately strong skills.

The proofs to the Theorems 1-3 are given below.

2.5 Analysis for Each Partition

Let us start analyzing the incentives to invest in human capital skills for agents in each partition. Depending on the realized outcome of uncertain terms of trade, the choice of occupation differs among different partitions. Therefore, the realized income may differ. Based on the expected choice of occupation and expected income level, the individuals choose to decide to invest in specific or general human capital investment.

2.5.1 Partition K_1 and K_8 :

Consider agents in partition K_1 . All agents in this partition will work in sector X in period 1 regardless of the realized terms of trade. Their expected utility can be written as follows:

$$\begin{cases} V(K_1, S_X) &= \frac{1}{2}v(p\beta\theta) + \frac{1}{2}v(\beta\theta) \\ V(K_1, G) &= \frac{1}{2}v(p\alpha\theta) + \frac{1}{2}v(\alpha\theta) \\ V(K_1, S_Y) &= \frac{1}{2}v(p\theta) + \frac{1}{2}v(\theta) \end{cases} . \quad (14)$$

In this case, it is straight forward to show

$$V(K_1, S_X) > V(K_1, G) > V(K_1, S_Y) \quad (15)$$

regardless of the shapes of Bernoulli utility function. Irrespective of the attitude toward risks, i.e. RN, RA1, or RA2, the Bernoulli utility functions are increasing monotonic functions, i.e. $v' > 0$. We know that $p\beta\theta > p\alpha\theta > p\theta$ and $\beta\theta > \alpha\theta > \theta$ for $p > \beta > \alpha > 1$ because of the assumptions of

the model. (15) shows that all agents in partition K_1 will make a specific investment in sector X. The same logic hold true for the agents in partition K_8 and they invest S_Y .

2.5.2 Partition K_2 and K_7 :

Next, consider agents in partition K_2 . They work in sector X unless they have invested in S_Y **and** the terms of trade turn out to be unfavorable to sector X, i.e. $P_1 = (1, p)$. Thus, their expected utility can be written as

$$\begin{cases} V(K_2, S_X) &= \frac{1}{2}v(p\beta\theta) + \frac{1}{2}v(\beta\theta) \\ V(K_2, G) &= \frac{1}{2}v(p\alpha\theta) + \frac{1}{2}v(\alpha\theta) \\ V(K_2, S_Y) &= \frac{1}{2}v(p\theta) + \frac{1}{2}v(p\beta\tau) \end{cases} . \quad (16)$$

From the analysis for K_1 , we know that

$$V(K_2, S_X) > V(K_2, G) \quad (17)$$

holds always true since $V(K_2, S_X) = V(K_1, S_X)$ and $V(K_2, G) = V(K_1, G)$ can be seen in (16). So we are left to compare the size between $V(K_2, S_X)$ and $V(K_2, S_Y)$. We should note that partition K_2 belongs to the region

$$\frac{1}{p\beta}\theta < \tau < \frac{1}{p}\theta. \quad (18)$$

We first look at the RN case: $v(M) = M$. By looking at

$$2 \{V(K_2, S_X) - V(K_2, S_Y)\} = (p\beta + \beta - p)\theta - p\beta\tau \quad (19)$$

we can conclude that the expression (19) is positive because

$$\frac{p\beta + \beta - p}{p\beta} = \frac{1}{p} + \left(1 - \frac{1}{\beta}\right) > \frac{1}{p}$$

together with (18).

Second, look at the case for RA1: $v(M) = \ln M$. The expression

$$2 \{V(K_2, S_X) - V(K_2, S_Y)\} = \ln \theta - \ln \tau + \ln \beta - \ln p \quad (20)$$

is positive because $\tau < \frac{\beta}{p}\theta$ always hold for agents in (18).

Third, look at the RA2 case: $v(M) = -\frac{1}{M}$. Let's look at the following expression:

$$2 \{V(K_2, S_X) - V(K_2, S_Y)\} = \frac{\theta - (p + 1 - \beta)\tau}{p\beta\theta\tau} \quad (21)$$

Because it is easy to verify that

$$\frac{1}{p} < \frac{1}{p+1-\beta} \quad (22)$$

and, by noting the inequality (18), we can say that expression (21) is always positive for agents in K_2 .

So we can conclude that

$$V(K_2, S_X) > V(K_2, S_Y) \quad (23)$$

for all three cases. Therefore, we can conclude that in partition K_2 , the specific investment for sector X, S_X , is always chosen for all RN, RA1 and RA2 cases. The same logic hold true for agents in partition K_7 and they invest S_Y .

