RIETI Discussion Paper Series 11-E-010

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RIETI Discussion Paper Series 11-E-010
February 2011

A Model of Financial Crises:[*]
Coordination failure due to bad assets

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Abstract

This paper constructs a model of financial crises that can explain characteristic features of the global financial crisis of 2008-2009, namely, the widespread freezing of asset transactions, the sharp contraction of aggregate output, and a deterioration in the labor wedge. This paper assumes that banks sell corporate bonds in the interbank market to raise money for short-term loans. The emergence of bad assets subsequent to the collapse of the asset-price bubble and asymmetric information among banks causes a freezing in the asset trading among banks (the market for lemons). Market freezing constrains the availability of bank loans as working capital for productive firms, causing output and the labor wedge to deteriorate. Given the market freeze, no proper incentives exist for banks to reveal their bad assets and dispose of them.

[*] I wish to thank Robert Lucas and Nancy Stokey for their helpful comments and encouragement. I would also like to thank Young Sik Kim, Tomoyuki Nakajima, Hak K. Pyo, Chi-Wa Yuen, Xiaodong Zhu and seminar participants at the SNU Conference in Honor of Lucas and Stokey for their comments and discussions. Masaru Inaba and Kengo Nutahara provided excellent research assistance. All remaining errors are mine.
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1 Introduction

The global financial crisis in 2008 and 2009 was characterized by the following features:

- a freezing of transactions in the asset markets;
- a sharp contraction in aggregate output; and
- a sharp deterioration in the labor wedge.

The purpose of this paper is to construct a "toy" model that can explain these features in a simple and clear logic and may serve as a possible building block for developing comprehensive theory of the current financial crisis.

The current global crisis could be understood as a severe (and possibly persistent) recession following the collapse of huge asset-price bubbles. This family of crises includes the US Great Depression in the 1930s and the persistent stagnation of the Japanese economy during 1991–2002, that is the Lost Decade of Japan. Some of the major economic downturns experienced by various countries and regions, which are precisely defined as the "great depressions" by Kehoe and Prescott (2002), share the same features. In these episodes as well as in the current crisis, enormous volumes of bad assets emerged subsequent to the bubble collapses, followed by the freeze in the asset markets and deteriorations of output (and as we discuss below the labor wedge). The bad asset problem is typified by the notorious nonperforming loans problem in Japan during the 1990s, and now represented by the "toxic" mortgage securities in the US market.

Labor Wedge: One common characteristic of the US Great Depression and the Lost Decade of Japan is the deterioration in the labor wedge. The labor wedge is defined as a wedge between the marginal rate of substitution between consumption and leisure for consumers (MRS) and the marginal product of labor for firms (MPL). More specifically, the labor wedge, $1-\tau$, is a market distortion expressed as an (imaginary) labor income tax in the following standard neoclassical growth model. As we do in the growth accounting, we assume that the real economy is described by a neoclassical growth model (with market distortion represented by $1 - \tau_t$), in which consumers maximize the discounted sum of the utility flow:

$$\sum_{t=0}^{\infty} \beta^t U(c_t, 1 - l_t)$$

subject to $c_t + k_{t+1} - (1-\delta)k_t \leq r_t^k k_t + (1 - \tau_t) w_t l_t$, where β (< 1) is the discount factor, c_t is consumption, l_t is labor, k_{t+1} is capital, r_t^k is the rent of capital and w_t is the wage rate; and firms maximize the period profit: $\pi_t = A_t k_t^{\alpha} l_t^{1-\alpha} - r_t^k k_t - w_t l_t$, where A_t is the productivity and α is the capital share in the Cobb-Douglas production technology. The labor wedge $1 - \tau_t$ is measured by

$$1 - \tau_t = \frac{-U_l/U_c}{(1-\alpha)A_t(k_t/l_t)^{\alpha}}. \tag{1}$$

In the case where $U(c, 1-l) = \ln c + \phi \ln(1-l)$, the labor wedge is

$$1 - \tau_t = \frac{\phi c_t}{(1-\alpha)(1-l_t)A_t(k_t/l_t)^{\alpha}} = \frac{\phi c_t}{(1-\alpha)(1-l_t)(y_t/l_t)}.$$

Using this equation, the labor wedge of any economy is measured from the macroeconomic data of consumption, labor, and output. In the literature of neoclassical studies on "great depressions of the 20th century," (Kehoe and Prescott 2002), it is shown that the labor-wedge deterioration has been a key driving force of downturns in the US Great Depression (Chari, Kehoe, and McGrattan 2007; Mulligan 2002) and in the 1990s in Japan (Kobayashi and Inaba 2007).[1] Shimer (2009) and Chari, Kehoe, and McGrattan (2009) emphasize that the labor wedge is a key factor not only in the great depressions but also in the usual business cycles. In the studies of the current global crisis, the labor wedge is not yet draw much attention of economic researchers. To my knowledge, the following figures are the first ones that show the drastic movement in the labor wedge in the United States during the crisis of 2008–2009.

[1] It should be noted that the literature found that another major factor of the downturns has been the deterioration of the total factor productivity (TFP) in the US Great Depression (Cole and Ohanian 1999; and Chari, Kehoe, and McGrattan 2007), in other great depression episodes (Kehoe and Prescott 2002) and in the 1990s in Japan (Hayashi and Prescott 2002).

Figure 1: U.S. Labor Wedge (1990–2009)

Figure 2: U.S. Labor Wedge (1964–2009)

As Figures 1 and 2 show, the labor wedge $1 - \tau_t$ drastically declined since the second quarter of 2008. Data shows that output and consumption declined sharply in 2008 and stop declining in 2009, while labor declined even more sharply in 2008 and continues declining in 2009. This sharp decline in the labor wedge is a bit puzzling because usual explanation for cyclical movements in the labor wedge is concerned with the labor market institutions. As Chari, Kehoe and McGrattan (2009) and Shimer (2009) argue, usual suspects of the labor wedge deterioration are

- a rise in the disutility of work,
- a rise in the labor and/or consumption taxes,
- a rise in the monopoly power of the labor unions, and
- a rise in the search frictions in the labor market.

All these factors are concerned with the labor market or the labor institutions. None of these factors seemed present in the current financial crisis. The fact that the labor wedge deteriorated as the financial turmoil deepened seems to indicate that financial frictions may be a primary factor that drives the labor wedge.[2] We pursue this possibility in our model below.

Related literature: Analyses and policy proposals for the current global crisis are found in Brunnermeier et al. (2009). The motivation of our paper is most close to Beaudry and Lahiri (2009) in that they intend to explain not only collapse in the credit market but also shrinkage of the aggregate output. Our focus on the labor wedge is different from them. Shreifer and Vishny (2009) is also close to our paper in showing

[2]Business cycle accounting results on Japan and Korea by Otsu and Pyo (2009) indicate that both the TFP and the labor wedge may be driven by the financial frictions.

that changes in asset prices may exacerbate real inefficiency. One feature of our model is that it is built on the standard neoclassical growth model, while both Beaudry and Lahiri (2009) and Shreifer and Vishny (2009) have difficulty in incorporating their models with the standard business cycle literature. Diamond and Rajan (2009) demonstrate a different mechanism for the credit freeze that a risk-shifting from investors to banks may cause the freezing of asset trade. Our market structure is close to that in Kiyotaki and Moore (2004, 2005), in which a certain asset works as inside money. Kiyotaki and Moore focus on the conditions for emergence of inside money, while we focus on those for the collapse of inside money due to emergence of new bad assets that cause the lemon problem.

The organization of this paper is as follows. In the next section, we construct the model. Section 3 analyzes the effect of the emergence of bad assets in financial crises and specifies the dynamics and steady states. Section 4 provides policy implications and Section 5 concludes.

2 Model

The model builds on the standard one-sector neoclassical growth model. Two key frictions are introduced: First friction in our model is the necessity of money as a medium of exchange in the labor market, which reduces to a constraint similar to the cash-in-advance constraint in Lucas and Stokey (1987). Second friction is the asymmetric information about assets among banks who trade the assets in the interbank market. The emergence of bad assets in the interbank market and asymmetric information causes freezing in asset trading among banks due to the same mechanism as Akerlof's (1970) market for lemons.

