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A Monetary Model of Banking Crises*

(Incomplete and preliminary)

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Abstract

We propose a new model for policy analysis of banking crises (or systemic bank runs) based on the monetary framework developed by Lagos and Wright (2005). If banks cannot enforce loan repayment and have to secure loans by collateral, a banking crisis due to coordination failure among depositors can occur in response to a sunspot shock, and the banks become insolvent as a result of the bank runs. The model is tractable and easily embedded into a standard business cycle model. The model naturally makes a distinction between money and goods, while most of the existing banking models do not. This distinction enables us to clarify further the workings of banking crises and crisis management policies. In particular, we may be able to use this framework to compare the efficacy of fiscal stimulus, monetary easing, and bank reforms as recovery efforts from the current global financial crisis.

Keywords: Monetary theory, bank runs, demand deposits, business cycles.

JEL Classifications: E32, E42, G01.

1 Introduction

The purpose of this paper is to construct a tractable model of banking crises, which can be embedded in standard dynamic stochastic general equilibrium (DSGE) models. In order

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to be compatible with a standard business cycle model, the model should distinguish nominal money (i.e., cash) from goods. It is difficult to incorporate the existing banking theory (such as Diamond-Dybvig framework) into standard DSGE models because the existing models do not distinguish between cash and goods in a tractable way. A model with a distinction between cash and goods is expected to enable us to clarify the workings of banking crises further. For example, in our model we can naturally explain the reason why a banking crisis decreases the value of the bank asset (see the incomplete loan enforcement model in Section 4), while the existing models exogenously assume that the early liquidation of investments by banks is costly. A model of banking crises that can be easily integrated in the standard DSGE models may enable the analysis of banking crises and ordinary business cycles in a unified framework. It may also enable the presentation of a seamless policy analysis on monetary and banking policies.

Our model builds on the monetary framework developed by Lagos and Wright (2005). Naturally, the bank deposit in our model is assumed to be primarily a substitute for money, which gives payment services to the depositors. The bank deposit in this paper represents not only bank deposit in reality, but also various debt liabilities of financial intermediaries, i.e., borrowings and bonds issued by banks, investment banks, and hedge funds.

In Section 2, we construct the basic model with loan enforcement and no bank insolvency shock. There are no bank runs in the basic model, when the deposit rate is positive. In Section 3, we introduce a macroeconomic shock which renders the banks insolvent. This is a model with a bank insolvency shock and loan enforcement. A banking crisis occurs when the bank insolvency shock hits the economy. Policy implications are (1) the suspension of convertibility may amplify the severity of the banking crisis, since this policy does not restore the payment activity of banks; (2) The bank reform to restore the solvency of the banking system is costly, but improves social welfare significantly; (3) Lender of Last Resort (LLR) lending by the central bank is not sufficient to stop a banking crisis. In Section 4, we construct a model with incomplete loan enforcement without a bank insolvency shock. In this model, bank runs due to coordination failure among depositors can occur even if there is no real shock that renders banks insolvent. Therefore, bank runs can occur as a sunspot equilibrium due to panic in the market, and the banks become insolvent as a result of the bank runs. In the incomplete loan enforcement model, it is shown that the bank reforms to restore the solvency of the banking system is the optimal policy for economic recovery from the banking crisis, while both fiscal stimulus and monetary easing are not sufficiently effective to resolve the crisis. Although the policy of bank reforms appears to be highly costly ex ante, its cost turns out to be zero ex post, precisely because the asset price responds positively to the policy.

Related Literature: There are several models of banking with distinctions between money and goods. Champ, Smith and Williamson (1996) analyze a model in which bank notes can be circulated as a means of payment or inside money. In their model, bank notes are circulated only if the government allows the issuance of bank notes, while we are interested in the case where the agents spontaneously determine not to accept bank notes as money even if the government allows their issuance. McAndrews and Roberds (1995, 1999) also analyze the payment intermediation by banks, but they do not formally analyze the disruption of payment intermediation. Allen and Gale (1998) show a welfare-improving role of fiat money during a banking crisis, while the money in their model works as a store of value, not as a means of payment. In the present paper, we focus on the payment services provided by the banking sector as a key driving factor of banking crises.

The organization of this paper is as follows. In Section 2, we show the basic model with loan enforcement and no bank insolvency shock. We show that there are no bank runs in the basic model. In Section 3, we introduce a bank insolvency shock that induces bank runs. We specify the equilibrium with bank runs and discuss policy implications. In Section 4, we present a modified model with incomplete loan enforcement and collateral constraint. We show that bank runs due to coordination failure can occur in response to a sunspot shock and the banks become insolvent as a result of the runs. We compare fiscal and monetary policies and bank reforms as recovery efforts from the current financial crisis in our framework. Section 5 concludes. In Appendix B we modify the incomplete loan enforcement model of Section 4 such that idiosyncratic shocks induce runs on individual banks due to coordination failure. We also show that individual bank runs due to idiosyncratic shocks are naturally contagious.

2 Basic Model

The model is a variant of the monetary economy with banks constructed by Berentsen, Camera and Waller (2007), which is based on the framework developed by Lagos and Wright (2005). The main difference of our model from theirs is that banks can perform *credit creation*, that is, the banks accept deposits and make loans such that the size of their balance sheets becomes several times larger than that of their cash reserves.

2.1 Environment

The model is a closed economy à la Lagos and Wright (2005), in which there are continua of two types of agents, sellers and buyers, who live forever. Time is discrete and continues from 0 to infinity: $t = 0, 1, \cdots, \infty$. The intertemporal discount factor for the utility flow is β for both agents, where $0 < \beta < 1$. The measures of sellers and buyers are n and $1 - n$ respectively. In each date t, there are two perfectly competitive markets that open sequentially: the day market and the night market. In the day market sellers and buyers trade the intermediate goods, taking the market price as given. A seller can produce and sell the intermediate goods. A seller who produces q units of intermediate goods incurs the utility cost of $c(q)$, where $c'(q) > 0$, $c''(q) > 0$ and $c(0) = 0$. The goods trade during the day market anonymously, but trades during the night market are not anonymous; an agent cannot identify her trading partner during the day market, but she can during the night market. Since all goods trade in the day market anonymously, trade credit between sellers and buyers is not available and sellers require immediate compensation, meaning payment with cash.[1] Consumption takes place only in the night market. There is also a

[1] In this paper, we exclude the possibility that bank deposits are accepted as a means of payment or as inside money. We assume that a buyer's transaction record with her bank is also private information

unit mass of banks that can accept deposits and make loans. The banks are one-period lived, that is, they are born in the date-$(t-1)$ night market and are liquidated in the date-t night market. The banks born in the date-$(t-1)$ night market accept deposits and make loans in that market, and they collect the loan repayments and pay out deposits in the date-t night market before liquidation. The banks eat all remaining profits when they are liquidated. The banks born in the date-$(t-1)$ night market have a record-keeping technology such that they can identify their depositors and borrowers in the date-t night market. This record-keeping technology is not available for sellers or buyers.

In the night market, all sellers and buyers produce consumption goods from labor input by a linear production technology which transforms h units of labor into h units of consumption goods, while the labor supply generates h units of disutility. All agents get utility $U(x)$ from consumption of x units, with $U'(x) > 0$, $U'(0) = +\infty$, $U'(+\infty) = 0$, and $U''(x) \leq 0$. This setting for the night market is standard in the Lagos-Wright framework, which makes the distribution of money holdings degenerate at the beginning of a period. On the other hand, at the beginning of the night market, buyers are endowed with one unit of the production machine that is used to produce the consumption goods from the intermediate goods. Buyers, sellers, and banks trade the machines and the intermediate goods in the competitive market, and the borrowers (i.e., buyers and sellers) repay their bank loans (in the form of the consumption goods). After the borrowers make loan repayments, y units of the consumption goods are produced from q units of the intermediate goods and k units of the machines by a Cobb-Douglas production technology: $y = Ak^{1-\theta}q^{\theta}$, where $0 < \theta < 1$. After the production of the consumption goods, the machines depreciate fully. Then bank deposits are paid out and banks are liquidated.

We assume that there exists a central bank in this economy. The central bank controls the supply of fiat currency. The amount of cash in the economy is given by $M_t = \gamma_{t-1}M_{t-1}$, where $\gamma_t(> 0)$ is the growth rate of money and also the inflation rate in

unavailable to a seller, and the seller cannot know the financial health of the buyer's bank. Consequently, the seller feels a risk that the buyer's bank may fail to transfer money to the seller's account. Therefore, a buyer's bank deposit is not accepted as a payment instrument by a seller.

a steady state equilibrium, which we focus on in this paper, and M_t denotes the per capita money stock on date t. All agents receive identical lump-sum transfers $(\gamma_{t-1} - 1)M_{t-1}$ at the beginning of the date-$(t-1)$ night market, in which sellers, buyers, and banks trade cash to determine their cash holdings that they carry over to the date-t day market.

Record keeping and demand deposits: We assume as Berentsen et al. (2007) that only the banks have a technology that allows record keeping of financial transactions but not trading histories in the goods market. Since the banks live for one period, they keep records of financial histories only for one period. We also assume that there is no cost for the banks to keep financial records. Thanks to this record-keeping technology, the banks can accept demand deposits and make loans. In this paper we assume that a demand deposit is an asset that is primarily designed to be a substitute for cash and to facilitate payment activities of depositors. An agent who holds cash enjoys the following two conveniences in payment activity: (1) She can use cash at anytime she wants, and (2) there is no uncertainty in the amount of money (i.e., cash) that she can use in the units of the fiat currency, because the amount of cash is by definition prefixed in the units of the fiat currency. Since these two features are very essential advantages of holding cash and the demand deposit is designed to be a substitute for cash, banks naturally offer these two features to their depositors. So we define the demand deposit as follows:

Definition 1 *The demand deposit contract is a contract such that (1) the bank commits itself to exchange the demand deposit for cash on demand at anytime; and (2) the exchange rate between the demand deposit and cash is prefixed by contract as a function of the time period of deposit.*

In our model, the demandability of a bank deposit is assumed as a necessary feature of the asset called bank deposit, while in the framework of Diamond and Rajan (2001), for example, it is a device to discipline the bankers in the principal-agent relationship.

Enforcement of loan repayment: We assume that in this basic model, the banks can completely enforce their borrowers (i.e., sellers or buyers) to repay their bank loans.

There is no risk of default of bank borrowers. In Section 4, we consider the case where the banks cannot enforce loan repayment completely and have to secure loans by collateral.

Sequential service constraint and bank failures: In the day market, some depositors withdraw their demand deposits and in the same market other depositors deposit additional cash into the banks. Although the banks are obliged by the demand deposit contract to pay the full amount of the deposit on demand, the banks may run out of their cash reserves under some circumstances. We say that the banks are "bankrupt" when they run out of cash. We assume the following process for the bankruptcy procedure for the banks.[2] When a bank runs out of cash, the remaining depositors who want to withdraw their deposits in the day market cannot withdraw, instead they can simply hold their deposits until the night market. In the night market, the bank is liquidated after it collects loans, and the bank assets are divided among the remaining depositors at a *pro rata* basis. In the basic model, deposits are fully repaid in the night market, since there is no possibility of impairment of bank assets. Definition 1 and this bankruptcy procedure naturally imply that the demand deposit contract is subject to the *sequential service constraint* (or the first come-first served constraint) proposed by Diamond and Dybvig (1983). That is, when a depositor who wants to withdraw comes to the bank teller, the bank must pay any amount of cash on demand up to a prefixed amount of the depositor's account as long as it has a cash reserve; and when the bank runs out of the cash reserve, the following withdrawers at the bank window cannot be paid cash, but

[2] Alternatively, we may be able to assume that the banks can sell their loan assets in the day market to obtain cash to meet the demand of withdrawals. We do not assume, however, that the asset market opens in the daytime for two reasons. First, because in this paper we focus on systemic banking crises in which all banks in the system are subject to bank runs, we need to assume asset buyers who are outsiders of the banking system. I suspect that assuming outside buyers would make the general equilibrium analysis of the model unnecessarily complicated. Second, the effect of asset sale is almost completely replicated by the monetary policy or the Lender-of-Last-Resort lending, in which the central bank lends cash to the banks taking their loan assets as collateral. Thus the behavior of the model with the asset market should be the same as that with monetary policy described in Sections 3.4 and 4.2. To modify the model so that banks can sell their loan assets is an agenda for future research.

they can still keep her deposit account until the night market. As we see in Section 3.3, the sequential services constraint is the key driver of banking crises in our model.