2.5.3 Partition K_3 and K_6 :

Now consider agents in partition K_3 who are in the region:

$$\frac{1}{p}\theta < \tau < \frac{\beta}{p}\theta. \quad (24)$$

They work in sector Y when the terms of trade are $(1, p)$ only if they had invested either G or S_Y . Otherwise, they work in sector X. Therefore, their expected utility can be written as

$$\begin{cases} V(K_3, S_X) &= \frac{1}{2}v(p\beta\theta) + \frac{1}{2}v(\beta\theta) \\ V(K_3, G) &= \frac{1}{2}v(p\alpha\theta) + \frac{1}{2}v(p\alpha\tau) \\ V(K_3, S_Y) &= \frac{1}{2}v(p\theta) + \frac{1}{2}v(p\beta\tau) \end{cases} \quad (25)$$

Note that $V(K_3, S_X) = V(K_2, S_X)$ and $V(K_3, S_Y) = V(K_2, S_Y)$, but $V(K_3, G) \neq V(K_2, G)$.

To conserve space, let \succ denote preference over the human capital investment $I^{HC} \in \{S_X, G, S_Y\}$ for a particular partition. If it is clear from the context, assume that the following notation can be used for any $i \in \{1, \dots, 8\}$:

$$I^{HC} \succ I^{HC'} \quad \text{for agents in } K_i \iff V(K_i, I^{HC}) > V(K_i, I^{HC'}).$$

By using the notation \succ , we look at the three cases for agents in K_3 and K_6 .

RN case for K_3 and K_6 : Let us first look at the RN case: $v(M) = M$. By looking at

$$2 \{V(K_3, S_X) - V(K_3, S_Y)\} = (p\beta + \beta - p)\theta - p\beta\tau \quad (26)$$

we know this is the same as (19). By looking at the following expression:

$$\frac{p\beta + \beta - p}{p\beta} - \frac{\beta}{p} = \frac{(p - \beta)(\beta - 1)}{p\beta},$$

we can conclude that

$$\frac{\beta}{p} < \frac{p\beta + \beta - p}{p\beta} \quad (27)$$

holds true since $\beta > 1$ and $p > \beta$. Then we can say that $S_X \succ S_Y$ for RN agents in K_3 . Now we need to compare S_X with G . Look at

$$2 \{V(K_3, S_X) - V(K_3, G)\} = (p\beta - p\alpha + \beta)\theta - p\alpha\tau. \quad (28)$$

The sign of (28) depends on the direction of the following inequality:

$$\tau \geq \frac{p\beta + \beta - p\alpha}{p\alpha}\theta$$

Considering the condition for partition (24),

$$\tau < \frac{p\beta + \beta - p\alpha}{p\alpha}\theta$$

always hold if

$$\frac{\beta}{p} < \frac{p\beta + \beta - p\alpha}{p\alpha} \Leftrightarrow \alpha < \frac{\beta(p + 1)}{p + \beta}$$

holds or

$$\alpha < \frac{\beta(p + 1)}{p + \beta} \Leftrightarrow S_X \succ G. \quad (29)$$

When the parameter α is larger, then there exist regions in which the reverse may occur.

$$\alpha > \frac{\beta(p + 1)}{p + \beta} \Leftrightarrow \begin{cases} S_X \succ G & \text{for } \frac{1}{p}\theta < \tau < \frac{p\beta - p\alpha + \beta}{p\alpha}\theta \\ G \succ S_X & \text{for } \frac{p\beta - p\alpha + \beta}{p\alpha}\theta < \tau < \frac{\beta}{p}\theta \end{cases} \quad (30)$$

The same logic can explain symmetrically the agents in K_6 .

RA1 case for K_3 and K_6 : Look at the case for RA1: $v(M) = \ln M$. We can conclude that $S_X \succ S_Y$ for RA1 agents in K_3 because

$$2\{V(K_3, S_X) - V(K_3, S_Y)\} = \ln \beta + \ln \theta - \ln p - \ln \tau \quad (31)$$

is positive for $\tau < \frac{\beta}{p}\theta$. If we now compare S_X and G , there are two cases depending on the parameter values for α and β . Look at

$$2\{V(K_3, S_X) - V(K_3, G)\} = 2 \ln \beta + \ln \theta - 2 \ln \alpha - \ln p - \ln \tau \quad (32)$$

and we find that the sign of this depends on

$$\tau \lesseqgtr \frac{\beta^2}{p\alpha^2}\theta$$

hence, the direction of inequality in $\beta \lesseqgtr \alpha^2$. We can easily see that

$$\beta > \alpha^2 \implies S_X \succ G \quad (33)$$

for the left side of the arrow in (33) implies

$$\frac{\beta}{p} < \frac{\beta^2}{p\alpha^2}.$$

Otherwise, the region in K_3 is divided into two in the following manner:

$$\beta < \alpha^2 \Leftrightarrow \begin{cases} S_X \succ G & \text{for } \frac{1}{p}\theta < \tau < \frac{\beta^2}{p\alpha^2}\theta \\ G \succ S_X & \text{for } \frac{\beta^2}{p\alpha^2}\theta < \tau < \frac{\beta}{p}\theta \end{cases} \quad (34)$$

Some workers in K_3 choose to make a specific investment in X while others make general investments. The same logic can explain symmetrically the agents in K_6 .