2.1 Baseline – A Neoclassical Growth Model

Before describing our model, let us review the standard neoclassical growth model as the baseline of our argument. In the baseline model, a representative consumer solves

$$\max_{c_t, l_t, k_{t+1}} \sum_{t=0}^{\infty} \beta^t U(c_t, 1 - l_t)$$

subject to $c_t + k_{t+1} - (1 - \delta)k_t \leq r_t^k k_t + w_t l_t$; and a representative firm solves

$$\max_{k_t, l_t} \pi_t = A_t k_t^{\alpha} l_t^{1-\alpha} - r_t^k k_t - w_t l_t.$$

The dynamics are determined by

$$-\frac{U_{l\ t}}{U_{c\ t}} = (1 - \alpha) A_t (k_t / l_t)^{\alpha}, \tag{2}$$

$$U_{c\ t} = \beta U_{c\ t+1} \{\alpha A_t (l_t / k_t)^{1-\alpha} + 1 - \delta\}, \tag{3}$$

$$c_t + k_{t+1} - (1 - \delta) k_t = A_t k_t^{\alpha} l_t^{1-\alpha}, \tag{4}$$

where $U_{c\ t} = \frac{\partial}{\partial c_t} U(c_t, 1 - l_t)$ and $U_{l\ t} = \frac{\partial}{\partial l_t} U(c_t, 1 - l_t)$.

2.2 Setup

The model is a closed one-sector economy with discrete time that continues from zero to infinity: $t = 0, 1, 2, \cdots, +\infty$. There are continua of consumers, firms, and banks, whose measures are normalized to one. These agents live forever. There is also the government (or the central bank) that can provide cash to banks and impose taxes on consumers. In this model all variables are described in the real term, that is, in terms of the consumption goods and we are not interested in nominal variables. So for simplicity, we fix the nominal price of the goods at one. This assumption implies that the gross rate of return on m_t is 1, where m_t is the real balance. (To modify our model so that the central bank conducts monetary policy that allows price changes is not difficult.)

The market structure in a representative period t is as follows. At the beginning of the period t, a representative consumer holds bank deposit (d_t) as her asset; a representative firm holds capital (k_t) as its asset, while it has outstanding corporate bonds (b_t) as its liability; and a representative bank holds corporate bonds and cash reserve (m_t) as its assets and bank deposits as its liability. Since the firm purchases the capital by issuing bonds, it is the case that

$$b_t = k_t,$$

for all t.[3] The balance sheet identity of the bank is that

$$d_t = b_t + m_t.$$

We assume that there is no asymmetric information between the firm and the lending bank. (As we posit later, asymmetric information exists only between banks.)

Labor market and interbank market: During the period t, the labor market and the interbank market open. We introduce a key market friction in the labor market, i.e., the anonymity of sellers (consumers) and buyers (firms) of the labor input. Because of the anonymity of the market, the sellers cannot trace the buyers after the trade (of labor input) is done. Therefore, trade by credit is impossible in the labor market, and the wage must be paid in cash.[4] Under the necessity of cash payment in the labor market, a firm who wants to hire labor input l_t must raise $w_t l_t$ units of cash in advance, where w_t is the wage rate. The firm requests a bank to make an intra-period loan $w_t l_t$ at interest rate x_t. (Note that x_t may be 0.) The bank in turn needs $w_t l_t$ units of cash to make an intra-period loan and it sells some portion of its bond holding b_t to other banks at the interbank price of the bonds (q_t) to raise money. The firm borrows the money and pays $w_t l_t$ to the consumer in cash. The consumer then deposits the wage $w_t l_t$ in her bank immediately. Although we may be able to assume that the timing of wage payment comes at random to each firm and so does the timing of short-term lending from each bank to a firm, we assume alternatively as follows in order to clarify the flow of cash:

[3]More precisely, if $b_0 = k_0$ in the initial period 0, the perfect competition among firms implies that profits are zero after wage payment and bond redemption. Therefore the purchase of k_{t+1} must be financed by issuing b_{t+1} for all $t \geq 0$.

[4]This logic that the anonymity of the market induces the necessity of cash payment is borrowed from monetary theory by Lagos and Wright (2005) and Berentsen, Camera and Waller (2007). The anonymity may be interpreted as the lack of memory as in Kocherlakota (1998).

Assumption 1 *During the period t, the (labor and interbank) market open twice sequentially. The market that open early is called the early market and the market that open late the late market. Consumers, firms and banks are divided into the two markets. An agent allocated to the early (late) market in period* 0 *is allocated to the early (late) market in all subsequent periods.*

A half of the consumers who are allocated to the early market are called the *early consumers*, and the other half who are allocated to the late market are called the *late consumers*. We define the *early firms*, the *early banks*, the *late firms*, and the *late banks* similarly. When the early market opens, the early banks raise money by selling some portion of their corporate bonds to the late banks, and they lend the money to the early firms. The early firms in turn pay wages in cash to the early consumers, who immediately deposit the wage in the early banks. Then the late market opens. the late banks raise money by selling their corporate bonds to the early banks, and they lend the money to the late firms, who pay wages in cash to the late consumers, who immediately deposit the wage in the late banks.

Goods market and asset market: At the end of the period t, the goods market and the asset market open. These markets are Walrasian market in which trade by credit is available and cash payment is not necessary. At this point, firms produce the consumption goods, $y_t = A_t k_t^{\alpha} l_t^{1-\alpha}$ and sell c_t to consumers and install k_{t+1} by issuing bonds, $b_{t+1} = k_{t+1}$. Corporate bonds b_t and the bank deposits d_t earn interest at the market rate and become $(1 + r_t)b_t$ and $(1 + r_t)d_t$ respectively. We assume for simplicity that the government conducts monetary policy such that the bond rate and the deposit rate become identical in equilibrium. See below.

Monetary policy: The government conduct the following monetary policy, which is financed by a lump-sum tax g_t imposed on the consumers.

- The government sets the money supply $\overline{m}_t$ at the beginning of period t (or at the end of period $t - 1$),

- the government makes cash injection $r_t^m m_t$ at the end of the period t to the bank that held cash reserve m_t at the beginning of the period t, and
- the government sets the rate of injection r_t^m such that banks' demand for reserve (m_t) equals the money supply: $m_t = \overline{m}_t$.

2.3 Equilibrium without Bad Assets

First, we specify the normal equilibrium where there is no bad assets on the bank balance sheets and therefore no asymmetric information emerges in the interbank market. It is shown that the normal equilibrium is exactly same as the equilibrium of the baseline neoclassical growth model described by equations (2)–(4).

Consumer: The consumer's problem is

$$\max_{c_t, l_t, d_{t+1}} \sum_{t=0}^{\infty} \beta^t U(c_t, 1 - l_t), \tag{5}$$

$$\text{subject to} \quad c_t + d_{t+1} \leq (1 + \tilde{r}_t) d_t + \tilde{w}_t l_t + \pi_t - g_t,$$

where $(\tilde{r}_t, \tilde{w}_t) = (r_t^e, w_t^e)$ if the consumer is the early consumer and $(\tilde{r}_t, \tilde{w}_t) = (r_t^l, w_t^l)$ if the consumer is the late consumer, π_t is the dividend from the firm, and g_t is the lump-sum tax.

Firm: The firm maximizes the discounted sum of the profit flows, which is discounted by the market rate. The firm's problem is as follows.

$$W(k_t, b_t) = \max_{l_t, k_{t+1}, b_{t+1}} \pi_t + \frac{1}{1 + r_{t+1}} W(k_{t+1}, b_{t+1}) \tag{6}$$

$$\text{subject to} \quad k_0 = b_0, \tag{7}$$

where $\pi_t = A_t k_t^{\alpha} l_t^{1-\alpha} + (1-\delta) k_t - k_{t+1} - (1 + \tilde{x}_t) \tilde{w}_t l_t - (1 + \tilde{r}_t) b_t + b_{t+1}$, where $(\tilde{x}_t, \tilde{r}_t, \tilde{w}_t) = (x_t^e, r_t^e, w_t^e)$ if the firm is the early firm and $(\tilde{x}_t, \tilde{r}_t, \tilde{w}_t) = (x_t^l, r_t^l, w_t^l)$ if the firm is the late firm. The condition $k_0 = b_0$ implies that in equilibrium under perfect competition, $k_t = b_t$ for all t (≥ 1).[5]

[5] If firms hold sufficient money (m_t^f) in advance, firms may be able to pay wages without borrowing from banks. Under the following assumptions, however, it is shown that $\forall t$, $m_t^f = 0$ even if firms are

Bank: The problem for a bank is

$$V(m_t, b_t, d_t) = \max\left[\tilde{\pi} + \frac{1}{1+r_t}V(m_{t+1}, b_{t+1}, d_{t+1})\right],$$

where $\tilde{\pi} = \pi^e$ if the bank is the early bank and π^l if it is the late bank. π^e and π^l are are specified below.