Limited participation and payment intermediation: We assume that the day market is divided by its nature into J submarkets that open sequentially. We call j-th submarket the j-submarket for $j = 1, 2, \cdots, J$. Buyers and sellers are divided equally into J groups and those in the j-th group can participate only in the j-submarket. Therefore, each seller and buyer are allowed to participate only in one submarket during one day. In each submarket, n/J sellers and $(1-n)/J$ buyers trade the intermediate goods. On the other hand, each bank can participate in all of the J submarkets. In addition to the record-keeping technology of the banks, this difference of market participation between the banks and the other agents gives the banks a superior technology to intermediate payment activities. If buyers hold cash during the day market instead of bank deposits, the cash in this economy would be paid only once during one day. But if all agents deposit their cash in their banks immediately after they obtain the cash, the cash is withdrawn from a bank by a buyer at the beginning of the j-submarket, paid to a seller, and deposited back in the bank (or another bank) by the seller at the end of the j-submarket. Therefore, in the case where the banks intermediate payment activities, the cash can be paid at most J times during one day. In the j-submarket, the buyers choose the amount of withdrawal and the sellers choose whether or not to deposit their cash revenue in their banks. Anticipating their choices, the banks decide their cash reserves in the date-$(t-1)$ night market.

2.2 Optimization problem for banks

A bank chooses the deposits, D_t, the bank loans, L_t, and the cash reserve, C_t, in the date-$(t-1)$ night market. D_t earns the interest during the night and becomes $(1+i_d)D_t$ at the beginning of the date-t day market. A bank takes $\{\xi_t^{(j)}\}_{j=1}^{J}$ as given, where if the bank holds $D_t^{(j)}$ units of deposits at the beginning of the j-submarket, the amount of withdrawal demand for the bank in the j-submarket is $\xi_t^{(j)} D_t^{(j)}$ units of cash. The values of $\{\xi_t^{(j)}\}_{j=1}^J$ are determined as an equilibrium outcome by the actions of the buyers and the sellers. We assume that a bank has no investment opportunities in the day market. Therefore, the additional cash deposited in a bank in the j-submarket, $\Delta^{(j)}$, should be held by the bank as its reserve. The remaining deposits at the end of the day market is $D^{(J+1)}$, which earns interest and becomes $(1+i_n)D_t^{(J+1)}$ at the beginning of the date-t night market. Now we introduce an indicator of a banking crisis, $\tilde{1}$, which satisfies that $\tilde{1}=1$ if depositors (i.e., buyers and sellers) decide to deposit their cash income immediately in the banks (no bank runs) and $\tilde{1}=0$ if they decide not to deposit (bank runs).

Given i, i_d, i_n, $\{\xi_t^{(j)}\}_{j=1}^J$, and $\tilde{1}$, a bank solves the following optimization problem:

$$\max_{L_t, C_t, D_t, \Delta_t^{(1)}, \cdots, \Delta_t^{(J)}} [\tilde{R}_t + C_t^{(J+1)} - (1+i_n)D_t^{(J+1)}]_+ \tag{1}$$

subject to

$$L_t + C_t \leq D_t, \tag{2}$$

$$(1+i_d)D_t \leq \frac{1}{\rho}C_t, \tag{3}$$

$$C_t^{(1)} = C_t, \tag{4}$$

$$D_t^{(1)} = (1+i_d)D_t, \tag{5}$$

$$\tilde{C}^{(j)} = \max\{C^{(j)} - \xi^{(j)}D^{(j)}, 0\}, \tag{6}$$

$$C^{(j+1)} = \tilde{C}^{(j)} + \Delta_t^{(j)}\tilde{1}, \tag{7}$$

$$\tilde{D}^{(j)} = D^{(j)} - \min\{\xi^{(j)}D^{(j)}, C^{(j)}\}, \tag{8}$$

$$D^{(j+1)} = \tilde{D}^{(j)} + \Delta_t^{(j)}\tilde{1}, \tag{9}$$

$$D^{(j+1)} \leq \frac{1}{\rho}C_t^{(j+1)}, \qquad \text{if } \tilde{1} = 1, \tag{10}$$

$$\tilde{R}_t = (1+i)L_t, \tag{11}$$

where $[a]_+ = a$ if $a \geq 0$ and $[a]_+ = 0$ if $a < 0$. The nonnegativity of the bank's objective function is due to limited liability of the banks. As banks live for one period, they naturally enjoy limited liability. If bank assets are impaired, banks default on their deposit liabilities in the night market. Conditions (3) and (10) say that the banks are constrained by the reserve requirement, ρ, which is imposed by the government. We consider a symmetric equilibrium where $\xi^{(j)} = \xi$ for all $j = 1, 2, \cdots, J$, where ξ is determined later by (60) as a result of optimizations of buyers and sellers in equilibrium.[3] We assume that the government determines the reserve requirement such that all withdrawers can get paid the amount of cash they want to withdraw. Therefore, the government sets $\rho = \xi$. Note that the government guarantees that the demandability of the deposits, that is the depositors can withdraw at anytime on demand. The banks do not care whether or not they can keep the promise of demandablility of deposits, but they maximize their expected profits. (It would be easy to modify the basic model such that the banks' objective includes keeping the promise of demandability of deposits and/or that the banks endogenously determine the ratio of cash reserves. We show an alternative model of the banking sector in Appendix A, which follows the structure of banking operations à la Freixas and Rochet [2008, section 8.2.1].)

Since the banks expect that $\xi^{(j)} = \xi$ for all $j = 1, 2, \cdots, J$, and (10) must be satisfied, they set that $\Delta_t^{(j)} = \xi D_t^j$. Therefore, if $\tilde{1} = 1$ (no bank runs), $C^{(J+1)} = C^{(J)} = \cdots = C^{(1)}$, $D^{(J+1)} = D^{(J)} = \cdots = D^{(1)}$, and the bank's problem is reduced to

$$\max_{L_t, C_t, D_t} [(1+i)L_t + C_t - (1+i_n)(1+i_d)D_t]_+ \tag{12}$$

subject to

$$L_t + C_t \leq D_t, \tag{13}$$

$$(1+i_d)D_t \leq \frac{1}{\rho}C_t. \tag{14}$$

It is easily shown that both (13) and (14) bind in equilibrium. The reduced form of this problem is

$$\max_{C_t} \left[(1+i)\left\{\frac{1}{(1+i_d)\rho} - 1\right\} + 1 - \frac{1+i_n}{\rho}\right]_+ C_t. \tag{15}$$

If $\left[(1+i)\left\{\frac{1}{(1+i_d)\rho} - 1\right\} + 1 - \frac{1+i_n}{\rho}\right]_+ > 0$, the bank can obtain an infinite amount of profits by setting $C_t = +\infty$. Since C_t cannot be infinite in equilibrium, it must be the

[3] We show in Section 2.8 that a banking crisis never occurs in the basic model as long as $i_n > 0$. If a banking crisis occurs, it would be the case that $\xi^{(j)} = 1$ for all j and $\tilde{1} = 0$.

case that

$$(1 + i_d)(1 + i_n) = 1 + \{1 - (1 + i_d)\rho\}i, \tag{16}$$

and the profit for the banks is zero in equilibrium. Given the loan rate, i, any combination of i_d and i_n can be the deposit rate as long as (16) is satisfied.[4] We assume that the government (or the central bank) determines i_d and the money supply, M_{t+1}. As shown below in (54), the loan rate i is determined as $\gamma_{+1} = M_{t+1}/M_t$ is set. Thus, we assume in effect that i and i_d are given by the central bank, whereas i_n is determined in competition among banks by (16).

Credit creation: Banks expand their balance sheets by credit creation. In the date-$(t-1)$ night market, a bank lends cash to a borrower and the borrower deposits the cash immediately into the bank or another bank; the cash redeposited is immediately lent to another borrower and the new borrower also deposits the cash into some bank. This process continues in the date-$(t-1)$ night market and the bank balance sheets expand until the reserve requirement binds.

Instability due to credit creation: Since $\rho = \xi \ll 1$, credit creation causes the cash reserve to be less than deposits outstanding: $C = \rho(1 + i_d)D \ll (1 + i_d)D$. Therefore, if depositors want to withdraw all their deposits and hold cash during the day market, only a small fraction ρ of depositors can withdraw cash. This is a bank run. The unequal distribution of cash among depositors resulting from the bank run disrupts the transactions of the intermediate goods and causes a huge loss of output, which reduces the social welfare.

Broader interpretation of the model: The banking system in this model may be interpreted as a simplified model of the financial system in reality which consists of broader financial intermediaries, such as commercial banks, insurance companies, investment banks and hedge funds, which raise their funds by issuing short-term debt to finance

[4]Competition among banks implies that i_d and i_n do not vary among banks; otherwise depositors change their banks in the middle of the day market.

illiquid projects. For this interpretation we have just to rephrase "banks" in our model as "investment banks" or "hedge funds" (or any other financial intermediaries); "deposits" as "short-term debt liabilities" of those financial firms; "cash" as "liquid assets" in general, such as cash, government bonds, and government-guaranteed bank deposits (from the depositors' point of view); and "bank loans" as "investments" in general, which are illiquid and risky. In this line of interpretation, the banking crisis (or bank runs) in this model may be regarded as a model of "flight to quality" that was observed during the 2007-2008 financial turmoil.

2.3 Social Welfare

Social welfare, W_t, is measured by the sum of the welfare of buyers and sellers. Note that a buyer is endowed with one unit of the machine, $k = 1$. In a steady state equilibrium, the social welfare is written as

$$(1-\beta)W = (1-n)Aq^{\theta} - nc\left(\frac{1-n}{n}q\right) + U(x) - x. \tag{17}$$

The first-best allocation is determined by

$$U'(x^*) = 1, \tag{18}$$

$$\theta A(q^*)^{\theta-1} = c'\left(\frac{1-n}{n}q^*\right), \tag{19}$$

where x^* is the first-best consumption and q^* is the first-best amount of the intermediate goods per a buyer.

2.4 Sequence of decisions

The sequence of decisions during a representative date t is as follows. Sellers enter the day market carrying cash, m^{ds}, and bank deposits, d^{ds}, as their assets, and bank loans, l^s, as their liabilities. Buyers enter the day market carrying cash, m^{db}, and bank deposits, d^{db}, as their assets, and bank loans, l^b, as their liabilities. Bank deposits earn interest so that sellers have the right to withdraw $(1+i_d)d^{ds}$ and buyers have the right to withdraw $(1+i_d)d^{db}$ at any time during the day market. In the day market, sellers and buyers are allocated to J submarkets randomly. In the j-submarket, a seller produces q^s units of the intermediate goods and sells them to the buyers. Then, she chooses cash, m^{ns}, and a bank deposit, d^{ns}, that she carries to the night market. A buyer buys q^b units of the intermediate goods and chooses cash, m^{nb}, and a bank deposit, d^{nb}, to carry to the night market. In the night market, bank deposits grow to $(1+i_n)d^{ns}$ for a seller and $(1+i_n)d^{nb}$ for a buyer, respectively. Bank loans grow to $(1+i)l^s$ for a seller and $(1+i)l^b$ for a buyer, respectively. Production, trade, and consumption of the consumption goods take place. Bank loans are repaid and bank deposits are paid out. All agents receive lump-sum transfers $(\gamma_t - 1)M_t$ and choose cash and bank deposits that they carry over to date $(t+1)$. In what follows, we look at a representative date t and explain backwards from the night market to the day market.

