Firm Heterogeneity and FDI with Matching Frictions

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Abstract

Firm-level data often show different modes of market access by firms with the same productivity levels, which is a mere knife-edge case in the basic firm heterogeneity model. This paper examines the foreign direct investment (FDI) decisions of individual firms with a simple framework, where firms and managers have to make matches for production. We find that predicted distributions of FDI firms are much more akin to real data than those suggested by the basic firm heterogeneity model, namely, there exists a range of firm productivities in which more productive firms may export while less productive firms may undertake FDI. Such a range of firm productivities becomes wider when either matching frictions increase or trade costs decline. Furthermore, matching frictions hurt production efficiency more for productive FDI firms than for less productive FDI firms.

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1 Introduction

Exploiting detailed firm-level data, a large body of empirical studies has revealed that firms engaged in international activities, such as FDI and exports, are rare, greater in their operation scale, and more productive than firms that do not enter international markets.¹ These empirical regularities are supported by theoretical contributions by Melitz (2003), Bernard, Eaton, Jensen, and Kortum (2003), Yeaple (2005), and others. In particular, for its clarity, the Melitz model is widely applied to explain the characteristics of internationalized firms. For example, extending the Melitz model, Helpman, Melitz, and Yeaple (2004) show a hierarchy among firms: only the more productive firms are internationalized and the most productive firms among this group choose FDI to serve the foreign markets.²

However, the hierarchy among firms is not necessarily so clear as the theory predicts, once we start to scrutinize empirical data. For example, Wakasugi, Toda, Sato, Nishioka, Matsuura, Ito, and Tanaka (2008) find that the productivity advantage of FDI firms over exporting firms is quite small although FDI firms tend to be more productive than exporters and firms serving the domestic market only. In particular, they observe that even in the group of the most productive firms, while quite a large number of firms choose FDI, an equally large number of firms choose exports.

In this paper, I examine the foreign direct investment decisions of individual firms in order to reconcile the empirical findings with the theoretical predictions of the pecking order of internationalization. In so doing, I extend the Melitz model by incorporating a simple search and matching framework. The model is based on the premise that firms have to search for managers who adroitly manage production with product expertise and knowledge about local business environments. It is not difficult for firms to collect information about potential managers in their home country. However, this does not necessarily hold when searching for managers in foreign countries. Matches in foreign countries tend to be associated with uncertainty about the quality of managers. As a result of matching with low quality managers, firms may not fully exert their intrinsic productivity level in foreign


²Bernard, Redding, and Schott (2007) extend the Melitz model by putting it in a Heckscher-Oline framework. Bustos (2007) incorporates technology adoption into the Melitz model and shows that exporters tend to adopt more advanced technology.
production (FDI).\textsuperscript{3} Worse, even highly productive firms may not find appropriate managers and be forced to choose exports rather than FDI.

This is not the first study that attempts to reconcile the implication derived from the standard Melitz model with the empirical fact that firms do not enter foreign markets according to an exact pecking order based on firms’ productivities. Indeed, Eaton, Kortum, and Kramarz (2008) modify the Melitz model by allowing firms to receive stochastic shocks over foreign demands and fixed market entry costs. Using the data of French firms, they estimate that firms’ underlying productivity heterogeneity explains about half the variation across firms in market entry and sales. Arkolakis (2008) introduces endogenous marketing costs into the Melitz model. Firms optimally choose their marketing costs, which are partly composed of fixed market entry costs, taking into account their underlying productivities. Thus, firms may have different levels of fixed costs, which result in a novel extensive margin caused by firms’ deepening market penetration.

I specify the search-match mechanism based on contributions of Rauch and Trindade (2003) and Grossman and Helpman (2005). Allowing for matching frictions between firms and managers is important in at least two respects. First, we find that the model can reproduce firm distributions much more akin to those observed in empirical data. The productivity of foreign affiliates is determined by both firms’ intrinsic productivity and the quality of the match with local managers. Thus, the foreign affiliates of firms with relatively high intrinsic productivity may fail if they have very low quality managers. Simultaneously, these matching frictions may provide relatively unproductive firms with a very high match quality and enable them to enter foreign markets through FDI. However, firms with low intrinsic productivity are considerably sensitive to match quality. As a result, firms very close to the threshold productivity level are highly unlikely to choose FDI. As firms’ intrinsic productivity rises, the chance of successful FDI progressively increases. This mechanism contributes to more realistic firm distributions, namely, there exists a range of firm productivities in which more productive firms may export while less productive firms may undertake FDI. Such a range of firm productivities becomes wider when either matching frictions increase or trade costs decline.

