RIETI Discussion Paper Series 11-E-011

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KOBAYASHI Keiichiro
RIETI

Research Institute of Economy, Trade & Industry, IAA
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http://www.rieti.go.jp/en/

RIETI Discussion Paper Series 11-E-011
February 2011

A Bad-Asset Theory of Financial Crises*

KOBAYASHI Keiichiro†
（RIETI／Hitotsubashi University）

Abstract

We propose a simple model of financial crises, which may be useful for the unified analysis of macro and financial policies implemented during the 2008-2009 financial crisis. A financial crisis is modeled as the disappearance of inside money due to the lemon problem à la Akerlof (1970), in a simplistic variant of Lucas and Stokey's (1987) Cash-in-Advance economy, where both cash and capital stocks work as media of exchange. The exogenous emergence of a huge amount of bad assets represents the occurrence of a financial crisis. Information asymmetry regarding the good assets (capital stocks) and the bad assets causes the good assets to cease functioning as inside money. The private agents have no proper incentive to dispose of the bad assets, and the crisis could be persistent, because the lemon problem is an external diseconomy. Macroeconomic policy (e.g., fiscal stimulus) provides outside money for substitution, and financial stabilization (e.g., bad-asset purchases) restores the inside money by resolving information asymmetry. The welfare-improving effect of the macro policy may be nonexistent or temporary, while the bad-asset purchases may have a permanent effect to shift the economy out of the crisis equilibrium.

* This paper is motivated by encouraging comments by Robert Lucas on the companion paper ("A Model of Financial Crises–Coordination Failure due to Bad Assets") presented at the Seoul National University Conference in Honor of Lucas and Stokey on September 15-18, 2009. I thank seminar participants at SNU and GRIPS for helpful comments.
† Hitotsubashi University; Research Institute of Economy, Trade, and Industry (RIETI); Canon Institute for Global Studies (CIGS); and Chuo University. E-mail: kobayashi-keiichiro@rieti.go.jp.

1 Introduction

The goal of this paper is to propose a simple model of financial crises that can reproduce stylized characteristics of the 2008–2009 global financial crisis and may serve as a building block of a theoretical framework for comparative analysis of macroeconomic policy (i.e., fiscal stimulus and monetary easing) and financial stabilization policy (e.g., bad-asset purchases). Our modeling strategy to achieve our goal is to view the financial crisis in 2008–2009 as a large-scale disappearance of inside money due to the lemon problem caused by the emergence of bad assets. There are following three features that appear to be characteristics of the 2008–2009 crisis:

1. a freezing of transactions of risky assets and a sharp increase in the demand for safe assets ("the flight to quality");

2. a sharp contraction in aggregate output and employment;

3. a sharp deterioration in the labor wedge.

The first two features may be obvious for any observer of the crisis. The third one may be new to the literature. The labor wedge, $1 - \tau^L$, is defined as a wedge between the marginal rate of substitution between consumption and leisure for consumers (MRS) and the marginal product of labor for producers (MPL): $1 - \tau^L = MRS/MPL$. As Shimer (2009) and Chari, Kehoe and McGrattan (2009) emphasize, the labor wedge moves procyclically and is increasingly regarded as a significant factor of business cycles. Figure 1, which is from Kobayashi (2009), shows the labor wedge in the US economy during the period of the first quarter of 1990 to the second quarter of 2009.

Insert Figure 1 here.

The US labor wedge experienced a drastic decline during the 2008–2009 crisis, which was the worst decline during the sample period. This figure indicates that the financial turmoil was associated with the labor wedge deterioration, while the existing literature usually assume that the factors related to the labor market are the major causes of the changes in the labor wedge: for example, labor-income tax, bargaining power of labor unions (Cole and Ohanian 2004), search frictions in the labor market (Shimer 2009). Therefore, the 2008–2009 crisis may indicate that we need a model of financial crisis in which financial frictions cause the labor wedge deterioration.

Outline of the model: We formalize the financial crisis as a large-scale disappearance of inside money. This view seems to be shared by increasing number of leading economists. Lucas (2009), Gorton and Metrick (2009), and Uhlig (2009), for example, describe their observations of the 2008–2009 financial crisis as the disappearance of inside money due to the similar mechanism as the bank runs that occurred during the Great Depression: the inside money in today's world is the various short-term debts of financial institutions, including primary dealer repos (Adrian and Shin 2009), while it was bank deposits in the 1930s when depositors ran on their bank deposits then uninsured; and this time around, financial institutions ran on their short-term loans[1] to other financial institutions, which were not government-insured. In the banking literature, the bank runs are driven either by the expectations in the multiple-equilibria setting (Diamond and Dybvig 1983) or by exogenous (productivity) shocks (Allen and Gale 2000). Our view is that the disappearance of inside money is not just an outcome of expectational changes or productivity shocks. Our hypothesis is that the emergence of bad assets causes the lemon problem à la Akerlof concerning the quality of the inside money (which is the risky financial assets), which leads to the disappearance of a large amount of inside money. To formalize this hypothesis, we build a simplistic model on a variant of the Cash-in-Advance economy described by Lucas and Stokey (1987). We assume that the agents can use both cash and capital stocks (or the ownership securities of them) as the instruments of payment.

Related literature: A considerable number of research papers have emerged in response to the 2008–2009 financial crisis. Most of them focus on the financial market

[1] Gorton and Metrick (2009) emphasize that the widely observed withdrawal of funds from the repo market is one form of modern "bank runs."

or banking system (see, for example, Diamond and Rajan 2009; Bolton, Santos and Scheinkman 2009; and Uhlig 2009), and not so many papers address both the macroeconomic recession and the financial turmoil. Beaudry and Lahiri (2009) and Shreifer and Vishny (2009) are the exceptions. But these models may not be suitable for comparative analysis of fiscal stimulus and financial stabilization policy. Kiyotaki and Moore (2004, 2005) posit a model of inside money, which is closely related to our model.

