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A Financial Crisis in a Monetary Economy

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Abstract

We generalize Lagos and Wright's (2005) framework for a monetary economy in a way that there exist two technologies, "high" and "low," for producing the goods in a decentralized matching market. The high technology is more productive than the low technology, while the agents who use the high technology cannot commit in advance to deliver the goods. The lack of commitment makes it infeasible to produce the goods with the high technology if trade is conducted via a simple cash payment. To use the high technology, private valuable assets, e.g., residential property, should be put up as a "hostage" à la Williamson (1983) in the transaction. In this setting, a deterioration in the balance sheet due to a financial crisis leads to the disappearance of residential assets which are not yet put up as collateral, and hinder the usage of the high technology, leading to a decline in aggregate productivity. In this case, monetary injections cannot restore productivity after a financial crisis.

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1 Introduction

There are many issues concerning financial crises. In this paper we focus on the following two specific issues, which we believe have important policy implications:

- **Long-term decline in the aggregate productivity after a financial crisis.** While a financial crisis is characterized by the liquidity shortage in the short run, it is often observed that the level or the growth rate of the productivity declines over a long period after a financial crisis. The Great Depression is an example. Ohanian (2001) shows that 13 percentage points in the 18 percent decline of the detrended TFP during the 1929–1933 period cannot be explained by the ordinary cyclical factors. Ohanian argues that the destruction of the "organization capital" could be the cause of the TFP decline during the Great Depression. We also observed the long-term slowdown of the TFP growth in Japan during the 1990s after the collapse of the land price in 1991 (see Hayashi and Prescott 2002). The causality between the productivity declines and the financial crises is a big research topic that may lead to an important policy implication for the financial crisis management. We try to formalize a mechanism that the balance-sheet deterioration of households or firms causes the destruction of a relation-specific production, which may be interpreted as a model of Ohanian's destruction of the organization capital. The idea that the destruction of specific types of transactions might have caused the aggregate productivity declines after financial crises is explored in Kobayashi and Inaba (2004) and Kobayashi (2006, 2007).

- **Whether or not monetary injection (or liquidity provision) can mitigate substantial damage of a financial crisis.** In standard models of financial crises, the robust policy implication is that sufficient monetary injection can mitigate the real damage of the financial crisis almost completely (e.g., Diamond and Rajan 2006, Allen and Gale 1998). The episodes of financial crises in reality indicate that the monetary policy may not be almighty as a tool of the crisis management. The questions we deal with in this paper are: Is the liquidity shortage the central factor in the financial crisis that damages the economy?; Is the Friedman's rule attains the first best outcome (in the financial crisis)? We show in this paper that if the balance-sheet problem emerges as a result of the financial crisis, the liquidity provision and/or the Friedman rule may not be able to attain the optimal.

As long as cash is the sole medium of exchange, the optimality of the Friedman rule robustly obtains (see Lucas and Stokey 1987). The main idea is that a certain kind of assets that have private values for the owners can mitigate the lack of commitment more efficiently than money. We consider two technologies for production of the goods that are traded in the matching market: the "high" technology and the "low" technology. The high technology has a higher productivity than the low technology, while the two technologies have difference in production processes that makes difference in the agents' ability to commit to deliver the goods. When a seller uses the high technology to produce the goods, she/he cannot commit to deliver the goods. In our setting, the problem due to the sellers' inability to commit to deliver the goods cannot be resolved by monetary exchange or a simple down payment. To utilize the high technology, the agents need to use a real asset, such as a residential property, that has a higher private value than its market value as a "hostage" to make a commitment. The use of specific assets as hostages in transactions is first pointed out by Williamson (1983) and is widely recognized in the contractual relationships among firms. Our idea is to combine Williamson's hostage model and the Lagos-Wright monetary model.

2 The Model

The model is a variant of Lagos and Wright (2005), in which time is discrete and each period is divided into two subperiods, day and night. There is a unit mass of agents who live forever with identical preference and discount factor β. During the day agents interact in a decentralized market (we denote it DM in what follows) with anonymous bilateral matching. At night agents trade in a centralized (Walrasian) market, which is denoted by CM in this paper.

In Section 2.1, we describe the matching process and production of the goods in the DM and characterize the partial equilibrium in the DM. In Section 2.2, we embed the partial equilibrium of the DM in the general equilibrium setting of Lagos and Wright (2005), that involve both the DM and the CM.