Second, the model has endogenous market entry costs in the form of payments toward local managers. Matched pairs of a firm and a manager make bargaining over FDI surplus.

\textsuperscript{3}I do not deal with organizational issues in this paper. Thus, what I call FDI in this paper can be arms-length transactions such as outsourcing or licensing.
The foreign manager demands their status-quo payoffs plus their share of net FDI surplus. In the model, while the status-quo payoffs are common among managers, the net FDI surplus varies across firms, depending on firms’ intrinsic productivity levels and match quality. On average, firms with low intrinsic productivity earn smaller FDI surplus. Thus, firms’ market entry costs (i.e., payments to local managers) are endogenous and firms with productivity close to threshold levels pay a smaller amount of market entry costs. Consequently, the model provides another view of the microeconomic structure of market entry costs and complements Arkolakis (2008).

A number of studies are related to this work. The closest are Eaton, Kortum, and Kramarz (2008) and Arkolakis (2008), which have already been mentioned above. In the literature on FDI, Nocke and Yeaple (2008, 2007) recently developed a general equilibrium model in which firms produce differentiated goods with combining two distinct capabilities: one is internationally mobile and the other is not. Using this framework, they consider two different modes of FDI, mergers and acquisitions (M&A) and greenfield FDI. They also consider the pecking order of modes of internationalization, but they do not examine the issue of overlapped productivity range, which is the main concern of this study. Grossman and Helpman (2005) study international matching between final good producers and intermediate good suppliers. In this sense, their study is very close to mine. However, their main focus is on the thick market effect. Also, they do not deal with firm heterogeneity.

The rest of the paper is organized as follows: in the next section, I describe the model, and Section 3 discusses its properties. Section 4 provides a numerical example of the model. Section 5 concludes and discusses issues that should be studied more deeply.

2 The Model

This section lays out a two-country model that contains two sectors and a continuum of potentially heterogeneous firms. One sector (sector $Z$) competitively produces a homogenous, numeraire good from labor. The other sector (sector $Y$) produces a continuum of differentiated varieties. In this sector, each firm has to search for a manager first. Then, the matched pairs of a firm and a manager produce differentiated goods in a monopolistically competitive manner.
2.1 Preferences

The world consists of two countries, home (H) and foreign (F) indexed by \( l, l' = H, F \) and \( l \neq l' \). Each country is populated by many identical households who own as a whole \( L_l \) units of labor, \( K_l \) units of capital, and \( S_l \) units of skill. Preferences are common across the two countries. The representative household maximizes the following utility function:

\[
U = \left[ \int_{i \in \Omega} q(i) \alpha di \right]^{\gamma/\alpha} q_0^{1-\gamma}, \quad \gamma \in (0, 1),
\]

where \( q(i) \) denotes the consumption of variety \( i \) of good \( Y \), \( \Omega \) the set of available varieties, and \( q_0 \) the consumption of the homogeneous good. The varieties of differentiated goods are substitutable from one another with the elasticity of substitution \( \sigma = 1/(1-\alpha) > 1 \).

These preferences yield the iso-elastic demand function for each variety \( i \) such that

\[
q(i) = \frac{\gamma E p(i)^{-\sigma}}{P^{1-\sigma}},
\]

where \( E \) represents the total expenditure, \( p(i) \) the price of variety \( i \), and \( P \) the aggregate price index in sector \( Y \) that is given by

\[
P = \left[ \int_{i \in \Omega} p(i)^{1-\sigma} di \right]^{1/(1-\sigma)}.
\]

2.2 Production Technology

The homogeneous good is produced with labor only under constant returns to scale and perfect competition. It is freely traded and taken as a numeraire. Home produces \( w_H \) units of the homogenous good per one unit of labor while foreign produces \( w_F \) units of the homogenous good per one unit of labor. For analytical clarity, I will focus on equilibria in which both countries produce the homogenous good, which implies that the wage rates are \( w_H \) for home and \( w_F \) for foreign, respectively. Without loss of generality, I also assume that \( w_H \geq w_F = 1 \).

