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The role of investment wedges in the Carlstrom-Fuerst economy and business cycle accounting*

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Abstract

Many researches that apply business cycle accounting (hereafter, BCA) to actual data conclude that models with investment frictions or investment wedges are not promising for modeling business cycle dynamics. In this paper, we apply BCA to artificial data generated by a variant model of Carlstrom and Fuerst (1997, *American Economic Review*), which is one of the representative models with investment frictions. Based on our findings, BCA leads us to conclude that models with investment wedges are not promising according to the criteria of BCA, even though the true model contains investment frictions.

Keywords: Business cycle accounting; investment wedge; investment friction; wedge decomposition

JEL Classification: C68, E13, E32

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1 Introduction

Chari, Kehoe, and McGrattan (2002, 2007) (hereafter, CKM) propose a simple method, business cycle accounting (hereafter, BCA), to investigate a promising class of frictional models. In BCA, the economy is assumed to be a prototype model with time-varying efficiency, labor, investment, and government wedges. Wedges, which are interepreted as distorions, are measured so that the prototype model perfectly accounts for the observed data. After the measurement of wedges, the importance of each wedge is evaluated through counterfactual simulations under an alternative sequence of wedges: wedge decompositions. The importance of wedges is judged by the similarity of output prediction to actual data.

Many papers including CKM deny the importance of investment wedges.[1] The models of Bernanke, Gertler, and Gilchrist (1999) and Carlstrom and Fuerst (1997) are often cited as representative models with investment frictions.

In this paper, we apply BCA to artificial data generated by a variant model of Carlstrom and Fuerst (1997). Based on our findings, BCA leads us to conclude that models with investment wedges are not promising according to the criteria of BCA, although the true model contains investment frictions. This is because BCA focuses only on determining whether or not investment wedges are the driving force of business cycles. In the Carlstrom-Fuerst economy, the role of investment wedges is to delay the propagation of technology shocks, which is consistent with empirical facts. BCA cannot capture such a role of investment wedges. We also find that (i) an investment wedge is negatively correlated with output; and (ii) an investment wedge accounts for some part of output if we introduce the adjustment costs of investment to the prototype model. These are consistent with the findings by CKM for the Great Depression. Therefore, the results from BCA do not deny the role of investment friction in the Carlstrom-Fuerst economy. The relevance of the Carlstrom-Fuerst economy is justified if the only exogenous shock is caused by technology. We show that, in the case of wealth shocks, the Carlstrom-Fuerst

[1] For the Japanese economy, Kobayashi and Inaba (2006) find that the investment wedge cannot account for the long stagnation during the 1990s.

economy is not consistent with empirical results.

The rest of this paper is organized as follows. In Section 2, we introduce the Carlstrom-Fuerst and the prototype economies. Section 3 applies BCA to artificial data generated in the Carlstrom-Fuerst economy. Section 4 contains some concluding remarks.

2 The model

2.1 Carlstrom-Fuerst economy

First, we provide a brief description about our Carlstrom-Fuerst economy.[2] There are two types of consumers: households and entrepreneurs. Households own capital stock and supply labor input and capital to competitive firms for the production of output. Entrepreneurs own capital stocks and supply capital to firms. Entrepreneurs also have investment technology, and they produce investment goods. Moreover, they have to borrow working capital to produce investment goods, however, the amount of borrowing is limited by their net worth because of the agency problem.

There are two modifications in our Carlstrom-Fuerst economy: utility function and the introduction of government consumption. We employ a log utility function $u(c_t, \ell_t) = \log(c_t) + \nu \log(1 - \ell_t)$. We also introduce constant government consumption g.[3] The

[2] See Carlstrom and Fuerst (1997) for details.