2.5 The night market

A seller solves the following program:

$$W^s(m^{ns}, d^{ns}, l^s) = \max_{x,h,m_{+1},d_{+1},l_{+1}} [U(x) - h + \beta V^s_{+1}(m^{ds}_{+1}, d^{ds}_{+1}, l^s_{+1})] \tag{20}$$

subject to

$$x + \phi(m^{ds}_{+1} + d^{ds}_{+1} - l^s_{+1}) = h + \phi\{m^{ns} + (1+i_n)d^{ns} - (1+i)l^s + (\gamma_t - 1)M_t\}, \tag{21}$$

where ϕ is the real value of cash in the units of the consumption goods. This program can be rewritten as

$$\begin{aligned} W^s(m^{ns}, d^{ns}, l^s) =& \phi\{m^{ns} + (1+i_n)d^{ns} - (1+i)l^s + (\gamma_t - 1)M_t\} \\ &+ \max_{x,m_{+1},d_{+1},l_{+1}} [U(x) - x - \phi(m^{ds}_{+1} + d^{ds}_{+1} - l^s_{+1}) + \beta V^s_{+1}(m^{ds}_{+1}, d^{ds}_{+1}, l^s_{+1})] \end{aligned}$$

The first-order conditions (FOCs) are $U'(x) = 1$ and

$$\phi \geq \beta V^s_m(+1), \qquad \text{where if } >, \text{ then } m^{ds}_{+1} = 0; \text{ if } =, \text{ then } m^{ds}_{+1} \geq 0; \tag{22}$$

$$\phi \geq \beta V^s_d(+1), \qquad \text{where if } >, \text{ then } d^{ds}_{+1} = 0; \text{ if } =, \text{ then } d^{ds}_{+1} \geq 0; \tag{23}$$

$$\phi \leq -\beta V^s_l(+1), \qquad \text{where if } <, \text{ then } l^s_{+1} = 0; \text{ if } =, \text{ then } l^s_{+1} \geq 0, \tag{24}$$

where $V_x^s(+1) \equiv \frac{\partial}{\partial x} V^s(m_{+1}^{ds}, d_{+1}^{ds}, l_{+1}^s)$ for $x = m_{+1}^{ds}, d_{+1}^{ds}, l_{+1}^s$. The envelope conditions imply that W^s can be written as

$$W^s(m^{ns}, d^{ns}, l^s) = \phi\{m^{ns} + (1+i_n)d^{ns} - (1+i)l^s\} + \overline{W}_t^s, \tag{25}$$

where $\overline{W}_t^s$ is independent from the state variables.

A buyer solves the following program:

$$W^b(q^b, m^{nb}, d^{nb}, l^b) = \max_{x,h,m_{+1},d_{+1},l_{+1}} [U(x) - h + \beta V_{+1}^b(m_{+1}^{db}, d_{+1}^{db}, l_{+1}^b)] \tag{26}$$

subject to

$$x + \phi(m_{+1}^{db} + d_{+1}^{db} - l_{+1}^b) = h + \phi\{ak + wq^b + m^{nb} + (1+i_n)d^{nb} - (1+i)l^b + (\gamma_t - 1)M_t\}, \tag{27}$$

where k is the number of the machines, q is the quantity of the intermediate goods, and a and w are the market prices of k and q, respectively. This program can be rewritten as

$$\begin{aligned} W^b(m^{nb}, d^{nb}, l^b) =& \phi\{ak + wq^b + m^{nb} + (1+i_n)d^{nb} - (1+i)l^b + (\gamma_t - 1)M_t\} \\ &+ \max_{x,m_{+1},d_{+1},l_{+1}} [U(x) - x - \phi(m_{+1}^{db} + d_{+1}^{db} - l_{+1}^b) + \beta V_{+1}^b(m_{+1}^{db}, d_{+1}^{db}, l_{+1}^b)] \end{aligned}$$

The first-order conditions are $U'(x) = 1$ and

$$\phi \geq \beta V_m^b(+1), \qquad \text{where if } >, \text{ then } m_{+1}^{db} = 0; \text{ if } =, \text{ then } m_{+1}^{db} \geq 0; \tag{28}$$

$$\phi \geq \beta V_d^b(+1), \qquad \text{where if } >, \text{ then } d_{+1}^{db} = 0; \text{ if } =, \text{ then } d_{+1}^{db} \geq 0; \tag{29}$$

$$\phi \leq -\beta V_l^b(+1), \qquad \text{where if } <, \text{ then } l_{+1}^{b} = 0; \text{ if } =, \text{ then } l_{+1}^{b} \geq 0. \tag{30}$$

The envelope conditions imply that W^b can be written as

$$W^b(q^b, m^{nb}, d^{nb}, l^b) = \phi\{ak + wq^b + m^{nb} + (1+i_n)d^{nb} - (1+i)l^b\} + \overline{W}_t^b, \tag{31}$$

where $\overline{W}_t^b$ is independent from the state variables. The buyers produce the consumption goods competitively from k and q with the Cobb-Douglas technology, $y = Ak^{1-\theta}q^\theta$.

Since k and q are competitively traded in the night market, the prices are determined in equilibrium by

$$\phi a = (1-\theta)A(q^b)^{\theta}, \tag{32}$$

$$\phi w = \theta A(q^b)^{\theta-1}, \tag{33}$$

since $k = 1$ and $q = q^b$ per buyer.

2.6 The day market

A seller solves the following program:

$$V^s(m^{ds}, d^{ds}, l^s) = \max_{q^s, m^{ns}, d^{ns}} -c(q^s) + W^s(m^{ns}, d^{ns}, l^s) \tag{34}$$

subject to

$$m^{ns} + d^{ns} = pq^s + m^{ds} + (1+i_d)d^{ds}, \tag{35}$$

$$m^{ns} \geq 0, \text{ and } d^{ns} \geq 0. \tag{36}$$

Equation (25) implies that this program can be rewritten as

$$V^s(m^{ds}, d^{ds}, l^s) = \max_{q^s, d^{ns}} \phi p q^s - c(q^s) + \phi\{m^{ds} + (1+i_d)d^{ds} + i_n d^{ns} - (1+i)l^s\} + \overline{W}_t^s$$

subject to $d^{ns} \leq pq^s + m^{ds} + (1+i_d)d^{ds}$. Given $i_n > 0$, the FOCs imply

$$\phi p = \frac{c'(q^s)}{1+i_n}, \tag{37}$$

$$d^{ns} = pq^s + m^{ds} + (1+i_d)d^{ds}, \tag{38}$$

$$m^{ns} = 0. \tag{39}$$

Therefore, sellers deposit all cash including the cash they receive from the buyers, pq^s, into their banks immediately. The envelope conditions are $V_m^s = \phi(1+i_n)$, $V_d^s = \phi(1+i_d)(1+i_n)$, and $V_l^s = -\phi(1+i)$. These conditions and the FOCs for the night market imply that

$$\phi \geq \beta\phi_{+1}(1+i_{n,+1}), \qquad \text{where if } >, \text{ then } m^{ds}_{+1}=0; \text{ if } =, \text{ then } m^{ds}_{+1} \geq 0; \tag{40}$$

$$\phi \geq \beta\phi_{+1}(1+i_{d,+1})(1+i_{n,+1}), \qquad \text{where if } >, \text{ then } d^{ds}_{+1}=0; \text{ if } =, \text{ then } d^{ds}_{+1} \geq 0; \tag{41}$$

$$\phi \leq \beta\phi_{+1}(1+i_{+1}), \qquad \text{where if } <, \text{ then } l^{s}_{+1}=0; \text{ if } =, \text{ then } l^{s}_{+1} \geq 0. \tag{42}$$

A buyer solves the following program:

$$V^b(m^{db}, d^{db}, l^b) = \max_{q^b, m^{nb}, d^{nb}} W^b(q^b, m^{nb}, d^{nb}, l^b) \tag{43}$$

subject to

$$m^{nb} + d^{nb} + pq^b = m^{db} + (1+i_d)d^{db}, \tag{44}$$

$$m^{nb} \geq 0, \text{ and } d^{nb} \geq 0. \tag{45}$$

Equation (31) implies that this program can be rewritten as

$$V^b(m^{db}, d^{db}, l^b) = \max_{q^b, m^{nb}, d^{nb}} \phi\{ak_t + wq^b + m^{nb} + (1+i_n)d^{nb} - (1+i)l^b\} + \overline{W}^b_t,$$

subject to $m^{nb} + d^{nb} + pq^b = m^{db} + (1+i_d)d^{db}$. In the case when $i_n > 0$, the following holds obviously:

$$d^{nb} = m^{db} + (1+i_d)d^{db} - pq^b, \tag{46}$$

$$m^{nb} = 0. \tag{47}$$

The reduced form of the buyer's program is

$$V^b(m^{db}, d^{db}, l^b) = \max_{q^b} \phi\{ak + wq^b - (1+i_n)pq^b + (1+i_n)m^{db} + (1+i_d)(1+i_n)d^{db} - (1+i)l^b\} + \overline{W}^b_t, \tag{48}$$

subject to

$$pq^b \leq m^{db} + (1+i_d)d^{db}. \tag{49}$$

The FOC is

$$(1+i_n+\lambda)p \geq w, \qquad \text{where if } >, \text{ then } q^b = 0; \text{ if } =, \text{ then } q^b \geq 0, \tag{50}$$

and the envelope conditions are $V_m^b = \phi(1+i_n+\lambda)$, $V_d^b = \phi(1+i_d)(1+i_n+\lambda)$, $V_l^b = -\phi(1+i)$, where λ is the Lagrange multiplier for (49). These conditions and the FOCs for the night market imply

$$\phi \geq \beta\phi_{+1}(1+i_{n,+1}+\lambda_{+1}), \qquad \text{where if } >, \text{ then } m_{+1}^{db} = 0; \text{ if } =, \text{ then } m_{+1}^{db} \geq 0; \tag{51}$$

$$\phi \geq \beta\phi_{+1}(1+i_{d,+1})(1+i_{n,+1}+\lambda_{+1}), \quad \text{where if } >, \text{ then } d_{+1}^{db} = 0; \text{ if } =, \text{ then } d_{+1}^{db} \geq 0; \tag{52}$$

$$\phi \leq \beta\phi_{+1}(1+i_{+1}), \qquad \text{where if } <, \text{ then } l_{+1}^{b} = 0; \text{ if } =, \text{ then } l_{+1}^{b} \geq 0. \tag{53}$$

2.7 Equilibrium

Conditions (42) and (53) imply that $\frac{\gamma_{+1}}{\beta} \leq 1+i_{+1}$, where $\gamma_{+1} = \phi/\phi_{+1}$ is the gross inflation rate. The inflation rate, γ_{+1}, is determined by the central bank. If the banks set the loan rate i_{+1} such that $\gamma_{+1}/\beta < 1+i_{+1}$, then $l^s = l^b = 0$ and the balance sheet identity of the banks imply that $D_t = C_t$. Thus assuming that the banks prefer making loans, the loan rate is determined by

$$\frac{\gamma_{+1}}{\beta} = 1+i_{+1}. \tag{54}$$

Since (16) implies that $1+i_n < 1+i$ and $(1+i_n)(1+i_d) < 1+i$, conditions (40), (41), and (54) imply that

$$m_{+1}^{ds} = 0, \text{ and } d_{+1}^{ds} = 0. \tag{55}$$

In the equilibrium where the banks are operative, it must be the case that $d^{db} > 0$. Therefore, (52) must hold with equality, implying that

$$\lambda_{+1} = \rho i_{+1} > 0, \tag{56}$$

while (51) implies that $m^{db} = 0$, if $i_{d,+1} > 0$. In this paper, we focus on the equilibrium allocation for the case where $i_d > 0$. So, $m^{db} = 0$ in the equilibrium. Since $i > 0$ when $i_d > 0$, it is the case that $\lambda_{+1} > 0$, and the liquidity constraint (49) binds. Therefore,

$$(1 + i_d)d^{db} = pq^b. \tag{57}$$

Therefore, $m^{nb} = d^{nb} = 0$, $d^{ns} = pq^s$, and $m^{ns} = 0$, where $q^s = (1 - n)q^b/n$. Conditions (33), (37), and (50) imply the following condition which determines the value of q^b:

$$\frac{\theta A(q^b)^{\theta-1}}{1+i} = \frac{c'(q^s)}{(1+i_d)(1+i_n)}. \tag{58}$$

Conditions (33), (50), and (57) imply that ϕd^{db} is determined by

$$\phi d^{db} = \frac{\theta A(q^b)^{\theta}}{1+i}. \tag{59}$$

Since $d^{ds} = 0$, the total amount of bank deposits is determined by $\phi D_t = (1 - n)\phi d^{db}$. Since $m^{ds} = m^{db} = 0$, all cash is held by banks as cash reserves: $C_t = M_t$. Condition (14) implies that the real value of cash, $z \equiv \phi M_t = \phi C_t$ is determined by $z = (1 + i_d)(1 - n)\rho\phi d^{db}$. Condition (13) implies that the total amount of loans is determined by $\phi L_t = \phi n l^s + \phi(1 - n)l^b = (1 - n)(1 - \rho)\phi d^{db}$, while ϕl^s and ϕl^b are indeterminate in this basic model. Finally, the banks' expectations on the withdrawals and redeposits in the j-submarket are determined as follows. In j-submarket, the buyers withdraw $\frac{1}{J}(1 + i_d)(1 - n)\phi d^{db} = \frac{1}{J}(1 + i_d)\phi D_t$, which equals $\xi(1 + i_d)\phi D_t$. The exact amount withdrawn is redeposited in the banks by the sellers in the same submarket. Therefore, in equilibrium,

$$\xi = \frac{1}{J}. \tag{60}$$

As we assumed that the government sets $\rho = \xi$, it is the case that $\rho = 1/J$.