(To be completed)

2 Model

The model is a simplified variant of the Cash-in-Advance economy in Lucas and Stokey (1987). We assume that the productive trees are endowed to the households and these trees do not deplete in the production process (we may call them the "Lucas trees"). We assume that the trees can work as the medium of exchange just like cash.[2] We consider the lemon problem à la Akerlof (1970), which is caused by the bad assets that are unproductive trees exogenously endowed to the private agents and are indistinguishable from the productive trees for all agents except for the owners of the bad assets.

2.1 Setup

The model is a closed economy with discrete time that continues from zero to infinity: $t = 0, 1, 2, \cdots, +\infty$. There is a unit mass of households, each of which consists of a husband (worker-seller) and a wife (buyer). At the beginning of the initial period $t = 0$, each household is endowed with a productive tree, which works as capital input in production of the consumption goods and does not deplete in the production process. We may call this tree a Lucas tree. The trees (or the ownership claims of them) are

[2]Since trees may not be portable, we may consider that paper of the ownership of trees can work as inside money. We can consider the following example of the paper: The owners of the trees issue bonds that are backed by the trees as collateral. The bonds may be interpreted as the ABSs (asset backed securities) that are backed by real assets (i.e., the trees) or just the securities of the ownership claims of the trees.

traded in the market every period t at the market price q_t, where q_t is in the unit of the consumption goods. Suppose that a household owns k_t units of the productive trees at the beginning of period t. In period t, the trees produce y_t units of consumption goods if the husband spends l_t units of his time taking care of the trees, with the Cobb-Douglas production technology: $y_t = Ak_t^{\alpha} l_t^{1-\alpha}$, where $0 < \alpha < 1$. In what follows, we set $\alpha = .3$. A household's momentary utility in period t is $U(c_t, 1 - l_t)$, which satisfies the standard neoclassical properties. In what follows we set $U(c_t, 1 - l_t) = \ln c_t + \phi \ln(1 - l_t)$. The household discounts the future utility at the rate of β, where $0 < \beta < 1$. In what follows, we set $\beta = .95$. The household cannot consume its own products and it can consume only the products of other households. Therefore the household need to buy the consumption goods in the market and, as in Lucas and Stokey (1987), the payment for purchase of the consumption goods is restricted to be in particular assets. In the original Lucas-Stokey economy, the payment in cash is required. In the present model, we assume that the payment should be either in cash or in trees. There is the government (or the central bank) in this economy that provides cash, $\overline{M}_t$, to the households, where $\overline{M}_t$ is the nominal amount of money supply.

Bad Assets: Households are endowed with bad assets every period. One unit of the bad asset is an unproductive tree that looks like a productive Lucas tree. The bad asset cannot be used as an input in production of the consumption goods. The bad assets in this paper represent nonperforming loans or toxic mortgage securities on the balance sheets of financial institutions in reality, which are generated as a result of collapses of asset-price bubbles and/or failures of businesses and households that borrowed money. Alternatively, the bad assets may be interpreted as "zombie firms" (Caballero, Hoshi, and Kashyap 2008) that lost productivity and are heavily indebted. In this model, we assume information asymmetry on bad asset among households as follows. On one hand, the household that owns a bad asset knows that it is the bad asset (or it is not a productive tree). On the other hand, the households cannot distinguish other households' bad assets from the productive trees. Only after a household obtains a tree in the market does the household know whether the tree is a bad asset (an unproductive tree) or a good asset (a productive tree). The law of motion of bad assets is given as follows. Suppose that a household owns n''_{t-1} units of bad assets at the end of period $t-1$. At the beginning of period t, the amount of bad assets becomes n_t, which is

$$n_t = Rn''_{t-1} + \epsilon_t, \tag{1}$$

where $n''_{-1} = 0$, $0 < R \leq 1$, and ϵ_t is a new endowment of bad assets, which may be positive or negative. Given n_t, the household can dispose of x_t units of the bad assets at the beginning of period t, with cost $\gamma(x_t) = \frac{\gamma}{2}x_t^2$, which is measured in the units of the Lucas trees. The household incurs the cost for maintenance of the remaining bad assets: $\delta(\epsilon_t - x_t) = \delta \times \max\{\epsilon_t - x_t, 0\}$, which is also measured in the units of the productive trees. We assumed that the bad assets inherited from the previous period, $R_t n''_{t-1}$, do not necessitate the maintenance cost, but that only the new endowment ϵ_t necessitates the maintenance cost if it is not disposed of at the beginning of period t.[3] The cost of disposition and maintenance of bad assets, $\gamma(x_t) + \delta(\epsilon_t - x_t)$, is a transfer to the government in the units of the productive trees. For simplicity of the analysis, we assume the following two assumptions on the cost:

- The household that disposes x_t units of bad assets must purchase $\gamma(x_t) + \delta(\epsilon_t - x_t)$ units of productive trees from other households and pay them to the government for costs of disposition and maintenance of the bad assets.

- There is a lump-sum transfer, τ_t, from the government to each household every period in the form of the trees. The endowment is given to wives in the market (see Time line below). In equilibrium $\tau_t = \gamma(x_t) + \delta(\epsilon_t - x_t)$, where the RHS is the social average of the sum of the costs for disposition and maintenance of the bad assets.

[3] Although this assumption seems to be a plausible description of the reality of bad assets, it can be relaxed. The qualitative results of our analysis do not essentially change but complexity of the exposition increases, if we assume that Rn''_{t-1} necessitates the maintenance costs.