RA2 case for K_3 and K_6 : Now let us look at the case for RA2: $v(M) = -\frac{1}{M}$. To provide the integrated analysis, we postpone the analysis for the RA2 case for agents in K_3 and K_6 to the section under the title: RA2 case: K_4 and K_5 .

2.5.4 Partition K_4 and K_5 :

Now consider agents in partition K_4 who are in the region:

$$\frac{\beta}{p}\theta < \tau < \theta. \quad (35)$$

These are the most flexible workers. So all of them work in sector X (respectively, Y) when the terms of trade is $(p, 1)$ (respectively, $(1, p)$), irrespective of their investment decisions. Therefore, their expected utility can be written as

$$\begin{cases} V(K_4, S_X) &= \frac{1}{2}v(p\beta\theta) + \frac{1}{2}v(p\tau) \\ V(K_4, G) &= \frac{1}{2}v(p\alpha\theta) + \frac{1}{2}v(p\alpha\tau) \\ V(K_4, S_Y) &= \frac{1}{2}v(p\theta) + \frac{1}{2}v(p\beta\tau) \end{cases} . \quad (36)$$

Note that $V(K_4, G) = V(K_3, G)$ and $V(K_4, S_Y) = V(K_3, S_Y)$, but $V(K_4, S_X) \neq V(K_3, S_X)$.

RN case for K_4 and K_5 : Let us first look at the RN case: $v(M) = M$. By looking at

$$2\{V(K_4, S_X) - V(K_4, S_Y)\} = p(\beta - 1)(\theta - \tau)$$

which is positive because of (35) and $p > 0$ and $\beta > 1$, we can conclude that

$$S_X \succ S_Y$$

for agents in K_4 . Now we have to compare S_X with G . Look at

$$2\{V(K_4, S_X) - V(K_4, G)\} = p[(\beta - \alpha)\theta - (\alpha - 1)\tau] \quad (37)$$

The sign of (37) depends on the direction of the following inequality:

$$\tau \geq \frac{\beta - \alpha}{\alpha - 1}\theta.$$

There are three possible parameter spaces:

1. All agents in K_4 choose S_X if

$$1 < \frac{\beta - \alpha}{\alpha - 1} \Leftrightarrow \alpha < \frac{\beta + 1}{2} \quad (38)$$

because (35) and (38) imply

$$\tau > \frac{\beta - \alpha}{\alpha - 1}\theta.$$

2. Some agents choose S_X and others choose G depending on the location if

$$\frac{\beta}{p} < \frac{\beta - \alpha}{\alpha - 1} < 1 \Leftrightarrow \frac{\beta + 1}{2} < \alpha < \frac{(p + 1)\beta}{p + \beta}$$

holds. Therefore,

$$\begin{cases} 0 < \tau < \frac{\beta - \alpha}{\alpha - 1} \theta & \iff \text{invest } S_X \\ \frac{\beta - \alpha}{\alpha - 1} \theta < \tau < \theta & \iff \text{invest } G \end{cases}$$

should hold.

3. All agents in K_4 choose S_X if

$$\frac{\beta - \alpha}{\alpha - 1} < \frac{\beta}{p} \iff \frac{(p + 1)\beta}{p + \beta} < \alpha \quad (39)$$

because (35) and (39) imply

$$\tau > \frac{\beta - \alpha}{\alpha - 1} \theta$$

for all agents in K_4 .

The last case must combine with the analysis on K_3 . The same logic can explain symmetrically the agents in K_5 .

RA1 case for K_4 and K_5 : Let us first look at the RA1 case: $v(M) = \ln M$. By looking at

$$2 \{V(K_4, S_X) - V(K_4, S_Y)\} = 0,$$

we must conclude that $S_X \sim S_Y$ for all agents in K_4 . To compare with general investments, look at

$$2 \{V(K_4, S_X) - V(K_4, G)\} = \ln \beta - 2 \ln \alpha$$

whose sign depends on the direction of inequality in

$$\beta \gtrless \alpha^2.$$

If $\beta > \alpha^2$ holds, then we should know that $S_X \sim S_Y \succ G$ for all agents in K_4 . Therefore, we can say that

$$S_X \sim S_Y \iff \frac{\beta}{p} \theta < \tau < \theta.$$

But if $\beta < \alpha^2$ holds, then we should know that

$$\frac{\beta^2}{p\alpha^2} < \frac{\beta}{p}$$

holds. Therefore, we know that $G \succ S_X \sim S_Y$ for all agents in K_4 . This last case must combine with the analysis of K_3 . The same logic can explain symmetrically about the agents in K_5 .

RA2 case for K_4 and K_5 : Let us look at the RA2 case: $v(M) = -\frac{1}{M}$. Here we want to analyze partitions K_3 and K_4 together. The case for partitions K_5 and K_6 is similar, so we omit the analysis.

