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Educational Support and Individual Ability with Endogenous Fertility[1]

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Abstract

In this paper, we present an OLG simulation model with transmission of individual ability and endogenous fertility in order to capture the effects that strengthening income redistribution, expansion of child benefit, and expansion of educational support have on economic disparity and economic growth.

Our simulation results show that expansion of educational support will achieve a reduction in inequality and maintenance or an increase in economic growth. In addition, the effects of expanded educational support are greater with a stronger correlation between parent and child ability.

On the other hand, our findings show that policies increasing child benefit or expanded minimum income cannot be expected to lead to reduction in inequality or improvement in economic growth.

Keywords: Overlapping generations (OLG); educational support; individual ability; endogenous fertility

JEL Classification: C68; D9; E62; H5; J13

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1. Introduction

In Japan, recent widening economic disparity is gradually reducing the opportunity of disadvantaged households to receive higher education.[2] In general, poor parents tend to have relatively more children and provide relatively less education. On the other hand, affluent parents tend to have fewer children and provide higher education. If the national average educational level decreases, the economy may not be able to maintain its growth in the future. This means that inequality, differential fertility, and growth have a certain relationship. This paper, therefore, asserts that the government should strengthen income distribution between rich and poor households.

Recent studies clarify the relationship between inequality, differential fertility, and growth, as Galor and Zang (1997) have shown. As the first example, Kremer and Chen (2000) examine the relationship between economic inequality and differential fertility by using cross-country data analysis. They find that inequality tends to have a positive relationship to differential fertility. Second, De La Croix and Doepke (2003) examine the relationship between economic inequality, differential fertility, and growth by using a growth regression with a differential-fertility variable. They find highly significant effects of differential fertility on growth. The same regression also reveals that the direct effect of inequality as measured by Gini coefficients is not significant, as long as differential fertility is included.

In addition, De La Croix and Doepke (2003) develop an overlapping generation (OLG) model with a channel from inequality to growth, showing that inequality affects growth through its effect on endogenous human capital and endogenous fertility. They find that economies with less equitable income distribution have higher differential fertility, accumulate less human capital, and have a lower rate of economic growth. Therefore, their study implicitly suggests the importance of income redistribution policies, i.e., wage tax and educational support, etc.

However, there also exists a separate, traditional approach to determining the quantity of education. Biologists distinguish “endowments” and “investment” by their source (genetic versus environmental factors). Becker (1967) captures these ideas. Families can bequeath human capital and financial assets. Parents choose the level of human capital investment in their children by comparing the return for the two investments (human capital versus financial assets). When the child’s ability is higher, the return on human capital investment rises.[3] Notice that in this framework without borrowing constraints, parental income and wealth play no role in determining child

[2] In this paper, “higher education” refers to college/university level.

[3] In this paper, “ability” indicates genetic influences transmitted from parent to child.

education or earnings. Only the child's ability matters. Extending this assumption, recent studies elucidate that the relationship between human capital and individual ability is more important. Clearly, the smaller the correlation between parent and child ability, the greater the earnings mobility. Han and Mulligan (2001) find that earnings mobility can be expected to be greater in economies with less variance in ability. In that case, the relationship between income redistribution policies and growth may be external while the true relationship may be that between individual ability and growth. Regarding this point, Hanushek, Leung, and Yilmaz (2004) analyze various educational supports such as those that exist in most public colleges and other institutions of higher learning by using an OLG model with uncertainty in college completion related to differences in ability. They find that these supports tend to improve the efficiency of the economy. However, fertility is not endogenous in their model. If fertility were endogenous, the result of Hanushek et al. (2004) might be different.

In addition, the Democratic Party of Japan (DPJ), the party of the current administration, has strongly proposed several policies in the field of "childrearing and education" in order to decrease inequality and increase economic growth. Briefly, they consist of 1) payment of a "child allowance" of ¥312,000 per annum for each child through completion of junior high school (compulsory education), 2) making public high school tuition effectively free of charge in order to create educational opportunity for all children, and 3) making university scholarships more inclusive, etc.[4] A simple policy traditionally meant to reduce inequality seems to be income redistribution. In this situation, we have to analyze the effect of the following policies: 1) strengthening of income redistribution, 2) increase in child benefit, and 3) expansion of educational support.

Therefore, we establish an OLG model with transmission of individual ability and endogenous fertility in order to capture the effects that strengthening income redistribution, increase in child benefit, and expansion of educational support have on inequality and economic growth.

This paper is organized as follows. In Section 2 we introduce a simple model for grasping an intuitive understanding. In Section 3, we will set the model for our main analysis. In Section 4, we describe simulation results, and Section 5 presents some concluding remarks.

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[4] See *Manifesto 2009* of the DPJ (http://www.dpj.or.jp/english/manifesto/manifesto2009.pdf)

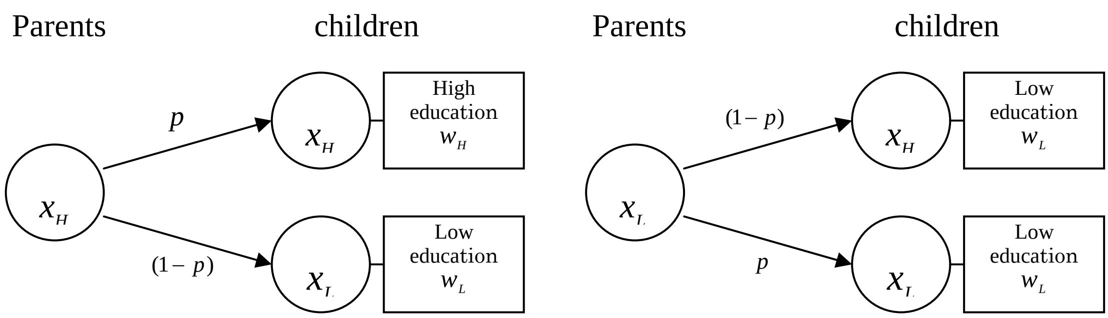
2. Simple Analysis

To keep the model very simple, we suppose that there exist only two generations, the parents' generation (generation 0) and their children's generation (generation 1), and each generation has two abilities, high ability (x_H) and low ability (x_L). Denote the number of parents with high (low) ability as N_0^H (N_0^L), and the wage rate per ability of the high (low) educated worker as w_H (w_L). We also assume that parents with high ability have children with high (low) ability at the possibility p ($1-p$). Similarly, parents with low ability have children with low (high) ability at the possibility p ($1-p$). Suppose, moreover, that 1) while children with affluent parents and individual high ability can receive high education, other children cannot receive it, 2) the worker's income with ability x_j (*j=L,H*) is determined as $w_i x_j$ (*i=H if worker with high education, and i=L if low education*), and 3) as an initial condition, the education of all parents with high (low) ability is high (low).

In this case, the rich parents who have children with high ability (x_H)—the number of such parents is pN_0^H—provide high education to their children, and, when the fertility rate is denoted as $n_H^{EDU=HIGH}(x_H)$, the number and income of their children is described as $N_1^H(1) \equiv n_H^{EDU=HIGH}(x_H) \times pN_0^H$ and $w_H x_H$. Moreover, rich parents who have children with low ability (x_L)—the number of such parents is $(1-p)N_0^H$—provide low education to their children, and, when the fertility rate is denoted as $n_H^{EDU=LOW}(x_L)$, the number and income of their children is described as $N_1^L(1) \equiv n_H^{EDU=LOW}(x_L) \times (1-p)N_0^H$ and $w_L x_L$.

Similarly, poor parents who have children with high (low) ability—the number of such parents is $(1-p)N_0^H$ (pN_0^L)—provide low education to their children, and, when the fertility rate is denoted as $n_L^{EDU=LOW}(x_H)$ ($n_L^{EDU=LOW}(x_L)$), the number and the income of their children is described as $N_1^L(2) \equiv n_L^{EDU=LOW}(x_H) \times (1-p)N_0^L$ ($N_1^L(3) \equiv n_L^{EDU=LOW}(x_L) \times pN_0^L$) and $w_L x_H$ ($w_L x_L$).

