RIETI Discussion Paper Series 11-E-012

Financial Crises and Assets as Media of Exchange

KOBAYASHI Keiichiro
RIETI

Research Institute of Economy, Trade & Industry, IAA
The Research Institute of Economy, Trade and Industry
http://www.rieti.go.jp/en/

RIETI Discussion Paper Series 11-E-012
February 2011

Financial Crises and Assets as Media of Exchange

KOBAYASHI Keiichiro*
（RIETI／Hitotsubashi University）

Abstract

We construct a monetary model of financial crises that can explain two characteristic features of the global financial crisis in 2008/2009, namely, the widespread freeze of asset transactions and a sharp contraction in aggregate output. We assume that the assets, such as real estate, work as media of exchange on a de facto basis in the goods market. In the financial crisis, excessively indebted investors hoard the assets hoping for a miraculous rise in their value (risk-shifting behavior), although the asset hoarding hinders the assets from working as media of exchange in the goods trading. Accordingly, the asset hoarding causes the disappearance of a significant portion of broad "money," which directly results in a contraction in aggregate production. Since the root of the problem is an external diseconomy caused by excessive indebtedness of investors, fiscal and monetary policies and debt reduction for investors have almost equivalent effects in terms of recovery efforts in a financial crisis.

* Hitotsubashi University; Research Institute of Economy, Trade, and Industry (RIETI); Canon Institute for Global Studies (CIGS); and Chuo University. E-mail: kobayashi-keiichiro@rieti.go.jp.

1 Introduction

The global financial crisis in 2008/2009 show the following features:

- Freeze of transactions in the asset markets,
- sharp contraction of the aggregate output.

See, for example, Brunnermeier et al. (2009) for the analysis of the global crisis. Diamond and Rajan (2009) show the risk-shifting effect can explain the first feature (i.e., the market freeze), given exogenous increase in the needs for liquidity. In this paper we develop a general equilibrium model in which both the first and the second features can be explained. The bottom line of our hypothesis is that this type of financial crises is a disappearance or vaporization from the marketplace of a significant portion of broad "money," which we vaguely define as general assets that can be very easily exchanged with cash so that they work as media of exchange on de facto basis. We formalize this notion of financial crises in our model, in which optimal behaviors of agents cause the disappearance of media of exchange from the market. Key ingredients are the risk-shifting effect à la Diamond and Rajan caused by risky debt and limited liability, and the assumption that the assets, such as real estate, work as media of exchange in our economy. This assumption is a shortcut to formalize the existence in reality of very liquid asset markets, in which the asset holders can easily obtain cash at anytime by selling the assets or obtaining loans secured by the assets as collateral. Although we assume in our model that the assets themselves work as inside money, we may be able to interpret this assumption as assuming the existence of banks that can issue bank deposits as inside money, and that the asset holders can easily obtain the deposit money from the banks by selling the assets or by borrowing collateral-secured loans.

There are three motivations to construct this model. First, we need a theory that consistently explain the above features of the global financial crisis in 2008/2009. Our hypothesis is that disappearance of significant portion of inside money may be the key factor. The second motivation is to provide a unified framework for policy analysis in which we are potentially enabled to compare and evaluate the efficacy of the various policy responses to the global financial crisis, in particular, fiscal stimulus, monetary easing and financial stabilization (e.g., capital injections and bad asset disposals), which crucially includes debt reduction of the borrowers. The third motivation is rather technical: we intend to construct a model of financial crises that can be easily embedded in the standard framework of the neoclassical growth theory and its variants for business cycle research, i.e., the dynamic stochastic general equilibrium (DSGE) models. As Aruoba, Waller and Wright (2007) demonstrate, the Lagos-Wright framework, which our model builds on, shares its basic structure with the neoclassical growth models so that it can be easily applied to quantitative business cycle research. In addition, nominal variables naturally arise and monetary policy issues can be easily analyzed. This feature of our model may enable us to analyze the (ordinary) business cycles and financial crises in a unified framework.

1.1 Intuition and a simple example

Intuition is outlined as follows. When an investors who hold, for example, real estate as their assets are overly indebted, they will choose to hoard their assets hoping for a miraculous rise in the price because they have nothing more to lose under the limited liability. While the hoarding of the assets is the optimal behavior for the investors with limited liability, it may be socially suboptimal and in that case excessive risk is shifted on the lenders to the investors. This is the market freeze due to the risk-shifting effect described by Diamond and Rajan (2009), which can be understood as a variant of moral hazard of overly indebted agents known as the *gambling for resurrection* in the banking literature (see, for example, Freixas and Rochet, 2008).

In addition, we assume that the assets are used as media of exchange just like money in trading of the goods. This assumption corresponds to the existence in reality of very liquid asset markets, in which the asset holders can sell the assets or obtain loans secured by the assets as collateral at anytime they want. Instead of formulating a liquid asset market, we assume for simplicity the asset-in-advance (AIA) constraint, (2), as an equivalence of the cash-in-advance (CIA) constraint in monetary models (see Lucas and

Stokey, 1987). [1] In this setting, the investors' hoarding of their assets have a significant external effect through hindering the assets from working as media of exchange in the goods market. This adverse effect can be translated as a disappearance of some portion of broad money or as a contraction of demand for the goods, which leads to a shrinkage of the aggregate output. This is the second feature of the 2008/2009 global financial crisis that we intend to describe in our model.

The following is a simple example that portrays the intuition. It shows that if debt is large, production and trades do not take place (a financial crisis), and that if debt is small, the goods are produced and traded.

There are two periods: $t = 1$ (Day) and $t = 2$ (Night). There are two agents: an investor and a producer. They are risk neutral and want to maximize their consumption at $t = 2$. At $t = 1$, the investor holds real estate, $k = 1$, as her asset and debt, L, as her liability. (We do not specify the lender of L.) The producer can produce the intermediate goods, q, with the utility cost at $t = 1$. The utility cost is $c(q) = c \cdot q$ in the unit of the consumption goods. The investor can buy q in exchange for k in the day market.

At $t = 2$, new endowment of the intermediate goods, q_e, are given to the investor. In this period, k and $Q = q + q_e$ are traded in a perfectly competitive market. The consumption goods, y, are produced from k and Q by the following technology: $y = Ak^{1-\alpha}Q^{\alpha} + Rk$ if the boom occurs and $y = Ak^{1-\alpha}Q^{\alpha}$ if the boom does not occur. The boom occurs with probability ϵ, where ϵ is close to zero. Since the market is competitive, the price of q in the night market is $w = \alpha A(k/Q)^{1-\alpha}$, and the price of k is $\tilde{a} = (1-\alpha)A(Q/k)^{\alpha}$ with probability $1-\epsilon$ and $\tilde{a} = (1-\alpha)A(Q/k)^{\alpha} + R$ with probability ϵ. We assume that $\alpha > 1/2$.

The producer's optimization problem is to choose q in the day market to maximize $pq - cq$, where p is the price of q (in the units of the consumption goods) in the day market. Assuming that the day market is competitive, we have $p = c$ in equilibrium.

[1] Although the AIA constraint says that the assets themselves work as inside money, we may be able to assume alternatively that there exist banks that can issue bank deposits as inside money and the asset holders can easily obtain the deposit money by selling the assets to the banks or by borrowing bank loans putting up the assets as collateral.