2.1 Partial Equilibrium in the Decentralized Market

2.1.1 The Environment

As in Lagos and Wright (2005), the day good comes in many varieties, of which each agent consumes only a subset. In the DM, each agent can produce one of these special goods that he himself does not consume. In the DM, each agent meet another agent at random with probability 1. In other words, the matching probability in the DM is 1. For two agents i and j drawn at random, there are three possible events: i consumes what j can produce but not vice versa (single coincidence of wants); j consumes what i can produce but not vice versa; and neither consumes what the other can produce (no coincidence).[1] The probability of a single coincidence is α, and the probability of no coincidence is $1 - 2\alpha$. In a single coincidence meeting, if i wants the special good j produces, we call i the buyer and j the seller. In the DM, agents are anonymous to one another and the trading history of each agent in the DM is not recorded. Therefore, the final payment for any transaction in the DM must be made by cash m, which is provided by the central bank.

Real asset: We assume that at the beginning of the day each agent has k units of real asset or residential property. The asset can be transferable in the DM. We assume that the value of k for the original owner is $(a + x)k$, while the value of k is ak for the other agents if k changes the ownership in the DM. (We justify these assumptions in the general equilibrium setting in Section 2.2, where k is endowed in the initial period and the amount of k does not increase over time.)

[1] For simplicity we assume away the possibility of the double coincidence of wants, i.e., i can consume what j can produce and vice versa.

Matching technology: Although the two agents that meet in the DM cannot be separated until the trading is finished in the original model of Lagos and Wright (2005), we assume the following for the matching technology:

Assumption 1 *Two agents who meet in the DM can be separated before the trading is finished and can meet again within the same DM if it is incentive compatible for both of them to meet again.*

For example, it may be the case that the two agents can specify the time and place of reunion within the day, but they cannot commit to show up. In this case, they can meet again only if the reunion is incentive compatible for both of them.

Two production technologies and timing of payment: We also assume that there exist two technologies for production of the goods traded in the DM: The "high" technology and the "low" technology. If the agent uses the high technology, he can transform l units of labor into Al units of the good, where $A > 1$, while if he uses the low technology, he can produce q units of the good from q units of labor. Supplying l units of labor incurs the utility cost $c(l)$ to the agent, where $c(0) = 0$, $c'(l) > 0$, and $c''(l) > 0$ for all $l \geq 0$. We assume the following for the two technologies:

Assumption 2 *If the agent (the seller) uses the low technology, he produces the good immediately during the meeting. In this case he has no chance to abscond without delivering the good after receiving payment. If the seller uses the high technology, he need to go home after he receives the specific order from the buyer and to produce the good at home. In this case the seller and the buyer need to meet again in the DM to make a delivery of the produced good. Therefore, in this case, the seller has a chance to abscond with money without producing the good if he receives the advance payment.*

There exists a trade-off between using the high technology and the low technology: The high technology is more productive than the low technology, but the seller cannot commit to deliver the good when he uses the high technology, while he can when using the low technology. We introduce one more assumption for deferred payment:

Assumption 3 *If the agents want to make a payment after production of the good is finished, the seller and the buyer renegotiate the terms of trade after the production. The renegotiation occurs under both the high and the low technologies as long as the payment is done after the production.*

The terms of trade must be renegotiation-proof if the agents want to make a payment after the production. There are three possibility for the timing of payment: (i) Advance payment before production is finished; (ii) Deferred payment after production is finished but before or at the delivery of the good; (iii) Deferred payment after the delivery of the good. Case (iii) is not feasible in our model because the buyer can freely abscond with the delivered good without paying for it. So the agents choose between (i) or (ii) as the timing of payment.

Preferences: We assume that the preference of the agents is linear in the goods, the real asset, and the cash, while it is non linear in the labor supply. The utility gain of the agent who obtains q units of the good that he can consume, m units of cash, and k units of the residential property from other agents, loses $\hat{k}$ units of his residential property, and supplies l units of labor, is

$$q + \phi m - (a + x)\hat{k} - c(l) + ak,$$

where ϕ is the real value of cash. The form of preference is justified in the general equilibrium setting in Section 2.2 and the prices a, x, and ϕ are also specified as equilibrium outcomes in the general equilibrium.

Under these environment, the trades of the goods under the low and the high technologies are characterized as follows.

2.1.2 Bargaining under the low technology

The bargaining outcome in the case where the buyer and the seller agree to use the low technology for production is the same as in Lagos and Wright (2005). It is shown below that the timing of payment, whether (i) or (ii), i.e., before or after production, is irrelevant for the bargaining outcome. If the seller and the buyer chooses the advance payment, the bargaining problem is exactly the same as that in Lagos and Wright, because under the low technology the buyer can monitor the production process directly and ensures the delivery of the good (Assumption 2):

$$\max_{q,d}\{q-\phi d\}^{\theta}\{\phi d-c(q)\}^{1-\theta}, \tag{1}$$

subject to

$$d \leq m_b, \tag{2}$$

where q is the amount of the good (and the labor supply), d is the cash payment for the good, m_b is the total amount of cash the buyer holds when he enters the DM, and θ is the bargaining power of the buyer. The first term, $q-\phi d$, is the gain for the buyer, and the second term, $\phi d - c(q)$, is the gain for the seller. We focus on the case where the constraint (2) is binding. As Lagos and Wright show, the solution is characterized by

$$d = m_b, \tag{3}$$

$$\phi m = z(q), \tag{4}$$

$$\text{where} \quad z(q) \equiv \frac{\theta c(q) + (1-\theta)c'(q)q}{\theta + (1-\theta)c'(q)}. \tag{5}$$