Rich families *Poor families*



Therefore, the average income of the parents' generation and the children's generation, $\bar{I}_0$ and $\bar{I}_1$, is given as follows:

$$\bar{I}_0 = \frac{x_H w_H N_0^H + x_L w_L N_0^L}{N_0^H + N_0^L}, \text{ and } \bar{I}_1 = \frac{x_H w_L N_1^H + x_L w_L N_1^L}{N_1^H + N_1^L}$$

where $N_1^H = N_1^H(1)$, and $N_1^L = N_1^L(1) + N_1^L(2) + N_1^L(3)$. The Gini-coefficient for income of the parents' generation and the children's generation, σ_0 and σ_1, is also given as follows:

$$\sigma_0 = \frac{(w_H x_H - w_L x_L)}{\left[\frac{N_0^H + N_0^L}{N_0^L} w_H x_H + \frac{N_0^H + N_0^L}{N_0^H} w_L x_L\right]}, \text{ and } \sigma_1 = \frac{(w_L x_H - w_L x_L)}{\left[\frac{N_1^H + N_1^L}{N_1^L} w_L x_H + \frac{N_1^H + N_1^L}{N_1^H} w_L x_L\right]}$$

In this situation, to assist poor parents of the first generation, government implements policies such as the expansion of educational support for higher education, child benefit, and minimum income. Thus, poor parents who have children with high ability—the number of such parents is $(1-p)N_0^L$—may be able to provide high education to their children, and the wage rate that their high ability children can obtain is expected to rise from low to high. Thus, by using the fertility rate ($n_L^{EDU=HIGH}(x_H)$), the number and income of their children is described as $N_1^H(2)\big|^{after} \equiv n_L^{EDU=HIGH}(x_H) \times (1-p)N_0^L$ and $w_H x_H$. As a result, the average income and the Gini-coefficient of the children's generation changes as follows:

$$\bar{I}_1\big|^{after} = \frac{x_H w_H N_1^H\big|^{after} + x_L w_L N_1^L\big|^{after}}{N_1^H\big|^{after} + N_1^L\big|^{after}}, \text{ and } \sigma_1\big|^{after} = \frac{(w_H x_H - w_L x_L)}{\left[\frac{N_1^H\big|^{after} + N_1^L\big|^{after}}{N_1^L\big|^{after}} w_H x_H + \frac{N_1^H\big|^{after} + N_1^L\big|^{after}}{N_1^H\big|^{after}} w_L x_L\right]}$$

where $N_1^H\big|^{after} = N_1^H(1)\big|^{after} + N_1^H(2)\big|^{after}$, and $N_1^L\big|^{after} = N_1^L(1)\big|^{after} + N_1^L(3)\big|^{after}$.

In this case, the following equations and relationships hold, if $N_1^H(1)\big|^{after} \approx N_1^H(1)$, $N_1^L(1)\big|^{after} \approx N_1^L(1)$, $N_1^H(2)\big|^{after} \approx N_1^L(2)$, $N_1^L(3)\big|^{after} \approx N_1^L(3)$ and $\frac{(w_H x_H - w_L x_L)N_1^H\big|^{after} N_1^L\big|^{after}}{(w_L x_H - w_L x_L)N_1^H N_1^L} < 1$ are satisfied.

$$N_1^H\big|^{after} > N_1^H, \quad N_1^L\big|^{after} < N_1^L, \text{ and } \frac{(w_H x_H - w_L x_L)N_1^H\big|^{after} N_1^L\big|^{after}}{(w_L x_H - w_L x_L)N_1^H N_1^L} < 1$$

$$\Rightarrow \quad \bar{I}_1\big|^{after} > \bar{I}_1, \text{ and } \sigma_1\big|^{after} < \sigma_1$$

The above relationships indicate that there exists the possibility that such policies (e.g. the expansion of educational support for higher education, child benefit, and minimum income, which is proportional to the average income before tax) could both improve economic growth and reduce inequality, and that this possibility further changes dramatically with changes in the parameter (p) of ability transmission. However, such policy effects depend on changes in the fertility rate, the choice of education, and the transmission of individual ability, etc. Therefore, it is too complex to analyze the effects mathematically. To solve such problems, in the next section we construct a model using the OLG model with transmission of individual ability and endogenous fertility to consider the effects.

3. The Model

In this section, we construct a model for considering the effect of 1) strengthening income redistribution, 2) increase in child benefit, and 3) expansion of educational support by using an OLG model with the transmission of individual ability and endogenous fertility. The detailed settings are shown in the following.

3.1. Household

Generation t lives for two periods: childhood and adulthood. All decisions are made in the adult period of life. Each family unit cares about consumption (c_t), the number of children (n_{t+1}), and the education of its children ($e_{t+1,i}$, *i=L or H*). The selection of education depends on the ability of the children. If each family selects low education ($e_{t+1,L}$), the wage rate per ability of its children becomes low ($w_{t+1,L}$). On the other hand, if each family selects high education ($e_{t+1,H}$), the wage rate per ability becomes high ($w_{t+1,H}$) by the probability of the children's ability (x_{t+1}) and becomes low ($w_{t+1,L}$) by the probability of failure $(1-x_{t+1})$. The budget constraint for generation t with ability x_t is expressed in the following equation:

$$n_{t+1}(\xi + e_{t+1,i} - \delta_t) + c_t = (1-\tau_t)kx_t w_{t,j}(1-\varsigma n_{t+1}) + m_t \tag{1}$$

where the index *i*, *j=L or H*, and the parameter ξ is the basic cost of childrearing. The parameter ς represents the opportunity cost which is the net lost income when parents bring up a child, $\delta_t \equiv \delta \overline{W}_t^A$ the subsidy for childrearing which is proportional to the average income after tax ($\overline{W}_t^A$), τ_t the wage tax rate, k the aggregate productivity, and $m_t \equiv m\overline{W}_t^B$ the guaranteed minimum income which is proportional to the average income before tax ($\overline{W}_t^B$). In addition, the utility function is given by:

$$U_t = \log(w_{t+1,i}) + \beta\log(n_{t+1}) + (1-\beta)\log(c_t) \tag{2}$$

where the parameter β refers to the weight of preference between the number of children, and the consumption and wage rate ($w_{t+1,i}$, *i=L or H*) correspond approximately to the human capital of their children. Thus, the expectation of the utility can be described as follows:

1) *if i=L (education=low)*:

$$E(U_t) = \log(w_{t+1,L}) + \beta\log(n_{t+1}(e_{t+1,L})) + (1-\beta)\log(c_t(e_{t+1,L})) \tag{3}$$

2) *if i=H (education=high)*:

$$E(U_t) = (1-x_{t+1})\left[\log(w_{t+1,L}) + \beta\log(n_{t+1}(e_{t+1,H})) + (1-\beta)\log(c_t(e_{t+1,H}))\right]$$
$$+ x_{t+1}\left[\log(w_{t+1,H}) + \beta\log(n_{t+1}(e_{t+1,H})) + (1-\beta)\log(c_t(e_{t+1,H}))\right] \quad (4)$$

Note that the second bracket term of equation (4) indicates the success of obtaining a high wage job by receiving a high education, and the first bracket term represents failure to do so.

3.2.Transmission of Individual Ability

Referring to the basic model of Hanushek et al. (2004), we provide the transmission mechanism of individual ability in the following form:

$$x_{t+1} = \min(\max(\sigma_1 + \sigma_2 x_t + \sigma_3 u_t, 0), 1) \quad (5)$$

where $x_t \in [0,1]$ refers to the parents' ability of generation t, $x_{t+1} \in [0,1]$ the children's ability, and u_t white noise which obeys the standard normal distribution at each period. Finally, the parameter σ_j (*j=1,2,3*) is assumed to be constant. In the case of $(\sigma_1, \sigma_2, \sigma_3) = (0,1,0)$, all of the parents' ability is completely transmitted to their children. In the other case of $(\sigma_1, \sigma_2, \sigma_3) = (-0.02, 0.5, 0.25)$, only part of the parents' ability is transmitted to their children and this transmission has a random factor of evolution or degeneration.