Time line: The events that take place during a representative period t are as follows. At the beginning of period t, new bad assets, ϵ_t, is realized. Therefore, representative household carries three assets at the beginning of period t: the productive tree, k_t; the bad assets, n_t $(= R_t n''_{t-1} + \epsilon_t)$; and cash, m_t. The husband produces the consumption goods $y_t = Ak_t^{\alpha} l_t^{1-\alpha}$ using his labor l_t. Then he (the seller) goes to the market to sell y_t. Then, the wife disposes of some amount (x_t) of bad assets. Then, she (the buyer) goes to the market carrying cash (m_t), trees (k_t), and bad assets $(n_t - x_t)$ if $n_t - x_t > 0$. In the market, she buys $\gamma(x_t) + \delta(\epsilon_t - x_t)$ units of trees at the price of q_t and loses them as the cost for disposition and maintenance of the bad assets. Then, she is immediately endowed with τ_t units of the trees. Then she buys the consumption goods by paying cash, productive trees, and bad assets. Note that the seller cannot distinguish the productive trees and the bad assets that he receives from the buyer. After the transaction, the husband and wife come back home and consume. At the end of period t, the government makes a lump-sum transfer of cash (μ_t), where μ_t may be positive or negative.

Monetary Policy: There may be various regimes for the conduct of monetary policy. In this paper we consider a deterministic variant of the one in the original Cash-in-Advance economy in Lucas and Stokey (1987) as a typical case.

- The initial money supply at $t = 0$ is given as $\overline{M}_0$ (> 0).
- The government sets the time-invariant money growth rate, g_m, where $\overline{M}_{t+1} = g_m \overline{M}_t$ for all $t \geq 0$.
- The government sets the cash injection at $\mu_t = (g_m - 1)\overline{M}_t$.

2.2 Optimization Problem

The representative household solves the optimization problem, described as the following Bellman equation:

$$V(p_t M_t, k_t, n_t, \epsilon_t) = \max_{c_t, l_t, x_t, M'_t, k'_t, n'_t, \hat{M}_t, \hat{k}_t} U(c_t, 1 - l_t) + \beta V(p_{t+1} M_{t+1}, k_{t+1}, n_{t+1}, \epsilon_{t+1}) \tag{2}$$

subject to

$$c_t = p_t M_t' + q_t[k_t' + n_t' - \gamma(x_t) - \delta(\epsilon_t - x_t) + \tau_t], \tag{3}$$

$$q_t \hat{k}_t + p_t \hat{M}_t \leq A k_t^{\alpha} l_t^{1-\alpha}, \tag{4}$$

$$k_t' \leq k_t, \tag{5}$$

$$p_t M_t' \leq p_t M_t, \tag{6}$$

$$n_t' \leq n_t - x_t, \tag{7}$$

$$M_{t+1} = M_t + \mu_t - M_t' + \hat{M}_t, \tag{8}$$

$$k_{t+1} = k_t - k_t' + (1 - \xi_t)\hat{k}_t, \tag{9}$$

$$n_t'' = \xi_t \hat{k}_t + n_t - x_t - n_t', \tag{10}$$

$$n_{t+1} = R n_t'' + \epsilon_{t+1}. \tag{11}$$

where the explanations of the above constraints are as follows. Constraint (3) is the liquidity constraint for the purchase of the consumption goods, in which p_t is the value of cash in terms of the consumption goods and M_t' is the amount of cash that the wife decides to pay for c_t, which must be no greater than her cash holdings, M_t. Therefore, we have (6). Note that p_t is the inverse of the nominal price of the consumption goods. $k_t' - \gamma(x_t) - \delta(\epsilon_t - x_t) + \tau_t$ in constraint (3) is the amount of the productive trees that the wife can pay for c_t. k_t' must be no greater than her holdings of the trees. Therefore, we have constraint (5). n_t' in (3) is the amount of the bad assets that the wife decides to pay for c_t, which must be no greater than her holdings of the bad assets, $n_t - x_t$. Naturally, we have (7). Constraint (4) is the budget constraint for the husband who sells output, $A k_t^{\alpha} l_t^{1-\alpha}$, in exchange for the productive trees, $\hat{k}_t$, at the market price q_t and cash, $\hat{m}_t$. The parameter ξ_t is the ratio of the bad assets in the total supply of the trees, which is perceived as an exogenous parameter for the private agents. Since the husband cannot distinguish the bad assets that are got mixed in $\hat{k}_t$, he just knows that $(1 - \xi_t)\hat{k}_t$ are the good assets (productive trees) and $\xi_t \hat{k}_t$ are the bad assets. Therefore, constraint (9) is the law of motion for the holdings of the productive trees, (8) is the law of motion for cash holdings, and (10) and (11) give the law of motion for the bad asset holdings.

The reduced form of the problem is

$$
\begin{aligned}
&V(p_t M_t, k_t, n_t, \epsilon_t) \\
&= \max U(c_t, 1-l_t) \\
&+ \beta V(p_{t+1}\{M_t + \mu_t - M_t' + \hat{M}_t\},\ k_t - k_t' + (1-\xi_t)\hat{k}_t,\ R[\xi_t \hat{k}_t + n_t - n_t' - x_t] + \epsilon_{t+1}, \epsilon_{t+1})
\end{aligned} \tag{12}
$$

subject to

$$c_t = p_t M_t' + q_t[k_t' + n_t' - \gamma(x_t) - \delta(\epsilon_t - x_t) + \tau_t], \tag{13}$$

$$q_t \hat{k}_t + p_t \hat{M}_t \leq A k_t^{\alpha} l_t^{1-\alpha}, \tag{14}$$

$$k_t' \leq k_t, \tag{15}$$

$$p_t M_t' \leq p_t M_t, \tag{16}$$

$$n_t' \leq n_t - x_t. \tag{17}$$

We now clarify the condition for (17) to be binding. If (17) holds with strict inequality in equilibrium, then the first-order conditions (FOCs) with respect to k_t' and n_t' imply that the following equation must hold in equilibrium:

$$(1-R)V_n(t) = -\lambda_t \alpha A k_t^{\alpha-1} l_t^{1-\alpha} - \beta^{-1}\eta_{t-1}^k, \tag{18}$$

where λ_t and η_t^k are the Lagrange multipliers for (14) and (15), respectively. Depending on the values of $\{R, \epsilon_t\}$, the above condition may be satisfied and (17) may become slack. If (17) is slack, the value of n_t' becomes strictly less than $n_t - x_t$ in equilibrium, and the equilibrium characterization becomes complicated. To avoid this complication of the analysis due to slackness of (17), we assume throughout in this paper that

$$0 < R \leq 1. \tag{19}$$

Under this assumption, equation (18) never holds for positive λ_t and nonnegative η_{t-1}^k and $V_n(t)$. Therefore we have