The investor's problem at $t = 1$ is to choose whether to hoard $k = 1$ or to buy the intermediate good, q, in exchange for k in order to solve the following problem:

$$\max\left\{E[\tilde{a}k - L]_+, \max_q E\left[wq - L + \tilde{a}\left[k - \frac{cq}{E[\tilde{a}]}\right]\right]_+\right\}, \tag{1}$$

$$\text{s.t.} \quad cq \leq E[\tilde{a}]k, \tag{2}$$

where the first term in the right-hand side is the gain from hoarding k and the second term is the gain from purchasing q in exchange for k, $E[\cdot]$ is the expectation operator, $\tilde{a}$ is the price of k at $t = 2$, $E[\tilde{a}]$ is the price of k at $t = 1$, and $[x]_+ = x$ if $x \geq 0$ and $[x]_+ = 0$ if $x < 0$. Note that the investor is under the limited liability. The constraint that $cq \leq E[\tilde{a}]k$ is the asset-in-advance (AIA) constraint in the goods market at $t = 1$. We assume the following for the parameter values:

$$c\left\{\left(\frac{\alpha A}{c}\right)^{\frac{1}{1-\alpha}} - q_e\right\} < (1-\alpha)A\left(\frac{\alpha A}{c}\right)^{\frac{\alpha}{1-\alpha}}, \tag{3}$$

which implies (2) does not bind in the equilibrium where the investor buys q in exchange for k. This economy has two types of equilibrium depending on the value of L: if $L < \overline{L}$, an equilibrium with $q = q^* > 0$ exists, and if $\underline{L} < L < R$, an equilibrium with $q = 0$ exists, where

$$\begin{aligned}
q^* &= \left(\frac{\alpha A}{c}\right)^{\frac{1}{1-\alpha}} - q_e, \\
\overline{L} &= (1-\alpha)A\left(\frac{\alpha A}{c}\right)^{\frac{\alpha}{1-\alpha}}, \\
(1-\epsilon)\underline{L} &= \frac{\alpha A q_e^{\alpha-1}}{c}[(1-\alpha)Aq_e^\alpha + \epsilon R] - \epsilon[(1-\alpha)Aq_e^\alpha + R].
\end{aligned}$$

Proof of the above claim is as follows. First, we show that if $L < \overline{L}$, the investors choose to buy q. We assume and justify that (2) does not bind in the equilibrium where $q > 0$. Since (2) does not bind, $w = p = c$ and $Q = Q^* \equiv \left(\frac{\alpha A}{c}\right)^{\frac{1}{1-\alpha}}$. The investor buys $q^* = Q^* - q_e$ in this equilibrium. Condition (3) implies that cq^* is strictly smaller than $E[\tilde{a}]k = (1-\alpha)A(Q^*)^\alpha + \epsilon R$. Therefore, it is justified that (2) does not bind in this equilibrium. It is easily shown that the investor does not strictly prefer hoarding k to purchasing q if and only if $L < \overline{L}$. Therefore, the investor is indifferent between hoarding k and purchasing q in this equilibrium, and they actually buys $q^* = Q^* - q_e$.

Second, similarly, the necessary and sufficient condition for the investor to strictly prefer hoarding k to buying q is that $\epsilon[(1-\alpha)AQ^\alpha + R - L]_+ > E[wq^s - L + \tilde{a}[k - cq^s/E[\tilde{a}]]]_+$, where q^s is the solution to $\max_q E[wq - L + \tilde{a}[k - cq/E[\tilde{a}]]]_+$ subject to $cq \le E[\tilde{a}]k$. Given $w = \alpha A q_e^{\alpha-1}$ and $E[\tilde{a}] = (1-\alpha)Aq_e^\alpha + \epsilon R$, this condition is equivalent to $\underline{L} < L < R$. In the case where the investor hoards k, the production becomes zero so that $q = 0$ and $Q = q_e$.

Note that if $0 \le L < \min\{\underline{L}, \overline{L}\}$, only the equilibrium with $q = q^*$ exists; and if $\max\{\underline{L}, \overline{L}\} < L < R$, only the equilibrium with $q = 0$ exists. Note also that if $\underline{L} < \overline{L}$, both the equilibrium with $q = q^*$ and one with $q = 0$ exist for $\underline{L} < L < \overline{L}$; and if $\underline{L} > \overline{L}$, there exists no equilibrium for $\overline{L} < L < \underline{L}$.

1.2 Related Literature (to be completed)

The structure of our model is similar to the monetary models by Lucas and Stokey (1987) and Lagos and Wright (2005). The motivation of our paper is most close to Beaudry and Lahiri (2009) in that they intend to explain not only collapse in the credit market but also shrinkage of the aggregate output. In their model, adverse selection due to information asymmetry on asset quality causes the credit freezes and the output declines. But the policy implications of Beaudry and Lahiri's model may be quite different from ours because the financial crisis in their model is one of multiple equilibria, the realization of which solely depends on pessimistic expectations. Shreifer and Vishny (2009) is also close to our paper in showing that changes in asset prices due to noise traders' sentiment may exacerbate real inefficiency in investments through securitization and leverage. Their model may not be appropriate, however, for analyzing and comparing fiscal, monetary and debt-restructuring policies, while our paper tries to provide a unified framework for comparing the efficacies of macro and financial policies.

The organization of this paper is as follows. In the next section, we construct a monetary general equilibrium model in which the example in Section 1.1 is embedded.

Section 3 provides policy analysis and Section 4 concludes.

2 Model

A key friction in our model is the necessity of media of exchange in the goods market, which is formulated as a constraint similar to the cash-in-advance constraint in Lucas and Stokey (1987). The model builds on a simplified variant of the monetary model developed by Lagos and Wright (2005). Although Lagos and Wright focus on search markets, the markets in our economy are competitive, just like in Berentsen, Camera and Waller (2007), and there are no search frictions. We use the Lagos-Wright framework in order to analyze the interaction between competing media of exchange, that is, cash and capital.

2.1 Setup

The model is a closed economy with discrete time that continues from zero to infinity: $t = 0, 1, 2, \cdots, +\infty$. In each date t, there are two competitive markets that open sequentially: the day market and the night market. There are continua of three types of agents in this economy: consumers, producers, and investors. The measure of each of the three agents is normalized to one. There is also a government (or a central bank) that can provide fiat currency and impose tax on the consumers. Consumers are infinitely lived. Producers are born in the date-t day market and die in the date-t night market after they consume their profits. Investors are born in the date-$(t-1)$ night market and die in the date-t night market after they consume their profits. There are two assets (cash and capital) and two goods (the intermediate goods and the consumption goods) traded in this economy. The consumption goods are the numeraire in this economy. All these assets and goods are divisible. The government injects cash, m, to the consumers in the night market. The cash is not depletable. Nature endows each consumer with one unit of capital, $k = 1$, in the date-$(t-1)$ night market. The producers and the investors have no endowment when they are born. The capital is used in production of the consumption goods in the date-t night market and is completely depleted to zero in the date-t night market after the production. The producers can produce the intermediate goods, q, in the day market, incurring the utility cost of $c(q)$ in the units of the consumption goods, where $c'(\cdot) > 0$ and $c''(\cdot) \geq 0$. The consumption goods, y, are produced from k and q in the night market by the following technology which varies depending on the macroeconomic environment: $y = Ak^{1-\alpha}q^{\alpha} + \tilde{R}k$, where $\tilde{R}$ is a random variable realized in the night market and

$$\tilde{R} = \begin{cases} 0 & \text{with probability } 1 - \tilde{\epsilon}, \\ R & \text{with probability } \tilde{\epsilon}. \end{cases}$$

$\tilde{R}$ is a macroeconomic variable that represents the boom in the night market if $\tilde{R} = R$, and otherwise if $R = 0$. The probability of the occurrence of the boom $\tilde{\epsilon}$ is also a random variable that is revealed at the beginning of the date-t day market:

$$\tilde{\epsilon} = \begin{cases} 1 & \text{with probability } \ 1 - \delta, \\ \epsilon \ (\ll 1) & \text{with probability } \ \delta. \end{cases} \tag{4}$$

This means that the agents in the date-$(t-1)$ night market have the prior that $\tilde{\epsilon} = 1$ with probability $1 - \delta$ and $\tilde{\epsilon} = \epsilon$ with probability δ. Later in this section, we consider the case of *equilibrium with euphoria*, in which the economic fundamental is $\delta = 1$, which means that the boom will come with a very small probability, ϵ, while the agents in date-$(t-1)$ night market are possessed by euphoria and mistakenly believe the prior that δ is close to zero: $\delta = \delta^e$ $(\ll 1)$. We define the *high state* as the state of the day market where $\tilde{\epsilon} = 1$, and the *low state* as that where $\tilde{\epsilon} = \epsilon$. We put subscript h on the variables in the high state and subscript l on those in the low state.