We first make it clear that agents belong to the regions:

$$\begin{cases} K_3 : \frac{1}{p}\theta < \tau < \frac{\beta}{p}\theta & \text{and} \\ K_4 : \frac{\beta}{p}\theta < \tau < \theta & . \end{cases}$$

We also list here the conditions for the preference of agents for investment types. Because $V(K_4, G) = V(K_3, G)$ and $V(K_4, S_Y) = V(K_3, S_Y)$ holds, the relationship between G and S_Y is the same between K_3 and K_4 . By checking the sign of the expression

$$2\{V(K_i, S_Y) - V(K_i, G)\} = \frac{(\beta - \alpha)\theta - \beta(\alpha - 1)\tau}{p\alpha\beta\theta\tau} \text{ for } i = 3, 4,$$

we can conclude that

$$\begin{cases} \tau > \frac{\beta - \alpha}{\beta(\alpha - 1)}\theta & \Leftrightarrow G \succ S_Y \text{ for } K_3 \text{ and } K_4 \\ \tau < \frac{\beta - \alpha}{\beta(\alpha - 1)}\theta & \Leftrightarrow S_Y \succ G \text{ for } K_3 \text{ and } K_4 \end{cases} \quad (40)$$

Now start looking at partition K_3 , to compare the two specific investments, we must look at the sign of the following expression:

$$2\{V(K_3, S_X) - V(K_3, S_Y)\} = \frac{\theta - (1 + p - \beta)\tau}{p\beta\theta\tau} \quad (41)$$

and we can conclude that

$$\begin{cases} \tau > \frac{1}{1 + p - \beta}\theta & \Leftrightarrow S_Y \succ S_X \text{ for } K_3 \\ \tau < \frac{1}{1 + p - \beta}\theta & \Leftrightarrow S_X \succ S_Y \text{ for } K_3 \end{cases} \quad (42)$$

If we compare S_X and G , we must check the sign of the following:

$$2\{V(K_3, S_X) - V(K_3, G)\} = \frac{\beta\theta - (\alpha + p\alpha - \beta)\tau}{p\alpha\beta\theta\tau} \quad (43)$$

and we can conclude that

$$\begin{cases} \tau > \frac{\beta}{\alpha + p\alpha - \beta}\theta & \Leftrightarrow G \succ S_Y \text{ for } K_3 \\ \tau < \frac{\beta}{\alpha + p\alpha - \beta}\theta & \Leftrightarrow S_Y \succ G \text{ for } K_3 \end{cases} \quad (44)$$

Look at partition K_4 . Let us first compare the two specific investments. Look at

$$2\{V(K_4, S_X) - V(K_4, S_Y)\} = \frac{\alpha(\beta - 1)(\tau - \theta)}{p\alpha\beta\theta\tau}$$

which is always positive because $\alpha > 0$, $\beta > 1$ and $\tau > \theta$ for all agents in K_4 . Therefore, we can conclude that

$$S_Y \succ S_X \quad \text{for all agents in } K_4 . \quad (45)$$

If we compare S_X and G for partition K_4 , we must check the sign of the following:

$$2 \{V(K_4, S_X) - V(K_4, G)\} = \frac{(\beta - \alpha)\tau - \beta(\alpha - 1)\theta}{p\alpha\beta\theta\tau}$$

and we can conclude that

$$\begin{cases} \tau > \frac{\beta(\alpha-1)\theta}{\beta-\alpha} \Leftrightarrow S_X \succ G \quad \text{for } K_4 \\ \tau < \frac{\beta(\alpha-1)\theta}{\beta-\alpha} \Leftrightarrow G \succ S_X \quad \text{for } K_4 \end{cases} . \quad (46)$$

To prepare the further analysis, we now claim the following two results:

Lemma 1 *The following relationship holds true.*

$$\frac{1}{p} < \frac{1}{p - (\beta - 1)} < \frac{\beta}{p} \quad (47)$$

Proof. The left side of (47) is obvious because $\beta > 1$. To prove the right side,

$$\frac{1}{p - (\beta - 1)} < \frac{\beta}{p} \Leftrightarrow \beta(\beta - 1) < p(\beta - 1)$$

which is true for $\beta < p$ by assumption. ■

The second result is given here.