Social welfare with and without banks: It is easily confirmed that the social welfare of the economy with the banking sector is identical to that of the cash economy without banks. This is because bank deposits serve as only a substitute for cash and provide no additional value to the economy. Thus our simplistic model does not offer a new theory for the raison d'être of bank deposits, such as the liquidity insurance proposed by Diamond and Dybvig (1983) or the optimal contractual design proposed by Diamond and Rajan (2001) that disciplines bankers.

2.8 Possibility of a bank run due to a herd behavior

In the basic model in this section, the banks never become insolvent because they can completely enforce loan repayment.

The case where $i_n > 0$ and $i_d > 0$: In this case, there are no bank runs. In a bank run, all buyers would want to withdraw deposits and all sellers would want to hold cash (No sellers would want to deposit their cash in the banks). For a seller, to hold the cash she received from buyers (and not to deposit the cash in the banks) is never optimal behavior. This is because the banks will surely repay the deposits in the night market with strictly positive interest $i_n > 0$, since the bank insolvency never occurs. Therefore, for a seller, depositing her revenue in a bank is strictly preferable to holding the revenue in the form of cash, no matter what the other agents do. Therefore, a bank run never occurs in this basic model with loan enforcement.

The case where $i_n = 0$ and $i_d > 0$: In this case, there is a possibility of bank runs due to herd behavior. It may be the case that after the goods trade in the day market, all sellers decide to hold cash rather than deposits. It is herd behavior but there is no strategic complementarity between the decisions of one seller and the other sellers. A seller, who chooses to hold bank deposits rather than cash, when all the other sellers choose to hold cash, is not worse-off compared to another seller who chooses to hold cash because both agents get exactly the same return in the night market. The nonexistence of strategic complementarity between the sellers' actions is a big difference of our basic model from the Diamond-Dybvig model. Since $i > 0$ and $i_d > 0$, $(1 + i_d)D_t = M_t/\rho = JM_t$. Withdrawal is J times larger than cash reserve. If all sellers decide to hold cash, buyers only in the 1-submarket successfully withdraw the full amount of their deposits. The other buyers can withdraw no cash. If all sellers decide to hold cash, transactions of the intermediate goods occur only in the 1-submarket, while the other submarkets are virtually shut down. The equilibrium output is basically the same as in the case of the banking crisis in the models of Sections 3 and 4.

In the basic model, the central bank can prevent the occurrence of bank runs by setting $i_n > 0$. There is no need of any other government intervention in our basic model where we assume complete loan enforcement and no bank insolvency shock. Incidentally, if $i = i_d = i_n = 0$, that is, if the Friedman rule is implemented, the first-best allocation can be attained in the basic model.

3 Bank Insolvency Shock

In this section we modify the basic model such that a macroeconomic shock hits the economy in the night market with a small probability and the shock renders all banks insolvent. In this section, we retain the assumption of complete loan enforcement, while we discard it in the next section. In this section, we assume that the macroeconomic shock destroys a portion of the bank assets after all bank loans are collected successfully. Note that in the text of this paper we only analyze a banking crisis due to a macroeconomic shock in which all banks are subject to runs. To modify our model for the case of individual bank runs, in which some banks are subject to runs and others are not, it is not difficult and changes our results only slightly. We will modify our model and analyze individual bank runs in Appendix B.

Macroeconomic shock: We assume that at the beginning of the date-t day market the value of a random variable $\tilde{\omega}$ is revealed:

$$\tilde{\omega} = \begin{cases} 1 & \text{with probability } 1 - \delta, \\ \omega\ (< 1) & \text{with probability } \delta. \end{cases} \tag{61}$$

The variable $\tilde{\omega}$ indicates the condition of the bank assets in the date-t night market. After the banks successfully collect loans, $(1 + i)L_t$, the banks are hit by the macroeconomic shock, $\tilde{\omega}$, and the bank assets, which are the consumption goods, are partially destroyed and become $\tilde{R}_t = \tilde{\omega}(1+i)L_t$. We will show that when $\tilde{\omega} = \omega < 1$, a banking crisis occurs. (Therefore, the probability of occurrence of a banking crisis is δ.)

Given α, which is the proportion of sellers who do not redeposit cash in the banks during a banking crisis, the banks solve

$$\max_{L_t, C_t, D_t} (1-\delta)[(1+i)L_t + C_t - (1+i_n)(1+i_d)D_t]_+ \\ + \delta[\omega(1+i)L_t + (1+\alpha i_n)C_t - (1+i_n)(1+i_d)D_t]_+ \tag{62}$$

subject to (13) and (14). Since the profit of a bank is no less than $(1-\delta)[(1+i)L_t + C_t - (1+i_n)(1+i_d)D_t]_+$, the requirement that the bank profit must be finite implies that (16) must hold in equilibrium. Since all sellers redeposit cash in the banks if and only if $\omega(1+i)L_t + C_t \geq (1+i_d)D_t$, it is easily shown that $\alpha = 0$ if $\overline{\omega} \leq \omega < 1$ and $\alpha = 1$ if $\omega < \overline{\omega}$, where $\overline{\omega} = (1-\rho)/(1-\rho+i_n)$. Since (16) implies that $i_n \leq (1-\rho)i$, it is the case that $\alpha = 1$ no matter what the value of i_n, when

$$\omega < \frac{1}{1+i}. \tag{63}$$

In what follows we assume that ω and i satisfy condition (63). In this case, when $\tilde{\omega} = \omega$, depositors in the date-t day market try to withdraw all their deposits and nobody redeposits cash in the banks. Thus we say that a bank run occurs when $\tilde{\omega} = \omega$ with probability δ.

Stochastic environment: Since $\tilde{\omega}$ is revealed at the beginning of the date-t day market, the sellers and the buyers in the date-$(t-1)$ night market must decide the amounts of cash, bank deposits, and bank loans that they carry over to date t without knowing the value of $\tilde{\omega}$. We define the following random variables: $\tilde{\Gamma}$ and $\tilde{\Lambda}$. $\tilde{\Gamma}$ is the probability that a depositor can successfully withdraw the full amount of her bank deposit in the date-t day market, which is common to all agents, i.e., sellers and buyers. $\tilde{\Lambda}$ is the proportion of demand deposits that are actually paid to a depositor in the date-t night market, that is, if a depositor holds $(1+i_n)(1+i_d)d_t$ units of deposit in her bank account, the amount she can ultimately obtain from her bank in the night market is

$(1+i_n)(1+i_d)\tilde{\Lambda}d_t$.

$$\tilde{\Gamma} = \begin{cases} 1 & \text{if } \tilde{\omega} = 1, \\ \Gamma\ (<1) & \text{if } \tilde{\omega} = \omega, \end{cases} \tag{64}$$

$$\tilde{\Lambda} = \begin{cases} 1 & \text{if } \tilde{\omega} = 1, \\ \Lambda\ (<1) & \text{if } \tilde{\omega} = \omega. \end{cases} \tag{65}$$

The values of Γ and Λ are determined as an equilibrium outcome. In what follows, we explain the decision problems of sellers and buyers backwards from the night market to the day market.

3.1 The night market

Note that $\tilde{\omega}$ and $\tilde{\Lambda}$ are already revealed in the night market. A seller solves the following program:

$$W^s(m^{ns}, d^{ns}, l^s; \tilde{\Lambda}) = \max_{x,h,m_{+1},d_{+1},l_{+1}} [U(x) - h + \beta V^s_{+1}(m^{ds}_{+1}, d^{ds}_{+1}, l^s_{+1})] \tag{66}$$

subject to

$$x + \phi(m^{ds}_{+1} + d^{ds}_{+1} - l^s_{+1}) = h + \phi\{m^{ns} + (1+i_n)\tilde{\Lambda}d^{ns} - (1+i)l^s + (\gamma_t - 1)M_t\}. \tag{67}$$

The FOCs are the same as those in the basic model. The envelope conditions imply that W^s can be written as

$$W^s(m^{ns}, d^{ns}, l^s; \tilde{\Lambda}) = \phi\{m^{ns} + (1+i_n)\tilde{\Lambda}d^{ns} - (1+i)l^s\} + \overline{W}^s_t. \tag{68}$$

A buyer solves the following program:

$$W^b(q^b, m^{nb}, d^{nb}, l^b; \tilde{\Lambda}) = \max_{x,h,m_{+1},d_{+1},l_{+1}} [U(x) - h + \beta V^b_{+1}(m^{db}_{+1}, d^{db}_{+1}, l^b_{+1})] \tag{69}$$

subject to

$$x + \phi(m^{db}_{+1} + d^{db}_{+1} - l^b_{+1}) = h + \phi\{\tilde{a}k + \tilde{w}q^b + m^{nb} + (1+i_n)\tilde{\Lambda}d^{nb} - (1+i)l^b + (\gamma_t - 1)M_t\}. \tag{70}$$

The FOCs are the same as those in the basic model. The envelope conditions imply that W^b can be written as

$$W^b(q^b, m^{nb}, d^{nb}, l^b; \tilde{\Lambda}) = \phi\{\tilde{a}k + \tilde{w}q^b + m^{nb} + (1 + i_n)\tilde{\Lambda}d^{nb} - (1 + i)l^b\} + \overline{W}_t^b. \tag{71}$$

The prices $\tilde{a}$ and $\tilde{w}$ are determined by (32) and (33), while q^b in these equations, which is the average amount of the intermediate goods per buyer, depends on the state of the economy, $\tilde{\omega}$.

3.2 The day market

The seller's value function $V^s(m^{ds}, d^{ds}, l^s)$ is determined by

$$V^s(m^{ds}, d^{ds}, l^s) = \sum_{i=n,s,f} \delta_i \max_{q_i^s, m_i^{ns}, d_i^{ns}} \{-c(q_i^s) + W^s(m_i^{ns}, d_i^{ns}, l^s; \Lambda_i)\}, \tag{72}$$

where i indicates the state of the seller: n is the case of no bank run, in which $\delta_n = 1 - \delta$ and $\tilde{\omega} = 1$; s is the case where the seller succeeds to withdraw during a bank run, in which $\delta_s = \delta\Gamma$ and $\tilde{\omega} = \omega$; and f is the case where the seller fails to withdraw during a bank run, in which $\delta_f = \delta(1 - \Gamma)$ and $\tilde{\omega} = \omega$.

If $i = n$, then $\tilde{\omega} = 1$ and $\Lambda_n = 1$. A seller solves the following program:

$$\max_{q_n^s, m_n^{ns}, d_n^{ns}} -c(q_n^s) + W^s(m_n^{ns}, d_n^{ns}, l^s; 1) \tag{73}$$

subject to

$$m_n^{ns} + d_n^{ns} = p_n q_n^s + m^{ds} + (1 + i_d)d^{ds}, \tag{74}$$

$$m_n^{ns} \geq 0, \text{ and } d_n^{ns} \geq 0, \tag{75}$$

where p_n is the price of the intermediate goods when $i = n$. Equation (68) implies that this program can be rewritten as

$$\max_{q^s, d^{ns}} \phi p q^s - c(q^s) + \phi\{m^{ds} + (1 + i_d)d^{ds} + i_n d^{ns} - (1 + i)l^s\} + \overline{W}_t^s$$

subject to $d^{ns} \leq pq^s + m^{ds} + (1+i_d)d^{ds}$. Given $i_n > 0$, the FOCs imply

$$\phi p_n = \frac{c'(q_n^s)}{1+i_n}, \tag{76}$$

$$d_n^{ns} = pq^s + m^{ds} + (1+i_d)d^{ds}, \tag{77}$$

$$m_n^{ns} = 0. \tag{78}$$

Therefore, sellers deposit all cash including the cash they receive from the buyers, pq^s, in their banks immediately.