We assume that the night market is a perfectly competitive Walrasian market, in which there are no frictions or information asymmetry, and that the day market is a competitive but anonymous market in which trade credit between the sellers and the buyers of the intermediate goods is not available and payment by "money" is necessary in trading of the goods. We assume that in the day market, the capital, k, works as a means of payment as well as cash, m. In other words, we assume that k is an inside money in our economy. This assumption is a shortcut to formalize the existence of a very liquid asset market in which agents who hold k can immediately obtain cash by selling k or by borrowing (from banks) against k as collateral. (While we assume in this paper that capital stock itself can work as inside money, we may be able to assume alternatively as in Kobayashi (2009a) that there exist banks that can issue bank deposits as inside money and the capital holders can easily obtain the deposit money by selling the capital stock to the banks or by borrowing bank loans putting up the capital stock as collateral.)

We assume that the consumers cannot maintain the capital that they are endowed with properly and the capital will depreciate to zero at the beginning of the date-t day market if the consumer keeps the capital from the date-$(t-1)$ night market to the date-t day market. Only the investors can preserve the capital from the date-$(t-1)$ night to the date-t day market. Because the investors have nothing to pay in exchange for k in the date-$(t-1)$ night market, all they can do is to borrow k from the consumers with a risky debt contract, in which an investor promises to pay a fixed amount in terms of the fiat currency in the date-t night market, and if she fails to pay the fixed amount, the consumer obtains all remaining things that the investor possesses in the date-t night market. It is well known that the risky debt contract is optimal under a certain set of assumptions on the information structure (see Gale and Hellwig [1985]) and we implicitly assume that these assumptions hold in our economy. We also assume that although the goods trading is anonymous in the day market the consumers can trace the borrowers (the investors) from the date-$(t-1)$ night market to the date-t night market so that inter-period debt contract between a consumer and an investor is feasible.

Under the risky debt contract the investors enjoy limited liability, which is the source of the risk-shifting effect (Diamond and Rajan [2009], Allen and Gale [2000]). In the date-t day market, the investors choose whether to hoard k until the night market or to buy the intermediate goods, q, from the producers in exchange for k. In the day market, the consumers too can buy the intermediate goods by paying cash, m. The producers produce and sell q to maximize their expected profits. We assume that the investors can maintain the capital k from the day market to the night market, and that the producers can maintain the capital k properly too until the night market, if they receive it as compensation for q in the day market. On the other hand, the producers cannot preserve the intermediate goods that they produce in the day market until the night market, while the investors and the consumers can.

In the date-t night market, k and q are traded competitively and the consumption goods, y, are produced competitively. The night market is Walrasian, i.e., trade credit is available for anyone, and there is no need for media of exchange.

2.2 Optimization problem

We first consider the producers' optimization problem. A producer has no endowments when she is born in the day market and she chooses the amount of the intermediate goods that she produces to maximize her expected profits. Thus the producer' problem is simply

$$\max_{Q} pQ - c(Q), \tag{5}$$

where Q is the amount of the intermediate goods produced and p is the price of Q in terms of the consumption goods. In equilibrium,

$$p = c'(Q). \tag{6}$$

The producer receives pQ in the form of cash or capital. Since as we show in the following the value of capital in the night market changes depending on whether the boom occurs in the night market, the amount of the consumption goods the producer ultimately obtains in the night market also changes depending on whether the boom occurs if she receive pQ in the form of capital.

The optimization problem for the consumers is similar to that for the representative agent in Lagos and Wright (2005). In the night market, the consumer's problem is

$$W(m, q, d) = \max_{x,h,m_{+1},d_{+1}} U(x) - h + \beta V(m_{+1}, d_{+1}), \tag{7}$$

subject to

$$x + \phi(m_{+1} + d_{+1}) = h + wq + \phi m + (1+i)\tilde{\omega}\phi d + (\gamma - 1)\phi M_{t-1}, \tag{8}$$

where $W(\cdot)$ is the value function at the beginning of the night market, m is cash holdings, q is the intermediate goods purchased, d is the loan to the investors made in the previous night market, x is the consumption, h is disutility from labor supply, $V(\cdot)$ is the value function at the beginning of the day market, β is the discount factor between date-t and date-$(t+1)$, ϕ is the value of cash in terms of the consumption goods, w is the price of q in the night market, and $(\gamma - 1)M_{t-1}$ is the lump-sum injection of cash from the government. As is standard in the Lagos-Wright framework, the consumer can produce h units of consumption goods from h units of labor supply, which causes h units of disutility. Lagos and Wright demonstrated that this quasi-linearity in the consumers' utility makes the analysis very tractable. The random variable $\tilde{\omega}$ represents the ratio of the remaining value to the initial value of the loan. In the high state, $\tilde{\omega}$ is revealed in the day market and $\tilde{\omega} = 1$. In the low state, $\tilde{\omega}$ is still a random variable in the day market, which realizes in the night market: $\tilde{\omega} = 1$ with probability ϵ and $\tilde{\omega} = \omega$ with probability $1 - \epsilon$, where ω is determined as an equilibrium outcome. This problem is reduced to $W(m, q, d) = \max_{x, m_{+1}, d_{+1}} = U(x) - x - \phi(m_{+1} + d_{+1}) + wq + \phi m + (1+i)\tilde{\omega}\phi d + (\gamma - 1)\phi M_{t-1} + \beta V(m_{+1}, d_{+1})$. The first-order conditions (FOCs) are $U'(x) = 1$ and

$$\phi = \beta V_m(m_{+1}, d_{+1}), \tag{9}$$

$$\phi = \beta V_d(m_{+1}, d_{+1}). \tag{10}$$

The envelope conditions imply

$$W(m, q, d) = wq + \phi m + (1+i)\tilde{\omega}\phi d + \overline{W}, \tag{11}$$

where $\overline{W}$ is independent of the state variables (m, q and d). In the day market, the consumer's problem is

$$V(m, d) = \max_{q_h, q_l} (1-\delta)E[W(m - \phi^{-1}p_h q_h, q_h, d)] + \delta E[W(m - \phi^{-1}p_l q_l, q_l, d)], \tag{12}$$

subject to

$$p_h q_h \leq \phi m, \tag{13}$$

$$p_l q_l \leq \phi m, \tag{14}$$

where $E[\cdot]$ is the expectation taken in the day market, and p_h (p_l) is the price of the intermediate goods in the day market when the economy is in the high (low) state. Conditions (13) and (14) are the cash-in-advance (CIA) constraints. Note that we implicitly assumed that the government conducts monetary policy such that ϕ takes on the same value in the high state and in the low state. (We can alternatively assume that $\phi_h \neq \phi_l$, where ϕ_h (ϕ_l) is the value of cash in the night market when the economy is in the high (low) state. In that case, the government must set the lending rate i in addition to the growth rate of money supply γ. We discuss this issue later in Section 3.2.) The condition (11) implies that this problem is reduced to

$$V(m,d) = \max_{q_h,q_l} (1-\delta)\{w_h q_h - p_h q_h\} + \delta\{w_l q_l - p_l q_l\} + \phi m + [1-(1-\epsilon)\delta + (1-\epsilon)\delta\omega](1+i)\phi d + \overline{V}, \tag{15}$$

subject to (13) and (14), where $\overline{V}$ is independent of the state variables. The FOCs are

$$(1-\delta)(w_h - p_h) = p_h \lambda_h, \tag{16}$$

$$\delta(w_l - p_l) = p_l \lambda_l, \tag{17}$$

where λ_h and λ_l are the Lagrange multiplier for (13) and (14), respectively. The envelope conditions are

$$V_m = \phi(1 + \lambda_h + \lambda_l), \tag{18}$$

$$V_d = [1-(1-\epsilon)\delta + (1-\epsilon)\delta\omega](1+i)\phi d. \tag{19}$$

These four conditions together with (9) and (10) imply that

$$\frac{\gamma}{\beta} = (1-\delta)\frac{w_h}{p_h} + \delta\frac{w_l}{p_l}, \tag{20}$$

$$\frac{\gamma}{\beta} = [1-(1-\epsilon)\delta + (1-\epsilon)\delta\omega](1+i), \tag{21}$$

where $\gamma = \phi_{-1}/\phi$ is the inflation rate or the money growth rate.