[3] We introduce government consumption to mitigate the volatility of the government wedge in the associated prototype model.

equilibrium system of our Carlstrom-Fuerst economy is summarized as follows:

$$\nu \frac{c_t}{1-\ell_t} = (1-\alpha) \cdot \frac{y_t}{\ell_t}, \tag{1}$$

$$\frac{q_t}{c_t} = \beta E_t \left[\frac{1}{c_{t+1}} \left\{ q_{t+1}(1-\delta) + \alpha \cdot \frac{y_{t+1}}{k_{t+1}+z_{t+1}} \right\} \right], \tag{2}$$

$$q_t = \beta\gamma E_t \left[\left\{ q_{t+1}(1-\delta) + \alpha \cdot \frac{y_{t+1}}{k_{t+1}+z_{t+1}} \right\} \frac{q_{t+1} f(\bar{\omega}_{t+1})}{1 - q_{t+1} g(\bar{\omega}_{t+1})} \right], \tag{3}$$

$$q_t = \left[1 - \Phi(\bar{\omega}_t)\mu + \phi(\bar{\omega}_t)\mu \frac{f(\bar{\omega}_t)}{f'(\bar{\omega}_t)} \right]^{-1}, \tag{4}$$

$$i_t^* = \frac{z_t \Big[q_t(1-\delta) + r_t \Big]}{1 - q_t g(\bar{\omega}_t)}, \tag{5}$$

$$y_t = A_t \Big[k_t + z_t \Big]^{\alpha} \ell_t^{1-\alpha}, \tag{6}$$

$$\Big[k_{t+1} + z_{t+1} \Big] = (1-\delta) \Big[k_t + z_t \Big] + i_t^* \Big[1 - \Phi(\bar{\omega}_t)\mu \Big], \tag{7}$$

$$e_t + q_t z_{t+1} = q_t i_t^* f(\bar{\omega}_t), \tag{8}$$

$$c_t + e_t + i_t^* + g = y_t, \tag{9}$$

$$\log(A_{t+1}) = \rho \log(A_t) + \varepsilon_{t+1}. \tag{10}$$

(1) is the intratemporal condition; (2), the Euler equation for households; (3), the Euler equation for entrepreneurs; (4) and (5), conditions for the optimal contract; (6), the production function; (7), the evolution of aggregate capital; (8), the budget constraint of entrepreneurs; (9), the resource constraint; and (10), the evolution of technology.

2.2 Prototype economy

The equilibrium system of the associated prototype economy is as follows:

$$\nu \frac{\tilde{c}_t}{1-\tilde{\ell}_t} = (1-\alpha) \cdot \frac{\tilde{y}_t}{\tilde{\ell}_t}, \tag{11}$$

$$\frac{1+\tau_{x,t}}{\tilde{c}_t} = \beta E_t \left[\frac{1}{\tilde{c}_{t+1}} \left\{ (1+\tau_{x,t+1})(1-\delta) + \alpha \cdot \frac{\tilde{y}_{t+1}}{\tilde{k}_{t+1}} \right\} \right], \tag{12}$$

$$\tilde{y}_t = \tilde{A}_t \tilde{k}_t^{\alpha} \tilde{\ell}_t^{1-\alpha}, \tag{13}$$

$$\tilde{c}_t + \tilde{g}_t + \tilde{i}_t = \tilde{y}_t, \tag{14}$$

$$\tilde{k}_{t+1} = (1-\delta)\tilde{k}_t + \tilde{i}_t, \tag{15}$$

where $\tilde{A}_t$ is the efficiency wedge; $1/(1+\tau_{x,t})$, the investment wedge; and $\tilde{g}_t$, the government wedge.[4] The evolution of $\boldsymbol{s}_t \equiv [\log(\tilde{A}_t), \tau_{x,t}, \log(\tilde{g}_t)]'$ is VAR(1):

$$\boldsymbol{s}_{t+1} = \boldsymbol{P}_0 + \boldsymbol{P}\boldsymbol{s}_t + \boldsymbol{\varepsilon}_{t+1}. \tag{16}$$

2.3 Equivalence result

If we interpret $i_t^* \left[1 - \Phi(\bar{\omega}_t)\mu \right]$ as (net) investment i_t of the Carlstrom-Fuerst economy and $k_t + z_t$ as the total capital K_t, the following equivalence result holds.[5]