Early bank: At the beginning of the period t, an early bank has m_t and b_t as its assets and d_t as its liability. In the early market, the early bank sells b^e units of bonds to late banks at price q^e to raise money for the short-term lending to an early firm. The bank lends $s^e = m_t + q^e b^e$ units of money to the firm at the intra-period interest rate of x^e. The early firms pay wages in cash to the early consumers and the early consumers deposits the cash in the early banks immediately. Therefore the early bank receives m^e units of cash as deposits. (In equilibrium, $s^e = m^e = w_t l_t$.) We assume that there is a regulation that prohibits the intra-period deposit m^e from earning interest. Thus the interest rate for m^e is zero. At the end of the early market, the early bank has m^e units of cash, s^e of short-term loans, and $b_t - b^e$ units of bonds as its assets and $d_t + m^e$ as its liability. Then the late market opens. In the late market, the early bank buys $\hat{b}^l$ units of bonds at the price of q^l from the late banks. At the end of the late market, the early bank has $m^e - q^l\hat{b}^l$ units of cash, s^e units of short-term loans, and $b_t - b^e + \hat{b}^l$ units of bonds as its assets, and $d_t + m^e$ as its liability. At the end of the period t, the bank settles the financial transactions. Since bonds $(b_t - b^e + \hat{b}^l)$, inter-period deposits (d_t), and the short-term loans (s^e) earn interest, they become $(1+r_t)(b_t - b^e + \hat{b}^l)$, $(1+r_t)d_t$, and $(1+x^e)s^e$, respectively. The bank receives the monetary injection $r_t^m m_t$ from the government. It chooses b_{t+1}, m_{t+1}, and d_{t+1} to carry over to the next period, subject to

allowed to hold cash. First, the initial value of m^f is zero: $m_0^f = 0$; and second, firms cannot hoard the proceeds of bond issuance as internal reserves. (This constraint may be imposed by the banks. If money can be easily diverted and consumed by the firm manager and the bank cannot verify that, banks demand the firms to secure b_{t+1} by collateral k_{t+1}.) Since no portion of b_{t+1} can be held as m_{t+1}^f, the perfect competition among firms makes that $b_{t+1} = k_{t+1}$ for all t and firms have no surpluses for internal reserves. Therefore, $m_t^f = 0$ for all t.

the balance sheet identity: $b_{t+1} + m_{t+1} \leq d_{t+1}$. The profit of the early bank is therefore

$$\pi^e = (1 + r_t)(b_t - b^e + \hat{b}^l) + m^e - q^l \hat{b}^l + r^m m_t + (1 + x^e)s^e - (1 + r_t)d_t - m^e - b_{t+1} - m_{t+1} + d_{t+1},$$

where $s_t = m_t + q^e b^e$. Using $d_t = b_t + m_t$, this equation is rewritten as

$$\begin{aligned}\pi^e =& (r^m + x^e - r_t)m_t + \{(1 + r_t) - q^l\}\hat{b}^l + \{(1 + x^e)q^e - (1 + r_t)\}b^e \\ & - b_{t+1} - m_{t+1} + d_{t+1},\end{aligned}$$

subject to $b_{t+1} + m_{t+1} \leq d_{t+1}$. To guarantee the inner solution it must be the case that $r^m_t = r_t - x^e$, $q^e = \frac{1+r_t}{1+x^e}$, and $q^l = 1 + r_t$.

Late bank: At the beginning of the period t, a late bank has m_t and b_t as its assets and d_t as its liability. In the early market, the late bank buys $\hat{b}^e$ units of bonds from the early banks. In the late market, the late bank sells b^l units of bonds to early banks to raise money for the short-term lending to a late firm. The bank lends $s^l = m_t - q^e \hat{b}^e + q^l b^l$ units of money to the firm at the intra-period interest rate of x^l. The late firms pay wages in cash to the late consumers and the late consumers deposits the cash in the late banks immediately. Therefore the late bank receives m^l units of cash as deposits. (In equilibrium, $s^l = m^l = w_t l_t$.) The interest rate for m^l is zero. At the end of the late market, the late bank has m^l units of cash, s^l units of short-term loans, and $b_t + \hat{b}^e - b^l$ units of bonds as its assets, and $d_t + m^l$ as its liability. At the end of the period t, the bank settles the financial transactions. Bonds, inter-period deposits, and the short-term loans earn interest. The bank receives the monetary injection $r^{ml}_t m_t$ from the government. It chooses b_{t+1}, m_{t+1}, and d_{t+1} to carry over to the next period, subject to the balance sheet identity: $b_{t+1} + m_{t+1} \leq d_{t+1}$. The profit of the late bank is therefore

$$\pi^l = (1 + r_t)(b_t + \hat{b}^e - b^l) + m^l + r^m m_t + (1 + x^l)s^l - (1 + r_t)d_t - m^l - b_{t+1} - m_{t+1} + d_{t+1},$$

where $s_t = m_t - q^e \hat{b}^e + q^l b^l$. Using $d_t = b_t + m_t$, this equation is rewritten as

$$\begin{aligned}\pi^l =& (r^{ml} + x^l - r_t)m_t + \{(1 + r_t) - (1 + x^l)q^e\}\hat{b}^e + \{(1 + x^l)q^l - (1 + r_t)\}b^l \\ & - b_{t+1} - m_{t+1} + d_{t+1},\end{aligned}$$

subject to $b_{t+1} + m_{t+1} \leq d_{t+1}$. To guarantee the inner solution it must be the case that $r_t^{ml} = r_t - x^l$, $q^e = \frac{1+r_t}{1+x^l}$, and $q^l = \frac{1+r_t}{1+x^l}$.

Normal Equilibrium: In the normal equilibrium where there is no bad asset, the above conditions for the inner solutions to the early and the late bank's problems imply that $x^e = x^l = 0$, $q^e = q^l = 1 + r_t$, $r^m = r^{ml} = r_t$. Since $x = 0$, the consumer's problem (5) and the firm's problem (6) imply that the allocations (c_t, l_t, k_{t+1}) in the early market and the late market are the same and that the model reduces to the baseline growth model, whose dynamics are determined by (2)–(4). The necessary condition for $x_t = 0$ is that the early and late banks can raise sufficient amount of money in the interbank market so that they can lend $w_t^* l_t^*$ to firms, where $w_t^* l_t^*$ is the wage in the baseline model. We put asterisk on the variables to denote the equilibrium values that solve (2)–(4). Thus the necessary and sufficient condition is that

$$\forall t, \quad w_t^* l_t^* \leq \overline{m}_t + \min\{\overline{m}_t, (1 + r_t^*) b_t^*\}, \tag{8}$$

where $b_t^* = k_t^*$ and $1 + r_t^* = \alpha A_t (l_t^*/k_t^*)^{1-\alpha} + 1 - \delta$. We simply assume that the model parameters and $\{\overline{m}_t\}_{t=0}^{\infty}$ are chosen such that (8) is satisfied. We also assume the following to make the equilibrium path different in the case of financial crisis:

$$\forall t, \quad w_t^* l_t^* \geq \overline{m}_t. \tag{9}$$

3 Financial Crisis

Thus far we have showed that the necessity of cash payment in the labor market is innocuous as long as b_t is exchanged for money in the interbank market. In this case, the model reduces to the baseline neoclassical growth model. We model a financial crisis as a time when b_t is not accepted in the interbank market because of the asymmetric information about bad assets among banks.

3.1 Bad Assets

We assume that (a huge amount of) bad assets emerge exogenously at the beginning of the period 0 and they are endowed to the banks equally. n units of bad assets emerge in period 0 and they stay in the economy unless the banks dispose of them. No more bad assets emerge in the subsequent periods. At the beginning of period 0, n units of bad assets are endowed to each bank. One unit of bad asset is durable paper that looks like a corporate bond that is promised in exchange for one unit of the consumption goods at the end of the current period. (The paper does not specify the exact date t when it pays out. For any given t, if the bad asset exists in period t, all agents except the holder of the bad asset regard it as a promise in exchange for the goods at the end of the period t.) However, the issuer of the bad asset is nonexistent. The bad asset appears as a claim on one unit of the goods, but actually it is not. The bad asset returns nothing at the end of the period t ($\forall t$). We assume information asymmetry on bad asset n among banks as follows. On one hand, banks know that n, which they possess, are the bad assets. On the other hand, banks cannot distinguish other banks' holdings of bad assets (n) from the good assets (b_t). Only after a bank buys paper in the interbank market does the bank know whether the paper is n or b_t.