Lemma 1 *Under the assumption that* $0 < R \leq 1$, *condition (17) is always binding.*

Since (17) is binding and the household chooses x_t to maximize c_t, condition (13) with binding (17) implies that x_t is determined by $x_t = x(\epsilon_t)$, which is defined by

$$x(\epsilon_t) = \min\left\{\frac{\delta - 1}{\gamma}, \max\{\epsilon_t,\ 0\}\right\}, \tag{20}$$

where we assumed that $\delta > 1$. We define

$$\hat{n}_t = n_t - x(\epsilon_t),$$

where $x(\epsilon_t)$ is determined by (20). Since ϵ_t affects the optimization problem only through $x(\epsilon_t)$, the problem is reduced to

$$\begin{aligned} V(p_t M_t, k_t, \hat{n}_t) = \max_{l_t, k'_t, M'_t, \hat{k}_t, \hat{M}_t} \ & U(q_t(k'_t + \hat{n}_t) + p_t M'_t,\ 1 - l_t) \\ & + \beta V(p_{t+1}\{M_t + \mu_t - M'_t + \hat{M}_t\},\ k_t - k'_t + (1 - \xi_t)\hat{k}_t,\ R\xi_t \hat{k}_t + \epsilon_{t+1} - x(\epsilon_{t+1})) \end{aligned} \tag{21}$$

subject to

$$q_t \hat{k}_t + p_t \hat{M}_t \leq A k_t^{\alpha} l_t^{1-\alpha}, \tag{22}$$

$$k'_t \leq k_t, \tag{23}$$

$$p_t M'_t \leq p_t M_t. \tag{24}$$

The FOCs are as follows:

$$-U_l(t) = (1 - \alpha) A k_t^{\alpha} l_t^{-\alpha} \lambda_t, \tag{25}$$

$$q_t U_c(t) - \beta V_k(t+1) - \eta_t^k \leq 0, \tag{26}$$

$$p_t U_c(t) - \beta p_{t+1} V_m(t+1) - p_t \eta_t^m \leq 0, \tag{27}$$

$$q_t \lambda_t \geq (1 - \xi_t)\beta V_k(t+1) + \xi_t \beta R_{t+1} V_n(t+1), \tag{28}$$

$$p_t \lambda_t \geq \beta p_{t+1} V_m(t+1), \tag{29}$$

where λ_t, η_t^k, and η_t^m are the Lagrange multipliers for (22), (23), and (24), respectively. The envelope conditions are

$$p_t V_m(t) = \beta p_{t+1} V_m(t+1) + p_t \eta_t^m, \tag{30}$$

$$V_k(t) = \lambda_t \alpha A (l_t / k_t)^{1-\alpha} + \beta V_k(t+1) + \eta_t^k, \tag{31}$$

$$V_n(t) = q_t U_c(t) \tag{32}$$

The resource constraints are

$$c_t = Ak_t^{\alpha} l_t^{1-\alpha}, \tag{33}$$

$$k_t = 1. \tag{34}$$

The equilibrium conditions are $k_t' = (1 - \xi_t)\hat{k}_t$; $M_t' = \hat{M}_t$; $M_t = \overline{M}_t$; $\mu_t = (g_m - 1)\overline{M}_t$; and $\tau_t = \gamma(x_t) + \delta(\epsilon_t - x_t)$. Note that $c_t = q_t(k_t' + \hat{n}_t) + p_t M_t'$ and constraint (33) hold because $\tau_t = \gamma(x_t) + \delta(\epsilon_t - x_t)$ in equilibrium.

2.3 Normal Equilibrium with a Small Amount of Bad Assets

In this subsection, we analyze the equilibrium in the case where only small amount of bad assets emerges every period. We assume that

$$0 \leq \epsilon_t \leq \frac{\delta - 1}{\gamma}, \qquad \text{for all } t \geq 0, \tag{35}$$

which implies that $\forall t,\ x_t = x(\epsilon_t) = \epsilon_t$, and

$$\xi_t = \hat{n}_t = 0 \qquad \text{for all } t \geq 0. \tag{36}$$

Since the bad assets are eliminated, the economy reduces to a standard model with the liquidity constraint, in which the representative household maximizes $\sum_{t=0}^{\infty} \beta^t U(c_t, 1 - l_t)$ subject to the production technology ($y_t = Ak_t^{\alpha} l_t^{1-\alpha}$), the budget constraint ($c_t + q_t k_{t+1} + p_t M_{t+1} \leq y_t + q_t\{k_t - \gamma(\epsilon_t) + \tau_t\} + p_t\{M_t + \mu_t\}$), and the liquidity constraint:

$$c_t \leq p_t M_t + q_t\{k_t - \gamma(\epsilon_t) + \tau_t\}, \tag{37}$$

with the equilibrium condition: $\tau_t = \gamma(\epsilon_t)$. We show in what follows that the first-best allocation is attained in this case because the liquidity constraint (37) does not bind in equilibrium, and that the demand for cash is zero in the first-best equilibrium.