3.3.Production Function

In our model, although the highly educated workers may be employed as a skilled labor force and obtain high wages, the lower educated workers, who are employed as unskilled, cannot obtain high wages and instead get jobs with a low-wage rate. Therefore, the production function of this model economy is assumed to be simple with a CES production function, which depends on the efficiency units of both high-wage labor and of low-wage labor.

$$Y_t = A\left[\varepsilon L_{t,H}{}^{\rho} + (1-\varepsilon)L_{t,L}{}^{\rho}\right]^{1/\rho}, \quad L_{t,j} \equiv \int_0^1 kx_t \times f_j(x_t, t)dx_t \quad (j=H, L) \quad (6)$$

where $L_{t,H}$ refers to the total efficiency unit of high-wage labor of generation t, $L_{t,L}$ the total efficiency unit of low-wage labor, $f_H(x,t)$ the distribution of ability with the population of high-wage labor at period t, $f_L(x,t)$ the distribution of ability with the population of low-wage labor at period t, and $0 < \varepsilon < 1$ is the weight parameter of productive difference between $L_{t,H}$ and $L_{t,L}$.

In the case of a competitive labor market, the wages are simply expected to be a marginal product:

$$w_{t,H} = \partial A\left[\varepsilon L_{t,H}{}^{\rho} + (1-\varepsilon)L_{t,L}{}^{\rho}\right]^{1/\rho} \Big/ \partial L_{t,H} = A\varepsilon L_{t,H}{}^{\rho-1} \Big/ \left[\varepsilon L_{t,H}{}^{\rho} + (1-\varepsilon)L_{t,L}{}^{\rho}\right]^{1/\rho-1} \tag{7}$$
$$w_{t,L} = \partial A\left[\varepsilon L_{t,H}{}^{\rho} + (1-\varepsilon)L_{t,L}{}^{\rho}\right]^{1/\rho} \Big/ \partial L_{t,L} = A(1-\varepsilon)L_{t,L}{}^{\rho-1} \Big/ \left[\varepsilon L_{t,H}{}^{\rho} + (1-\varepsilon)L_{t,L}{}^{\rho}\right]^{1/\rho-1}$$

3.4.Determination of Education

We assume that the cost of education ($e_{t+1,i}$, *i=L or H*) is proportional to the average income of households as follows:

$$\begin{cases} e_{t+1,H} \equiv \tilde{e}_H \overline{W}_t{}^A - \Delta_{t+1} \\ e_{t+1,L} \equiv \tilde{e}_L \overline{W}_t{}^A \quad \text{(where } \overline{W}_t{}^A \equiv (1-\tau_t)\dfrac{\int_0^1 kxw_{t,H} f_H(x_t,t)dx_t + \int_0^1 kxw_{t,L} f_L(x_t,t)dx_t}{\int_0^1 f_H(x_t,t)dx_t + \int_0^1 f_L(x_t,t)dx_t} + m_t \text{)} \end{cases} \tag{8}$$

where $\Delta_{t+1} \equiv \Delta(\tilde{e}_H - \tilde{e}_L)\overline{W}_t$ refers to the educational support from government, W_t^A the average income after tax at period t, and the parameter $\tilde{e}_i$ (*i=L or H*) and Δ is constant.[5] It implies that the cost of education is fixed and does not depend on the parents' wage. Higher education is therefore relatively expensive for poor households. In contrast, in the equation (1), the opportunity cost is higher for households who have high incomes. Therefore, parents with high education and high incomes substitute child quality for child quantity and choose to have fewer children with more education.

The education of each household can be determined by using the optimal control approach. From equations (1) to (3), the lagrangian and the expectation of utility can be described as follows:

1) *if Household decides on low education*:

Because the children's wage becomes low ($w_{t+1,L}$) if households select low education ($e_{t+1,L}$), the lagrangian is defined as follows (*j=L or H*):

$$\Omega_t^{EDU=LOW} \equiv \log\left(w_{t+1,L} n_{t+1}{}^{\beta} c_t{}^{1-\beta}\right) - \lambda\left[n_{t+1}(\xi + e_{t+1,L} - \delta_t) + c_t - (1-\tau_t)kx_t w_{t,j}(1-\varsigma n_{t+1}) - m_t\right]$$

From this lagrangian, the following first order conditions are driven:

$$c_t = (1-\beta)\left[(1-\tau_t)kx_t w_{t,j} + m_t\right], \quad n_{t+1} = \frac{\beta\left[(1-\tau_t)kx_t w_{t,j} + m_t\right]}{(\xi + e_{t+1,L} - \delta_t) + (1-\tau_t)kx_t w_{t,j}\varsigma}$$

Thus, the indirect utility function is given as follows (*j=L or H*):

$$U_t^{EDU=LOW}(j) = \log\left[\beta^{\beta}(1-\beta)^{1-\beta}\frac{w_{t+1,L}\left[(1-\tau_t)kx_t w_{t,j} + m_t\right]}{\left((\xi + e_{t+1,L} - \delta_t) + (1-\tau_t)kx_t w_{t,j}\varsigma\right)^{\beta}}\right]$$

In addition, from equation (3), the expectation of utility is described as the following form (*j=L or H*):

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[5] We also define the average income before tax (W_t^B), by setting $\tau_t = 0$ and $m_t = 0$ on the average income after tax (W_t^A) in equation (8).

$$E(U_t^{EDU=LOW}(j)) = U_t^{EDU=LOW}(j) \tag{9}$$

2) *if Household decides on high education*:

In this case, the lagrangian is defined as follows (*i*, *j=L or H*):

$$\Omega_t^{EDU=HIGH} \equiv \log\left(w_{t+1,i} n_{t+1}{}^{\beta} c_t{}^{1-\beta}\right) - \lambda\left[n_{t+1}(\xi + e_{t+1,H} - \delta_t) + c_t - (1-\tau_t)kx_t w_{t,j}(1-\varsigma n_{t+1}) - m_t\right]$$

From this lagrangian, the following first order conditions are driven:

$$c_t = (1-\beta)\left[(1-\tau_t)kx_t w_{t,j} + m_t\right], \quad n_{t+1} = \frac{\beta\left[(1-\tau_t)kx_t w_{t,j} + m_t\right]}{(\xi + e_{t+1,H} - \delta_t) + (1-\tau_t)kx_t w_{t,j}\varsigma}$$

Thus, the indirect utility function is given as follows (*i*, *j=L or H*):

$$U_t^{EDU=HIGH}(i,j) = \log\left[\beta^{\beta}(1-\beta)^{1-\beta} \frac{w_{t+1,i}\left[(1-\tau_t)kx_t w_{t,j} + m_t\right]}{\left((\xi + e_{t+1,H} - \delta_t) + (1-\tau_t)kx_t w_{t,j}\varsigma\right)^{\beta}}\right]$$

Therefore, from equation (3), the expectation of utility which depends on the children's ability (x_{t+1}) is described as the following form (*j=L or H*):

$$E(U_t^{EDU=HIGH}(j)) = (1-x_{t+1})U_t^{EDU=HIGH}(L,j) + x_{t+1}U_t^{EDU=HIGH}(H,j) \tag{10}$$

On the decision-making of education, each household also considers the children's ability (x_{t+1}). From equations (9) and (10), the condition in which households select high education is the following (*j=L or H*):

$$E(U_t^{EDU=HIGH}(j)) > E(U_t^{EDU=LOW}(j)) \tag{11}$$

$$\Leftrightarrow \quad x_{t+1} > \bar{x}_{t+1}(j) \equiv \frac{\beta \log\left(\dfrac{(\xi + e_{t+1,H} - \delta_t) + (1-\tau_t)kx_t w_{t,j}\varsigma}{(\xi + e_{t+1,L} - \delta_t) + (1-\tau_t)kx_t w_{t,j}\varsigma}\right)}{\log\left[\dfrac{w_{t+1,H}}{w_{t+1,L}}\right]}$$

We assume that parents can observe their children's ability. Equation (10) therefore means that, if $x_{t+1} > \bar{x}_{t+1}(j)$, parents select high education. On the other hand, low education is selected if $x_{t+1} < \bar{x}_{t+1}(j)$. Further, the relationship $\partial\bar{x}_{t+1}(j)/\partial w_{t,j} < 0$ and $\partial\bar{x}_{t+1}(j)/\partial x_t < 0$ hold. The income ($kx_t w_{t,j}$, *j=L or H*) of each household is different. It indicates that, even if children have the same ability, their education depends on the environment of their household.

