Asset Prices and Monetary Policy in a Sticky-Price Economy with Financial Frictions

NUTAHARA Kengo
Senshu University
Asset Prices and Monetary Policy in a Sticky-Price Economy with Financial Frictions

Kengo NUTAHARA
Department of Economy, Senshu University

Abstract

A recent study shows that equilibrium indeterminacy arises if monetary policy responds to asset prices, especially share prices, in a sticky-price economy. We show that equilibrium indeterminacy never arise if the working capital of firms is subject to their asset values by financial frictions.

Keywords: asset prices; financial frictions; equilibrium indeterminacy; monetary policy
JEL classification: E10; E32
1 Introduction

Should monetary policy respond to asset prices? There is a large number of the literature on this question. For example, the unimportance of responding to asset prices is reminiscent of the findings of Bernanke and Gertler (2001) and Gilchrist and Leahy (2002). Iacoviello (2005) shows that, if the central bank wants to minimize output and inflation fluctuations, a little is gained by responding to asset prices. Faia and Monacelli (2007) find that there is a case where monetary policy should respond to increases in asset prices by lowering nominal interest rate.

A recent paper by Carlstrom and Furest (2007) provide a negative answer; equilibrium indeterminacy arises if monetary policy responds to asset prices in a sticky-price economy. While many previous studies employ prices of capital as asset prices, Carlstrom and Furest (2007) focus on share prices that reflect firms’ profits. In their model, an increase in inflation reduces firms’ profits, and asset price declines. Then, monetary policy responding asset prices, or share prices, implicitly weakens overall reactions to inflation. This is a source of equilibrium indeterminacy in their model.

In this paper, we show that equilibrium indeterminacy never arise if there is a credit market imperfection. We introduce a collateral constraint to the economy of Carlstrom and Furest (2007). The working capital, or wage payment, of firms is subject to a collateral constraint in our economy. In our economy, an increase in inflation reduces firms’ profits as in the economy of Carlstrom and Furest (2007). However, share price does not change since the inefficiency of the collateral constraint increases and the premium of share as a collateral increases.

Our result implies that, under the credit market imperfection, there is no negative aspect of monetary policy responding asset prices pointed out by Carlstrom and Furest (2007). Since the discussion on monetary policy responding to asset prices often arise in recessions associated with financial crisis like the Japan’s lost decade during the 1990s and the recent financial crisis in the U.S. economy, our result in the economy with financial frictions would have a certain implication on the literature of monetary policy.

Collateral constraints are often employed to account for the observed facts of business

The rest of this paper is organized as follows. Section 2 introduces our basic economy with a collateral constraint. In Section 3, our main results are shown. Equilibrium indeterminacy never arise even if monetary policy responds to asset price fluctuations under the credit market imperfection. Section 4 draws a certain conclusion.

2 The model

Our model is based on one employed by Carlstrom and Fuerst (2007). One departure from their model is that there is a collateral constraint on working capital. In order to introduce the collateral constraint, the environment of our economy is slightly different from that of Carlstrom and Fuerst (2007). However, the equilibrium system is identical to that of Carlstrom and Fuerst (2007) if the collateral constraint never binds.

2.1 Households: workers and managers

We consider households that consist of workers and managers. The household begins period $t$ with $M_t$ cash balances, $B_t$ one-period nominal bonds that pay $R_{t-1}$ gross interest rate, $S_t$ shares of stock of retailers that sell at price $Q_t$ and pay dividend $D_t$.

The utility function is

$$U(C_t, L_t, M_{t+1}/P_t) = C_t^{1-\sigma} \frac{1}{1-\sigma} + \frac{L_t^{1+\gamma}}{1+\gamma} + V(M_{t+1}/P_t),$$

where $\sigma > 0, \gamma > 0, V$ is increasing and concave, $C_t$ denotes consumption, $L_t$ denotes labor supply, and $M_{t+1}/P_t$ denotes real cash balances at the end of period $t$. 