The investors are born in the date-$(t-1)$ night market and borrow k from the consumers with risky debt contract, while they incur the utility cost for asset management, $e(k)$, to maintain k from the date-$(t-1)$ night market to the date-t day market, where $e(0) = 0$, $e'(\cdot) > 0$ and $e''(\cdot) > 0$. We borrow this setup from Allen and Gale (2000). The optimization problem for the investors in the date-$(t-1)$ night market is

$$\max_k \Phi(k) - e(k), \tag{22}$$

where $\Phi(k)$ is the value of holding k units of capital in the date-$(t-1)$ night, which is determined by the following optimization problem in the date-t day market:

$$\Phi(k) = \max\left\{E\left[\tilde{a}k - (1+i)\phi bk\right]_+, \max_{q^i} E\left[wq^i - (1+i)\phi bk + \tilde{a}\left[k - \frac{pq^i}{E[\tilde{a}]}\right]\right]_+\right\}, \tag{23}$$

subject to

$$pq^i \leq E[\tilde{a}]k, \tag{24}$$

where $(1+i)$ is the loan rate, b is the nominal price of k in the date-$(t-1)$ night market, q^i is the amount of the intermediate goods the investor purchases, $\tilde{a}$ is the price of capital in the night market, and $E[\tilde{a}]$ is the price of capital in the day market. The first term in the right-hand side of (23) is the expected gain from hoarding k and the second term is the expected gain from buying q in exchange for k. Constraint (24) is the asset-in-advance (AIA) constraint. The FOC with respect to q^i is $w = (1+\mu)p$, where μ is the Lagrange multiplier for (24). If $1 + \mu = w/p > 1$, $\Phi(k)$ is reduced to

$$\Phi(k) = \max\{E[\tilde{a}k - (1+i)\phi bk]_+, [(1+\mu)E[\tilde{a}]k - (1+i)\phi bk]_+\}, \tag{25}$$

and $q = E[\tilde{a}]k/p$. If the first term in the right-hand side of (25) is smaller than or equal to the second term, the investors in the day market choose to buy q^i in exchange for k, otherwise they choose to hoard k until the night market. Appendix A shows that in the case where $1 + \mu = w/p = 1$, the reduced form of $\Phi(k)$ is still (25) and either $q^i = 0$ or q^i

is indeterminate, that is, the investors' expected gain is the same for all $q^i \in [0, E[\tilde{a}]k/p]$. In the night market, perfect competition in trading of k and q implies

$$w = \alpha A K^{1-\alpha} Q^{\alpha-1}, \tag{26}$$

$$\tilde{a} = \begin{cases} (1-\alpha)AK^{1-\alpha}Q^{\alpha} & \text{with probability } 1-\tilde{\epsilon}, \\ (1-\alpha)AK^{1-\alpha}Q^{\alpha} + R & \text{with probability } \tilde{\epsilon}, \end{cases} \tag{27}$$

where K and Q are the total amounts of the capital and the intermediate goods in this economy, respectively. Note that $K = 1$. Note also that $\tilde{\epsilon}$ is revealed in the day market.

2.3 Baseline equilibrium

We define the baseline equilibrium as that with $\delta = 1$ and all agents rationally believe that δ is 1. This is the equilibrium without euphoria. We assume for parameter values that

$$\gamma > \beta. \tag{28}$$

Note that (28) just says that the inflation rate (or money growth rate) must be larger than the consumers' discount factor. We assume and justify later that there is no default in equilibrium. Since there is no high state in this equilibrium we omit the subscript l on the variables. We put subscript b to stand for "baseline" instead. The equilibrium prices and allocation are determined by the following five equations for five unknowns (p_b, Q_b, q_b, q_b^i, ϕ_b), given γ and m:

$$p_b = c'(Q_b), \tag{29}$$

$$p_b q_b = \phi_b m, \tag{30}$$

$$p_b q_b^i = (1-\alpha)AQ_b^{\alpha} + \epsilon R, \tag{31}$$

$$\frac{\gamma}{\beta} = \frac{\alpha A}{p_b} Q_b^{\alpha-1}, \tag{32}$$

$$Q_b = q_b + q_b^i \tag{33}$$

The investors purchase the intermediate goods in exchange for k, and the AIA constraint binds in equilibrium, since (28) and (32) imply that $1 + \mu = w_b/p_b = \gamma/\beta > 1$. The investors' optimization in the date-$(t-1)$ night market is (22) and the FOC with respect to k at $k = 1$ implies that

$$\begin{aligned}(1+i)\phi b &= (1+\mu)E[\tilde{a}] - e'(1) \\ &= \frac{\alpha A Q_b^{\alpha-1}}{c'(Q_b)}\{(1-\alpha)AQ_b^{\alpha} + \epsilon R\} - e'(1). \qquad (34)\end{aligned}$$

Since $e'(1) > 0$ and the income of the investor is $(1 + \mu)E[\tilde{a}]$, there is no default on the debt $(1 + i)\phi b$ in this equilibrium. Therefore, we have $\omega = 1$, and condition (21) implies that $\gamma/\beta = 1 + i$, which together with (34) determines i, b, and $d = b$.

Note that if the government sets $\gamma = \beta$, (28) implies that the monetary friction can be eliminated and the first-best allocation is attained. Thus when there is no euphoria, the Friedman rule is the optimal policy in this model.

2.4 Financial crisis – equilibrium with euphoria

We describe a financial crisis as an outcome of the following equilibrium with euphoria: Although the economic fundamental is $\delta = 1$, all agents in the date-$(t-1)$ night market are possessed by euphoria and mistakenly believe that δ is very small; and in the date-t day market, the value of $\tilde{\epsilon}$ is revealed to be ϵ ($\ll 1$), despite of the agents' expectations that $\tilde{\epsilon}$ would be 1 with large probability. A financial crisis corresponds to the state of the day market in which $\tilde{\epsilon} = \epsilon$. Therefore, in the equilibrium with euphoria, the financial crisis occurs with probability 1 in the date-t day market.[2] We can show that if the agents' belief δ^e is less than 1 and R is sufficiently large, the equilibrium amount of debt, $(1 + i)\phi b$, becomes so large that the investors hoard k and do not buy the intermediate goods in the day market when $\tilde{\epsilon}$ turns out to be ϵ. We assume and justify later that $q_l^i = 0$ in the low state where $\tilde{\epsilon}$ turns out to be ϵ, and that in the high state $w_h = p_h$

[2] Alternatively, we can think of the financial crisis as a bad realization in the rational expectations equilibrium: Suppose that $0 < \delta < 1$ and all agents know the true value of δ. In this setting, the financial crisis occurs when $\tilde{\epsilon}$ turns out to be ϵ with probability δ.

and (24) does not bind. Given the expected value δ^e, γ, and m, the equilibrium prices and allocation are determined by the following eight equations for eight unknowns (p_h, p_l, Q_h, Q_l, q_h, q_l, q^i, ϕ):

$$p_h = c'(Q_h), \tag{35}$$

$$p_l = c'(Q_l), \tag{36}$$

$$p_h q_h = \phi m, \tag{37}$$

$$p_l q_l = \phi m, \tag{38}$$

$$p_h = w_h = \alpha A Q_h^{\alpha-1}, \tag{39}$$

$$\frac{\gamma}{\beta} = 1 - \delta^e + \delta^e \frac{\alpha A}{p_l} Q_l^{\alpha-1}, \tag{40}$$

$$Q_h = q_h + q^i, \tag{41}$$

$$Q_l = q_l. \tag{42}$$

Equation (39) implies that $Q_h = Q_*$, where Q_* is defined by $c'(Q_*) = \alpha A Q_*^{\alpha-1}$. The variables: $(1+i)$, b, ω and d are yet to be determined below. Before that, we should clarify the condition on parameter values for that the above equilibrium allocation is consistent with the optimization by the investors. The solution to (25) should be hoarding of k in the low state and purchase of q^i in the high state. A sufficient condition for that is

$$R > \max\{\underline{R}, \quad c'(Q_*)q^i - (1-\alpha)AQ_*^{\alpha}\} \tag{43}$$

where

$$\underline{R} \equiv \frac{e'(1)}{(1-\hat{\epsilon})(1-\delta^e)} + \frac{(1-\alpha)\alpha A^2 Q_l^{2\alpha-1}}{(1-\hat{\epsilon})c'(Q_l)},$$

where $\hat{\epsilon} \equiv \epsilon \alpha A Q_l^{\alpha-1}/c'(Q_l)$. Note that the value of the right-hand side of (43) is independent from R. See Appendix B for the proof. Therefore, we have the following proposition.