3.5.Government

We focus on the effect that the strengthening of income redistribution, the increase in child benefit expansion, and the expansion of educational support have on inequality and economic growth through the transmission of individual ability. In our model, we assume that government collects its revenue by taxes only and does not issue public bonds. From (1) and (6), government budget constraint is therefore driven as follows:

$$T_t = G_t^{CON} + G_t^{CB} + G_t^{MIN} + G_t^{EDU} \tag{12}$$

Where $T_t \equiv \tau_t (L_{t,L} + L_{t,H})$ refers to the tax revenue, $G_t^{CON} \equiv cY_t$ the government consumption, $G_t^{CB} \equiv \delta_t N_{t+1}$ the government subsidy to child bearing, $G_t^{MIN} \equiv m_t N_t^{TOTAL}$ the lump-sum grant as minimum income, $G_t^{EDU} \equiv \Delta_{t+1} N_{t+1}^{EDU=HIGH}$ the government subsidy to higher education at the period t, N_t^{TOTAL} the total population of generation t, $N_t^{EDU=HIGH}$ the population of generation t with higher education, and the parameter of government consumption (c) is constant.

3.6.Macro Dynamics of the Model and Initial conditions

The aggregate dynamics of our model are determined by equations (1) to (12) and the following initial conditions:

1) *the wage rate* :

$$w_{0,L} = (1-\varepsilon), \quad w_{0,H} = \varepsilon \tag{13}$$

2) *the distribution of ability*:

$$f_j(x,0) = \frac{N_0}{\sqrt{2\pi s^2}} \exp\left[-\frac{(x-\mu)^2}{s^2} \right] \qquad (j{=}L \text{ or } H) \tag{14}$$

These dynamics drive the economics path which depends on government policies. However, it is difficult to analyze the paths as functional solutions directly, because the transmission of individual ability is complex. Therefore, we use a numerical simulation method in Section 4.

4 Simulation Scenarios and Results

4.1. Simulation Scenarios and Parameter Setting

In this section, first we present several simulation scenarios in order to achieve our aim. The scenarios are classified into two cases and four policies. Case 1 assumes that the parameter in equation (5) which expresses the transmission of individual ability is $(\sigma_1, \sigma_2, \sigma_3) = (0,1,0)$. This case is that all of the parents' ability is completely transmitted to their children. Moreover, Case 2 assumes that the parameter is $(\sigma_1, \sigma_2, \sigma_3) = (-0.05, 0.4, 1)$. This case is that only part of the parents' ability is transmitted to their children. In each case, we analyze the effects of the following four policies.

	Δ	δ	m
1) Policy 1 (Baseline)	0	0.0225	0.2
2) Policy 2 (Expansion of educational support)	0.5	0.0225	0.2

3) Policy 3 (Expansion of child benefit)	0	0.0225+ α_δ	0.2
4) Policy 4 (Expansion of minimum income)	0	0.0225	0.2+ α_m

Note: Δ, δ and m are the parameters of equations (1) and (8).

Next, in order to compare the effect of Policies 2 to 4 with each other, it is necessary to maintain the government budget of the equation (12) neutrally. Therefore, in Policies 3 and 4, we correspond the government expenditure to GDP, which is driven from the calculation that the right hand side of the equation (12) is divided by the left hand side of the equation (6), with that of Policy 2, by controlling the parameters (α_δ and α_m) on the above table. Thus, the wage tax rate (τ_t) is also endogenously determined, satisfying equation (12).

Finally, we set the other parameters of the equations (6), (12) and (16) as follows: 1) $\varepsilon = 0.55$, $\rho = 1$, 2) $c = 0.02$, and 3) $\mu = 0.4$, $s = 0.2$, $N_0 = 10^4$.

4.2. Simulation Results

We now turn to describe the simulation results reported in Figures 1 to 6 and Tables 1 and 2. Here we present the scenarios of results of Policies 1 to 4 in Cases 1 and 2. In each figure, the plot denoted as "+" shows the result of Policy 1 (baseline), the plot denoted as "●" the result of Policy 2 (Expansion of educational support), the plot denoted as "△" the result of Policy 3 (Expansion of child benefit), and the plot denoted as "○" the result of Policy 4 (Expansion of minimum income).

(1) Government expenditure and control parameters

Table 1 (Table 2) shows the government expenditure to GDP and the control parameters in Case 1 (Case 2). In Cases 1 and 2, the government expenditure to GDP of Policies 3 and 4 completely corresponds to that of Policy 2. This means that our simulation results achieve the neutrality of government budget. Thus, the control parameters in Case 1 (Case 2) are α_δ =0.0116 and α_m =0.0109 (α_δ =0.0125 and α_m =0.0115).

(2) Distribution density

Figure 1 shows the distribution density of the 10th generation's ability in Case 1 and 2. On the ability range (0.1, 0.2) in Case 1, the distribution density of Policy 2 (Expansion of educational support) is lower than that of other Policies. However, on the ability range (0.3, 0.5) in Case 1, the distribution density of Policy 2 is higher than that of other Policies. This means that Policy 2 shifts the distribution density towards higher ability. On the other hand, we cannot clearly see such effect in the distribution density of Case 2. But as we explain at the following, this effect can be seen in Figure 2, which has more detailed information.

(3) Ratio of low-wage rate population, etc

Figure 2 shows the ratio of low-wage rate population to the total population with the same ability in Cases 1 and 2. In Case 1, there are two line groups. The first line group is in the ability range (0, 0.2). This group has no slope and the value of the ratio is 100%. This means that the ability range (0, 0.2) is dominated by only the workers with a low-wage rate. On the other hand, the second line group is in the ability range (0.1, 0.7). This group has a downward slope from left to right. This means that number of workers with a low-wage rate gradually decreases in the higher ability area. These groups of each policy are almost the same, but have a difference in the ability range (0.1, 0.2). In this range, the low-wage rate population ratio of Policy 2 (expansion of educational support) is the only one under 100%. This represents that Policy 2 shifts the distribution density towards higher ability.

Moreover, even in Case 2, we can see such effect. In this case, the low-wage rate population ratio of Policy 2 is lower than that of other policies. It also means that Policy 2 shifts the distribution density towards higher ability in Case 2.

In addition, Figure 3 shows the normalized population (1^{st} period=100). In Cases 1 and 2, the population of Policies 2 and 3 is higher than that of Policy 1.

(4) GDP per capita and Gini-coefficient for income

First, Figure 4 shows GDP per capita during periods 1 to 10 in Case 1 and Case 2. In Case 1, the GDP per capita of Policy 2 (expansion of educational support) is largest. That is why the distribution density of Policy 2 in Case 1 has more workers with high ability, as we have already described. However, in Case 2, this effect disappears and the GDP per capita of each policy is almost the same. In addition, compared with Policy 1 (baseline), Policies 2 to 4 do not clearly change GDP per capita. This reason is that the distribution density of each policy in Case 2 is almost the same.

Next, Figure 5 shows the Gini-coefficient for the income of each household after policies during periods 1 to 10 in Cases 1 and 2. This figure indicates that the Gini-coefficient for income after Policy 2 (expansion of educational support) is smallest in Case 1. Moreover, even in Case 2, such effect can be seen as an overall tendency. This suggests that Policy 2 has the effect of reducing inequality and maintaining or increasing GDP per capita.

(5) Average utility and Gini-coefficient for the utility

Figure 6 shows the average utility of each generation in Cases 1 and 2. In Case 1, Policy 2 (expansion of educational support), compared with Policy 1 (baseline), improves only the average utility. In contrast, Policies 3 and 4 worsen the average utility. Moreover, Figure 7 shows the Gini-coefficient for the utility of each generation in Cases 1 and 2. In Case 1, Policy 2 (expansion of educational support), compared with Policy 1 (baseline), improves only the Gini-coefficient for utility.

In Case 2, such effects on the average utility and the Gini-coefficient for utility can be clearly seen as an overall tendency. These facts suggest that Policy 2 (expansion of educational support), compared with Policy 3 (expansion of child benefit) or Policy 4 (expansion of minimum income), should be carried out in order to reduce inequality and increase economic growth under government budget neutrality.

5 Concluding Remarks

In this paper, we present an OLG simulation model with transmission of individual ability and endogenous fertility in order to capture the effects that strengthening income redistribution, expansion of child benefit, and expansion of educational support have on economic disparity and economic growth.