We can show that when the agents want to set the final payment after production (but before or at the delivery) the agents can set up a renegotiation-proof scheme of transaction, which generates the identical bargaining outcome to (3)–(5). The agents bargain over (q, d', e), where d' is the final payment and e is the down payment to be made before production. There are two stages of bargaining: the ex-ante bargaining to determine q and e before production starts, and the renegotiation (ex-post bargaining) to determine d' after production. We first examine the renegotiation stage and then go back to the ex-ante bargaining stage.

In the renegotiation, the agents bargain over d', since q and e are already realized:

$$\max_{d'}\{q-\phi d'\}^{\theta}\{\phi d'\}^{1-\theta}, \tag{6}$$

subject to

$$d' \leq m_b - e. \tag{7}$$

The solution on the premise that (7) is not binding is

$$\phi d' = (1-\theta)q. \tag{8}$$

Given the solution of the renegotiation stage, (8), the ex-ante bargaining problem is that

$$\max_{q,d,e}\{q - \phi d\}^{\theta}\{\phi d - c(q)\}^{1-\theta}, \tag{9}$$

subject to

$$\phi d = (1-\theta)q + \phi e \leq \phi m_b, \tag{10}$$

which reduces to

$$\max_{q,e}\{\theta q - \phi e\}^{\theta}\{(1-\theta)q + \phi e - c(q)\}^{1-\theta}, \tag{11}$$

subject to

$$(1-\theta)q + e \leq m_b. \tag{12}$$

It is easily shown that the solution (q and $\phi d = (1-\theta)q + \phi e$) is characterized by (3)–(5).

This exercise shows that in the case where the seller can precommit to deliver the product the trade of the good is feasible by the cash payment, whether it is advance payment or deferred payment with partial down payment. The renegotiation is irrelevant for the bargaining outcome and the agents can get the Lagos-Wright bargaining outcome by the cash payment.

2.1.3 Bargaining under the high technology

We consider the bargaining problem in the case where the seller and the buyer want to use the high technology. First, we show that they cannot implement the transaction of the good if they are restricted to use only cash as a trading tool. Second, we show that they can attain the Lagos-Wright outcome, i.e, the ex-ante bargaining outcome, using the residential property as a means of down payment or "hostage" in the spirit of Williamson (1983). We assume the following assumption for the parameter values and the cost function of labor supply, $c(l)$:

Assumption 4

$$(1-\theta)A < c'(0) < 1.$$

Since $c(0) = 0$, $c'(l) > 0$, and $c''(l) > 0$, this assumption directly implies that for all l (> 0), $c(l) - (1-\theta)Al > 0$. Given this result, we can show the following proposition:

Propositon 1 *The seller and the buyer cannot trade the good produced by the high technology if they are restricted to use only cash as a medium of exchange.*

(Proof) Since the seller cannot commit beforehand to deliver the product (see Assumption 2), it is impossible to implement the trade with advance payment. That is, if the seller obtain the payment before he produces the good, he goes home, does not produce the good, and never show up at the place of reunion for delivery. Anticipating this result, the buyer never agree to pay in advance.

Thus for the proof of this proposition, it is sufficient to show that the transaction is not feasible even with deferred payment with partial down payment. We show that any set of total payment, d, and down payment, e, does not satisfy the seller's incentive compatibility condition for delivery of the product. It is shown that for any d and e the seller never show up for delivery, once he receives down payment e. Suppose that the agents can set up a renegotiation-proof scheme of transaction, (Al, d', e), where l is the labor, Al is the amount of the good, d' is the final payment after production, and e is the down payment before production. In the renegotiation, the agents bargain over d', since Al and e are already realized:

$$\max_{d'} \{Al - \phi d'\}^{\theta} \{\phi d'\}^{1-\theta}, \tag{13}$$

subject to

$$d' \leq m_b - e. \tag{14}$$

The solution is

$$\phi d' = \min\{(1-\theta)Al, \phi(m_b - e)\}. \tag{15}$$

We focus on the case where $d = d' + e \leq m_b$ is binding. Therefore, $\phi e = \phi m_b - \phi d' \geq \phi m_b - (1 - \theta)Al$. On the other hand, total gain for the seller in this transaction is $\phi d - c(l) = \phi m_b - c(l)$. The incentive compatibility condition for the seller to deliver the good is

$$\phi e < \phi d - c(l), \tag{16}$$

which means that the down payment is smaller than the total gain that the seller can obtain by producing and delivering the good to the buyer. Condition (16) is violated if $(1-\theta)Al - c(l) < 0$, where l is the amount of the good to be produced and so $l \geq 0$. Since Assumption 4 implies that $(1 - \theta)Al - c(l) < 0$ for all $l > 0$, we have the result that condition (16) cannot hold for any $l > 0$. Therefore, the good produced by the high technology cannot be traded with cash. (End of Proof)