If $i = s$, then $\tilde{\omega} = \omega$ and $\Lambda_s = \Lambda < 1$. A seller solves the following program:

$$\max_{q_s^s, m_s^{ns}, d_s^{ns}} -c(q_s^s) + W^s(m_s^{ns}, d_s^{ns}, l^s; \Lambda) \tag{79}$$

subject to

$$m_s^{ns} + d_s^{ns} = p_\omega q_s^s + m^{ds} + (1+i_d)d^{ds}, \tag{80}$$

$$m_s^{ns} \geq 0, \text{ and } d_s^{ns} \geq 0, \tag{81}$$

where p_ω is the price when $\tilde{\omega} = \omega$. Equation (68) implies that this program can be rewritten as

$$\max_{q^s, d^{ns}} \phi pq^s - c(q^s) + \phi\{m^{ds} + (1+i_d)d^{ds} + \{(1+i_n)\Lambda - 1\}d^{ns} - (1+i)l^s\} + \overline{W}_t^s$$

subject to $d^{ns} \leq pq^s + m^{ds} + (1+i_d)d^{ds}$. If a proportion α of sellers decide not to redeposit their cash revenue, $\Lambda_t = \{(1+i)\omega L + C - \alpha C\}/\{(1+i_n)(1+i_d)D - (1+i_n)\alpha C\} \leq \{(1+i)\omega L + C\}/\{(1+i_n)(1+i_d)D\}$. Since we assumed that $(1+i)\omega L + C < (1+i_d)D$, it is the case that $(1+i_n)\Lambda < 1$. Therefore, the FOCs imply

$$\phi p_\omega = c'(q_s^s), \tag{82}$$

$$d_s^{ns} = 0, \tag{83}$$

$$m_n^{ns} = p_\omega q_s^s + m^{ds} + (1+i_d)d^{ds}. \tag{84}$$

If $i = f$, a seller solves the following program:

$$\max_{q_f^s, m_f^{ns}, d_f^{ns}} -c(q_f^s) + W^s(m_f^{ns}, d_f^{ns}, l^s; \Lambda) \tag{85}$$

subject to

$$m_f^{ns} \leq p_\omega q_f^s + m^{ds}, \tag{86}$$

$$d_f^{ns} = (1+i_d)d^{ds} + p_\omega q_f^s + m^{ds} - m_f^{ns}, \tag{87}$$

$$m_s^{ns} \geq 0, \text{ and } d_s^{ns} \geq 0. \tag{88}$$

The solution is

$$\phi p_\omega = c'(q_f^s), \tag{89}$$

$$m_f^{ns} = p_\omega q_f^s + m^{ds}, \tag{90}$$

$$d_f^{ns} = (1+i_d)d^{ds}, \tag{91}$$

The envelope conditions are $V_m^s = \phi\{1+(1-\delta)i_n\}$, $V_d^s = \phi(1+i_d)[(1+i_n)\{1-\delta+(1-\Gamma)\delta\Lambda\}+\delta\Gamma]$, and $V_l^s = -\phi(1+i)$. These conditions and the FOCs for the night market imply that

$$\phi \geq \beta\phi_{+1}\{1+(1-\delta)i_{n,+1}\}, \qquad \text{where if } >, \text{ then } m_{+1}^{ds} = 0; \quad \text{if } =, \text{ then } m_{+1}^{ds} \geq 0; \tag{92}$$

$$\begin{aligned} &\phi \geq \beta\phi_{+1}(1+i_{d,+1})[(1+i_{n,+1})\{1-\delta+(1-\Gamma)\delta\Lambda\}+\delta\Gamma], \\ &\qquad \text{where if } >, \text{ then } d_{+1}^{ds} = 0; \quad \text{if } =, \text{ then } d_{+1}^{ds} \geq 0, \end{aligned} \tag{93}$$

and (42).

A buyer solves the following program:

$$V^b(m^{db}, d^{db}, l^b) = \sum_{i=n,s,f} \max_{q_i^b, m_i^{nb}, d_i^{nb}} \delta_i W^b(q_i^b, m_i^{nb}, d_i^{nb}, l^b; \Lambda_i), \tag{94}$$

subject to budget and liquidity constraints for the respective states, where i indicates the state of the buyer: n is the case of no bank run, in which $\delta_n = 1-\delta$ and $\Lambda_n = 1$; s is the case where the buyer succeeds to withdraw during a bank run, in which $\delta_s = \delta\Gamma_t$ and $\Lambda_s = \Lambda < 1$; and f is the case where the buyer fails to withdraw during a bank run, in which $\delta_f = \delta(1-\Gamma_t)$.

If $i = n$, a buyer solves the following program:

$$\max_{q_n^b, m_n^{nb}, d_n^{nb}} W^b(m_n^{nb}, d_n^{nb}, l^b; 1) \tag{95}$$

subject to

$$m_n^{nb} + d_n^{nb} + p_n q_n^b \leq m^{db} + (1 + i_d) d^{db}, \tag{96}$$

$$m_n^{nb} \geq 0, \text{ and } d_n^{nb} \geq 0. \tag{97}$$

Equation (71) implies that

$$d_n^{nb} = m^{db} + (1 + i_d) d^{db} - p_n q_n^b, \tag{98}$$

$$m_n^{nb} = 0. \tag{99}$$

Therefore, buyers deposit all remaining cash in their banks immediately.

Similarly, if $i = s$, equation (71) and the fact that $(1 + i_n)\Lambda < 1$ imply that

$$m_s^{nb} = m^{db} + (1 + i_d) d^{db} - p_\omega q_s^b, \tag{100}$$

$$d_s^{nb} = 0, \tag{101}$$

and if $i = f$,

$$m_s^{nb} = m^{db} - p_\omega q_f^b, \tag{102}$$

$$d_s^{nb} = (1 + i_d) d^{db}. \tag{103}$$

Therefore, buyers never redeposit their remaining cash in the banks when the bank insolvency shock hits the economy.

Using these results and (71), the buyer's program is reduced to

$$\begin{aligned} V^b(m^{db}, d^{db}, l^b) = \max_{q_n^b, q_s^b, q_f^b} & (1 - \delta)\phi\{a_n k + w_n q_n - (1 + i_n) p_n q_n\} + \delta\Gamma\phi\{a_\omega k + w_\omega q_s - p_\omega q_s\} \\ & + \delta(1 - \Gamma)\phi\{a_\omega k + w_\omega q_f - p_\omega q_f\} + \{1 + (1 - \delta) i_n\}\phi m^{db} \\ & + [\{1 - \delta + (1 - \Gamma)\delta\Lambda\}(1 + i_n) + \delta\Gamma](1 + i_d)\phi d^{db} - (1 + i)\phi l^b + \overline{W}_t^b, \end{aligned} \tag{104}$$

subject to

$$p_n q_n^b \leq m^{db} + (1 + i_d) d^{db}, \tag{105}$$

$$p_\omega q_s^b \leq m^{db} + (1 + i_d) d^{db}, \tag{106}$$

$$p_\omega q_f^b \leq m^{db}, \tag{107}$$

where a_n and w_n are the prices when $\tilde{\omega} = 1$, and a_ω and w_ω are those when $\tilde{\omega} = \omega$. Denoting the Lagrange multipliers for (105), (106) and (107) by λ_n, λ_s and λ_f, respectively, the FOCs are

$$\lambda_n = (1-\delta)\left(\frac{w_n}{p_n} - 1 - i_n\right), \tag{108}$$

$$\lambda_s = \delta\Gamma\left(\frac{w_\omega}{p_\omega} - 1\right), \tag{109}$$

$$\lambda_f = \delta(1-\Gamma)\left(\frac{w_\omega}{p_\omega} - 1\right), \tag{110}$$

and the envelope conditions are $V_m^b = \phi\{1+(1-\delta)i_n + \lambda_n + \lambda_s + \lambda_f\}$, $V_d^b = \phi(1+i_d)[\{1-\delta+(1-\Gamma)\delta\Lambda\}(1+i_n) + \delta\Gamma + \lambda_n + \lambda_s]$, $V_l^b = -\phi(1+i)$. These conditions and the FOCs for the night market imply

$$\phi \geq \beta\phi_{+1}\{1+(1-\delta)i_{n,+1} + \lambda_{n,+1} + \lambda_{s,+1} + \lambda_{f,+1}\},$$
$$\text{where if } >, \text{ then } m_{+1}^{db} = 0; \quad \text{if } =, \text{ then } m_{+1}^{db} \geq 0; \tag{111}$$

$$\phi \geq \beta\phi_{+1}(1+i_{d,+1})[\{1-\delta+(1-\Gamma)\delta\Lambda\}(1+i_{n,+1}) + \delta\Gamma + \lambda_{n,+1} + \lambda_{s,+1}],$$
$$\text{where if } >, \text{ then } d_{+1}^{db} = 0; \quad \text{if } =, \text{ then } d_{+1}^{db} \geq 0, \tag{112}$$

and (53).

3.3 Equilibrium

We assume that δ is sufficiently small and satisfies

$$0 < \delta < \frac{\rho}{1-\rho} i_d. \tag{113}$$

We assume and justify later that under condition (113), (111) holds with strict inequality and (112) holds with equality.[5] With this assumption, $m^{db} = 0$ in equilibrium, which directly implies that $q_f^b = 0$, since $p_\omega q_f^b \leq m^{db}$. That is, when a banking crisis occurs,

[5]In general, either (111) or (112) should hold with strict inequality in equilibrium. This is because in equilibrium four conditions: (111), (112), (114), and (115) must be satisfied, while there are three unknowns: q_n^b, q_s^b, and ϕd^{db}. Only when δ takes on a particular value, both (111) and (112) are satisfied with equality.

only buyers who successfully withdraw their deposits can buy the intermediate goods. Therefore, total production of the intermediate goods in the case of a banking crisis is $\Gamma(1-n)q_s^b$. The competition in production of the consumption goods implies $\phi a_n = (1-\theta)A(q_n^b)^\theta$, $\phi w_n = \theta A(q_n^b)^{\theta-1}$, $\phi a_\omega = (1-\theta)A(\Gamma q_s^b)^\theta$, and $\phi w_\omega = \theta A(\Gamma q_s^b)^{\theta-1}$. The seller's optimization in the day market implies $\phi p_n = c'(\frac{1-n}{n}q_n^b)$ and $\phi p_\omega = c'(\frac{1-n}{n}\Gamma q_s^b)$. The liquidity constraints (105) and (106) can be rewritten as

$$c'\left(\frac{1-n}{n}q_n^b\right)q_n^b = (1+i_d)\phi d^{db}, \tag{114}$$

$$c'\left(\frac{1-n}{n}\Gamma q_s^b\right)q_s^b = (1+i_d)\phi d^{db}, \tag{115}$$

which determine q_n^b and q_s^b as functions of ϕd^{db}. (112) can be rewritten as

$$\frac{\gamma_{+1}}{\beta} = \delta(1-\Gamma)\Lambda(1+i_{n,+1})(1+i_{d,+1}) + \frac{(1-\delta)\theta A(q_{n,+1}^b)^\theta + \delta\theta A(\Gamma q_{s,+1}^b)^\theta}{\phi d_{+1}^{db}}. \tag{116}$$

The equilibrium value of Γ is determined by $\Gamma = C/\{(1+i_d)D\} = \rho$ from (14). In the steady state equilibrium, (42) or (53) hold with equiality. Thus the central bank determines i by setting $\gamma = M_{t+1}/M_t = \phi/\phi_{+1}$, since $\gamma/\beta = 1+i$. As we assumed the central bank determines i_d, i_n is also given by (16). Given these interest rates, conditions (114), (115), and (116) determine ϕd^{db}, q_n^b, and q_s^b. The total amount of bank deposits is $\phi D = (1-n)\phi d^{db}$. Since all cash is held by the banks, the real balance is $z = \phi M_t = (1-n)(1+i_d)\rho\phi d^{db}$. The total amount of bank loans is $\phi L = \{(1+i_n)\rho^{-1}-1\}z/(1+i)$ The value of Λ is determined by $\Lambda = (1+i)\omega L/(1+i_n)\{(1+i_d)D-C\} = (1-\rho+i_n)\omega/\{(1-\rho)(1+i_n)\}$. Condition (63) guarantees that $(1+i_n)\Lambda < 1$. Given the equilibrium values of these variables, we can prove that (113) is a sufficient condition for (111) holding with strict inequality when (112) holds with equality. The proof is given in Appendix C.