Our simulation results show that expansion of educational support will achieve a reduction in inequality and maintenance or an increase in economic growth. In addition, the effects of expanded educational support are greater with a stronger correlation between parent and child ability.

On the other hand, our findings show that policies increasing child benefit or expanded minimum income cannot be expected to lead to reduction in inequality or improvement in economic growth.

The weakness of our study is that our model does not include the following points: 1) the effect of heterogeneous households with different preferences such as weight of preference between the number of children and consumption, 2) the effect of endogenous labor supply, 3) the effect of the existence of a social security system (e.g., pay-as-you-go type public pension), and 4) the effects that physical capital or the global capital market have on economic growth, etc. These points remain subjects for future study.

In addition, the Japanese government is currently trying to reduce inequality and increase economic growth by implementing several policies such as expansion of child benefit and educational support, etc. Therefore, if our model can be made more robust, it may be of use in the evaluation of such policies.

Finally, our analysis provides a new perspective on the relationship between economic inequality and economic growth. Existing studies (e.g., De La Croix et al. (2003)) have suggested the importance of income redistribution policies. However, the results in this paper suggest that among these policies, educational support may be the most important.

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Table 1: Government expenditure and controlling parameters in Case 1

	Policy 1 (Base line)	Policy 2 (Educational Support)	Policy 3 (Child Benefit)	Policy 4 (Minimum Income)
Δ	0	0.5	0	0
δ	0.0225	0.0225	0.0341	0.0225
m	0.2	0.2	0.2	0.2109

Period	Wage Tax Rate (τ)			
	Policy 1 (Base line)	Policy 2 (Educational Support)	Policy 3 (Child Benefit)	Policy 4 (Minimum Income)
1	24.1%	25.2%	25.2%	25.2%
2	24.0%	25.2%	25.2%	25.2%
3	24.1%	25.2%	25.2%	25.2%
4	24.1%	25.2%	25.2%	25.2%
5	24.1%	25.2%	25.2%	25.2%
6	24.1%	25.2%	25.2%	25.2%
7	24.1%	25.2%	25.2%	25.2%
8	24.1%	25.2%	25.2%	25.2%
9	24.1%	25.2%	25.2%	25.2%
10	24.1%	25.2%	25.2%	25.2%

Period	Government Expenditure to GDP			
	Policy 1 (Base line)	Policy 2 (Educational Support)	Policy 3 (Child Benefit)	Policy 4 (Minimum Income)
1	24.1%	25.2%	25.0%	25.3%
2	24.0%	25.2%	25.2%	25.2%
3	24.1%	25.2%	25.2%	25.2%
4	24.1%	25.2%	25.2%	25.2%
5	24.1%	25.2%	25.2%	25.2%
6	24.1%	25.2%	25.2%	25.2%
7	24.1%	25.2%	25.2%	25.2%
8	24.1%	25.2%	25.2%	25.2%
9	24.1%	25.2%	25.2%	25.2%
10	24.1%	25.2%	25.2%	25.2%

Table 2: Government expenditure and controlling parameters in Case 2

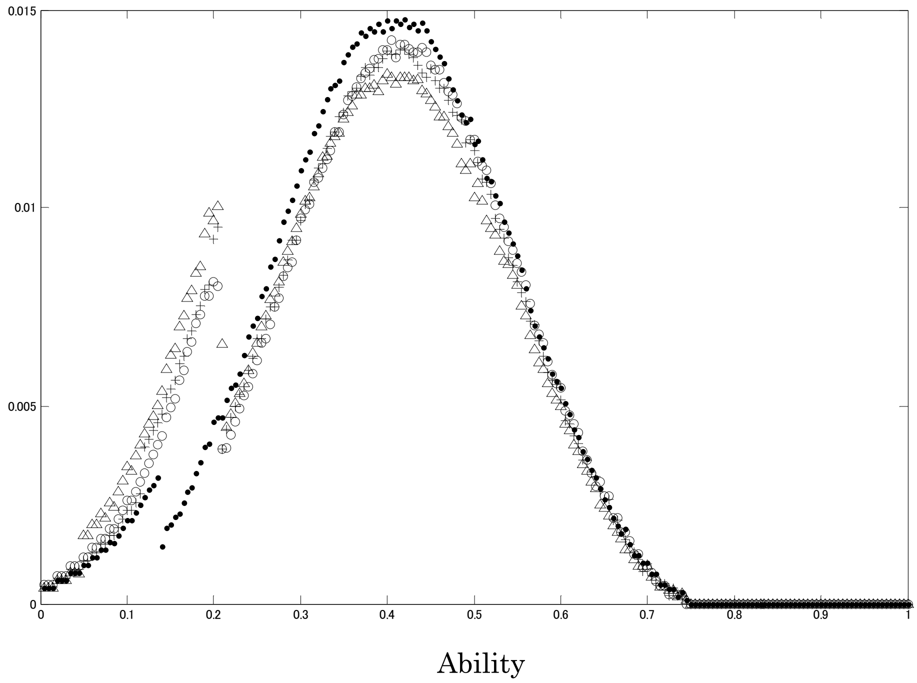
	Policy 1 (Base line)	Policy 2 (Educational Support)	Policy 3 (Child Benefit)	Policy 4 (Minimum Income)
Δ	0	0.5	0	0
δ	0.0225	0.0225	0.035	0.0225
m	0.2	0.2	0.2	0.2115

Period	Wage Tax Rate (τ)			
	Policy 1 (Base line)	Policy 2 (Educational Support)	Policy 3 (Child Benefit)	Policy 4 (Minimum Income)
1	24.1%	25.2%	25.2%	25.2%
2	24.0%	25.2%	25.2%	25.2%
3	24.0%	25.2%	25.2%	25.2%
4	24.0%	25.2%	25.2%	25.2%
5	24.0%	25.2%	25.2%	25.2%
6	24.0%	25.2%	25.2%	25.2%
7	24.0%	25.2%	25.2%	25.2%
8	24.0%	25.2%	25.2%	25.2%
9	24.0%	25.2%	25.2%	25.2%
10	24.0%	25.2%	25.2%	25.2%

Period	Total Government Expenditure to GDP			
	Policy 1 (Base line)	Policy 2 (Educational Support)	Policy 3 (Child Benefit)	Policy 4 (Minimum Income)
1	24.1%	25.2%	25.1%	25.3%
2	24.0%	25.2%	25.2%	25.2%
3	24.0%	25.2%	25.2%	25.2%
4	24.0%	25.2%	25.2%	25.2%
5	24.0%	25.2%	25.2%	25.2%
6	24.0%	25.2%	25.2%	25.2%
7	24.0%	25.2%	25.2%	25.2%
8	24.0%	25.2%	25.2%	25.2%
9	24.0%	25.2%	25.2%	25.2%
10	24.0%	25.2%	25.2%	25.2%