Although the high technology goods cannot be traded when cash is the sole medium of exchange, the agents can use the residential property, k, as a tool of transaction. As we assume, the anonymity and the lack of record-keeping of transaction history imply that the final payment in the DM must be in cash. But the agents can transfer k as a hostage to make sure the delivery of the goods.[2]

We show in what follows that if the buyer and the seller use the asset k as a hostage in their transaction, they can implement the Lagos-Wright bargaining outcome in the DM under a certain parameter region. The Lagos-Wright bargaining problem under the high technology is a simple Nash bargaining: $\max_{l,d}\{Al - \phi d\}^{\theta}\{\phi d - c(l)\}^{1-\theta}$ subject to $d \leq m_b$, where Al is the amount of the good traded and d is the final monetary payment from the buyer to the seller. The Lagos-Wright bargaining outcome is determined by $\phi m_b = \phi d = z(l, A)$, where $z(l, A) \equiv \frac{\theta Ac(l)+(1-\theta)c'(l)Al}{\theta A+(1-\theta)c'(l)}$.

We consider the bargaining scheme in which the buyer transfers a certain amount of his residential asset k to the seller as a hostage, and he takes it back by paying cash at the renegotiation stage. The scheme is characterized by (Al, d, k_e), where Al is the amount

[2]Note that in the following transaction scheme the buyer offers the hostage to the seller and not vice versa. The hostage that the buyer puts up makes sure that the seller can get a sufficient gain in the renegotiation stage, which makes sure that the incentive compatibility for the seller to deliver the goods is satisfied.

of the good, l is the labor, d' is the final monetary payment, and k_e is the residential asset to be transfered as the hostage. There are two bargaining stages in the scheme: the ex-ante bargaining before production to decide (Al, k_e), and the renegotiation after production to decide d.

The bargaining problem at the renegotiation stage, in which the buyer pays d units of cash to the seller in exchange for Al units of the good and k_e units of the asset that was transferred to the seller as a hostage, is as follows:

$$\max_{d} \{Al + (a + x)k_e - \phi d\}^{\theta} \{\phi d - ak_e\}^{1-\theta}, \tag{17}$$

subject to

$$d \leq m_b. \tag{18}$$

Note that the value of the asset k_e is $(a + x)k_e$ for the original owner, i.e., the buyer, while the value for the seller is ak_e, because there exists a private value for the buyer, xk_e. Solving this problem on the premise that condition (18) does not bind, we obtain that

$$\phi d = (1 - \theta)(Al + xk_e) + ak_e. \tag{19}$$

Given this solution to the renegotiation stage, the ex-ante bargaining problem is as follows:

$$\max_{d,k_e,l} \{Al - \phi d\}^{\theta} \{\phi d - c(l)\}^{1-\theta} \tag{20}$$

subject to

$$\phi d = (1 - \theta)(Al + xk_e) + ak_e \leq \phi m_b, \tag{21}$$

$$k_e \leq k. \tag{22}$$

We focus on the case where (21) is binding. We also assume for simplicity of the analysis that each agent owns the sufficient amount of the residential asset so that (22) is not binding. The solution to the above problem is characterized by

$$\phi m_b = \phi d = z(l, A), \tag{23}$$

$$k_e = \frac{m_b - (1-\theta)Al}{(1-\theta)x + a}. \tag{24}$$

We assume the following for x, a, and other parameter values:

Assumption 5 *The private value x and the market value a of the asset k satisfies*

$$\frac{x}{a} > \sup_{0 \le l \le l^*} G(l), \tag{25}$$

where l^* is the solution to $A = c'(l)$ and

$$G(l) = \frac{\{\theta A + (1-\theta)c'(l)\}\{c(l) - (1-\theta)Al\}}{(1-\theta)^2 c'(l)\{Al - c(l)\}}. \tag{26}$$

For example, if the utility cost of labor is a quadratic function, i.e., $c(l) = c_1 l + c_2 l^2$, where $c_1 > (1-\theta)A$, it is easily shown that $\sup_{0 \le l \le l^*} G(l) = \max\{G(0), G(l^*)\}$. Under this assumption, we can show the following proposition:

Propositon 2 *Under Assumption 5, the ex-ante bargaining solution that is characterized by (23)–(24) is incentive compatible for the seller, that is, the seller is willing to deliver Al units of the good and k_e units of the asset in exchange for d units of the cash. Therefore the buyer and the seller can successfully implement the Lagos-Wright bargaining outcome for the high technology good by solving the ex-ante bargaining (20)–(22) and the renegotiation (17)–(18).*