We also assume that there is the following costly revelation technology of bad assets. A bank can reveal by paying the real cost γ that one unit of its own bad asset is *not* a genuine corporate bond. So if the bank pays γn, it can reveal all bad assets on its balance sheet. (γn is the dead weight loss.) Once γn is paid by a bank, it becomes the public information that the bank's n are not corporate bonds. Note however that a bank *cannot* reveal that a genuine bond that it possesses is not a bad asset. Therefore even after the bank reveals all its own bad assets n, other banks are still uncertain that the bank's remaining assets (b_t) may include bad assets.

We regard revelation of bad assets and disposal of bad assets as almost the same event. We assume that banks can dispose of n only after revelation. If they don't pay $\gamma\ n$ in period t, the banks hold the bad assets n in period $t+1$.

3.2 Optimization for Banks with Bad Assets

In this subsection we describe the optimizations of agents under the existence of the bad assets. The problems for consumers and firms are identical to those in the case without bad assets: The consumer solves (5) and the firm solves (6).

Early bank: At the beginning of the period t, an early bank has m_t, b_t, and n_t as its assets and d_t as its liability. In the early market, the early bank sells $n^e (\leq n_t)$ units of bad assets and b^e units of bonds to late banks at price q^e to raise money for the short-term lending to an early firm. The bank lends $s^e = m_t + q^e(n^e + b^e)$ units of money to the firm at the intra-period interest rate of x^e. The early firms pay wages in cash to the early consumers and the early consumers deposits the cash in the early banks immediately. Therefore the early bank receives m^e units of cash as deposits. (In equilibrium, $s^e = m^e = w_t l_t$.) The interest rate for m^e is zero. At the end of the early market, the early bank has m^e units of cash, s^e of short-term loans, $b_t - b^e$ units of bonds, and $n_t - n^e$ units of bad assets as its assets and $d_t + m^e$ as its liability. Then the late market opens. In the late market, the early bank buys $\hat{b}^l$ units of bonds at the price of q^l from the late banks. But $\xi^l \hat{b}^l$ turns out to be bad assets, where ξ^l is the ratio of bad assets in total bond supply of the late interbank market. ξ^l is taken as exogenous by each early bank. (ξ^l is determined as an equilibrium outcome.) At the end of the late market, the early bank has $m^e - q^l \hat{b}^l$ units of cash, s^e units of short-term loans, $b_t - b^e + (1 - \xi^l)\hat{b}^l$ units of bonds, and $n_t - n^e + \xi^l \hat{b}^l$ units of bad assets as its assets, and $d_t + m^e$ as its liability. At the end of the period t, the bank settles the financial transactions. The bonds, the inter-period deposits, and the short-term loans earn interest. The bank receives the monetary injection $r_t^m m_t$ from the government. It chooses b_{t+1}, m_{t+1}, and d_{t+1} to carry over to the next period, subject to the balance sheet identity: $b_{t+1} + m_{t+1} \leq d_{t+1}$.[6] The profit of the early bank is therefore

$$\pi^e = (1+r_t)\{b_t - b^e + (1-\xi^l)\hat{b}^l\} + m^e - q^l\hat{b}^l + r^m m_t + (1+x^e)s^e - (1+r_t)d_t - m^e$$
$$- b_{t+1} - m_{t+1} + d_{t+1},$$

where $s_t = m_t + q^e(n^e + b^e)$. Note that the bad assets $(n_t - n^e + \xi^l\hat{b}^l)$ do not appear in π^e because they don't yield any return in the form of the goods. Using $d_t = b_t + m_t$, this equation is rewritten as

$$\pi^e = (r^m + x^e - r_t)m_t + \{(1+r_t)(1-\xi^l) - q^l\}\hat{b}^l + \{(1+x^e)q^e - (1+r_t)\}b^e$$
$$+ (1+x^e)q^e n^e - b_{t+1} - m_{t+1} + d_{t+1}. \tag{10}$$

The Bellman equation for the early bank is

$$V(n_t, b_t, m_t, d_t) = \max_{n^e, b^e, \hat{b}^l, n_{t+1}, b_{t+1}, m_{t+1}, d_{t+1}} \pi^e + \frac{1}{1+r_{t+1}} V(n_{t+1}, b_{t+1}, m_{t+1}, d_{t+1}), \tag{11}$$

$$\text{s.t. } n^e \leq n_t, \tag{12}$$
$$n_{t+1} = n_t - n^e + \xi_t^l \hat{b}_t^l,$$
$$b_{t+1} + m_{t+1} \leq d_{t+1}.$$

Late bank: At the beginning of the period t, a late bank has m_t, b_t, and n_t as its assets and d_t as its liability. In the early market, the late bank buys $\hat{b}^e$ units of bonds from the early banks. But $\xi^e\hat{b}^e$ units turn out to be bad assets, where ξ^e is the ratio of bad assets in total bond supply of the early interbank market. ξ^e is taken as exogenous by each late bank. In the late market, the late bank sells b^l units of bonds and $n^l(\leq n_t + \xi^e\hat{b}^e)$ units of bad assets at price q^l to early banks to raise money for the short-term lending to a late firm. The bank lends $s^l = m_t - q^e\hat{b}^e + q^l(b^l + n^l)$ units of money to the firm at the intra-period interest rate of x^l. The late firms pay wages in cash to the late consumers

[6] This balance sheet identity says that the banks use the deposits (d_{t+1}) to purchase the bonds (b_{t+1}) from the firms and the cash (m_{t+1}) from other banks and consumers. Since we assume that there is no information asymmetry between the firms and the lending banks (page 7), it is the case that b_{t+1} do not contain the bad assets. Since cash is distinguishable from the bad assets, m_{t+1} do not contain the bad assets either.

and the late consumers deposits the cash in the late banks immediately. Therefore the late bank receives m^l units of cash as deposits. (In equilibrium, $s^l = m^l = w_t l_t$.) The interest rate for m^l is zero. At the end of the late market, the late bank has m^l units of cash, s^l units of short-term loans, $b_t + (1-\xi^e)\hat{b}^e - b^l$ units of bonds, and $n_t + \xi^e b^e - n^l$ units of bad assets as its assets, and $d_t + m^l$ as its liability. At the end of the period t, the bank settles the financial transactions. Bonds, inter-period deposits, and the short-term loans earn interest. The bank receives the monetary injection $r_t^{ml} m_t$ from the government. It chooses b_{t+1}, m_{t+1}, and d_{t+1} to carry over to the next period, subject to the balance sheet identity: $b_{t+1} + m_{t+1} \leq d_{t+1}$. The profit of the late bank is therefore

$$\begin{aligned}\pi^l =& (1+r_t)\{b_t + (1-\xi^e)\hat{b}^e - b^l\} + m^l + r^m m_t + (1+x^l)s^l - (1+r_t)d_t - m^l \\ &- b_{t+1} - m_{t+1} + d_{t+1},\end{aligned}$$

where $s_t = m_t - q^e \hat{b}^e + q^l(b^l + n^l)$. Using $d_t = b_t + m_t$, this equation is rewritten as

$$\begin{aligned}\pi^l =& \{(1+r_t)(1-\xi^e) - (1+x^l)q^e\}\hat{b}^e + \{(1+x^l)q^l - (1+r_t)\}b^l + (1+x^l)q^l n^l \\ &+ (r^{ml} + x^l - r_t)m_t - b_{t+1} - m_{t+1} + d_{t+1}. \end{aligned} \tag{13}$$

The Bellman equation for the late bank is

$$V(n_t, b_t, m_t, d_t) = \max_{n^l, b^l, \hat{b}^e, n_{t+1}, b_{t+1}, m_{t+1}, d_{t+1}} \pi^l + \frac{1}{1+r_{t+1}} V(n_{t+1}, b_{t+1}, m_{t+1}, d_{t+1}), \tag{14}$$

$$\text{s.t. } n^l \leq n_t + \xi^e \hat{b}^e, \tag{15}$$

$$n_{t+1} = n_t - n^l + \xi_t^e \hat{b}_t^e,$$

$$b_{t+1} + m_{t+1} \leq d_{t+1}.$$

Two Types of Equilibrium: It will be shown that for small enough n, there exists the *active equilibrium* in which $q_t < 1 + r_t$ and the bonds and the bad assets are actively traded in the interbank market. It is also shown that for any positive n, there exists the *crisis equilibrium* in which $q_t = 0$ and the interbank trading of bonds and bad assets is shut down. In this case, trading of the corporate bonds freezes and the banks can lend only their own cash reserves (m_t) to the firms for wage payment. Therefore, it will be shown that the dynamics of the crisis equilibrium reduce to the consumer's problem (5) and the firm's problem (6) with equilibrium condition that $w_t l_t \leq \overline{m}_t$.