Proposition 1 *We assume that R is sufficiently large such that (43) is satisfied. The investors hoard capital and do not purchase the intermediate goods in the low state, and the equilibrium allocation and the equilibrium prices are determined by (35)–(42).*

It is also necessary that $q^i \geq 0$ for the solutions to (35)–(42) to be the equilibrium outcome. We can show the following claim.

Claim 1 *The solutions to (35)–(42) satisfy* $q^i \geq 0$.

The proof is as as follows. Equations (39) and (40) imply that $Q_l < Q_h = Q_*$. On the other hand, (37) and (38) imply $c'(Q_l)Q_l = c'(Q_*)q_h$, which imply $Q_l > q_h$. Since $Q_l < Q_h = q_h + q^i$, it is the case that $q^i > 0$. (End of Proof)

The FOC for (22) with respect to k at $k = 1$ implies that the equilibrium value of $(1+i)\phi b$ is determined by

$$(1+i)\phi b = \frac{\delta^e \epsilon\{(1-\alpha)AQ_l^\alpha + R\} + (1-\delta^e)\{(1-\alpha)AQ_*^\alpha + R\} - e'(1)}{1-\delta^e+\delta^e\epsilon}. \tag{44}$$

Since all agents believe that the investors ultimately default with probability $(1-\epsilon)\delta^e$, (21) implies

$$\frac{\gamma}{\beta} = [1-(1-\epsilon)\delta^e + (1-\epsilon)\delta^e\omega](1+i), \tag{45}$$

$$\omega = \frac{(1-\alpha)AQ_l^\alpha}{(1+i)\phi b}. \tag{46}$$

Given γ, these two equations and (44) determine $(1+i)$, b, and ω. The amount of loan for the consumer is simply $d = b$. For convenience in the next section, we define the equilibrium $\mathbf{E_0}$ as follows:

Definition 1 *The equilibrium* $\mathbf{E_0}$ *is an equilibrium in which a financial crisis occurs in the day market and there is no policy response, which is determined by (35)–(42) and (44)–(46).*

3 Discussion

In our model, a financial crisis is described as a plunge of output of the intermediate goods due to disappearance of media of exchange, that is, capital, which is caused by hoarding of capital by the debt-ridden investors. Both the contraction of output and the freeze in asset trading are present in the financial crisis in our model, and there are causality between these two features. The contraction of the aggregate output is caused by the freeze in the asset market, which is caused by the risk-shifting behavior (or the gambling for resurrection) of the debt-ridden investors.

3.1 Welfare loss due to a financial crisis

In this model, we formalize a financial crisis as an equilibrium realized in the day market after the agents are possessed by euphoria in the previous night market: A financial crisis is the low state in the day market which is realized with probability $\delta = 1$, while the agents believed in the previous night market that the low state would be realized with a very low probability δ^e ($\ll 1$) and the investors were overly indebted based on the overly optimistic expectations. In the financial crisis, the investors hoard k and do not purchase the intermediate goods, leading to a plunge of the output of the intermediate goods in the day market. We define the welfare cost due to a financial crisis as the gap between the *social surplus* in the baseline equilibrium and that in the financial crisis. The social surplus can be defined as the utility gain from consumption of Q for all agents minus the utility cost of producing Q for the producers. The quasi-linearity of the consumers' utility and the linearity of the producers' and the investors' utilities imply that the social surplus from production of Q equals $AQ^\alpha - c(Q)$. Here, we just compare the social surplus between the baseline equilibrium and the financial crisis and omit any effects of redistribution of wealth between different types of agents. Equations (32) and (40) imply that

$$\frac{w_l}{p_l} - \frac{w_b}{p_b} = \left(\frac{1}{\delta^e} - 1\right)\left(\frac{w_b}{p_b} - 1\right) > 0, \tag{47}$$

where $w_l/p_l = \alpha A Q_l^{\alpha-1}/c'(Q_l)$, $w_b/p_b = \alpha A Q_b^{\alpha-1}/c'(Q_b)$ and Q_l is the production in the financial crisis (the low state) and Q_b is the production in the baseline equilibrium. Since $\alpha A Q^{\alpha-1}/c'(Q)$ is decreasing in Q, the above inequality implies that $Q_l < Q_b$. Since the social surplus $AQ^\alpha - c(Q)$ is concave and $Q_l < Q_b < Q_*$, where Q_* is the socially optimal level of production, it is obvious that the financial crisis is socially costly. Condition (47) implies that the social cost of the financial crisis becomes larger as δ^e becomes smaller.

In other words, our model implies that the cost of a financial crisis becomes more severe as the preceding euphoria is more excessive.

It is also obvious that the financial crisis is associated with the lower price of the intermediate goods than that in the baseline equilibrium. We assume that $c'(Q)$ is strictly increasing in Q. Since $Q_l < Q_b$,

$$p_l = c'(Q_l) < c'(Q_b) = p_b.$$

3.2 Varying nominal prices and deflation

In solving the model, we implicitly assumed that ϕ, the value of money in terms of the consumption goods, is invariant between the high state and in the low state. We can relax this assumption so that the value of money becomes ϕ_h in the high state and ϕ_l in the low state, where $\phi_h \neq \phi_l$. We analyze the model with varying ϕ in Appendix C. As shown in Appendix C, the government has to determine two policy variables because there are two macroeconomic states that can be realized in the date-t day market. For example, the government sets the inflation rate contingent on the realization of the high state, $\gamma_h \equiv \phi_{-1}/\phi_h$, and the loan rate, i. The contingent inflation rate, γ_h, is implicitly determined by setting the money growth rate M_t/M_{t-1}. By setting γ_h and i, the government can set ϕ_h and ϕ_l, which in turn determine Q_h and Q_l. Therefore, in our model, when the government sets γ_h and i, it implicitly determines Q_h and Q_l, either intentionally or unintentionally. Suppose that the government wants to set the production in the low state at $Q_l = Q_*$. This targeted level of production is socially optimal and this is attainable under appropriate parameter values. Note, however, that attaining $Q_l = Q_*$ through setting γ_h and i may not be a realistic policy recommendation. This is because equation (61) implies that if $\phi_h > \beta$, $\phi_l = \xi\phi_h$ must be strictly greater than ϕ_h in order to attain $Q_l = Q_*(= Q_h)$. That $\phi_l > \phi_h$ seems to correspond to deflationary environment in reality, in which rich people become more rich and buy more in the financial crisis.

3.3 Policy analysis – Equivalence of macro and financial policies

In this subsection, we show that under certain conditions the following three policy options have almost equivalent effects in the financial crisis: The policy options concerned are fiscal policy (subsidy to the consumers), monetary policy (liquidity provision to the investors) and financial stabilization (debt reduction for the investors). We assume that these policies can be undertaken in the day market, in which the economy falls into the financial crisis. We compare the equilibria in which respective policies are perfectly foreseen before the crisis.