Proposition 1 (Equivalence result) *The equilibrium allocation of the Carlstrom-Fuerst economy* $\{c_t, \ell_t, i_t, K_{t+1}, y_t\}_{t=0}^{\infty}$ *coincides with that of the prototype model* $\{\tilde{c}_t, \tilde{\ell}_t, \tilde{i}_t, \tilde{k}_{t+1}, \tilde{y}_t\}_{t=0}^{\infty}$ *if* $A_t = \tilde{A}_t$, $q_t = 1 + \tau_{x,t},$, *and* $e_t + i_t^* \Phi(\bar{\omega}_t)\mu + g = \tilde{g}_t$.

The proof is simple. It is obtained in a straightforward manner by comparing two equilibrium systems. The remaining problem is the VAR(1) specification for wedges, as discussed by Nutahara and Inaba (2008). It is easily verified that conditions of Theorems 1 and 2 of Nutahara and Inaba (2008) are satisfied in this case. This proposition states that the equilibrium allocation of the Carlstrom-Fuerst economy is consistent with that of the prototype model through adjustments of the investment and government wedges.

[4] We eliminate the labor wedge following CKM.

[5] This equivalence result is slightly different from that of CKM. They construct a prototype economy with adjustment costs of investment.

3 BCA in the Carlstrom-Fuerst economy

3.1 Main result

First, we generate artificial data from our Carlstrom-Fuerst economy. The parameter values of the Carlstrom-Fuerst economy are as follows. We employ the same parameter values employed by Carlstrom and Fuerst (1997) except for the steady-state ratio of g/y. We employ $g/y = 0.1$. We approximate the equilibrium system by log-linearization and generate 1000 long-period artificial data.

Following the standard method of BCA, we measure efficiency, labor, investment, and government wedges so that the prototype model can perfectly account for data of consumption, investment, labor, and output. Then, we obtain wedge decompositions by providing only one wedge. Figure 1 shows the actual output and output prediction with only one wedge.

[Insert Figure 1]

The contribution of investment wedge to output is rather small and negative. Hence, by the criteria of BCA, the investment wedge is not promising. However, our data-generating model is the Carlstrom-Fuerst economy, which is one of the representative models with investment frictions.

Why does BCA lead us to such a conclusion? It is due to the role of investment wedges. Figure 2 shows the impulse responses to the one percent technology shock in the Carlstrom-Fuerst economy.

[Insert Figure 2]

When technology shocks hit the economy, output increases and the investment wedge $1/(1 + \tau_{x,t})$ decreases, or the distortion in the investment process increases. The important feature of the Carlstrom-Fuerst economy is the hump-shaped impulse response of output to technology shocks, which is consistent with the finding of Cogley and Nason (1995). The role of investment wedges is to delay the propagation of shocks. Then, the main driving force of output is the efficiency wedge, and the investment wedge explains deviations from the simple real business cycle model. BCA cannot capture this role of investment wedges. As in Figure 2, the investment wedge is negatively correlated with output and this is consistent with empirical findings by CKM for the Great Depression. Therefore, CKM's results of BCA cannot deny the importance of the Carlstrom-Fuerst economy.

3.2 Adjustment costs of investment

CKM report that the investment wedge can explain some parts of output fluctuations if the model has adjustment costs of investment such as

$$\tilde{k}_t = (1-\delta)\tilde{k}_{t-1} + \tilde{i}_t - \frac{a}{2}\left(\frac{\tilde{i}_t}{\tilde{k}_{t-1}} - \delta\right)^2 \tilde{k}_{t-1}. \tag{17}$$

Their results are also consistent with the Carlstrom-Fuerst economy. Responses of the investment wedgeto technology shocks, in the case of the prototype model with adjustment costs of investment, are in Figure 3.

[Insert Figure 3]

We set $a = 0$ for no cost, $a = 3.22$ for BGG cost, and $a = 12.88$ for extremum cost following CKM. The investment wedge positively comoves with output if there is an adjustment cost. Then, the investment wedge explain some parts of output as found by CKM if there are adjustment costs in the prototype model.