There is a continuum of (atomless) firms that differ in their productivity levels. Each firm invents a new design of differentiated goods by investing one unit of capital. Thus, the measure of firms is given by \( K_l \). I assume that firms (i.e., product designers) cannot manufacture their products themselves. They must employ managers who operate the manufacturing process of differentiated goods. The production of each variety is, thus, a joint venture by the way of pairing of a firm and a manager. Managers obtain management
skills by investing one unit of skill. Thus, country \( l \) has \( S_l \) managers. I assume that \( S_l \) is sufficiently greater than \( K_l \).

Managers are also heterogeneous. Each manager has some specialization for a certain product and cannot operate the production of each differentiated good equally well. Thus, the quality of matching between a firm and a manager affects the productivity of the differentiated good. Specifically, it is assumed that the productivity level of a variety is given by 
\[
A_l \varphi z^{1/(\sigma - 1)}
\]
where \( A_l \) denotes the effectiveness of one unit of labor in sector \( Y \) in country \( l \), \( \varphi \) firm-specific (relative) productivity levels, and \( z \in [0, 1] \) the quality index of matching between a firm and a manager. The total cost function of variety \( i \) produced in country \( l \), thus, takes the form of
\[
TC = \frac{w_l A_l \varphi z^{1/(\sigma - 1)} q(i)}{q(i)}.
\]
In what follows, I will call \( A_l \varphi \) as firms’ “intrinsic” productivity in order to distinguish it from the “realized” productivity, \( A_l \varphi z^{1/(\sigma - 1)} \).

Firms’ intrinsic productivity level, \( \varphi \), is randomly drawn from a distribution with the cdf of \( G \) and the pdf of \( g \). This distribution is common across the countries. As Helpman, Melitz, and Yeaple (2004) and others, I specify the distribution as a Pareto distribution with \( \varphi \in [1, \infty) \) and the shape parameter \( k > \sigma - 1 \). The cdf is given by
\[
G(\varphi) = 1 - \varphi^{-k}.
\]

I assume that firms are internationally mobile but managers are not. Thus, firms have to employ local managers for local production, which implies that when a firm sets up a foreign plant, it has to search for a manager in the foreign country.

### 2.3 Matching and Bargaining

Events proceed sequentially and the timing of events is as follows (see Figure 1). After knowing the intrinsic productivity level of \( \varphi \), each firm starts to search for an appropriate

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4This assumption about \( k \) is necessary for a finite mean of productivity levels.
manager. I assume that international matching occurs first. If FDI is more profitable than exports, firms in country $l$ start to search for managers in country $l'$. As will be shown soon, international matching is associated with informational uncertainty. Hence, some firms successfully spot appropriate managers while some firms fail to do so (Stage 1).

Once the international matching stage is finished, then, firms start to search for local managers for domestic production (Stage 2). After the domestic matching stage is completed, production and sales occur and revenues are distributed to all related economic agents (Stage 3).

Each firm (each variety of differentiated goods) has its ideal manager. The determination of match quality, $z$, is based on the “ideal” variety approach. Specifically, I borrow the framework of the matching process between a firm and a manager from Rauch and Trindade (2003) and Grossman and Helpman (2005). Suppose that firms are equally spaced around a circle of the circumference of 2. Managers are also equally spaced around the circumference. For each firm, there exists the ideal manager at the farthest point on the arc. In Figure 2, when the location of firm $i$ is represented by point $i$, firm $i$’s ideal manager is located at the farthest point on the arc, point $j$. Match quality $z$ is measured by the shortest arc-distance between a firm and a manager. For example, when firm $i$ matches with manager $j'$, the match quality is $z_{ij'}$ as shown in Figure 2. Thus, the distance between a firm and a manager, $z$, may take any value between $[0, 1]$: matching with the best manager results in $z = 1$ and the worst in $z = 0$.