(Proof) The incentive compatibility for delivery is

$$ak_e < \phi m_b - c(l), \tag{27}$$

where the left-hand side is the gain that the seller can get by absconding with the hostage asset (k_e) without producing the good, and the right-hand side is the gain that he can receive by delivering the good, Al, and k_e. Using (23) and (24), we can rewrite (27) as

$$\frac{x}{a} > G(l(m_b)), \tag{28}$$

where $l(m_b)$ is the solution to (23). Since $0 \le l(m_b) \le l^*$, we obtain that (25) is the sufficient condition for (27). Assumption 5 guarantees that (27) is satisfied. (End of Proof)

2.1.4 Disappearance of the assets and the productivity declines

The above arguments directly imply that the high technology is used in production of the goods in the DM only if the agents hold a sufficient amount of the residential asset, k, which can be used as a hostage in the bargaining. If the asset disappears, the transaction that involves the high technology becomes infeasible and the agents produce the goods in the DM only with the low technology. Therefore, the aggregate productivity of this economy is high (low) when there exist the large (small) amount of the real asset that can be used as a hostage in the bargaining.

This result may be one possible explanation for the productivity declines observed after the financial crises, since the amount of the real assets that can be put up as a hostage must decrease during and after a financial crisis. There are at least the following two possibilities for k to disappear as a result of a financial crisis: the lemon problem in the asset market and the excessive debt secured by k as a collateral.

The lemon problem: Suppose that a financial crisis is an event in which once worthless real estates are mistakenly recognized and traded by all agents as highly valuable residential property (the *emergence of an asset bubble*), and then all agents suddenly recognize that these assets are in fact worthless (the *collapse of the asset bubble*). After the collapse of the bubble, agents recognize that some portion of their holdings of real estates is valuable residential assets, while the other portion is worthless *lemons* in the sense of Akerlof (1970). In this situation, the seller and the buyer cannot implement the bargaining scheme in which the buyer transfers k_e units of the real asset to the seller as a hostage if the seller cannot tell whether the hostage that the buyer offers is valuable residential property or a worthless lemon. This is because the buyer has an incentive to give the seller k_e units of worthless lemon as the hostage, since the buyer can get a larger gain in the renegotiation stage if the hostage is worthless (for himself and/or for the other agents). Anticipating that the buyer will offer the lemon as a hostage, the seller never accept to produce and sell the good using the high technology. Therefore, when there emerges a substantial lemon problem in the asset market, the agents become unable to use the high technology and they are forced to use the low technology, leading to the declines in the aggregate productivity of the economy.

The excessive debt: If the real asset has been already put up as collateral for consumption loans in the night market (the CM), the asset cannot be used as a hostage in the bargaining in the day market (the DM). We assume in the normal equilibrium the amount of the consumption loans is small so that there are sufficient amount of the real asset to be used as a hostage in the bargaining and the high-technology production is implemented. Suppose now that the asset price bubble emerged and then collapsed in the night market of k. Suppose also that in the bubble period the amount of the consumption loans covered by collateral (k) increased drastically because the value of k also increased as a result of the bubble. When the bubble collapsed, the value of the consumption loans did not change, while the collateral for those loans decreased. After the bubble collapse, there emerges the situation that all real assets k are unintentionally put up as collateral for the consumption loans. In this situation, the agents cannot use k as a hostage for the bargaining, leading to the declines in the aggregate productivity in the economy. We will formalize this excessive debt story in the following general equilibrium setting.

Policy implication: This model implies that the productivity decline after a financial crisis may be caused by disappearance of the hostage-able assets due to the lemon problem (or the bad asset problem) or the excessive accumulation of the collateral loans. A policy implication from this model is that the problem of the asset disappearance cannot be resolved by money injection. The problem is not monetary but real that is associated with the balance-sheet deteriorations of the economic agents. The monetary policy is not sufficient to resolve the productivity declines caused by the mechanism described in this model. If this model describes the major mechanism of the productivity declines after the financial crises, it can be said that in addition to the monetary injections we may need other policy measures for financial crisis management, such as disposition of the bad assets and reduction of excessive debts, in order to restore the aggregate productivity of the economy.

2.2 General equilibrium

We embed the bargaining in the DM into the general equilibrium setting similar to Lagos and Wright (2005). We show that there exist multiple steady-state equilibria in one of which the amount of consumption loans is small so that there are sufficient amount of the residential assets that can be used as a hostage in the bargaining and the high-technology production is implemented. In the other equilibrium, the amount of consumption loans is very large and all the residential assets are put up as collateral for the consumption loans so that the assets cannot be used as a hostage and only the low-technology production is implemented.