At the beginning of periods, a household splits into a worker and a manager. A worker supply labor \( L_t \) and earns wage income \( P_l W_t L_t \) where \( P_l \) denotes aggregate price level. A manager produces homogenous goods employing labor them to retailers at price \( P_t Z_t \).

The production technology of managers is

\[
Y_t = H_t,
\]

where \( H_t \) denotes labor demand. We assume that managers have to pay wage to workers in advance and they borrow working capital from banks. A bank can issue bank notes that can be circulated in our economy. Letting \( N_t \) be the amount that the manager borrows, the manager’s choice of \( H_t \) is constrained by

\[
P_t W_t H_t \leq N_t.
\]

Since this borrowing and lending are intra-period, gross interest rate of this is zero in equilibrium. As in Kiyotaki and Moore (1997), the manager cannot fully commit himself to repay the debt. Then, the borrowing of a manager is subject to a collateral constraint:

\[
N_t \leq \varphi P_t Q_t S_t.
\]

where \( 0 < \varphi \leq 1 \). In order to consider a collateral constraint, we assume that a worker cannot supply to a manager from the same agent.

After the production of goods, a worker and a manager go back to home and decide consumption and holdings of money and bond as single agent: household. The budget constraint of household is

\[
P_t C_t + M_{t+1} + P_t Q_t N_{t+1} + B_{t+1} + P_t W_t H_t \\
\leq P_t Z_t Y_t + P_t W_t L_t + M_t + P_t Q_t S_t + R_{t-1} B_t + P_t D_t S_t + X_t,
\]

where \( Z_t \) denotes the relative price of goods produced by managers and \( X_t \) denotes monetary injection.

\(^1\)Similar setting of the credit market imperfection is employed by Kobayashi, Nakajima, and Inaba (2007), Kobayashi and Nutahara (2007), and Harrion and Weder (2010).
The first order conditions of households are

\[ C_t^\sigma L_t^\gamma = W_t, \quad (6) \]
\[ C_t^{-\sigma} = \beta C_{t+1}^{-\sigma} \frac{R_t}{\Pi_{t+1}}, \quad (7) \]
\[ C_t^{-\sigma} Q_t = \beta C_{t+1}^{-\sigma} [Q_{t+1}(1 + \varphi \Theta_{t+1}) + D_{t+1}], \quad (8) \]
\[ W_t(1 + \Theta_t) = Z_t, \quad (9) \]
\[ (W_t H_t - \varphi Q_t S_t) \Theta_t = 0, \quad \Theta_t \geq 0, \quad (10) \]

where \( \Pi_{t+1} = P_{t+1}/P_t \) and \( \Theta_t \) denotes the ratio of the Lagrange multiplier of the collateral constraint to that of the budget constraint, and it can be interpreted as the inefficiency of collateral constraint. (6) is the intratemporal optimization condition, (7) is the Euler equation of consumption, (8) is the Euler equation of assets, (9) is the marginal productivity condition of labor, and (10) is on the condition on the collateral constraint.

By (7) and (8), we have the more familiar asset price relationship:

\[ Q_t = [Q_{t+1}(1 + \varphi \Theta_{t+1}) + D_{t+1}] \frac{\Pi_{t+1}}{R_t}. \quad (11) \]

Note that in the case with a binding collateral constraint, asset price is affected by the inefficiency of collateral constraint, \( \Theta \). If a shock tightens the collateral constraint, the premium of asset as a collateral increases, and then it has a positive effect on asset prices.

### 2.2 Retailers

We assume that monopolistically competitive retailers as employed by Bernanke, Gertler, and Ghilchrist (1999). Retailers buy goods at price \( P_t Z_t \) from managers, produce differentiated goods using linear technology, and set prices. Under the standard Calvo-type sticky-price setting, the New Keynesian Phillips curve is

\[ \pi_t = \lambda z_t + \beta \pi_{t+1}. \quad (12) \]

where lower-case letters denote log deviations from the steady state. Note that the real whole sell price \( Z_t \) can be interpreted as the real marginal cost of retailers. The retailers’
profits are paid out as dividends. Then, we have

\[ D_t = (1 - Z_t)Y_t. \]  

(13)

### 2.3 Monetary policy

We assume that monetary authority follows a simple Taylor rule:

\[ r_t = \tau \pi_t + \tau_q q_t, \]  

(14)

where lower letters, \( r_t \) and \( q_t \), denote the log-deviations from a steady state of \( R_t \) and \( Q_t \), respectively.