Simplifying assumptions on preference, technology, and money supply: To show simple solution explicitly, we assume in what follows that productivity is time invariant: $\forall t,\ A_t = A$; and

$$U(c_t, 1 - l_t) = \ln c_t + \phi \ln(1 - l_t).$$

We also assume that the money supply is time invariant: $\forall t,\ \overline{m}_t = \overline{m}$. Since we assumed that the money supply satisfies (8) and (9), the following constraint for $\overline{m}$ is required:

$$\overline{m} \leq w^* l^* \leq \overline{m} + \min\{\overline{m}, (1 + r^*) b^*\}, \tag{16}$$

where the variables with asterisk are the steady state values of the normal equilibrium. Equations (2)–(4) imply that the steady state of the normal equilibrium (c^*, l^*, k^*) is determined by

$$c^* = c(k^*) = [\alpha^{-1}(\beta^{-1} - 1 + \delta) - \delta] k^*, \tag{17}$$

$$l^* = l(k^*) = [\alpha^{-1} A^{-1} (\beta^{-1} - 1 + \delta)]^{1/(1-\alpha)} k^*, \tag{18}$$

$$\frac{\phi c(k^*)}{\{1 - l(k^*)\}} = (1 - \alpha) A \left(\frac{\alpha A}{\beta^{-1} - 1 + \delta} \right)^{\frac{\alpha}{1-\alpha}}. \tag{19}$$

Why money supply is constrained? The government should be able to control the amount of money supply to some extent. But we do not assume that the government can freely set the real amount of money supply. We should interpret $\overline{m}$ as the upper bound of the money supply, and (16) is a technological constraint imposed on the government. To justify the constraint on money supply, we borrow the logic of the fiscal theory of price level (see for example, Woodford 2001): Although the government can freely set the nominal amount of money supply, the price level adjusts such that the real value of the money supply becomes less than or equal to the expected value of the discounted sum of the future tax revenues. The real value of the tax revenue is determined by the tax technology and political constraints, both of which can be plausibly considered as exogenous factors to our model. Therefore, we consider that the real money supply has a natural upper bound and (16) is satisfied.

3.3 Active Equilibrium

In this subsection we show that for small n there exists the active equilibrium in which $q_t < 1 + r_t$ and the bonds and the bad assets are actively traded in the interbank market. We also show that for sufficiently large n, the active equilibrium does not exist.

The FOCs with respect to n^e and the envelope condition with respect to n_t for (11) imply that the constraint (12) strictly binds if $r_t > 0$, which is the case in our model. Similarly, the FOC with respect to n^l and the envelope condition with respect to n_t for (14) imply (15) is binding. Therefore, the Bellman equation for the early bank becomes as follows (we omit the state variables b_t, d_t, m_t)

$$\begin{aligned} V(n_t) =& (r_t^m + x^e - r_t)m_t + \{(1+x^e)q^e - (1+r_t)\}b^e + \{(1-\xi_t^l)(1+r_t) - q^l\}\hat{b}^l \\ &+ q^e(1+x^e)n_t + \frac{1}{1+r_{t+1}}V(\xi_t^l\hat{b}_t^l). \end{aligned} \tag{20}$$

The Bellman equation for the late bank becomes

$$\begin{aligned} V(n_t) =& (r_t^m + x^e - r_t)m_t + \{(1+r_t)(1-\xi_t^e) - (1+x_t^l)(q^e - q^l\xi^e)\}\hat{b}^e + \{(1+x_t^l)q^l - (1+r_t)\}b^l \\ &+ q^l(1+x^l)n_t + \frac{1}{1+r_{t+1}}V(0). \end{aligned} \tag{21}$$

Note that all bad assets go back and forth between the early and the late banks, because constraints (12) and (15) are binding. At the beginning of period 0, each bank is endowed with n units of bad assets. In the early market, the early banks sell all $n(= \xi_0^e\hat{b}_0^e)$ to the late banks, and in the late market, the late banks sell all $n + \xi_0^e\hat{b}_0^e = 2n$ to the early banks. Therefore, at the beginning of the period 1, the early banks hold $2n$ units of bad assets and the late banks hold 0 unit of bad assets. In period t (≥ 1), the early banks sell $2n(= \xi_t^e\hat{b}_t^e)$ units of bad assets to the late banks in the early market, and the late banks sell $0 + \xi_t^e\hat{b}_t^e = 2n$ units of bad assets to the early banks in the late market. We solve these equations on the premise that the solutions $(b^e, b^l, \hat{b}^e, \hat{b}^l, m_t)$ are inner solutions. The FOCs and the envelope condition for the early bank imply $r_t^m = r_t - x_t^e$,

$q_t^e = (1+r_t)/(1+x_t^e)$, and $q_t^l = (1+r_t)\left(1 - \frac{r_t}{1+r_t}\xi_t^l\right)$. The FOCs for the late bank imply $r_t^m = r_t - x_t^l$, $q_t^e = (1+r_t)/(1+x^l) = q^l$. Therefore, in this equilibrium $x_t^e = x_t^l = x_t$, $q_t^e = q_t^l = (1+r_t)/(1+x_t)$ and $\frac{1}{1+x_t} = 1 - \frac{r_t}{1+r_t}\xi_t^l$. Note that $x_t^e = x_t^l = x_t$. Since the short-term rate is equal in the early and the late markets, the consumers and the firms face the same prices in the early and the late markets: $r_t^e = r_t^l = r_t$ and $w_t^e = w_t^l = w_t$. Therefore, the allocation (c_t, l_t, k_{t+1}) must be the same in the early and the late markets. Therefore, the intra-period lending by the early bank in period t (≥ 1) can be written as

$$s_t = w_t l_t = m_t + q_t^e(b_t^e + 2n),$$

while that by the late bank is

$$s_t = w_t l_t = m_t - q_t^e(b_t^e + 2n) + q_t^l(b^l + 2n).$$

Adding up these equations and using $\xi_t^l = \frac{2n}{b^l+2n}$ and $w_t = -U_{l\ t}/U_{c\ t}$, we obtain

$$-\frac{U_{l\ t}}{U_{c\ t}} l_t = m_t + q^l \frac{n}{\xi_t^l}. \tag{22}$$

The FOCs and the envelope condition for (20) imply that

$$1 - \frac{r_t}{1+r_t}\xi_t^l = \frac{1}{1+x_t}. \tag{23}$$

In the active equilibrium, if it exists at all, the model reduces to (5) and (6) with equilibrium conditions (22) and (23). Therefore, the equilibrium dynamics $\{c_t, l_t, k_{t+1}, x_t, \xi_t^l\}_{t=0}^{\infty}$ should be determined by (22), (23), and

$$-\frac{U_{l\ t}}{U_{c\ t}} = \frac{(1-\alpha)A_t(k_t/l_t)^\alpha}{1+x_t}, \tag{24}$$

$$U_{c\ t} = \beta U_{c\ t+1}\{\alpha A_t(l_t/k_t)^{1-\alpha} + 1 - \delta\}, \tag{25}$$

$$c_t + k_{t+1} - (1-\delta)k_t = A_t k_t^\alpha l_t^{1-\alpha}, \tag{26}$$

if the active equilibrium exists. The following proposition establishes the existence.