Figure 1: Distribution density of 10th generation's ability

1) Case1



2) Case 2

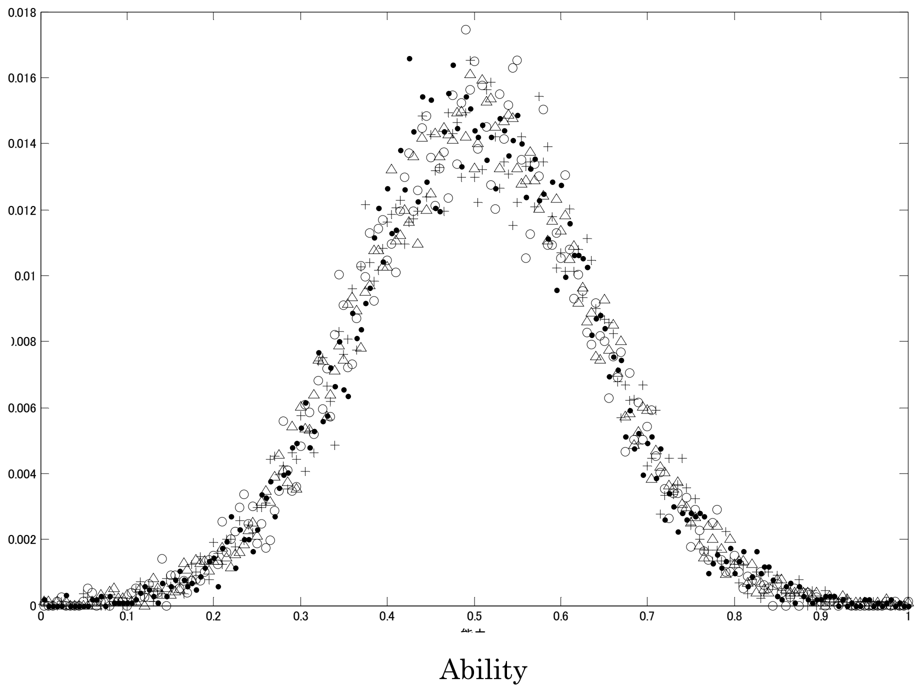
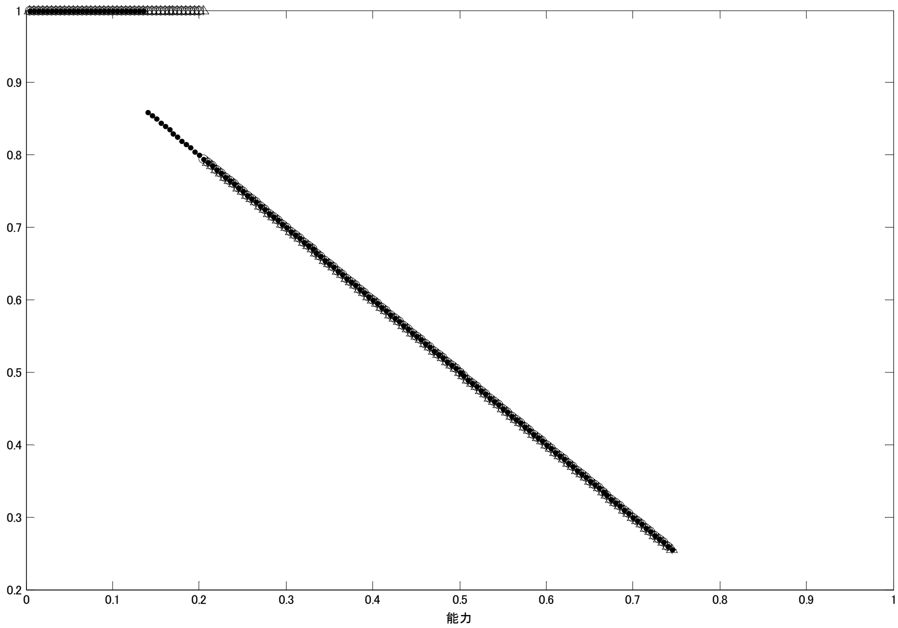


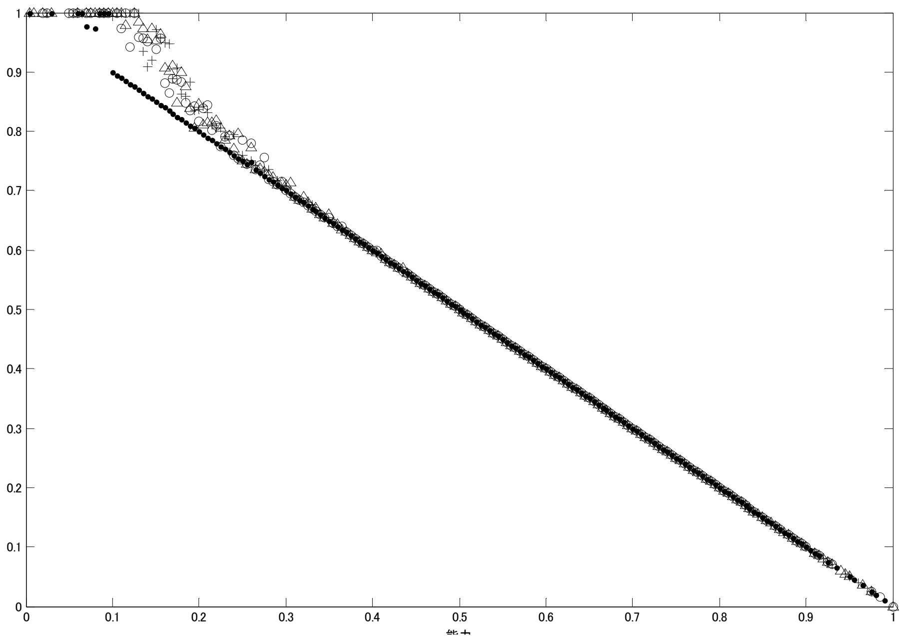
Figure 2: Ratio of low wage rate population

1) Case1



Ability

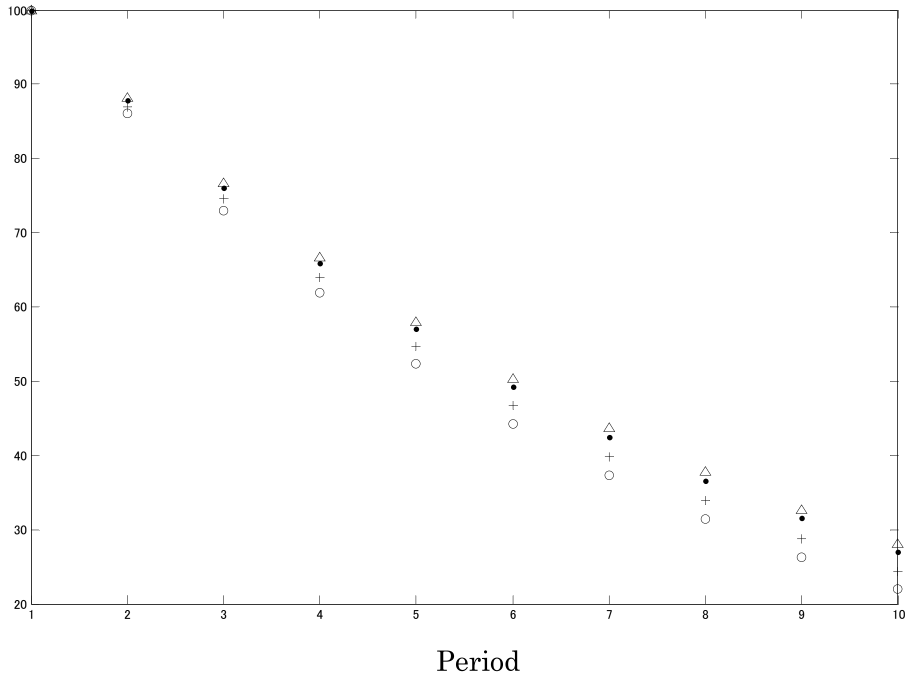
2) Case 2



Ability

Figure 3: The normalized population (1st period=100)