Lemma 2 *The following relationship holds true.*

$$\frac{2\beta}{p+1} < \frac{p+\beta}{p+1} < \frac{2\beta}{\beta+1} < \frac{\beta(\beta+p)}{\beta^2+p} \quad (48)$$

Proof. From the left side, because $p > \beta$, it is obvious that

$$\frac{\beta+\beta}{p+1} < \frac{p+\beta}{p+1}.$$

Next, look at the mid inequality,

$$\frac{p+\beta}{p+1} < \frac{2\beta}{\beta+1} \Leftrightarrow (\beta-1)(\beta-p) < 0$$

which is true because $\beta > 1$ and $p > \beta$. From the right side,

$$\frac{2\beta}{\beta+1} < \frac{\beta(\beta+p)}{\beta^2+p} \Leftrightarrow (\beta-1)(\beta-p) < 0$$

which is also true. This concludes the proof. ■

Now, given the relative size in (48), we can analyze five cases depending on the size of the parameter α :

Case 1: $\alpha < \frac{2\beta}{p+1}$

Case 2: $\frac{2\beta}{p+1} < \alpha < \frac{p+\beta}{p+1}$

Case 3: $\frac{p+\beta}{p+1} < \alpha < \frac{2\beta}{\beta+1}$

Case 4: $\frac{2\beta}{\beta+1} < \alpha < \frac{\beta(\beta+p)}{\beta^2+p}$

Case 5: $\frac{\beta(\beta+p)}{\beta^2+p} < \alpha$

2.5.5 Case 1: $\alpha < \frac{2\beta}{p+1}$.

When $\alpha < \frac{2\beta}{p+1}$ holds, the following relationship holds true:

$$\frac{1}{p} < \frac{1}{1+p-\beta} < \frac{\beta}{p} < 1 \quad (49)$$

and

$$\left\{ \begin{array}{l} \frac{\beta(\alpha-1)}{\beta-\alpha} < \frac{\beta}{p} \\ 1 < \frac{\beta-\alpha}{\beta(\alpha-1)} \\ 1 < \frac{\beta}{\alpha+p\alpha-\beta} \end{array} \right. . \quad (50)$$

This is because $\alpha < \frac{2\beta}{p+1}$ automatically imply $\alpha < \frac{p+\beta}{p+1}$ and $\alpha < \frac{2\beta}{\beta+1}$, and we can derive a few results:

$$\left\{ \begin{array}{l} \alpha < \frac{2\beta}{p+1} \Leftrightarrow 1 < \frac{\beta}{\alpha+p\alpha-\beta} \\ \alpha < \frac{p+\beta}{p+1} \Leftrightarrow \frac{\beta(\alpha-1)}{\beta-\alpha} < \frac{\beta}{p} < \frac{\beta}{\alpha+p\alpha-\beta} \\ \alpha < \frac{2\beta}{\beta+1} \Leftrightarrow \frac{\beta(\alpha-1)}{\beta-\alpha} < 1 < \frac{\beta-\alpha}{\beta(\alpha-1)} \end{array} \right.$$

Together with (47), (49) and (50) can be shown. If we summarize all conditions (40)-(46), we can state the following results for the case $\alpha < \frac{2\beta}{p+1}$.

$$\left\{ \begin{array}{l} \frac{1}{p}\theta < \tau < \frac{1}{1+p-\beta}\theta \Leftrightarrow S_X \succ S_Y \succ G \\ \frac{1}{1+p-\beta}\theta < \tau < \frac{\beta}{p}\theta \Leftrightarrow S_Y \succ S_X \succ G \\ \frac{\beta}{p}\theta < \tau < \theta \Leftrightarrow S_Y \succ S_X \succ G \end{array} \right.$$

2.5.6 Case 2: $\frac{2\beta}{p+1} < \alpha < \frac{p+\beta}{p+1}$.

When $\alpha < \frac{p+\beta}{p+1}$ holds, then the following relationship holds true:

$$\frac{1}{p} < \frac{1}{1+p-\beta} < \frac{\beta}{p} < \frac{\beta}{\alpha+p\alpha-\beta} < 1 \quad (51)$$

and

$$\begin{cases} \frac{\beta(\alpha-1)}{\beta-\alpha} < \frac{\beta}{p} \\ 1 < \frac{\beta-\alpha}{\beta(\alpha-1)} \end{cases}.$$

Because $\alpha < \frac{p+\beta}{p+1}$ automatically imply $\alpha < \frac{2\beta}{\beta+1}$, we can derive a few results:

$$\begin{cases} \frac{2\beta}{p+1} < \alpha \Leftrightarrow \frac{\beta}{\alpha+p\alpha-\beta} < 1 \\ \alpha < \frac{p+\beta}{p+1} \Leftrightarrow \frac{\beta(\alpha-1)}{\beta-\alpha} < \frac{\beta}{p} < \frac{\beta}{\alpha+p\alpha-\beta} \\ \alpha < \frac{2\beta}{\beta+1} \Leftrightarrow \frac{\beta(\alpha-1)}{\beta-\alpha} < 1 < \frac{\beta-\alpha}{\beta(\alpha-1)} \end{cases}$$

Together with (47), (51) can be shown. If we summarize all conditions (40)-(46), we can state the following results for the case $\frac{2\beta}{p+1} < \alpha < \frac{p+\beta}{p+1}$.

$$\begin{cases} \frac{1}{p}\theta < \tau < \frac{1}{1+p-\beta}\theta \Leftrightarrow S_X \succ S_Y \succ G \\ \frac{1}{1+p-\beta}\theta < \tau < \frac{\beta}{p}\theta \Leftrightarrow S_Y \succ S_X \succ G \\ \frac{\beta}{p}\theta < \tau < \theta \Leftrightarrow S_Y \succ S_X \succ G \end{cases}$$