3.3 Wealth shock

In addition to the technology shock, Carlstrom and Fuerst (1997) consider the wealth shock, which is a redistribution of capital stock from households to entrepreneurs. This wealth shock can be interpreted as Irving Fisher's debt-deflation hypothesis. Figure 4 shows the impulse responses to the one-percent wealth shock.

[Insert Figure 4]

Contrary to the case of technology shocks, the investment wedge comoves with output even if there are no adjustment costs of investment. An increase in an entrepreneur's net worth loosens the borrowing limit, and implies the reduction of distortions in investment. Then, if a wealth shock was an important source of business cycle fluctuations, the investment wedge should have accounted for some part of output. Thus, the wealth shock hypothesis of the Carlstrom-Fuerst economy is not consistent with BCA results.

4 Concluding remarks

In this paper, we applied BCA to artificial data generated by a model with investment frictions à la Carlstrom and Fuerst (1997). We found that if the only exogenous shock is related to technology, the Carlstrom-Fuerst economy is consistent with empirical results by CKM and other applications of BCA. We also found that BCA results deny the role of a wealth shock, or debt-deflation hypothesis, in the Calstrom-Fuerst economy. Our findings imply that empirical findings of BCA do not deny the importance of investment frictions.

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Figure 1: Output decomposition with one wedge