1) Case1



2) Case 2

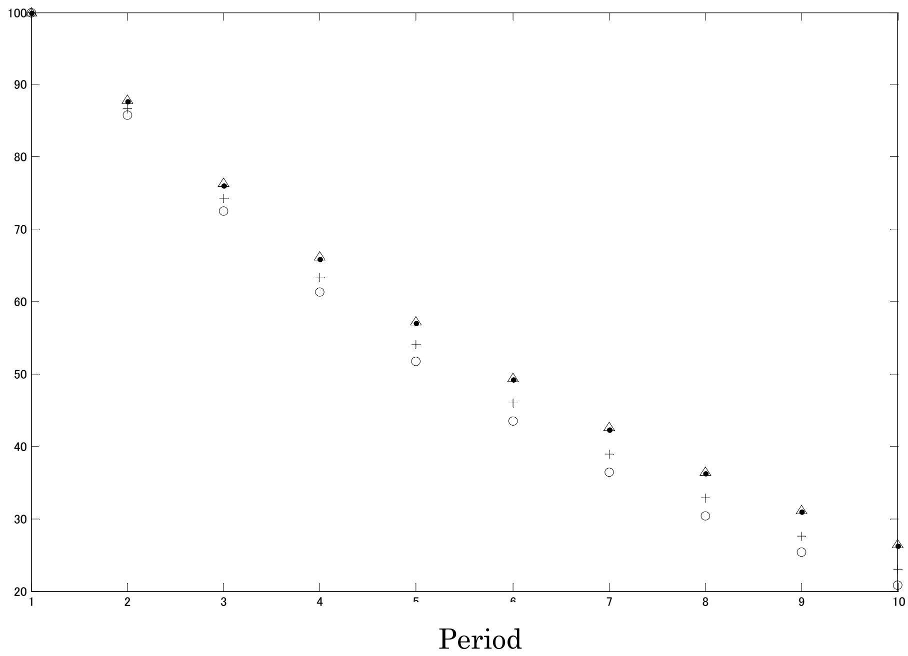
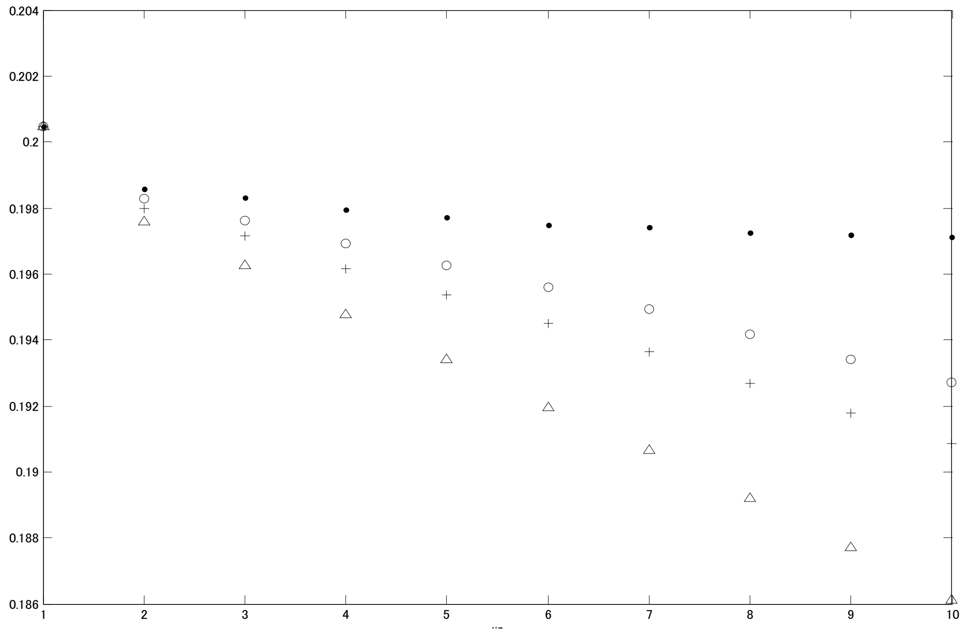