3.3.1 Fiscal policy

We define the fiscal policy as giving subsidy (cash) to the consumers, which is financed later in the night market by a lump-sum tax on the consumers. We assume as a crucial assumption that there is an upper limit for the nominal amount of the subsidy, $\overline{M}$. This upper limit is exogenously given by some political or technological constraints. We consider the equilibrium in which all agents in the date-$(t-1)$ night market expect that the government would undertake the above fiscal policy in the date-t day market if the economy fall into the financial crisis. In this equilibrium, the CIA constraint for the consumers changes from (38) to

$$p_l q_l = \phi(m + \overline{M}). \tag{48}$$

The equilibrium with fiscal policy, $\mathbf{E_f}$, is defined as follows:

Definition 2 *The equilibrium* $\mathbf{E_f}$ *is an equilibrium in which a financial crisis occurs in the day market and the government responds to the crisis by the fiscal policy, which is determined by (35)–(37), (39)–(42), (44)–(46), and (48).*

Comparing $\mathbf{E_0}$ and $\mathbf{E_f}$, we can easily show from equation (40) that the aggregate production, Q_l, is the same for both equilibria, and that the consumption profile and labor supply of the agents are also the same for both equilibria. Therefore, the social welfare in $\mathbf{E_f}$ is equal to that in $\mathbf{E_0}$. We can interpret this result as the ineffectiveness of the fiscal policy in the case when the policy response is perfectly foreseen before the crisis occurs. Alternatively, it can be said that if the fiscal policy is not perfectly foreseen by the agents beforehand, the fiscal policy is effective to increase the aggregate production in the day market. Note that the effectiveness of the fiscal policy comes from the fact that the government subsidy loosens the CIA constraint for the consumers and therefore the fiscal policy is effective even though the Ricardian equivalence precisely holds in this model. Note that since R is very large and satisfy (43), the debt-ridden investors never choose to buy the intermediate goods for any level of aggregate production, Q_l $(\leq Q_*)$.

3.3.2 Monetary Policy (liquidity provision)

We define the liquidity provision during a financial crisis as follows: the government issue new cash and lends cash to the debt-ridden investors for purchasing the intermediate goods. The government loans to the investors are secured by the collateral k, with the gross (nominal) interest rate of 1. (There are arbitrariness in the choice of the interest rate of the government loans. We set it at 1 so that there is no resource transfer to the government sector.) The government loans must be repaid in the night market and are senior debts that are prior to the existing loans from the consumers. The investors can hoard the cash they borrow from the government or use the cash to purchase the intermediate goods, and repay the government loans in the night market. Although the investors are ultimately insolvent unless the boom occurs ($\tilde{R} = R$), they weakly prefer purchasing q to hoarding cash as long as $w \geq p$. We consider the equilibrium in which all agents in the date-$(t-1)$ night market expect that the government would undertake the above monetary policy in the date-t day market if the economy fall into the financial crisis. In this equilibrium, the aggregate production in the low state is determined by

$$Q_l = q_l + \hat{q}^i, \tag{49}$$

instead of equation (42), where

$$p_l \hat{q}^i = (1-\alpha) A Q_l^{\alpha} + \epsilon R. \tag{50}$$

The equilibrium with monetary policy, $\mathbf{E_m}$, is defined as follows:

Definition 3 *The equilibrium* $\mathbf{E_m}$ *is an equilibrium in which a financial crisis occurs in the day market and the government responds to the crisis by the monetary policy, which is determined by (35)–(41), (44)–(46), (49), and (50).*

There is a necessary and sufficient condition for the existence of $\mathbf{E_m}$, which is

$$c'(Q_l)Q_l > (1-\alpha)AQ_l^\alpha + \epsilon R, \tag{51}$$

where Q_l is determined by (40).[3] The sufficient condition for (51) is $c'(Q_l)Q_l \leq (1-\alpha)AQ_l^\alpha$, which is equivalent to

$$\delta^e > \frac{1-\alpha}{2\alpha-1}\left(\frac{\gamma}{\beta}-1\right). \tag{52}$$

This condition implies that if the agents believe that a financial crisis occurs with not-so-small probability, δ^e, there exists a monetary equilibrium in which the government undertake the monetary policy in the financial crisis. (Note that the same logic as that in Claim 1 implies that q_h and q^i are both strictly positive in the high state.)

It is easily shown from (40) that Q_l in $\mathbf{E_m}$ is equal to that in $\mathbf{E_0}$. Since we assume that the repayment of the government debt is $p_l\hat{q}^i$, equations (44) and (46) change to

$$(1+i)\phi b = \frac{\delta^e\epsilon\{(1-\alpha)AQ_l^\alpha + R + (w_l - p_l)E[\tilde{a}_l]/p_l\} + (1-\delta^e)\left\{(1-\alpha)AQ_*^\alpha + R\right\} - e'(1)}{1-\delta^e+\delta^e\epsilon}, \tag{53}$$

$$\omega = \frac{(1-\alpha)AQ_l^\alpha + (w_l - p_l)E[\tilde{a}_l]/p_l}{(1+i)\phi b}, \tag{54}$$

respectively. There are slight difference in consumption profile and labor supply between $\mathbf{E_m}$ and $\mathbf{E_0}$. The consumption of the investors becomes $(1-\alpha)AQ_l^\alpha + R + (w_l - p_l)E[\tilde{a}_l]/p_l$ when $\tilde{R} = R$ in $\mathbf{E_m}$, while it is $(1-\alpha)AQ_l^\alpha + R$ when $\tilde{R} = R$ in $\mathbf{E_0}$. But this difference is offset by the difference in labor supply, h, by the consumer in the night market because

[3] If $c'(Q_l)Q_l \leq (1-\alpha)AQ_l^\alpha + \epsilon R$ for Q_l determined by (40), cash has no value in the equilibrium: $\phi = q_h = q_l = 0$. The aggregate production, Q, in this case is not determined by (40), but Q is determined by $c'(Q)Q = (1-\alpha)AQ^\alpha + \epsilon R$. (40) is no longer an equilibrium condition since $\phi = 0$. To make the equilibrium with $\phi = 0$ exist, we need additional assumption that the government can provide loans in the real term and the real loans can be used as media of exchange.

of the linearity of the utilities of the agents. Therefore, the social welfare in $\mathbf{E_m}$ is equal to that in $\mathbf{E_0}$. We can interpret this result as the ineffectiveness of the monetary policy in the case when the policy response is perfectly foreseen before the crisis occurs. Alternatively, it can be said that if the monetary policy is not perfectly foreseen by the agents beforehand, the monetary policy is effective to increase the aggregate production in the day market.

We made a strong assumption to derive the effectiveness of the monetary policy. It is that the loans from the government have perfect seniority over the loans from consumers, which is not plausible in the real world. If the government seniority is incomplete, the government incurs a significant loss that should be tax financed. In this case, the monetary policy reduces to a fiscal policy. [4]

3.3.3 Debt reduction

We define the debt reduction policy as reduction of the investors' debt from $(1+i)\phi bk$ to $(1+i)\phi b'k$, which is a sufficiently small amount such that the investors never default on the debt for all realization of $\tilde{R}$ no matter whether they hoard the capital or purchase the intermediate goods. In this case, the investors choose to purchase the intermediate goods in the low state rather than to hoard the capital as long as $w \geq p$. The government can implement the debt reduction by just nullifying the debt obligation of the investors partially such that the remaining debt becomes $(1+i)\phi b'k$. This policy may be regarded as a simplification of bank closure policy through deposit cut.[5] We consider the equilibrium in which all agents in the date-$(t-1)$ night market expect that the government would undertake the debt reduction policy in the date-t day market if the economy fall into the financial crisis. In this equilibrium, the production in the low state satisfies (49) and

[4]If we explicitly introduce the banking sector in this model, we could have the "credit trap" in which the bank lending to the insolvent investors do not increase even thought the government (or the central bank) injects base money into the banking sector and the banks hoard the injected cash. Monetary policy becomes ineffective in this situation. See Benmerech and Bergman (2009) for a model of credit trap.

[5]Alternatively, the government can give subsidy $(1+i)\phi(b-b')k$ to the investors in the date-t night market, while the subsidy is financed by a lump-sum tax on the consumers.