Firms lack information about the precise location of their ideal manager. However, by undertaking a search, firms can symmetrically narrow down the arc where their ideal managers exist. I assume that this search does not need any tangible inputs. Figure 2 illustrates that firm $i$ can narrow down the scope of $z$ by eliminating the part of the arc of solid line $2\lambda$ before selecting a manager. Then, firms randomly choose a manager with $z \in [\lambda, 1]$. Thus, parameter $\lambda \in [0, 1]$ is an index of search efficiency. Many factors, such as geographical proximity, cultural similarity (e.g. language), and telecommunication technology, may affect $\lambda$. It is natural that firms can find suitable managers more easily in their origin country than in foreign countries, exploiting the familiarity of the business environment in their origin countries. Based on this premise, I assume that $\lambda$ equals to 1

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5 The ideal variety approach is originated by Lancaster (1979) and applied to the Heckscher-Ohlin framework by Helpman (1981).

6 As Grossman and Helpman (2005) note, consumers regard differentiated goods located at the same location on the circle as differentiated while they are the same type from the point of view of managers.
for domestic matching. In other words, firms can always match with the best managers for domestic production.

Once a firm and a manager makes a match, they immediately know the match quality \( z \), and then decide whether or not they will hold the match. If they hold the match, they make an arrangement for production and profit sharing. Otherwise, the match is aborted. Firms that could not match with managers of acceptable quality in the international matching stage serve the foreign markets as exporters instead of multinationals.7

I assume that once they proceed the arrangement stage, they can reach an efficient agreement where joint surplus is maximized. The successful match of firm \( i \) and manager \( j \) in country \( l \), hence, sets the price at \( p(i) = w_l/[\alpha A_l \varphi z^{1/(\sigma - 1)}] \), facing with the iso-elastic demand in (2). The gross match surplus \( \Pi(\varphi, z) \) generated by this pair is given by

\[
\Pi(\varphi, z) = zw_l^{1-\sigma}[A_l \varphi]^{\sigma-1},
\]

where \( M_l \equiv \gamma E_l P_l^{\sigma-1}/(\sigma \alpha^{1-\sigma}) \) is the mark-up adjusted residual demand.

Firms and managers that formed matches bargain over their match surplus, following the Nash bargaining rule. Without loss of generality, it is assumed that any pair of firms and managers will evenly share the match surplus.

2.4 Domestic Production

Given the sequence of events, I start the description of matching at domestic production. Since it is assumed that firms can find their ideal managers without any friction for domestic production, \( z = 1 \) is always realized. A domestic match generates (gross) profits from the local market, \( \Pi_{D_l} \),

\[
\Pi_{D_l}(\varphi) = M_l w_l^{1-\sigma}[A_l \varphi]^{\sigma-1}.
\]

In order to focus on the foreign direct investment decisions of individual firms, I abstract fixed costs for exports from the model. Any firms located in country \( l \) can export the differentiated goods to country \( l' \), incurring iceberg-type transportation costs: \( \tau_l > 1 \) units need to be shipped for one unit to arrive in country \( l' \). When the good produced in country

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7I do not consider the possibility that firms cannot meet any managers. This possibility introduces additional matching frictions, that is, unemployed managers and vacancy firms. The introduction of such matching frictions into the model may be an interesting extension. However, it seems unnecessary for the current purpose of the model. In order to avoid the issue of unemployment managers and vacant firms, I simply assume a hypothetical matching market maker who can arbitrarily adjust the mass of managers matching with firms.
Figure 2: Matching of a firm and a manager

\(l\) is shipped to country \(l'\), the marginal cost of serving country \(l'\) is \(\tau_{l'}w_1/\varphi\). The match surplus from exports from country \(l\) to country \(l'\), \(\Pi_{XL}\), is given by

\[
\Pi_{XL}(\varphi) = M_{l'l}w_1^{1-\sigma}[A_{l'l}]^{\sigma-1} - \sigma \left[ A_{l'l} \varphi \right]^{-1},
\]

where \(T_{l'} \equiv \tau_{l'}^{1-\sigma}\) is a transformed measure of the transportation costs.