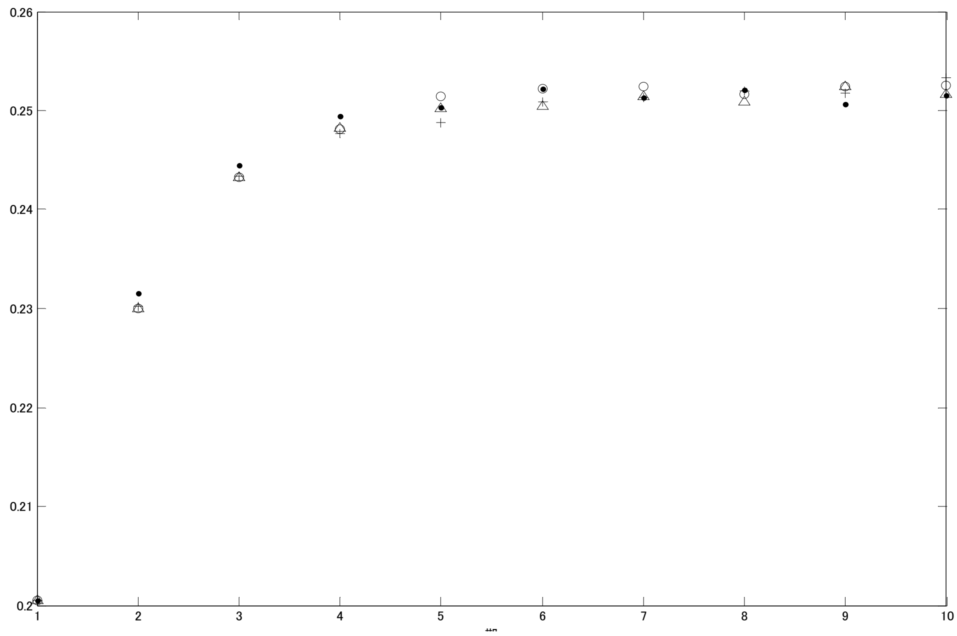
Figure 4: GDP per capita

1) Case 1



Period

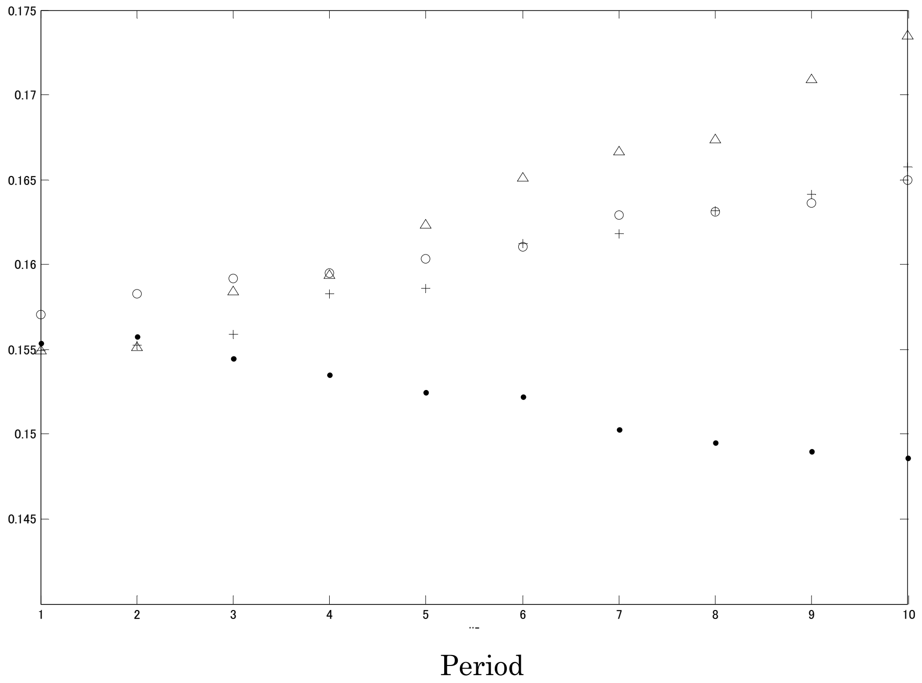
2) Case 2



Period

Figure 5: Gini-coefficient for income

1) Case 1



2) Case 2

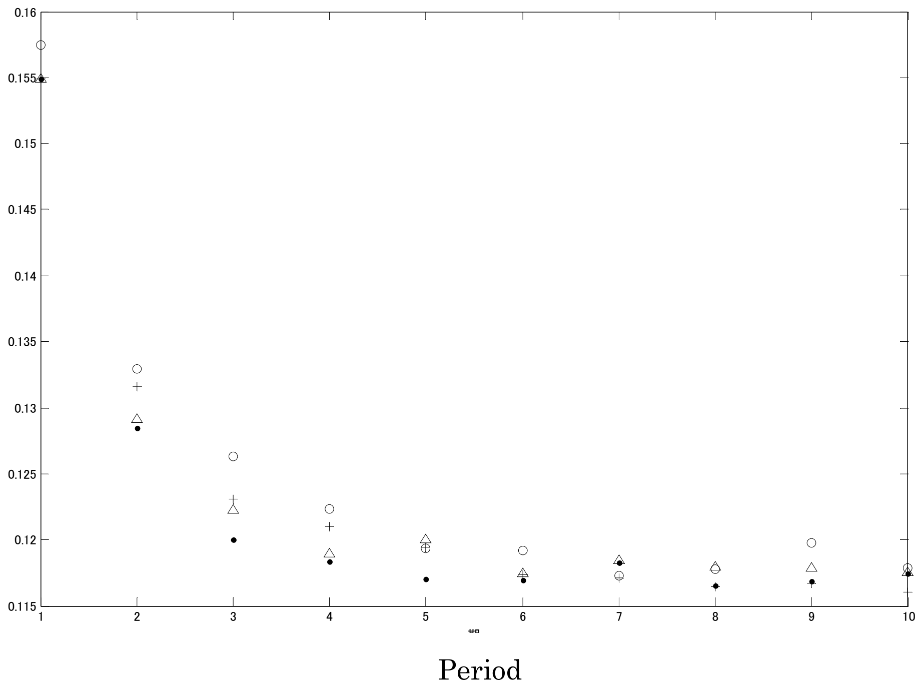
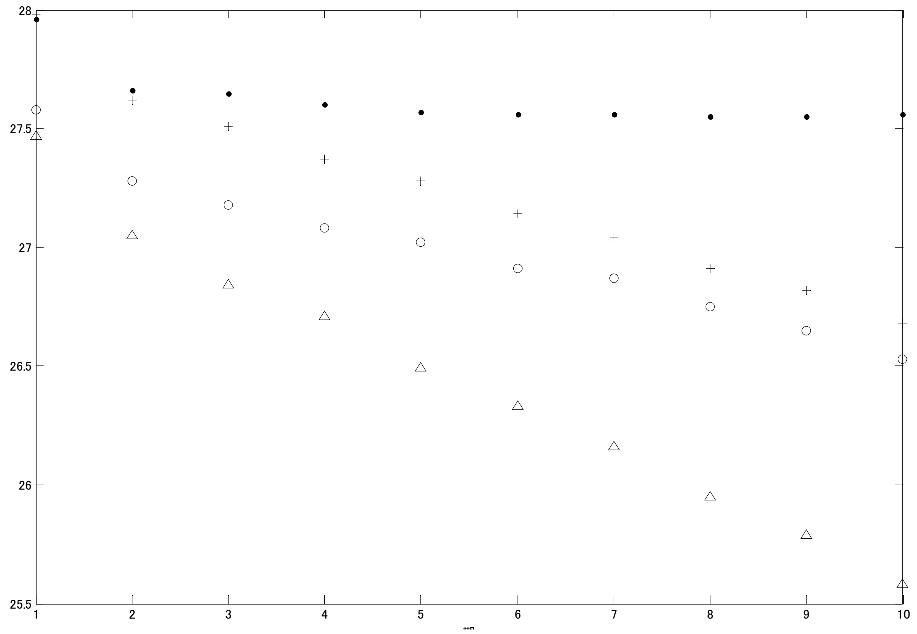


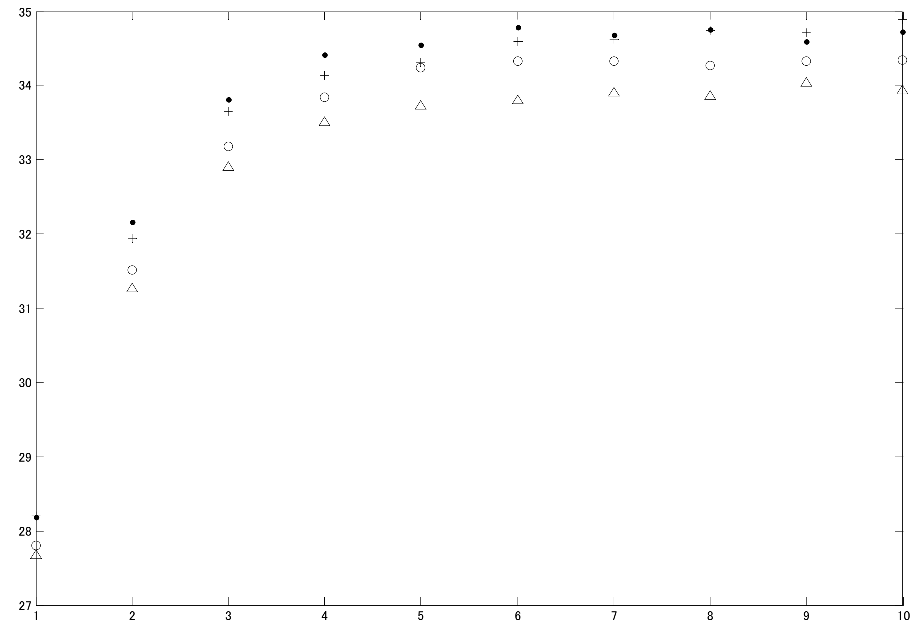
Figure 6: Average utility

1) Case 1



Period

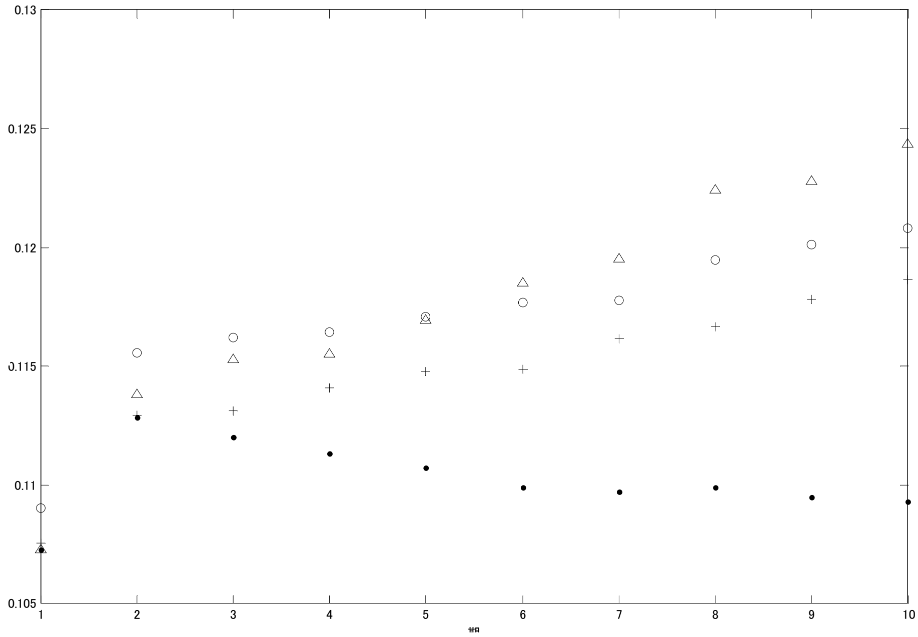
2) Case 2



Period

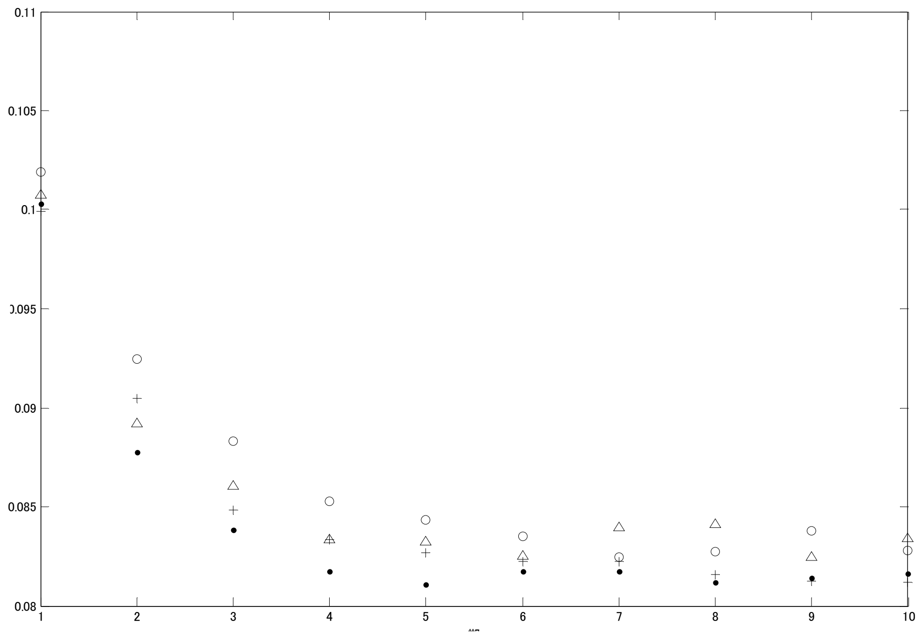
Figure 7: Gini-coefficient for utility

1) Case 1



Period

2) Case 2



Period