(50). Therefore, the equilibrium allocation is determined by (35)–(41), (49) and (50). The aggregate production in the low state, Q_l, is determined by (40), and therefore it is the same as that in $\mathbf{E_0}$. There is some arbitrariness in the amount of the debt reduction. We set a sufficiently small amount for b', which the investors never default on as follows:

$$(1+i)\phi b' = (1-\alpha)AQ_l^\alpha, \tag{55}$$

where ϕ and Q_l are determined by (35)–(41), (49) and (50), and i is yet to be determined. Given the expectations that the debt reduced in the financial crisis, the gain for the investors of holding k in the date-$(t-1)$ night market is

$$\Phi(k) = (1-\delta^e)[(1+\mu_h)E[\tilde{a}_h]k - (1+i)\phi bk] + \delta^e[(1+\mu_l)E[\tilde{a}_l]k(1+i)\phi b'k], \tag{56}$$

where $(1+\mu)E[\tilde{a}] = \frac{\alpha AQ^{\alpha-1}}{c'(Q)}[(1-\alpha)AQ^\alpha + \epsilon R]$ for $Q = Q_h, Q_l$. The FOC for (22) with respect to k at $k=1$ implies that

$$(1+i)\phi b = (1+\mu_h)E[\tilde{a}_h] + \frac{\delta^e}{1-\delta^e}[(1+\mu_l)E[\tilde{a}_l] - (1+i)\phi b'] - \frac{e'(1)}{1-\delta^e}. \tag{57}$$

Since the debt reduction occurs in the low state, (19) changes to $V_d = [1-\delta^e+\delta^e\omega](1+i)\phi d$, and therefore, (45) and (46) change to

$$\frac{\gamma}{\beta} = [1-\delta^e+\delta^e\omega](1+i), \tag{58}$$

$$\omega = b'/b. \tag{59}$$

Therefore, the equilibrium with debt reduction policy, $\mathbf{E_d}$, is defined as follows:

Definition 4 *The equilibrium* $\mathbf{E_d}$ *is an equilibrium in which a financial crisis occurs in the day market and the government responds to the crisis by the debt reduction policy, which is determined by (35)–(41), (49), (50), (55) and (57)–(59).*

As is similar to the case of $\mathbf{E_m}$, parameters must satisfy the condition (51) and (52) for $\mathbf{E_d}$ to exist. As stated above, the aggregate production, Q_l, is the same for both $\mathbf{E_0}$ and $\mathbf{E_d}$. The social welfare is also the same in both equilibria.[6] We can interpret this result

[6]There are slight difference in consumption profile and labor supply between the two equilibria. The consumption of the investors becomes strictly positive even when $\tilde{R} = 0$ in $\mathbf{E_d}$, while it is zero in $\mathbf{E_0}$. But this difference is offset by the difference in labor supply, h, by the consumer in the night market because of the linearity of the utilities of the agents.

as the ineffectiveness of the debt reduction policy in the case when the policy response is perfectly foreseen before the crisis occurs. Alternatively, it can be said that if the debt reduction policy is not perfectly foreseen by the agents beforehand, the debt reduction is effective to increase the aggregate production in the day market. The effectiveness of the debt reduction comes from the fact that this policy eradicates the negative macroeconomic externality that the large debt burden of the investors exerts, that is, the large debt makes the almost insolvent investors hoard their capital as a result of their rational choice, while the hoarding of capital causes an unintentional reduction of the media of exchange in the goods market.

The debt reduction is equivalent to a lump-sum transfer from the consumers to the investors. If the lump-sum transfer is not available, the tax distortion associated with this policy may be large.

4 Conclusion

Our experience of the global financial crisis in 2008/2009 indicates that we should formalize a major financial crisis as an event associated with

- freeze of transactions in the asset markets,
- sharp contraction of the aggregate output.

Our interpretation of this type of financial crises is a disappearance or vaporization from the marketplace of a significant portion of broad "money," which we vaguely define as general assets that can be very easily exchanged with cash so that they work as media of exchange on de facto basis. We formalized this notion of financial crises in our model, where optimal behaviors of agents under risky debt contract and limited liability cause the disappearance of media of exchange from the market. Market freeze is known to be explained by the risk-shifting effect (Diamond and Rajan, 2009): If the asset holders with limited liability are excessively indebted, they choose to hoard the assets waiting for a miraculous rise in their values. Asset hoarding that causes the market freeze can be understood as a risk-shifting behavior of debt-ridden investors. If the assets are used as media of exchange in the goods market, the risk-shifting behavior exerts a significant externality that overly decreases the aggregate production. Our notion that the assets, such as real estates, are used as de facto money is a simplification of the reality in which the asset holders can obtain cash at any time by selling the assets or by borrowing money by putting up the assets as collateral. In the economy where the assets work as media of exchange, hoarding of the assets is translated as disappearance of significant portion of the broad money, which directly causes contraction of the aggregate output.[7]

Our model features the externality of *debt overhang* (Lamont 1995) that causes a sharp contraction of the aggregate output. It was shown that to eradicate this externality, macroeconomic policies, i.e., fiscal and monetary policies, and financial policy (or debt reduction of excessively indebted agents) have almost equivalent effect. Therefore, this externality may justify a government policy that facilitates debt reduction during a financial crisis. A possible policy scheme may be to make the bankruptcy procedure contingent on the occurrence of a financial crisis (the *crisis-contingent bankruptcy procedure*). Debt reduction through the bankruptcy procedure should be quick and drastic during a financial crisis in order to eradicate the adverse external effect. For example, we may provide the procedure in which the government has the right, which becomes effective only during a financial crisis, to urge borrowers (or lenders) to file for bankruptcy if their debts exceed a certain threshold. After the global financial crisis, economists and policy makers enthusiastically argue for a new architecture of financial regulation, namely, macroprudential regulation (see Borio [2003], Brunnermeier et al. [2009]). The macroprudential regulation is financial regulation that varies contingent on changes in macroeconomic environment. Our notion of crisis-contingent bankruptcy procedure may form a pair with macroprudential financial regulation. Both seeks varying regulations contingent on macroeconomic fluctuations, while the latter is for lenders' financial health and the former is for borrowers' debt reduction.

[7]In the companion paper, Kobayashi (2009), we formalize bank runs as a disappearance of broad money. In that paper, bank deposits work as a medium of exchange in the normal times, whereas when a crisis occurs a significant portion of bank deposits become frozen as a result of the bank runs and the output shrinks.

A lesson from the global financial crisis must be at least that we need to pay more attention to the adverse effects on macroeconomic performance caused by excessive debts in the private sector, which necessarily accumulate before and during a financial crisis.

References

Allen, F., and D. Gale (2000). "Bubbles and Crises." *Economic Journal* 110: 236–55.

Aruoba, S. B., C. Waller and R. Wright (2007). "Money and Capital." Working paper 07-14, Federal Reserve Bank of Cleveland.

Benmerech, E., and N. K. Bergman (2009). "Credit Traps." mimeo. Harvard Univ.

Berentsen, A., G. Camera and C. Waller (2007). "Money, credit and banking." *Journal of Economic Theory* 135: 171–195.

Beaudry, P., and A. Lahiri (2009). "Risk Allocation, Debt Fueled Expansion and Financial Crisis." NBER Working Paper No. 15110.

Borio, C. (2003). "Towards a Macroprudential Framework for Financial Supervision and Regulation?" CESifo Economic Studies, 49(2):181-215.

Brunnermeier, M., A. Crockett, C. Goodhart, A.D. Persaud and H. Shin (2009). *The Fundamental Principles of Financial Regulation.* Geneva Reports on the World Economy 11 (www.voxeu.org/reports/Geneva11.pdf).

Diamond, D. W., and R. G. Rajan (2009). "Fear of Fire Sales and the Credit Freeze." NBER Working Paper No. 14925.

Freixas, X., and J.-C. Rochet (2008). *Microeconomics of banking*, second edition. Cambridge, Massachusetts: MIT Press.

Gale, D., and M. Hellwig (1985). "Incentive-compatible debt contracts: the one-period problem." *Review of Economic Studies* 52: 647–63.

Kobayashi, K. (2009a). "Asset-price Collapse and Market Disruptions." RIETI Discussion Paper Series ***.

Kobayashi, K. (2009b). "A Monetary Model of Banking Crises." RIETI Discussion Paper Series ***.

Lagos, R., and R. Wright (2005). "A unified framework for monetary theory and policy analysis." *Journal of Political Economy* 113 (3): 463–484.

Lamont, O. (1995). "Corporate-Debt Overhang and Macroeconomic Expectations." *American Economic Review*, 85(5): 1106-17.

Lucas, L. E.,Jr., and N. L. Stokey (1987). "Money and Interest in a Cash-in-Advance Economy." *Econometrica*, 55(3): 491–513.

Shreifer, A., and R. W. Vishny (2009). "Unstable Banking." NBER Working Paper No. 14943.