Following the Nash bargaining rule, the match surplus generated by domestic production is evenly split between the partners. Since the international matching market is closed at this stage, each party’s status-quo payoff is zero. Each partner, thus, obtains \([\Pi_{Dl}(\varphi) + \Pi_{XL}(\varphi)]/2\) from domestic production.

2.5 International Matching for FDI

I turn now to international matching for FDI. Firms may choose FDI for saving the transportation cost (horizontal FDI) or for exploiting inexpensive production factors (vertical FDI). The model may include these two motivations. However, vertical FDI, namely firms’ setting up plants abroad with the shutdown of the domestic plants, does not add particularly interesting insights in this framework. Thus, I will focus on the case of horizontal FDI. The model differentiates FDI from exports by emphasizing that firms have to search for appropriate local managers to run foreign affiliates.\(^8\)

Given that the match between a firm from country \(l\) and a manager in country \(l'\)

\(^8\)In reality, it is observed that firms send managerial-class employees to foreign affiliates in stead of hiring those locally. However, these behaviors seem to be limited only at early stage of FDI.
generates quality $z$, the gross profits from FDI, $\Pi_{II}$, are given by

$$\Pi_{II}(\varphi, z) = M_I z w^{1-\sigma}_I [A_I(\varphi)]^{\sigma-1},$$

(8)

where I assume that multinational enterprises (MNE) bring their own technologies across the borders.

The same Nash bargaining rule and the share apply to international matching. Since any firm can export, the firm’s status-quo payoff is $\Pi_{XI}(\varphi)/2$. At this stage, the manager can expect matching with a domestic firm in the next stage. In domestic matching, the best match is assured (i.e., $z = 1$), but matched firm’s intrinsic productivity level is random. Thus, the manager’s status-quo payoff is $[\Pi_{XI}(\tilde{\varphi}) + \Pi_{DI}(\tilde{\varphi})]/2$ where $\tilde{\varphi}$ is the average relative productivity level such that

$$\tilde{\varphi} = \left[ \int_1^{\infty} \varphi^{\sigma-1} g(\varphi) d\varphi \right]^{1/(\sigma-1)} = \left[ \frac{k}{k + 1 - \sigma} \right]^{1/(\sigma-1)}.$$  (9)

International matches will be maintained only when the match quality $z$ is sufficiently high for generating net surplus. Otherwise, firms choose exports, forming matches with local managers in the next stage.\(^9\)

\section{2.6 Market Clearing}

In order to close the model, we impose the requirement that a country’s nominal income equals the value of the production of the numeraire good and the differentiated goods. Since the model does not require free entry, all generated profits must be distributed to the households. For this purpose, I assume a hypothetical fund that collects profits from all firms and managers and redistributes them to the households as shareholders of firms and managers.\(^{10}\) Then, total expenditure $E_l$ can be expressed by the sum of labor income, total profits from domestic production, and total payoffs earned by managers who work for multinationals. Namely,

$$E_l = L_l + K_l \int_1^{\infty} \Pi_l(\varphi) g(\varphi) d\varphi + K_I \int_1^{\infty} \tilde{s}_l(\varphi) \delta(\varphi) g(\varphi) d\varphi,$$

(10)

where $\tilde{s}(\varphi)$ is the average payoff to managers who work for firms from country $l'$ and $\delta(\varphi)$ represents the probability that international matching succeed, given $\varphi$.

\(^9\)As will be shown soon, high-productivity firms will choose FDI in equilibrium. Thus, the threat point of managers, $[\Pi_{XI}(\tilde{\varphi}) + \Pi_{DI}(\tilde{\varphi})]/2$, appears to be overstated. However, firms from country $l$ use $\Pi_{XI}(\varphi)/2$. As a mirror image, firms from country $l'$ simultaneously use $\Pi_{XI}(\varphi)/2$ as their threat point. Thus, managers using $[\Pi_{XI}(\tilde{\varphi}) + \Pi_{DI}(\tilde{\varphi})]/2$ as their threat point is consistent.

\(^{10}\)For example, each household owns $K_I/L_l$ shares of all firms and $S_l/L_l$ shares of all managers along with one unit of labor.
3 Properties of the Model

This section examines properties of the model. In what follows, I will focus on home firms’ FDI (foreign firms’ FDI is a mirror image of home firms’ FDI).