2.5.7 Case 3: $\frac{p+\beta}{p+1} < \alpha < \frac{2\beta}{\beta+1}$.

When $\frac{p+\beta}{p+1} < \alpha < \frac{2\beta}{\beta+1}$ holds, then the following relationship holds true:

$$\frac{1}{p} < \frac{1}{1+p-\beta} < \frac{\beta}{\alpha+p\alpha-\beta} < \frac{\beta}{p} < \frac{\beta(\alpha-1)}{\beta-\alpha} < 1 < \frac{\beta-\alpha}{\beta(\alpha-1)} \quad (52)$$

We can derive a few results:

$$\begin{cases} \frac{p+\beta}{p+1} < \alpha \Leftrightarrow \frac{\beta}{\alpha+p\alpha-\beta} < \frac{\beta}{p} < \frac{\beta(\alpha-1)}{\beta-\alpha} \\ \alpha < \frac{2\beta}{\beta+1} \Leftrightarrow \frac{\beta(\alpha-1)}{\beta-\alpha} < 1 < \frac{\beta-\alpha}{\beta(\alpha-1)} \end{cases}$$

We also have to check if

$$\frac{\beta}{\alpha+p\alpha-\beta} \leq \frac{1}{1+p-\beta} \quad (53)$$

cannot occur. Suppose it does. Then we will encounter a contradiction because there must exist agents within

$$\frac{\beta}{\alpha + p\alpha - \beta}\theta \leq \tau \leq \frac{1}{p - (\beta - 1)}\theta \quad (54)$$

whose preference can be represented by the following three:

$$\begin{aligned} S_Y &\succ G \\ S_X &\succeq S_Y \\ S_X &\succeq G \end{aligned} \quad (55)$$

which violates the transitivity of the preference.

First, $S_Y \succ G$ because all agents in (54) must satisfy

$$\tau < \frac{\beta - \alpha}{\beta(\alpha - 1)}\theta$$

since (52) and (53). $S_X \succeq S_Y$ because of the right side in (54) and whose preference can also be represented by $S_X \succeq G$ because of the left side of (54). This leads to (55).

If we summarize all conditions (40)-(46), we can state the following results for the case 3:

$$\left\{ \begin{array}{ll} \frac{1}{p}\theta < \tau < \frac{1}{1+p-\beta}\theta & \Leftrightarrow S_X \succ S_Y \succ G \\ \frac{1}{1+p-\beta}\theta < \tau < \frac{\beta}{\alpha+p\alpha-\beta}\theta & \Leftrightarrow S_Y \succ S_X \succ G \\ \frac{\beta}{\alpha+p\alpha-\beta}\theta < \tau < \frac{\beta(\alpha-1)}{\beta-\alpha}\theta & \Leftrightarrow S_Y \succ G \succ S_X \\ \frac{\beta(\alpha-1)}{\beta-\alpha}\theta < \tau < \theta & \Leftrightarrow S_Y \succ S_X \succ G \end{array} \right.$$

2.5.8 Case 4: $\frac{2\beta}{\beta+1} < \alpha < \frac{\beta(\beta+p)}{\beta^2+p}$.

When $\frac{2\beta}{\beta+1} < \alpha < \frac{\beta(\beta+p)}{\beta^2+p}$ holds, then the following relationship holds true:

$$\frac{1}{p} < \frac{1}{1+p-\beta} < \frac{\beta}{\alpha+p\alpha-\beta} < \frac{\beta}{p} < \frac{\beta-\alpha}{\beta(\alpha-1)} < 1 < \frac{\beta(\alpha-1)}{\beta-\alpha} \quad (56)$$

Because $\frac{2\beta}{\beta+1} < \alpha$ automatically implies $\frac{p+\beta}{p+1} < \alpha$, we can derive a few results:

$$\left\{ \begin{array}{ll} \frac{p+\beta}{p+1} < \alpha & \Leftrightarrow \frac{\beta}{\alpha+p\alpha-\beta} < \frac{\beta}{p} < \frac{\beta(\alpha-1)}{\beta-\alpha} \\ \frac{2\beta}{\beta+1} < \alpha & \Leftrightarrow \frac{\beta-\alpha}{\beta(\alpha-1)} < 1 < \frac{\beta(\alpha-1)}{\beta-\alpha} \\ \alpha < \frac{\beta(\beta+p)}{\beta^2+p} & \Leftrightarrow \frac{\beta}{p} < \frac{\beta-\alpha}{\beta(\alpha-1)} \end{array} \right. \quad (57)$$

In addition to the above, we must check if

$$\frac{\beta}{\alpha + p\alpha - \beta} \leq \frac{1}{1 + p - \beta} \quad (58)$$

cannot occur. Suppose it does. Then we will encounter a contradiction because there must exist agents within

$$\frac{\beta}{\alpha + p\alpha - \beta}\theta \leq \tau \leq \frac{1}{p - (\beta - 1)}\theta \quad (59)$$

whose preferences can be represented as follows:

$$\begin{aligned} S_Y &\succ G \\ S_X &\succeq S_Y \\ S_X &\succeq G \end{aligned} \quad (60)$$

which violates the transitivity of the preference.