Real damage due to banking crisis: Since the bank insolvency shock ω is an exogenous shock, the loss of bank asset: $(1-\omega)(1+i)\phi L$ is an exogenous loss to the economy, which is unavoidable once ω is realized. There is additional damage due to a banking crisis. If a banking crisis occurs, only the proportion $\Gamma = \rho = 1/J$ of the total buyers can buy the intermediate goods and the total amount of the intermediate goods produced becomes $(1-n)\rho q_s^b$ instead of $(1-n)q_n^b$ in normal times. The liquidity constraints, (114) and (115), imply

$$c'\left(\frac{1-n}{n}q_n^b\right)q_n^b = c'\left(\frac{1-n}{n}\Gamma q_s^b\right)q_s^b. \tag{117}$$

For $\Gamma < 1$, it is easily shown that $\Gamma q_s^b < q_n^b$ (and $q_s^b > q_n^b$), when $c'(q)$ is strictly increasing in q. For example, if $c(q) = q^2$ and $\Gamma = \rho = 1/9$, (117) implies that $\frac{(1-n)\rho q_s^b}{(1-n)q_n^b} = 1/3$, that is, the total production of the intermediate goods during the banking crisis becomes one third of that in normal times. The subsequent gap in consumption goods production is $(1-n)A\{(q_n^b)^\theta - (\rho q_s^b)^\theta\}$. This is the real damage of a banking crisis. In the case where $\theta = 1/2$, $\rho = 1/9$, and $c(q) = q^2$, the real damage of a banking crisis is about 42% of the consumption goods produced in normal times. Note that the key factor that generates this real damage of a banking crisis is the sequential service (first come-first serve) constraint on depositors. If, as in Allen and Gale (1998), the banks can suspend the withdrawals and give an equal amount of cash to all depositors (i.e., buyers), the disruption in the goods market does not occur. This is because all buyers hold cash and therefore they can buy the normal amount of the intermediate goods under flexible prices.[6]

Deflation: Since $\Gamma q_s^b < q_n^b$ and $q_s^b > q_n^b$, it is easily shown from (117) that the price level in the banking crisis is lower than that in normal times: $\phi p_\omega = c'\left(\frac{1-n}{n}\Gamma q_s^b\right) < c'\left(\frac{1-n}{n}q_n^b\right) = \phi p_n$. This result indicates that a banking crisis may induce deflation of nominal prices. This is consistent with the historical episodes of banking crises, such as Japan's decade-long deflation and bank distress in the late 1990s and early 2000s. Boyd et al. (2001) show empirical evidence from data of banking crises all over the world that the inflation rate is lowered significantly during a banking crisis. Their finding also support our theoretical prediction that a banking crisis may lower the price level.

Friedman's Rule: Note that the real damage due to banking crisis is made even in the case where $i = i_d = i_n = 0$. This is because (117) implies that $\Gamma q_s^b < q_n^b$ as long

[6]I thank Franklin Allen for pointing out this feature of my model.

as $\Gamma < 1$. The production of intermediate goods in normal times, $(1-n)q_n^b$, is larger than the first-best level and that in the banking crisis, $(1-n)\Gamma q_s^b$, is smaller than the first-best level. Thus under the threat of banking crisis, Friedman's rule cannot attain the optimal allocation.

Lower loan demand during a banking crisis: It is often observed that the loan demand of firms decreases during a banking crisis. Even though the nominal interest rate had been set at almost zero for nearly a decade since 1995 in Japan, the banks had difficulty finding (good) borrowers. We can explain this phenomenon by a slightly modified version of our model. Suppose that there is a unit mass of merchants who live only for one period: A merchant is born at the beginning of the date-t day market and dies in the date-t night market. They maximize the profit by buying the intermediate goods at the price of p from the sellers and selling them at the price of p' to the buyers. We assume a technological constraint that sellers and buyers cannot trade directly and that sellers can sell only to merchants and buyers can buy only from merchants. The merchants are not endowed with cash or deposits, and they need to borrow cash, l^m, at the beginning of the day market. For simplicity, we assume that the merchants borrow cash from the central bank at the loan rate $\hat{i}$, so that the loan supply to the merchants is not affected by the bank runs. The problem for a merchant is $\max_{q,l^m} p'q - (1+\hat{i})l^m$ subject to $pq \leq l^m$. In equilibrium, $p = (1+\hat{i})p'$ and $l^m = pq$. The nominal amount of total loans to merchants is $L_\omega^m \equiv (1-n)p_\omega \rho q_s^b$ in a banking crisis and is $L_n^m \equiv (1-n)p_n q_n^b$ in normal times. Since the collateral constraint for the buyers implies that $(1+\hat{i})pq = (1+i_d)d^{db}$, it is easily shown that $L_\omega^m / L_n^m = (1+\hat{i}_n)\rho/(1+\hat{i}_\omega)$, where $\hat{i}_n$ and $\hat{i}_\omega$ are the short-term rates in normal times and in a banking crisis, respectively. If ρ is small, even when the central bank sets $\hat{i}_\omega = 0$, the loan demand decreases during a banking crisis, that is, $L_\omega^m < L_n^m$. This result can be interpreted as follows: When a banking crisis occurs, the buyers cannot buy the intermediate goods because they run short of cash and the production of the intermediate goods decreases; and the decrease in the demand of the intermediate goods reduces the borrowing by the merchants for their working capital in the day market, which is perceived by the central bank as an overall reduction of loan demand.[7]

3.4 Policy implications

There are several policies that are relevant to the banking crises.

Deposit insurance: Suppose that the government imposes a tax, which is proportional to the amount of deposits, on the banks in the date-$(t-1)$ night market and the tax revenue is kept as the deposit insurance fund. (We do not specify how the tax proceeds are kept or invested during the night market.) When a banking crisis occurs in the date-t day market, the government pays subsidies to the banks from the deposit insurance fund. This is a simplified model of deposit insurance. This policy is not effective to stop the bank runs as long as the negative shock is large, that is, ω is so small that the amount of subsidy necessary to restore the solvency of the banks exceeds the amount of the deposit insurance fund.[8] In addition, there should be a tax distortion due to the proportional tax on bank deposits.

Fiscal policy: When the buyers fail to buy the intermediate goods because of the shortage of cash during a banking crisis, the government itself may be able to buy the goods in the day market and sell them in the night market. This fiscal policy can support the production of the intermediate goods, but it cannot restore the solvency of the banks and cannot stop the bank runs. Thus the banking crisis continues and the banks run out

[7] I leave the construction of a full model with merchants for future research. It may be possible to make a model such that the merchants live for infinite periods and borrow from the private banks. The similar declines of the loan demand in the banking crisis would be shown if we assume that when the merchants buy the intermediate goods in the day market, they can buy them by credit with probability π and pay cash for the material goods with probability $1-\pi$. If π is large, it would be shown that the merchants do not carry cash or bank deposits to the day market from the previous night market.

[8] If the deposit insurance fund is subsidized sufficiently by the government, the bank runs may be effectively stopped. We classify, however, the injection of taxpayer money into the deposit insurance fund as one form of the bank reforms to restore the solvency of banks, which we discuss later.

of cash reserves. The government can issue cash to finance the fiscal policy in the day market, which can be redeemed in the night market by selling the intermediate goods to the buyers. No tax is necessary to implement this policy.[9] The real damage of a banking crisis is completely avoided by this policy, while almost all of the dead weight cost, $(1-\omega)\phi(1+i)L$, is born by the unlucky buyers who could not withdraw cash during the day market.

Suspension of convertibility: This policy may be interpreted in our model as prohibiting withdrawal in the day market. This policy effectively stops the runs of depositors, while no buyers can buy the intermediate goods, since they cannot have cash and the sellers only accept cash as a payment instrument. Thus $p_\omega = q_s^b = q^s = 0$ under the policy of suspension of convertibility. Therefore, this policy may amplify the real damage of the banking crisis.

Monetary policy or Lender of Last Resort (LLR) lending: Suppose that in the day market the central bank lends cash to the banks up to the value of the bank assets, $(1+i)\omega L$. This policy improve welfare by facilitating the trade of the intermediate goods, but only partially. This is because LLR lending is lending of cash up to $(1+i)\omega L$, while the cash demand in the day market is $(1+i_d)D - C$, where we assumed that $(1+i)\omega L < (1+i_d)D - C$ (see the arguments that derive [63]). Since the LLR lending does not restore the solvency of the banks, this policy cannot stop the bank runs and all banks ultimately run out of cash reserves. At the beginning of the night market, the banking sector has the remaining deposit liabilities of $(1-\omega)(1+i)L$ and the liability to the central bank, $\omega(1+i)L$, while it has the loan asset of $\omega(1+i)L$. (Lucky depositors withdraw $(1+i)\omega L + C$ in total during the day market.)

[9] Note that $p = w$. This is because the government is not subject to a liquidity constraint and the interest rate, i_n, is effectively zero during the bank runs, since nobody wants to deposit money in the banks.

Bank reforms to restore the solvency of banks: If the government guarantees that the bank deposits (or all bank liabilities in reality) are fully repaid in the night market, there are no bank runs and the intermediate goods are produced just as much as in normal times. Thus, the real damage of the banking crisis can be completely eliminated by this policy. But the government incurs the cost to guarantee the bank liability, which amounts to $(1-\omega)(1+i)\phi L$ plus the associated tax distortion if it is financed by distortionary taxes. This cost of the policy implementation may be much smaller than the real damage of the crisis, especially when ρ is a small number. So, government guarantee of bank liabilities may improve the social welfare significantly. There are several methods to implement this policy: for example, deposit insurance without an upper limit, the blanket guarantee of the bank liabilities, and taxpayer-funded capital injections to the banks. Whatever methods are taken, the government should restore the public's expectations in the solvency of the banks. This expectation would be formed only if the true values of the bank assets are publicly known and the gap between the banks' liabilities and their assets is explicitly guaranteed or made up by either government policy or capital augmentation in the market. Note that the counterpart, in reality, of the bank deposit in our model may be not only the bank deposits but also bonds and general debts issued by financial institutions, such as investment banks and hedge funds.

4 Incomplete Loan Enforcement and Collateral Constraint

In this section, we modify the basic model and assume that the banks cannot enforce the repayment of the bank loans. We show that in the incomplete loan enforcement model a banking crisis due to coordination failure can occur without any real insolvency shock.

Incomplete loan enforcement: We assume that in the night market, a borrower can walk away without repaying the bank loan, and that the bank can seize the borrower's machine, k, if the repudiating borrower owns a machine, and sell the machine in the market to recover (a part of) the loan. We also assume that the banks cannot impose any other penalty to repudiators than to seize their machines, if they own the machines. Therefore, the bank loan must be secured by collateral, and the machines endowed to the buyers are the only asset that can serve the role of the collateral. Under this environment of incomplete loan enforcement, the bank loans of sellers and buyers made in the date-$(t-1)$ night market must satisfy the following collateral constraint:

$$(1+i)l_t^s = 0, \tag{118}$$

$$(1+i)l_t^b \leq E_{t-1}[a_t k_t], \tag{119}$$

where $E_{t-1}[\ \cdot\ ]$ is the expectation as of date-$(t-1)$. Since the sellers have no collateralizable assets, they cannot borrow the bank loans.

Sunspot shock: Similar to the model in Section 3, we assume that there is a macroeconomic random variable $\tilde{\omega}$ and at the beginning of the date-t day market the value of $\tilde{\omega}$ is revealed:

$$\tilde{\omega} = \begin{cases} 1 & \text{with probability } 1-\delta, \\ \omega\ (<1) & \text{with probability } \delta. \end{cases} \tag{120}$$

The difference from the model of Section 3 is that $\tilde{\omega}$ does not destroy any real resources but it affects the depositors' expectations on the other depositors' withdrawal decisions. If $\tilde{\omega} = 1$, all depositors believe that all sellers immediately deposit their cash revenue in the banks in the day market, while if $\tilde{\omega} = \omega(<1)$, all depositors believe that no sellers deposit their cash revenue in the banks and the banks run out of cash reserves in the 1-submarket. Since $\tilde{\omega}$ affects only the macroeconomic expectations, we can call it a sunspot shock.