We assume and justify later that $\eta_t^k = 0$ in the equilibrium. We also assume and justify that $k_t' > 0$ and $\hat{k}_t > 0$. The real variables (c_t and l_t) are determined as follows. Since k_t' and $\hat{k}_t$ are assumed to be strictly positive, (26) and (28) imply $\lambda_t = U_c(t)$. Then (25) and $k_t = 1$ imply

$$-\frac{U_l(t)}{U_c(t)} = (1 - \alpha) A l_t^{-\alpha}. \tag{38}$$

This condition and (33) determines the real variables (c^*, l^*) uniquely. Note that (c^*, l^*) are time invariant and socially optimal because the outcome is identical to that in the case with no monetary frictions. Since we assumed $\eta_t^k = 0$, (26) and (31) imply that $q_t = q^* = (1-\alpha)c^*/(\beta^{-1}-1)$. Given $q_t = q^*$, it is justified that $\eta_t^k = 0$: Since $\alpha = .3$ and $\beta = .95$, it is the case that $c_t = c^* < q^* = q_t k_t$, which implies that k_t' is strictly smaller than $k_t = 1$. That $k_t' < k_t$ justifies that $\eta_t^k = 0$ in this equilibrium. Monetary variable (p_t) is determined as follows. First, it is shown that $p_t \eta_t^m = 0$. (Proof is as follows: Suppose on the contrary that $p_t \eta_t^m > 0$. It must be the case that $M_t' = M_t$ (≥ 0) and (27) holds with equality. Therefore, $\lambda_t = U_c(t)$ implies that (29) holds with strict inequality, meaning that $\hat{M}_t = 0$. Since $M_t' = \hat{M}_t$ ($= 0$) must hold in (symmetric) equilibrium, it must be the case that the money demand is always zero, i.e., $M_t = 0$ for all $t \geq 0$. This is a contradiction because $M_t = \overline{M}_t$ must hold and by assumption $\overline{M}_t > 0$. Proof ends.) Therefore, either $\eta_t^m = 0$ or $p_t = 0$. If $\eta_t^m = 0$, (27) implies that $p_t U_c(t) = p_{t+1}\beta V_m(t+1)$. Since the CIA constraint does not bind, it must be the case that $V_m(t+1) = 0$ and therefore, $p_t = 0$. So in any case, $p_t = 0$ in equilibrium.

Observations: In the normal equilibrium without bad assets, the real allocation (c^*, l^*) is the first best and the labor wedge, $1-\tau^L$, is 1, because (38) implies that

$$1 - \tau^L \equiv \frac{-\frac{U_l(t)}{U_c(t)}}{(1-\alpha)A l_t^{-\alpha}} = 1.$$

The money demand is zero, i.e., $p_t M_t = 0$ for all $t \geq 0$, which may be interpreted as a simplification of the reality, where the demand for hard currency or safe assets is low when the financial intermediation is well functioning and there is an abundant supply of inside money in the economy.[4]

[4] With a cost of complication, we can modify our model such that the money demand becomes strictly positive in the normal equilibrium. For example, if we assume that cash is a necessary input for production of inside money (or financial intermediation services), the aggregate demand for cash cannot be zero in equilibrium.

2.4 Crisis Equilibrium with a Large Amount of Bad Assets

In this subsection, we consider the case that ϵ_0 is large enough such that $\hat{n}_t > 0$ for all $t \geq 0$. (We also assume for simplicity that $\epsilon_t = 0$ for $t \geq 1$.) First, we show that the market collapse due to the lemon problem can occur if $\hat{n}_t > 0$ for all $t \geq 0$.

Lemma 2 *If $\hat{n}_t > 0$ for all $t \geq 0$, it is consistent with the households' optimization that $\xi_t = 1$ and $q_t = 0$ for all t.*

(Proof) When $\xi_t = 1$, the FOC with respect to $\hat{k}$, (28), which is relevant to the husband, becomes

$$q_t \lambda_t \geq \beta R q_{t+1} U_c(t+1).$$

The left-hand side is the marginal cost for buying $\hat{k}$ on date t and the right-hand side is the marginal gain by selling $\hat{k}$ on date $(t+1)$. (Note that since $\xi_t = 1$, $\hat{k}_t$ are all bad assets.) Anticipating the selling price at $t+1$ is zero, i.e., $q_{t+1} = 0$, the husband never bid up q_t (i.e., the buying price at t) to a positive value. Therefore as long as $\xi_t = 1$, the competition among the husbands never bid up q_t, and q_t stays at 0. The wives choose the supply of k' by the FOC, (26):

$$q_t U_c(t) \leq \beta V_k(t+1) + \eta_t^k.$$

Since RHS is strictly positive, $q_t = 0$ implies that the wife sets $k'_t = 0$. On the other hand, as Lemma 1 shows, the wife chooses to sell $n'_t = \hat{n}_t$ even when $q_t = 0$. Therefore, all assets sold in the market are the bad assets, n'_t. Therefore, $\xi_t = 1$. This argument proves the claim of the lemma. (Proof ends)

There may be the *active* equilibria in which $q_t > 0$ even though $\hat{n}_t > 0$. We see in Appendix that the active equilibrium cannot exist if the amount of the bad assets are time-invariant, i.e., $\hat{n}_t = \hat{n}$ for all t. In this subsection, we focus on the equilibrium in which $q_t = 0$ and $\xi_t = 1$, which we call the *crisis equilibrium.* The crisis equilibrium is basically the Cash-in-Advance equilibrium, in which the households can buy consumption goods only in exchange for cash. Therefore,

$$c_t = p_t M_t, \quad \text{and } l_t = \left(\frac{p_t M_t}{A} \right)^{\frac{1}{1-\alpha}}. \tag{39}$$

The FOCs of the optimization problem (21) with respect to M'_t and l_t imply that

$$\frac{-\frac{U_l(t)}{U_c(t)}}{(1-\alpha)Ak_t^{\alpha}l_t^{-\alpha}} = \frac{p_{t+1}\beta}{p_t}\frac{U_c(t+1)}{U_c(t)} \tag{40}$$