3.1 Threshold Match Quality

For successful international match, the profits of FDI are not less than the sum of a firm’s status-quo payoffs manager’s status-quo payoffs: i.e.,

\[ \Pi_{IH}(\varphi, z) \geq \frac{\Pi_{XH}(\varphi)}{2} + \frac{\Pi_{XF}(\tilde{\varphi}) + \Pi_{DF}(\tilde{\varphi})}{2}. \]

(11)

From this condition, for given \( \varphi \), the threshold match quality of \( z^*_H(\varphi) \) below which home firms prefer exports to FDI is expressed by

\[ z^*_H(\varphi) = \frac{1}{2} \left( T_F \omega + \frac{1}{2} T_H m_H \left[ \frac{A_H \varphi}{A_F \tilde{\varphi}} \right]^{1-\sigma} \right), \]

(12)

where \( \omega \equiv (w_F/w_H)^{\sigma-1} = w_H^{1-\sigma} \) is a transformed measure of the relative foreign wage and \( m_H \equiv M_H/M_F \) is the relative home market size.

Equation (12) identifies two effects that govern threshold match quality: the profitability of FDI relative to exports (“FDI-profitability effect”) and the relative bargaining power between the firm and the manager (“relative bargaining-power effect”). The first term of the right-hand side of (12) represents the FDI-profitability effect. This is simply the ratio of the marginal production costs of exporting to FDI. As foreign tariff \( \tau_F \) and/or home wage \( w_H \) rises, FDI becomes more profitable than exports, which leads to a lower threshold match quality.

The next term represents the relative bargaining-power effect since this term is the ratio of the foreign manager’s status-quo payoff to FDI (gross) surplus. As either home tariff \( \tau_H \) rises, the relative home maker size \( m_H \) falls, or the average productivity level in foreign \( A_F \tilde{\varphi} \) falls, foreign managers lose their bargaining power since the status-quo payoffs decline. Thus, in either case, the threshold match quality falls. It should be noted that firm heterogeneity affects the threshold match quality of \( z^*_H(\varphi) \) not through the FDI-profitability effect but through the relative bargaining-power effect. Intuitively, when matches with a high-productivity home firm are realized, it becomes less attractive for a foreign manager to wait for opportunities of working with a local firm. As a result, the foreign manager is
willing to accept a relatively lower share of FDI surplus, which lowers the threshold match quality.

Letting \( s(\varphi, z) \) denote a foreign manager’s payoffs, the matched home firm obtains \( \pi_{IH}(\varphi, z) = \Pi_{IH}(\varphi, z) - s(\varphi, z) \) from FDI. The Nash solution gives \( \pi_{IH}(\varphi, z) \) and \( s(\varphi, z) \), respectively, as follows:

\[
\pi_{IH}(\varphi, z) = \left[ \frac{2z + T_F \omega}{4} \right] M_F [A_H \varphi]^{\sigma-1} - \frac{[M_H T_H + M_F] [A_F \tilde{\varphi}]^{\sigma-1}}{4},
\]

(13)

\[
s(\varphi, z) = \left[ \frac{2z - T_F \omega}{4} \right] M_F [A_H \varphi]^{\sigma-1} + \frac{[M_H T_H + M_F] [A_F \tilde{\varphi}]^{\sigma-1}}{4}.
\]

(14)

Since home firms’ net profits from exports are given by \( \pi_{XH}(\varphi) = M_F T_F \omega [A_H \varphi]^{\sigma-1}/2 \), we can immediately establish the following result from (13).

**Lemma 1.** As long as \( \lambda > T_F \omega/2 \), FDI is always viable, and FDI is more profitable than exports for high-productivity firms.

**Proof.** See Appendix.

The result that FDI is more profitable than exports for high-productivity firms itself is not new. However, the model provides a novel perspective on the FDI fixed costs that would be deeply related to local managers’ status-quo payoffs. In particular, the model emphasizes the profitability of local firms where local managers may alternatively work.\(^{11}\) Notice that the foreign manager’s payoffs include the fixed payment (the second term of the right-hand side of (14)). The source of this fixed payment is, of course, the outside option of managers: they may work with domestic firms instead of multinational enterprises (MNE). Thus, any changes that will raise the value of the outside option, such as an improvement of the average productivity of foreign firms \( (A_F \tilde{\varphi} \uparrow) \), a lower trade cost for exporting to home \( (T_H \downarrow) \), and an increase in the relative market size \( (m_H = M_H / M_F \uparrow) \), lead to an increase in the fixed costs for FDI and have FDI difficult for home firms with low intrinsic productivity levels.