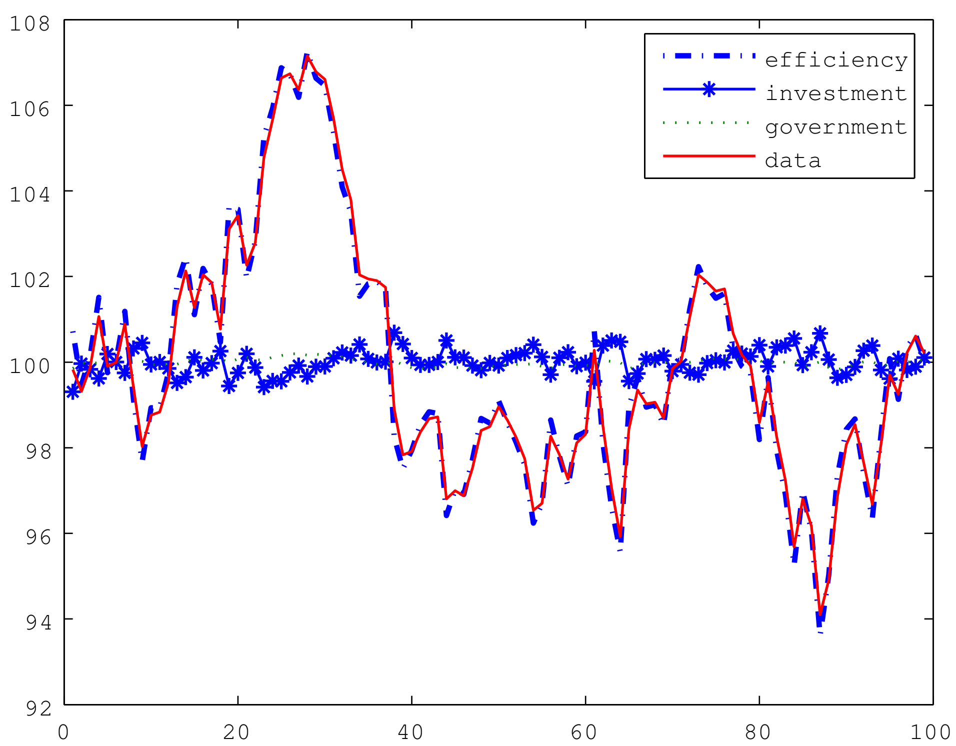

Figure 2: Impulse responses to a one-percent technology shock

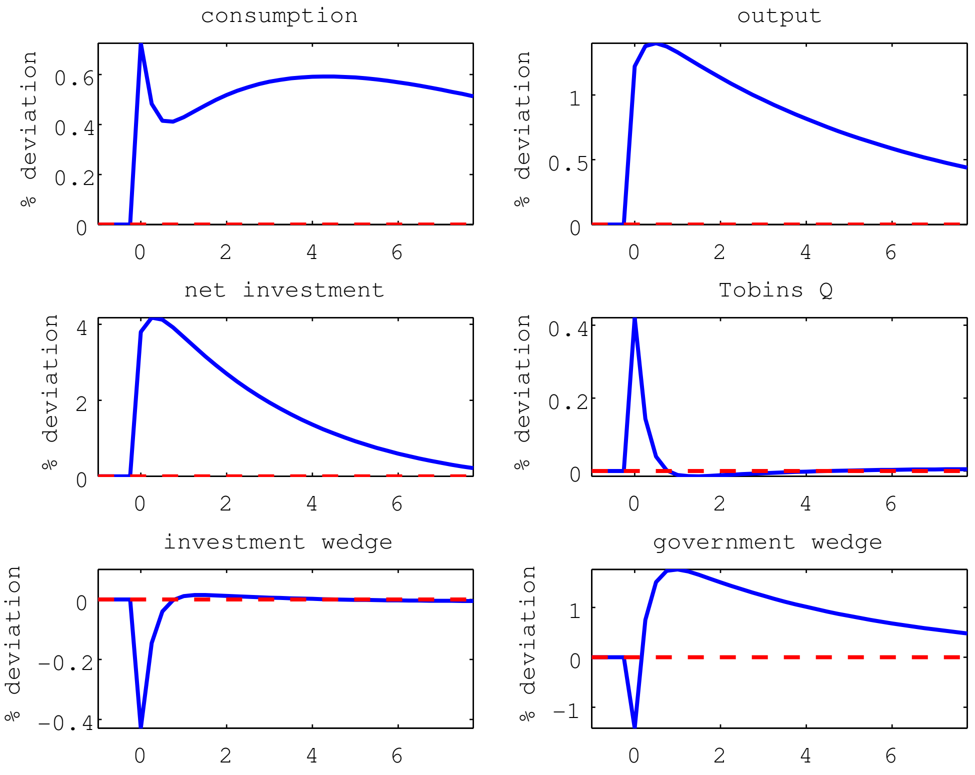

Figure 3: Impulse responses of the investment wedge: with adjustment costs of investment

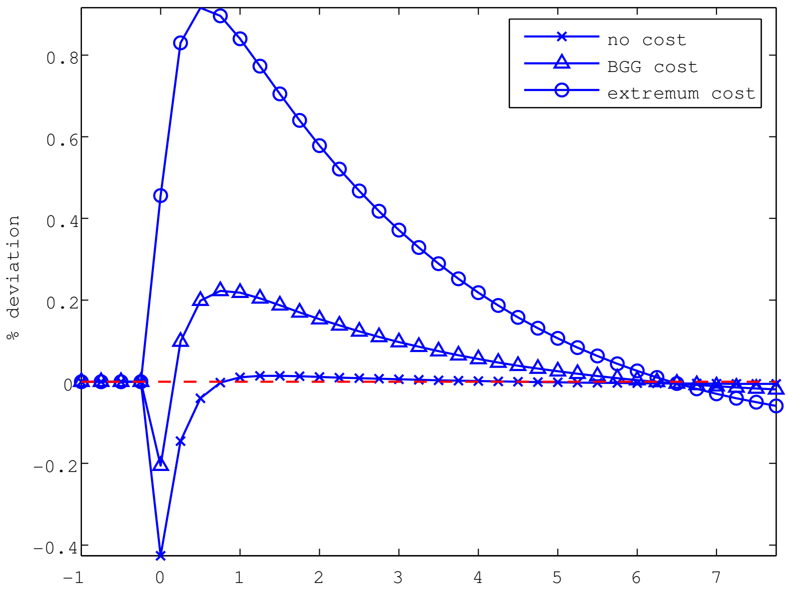

Figure 4: Impulse responses to a one-percent wealth shock