2.2.1 Environment

We need to redefine the environment in the general equilibrium setting. We assume that an agent cannot consume during the day but he can consume at night. Therefore, an agent who obtains the good in the DM brings the good into the CM and he consume it in the CM. In addition to the good that he obtains in the DM, the agent can produce the consumption good that he himself can consume (or sell in the market) from his own labor. The labor h in the CM can be transformed into the good one-for-one and incurs the utility cost h to the agent. The consumption c in the CM gives the utility $U(c)$ to the agent, where $U'(c) > 0$ and $U''(c) < 0$.

There are two kinds of assets that can be traded in the CM: the residential property, k, and the consumption loan, b. The residential property generates ω units of the general good in the CM that can be consumed by all agents and δ units of the special good in the CM that only the original owner of k can consume. Agents make and borrow the consumption loans with each other. So, in a symmetric equilibrium each agent have the same amount of the consumption loan both as his asset and as liability. The consumption loan b_{+1} made in the current CM will generate the gross return $(1 + r_{+1})b_{+1}$ in the CM in the next period. The consumption loan must be secured by the residential assets of the borrower and the collateral constraint for the borrower is written as $(1 + r_{+1})b_{+1} \leq a_{+1}k_{+1}$, where a_{+1} is the market price of the residential property and k_{+1} is the holdings of the residential property at the end of the current period. We will show shortly that in the steady-state equilibrium the asset prices are given by

$$a = \frac{\beta\omega}{1-\beta}, \tag{29}$$

$$x = \frac{\beta\delta}{1-\beta}. \tag{30}$$

To ensure the multiple equilibria, we assume that an agent need to pay a fixed cost, κ, at the beginning of the DM in each period to obtain the capability to use the high technology, where κ is a dead weight loss. If an agent does not pay κ in period t, he can use only the low technology for production in the DM in that period. We assume that whether an agent is capable to use the high technology is observable for the other agents.

2.2.2 Optimization Problem

We will denote the value function for entering the DM by $V(m, k, \hat{k}, b, b')$ and for entering the CM by $W(q, m, k, \hat{k}, b, b')$, where m is the cash holdings, k is the amount of the residential property which the agent purchased from other agents in the market, $\hat{k}$ is the amount of the residential property which the agent has owned since the initial period, b is the consumption loan outstanding that the agent borrowed, b' is the consumption loans outstanding that the agent lent, q is the amount of the good that the agent obtained in the DM. Note that $\hat{k}$ generates $(\omega + \delta)\hat{k}$ units of the consumption for the owner and k generates $\omega(k - \hat{k})$ units of consumption.

The Bellman equation for the agent entering the DM is

$$
\begin{aligned}
&V(m,k,\hat{k},b,b') \\
&= (1-2\alpha)W(0,m,k,\hat{k},b,b') + \alpha\hat{\sigma}W(Al_h',m-d_h,k,\hat{k},b,b') + \alpha(1-\hat{\sigma})W(l_l',m-d_l,k,\hat{k},b,b') \\
&+ \alpha[-c(l_l) + W(0,m+d_l',k,\hat{k},b,b')] \\
&+ \max\{-\kappa + \alpha\hat{\xi}[-c(l_h) + W(0,m+d_h',k,\hat{k},b,b') + c(l_l) - W(0,m+d_l',k,\hat{k},b,b')],\ 0\},
\end{aligned}
\tag{31}
$$

where $\hat{\sigma}$ is the ratio of agents who can produce the goods with the high technology, Al_h' is the purchased amount of the good produced by the high technology, d_h is the cash payment for the good produced with the high technology, l_h' is the purchase of the good produced by the low technology, d_l is the cash payment for the good produced by the high technology, l_l is the labor supply for the low-technology production, d_l' is the cash revenue by selling the good produced by the low technology, $\hat{\xi}$ is the ratio of agents who has the sufficient amount of the residential assets to be used as a hostage in the bargaining and $1-\hat{\xi}$ is the ratio of agents who has no amount of the asset that can be used as a hostage,[3] l_h is the labor supply for the high-technology production, d_h' is the cash revenue by selling the good produced by the high technology. The trade of the good produced with the low technology in the DM, $(d_l, l_l') = (d_l', l_l)$, is determined by the bargaining between the buyer and the seller. Subscript b (s) represents the buyer (seller) in the following bargaining problem:

$$
\begin{aligned}
\max_{d_l, l_l} \quad & \{W(l_l, m_b - d_l, k_b, \hat{k}_b, b_b, b_b') - W(0, m_b, k_b, \hat{k}_b, b_b, b_b')\}^{\theta} \\
& \times \{-c(l_l) + W(0, m_s + d_l, k_s, \hat{k}_s, b_s, b_s') - W(0, m_s, k_s, \hat{k}_s, b_s, b_s')\}^{1-\theta},
\end{aligned}
\tag{32}
$$

subject to

$$
d_l \leq m_b. \tag{33}
$$

[3]For simplicity, we assume that the agents have either zero amount of the asset for a hostage or sufficient amount that makes condition (22) nonbinding. This assumption can be justified by showing that the bargaining outcome in the case where (22) is binding cannot be an equilibrium outcome, though we do not argue in the detail here.