Note that the LHS equals the definition of the labor wedge. Using (33) and (39), the above equation reduces to

$$-U_l(t)l_t = \frac{1-\alpha}{g_m}\beta U_c(t+1)c_{t+1}, \tag{41}$$

under the time-invariant monetary policy, $M_{t+1} = g_m M_t$. In the steady state equilibrium where $c_{t+1} = c_t$, the above equation (41) determines the real balance, $p_t M_t \equiv m(g_m)$; consumption, $c = m(g_m)$; labor, $l = (m(g_m)/A)^{\frac{1}{1-\alpha}}$; and prices, $p_t = g_m^{-t} m(g_m)/\overline{M}_0$. If the Friedman Rule ($g_m = \beta$) is adopted, the steady-state version of (41) reduces to (38), and the steady-state allocation becomes the first best: (c^*, l^*), which is the same as that in the normal equilibrium. If $U(c, 1-l) = \ln c + \phi \ln(1-l)$, (41) can be rewritten as

$$\frac{\phi l}{1-l} = (1-\alpha)\frac{\beta}{g_m}, \tag{42}$$

which implies that the only equilibrium is the steady state equilibrium, which is suboptimal ($l < l^*$) if $g_m > \beta$, and optimal ($l = l^*$) if $g_m = \beta$.

Observations: The crisis equilibrium is consistent with the three stylized facts of the 2008–2009 crisis, that we saw in Section 1. First, in the crisis equilibrium the asset price collapses to zero ($q_t = 0$) and trading of assets is frozen, and there emerges the positive money demand ($p_t M_t > 0$), which may represent the flight to quality. This feature is consistent with the fact 1. The output and the labor become smaller than those in the normal equilibrium ($c < c^*$ and $l < l^*$) if $g_m > \beta$. This is consistent with the fact 2. The labor wedge is also less than 1 if $g_m > \beta$, as is clear from (40) with $c_t = c_{t+1}$ and $p_t/p_{t+1} = g_m > \beta$. This is consistent with the fact 3.

Persistence of the crisis: Our analysis implies that the financial crisis may not be a temporary phenomenon but may be a permanent shift of the equilibrium from the optimal equilibrium to the suboptimal equilibrium. In this model, once $\hat{n}_t$ becomes strictly positive and the economy falls into the crisis equilibrium, there is no proper incentive for the private agents to dispose of the bad assets. In other words, the collapse of the asset market (i.e., $q_t = 0$) due to the lemon problem is an external diseconomy for the households and they have no private incentive to resolve this externality. Therefore, the households continue to hold the bad assets $\hat{n}_t$ as long as the disposition of the bad assets is costly for the households. As a result, the economy may be stuck in the crisis equilibrium permanently. The persistence of the crisis equilibrium due to the externality may potentially explain the lengthy duration of the Great Depression in the 1930s and the Lost Decade of Japan during 1991–2002. Our view is related to but essentially different from the notion that "zombie lending" prolonged the recession in Japan by distorting the resource allocation (Caballero, Hoshi, and Kashyap 2008). While the externality in the level of corporate finance and investment is argued in Caballero et al., we argue that the existence of the bad assets (or zombie firms) may destroy the inside money (or liquidity) by causing the macroeconomic information asymmetry.

3 Discussion

3.1 Policy Implications

We can compare and assess the efficacy of macroeconomic policy and bad-asset purchase in the crisis equilibrium. We define the two policies as follows.

- *Macroeconomic Policy:* When the economy is in the crisis equilibrium where the money growth rate is g_m, the government issues additional cash (unexpectedly for the households) and gives it to the households as a surprise subsidy. We consider two policy schemes of macroeconomic policy.

  1. (Surprise at level of money supply)
     The government issues additional cash Δ in period 0 and gives it to the households equally as subsidy. The money supply becomes $\overline{M}_0' = \overline{M}_0 + \Delta$ in period 0. Then the government sets money supply following the constant growth rule: $\overline{M}_1 = g_m \overline{M}_0'$ and $\overline{M}_{t+1} = g_m \overline{M}_t$ for $t \geq 1$.

2. (Surprise at growth rate of money supply)

   The government issues additional cash Δ in period 0 and gives it to the households equally as subsidy. The money supply becomes $\overline{M}'_0 = \overline{M}_0 + \Delta$ in period 0. Then the government sets money supply following the constant growth rule: $\overline{M}_1 = g_m \overline{M}_0$ and $\overline{M}_{t+1} = g_m \overline{M}_t$ for $t \geq 1$. Therefore, $M_1/M'_0 < g_m$ and $M_{t+1}/M_t = g_m$ for $t \geq 1$.

- *Bad-Asset Purchases:* The government purchases all the bad assets $\hat{n}_t$ in the market at a positive price $\underline{q}$ in period t, while the government redeems $\underline{q}\hat{n}_t$ at the end of period t by imposing the lump-sum tax on the households. A very low $\underline{q}$ can be incentive-compatible for the households, since the value of the bad assets is zero for the household in the crisis equilibrium.[5] Note that if the bad assets are generated only in period 0 and no additional bad assets emerge in the subsequent periods, the government needs to purchase the bad assets only once in period 0, to eliminate them from the market permanently.

No or Temporary effect of the macro policy: Under flexible prices, the macro policy 1 have no effect because the price level in period 0 immediately changes in response to the cash injection Δ, such that the real balance in period 0 stays at $m(g_m)$. In this case, the real allocation does not change from the original crisis equilibrium. The macro policy 2 can increase the output (and the labor) only in period 0. First, since the money grows at the rate of g_m from $t = 1$ on under the macro policy 2, the economy goes back to the crisis equilibrium from $t = 1$ on. Second, the condition (41) for $t = 0$ changes to

$$-U_l(0)l_0 = \frac{1-\alpha}{g'_m}\beta U_c(1)c_1,$$

where $g'_m = M_1/M'_0$ and $c_1 = c$, where c is the consumption in the crisis equilibrium. Since $g'_m < g_m$ and $-U_l(0)l_0$ is an increasing function of l_0, it must be the case that $l_0 > l$, where l is the labor input in the crisis equilibrium. The prices are determined by

[5] In reality, where the book values of bad assets are high, the government needs to offer high prices that are close to the book values, in order to induce the financial institutions to sell the bad assets.