### 2.4 Equilibrium

We focus on a symmetric equilibrium with \( H_t = L_t \). The total supply of share is one: \( S_t = 1 \), and total supply of nominal bond is zero: \( B_t = 0 \).

The definition of a competitive equilibrium is as follows.

**Definition 1.** Given monetary policy rule (14), a competitive equilibrium is a sequences of prices \( \{\pi_t, Q_t, W_t, Z_t, R_t\} \) and quantities \( \{C_t, H_t, L_t, Y_t, B_t, M_t, D_t, \Theta_t\} \) such that (i) households maximize their utilities, (ii) retailers maximize their profits, and (iii) all markets clear.
The equilibrium system of this economy is

\[
C_t^\sigma H_t^\gamma = W_t, \tag{15}
\]

\[
C_t^{\sigma-\sigma} = \beta C_{t+1}^{\sigma-\sigma} \frac{R_t}{\Pi_{t+1}}, \tag{16}
\]

\[
Q_t = \left[ Q_{t+1}(1 + \varphi \Theta_{t+1}) + D_{t+1} \right] \frac{\Pi_{t+1}}{R_t}, \tag{17}
\]

\[
W_t(1 + \Theta_t) = Z_t, \tag{18}
\]

\[
(W_t H_t - \varphi Q_t) \Theta_t = 0, \quad \Theta_t \geq 0, \tag{19}
\]

\[
D_t = (1 - Z_t) Y_t, \tag{20}
\]

\[
Y_t = H_t = C_t, \tag{21}
\]

\[
\pi_t = \lambda z_t + \beta \pi_{t+1}, \tag{22}
\]

\[
r_t = \tau \pi_t + \tau_q q_t. \tag{23}
\]

3 Main results

3.1 Main results

The following condition is necessary and sufficient for a binding collateral constraint at a steady state.

**Proposition 1.** A collateral constraint (4) is binding at a steady state if and only if

\[
\varphi < \frac{1 - \beta}{\beta} \cdot \frac{Z}{1 - Z}. \tag{24}
\]

**Proof.** By the steady-state equilibrium system, we obtain

\[
W = C^{\sigma+\gamma},
\]

\[
C = \left[ \frac{Z}{1 + \Theta} \right]^{1/(\sigma+\gamma)},
\]

\[
Q = \frac{(1 - Z) \left[ \frac{Z}{1 + \Theta} \right]^{1/(\sigma+\gamma)}}{1/\beta - (1 + \varphi \Theta)}.
\]

Inserting these into a collateral constraint, \( WC = \varphi Q \), yields

\[
\Theta = \frac{Z [1 - \beta (1 - \varphi)]}{\beta \varphi} - 1.
\]
\( \Theta \) is greater than zero if and only if (24) holds.

It is easily shown that this economy is identical to that of Carlstrom and Fuerst (2007) if a collateral constraint never binds, \( \Theta_t = 0 \). Then, the following proposition holds.

**Proposition 2.** Assume that (24) does not hold and a collateral constraint never binds.

(i) If \( \tau_q = 0 \), a necessary and sufficient condition for equilibrium determinacy is \( \tau > 1 \).

(ii) If \( \tau > 1 \), a necessary and sufficient condition for equilibrium determinacy is

\[
\tau_q < \frac{\lambda(\tau - 1)}{(1 - \beta)A},
\]

where \( A = \frac{Z(1+\sigma+\gamma)-1}{(\sigma+\gamma)(1-Z)} \).