The trade of the good produced with the high technology in the DM, $(d_h, l'_h) = (d'_h, l_h)$, is determined by the following renegotiation-proof bargaining. From the reasoning in Section 2.1, we know that only $\hat{k}$ can be a hostage in the bargaining, while k cannot be a hostage:

$$\max_{d_h, l_h, k_e} \{W(Al_h, m_b - d_h, k_b, \hat{k}_b, b_b, b'_b) - W(0, m_b, k_b, \hat{k}_b, b_b, b'_b)\}^{\theta}$$
$$\times \{-c(l_h) + W(0, m_s + d_h, k_s, \hat{k}_s, b_s, b'_s) - W(0, m_s, k_s, \hat{k}_s, b_s, b'_s)\}^{1-\theta}, \quad (34)$$

subject to

$$k_e \leq \hat{k}_b - \max\left\{\frac{(1+r)b_b}{a} - k_b, 0\right\}, \quad (35)$$

$$d_h \text{ is the solution to the renegotiation (37),} \quad (36)$$

where k_e is the amount of the residential property put up as a hostage. Note that the asset already put up as collateral for b_b cannot be used as a hostage in the bargaining. The renegotiation of this transaction is as follows: Given k_e and l_h,

$$\max_{d_h} \{W(Al_h, m_b - d_h, k_b, \hat{k}_b, b_b, b'_b) - W(0, m_b, k_b, \hat{k}_b - k_e, b_b, b'_b)\}^{\theta}$$
$$\times \{W(0, m_s + d_h, k_s, \hat{k}_s, b_s, b'_s) - W(0, m_s, k_s + k_e, \hat{k}_s, b_s, b'_s)\}^{1-\theta}, \quad (37)$$

subject to

$$d_h \leq m_b. \quad (38)$$

The Bellman equation for the agent entering the CM is

$$W(q, m, k, \hat{k}, b, b') = \max_{c, h, m_{+1}, k_{+1}, \hat{k}_{+1}, b_{+1}, b'_{+1}} U(c) - h + \beta V(m_{+1}, k_{+1}, \hat{k}_{+1}, b_{+1}, b'_{+1}), \quad (39)$$

subject to

$$c = h + q + \omega k + (\omega + \delta)\hat{k} + \phi(m - m_{+1}) + a(k + \hat{k} - k_{+1} - \hat{k}_{+1}) + (1+r)(b' - b) + (b_{+1} - b'_{+1}), \quad (40)$$

$$(1 + r_{+1})b_{+1} \leq a_{+1}(k_{+1} + \hat{k}_{+1}), \quad (41)$$

$$\hat{k}_{+1} \leq \hat{k}. \quad (42)$$

We follow the technique in Lagos and Wright (2005) to characterize the general equilibrium: We focus on the steady-state equilibrium where real variables do not change over time but money growth rate (or the inflation rate) is constant; and we first analyze the Bellman equation for the CM and go back to the Bellman equation for the DM. In this paper we focus on the case where the Lagrange multiplier for (41) is zero. The FOCs for (39) with respect to k_{+1} and $\hat{k}_{+1}$ are $a = \beta V_k(+1)$ and $a + \eta = \beta V_{\hat{k}}(+1)$, respectively, where η is the Lagrange multiplier for (42). We guess and verify later that η is a constant, that is, η is not a function of the state variables $(q, m, k, \hat{k}, b, b')$. Under the assumption that η is a constant, the usual arguments in the Lagos-Wright framework lead to the following reduced form of the value function:

$$W(q, m, k, \hat{k}, b, b') = q + (\omega + a)k + (\omega + \delta + a + \eta)\hat{k} + \phi m + (1 + r)(b' - b) + W_0, \tag{43}$$

where W_0 is a constant. Changing the notation from η to x, equation (43) directly implies that the bargaining problems for the DM described in this subsection also reduce to the corresponding bargaining problems in Section 2.1. Equation (43) also implies that (31) can be rewritten as

$$\begin{aligned} & V(m, k, \hat{k}, b, b') \\ & \quad = \phi m + (\omega + a)k + (\omega + \delta + a + \eta)\hat{k} + (1 + r)(b' - b) \\ & \quad + \alpha\hat{\sigma}\{Al_h(m) - \phi d_h(m)\} + \alpha(1 - \hat{\sigma})\{l_l(m) - \phi d_l(m)\} + \alpha\{-c(l_l(m')) + \phi d_l(m')\} \\ & \quad + \max\{-\kappa + \alpha\hat{\xi}[-c(l_h(m')) + \phi d_h(m') + c(l_l(m')) - \phi d_l(m')], 0\}, \end{aligned} \tag{44}$$