$p_0 = c_0/M_0'$ and $p_t = c/M_t$ for $t \geq 1$. Therefore, the macro policy 2 has a temporary effect to improve the social welfare by relaxing the CIA constraint at $t = 0$. These macro policies do not resolve the lemon problem due to bad assets and therefore the market freeze in asset trading (i.e., $q_t = 0$) continues permanently despite of the (repeated surprises of) fiscal stimulus or monetary easing.

Permanent effect of the bad-asset purchases: The effect of bad-asset purchases is to eliminate the bad assets from the market and resolve the lemon problem. If the households have the expectations that the government purchases all bad assets and they do not remain in the market, then the trading of trees is restored and the price of trees becomes q^*. As a result, the economy shifts to the optimal equilibrium, in which the liquidity constraint (37) is nonbinding and the allocation is the first best, i.e., $(c_t, l_t) = (c^*, l^*)$. Note that the effect of the bad-asset purchases may be permanent and one shot of the bad-asset purchase can shift the economy from the crisis equilibrium to the optimal equilibrium in which $q_t = q^* > 0$. This is the case if all the bad assets are generated in period 0 and no additional bad assets are generated in the subsequent periods (or only small amounts of bad assets are generated in the subsequent periods) such that $\hat{n}_0 > 0$ and $\hat{n}_{t+1} = R\hat{n}_t > 0$ for all $t \geq 0$.

3.2 Some Ideas for Future Research

We can speculate ideas for possible developments of our model in the future research.

On business cycles: The mechanism that generates the crisis equilibrium in this model may be general and can potentially explain the ordinary business cycles as well as financial crises. It seems quite natural that a class of assets begins and then ceases to function as inside money, as the lemon problem emerges and disappears due to the changes in the amount of the bad assets in the particular class of assets. Therefore, it may be a possible hypothesis for the causes of the business cycles that freezing and unfreezing of the market for certain asset classes, which may be driven by appearance and disappearance of the bad assets, may cause money shocks (emergence and disappearance of inside money) that drive fluctuations of output, labor, and investment in the business cycle frequencies. We may be able to develop an analytical and quantitative model of business cycles based on this hypothesis.

Toward a new monetary policy rule: The above development of business cycle theory might naturally call for a new monetary policy rule that the central bank should cut the nominal interest rate in response to an unexpected increase in the ratio of bad assets in total bank assets (or the ratio of bad loans in total bank loans). Since the increase (and decrease) in the bad assets causes a surprise decrease (and increase) in the amount of inside money through the lemon problem, in a model with price rigidities it may be welfare improving that the central bank increases money supply when the bad assets increase. For example, we may be able to come up with the following augmented version of the Taylor rule as an approximation for the optimal monetary policy:

$$i_t = \bar{i} + \alpha(\pi_t - \overline{\pi}) + \beta x_t - \gamma d_t,$$

where i_t is the policy rate of nominal interest, $\bar{i}$ is the target rate of nominal interest, π_t is the inflation rate, $\overline{\pi}$ is the target rate of inflation, x_t is the demand gap, and d_t is the bad-asset ratio of whole banks or broader financial institutions in the economy.

On policy coordination for bad-asset dispositions: Although this model is a closed economy model and we did not analyze any international aspect of the financial crisis, there may be the case for international policy coordination to accelerate the bad asset dispositions. Because the external diseconomy due to the lemon problem in the asset trading that leads to the disappearance of inside money affects the global financial markets and the global economy, it can possibly be justified that the international policy coordination to resolve the lemon problem may be welfare improving for all countries. The policy coordination may include the international harmonization of bankruptcy procedures and/or setting up an international organization that is to buy up the toxic securities that were scattered around the world.

4 Conclusion

We proposed a simple model of financial crises that can reproduce the basic features of the 2008–2009 global financial crisis and may serve as a building block for a theoretical framework for comparative analysis of macro policy (e.g., fiscal stimulus) and financial stabilization policy (e.g., bad-asset purchases). We view the financial crisis as a large-scale disappearance of inside money, which is caused by the information asymmetry (the lemon problem) concerning the quality of financial assets. The lemon problem is caused by the emergence of bad assets, resulting from the collapse of the asset-price bubbles. The shrinkage of inside money tightens the liquidity constraints for private agents and causes sharp declines in output and employment. The crisis may be persistent because the private agents have no proper incentive to dispose of the bad assets under the externality of the lemon problem. This externality calls for a policy intervention. Macroeconomic policy may be able to provide outside money for substitution and relax the liquidity constraints for the private agents. Relaxing the liquidity constraint has the effect to improve welfare. Financial stabilization policy (especially, bad-asset purchases) can resolve the lemon problem by eliminating bad assets from the market. If the policy intervention successfully resolve the lemon problem, then trading of financial assets is restored and inside money emerges again. Our simplistic model implies that the welfare improving effect of macro policy may be nonexistent or temporary, while the bad-asset purchases may be able to shift the economy from the crisis equilibrium to the normal equilibrium permanently. We may be able to compare and assess macro and financial policies in a more realistic setting by elaborating a detailed model appropriately based on our simple framework.

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A Non Existence of Active Equilibria with Bad Assets

In this Appendix we show that if $\hat{n}_t > 0$ is time-invariant, there do not exist the active equilibria in which the asset price is positive, i.e., $q_t > 0$.

We consider the possibility of existence of the following two types of active equilibria: (i) an equilibrium in which $\eta_t^k = 0$ and $q_t > 0$; and (ii) an equilibrium in which $\eta_t^k > 0$ and $q_t > 0$. We show that neither of these two active equilibria exists.