- When $\tilde{\omega} = \omega$, a banking crisis occurs and the production of the intermediate goods is severely disrupted. We assume and justify later for a standard set of parameter values that if $\tilde{\omega} = \omega$, the collateral value $a_\omega k$ becomes much smaller than the outstanding debt, $(1+i)l^b$, in the night market and that the banks cannot collect the full amount of the loans. In this case, the bank asset becomes $(1-n)a_\omega k$, while the following relationship holds:

$$(1-n)a_\omega k < (1+i)L. \tag{121}$$

- We assume and justify later for a standard set of parameter values that if $\tilde{\omega} = 1$, the collateral value $a_n k$ becomes larger than the outstanding debt, $(1+i)l^b$, in the night market and that the banks can collect the full amount of the loans. In this case, the bank asset becomes $(1+i)L$.

- We assume and justify later for a sufficiently small value of δ that the collateral constraint (119) does not bind in the date-$(t-1)$ night market.

- Unlike the basic model in Section 2, the withdrawal decisions among depositors have the strategic complementarity à la Diamond and Dybvig (1983), because the banking crisis induces the impairment of the bank assets, through declines of the asset price from a_n to a_ω. Therefore, a depositor is worse-off if she redeposits cash in a bank in the day market when all the other depositors withdraw cash. That is, she can get paid only a small part of her bank deposit in the night market because the bank asset decreases to $(1-n)a_\omega k$ $(< (1+i)L)$ as a result of the banking crisis.

Given the sunspot shock $\tilde{\omega}$, the banks solve

$$\begin{aligned}\max_{L_t, C_t, D_t} &(1-\delta)[(1+i)L_t + C_t - (1+i_n)(1+i_d)D_t]_+ \\ &+ \delta[(1-n)a_\omega k + (1+i_n)C_t - (1+i_n)(1+i_d)D_t]_+ \end{aligned} \tag{122}$$

subject to (13) and (14). Since the profit of a bank is no less than $(1-\delta)[(1+i)L_t + C_t - (1+i_n)(1+i_d)D_t]_+$, the requirement that the bank profit must be finite in equilibrium implies that (16) must hold in equilibrium.

4.1 Equilibrium

The optimization problems for sellers and buyers are identical to those in the model of Section 3. Most of the conditions that determine the equilibrium values of cash holdings, deposits, and the quantity of the intermediate goods are identical to those in Section 3. The only difference is that the value of Λ is endogenized in the model of incomplete loan enforcement, while it is basically exogenously given by the value of ω in the model of Section 3.

Λ is determined as follows: When $\tilde{\omega} = \omega$, the bank run occurs and the asset price declines such that the collateral value, $a_\omega k$, becomes less than the outstanding bank debt, $(1+i)l^b$. The bank asset becomes $(1-n)a_\omega k$, while the remaining bank liability is $(1+i_n)\{(1+i_d)D - C\} = (1+i_n)(1+i_d)(1-\rho)(1-n)d^{db}$. Therefore, using $\Gamma = \rho$ and $\phi a_\omega = (1-\theta)A(\Gamma q_s^b)^\theta$, we have

$$\Lambda = \frac{(1-n)a_\omega k}{(1+i_n)(1+i_d)(1-\rho)(1-n)d^{db}} = \frac{(1-\theta)A\rho^\theta (q_s^b)^\theta}{(1+i_n)(1+i_d)(1-\rho)\phi d^{db}}. \tag{123}$$

Conditions (116) and (123) imply

$$\frac{\gamma_{+1}}{\beta} = \frac{(1-\delta)\theta A(q_{n,+1}^b)^\theta + \delta A(\rho q_{s,+1}^b)^\theta}{\phi d_{+1}^{db}}. \tag{124}$$

Conditions (114), (115), and (124) determine ϕd^{db}, q_n^b, and q_s^b, which in turn determine Λ by (123). The other variables are determined similarly as those in Section 3.3.

Parameter values that justify our assumptions: We assumed that the collateral constraint (119) does not bind in the date-$(t-1)$ night market and (121) holds if a bank run occurs. We can show that if δ is sufficiently small, these assumptions hold in the case where $\theta = 1/2$, $\rho = 1/9$, and $c(q) = q^2$. Since δ is small, the value of ϕd^{db} is approximated by (59), the value in the basic model. Thus, $(1+i)\phi L = (1-n)(1-\rho)\phi d^{db} \approx (1-n)(1-\rho)\theta A(q_n^b)^\theta$. The value of the collateralized asset is also approximated by $E_{t-1}[(1-n)\phi a_t k] = (1-n)(1-\theta)[(1-\delta)A(q_n^b)^\theta + \delta A(\rho q_s^b)^\theta] \approx (1-n)(1-\theta)A(q_n^b)^\theta$. The condition for (119) to be nonbinding is approximated by $(1-\rho)\theta < 1-\theta$, which holds for $\theta = 1/2$ and $\rho = 1/9$. The right-hand side of (121) is $(1-n)\phi a_\omega k = (1-n)(1-\theta)A(\rho q_s^b)^\theta$, where $q_s^b = q_n^b/\sqrt{\rho}$ in the case where $c(q) = q^2$ (see Section 3.3). Therefore, the condition for (121) to hold is approximated by $(1-\rho)\theta > (1-\theta)\rho^{\theta/2}$, which also holds for $\theta = 1/2$ and $\rho = 1/9$.

On the fire sale of assets: In our model, there is no fire sale of collateralized assets (i.e., the machines, k) during a banking crisis. The downward spiral of asset prices due to the fire sale by financial institutions is called the fire-sale externality, which is arguably the main rationale for banking regulation (see Brunnermeier, et al. [2009]). Although the fire-sale externality is not present in our model, it should be easily incorporated in our model by some modifications, which we leave for future research. On the other hand, as Brunnermeier, et al. (2009) point out, fire sales should have been regarded as good buying opportunities for professional investors and, therefore, should have ceased soon spontaneously due to increased demand for the excessively cheap assets. Thus it is a puzzle that the downward spiral of a fire sale can continue to a considerable extent and cause extensive damage to the economy. The mechanism of the asset-price decline present in our model may give one possible explanation for why the agents expect that the asset price will not pick up soon, but that it will continue to decline. (In the present paper, the asset price declines during the banking crisis due to disruptions of trading of the intermediate goods and the resulting decreases in productions of the consumption goods, which are caused by bank runs.)

4.2 Policy implications

In the incomplete loan enforcement model, we have similar policy implications for the banking crises as those in Section 3. A notable lesson of the incomplete loan enforcement model is that the cost of bank reform to restore solvency may turn out to be small after the policy is implemented, while it appears to be huge before the policy takes place. This is precisely because the asset price responds positively to the policy.

Deposit insurance and suspension of convertibility: Similar arguments hold as those in Section 3.4. The suspension of convertibility may amplify the severity of the banking crisis.

Monetary policy or Lender of Last Resort (LLR) lending: Suppose that when a bank run occurs in the day market the central bank lends cash to the banks up to the value of the observed bank asset, $(1-n)a_{\omega}k$. This upper limit is clearly insufficient to restore the normal production of the intermediate goods. The expected value of $(1+i_n)\Lambda$ when the policy is implemented stays below 1 and the depositors continue running on the banks: At the beginning of the night market, the banking sector has the remaining deposit of $(1+i)L-(1-n)a_{\omega}k$ and the liability to the central bank, $(1-n)a_{\omega}k$, and the bank assets of $(1-n)a_L k$, where $(1+i)L > (1-n)a_L k$ and a_L is the price of machines under LLR lending. (It is the case that $a_L > a_{\omega}$, since the production of the intermediate goods is increased by LLR lending.) Therefore, as long as the central bank limits its lending to the value of bank assets, the bank runs cannot be stopped by the LLR lending. Alternatively, if the central bank internalizes the positive effect of the LLR lending on the asset price and commits itself to lend up to $(1-n)a_e k$, where a_e is the equilibrium price of the machines, then it is easily shown that in equilibrium a_e becomes a_n and the solvency of banks and the production are restored. In this case, the real damage of the banking crisis is completely eliminated.

Bank reforms to restore the solvency of banks: If the government guarantees that the bank deposits (or all bank liabilities in reality) are fully repaid in the night market, there is no bank runs and the intermediate goods are produced just as much as in normal times. Thus, the real damage of the banking crisis can be completely eliminated by this policy. Moreover, the asset price rises in response to the increase of the production of the intermediate goods. Before the policy is implemented, the observed (or expected) asset price is a_{ω} and the banks appear to be insolvent. Therefore, the cost for the government to guarantee the bank liabilities appears to be huge, which is $(1+i)L-(1-n)a_{\omega}k$. If the guarantee is implemented, however, the value of the collateral rises to a_n, which satisfies $(1-n)a_n k > (1+i)L$. Thus, the banks restore their ability to collect the full amount of their loans. Therefore, by the guarantee of bank liabilities, the solvency of the banking system is restored and the government incurs no cost to implement the guarantee ex post. This seems to be a relevant lesson for episodes of banking crises. In many episodes of banking crises, the cost of the bank reform appeared to be incredibly huge in the midst of the crises, while the cost turned out ex post to be considerably smaller than expected.

Fiscal policy: We can consider the same fiscal policy that we argued in Section 3.4. If the government commits itself to this policy, a banking crisis never occurs. This is because all agents expect that the banks will never be insolvent, even when the sunspot shock hits the economy, since the asset price would become a_n even after a banking crisis in response to the fiscal policy due to the same mechanism as above. Since a banking crisis never occurs, there is no loss of social welfare compared with the case of no sunspot shock. Therefore, the fiscal policy is a good policy to resolve the financial crisis in this model where the government can work as a perfect substitute for the buyers in the day market. However, if we change the setting of the model slightly such that the government can substitute for the buyers only imperfectly, it is shown that the fiscal policy cannot restore the solvency of banks and cannot stop the bank runs. For example, suppose that the government cannot maintain the intermediate goods properly, while the buyers can, and therefore the intermediate goods purchased by the government perish completely at the beginning of the night market. In this setting, the price of machines stays at a_ω regardless of the fiscal policy. The fiscal policy cannot restore solvency of the banks nor the production of the consumption goods, while the amount of the intermediate goods produced can be restored completely. In this case the fiscal policy cannot improve the social welfare once the economy is hit by a banking crisis.[10]

Implications for the global financial crisis: In reaction to the current crisis, which began in the US in early 2007 and then spread all over the world, policy debates

[10] More precisely, the fiscal policy does not improve the total amount of social welfare, but it redistributes wealth from buyers to sellers: Since the government cannot preserve the intermediate goods, it cannot sell them in the night market and needs to finance the fiscal policy by taxes. Suppose that the lump-sum tax is available and the government imposes the same amount on each buyer and seller. In this case the government gives wealth to sellers by purchasing the intermediate goods in the day market, while it takes away the cost from both buyers and sellers in the night market. Thus in effect, the fiscal policy just transfers wealth from buyers to sellers.

are now dominated by arguments about the efficacy of recovery efforts that have been and are now undertaken: that is, extraordinary monetary easing, massive fiscal stimulus, and bank reforms aimed at restoring the solvency of the financial system. The policy implications from our stylized model give some basis for judging these policy options. As we saw above, monetary easing may not be able to stop the financial crisis as long as central banks provide only liquidity but do nothing to restore the solvency of the financial system. The demand stimulus from the fiscal measures may be a good policy to stop the crisis and restore market confidence and the solvency of the banking system only if governments can efficiently work as substitutes for the liquidity-constrained firms (i.e., buyers) in the chains of production in private economies. As is most likely, if governments are inefficient substitutes for the private buyers, neither market confidence nor solvency of the financial system can be restored and the fiscal stimulus will fail to stop the further deterioration of the crisis. What may be most necessary are the bank reforms aimed explicitly at restoring the solvency of the financial system, which entail decisive policy initiatives for stringent asset evaluations of financial institutions, all-out disposals of bad assets, and sufficient capital augmentations by either private investors, taxpayer money, or both.