We can explicitly see the relationship between FDI difficulty and the bargaining position of foreign managers by considering the threshold (relative) productivity levels. The

\(^{11}\) The literature of firm heterogeneity and international trade typically assumes that \( f_I > \tau^{\sigma-1} f_X \) where \( f_I \) and \( f_X \) are fixed costs for FDI and exports, respectively, and \( \tau \) is a usual iceberg-type transportation cost. See for example Helpman, Melitz, and Yeaple (2004). In reality, there exists various types of fixed costs for MNEs to run foreign affiliates. The model obviously abstracts many of them. However, adding these fixed costs to the model does not alter the model in essential manners.
threshold match quality \( z_H^*(\varphi) \) in (12) is decreasing in \( \varphi \). Since the worst match quality is \( \lambda \), home firms with \( z_H^*(\varphi) \leq \lambda \) always choose FDI irrespective of match quality. In contrast, some home firms with very low productivity levels will not be able to undertake FDI even if they match with the best managers (i.e., \( z_H = 1 \)). Thus, two threshold productivity levels, \( \underline{\varphi} \) and \( \bar{\varphi} \), can be established by setting \( z_H^* = 1 \) and \( z_H^* = \lambda \) in (12), respectively: i.e.,

- home firms with \( \varphi \leq \underline{\varphi} \) always export;
- Firms with \( \varphi \in (\underline{\varphi}, \bar{\varphi}) \) can undertake FDI only when matching with foreign managers yields sufficient match quality \( z \geq z_H^*(\varphi) \). Otherwise, they choose exports;
- Firms with \( \varphi \geq \bar{\varphi} \) always undertake FDI,

where

\[
\underline{\varphi}^{-1} = \frac{1 + T_H m_H}{2 - T_F \omega} \left[ \frac{A_F \bar{\varphi}}{A_H} \right]^{-1} \quad \text{and} \quad \bar{\varphi}^{-1} = \frac{1 + T_H m_H}{2 \lambda - T_F \omega} \left[ \frac{A_F \bar{\varphi}}{A_H} \right]^{-1}.
\]

The size of the productivity range \((\underline{\varphi}, \bar{\varphi})\) where firms’ FDI decision making depends on match quality \( z_H \) is measured by the relative threshold productivity level of \( \bar{\varphi}/\underline{\varphi} \). This is simply given by

\[
\frac{\bar{\varphi}}{\underline{\varphi}} = \left[ \frac{2 - T_F \omega}{2 \lambda - T_F \omega} \right]^{1/(\sigma - 1)}.
\]

The properties of the two threshold productivity levels \( \underline{\varphi} \) and \( \bar{\varphi} \) are recorded in the following proposition.

**Proposition 1.** There exist two threshold productivity levels, \( \underline{\varphi} \) and \( \bar{\varphi} \). Firms with intrinsic productivity levels below \( \underline{\varphi} \) serve the foreign market via exports while firms with intrinsic productivity levels above \( \bar{\varphi} \) serve the foreign market via FDI. In the middle range of \((\underline{\varphi}, \bar{\varphi})\), firms may serve the foreign market via either exports or FDI.

The two threshold productivities show the following properties:

1. They are increasing in the relative home market size \( m_H \), the average foreign firms’ productivity \( A_F \bar{\varphi} \), the inverse of trade costs \( T_H \) and \( T_F \), and the relative foreign wage \( \omega \).