Lemma 3 *If $\hat{n}_t$ is strictly positive and time-invariant, i.e., $\hat{n}_t = \hat{n} > 0$ for all t, there is no equilibrium in which $\eta_t^k = 0$ and $q_t > 0$ for all t.*

(Proof) Suppose that there exists an equilibrium in which $\eta_t^k = 0$ and $q_t > 0$. In this equilibrium, the real allocation must be time-invariant, since $\hat{n}_t$ is time-invariant. In this equilibrium, it must be the case that $(c_t, l_t) = (c^*, l^*)$. Otherwise the household can increase c_t by increasing k_t', since k_t' is strictly less than k_t as $\eta_t^k = 0$ implies. Therefore, in this equilibrium, $(c_t, l_t) = (c^*, l^*)$ if it exists at all. (25) implies that $\lambda_t = -U_l^*/[(1-\alpha)A(l^*)^{-\alpha}] = U_c^*$. Thus (26) and (31) imply that $q_t = q^*$. (28) with binding (26) implies that $q_t U_c(t) = \beta R q_{t+1} U_c(t+1)$. Since $q_t = q^*$ and $U_c(t) = U_c^*$, this equation implies that $\beta R = 1$. Since we assumed that $R \leq 1 < \beta^{-1}$, equation $\beta R = 1$ never holds. Therefore, the equilibrium with $q_t > 0$ and $\eta_t^k = 0$ cannot exist. (Proof ends.)

Lemma 4 *If $\hat{n}_t$ is strictly positive and time-invariant, i.e., $\hat{n}_t = \hat{n} > 0$ for all t, there* *is no equilibrium in which* $\eta_t^k > 0$ *and* $q_t > 0$ *for all* t.

(Proof) Suppose that there exists an equilibrium in which $\eta_t^k > 0$ and $q_t > 0$. In this equilibrium, the real allocation must be time-invariant, since $\hat{n}_t$ is time-invariant. Since $\xi = \hat{n}/(k_t + \hat{n}) = \hat{n}/(1+\hat{n})$ and $U_c(t) = 1/c_t$, (28) implies that

$$\eta^k = (1+\hat{n})q\lambda - (1+\beta\hat{n}R)\frac{q}{c}. \tag{43}$$

Since $\eta^k > 0$, it must be the case that

$$\lambda < \frac{1+\beta\hat{n}R}{(1+\hat{n})c}. \tag{44}$$

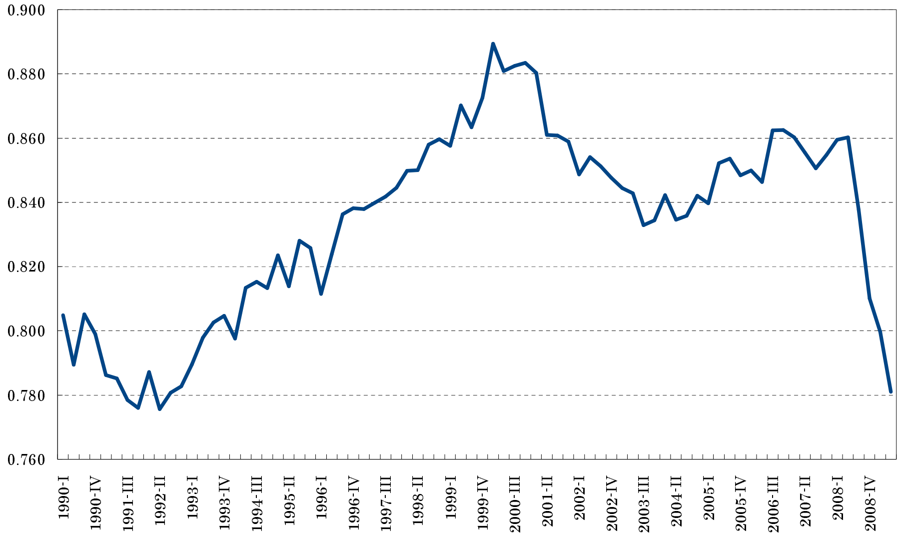
(31) with binding (26) implies that $\frac{q}{c} - \eta^k = \alpha\beta\lambda c + \beta\frac{q}{c}$. This equation and (43) imply

$$\lambda = \frac{(1+\hat{n}R)\beta q}{[(1+\hat{n})q - \alpha\beta c]c}. \tag{45}$$

Since $\eta^k > 0$, $c = q(1+\hat{n}) + pM \geq q(1+\hat{n})$. Therefore, $c/q \geq 1+\hat{n}$ in equilibrium if it exists. This condition together with (44) and (45) implies that the necessary condition for (44) is $(1+\alpha+\hat{n}R\alpha\beta)\beta < 1$. Since $\hat{n} > 0$, the necessary condition is $(1+\alpha)\beta < 1$, which is not satisfied for $\alpha = .3$ and $\beta = .95$. Therefore, the equilibrium with $\eta^k > 0$ and $q > 0$ does not exist. (Proof ends.)

A caveat follows. If (5) is replaced with $k_t' \leq \theta k_t$ and θ is sufficiently smaller than 1, where θ is the collateral ratio, there can be an equilibrium in which $\eta^k > 0$ and $q > 0$ even if $\hat{n}_t = \hat{n} > 0$ for all t. It can be shown that the condition for the money demand to be positive in such an equilibrium is that $\beta < \pi < 1 - \alpha\beta$, which never holds with $\alpha = .3$ and $\beta = .95$. Therefore, $pM = 0$ in the equilibrium where $\eta^k > 0$ and $q > 0$.

Figure 1: U.S. Labor Wedge (1990-2009)



Following Chari, Kehoe, and McGrattan (2007), the labor wedge is defined as

$$(\text{labor wedge}) = \frac{\phi}{1-\alpha} \times \frac{c_t}{y_t} \times \frac{l_t}{1-l_t}.$$

We set $\phi = 2$, and $\alpha = .36$. The data of the consumption-output ratio (c_t/y_t) is from the Breau of Economic Analysis. The data of hour (l_t) is taken from Cociuba, Prescott, and Ueberfeldt (2009).