4.3 Extension of the model: Productivity shock and the business cycle

As the Lagos-Wright monetary model is embedded in a standard business cycle model by Aruoba, Waller, and Wright (2006), the incomplete loan enforcement model in this section can be also embedded in a standard business cycle model. In the integrated model, a slight change in the productivity can generate a large economic downturn due to the occurrence of a banking crisis: Suppose, as usual in standard business cycle models, that the productivity parameter A of the production function of the consumption goods is subject to stochastic shocks. Suppose also that the shock to A is revealed at the beginning of the day market. If a macroeconomic shock lowers A below a certain threshold value with probability δ, the banks become insolvent and the banking crisis occurs. The banking crisis abruptly reduces the production of the intermediate goods and does the economy extensive damage. Therefore, a (small) shock in the productivity, A, can induce a considerable fluctuation in the economic activities through the occurrence of a banking crisis. This feature of the model may be useful for further understanding of the amplification mechanism of business cycles and the current global financial crisis.

5 Conclusion

We proposed a new model for policy analysis of banking crises based on the monetary framework developed by Lagos and Wright (2005). The model is tractable and easily embedded into a standard business cycle model. The model naturally makes a distinction between money and goods, while most of the existing banking models do not. This distinction enables us to clarify further the workings of banking crises and crisis management policies.

The bank deposit is defined as an asset that substitutes for cash, which is necessarily demandable. We assume that in the model only cash is accepted as a payment instrument in transactions of the goods in the day market. The bank deposits accelerate the circulation of cash and facilitate payment activities and transactions of the goods. A bank run disrupts the circulation of cash and consequently reduce the output severely through a reduction in transactions of the goods.

We may be able to use this framework to compare the efficacy of fiscal stimulus, monetary easing, and bank reforms as recovery efforts from the current global financial crisis. Our model shows that monetary easing is not sufficiently effective to stop bank runs unless the policy resolves the bank insolvency, and that fiscal stimulus can neither restore market confidence nor solvency of the banking system as long as the government is an inefficient substitute for liquidity-constrained buyers. A notable policy lesson is that the bank reforms to restore the solvency of the banking system (through, e.g., taxpayer-funded capital injections, or the blanket guarantee of bank liabilities by the government) may be optimal as a recovery policy from a banking crisis. Moreover, although the cost of bank reforms appears to be incredibly high in the midst of the banking crisis, it may turn out to be considerably small once the policy is implemented because the asset price responds positively to the policy and the higher asset price may restore the solvency of the banking system.

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A Alternative specification of banking sector

In this appendix we outline a model of the banking sector that can be regarded as an alternative to the specification in Section 2.2. For simplicity, we assume that $i_n = 0$ and that the bank's objective function is zero when a bank run occurs: Banks maximize their profits only in the state of no bank runs and they do not care about the state of bank runs.

Following Section 8.2.1 of Freixas and Rochet (2008), we assume that in the case of no bank runs the net amount of withdrawal during the day market (i.e., the sum of withdrawals in J submarkets minus the sum of new deposits in J submarkets) is an idiosyncratic random variable for a bank and when it runs short of reserve, the bank can borrow cash from the central bank or the interbank market at a penalty rate, i_p. We assume that the bank can earn interest at the rate of i_r ($< i_d$) by depositing the reserve with the central bank. These assumptions imply that the profit of a bank may be strictly positive. Thus we also assume in order to limit the size of the bank assets that the banks incur convex cost for managing the deposits, $c(\phi D)$, where $c'(\cdot) > 0$ and $c''(\cdot) > 0$. We assume that if the bank profit is strictly positive, the bank eats all the profit when it is liquidated in the night market. In this case, the banks solve the following problem and endogenously determine the amount of reserves, C, given i, i_d, i_r, and i_p:

$$\max_{L,C,D}(1+i)\phi L + (1+i_r)\phi C - (1+i_d)\phi D - i_p\phi D\left[\int_{\rho}^{1}(x-\rho)f(x)dx\right] - c(\phi D),$$

subject to $L + C \leq D$, where $\rho \equiv C/D$, x is the net withdrawal divided by the deposit, and $f(x)$ is the probability density function of x with the support of $-\infty < x \leq 1$. Note that $x \leq 1$ because a withdrawal cannot exceed the outstanding deposit. The problem is reduced to

$$\max_{\phi D,\rho}\Phi(\rho)\phi D - c(\phi D),$$

where $\Phi(\rho) = i - i_d - (i_d - i_r)\rho - i_p\left[\int_{\rho}^{1}(x-\rho)f(x)dx\right]$. As Freixas and Rochet (2008) show, this is a convex problem and the solutions are determined by

$$\int_{\rho}^{1} f(x)dx = \frac{i_d - i_r}{i_p},$$

$$c'(\phi D) = \Phi(\rho).$$

The central bank can implicitly determine ρ (> 0) by deciding i_r and i_p such that $0 \leq i_r < i_d < i_p$. If the total amount of cash in the economy (deposited with the central bank) is determined by the cental bank, the deposit rate, i_d, is determined in equilibrium such that the demand for deposits by depositors equals the supply of deposits by banks. As we argue in the text, the loan rate, i, is determined in equilibrium by monetary policy, i.e., the money growth rate. This alternative model of banking sector is compatible with the optimizations by sellers and buyers in the text.

B Idiosyncratic shock, bank runs, and contagion

In this appendix, we modify the model of Section 4 (The incomplete loan enforcement model) such that idiosyncratic shocks can induce bank runs. In this model we also show that a run on one bank can naturally induce a contagion of bank runs.

B.1 Setting

We modify the model of Section 4 as follows. There are three major changes.

First, the shock $\tilde{\omega}_t$ is an idiosyncratic shock to banks. Each bank i receives an independent shock $\tilde{\omega}_{i,t}$, which takes on the value of 1 with probability $1-\delta$ and ω (< 1) with probability δ. Therefore, the ratio $1-\delta$ of banks are not subject to runs, while the ratio δ of banks are hit by bank runs in equilibrium.

Second, we assume that the borrowers of bank i are the depositors of the same bank. That is, a buyer who borrows from a bank deposits the borrowed money into the lender bank. (If a borrower deposits the borrowed money into another bank, idiosyncratic shock can induce contagion of bank runs. See Section B.3.)

Third, we assume that the intermediate goods, q, must be installed and combined with the machine during the day market. A buyer who bought q units of the intermediate goods can install only q units of the goods into her machine. Therefore, in the night market, a buyer can sell her machine together with the installed intermediate goods, q. The intermediate goods, q, and the machines, k, cannot be sold separately in the night market. We also assume that there exists $\underline{q}$ (> 0) such that the production of the consumption goods from the machine is $y = Aq^{\theta}$ if $q \geq \underline{q}$ and $y = 0$ if $q < \underline{q}$. The value of a machine with q is y in the night market. If a buyer fails to buy the intermediate goods in the day market, then her machine can produce nothing in the night market. The most important consequence is that a bank cannot recover its bank loan from the borrower if she is a buyer who failed to buy q ($\geq \underline{q}$) in the day market, because the value of her collateral is zero and she can walk away leaving the worthless collateral in the hands of the bank. The collateral constraint (119) is changed to

$$(1+i)l_t^b \leq E_{t-1}[y] \tag{125}$$

B.2 Equilibrium

When a shock hits a bank, ρ ($= 1/J$) depositors of the bank can withdraw and buy q, while $1-\rho$ depositors cannot withdraw and fail to buy q. Because of the third assumption we made above, the value of the machines becomes zero for the depositors who failed to withdraw. Since the depositors are the borrowers of the same bank (the second assumption) and the machines are the collateral for their bank loans, the bank becomes insolvent due to the bank run. This is because the collateral value of the machines becomes zero for $1-\rho$ depositors and the bank cannot recover their loans in excess of the value of their collateral. Therefore, the sunspot shock, $\tilde{\omega}$, induces a self-fulfilling bank run that renders the bank insolvent.

The optimization problems and the equilibrium conditions are quite similar to those in Sections 3 and 4. There are only several changes: Replace $\tilde{a}k+\tilde{w}q^b$ in (70) and (71) with $A(q^b)^\theta$; Replace $a_nk+w_nq_n$ with $\phi^{-1}Aq_n^\theta$, $a_\omega k+w_\omega q_s$ with $\phi^{-1}Aq_s^\theta$, and $a_\omega k+w_\omega q_f$ with $\phi^{-1}Aq_f^\theta$ in (104); Replace $\frac{w_n}{p_n}$ with $\frac{\theta Aq_n^\theta}{\phi p_n q_n}$ in (108), and $\frac{w_\omega}{p_\omega}$ with $\frac{\theta Aq_s^\theta}{\phi p_\omega q_s}$ in (109). Note that (110) implies that the following relationship holds

$$\lambda_f \leq \delta(1-\Gamma)\left(\frac{\theta A\underline{q}^\theta}{\phi p_\omega \underline{q}}-1\right). \tag{126}$$

For a sufficiently small δ, (111) holds with strict inequality, and (112) holds with equality. Therefore, $m^{db}=q_f^b=0$. Since shocks are idiosyncratic, the equilibrium price p is unique and the liquidity constraints imply that $q_s^b=q_n^b\equiv q^b$ (and $q_f^b=0$). Λ is determined by $\Lambda=\frac{\rho A(q^b)^\theta}{(1+i_n)(1+i_d)(1-\rho)\phi d^{db}}$. Therefore, (124) is replaced by

$$\frac{\gamma_{+1}}{\beta}=\frac{[(1-\delta)\theta+(1+\theta)\delta\rho]A(q_{+1}^b)^\theta}{\phi d_{+1}^{db}}. \tag{127}$$

The values of variables are determined similarly as those in Section 4 by these conditions.

B.3 Contagion

By relaxing the second assumption in Section B.1, we can easily show that bank runs are intrinsically contagious, that is, a run on one bank naturally causes a run on another bank. We assume for simplicity of exposition that there are only two competitive banks in the economy, bank 1 and bank 2. We consider a symmetric case in which the sizes of bank loans, deposits and cash reserves are identical between the two banks. We change the second assumption in Section B.1 to one where all depositors of bank 1 are the borrowers of bank 2 and vice versa. Suppose the sunspot shock hits bank 1 and depositors start running on bank 1. The public expectation is that only ρ $(=1/J)$ depositors can successfully withdraw and buy q, while $1-\rho$ depositors fail to withdraw and cannot buy q. Since the depositors of bank 1 are the borrowers of bank 2, the run on bank 1 generates the public expectation that bank 2 becomes insolvent. This is because the collateral value becomes zero for $1-\rho$ borrowers of bank 2. Anticipating the insolvency of bank 2, the depositors of bank 2 also start running on bank 2. The bank run on bank 2 in turn renders bank 1 insolvent because the depositors of bank 2 are the borrowers of bank 1. Thus, the sunspot shock on bank 1 induces bank runs on both banks and makes both banks insolvent in a self-fulfilling way.

The above case is a very stylized example of contagion among bank runs. This model implies that in general a run on one bank can trigger various types of contagion leading to other bank runs, depending on the structure of the financial network or the way in which the borrowers of a particular bank deposit their borrowed money in that bank or other banks.

C On the condition for δ

We prove that (113) is a sufficient condition that (111) holds with strict inequality when (112) holds with equality. It is sufficient to show that the right-hand side (RHS) of (111) is strictly smaller than the RHS of (112). The RHS of (111) is written as $(1-\delta)(w_n/p_n)+\delta(w_\omega/p_\omega)$, while the RHS of (112) is $(1+i_d)[(1-\rho)(1+i_n)\delta\Lambda+(1-\delta)(w_n/p_n)+\delta\rho(w_\omega/p_\omega)]$. Since $(1-\rho)(1+i_n)\delta\Lambda > 0$, a sufficient condition is

$$(1+i_d)\left[(1-\delta)\frac{w_n}{p_n}+\delta\rho\frac{w_\omega}{p_\omega}\right] > (1-\delta)\frac{w_n}{p_n}+\delta\frac{w_\omega}{p_\omega}. \tag{128}$$

Note that this condition does not depend on the value of Λ and therefore it is the sufficient condition for both models of Sections 3 and 4. Since $\phi w_n = \theta A(q_n^b)^{\theta-1}$, $\phi w_\omega = \theta A(\rho q_s^b)^{\theta-1}$, and $\phi p_n q_n^b = \phi p_\omega q_s^b = (1+i_d)\phi d^{db}$, condition (128) is rewritten as

$$\left(\frac{\rho q_s^b}{q_n^b}\right)^\theta < \frac{\rho i_d}{1-(1+i_d)\rho}\frac{1-\delta}{\delta}. \tag{129}$$

Since $\rho q_s^b < q_n^b$, a sufficient condition for (129) is that the RHS of (129) is strictly larger than 1, which is equivalent to (113).