First, $S_Y \succ G$ because all agents in (59) must satisfy

$$\tau < \frac{\beta - \alpha}{\beta(\alpha - 1)}\theta$$

since (56). $S_X \succeq S_Y$ because of the right side in (59) and whose preference can also be represented by $S_X \succeq G$ because of the left side of (59). This leads to (60).

If we summarize all conditions (40)-(46), we can state the following results for case 4.

$$\left\{ \begin{array}{ll} \frac{1}{p}\theta < \tau < \frac{1}{1+p-\beta}\theta & \Leftrightarrow S_X \succ S_Y \succ G \\ \frac{1}{1+p-\beta}\theta < \tau < \frac{\beta}{\alpha+p\alpha-\beta}\theta & \Leftrightarrow S_Y \succ S_X \succ G \\ \frac{\beta}{\alpha+p\alpha-\beta}\theta < \tau < \frac{\beta-\alpha}{\beta(\alpha-1)}\theta & \Leftrightarrow S_Y \succ G \succ S_X \\ \frac{\beta-\alpha}{\beta(\alpha-1)}\theta < \tau < \theta & \Leftrightarrow G \succ S_Y \succ S_X \end{array} \right.$$

2.5.9 Case 5: $\frac{\beta(\beta+p)}{\beta^2+p} < \alpha$.

When $\frac{\beta(\beta+p)}{\beta^2+p} < \alpha$ holds, then the following relationship holds true:

$$\frac{1}{p} < \frac{1}{1+p-\beta} < \frac{\beta}{\alpha+p\alpha-\beta} < \frac{\beta-\alpha}{\beta(\alpha-1)} < \frac{\beta}{p} < 1 < \frac{\beta(\alpha-1)}{\beta-\alpha} \quad (61)$$

Because $\frac{\beta(\beta+p)}{\beta^2+p} < \alpha$ automatically imply $\frac{p+\beta}{p+1} < \alpha$ and $\frac{2\beta}{\beta+1} < \alpha$, we can derive a few results:

$$\left\{ \begin{array}{l} \frac{p+\beta}{p+1} < \alpha \Leftrightarrow \frac{\beta}{\alpha+p\alpha-\beta} < \frac{\beta}{p} < \frac{\beta(\alpha-1)}{\beta-\alpha} \\ \frac{2\beta}{\beta+1} < \alpha \Leftrightarrow \frac{\beta-\alpha}{\beta(\alpha-1)} < 1 < \frac{\beta(\alpha-1)}{\beta-\alpha} \\ \frac{\beta(\beta+p)}{\beta^2+p} < \alpha \Leftrightarrow \frac{\beta-\alpha}{\beta(\alpha-1)} < \frac{\beta}{p} \end{array} \right. \quad (62)$$

In addition to the above, we must check if

$$\frac{\beta-\alpha}{\beta(\alpha-1)} \leq \frac{\beta}{\alpha+p\alpha-\beta} \quad (63)$$

cannot occur. Suppose it does. Then we will encounter a contradiction because there must exist the agents within

$$\frac{\beta-\alpha}{\beta(\alpha-1)}\theta \leq \tau \leq \frac{\beta}{\alpha+p\alpha-\beta}\theta \quad (64)$$

whose preference can be represented by the following three:

$$\begin{array}{l} G \succeq S_Y \\ S_Y \succ S_X \\ S_X \succeq G \end{array} \quad (65)$$

which violates the transitivity of the preference.

First, $S_Y \succ S_X$ because all agents in (64) must satisfy

$$\tau > \frac{1}{1+p-\beta}\theta$$

since (61). $G \succeq S_Y$ because of the left side in (64) and whose preference can also be represented by $S_X \succeq G$ because of the right side of (64). This leads to (65).

If we summarize all conditions (40)-(46), we can state the following results for the case 5:

$$\left\{ \begin{array}{l} \frac{1}{p}\theta < \tau < \frac{1}{1+p-\beta}\theta \Leftrightarrow S_X \succ S_Y \succ G \\ \frac{1}{1+p-\beta}\theta < \tau < \frac{\beta}{\alpha+p\alpha-\beta}\theta \Leftrightarrow S_Y \succ S_X \succ G \\ \frac{\beta}{\alpha+p\alpha-\beta}\theta < \tau < \frac{\beta-\alpha}{\beta(\alpha-1)}\theta \Leftrightarrow S_Y \succ G \succ S_X \\ \frac{\beta-\alpha}{\beta(\alpha-1)}\theta < \tau < \theta \Leftrightarrow G \succ S_Y \succ S_X \end{array} \right.$$

2.5.10 Summary of Cases

Case 1-3 can be summarized to be

$$\left\{ \begin{array}{ll} \frac{1}{p}\theta < \tau < \frac{1}{1+p-\beta}\theta & \iff \text{invest } S_X \\ \frac{1}{1+p-\beta}\theta < \tau < \theta & \iff \text{invest } S_Y \end{array} \right.$$

for the agents in K_3 and K_4 with $\alpha < \frac{2\beta}{\beta+1}$.