**Proof.** See the proof of Proposition 1 of Carlstrom and Fuerst (2007).

Proposition 2 implies that there is equilibrium indeterminacy if \( \tau_q \) is larger than a threshold.

In this paper, we focus on a case where a collateral constraint is binding. It is convenient to log-linearize our equilibrium system for the analyses. The linearized system with a binding collateral constraint is as follows.

\[
(\sigma + \gamma)c_t = w_t, \quad (25)
\]
\[
\sigma(c_{t+1} - c_t) = r_t - \pi_{t+1}, \quad (26)
\]
\[
q_t = \beta(1 + \varphi \Theta) \left[ q_{t+1} + \frac{\varphi \Theta}{1 + \varphi \Theta} \theta_{t+1} \right] + [1 - \beta(1 + \varphi \Theta)]d_{t+1} + (\pi_{t+1} - \pi_t), \quad (27)
\]
\[
w_t + c_t = q_t, \quad (28)
\]
\[
d_t = c_t - \frac{Z}{1 - Z} z_t, \quad (29)
\]
\[
z_t = w_t + \frac{\Theta}{1 + \Theta} \theta_t, \quad (30)
\]
\[
\pi_t = \beta \pi_{t+1} + \lambda z_t, \quad (31)
\]
\[
r_t = \tau \pi_t + \tau_q q_t, \quad (32)
\]
where lower case letters denote log deviations from the steady state and
\[ \Theta = \frac{Z \left[ 1 - \beta(1 - \varphi) \right]}{\beta \varphi} - 1. \quad (33) \]

This system is reduced to the following matrix form:
\[
\begin{bmatrix}
1 & 0 & \Phi_1 \\
1 & 0 & \Phi_2 \\
\beta & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\pi_{t+1} \\
z_{t+1} \\
q_{t+1}
\end{bmatrix}
= \begin{bmatrix}
\tau & 0 & \Phi_1 + \tau_q \\
\tau & 0 & 1 + \tau_q \\
1 & -\lambda & 0
\end{bmatrix}
\begin{bmatrix}
\pi_t \\
z_t \\
q_t
\end{bmatrix},
\]
where
\[ \Phi_1 = \frac{\sigma}{1 + \sigma + \gamma}, \]
\[ \Phi_2 = \frac{1 + \beta(1 - \varphi)(\sigma + \gamma)}{1 + \sigma + \gamma}. \]

The first equation is the Euler equation of consumption (26). The second one is the Euler equation of asset price (27). The last one is the New Keynesian Phillips curve (31). Note that this system is closed by only first and second equations with \( \pi_t \) and \( q_t \).

The main result in this paper is as follows.

**Proposition 3.** Assume \( \beta \geq Z \) (24), and a collateral constraint is always binding. A necessary and sufficient condition for equilibrium determinacy is \( \tau > 1 \).

**Proof.** Let \( x_1, x_2, \) and \( x_3 \) denote three eigenvalues. It is obvious that one of them, \( x_1 \), is infinity. The characteristic equation for \( x_2 \) and \( x_3 \) is
\[ F(x) = \frac{\lambda}{1 + \sigma + \gamma} (x - \tau) \left\{ [1 - \sigma + \beta(1 - \varphi)(\sigma + \gamma)] x - (1 + \gamma) \right\}. \]

Then, the eigenvalues are \( x_2 = \tau \) and \( x_3 = \frac{1 + \gamma}{1 - \sigma + \beta(1 - \varphi)(\sigma + \gamma)} \). The numerator, \( 1 + \gamma \), of \( x_3 \) is strictly positive. The denominator is
\[ 1 - \sigma + \beta(1 - \varphi)(\sigma + \gamma) > 1 - \sigma + \beta \left( 1 - \frac{1 - \beta}{\beta} \cdot \frac{Z}{1 - Z} \right) (\sigma + \gamma) \]
\[ = 1 + \frac{1 - \beta}{1 - Z} \sigma + \frac{\beta - Z}{1 - Z} \gamma > 0, \]
by (24) and \( \beta \geq Z \). Then, it is shown that \( x_3 > 1 \) since
\[ (1 + \gamma) - [1 - \sigma + \beta(1 - \varphi)(\sigma + \gamma)] = [1 - \beta(1 - \varphi)](\sigma + \gamma) > 0. \]

Finally, \( \tau > 1 \) is necessary and sufficient for equilibrium determinacy. \( \square \)
Proposition 3 implies that the stance of a central bank to asset price fluctuations does not matter for equilibrium determinacy if a collateral constraint is binding.