Proposition 1 *If n is small enough, there exists the active equilibrium $\{c_t, l_t, k_{t+1}, x_t, \xi_t^l\}_{t=0}^{\infty}$ that solves (22), (23), (24), (25), and (26).*

(Proof) Let $\xi^{(i)} = \{\xi_t^{(i)}\}_{t=0}^{\infty}$ be a Cauchy sequence on $[0,1]$. Define a mapping $T(n)$ from $\xi^{(i)}$ to $\xi^{(i+1)} = T(n)\xi^{(i)}$ as follows. First, solve (23), (24), (25), and (26), under the condition that $\forall t,\ \xi_t = \xi_t^{(i)}$. The solution can be denoted as $\{x_t^{(i)}, c_t^{(i)}, l_t^{(i)}, k_{t+1}^{(i)}\}_{t=0}^{\infty}$. Second, determine $b_t^{(i)}$ by $w_t^{(i)} l_t^{(i)} = \overline{m} + q_t^{(i)} \left(\frac{b_t^{(i)}}{2} + n \right)$, where $q_t^{(i)} = (1 + r_t^{(i)})/(1 + x_t^{(i)})$. Third, determine $\xi^{(i+1)}$ by $\xi_t^{(i+1)} = 2n/(2n + b_t^{(i)})$. If n is small enough, it is the case that $\forall t, b_t^{(i)} \geq 0$. In this case, $\xi^{(i+1)} = \{\xi_t^{(i+1)}\}_{t=0}^{\infty}$ is also a Cauchy sequence on $[0,1]$. This mapping $\xi^{(i+1)} = T(n)\xi^{(i)}$ is a continuous mapping from the set of Cauchy sequences on $[0,1]$ to itself. Therefore, the Schauder Fixed-Point Theorem implies that there exists a fixed point of $T(n)$ that satisfies $\xi = T(n)\xi$. See Section 17.4 of Stokey and Lucas with Prescott (1989) for the Schauder Fixed-Point Theorem. The values of $\{c_t, l_t, k_{t+1}, x_t\}$ that correspond to $\xi = \{\xi_t\}_{t=0}^{\infty}$ determine the equilibrium path. (Proof ends.)

The equilibrium may not be unique. Since there exist two steady states for sufficiently small n (see Section 3.4), there may be two different equilibrium paths for a small n that converge on the different steady states. The following proposition establishes the nonexistence of the active equilibrium for a large enough n.

Proposition 2 *If n is large enough, the active equilibrium does not exist.*

(Proof) We assume that n satisfies $n > W(k_0) - \overline{m}$, where k_0 is the initial value of capital stock and $W(k_0) = \max_t w_t^* l_t^*$, where the variables with asterisk are those in the normal equilibrium (2)–(4). Suppose that there exists the active equilibrium for a large n. In the active equilibrium, it must be the case that $q_t = \left(1 - \frac{r_t}{1+r_t}\xi_t\right)(1 + r_t) \geq 1$, since $\xi_t \leq 1$. In this case, $w_t l_t \leq w_t^* l_t^*$, where $w_t l_t$ are the values in the active equilibrium. Therefore, $n > W(k_0) - \overline{m}$ implies

$$w_t l_t < \overline{m} + n. \tag{27}$$

Condition (27) implies that for any $q_t \geq 1$, the banks can raise enough money for the intra-period lending ($w_t l_t$) by selling only a strictly smaller amount of the bad assets than n. (Moreover, banks need not sell genuine bonds to raise money.) This means that the condition (12) and (15) do not bind in the bank's optimization. Since (12) and (15) must be binding in the active equilibrium, this is the contradiction. Therefore, the active equilibrium does not exist. (Proof ends.)

3.4 Steady State of the Active Equilibrium

The steady state of the active equilibrium $(c^a, l^a, k^a, x^a, \xi^a)$ is determined as the solution to the following system of equations (note that $1 + r^a = \beta^{-1}$ in the steady state):

$$
\begin{aligned}
&c^a = c(k^a) = [\alpha^{-1}(\beta^{-1} - 1 + \delta) - \delta]k^a, \\
&l^a = l(k^a) = [\alpha^{-1}A^{-1}(\beta^{-1} - 1 + \delta)]^{1/(1-\alpha)}k^a, \\
&\frac{\phi c(k^a)}{\{1 - l(k^a)\}} = \frac{1}{1 + x^a}(1 - \alpha)A\left(\frac{\alpha A}{\beta^{-1} - 1 + \delta}\right)^{\frac{\alpha}{1-\alpha}}, \\
&\frac{\phi c^a l^a}{1 - l^a} = m + \frac{\beta^{-1}}{1 + x^a}\frac{n}{\xi^a}, \\
&1 - (1 - \beta)\xi^a = \frac{1}{1 + x^a}.
\end{aligned}
$$

It is shown from these equations that k and x are determined by

$$
k = \frac{B}{C + D(1 + x)}, \tag{28}
$$

$$
k = E\left(m + \frac{(\beta^{-1} - 1)n}{x}\right)(1 + x), \tag{29}
$$

where

$$
\begin{aligned}
B &= \frac{(1 - \alpha)\alpha^{\alpha/(1-\alpha)}A^{1/(1-\alpha)}}{(\beta^{-1} - 1 + \delta)^{-\alpha/(1-\alpha)}}, \\
C &= \phi[\alpha^{-1}(\beta^{-1} - 1 + \delta) - \delta], \\
D &= B[\alpha^{-1}A^{-1}(\beta^{-1} - 1 + \delta)], \\
E &= \frac{\alpha}{(1 - \alpha)(\beta^{-1} - 1 + \delta)}.
\end{aligned}
$$

It is easily confirmed that if n is sufficiently small, the system of equations (28) and (29) has two solutions, while there is no solution if n is large. Therefore, the active equilibrium has two steady states if n is sufficiently small and no steady state if n is sufficiently large.

In what follows, we focus on the equilibrium where the interbank market is shut down.

3.5 Crisis Equilibrium

If bad assets n are endowed to banks at the beginning of period 0, exactly the same reasoning as the market for lemons (Akerlof 1970) shows that for any positive value of n there exists an equilibrium path in which the interbank market is shut down. We call it the *crisis equilibrium.*

Proposition 3 *For any n (> 0), there exists the crisis equilibrium in which the price of bonds in the interbank market is 0 and banks never trade corporate bonds.*

(Proof) Suppose that the prices of corporate bonds in the early and late interbank market are zero: $q^e = q^l = 0$. Consider the late market. Equation (21) implies that the late banks never sell the corporate bonds if $q^l < \frac{1+r_t}{1+x^l}$, because the marginal gain from selling b^l is $\{(1+x_t^l)q_t^l - (1+r_t)\}$. Since $q^l = 0$, the late banks surely offer to sell the bad assets to the early banks. It is shown as follows that the early banks have no incentive to bid up q^l: because each early bank is infinitesimally small, the early bank can buy only the bad assets for any bid price q^l, implying $\xi^l = 1$; since $\xi^l = 1$, (20) implies that the early bank's marginal gain from buying $\hat{b}_t^l$ (which are surely bad assets) is $(1-\xi_t^l)(1+r_t) - q^l = -q^l$, which is negative for all q^l (> 0); therefore the early bank has no incentive to bid up q^l. Consider next the early market. Equation (20) implies that the early banks never sell the corporate bonds b_t^e if $q^e < \frac{1+r_t}{1+x^e}$. Since $q^e = 0$, the early banks surely offer to sell the bad assets n to the late banks. It is shown as follows that the late banks have no incentive to bid up the price: because each late bank is infinitesimally small, the late bank can buy only the bad assets for any bid price q^e, that is $\xi^e = 1$; in this case, since $\xi^e = 1$ and $q^l = 0$, (21) implies that the late bank's marginal gain from buying $\hat{b}_t^e$ (which are surely bad assets) is $(1+r_t)(1-\xi_t^e) - (1+x_t^l)(q^e - q^l\xi^e) = -(1+x^l)q^e$, which is negative for any q^e (> 0); therefore, the late bank has no incentive to bid up q^e. Therefore, $q^e = q^l = 0$ can be equilibrium prices and in this case $\hat{b}_t^e = \hat{b}_t^l = 0$. That is, the banks never trade corporate bonds. (Note that if the infinitesimally small banks collectively bid up prices, the economy can shift to the active equilibrium, where $q^e > 0$ and $q^l > 0$.) (Proof ends.)
In the crisis equilibrium, the interbank market is shut down and therefore the banks can

lend at most m_t, their own cash reserves, to the firms for wage payment. Therefore, the model reduces to (5) and (6) with the equilibrium condition that $w_t l_t \leq \overline{m}_t$. The dynamics of the reduced-form model are determined by (24), (25), (26), and

$$-\frac{U_{l\ t}}{U_{c\ t}} l_t \leq \overline{m}_t, \tag{30}$$

Figure 3 shows the dynamics of the economy, which is initially in the steady state of the normal equilibrium (17)–(19) and the bad assets emerge at $t = 0$. The economy converges on the steady state of the crisis equilibrium that is specified in Section 3.6.