Case 4-5 can be summarized to be

$$\left\{ \begin{array}{ll} \frac{1}{p}\theta < \tau < \frac{1}{1+p-\beta}\theta & \iff \text{invest } S_X \\ \frac{1}{1+p-\beta}\theta < \tau < \frac{\beta-\alpha}{\beta(\alpha-1)}\theta & \iff \text{invest } S_Y \\ \frac{\beta-\alpha}{\beta(\alpha-1)}\theta < \tau < \theta & \iff \text{invest } G \end{array} \right.$$

for the agents in K_3 and K_4 with $\frac{2\beta}{\beta+1} < \alpha$.

The same logic can explain symmetrically the agents in K_5 and K_6 . And together with the analysis for K_1 and K_2 (K_7 and K_8), this proves the result in Theorem 3.

3 Conclusion

In this paper, I extended the model of occupational choice to a two-period case. I now allow human capital skills investments by individual agents and will examine the incentives by individuals when they start out from different combinations of multi-dimensional innate capabilities. In general, specialization enhances the productive capabilities of society through the division of labor. Investing in specific skills, however, may be a risky strategy for an individual agent if there is no insurance market under uncertainty. Human capital investment in general skills is usually considered to be a form of self-insurance. Previous work in this field dealt with a case where every individual is identical before his human capital investment. The model introduced in this paper analyzed the investment decision problem when individual agents are heterogeneous in the sense of both absolute and comparative advantages in different sectors. The paper analyzed a case in which we encounter terms of trade uncertainty.

I identify the conditions determining when agents will invest in general skills and when they specialize in a particular sector. Depending on parameter values: α , β , and p , it is quite possible for all individuals to invest in their innately strong skills even if there is no insurance market. For the agents with a narrower comparative advantage (those who are close to the 45-degree

line in the unit square), general investment may occur when parameter α is relatively stronger. For risk neutral agents, I did not find anyone who invested in human capital skills in the direction of their innate comparative disadvantage.

Our RN (risk neutral) case results are favored toward specific investment in skills where agents have innate comparative advantages, probably for the following reasons. First, the agents are assumed to be risk neutral rather than risk averse. Risk neutral agents simply try to maximize expected income rather than expected utility. Therefore, given the set up of the model in this paper, agents find it worthwhile to enhance their strong skills rather than bolstering their weaker skills. Second, investment technology is assumed to be one of constant returns to scale. The investment coefficients α and β are multiplied proportionately by θ and τ , the agents' skill levels prior to investment. Thus, agents who already had a high θ (before investment) will derive more benefit from training through either general or specific investment than agents with a lower value of θ . Agents with stronger comparative advantage in one sector will find themselves better off when they invest in the skills that deliver the best return on investment. Third, the structure of uncertainty in this paper is limited to a very specific case of equal probabilities over two states of nature. The choice of this uncertainty structure is done in favor of simplicity, but it is true that individual incentives will change if we change the probability distribution. Future research can address these extensions.

When we look at cases of risk averse agents, things become very different. The relative relationship between the specific versus general investments is similar: general investments are made only when the parameter α is larger and they occur only for those who are close to the 45-degree line. However, the analysis for specific investments for risk averse agents shows quite different results from that of risk neutral agents. We found cases where agents invest in specific skills in the opposite direction. Some agents try to specifically invest in their weak skills. In the end, for some agents, there is a reversal of comparative advantage if we compare the ex ante and ex post of the agent's human capital investment.

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A The Relationship between CPI and Welfare for Homothetic Demand

When the utility function $u(x, y)$ is homothetic and strictly monotone, then we can write its indirect utility function as separable in the price part $C(P_X, P_Y)$ and the income $v(M)$ part, i.e.,

$$V(P_X, P_Y, I) = C(P_X, P_Y) \cdot v(M) \quad (66)$$

where $v(\cdot)$ is an strictly increasing function.

Claim 1 $\frac{1}{C(P_X, P_Y)}$ is a measure of consumer price index.

When we want to compare welfare across different times, the change in relative prices is always a nuisance. But if we assume that $C(P_X, P_Y)$ is symmetric in P_X and P_Y , then we can conclude that

$$C(p, 1) = C(1, p) \quad (67)$$

Thus, when equation (67) holds, then the comparison of income function $v(M)$ is a one-to-one relationship with the comparison of welfare.