3.2 Interpretations

Why equilibrium indeterminacy never arise if monetary policy responds to asset prices in the case where collateral constrain is binding?

Carlstrom and Fuerst (2007) explain that, if the inflation increases permanently by one percentage and the central bank follows a policy rule \( (14) \), the nominal interest rate increases by

\[
\tau - \frac{A(1 - \beta)}{\lambda} \tau_q. \tag{35}
\]

Their result is shown as follows. By the New Keynesian Phillips curve implies that a permanent increase of inflation increase real marginal cost \( Z \). By the steady state equilibrium system where a collateral constraint never binds, we have

\[
Q = \frac{(1 - Z)Z^{1/(\sigma + \gamma)}}{1/\beta - 1}. \tag{36}
\]

Under reasonable calibration, it is shown that asset price, \( Q \), is decreasing in \( Z \). Then, high inflation means low asset prices. Monetary policy responding asset prices implicitly weakens its overall response to inflation, and this is a source of equilibrium indeterminacy in their model. This is an example of the celebrated “Taylor Principle”: a permanent increase in the inflation rate leads to a more than proportionate increase in the inflation rate. If (35) exceeds one, monetary policy rule satisfies the Taylor Principle.

On the contrary, if a collateral constraint is binding, we have

\[
Q = \frac{(1 - Z)\left[\frac{Z}{1 + \Theta}\right]^{1/(\sigma + \gamma)}}{1/\beta - (1 + \varphi \Theta)} \tag{37}
\]

and

\[
\Theta = \frac{Z[1 - \beta (1 - \varphi)]}{\beta \varphi} - 1. \tag{38}
\]
These conditions imply that

\[ Q = \varphi^{1/(\sigma + \gamma)} \left[ \frac{\beta}{1/\beta - (1 + \varphi)} \right]^{(1+\sigma+\gamma)/\sigma+\gamma}. \]  

(39) implies that asset price does not change if there is a permanent increase of inflation! This is because the inefficiency of collateral constraint, \( \Theta \), absorbs effects of an increase in inflation. Then, the nominal interest rate increases by \( \tau \) in the economy with a binding collateral constraint. Finally, the stance of a central bank to asset price fluctuations does not matter for equilibrium determinacy in our model with a binding collateral constraint.

\section{4 Concluding remarks}

A recent paper by Carlsrom and Furest (2007) found that equilibrium indeterminacy arises if monetary policy responds to asset prices in a sticky-price economy where asset prices reflect firms’ profits.

Since monetary policy responding to asset prices is often discussed after the recession associated with financial crash, by introducing a collateral constraint into their model, we showed that equilibrium indeterminacy never arise even if monetary policy responds to asset prices. An increase in inflation reduces firms’ profits, and asset price declines in a standard sticky-price model. However, under the credit market imperfection, asset price does not change since the inefficiency of the collateral constraint increases and the premium of share as a collateral increases.

Our result implies that a negative aspect of monetary policy responding to asset prices, equilibrium indeterminacy, never arises under the credit market imperfection. Of course, in order to answer the question whether monetary policy should respond to asset prices, it is a future task to investigate optimal policy in the economy with credit frictions. However, since the question of monetary policy responding to asset prices often arise in recessions associated with financial crisis like the Japan’s lost decade during the 1990s and the recent financial crisis in the U.S. economy, our result would have a certain implication on the literature of monetary policy.
References