Figure 3: Simulation Result

Banks have no incentive to dispose of bad assets: In the crisis equilibrium, banks have no incentive to reveal their own bad assets even though the costly revelation technology is available. This is shown as follows. Suppose that an early bank reveal n by paying γn, but the other early banks do not. In this case, late banks know that bad assets are still in the early interbank market. Because of the asymmetric information, the late banks don't know who has the bad assets and who does not. Therefore, the late banks still face $\xi_t^e = 1$, because the early bank who disposed of the bad assets is infinitesimally small. The same arguments as Proposition 3 hold and therefore $q_t^e = 0$. In the end, the early bank who paid γn cannot sell the corporate bonds in the interbank market. So if the bank reveals its bad assets, it pays γn for nothing. Thus there is no incentive for banks to reveal their bad assets in the crisis equilibrium.

In this model, once bad assets n emerge and the economy falls into the crisis equilibrium, the banks hold n forever and the equilibrium path shifts from what is described by (2)–(4) to what is described by (24)–(26) and (30).

3.6 Steady State of the Crisis Equilibrium

Since the banks have no proper incentive to dispose of bad assets, once bad assets emerge and the economy falls into the crisis equilibrium, the economy converges on the steady state with the bad assets. Equations (24)–(26) and (30) imply the economy converges on the steady state (c^c, l^c, k^c, x^c), which is determined by

$$c^c = c(k^c) = [\alpha^{-1}(\beta^{-1} - 1 + \delta) - \delta]k^c, \tag{31}$$

$$l^c = l(k^c) = [\alpha^{-1}A^{-1}(\beta^{-1} - 1 + \delta)]^{1/(1-\alpha)}k^c, \tag{32}$$

$$\frac{\phi c(k^c)l(k^c)}{\{1 - l(k^c)\}} = \overline{m}, \tag{33}$$

$$\frac{\phi c(k^c)}{\{1 - l(k^c)\}} = \frac{1}{1 + x^c}(1 - \alpha)A\left(\frac{\alpha A}{\beta^{-1} - 1 + \delta}\right)^{\frac{\alpha}{1-\alpha}}. \tag{34}$$

Equation (33) is the liquidity constraint for the intra-period bank loans, i.e., $w^c l^c = \overline{m}$.

We now compare the steady states of the crisis equilibrium and the normal equilibrium. Let us define $f(k) \equiv \frac{\phi c(k)l(k)}{\{1-l(k)\}}$, where $c(k) = [\alpha^{-1}(\beta^{-1} - 1 + \delta) - \delta]k$ and $l(k) = [\alpha^{-1}A^{-1}(\beta^{-1} - 1 + \delta)]^{1/(1-\alpha)}k$. Condition (33) is rewritten as $f(k^c) = \overline{m}$ and the first inequality in (16) implies that $f(k^*) > \overline{m}$. Since $f(k)$ is strictly increasing in k, these conditions imply $k^c < k^*$, which directly implies $c^c < c^*$ and $l^c < l^*$. Equations (34) and (19) then imply $x^c > 0$. Since the output is also proportional to capital, $k^c < k^*$ implies that the output in the crisis equilibrium is smaller than the output in the normal equilibrium. The labor wedge $1 - \tau$ is defined by (1). Therefore, the labor wedge in the crisis equilibrium $1 - \tau^c$ is

$$1 - \tau^c = \frac{1}{1 + x^c} < 1,$$

while the labor wedge in the normal equilibrium $1 - \tau^* = 1$. We have the labor wedge deterioration in the crisis equilibrium.

4 Discussion

In our model, wages must be paid in cash due to anonymity in the labor market. The firms need to borrow money from the banks, and the banks in turn need to raise money for lending to the firms. When the interbank market functions well necessary money for wage payments are raised without frictions and the optimal allocation is attained. If bad assets emerge the asymmetric information about the assets among banks causes malfunction of the interbank market, that is, asymmetric information freezes interbank asset trading.[7] As a result, the amount of money available for working capital loans is constrained. This coordination failure causes a structural change of the economy from the normal equilibrium in which the liquidity constraint (8) is nonbinding to the crisis equilibrium in which the constraint (30) is binding. Output and the labor wedge persistently deteriorate in the crisis equilibrium. Because of the asymmetric information among banks, no proper incentive exists for banks to individually reveal the bad assets (or to remove the bad assets).

4.1 Policy Implication

If all of the bad assets n are revealed, the market for b_t is restored. If the cost of revelation $\gamma\, n$ is not excessive, the revelation is welfare improving. (We assume that banks rationally expect the values of ξ_t^e and ξ_t^l, both of which become 0, if all of n are revealed.) As a result of the coordination failure, each bank, that is infinitesimally small, has no incentive to reveal its own n individually. Therefore, intervention by the government that accelerates the bad asset disposals may be justified. The policy options are, for example, stringent asset evaluations ("stress test"), which should be done repeatedly; government purchases of the bad assets; reintroduction of stringent accounting rules for banks; and provision of policy scheme for recapitalization (or temporary nationalization) of banks.

In this model, properly specified macroeconomic policy is also effective for relaxing the financial constraints (30) in the market. Let us consider the following fiscal policy:

[7]The emergence of bad assets and asymmetric information about the asset quality may directly cause a decrease in bank lending to productive firms. We may consider the following model which is slightly different from the model in this paper: Firms (or entrepreneurs) own the capital stock and they need to put up the capital as collateral when they borrow the working capital (for wage payment) from the banks; and the bad assets are endowed to the firms and banks cannot distinguish the bad assets and the productive capital. In this setting, there may exist an equilibrium in which the banks do not lend the working capital to the firms because of the same mechanism as Akerlof's market for lemons. Notable feature is that if the lemon problem occurs in the bank lending, cash injection into the banking sector cannot increase the amount of bank lending to the firms. This phenomenon that bank lending decreases despite of the central bank's huge cash injection into the banks is called the *credit trap*. See Benmelech and Bergman (2009) for a model of credit trap.

At the beginning of the period t, the government gives banks (or firms) a subsidy in the form of cash, m_t^g; and at the end of the period t, the government imposes a tax on consumers, τ_t^g, where $\tau_t^g = m_t^g$. Though the government budget is balanced within the period t, this fiscal policy is still welfare improving because it relaxes the liquidity constraint (30) to

$$w_t l_t \leq \overline{m}_t + m_t^g.$$

Monetary policy (or liquidity provision) can be designed as follows: At the beginning of the period t, the government lends m_t^g units of cash to banks and collect it at the end of the period t. This policy also relaxes (30). As we assumed in (16), however, the real money supply has the upper bound that is determined by tax technology and/or some political factors. Once the money supply ($\overline{m}_t + m_t^g$) hits the upper bound, the government cannot increase it beyond the bound. In this sense, the relaxing effect of the above fiscal and monetary policies should be temporary and these macroeconomic policies cannot change the ultimate steady state where the economy converges on. This is because fiscal and monetary policies do not resolve the adverse selection in the interbank market and therefore do not restore interbank trading of the corporate bonds.

4.2 Business Cycles

This model show that freezing of the market for a certain asset class may cause output declines and the labor-wedge deteriorations by reducing available money for working capital loans. The model may be useful too to explain productivity changes in the business cycle frequencies. As Chari, Kehoe and McGrattan (2007) argue, financial constraints on financing the purchase of the intermediate goods can appear as TFP changes. If the production technology for the gross output is described by $y_t + z_t = Ak_t^{\alpha\theta} l_t^{(1-\alpha)\theta} z_t^{\theta}$, where z_t is the intermediate goods and y_t is the net output, changes in financial constraints on the purchase of z_t are observed as changes in TFP in the production function of the net output ($y_t = A_t k_t^{\alpha} l_t^{1-\alpha}$). See also Kobayashi, Nakajima and Inaba (2007) for the details. In this case, the productivity changes can be driven by asset market freezing (due to coordination failure), because the market freeze may tighten the financial constraints on the intermediate inputs through the same mechanism in our model. Therefore, we can come up with a possible hypothesis for the causes of the business cycles: That is, freezing and unfreezing of the market for a certain asset class may drive fluctuations of productivity, output, and the labor wedge in the business cycle frequencies.

5 Conclusion

Our experience of the global financial crisis in 2008 and 2009 suggests that we should formalize a major financial crisis as an event associated with

- a freezing of transactions in the asset markets; and
- a sharp contraction in aggregate output.

In addition to them, we find that a notable characteristic of the current crisis, which is common to the US Great Depression and the 1990s in Japan, is

- a sharp deterioration in the labor wedge.

In this paper we constructed a toy model that can explain these features. Our interpretation of this type of financial crises is a decrease in availability of working capital loans due to freezing of interbank trading of financial assets.