A The investor's optimization when $w = p$

We show the investor's optimization in the date-t day market is (25) even in the case where $1 + \mu = w/p = 1$. Note that if $\tilde{\epsilon} = 1$, it is obvious from $\tilde{a} = E[\tilde{a}]$ that (25) holds and q^i is indeterminate. So we focus on the low state where $\tilde{\epsilon} = \epsilon$ ($\ll 1$). For brevity we define $\pi(q_*^i) = \max_{q^i} E[\tilde{\pi}(q^i)]_+$, where q_*^i is the solution to the maximization of the right-hand side and $\tilde{\pi}(q^i) = wq^i - (1+i)\phi bk + \tilde{a}\left[k - \frac{pq^i}{E[\tilde{a}]}\right]$. There are three cases that we should consider: (1) a case in which $\tilde{\pi}(q^i_*) < 0$ only when $\tilde{R} = 0$; (2) a case in which $\tilde{\pi}(q^i_*) < 0$ only when $\tilde{R} = R$; and (3) a case in which $\tilde{\pi}(q^i_*) \geq 0$ regardless of the realized value of $\tilde{R}$. In case (1), $\tilde{\pi}(q^i)$ is nonnegative only when $\tilde{R} = R$. Therefore, $\pi(q^i_*) = \max_{q^i} \epsilon \left\{\left[1 - \frac{(1-\alpha)AQ^\alpha + R}{E[\tilde{a}]}\right] pq + \{(1-\alpha)AQ^\alpha + R\}k - (1+i)\phi bk\right\}$. Since $E[\tilde{a}] = (1-\alpha)AQ^\alpha + \epsilon R < (1-\alpha)AQ^\alpha + R$, the solution should be $q^i_* = 0$ and $\pi(0) = E[ak - (1+i)\phi bk]_+$. Next, we show case (2) never happens. Suppose there exists case (2). In this case, $\pi(q^i_*) = \max_{q^i}(1-\epsilon)\left\{\left[1 - \frac{(1-\alpha)AQ^\alpha}{E[\tilde{a}]}\right] pq + \{(1-\alpha)AQ^\alpha + R\}k - (1+i)\phi bk\right\}$. Since $E[\tilde{a}] = (1-\alpha)AQ^\alpha + \epsilon R > (1-\alpha)AQ^\alpha$, the solution should be $q^i_* = E[\tilde{a}]k/p$. But then $\tilde{\pi}(q^i_*)$ takes on the same value regardless of the realized value of $\tilde{R}$, meaning that $\tilde{\pi}(q^i_*) > 0$ when $\tilde{R} = R$, which is a contradiction. Therefore, case (2) never happens. Case (3) is divided into the following two cases: (3-1) a case in which $q^i_* = E[\tilde{a}]k/p$; and (3-2) a case in which $q^i_* < E[\tilde{a}]k/p$. In general, however, case (3-1) cannot happen in equilibrium. Since $Q = q + q^i_*$, where Q is the total supply of the intermediate goods and q is the amount purchased by the consumers, $w = p$ implies $\alpha A(q + q^i_*)^{\alpha-1} = c'(q + q^i_*)$, which uniquely determines q^i_*, given q. The condition $q^i_* = E[\tilde{a}]k/p$ is equivalent to $q^i_* = \{(1-\alpha)A(q + q^i_*)^\alpha + \epsilon R\}/c'(q + q^i_*)$, which is not compatible with $w = p$ in general. So we focus on case (3-2). In this case, $\pi(q^i_*) = E\left[pq^i_* - (1+i)\phi bk + \tilde{a}\left[k - \frac{pq^i_*}{E[\tilde{a}]}\right]\right] = E[a]k - (1+i)\phi bk$. Since $\pi(q^i_*)$ takes on the same value for all $q^i_* \in [0, E[\tilde{a}]k/p]$, q^i_* is indeterminate. The analysis of cases (1) and (3-2) implies that $\Phi(k)$ is determined by (25) and either $q^i_* = 0$ or q^i_* is indeterminate in equilibrium.

B Condition for hoarding of capital

In this appendix we prove that (43) is a sufficient condition for that the investors hoard k in the low state and purchase q^i in the high state and that the equilibrium prices and allocation are determined by (35)–(42). Proof is by contradiction. Suppose that the investors purchase q^i and do not hoard k in the low state. In this case the gain for the investors from purchasing the intermediate goods should be $\frac{w_l}{p_l}E_l[\tilde{a}]$ in the low state.

Therefore, the FOC for (22) with respect to k at $k = 1$ implies

$$(1+i)\phi b = \delta^e \frac{w_l}{p_l} E_l[\tilde{a}] + (1-\delta^e)\{(1-\alpha)AQ_*^\alpha + R\} - e'(1). \tag{60}$$

The assumption that $R > \underline{R}$ implies that $\frac{w_l}{p_l} E_l[\tilde{a}] - (1+i)\phi b < 0$ if$(1+i)\phi b$ is determined by (60). Thus the investors hoard capital in this case. This is a contradiction. Therefore, $(1+i)\phi b$ is determined by (44) and the investors hoard capital in the low state. It is easily shown that $(1-\alpha)AQ_*^\alpha + R - (1+i)\phi b > 0$ and the investors actually buy the intermediate goods in the high state. The assumption that $R > c'(Q_*)q^i - (1-\alpha)AQ_*^\alpha$ implies that the AIA constraint (24) does not bind in the high state.

C Generalized model with varying value of money

In the text, we assumed that the value of ϕ is equal in the high state and in the low state. Under this constraint, it has been shown in the text that the loan rate, i, is uniquely determined as an equilibrium outcome. Generally, we can relax this constraint and assume that the value of money is ϕ_h in the high state and ϕ_l in the low state, where ϕ_h may not be equal to ϕ_l. We can solve the model allowing that $\phi_h \neq \phi_l$. Denoting $\xi = \phi_l/\phi_h$ and $\gamma_h = \phi_{-1}/\phi_h$, the equilibrium is determined by the following set of equations:

$$\frac{\gamma_h}{\beta} = 1 - \delta^e + \delta^e \xi \frac{\alpha A Q_l^{\alpha-1}}{c'(Q_l)}, \tag{61}$$

$$c'(Q_h)q_h = \phi_h m, \tag{62}$$

$$c'(Q_l)Q_l = \xi \phi_h m, \tag{63}$$

$$c'(Q_h) = \alpha A Q_h^{\alpha-1}, \tag{64}$$

$$Q_h = q_h + q^i, \tag{65}$$

$$\frac{\gamma_h}{\beta} = \{1 - (1-\xi\epsilon)\delta^e\}(1+i) + (1-\epsilon)\delta^e \frac{(1-\alpha)AQ_l^\alpha}{\phi_h b}, \tag{66}$$

$$(1+i)\phi_h b = \frac{\delta^e \epsilon \{(1-\alpha)AQ_l^\alpha + R\} + (1-\delta^e)\frac{\alpha A Q_h^{\alpha-1}}{c'(Q_h)}\{(1-\alpha)AQ_h^\alpha + R\} - e'(1)}{\delta^e \xi \epsilon + 1 - \delta^e}. \tag{67}$$

This is a set of seven equations, which include nine variables: γ_h, ϕ_h, ξ, Q_h, Q_l, q_h, q^i, i, and $\phi_h b$. Therefore, the government (or the central bank) should set two variables to pin down the equilibrium uniquely. What the government can set are ϕ_h, through setting the quantity of money supply, and i. If the government sets γ_h and i appropriately, the government can control productions, Q_h and Q_l, at least to some extent. (In the text we implicitly assumed that the government chooses the value of i such that $\xi = 1$.)

Note, however, that this generalized model has a peculiar implication: $\phi_l > \phi_h$ is desirable, meaning that deflation is desirable during a financial crisis. For example, suppose that the government wants to set Q_l at Q_* $(= Q_h)$, which is the socially optimal level of production. In this case, (61) implies that ξ must be greater than 1, if $Q_l = Q_*$. In other words, the necessary condition for $Q_l = Q_*$ is $\phi_l > \phi_h$, which is deflation during a financial crisis.