where $l_h(m)$, $d_h(m)$, $l_l(m)$, $d_l(m)$ are the bargaining solutions on the premise that the buyer's cash-in-advance conditions are binding. Note that m' is the cash holding of the trading partner and therefore it is not a choice variable for the agent whose cash holding is m in (44). Now the FOCs for (39) with respect to k and $\hat{k}$ become $a = \beta(\omega + a_{+1})$ and $a + \eta = \beta(\omega + \delta + a_{+1} + \eta)$, respectively. Therefore, $\eta = \beta\delta/(1 - \beta)$. Therefore our guess that η is a constant is verified. In the steady state where $a = a_{+1}$, the FOC with respect to k implies that $a = \beta\omega/(1 - \beta)$, which is (29). Changing the notation from η

to x we obtain (30). Since $\eta = x$ is the Lagrange multiplier for (42), $\eta = x > 0$ implies that $\hat{k}$ is constant over time, and therefore in equilibrium $\hat{k}_t = \hat{k}_0$ and $k_t = 0$, where $\hat{k}_0$ is the initial endowment of the residential property.

2.2.3 Multiple equilibria

As in the Lagos-Wright model, if the government (or the central bank) gives the money growth rate ($\pi = \phi/\phi_{+1}$), the real balance (ϕm) is determined as an equilibrium outcome and other variables are also determined as functions of ϕm so that the equilibrium is completely characterized. The real balance is determined by the envelope condition for (44) with respect to m:

$$V_m(m, k, \hat{k}, b, b') = \phi + \alpha\hat{\sigma}\{Al_h'(m) - \phi d_h'(m)\} + \alpha(1 - \hat{\sigma})\{l_l'(m) - \phi d_l'(m)\}, \qquad (45)$$

and the FOC for (39) with respect to m_{+1}: $\phi = \beta V_m(m_{+1}, k_{+1}, \hat{k}_{+1}, b_{+1}, b'_{+1})$.

There are two steady-state equilibria in this economy, which are characterized by $(\hat{\sigma}, \hat{\xi})$: In one equilibrium $(\hat{\sigma}, \hat{\xi}) = (1, 1)$, and in the other $(\hat{\sigma}, \hat{\xi}) = (0, 0)$.

If $\hat{\xi} = 1$, it is the case that $-\kappa + \alpha[-c(l_h(m')) + \phi d_h(m') + c(l_l(m')) - \phi d_l(m')] > 0$ under appropriate parameter values. Therefore, all agents choose to pay κ, leading to $\hat{\sigma} = 1$. If $\hat{\sigma} = 1$, to use the residential property as a hostage in the bargaining is valuable for the owner. Therefore, all agents choose to set b at sufficiently low such that constraint (41) becomes nonbinding and they have sufficient amount of $\hat{k}$ for a hostage, leading to $\hat{\xi} = 1$. If $\hat{\xi} = 0$, no one pays κ and therefore $\hat{\sigma} = 0$. If $\hat{\sigma} = 0$, there is no chance for an agent to use k as a hostage in the bargaining. Therefore, the agents are indifferent to the amount of the consumption loans b. We assume that the agents set b at a largest possible amount if they are indifferent to the amount of b. (** We set this assumption for simplicity of exposition though it is unnatural.**) Therefore, all agents choose to set b at the largest amount so that all $k + \hat{k}$ are used as collateral for b, leading to $\hat{\xi} = 0$.

3 Conclusion

In this paper we showed that the economic agents need to use the assets that are privately valuable, e.g., residential property, as hostages in bargaining in order to utilize a certain production technology, which entails the lack of commitment. If the agents are restricted to trade the goods with simple cash payment, the lack of commitment problem cannot be resolved and as a consequence the specific production technology cannot be used.

We conceptualize a financial crisis as an event that causes the disappearance of the assets that can be used as hostages in the bargaining: The lemon problem due to emergence of bad assets in the real estate market may make the agents unable to use the assets as hostages; and if the amount of the consumption loans becomes too large as a consequence of the emergence and collapse of the asset bubbles, then all real property becomes collateral for the consumption loans and there remains no property that can be used as hostages in the bargaining. The disappearance of the hostage-able assets hinders the usage of the specific production technology, leading to a persistent decline of the aggregate productivity of the economy.

This mechanism implies that the productivity declines after a financial crisis may be caused by a real problem, such as the balance-sheet deterioration of the economic agents, and that the productivity cannot be restored only with monetary policy measures such as a liquidity injection or lowering the nominal interest rates. Our model implies that for the financial crisis management necessitates *real* policy measures that may entail fiscal outlays, such as the government purchases of the bad assets or the rehabilitation of the debt-ridden borrowers through subsidies and bankruptcy procedures.

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