The firms need to borrow money from the banks, and the banks in turn need to raise money for lending to the firms. When the interbank market functions well necessary money for working capital loans are raised without frictions and the optimal allocation is attained. If bad assets emerge the asymmetric information about the assets among banks causes malfunction of the interbank market, that is, asymmetric information freezes interbank asset trading. As a result, the amount of money available for working capital loans is constrained. This coordination failure causes a structural change of the economy from the normal equilibrium in which the liquidity constraint is nonbinding to the crisis equilibrium in which it is binding. Output and the labor wedge persistently deteriorate in the crisis equilibrium. Because of the asymmetric information among banks, no proper incentive exists for banks to individually reveal the bad assets (or to remove the bad assets). In this model, the government intervention to accelerate bad asset disposals may improve social welfare.

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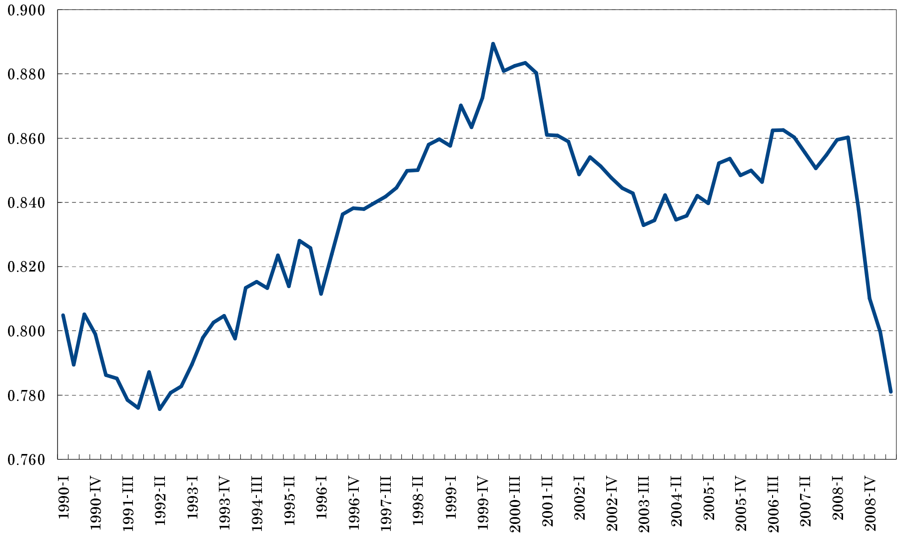
Figure 1: U.S. Labor Wedge (1990-2009)

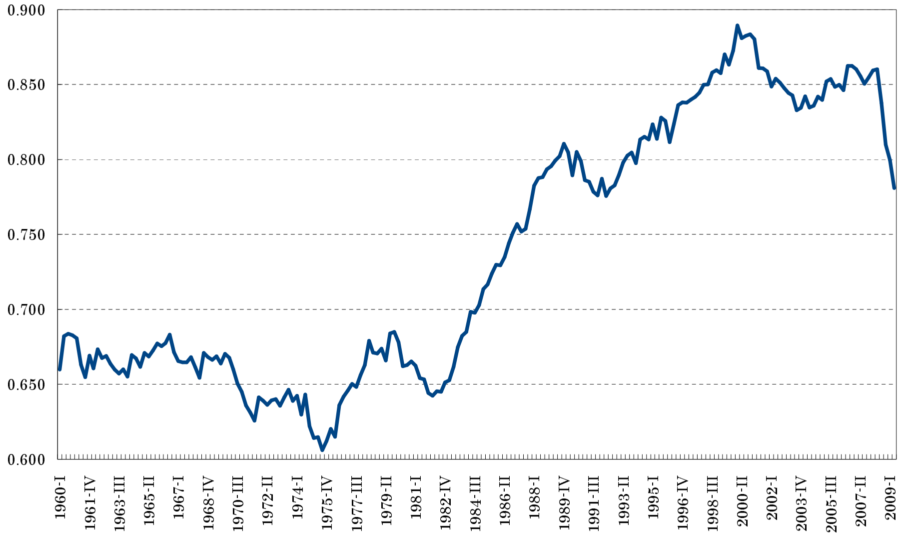


Following Chari, Kehoe, and McGrattan (2007), the labor wedge is defined as

$$(\text{labor wedge}) = \frac{\psi}{1-\alpha} \times \frac{c_t}{y_t} \times \frac{h_t}{1-h_t}.$$

We set $\psi = 2$, and $\alpha = .36$. The data of the consumption-output ratio (c_t/y_t) is from the Breau of Economic Analysis. The data of hour (h_t) is taken from Cociuba, Prescott, and Ueberfeldt (2009).

Figure 2: U.S. Labor Wedge (1960-2009)



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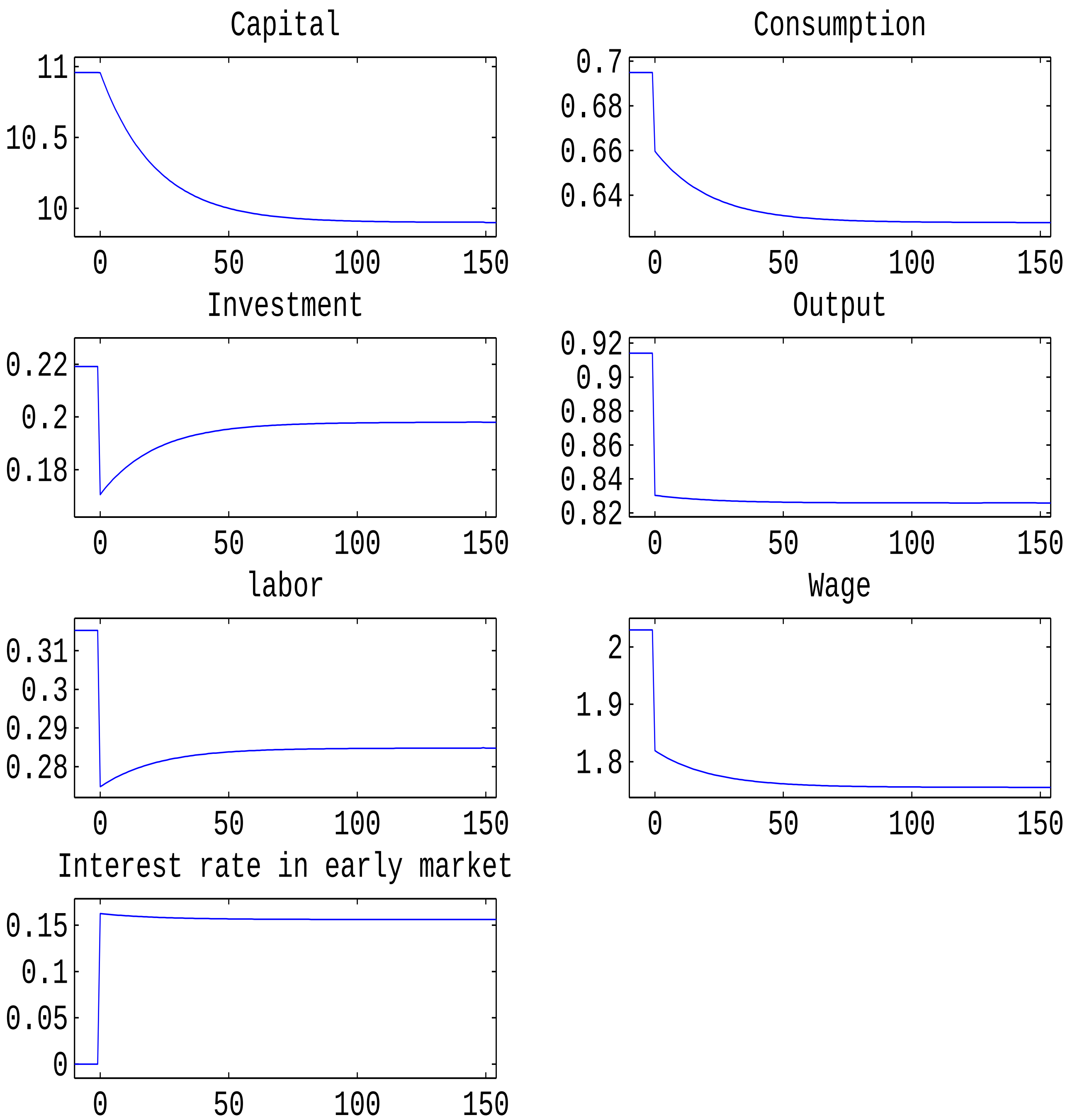

Figure 3: Simulation result