2. The distance between these two threshold productivity levels becomes wider when (i) matching efficiency falls \( \lambda \downarrow \), (ii) the trade cost for exporting to foreign falls \( T_F \uparrow \), and the relative foreign wage rises \( \omega \uparrow \).
Since it is straightforward to obtain these results from (15) and (16), the proof is omitted. These results are rather intuitive. When \( \lambda \) declines, it becomes more difficult for firms to find acceptable managers for FDI. Thus, even relatively productive firms may fail to undertake FDI, which leads to a wider productivity scope where firms with higher productivity levels may export while those with lower productivity levels may undertake FDI. A lower trade cost or a lower relative home wage decreases the profitability of FDI relative to exports. Again, matching becomes difficult even for relatively high productive firms, which also results in a wider productivity scope where high productive firms may export while low productive firms may choose FDI.

As a simple application of the model, it may be interesting to consider FDI between developed and developing countries. In such FDI, firms in developed countries set up foreign affiliates for exploiting the inexpensive production factor in developing countries. It is simple to presume that \( w_H > w_F \) where home is developed and foreign is developing. However, the model suggests that if we assume that foreign local firms are technologically behind those in home, it may lower the level of status-quo payoffs for foreign managers, which encourages relatively unproductive home firms to undertake FDI. This prediction seems consistent with empirical regularities.

Foreign managers are uniformly distributed on the circumference of the circle. Hence, for range \( (\bar{\varphi}, \bar{\varphi}) \), the probability of a successful match for FDI is expressed by

\[
Prob(z \geq z^*_H(\varphi)) \equiv \delta_H(\varphi) = \frac{1 - z^*_H(\varphi)}{1 - \lambda}.
\]

(17)

For a given \( \varphi \), the average match quality \( \tilde{z}_H(\varphi) \) is simply expressed by \( \tilde{z}_H(\varphi) = [1 + z^*_H(\varphi)]/2 \). Since \( z^*_H(\varphi) \) is decreasing in \( \varphi \), the probability of a successful match increases as \( \varphi \) rises while the average match quality \( \tilde{z}_H(\varphi) \) falls. I record these results as a following proposition.

**Proposition 2.** For home firms with \( \varphi \in (\bar{\varphi}_H, \bar{\varphi}_H) \), the probability of successful matching is increasing in \( \varphi \). The average quality of international matches declines as the firm’s intrinsic productivity level rises until it reaches \( \bar{\varphi}_H \). Then, the average quality of international matches is constant at \( (1 + \lambda)/2 \) for firms with not less than \( \bar{\varphi}_H \).

Plugging the average match quality \( \tilde{z}_H(\varphi) \) into equations (13), the average FDI payoff to home firms with \( \varphi \), \( \tilde{\pi}_H(\varphi) \), can be expressed by
\[
\bar{\pi}_H(\varphi) = \begin{cases} 
\left[ \frac{2 + 3T_F \omega}{8} - \frac{[M_H T_H + M_F][A_H \varphi]^{\sigma - 1}}{4} \right] M_F[A_H \varphi]^{\sigma - 1} \quad &\text{if } \varphi \in [\underline{\varphi}_H, \bar{\varphi}_H], \\
\left[ \frac{1 + \lambda - T_F \omega}{4} \right] M_F[A_H \varphi]^{\sigma - 1} \quad &\text{if } \varphi \in [\bar{\varphi}_H, \infty),
\end{cases}
\]

(18)

In a similar vein, from (14), the average FDI payoff to foreign managers that match with home firms with \(\varphi\) is given by

\[
\tilde{s}_F(\varphi) = \begin{cases} 
\left[ \frac{2 - T_F \omega}{8} + \frac{3[M_H T_H + M_F][A_F \varphi]^{\sigma - 1}}{4} \right] M_F[A_H \varphi]^{\sigma - 1} \quad &\text{if } \varphi \in [\underline{\varphi}_H, \bar{\varphi}_H], \\
\left[ \frac{1 + \lambda - T_F \omega}{4} \right] M_F[A_H \varphi]^{\sigma - 1} \quad &\text{if } \varphi \in [\bar{\varphi}_H, \infty).
\end{cases}
\]

(19)

From these expressions on the average payoffs, the following statement is recorded as a proposition.

**Proposition 3.** As firms’ intrinsic productivity \(\varphi\) rises, the average FDI payoffs for firms rise more rapidly than those for managers, which implies that on average, productive firms’ FDI profit share is greater than unproductive firms’.

The intuition of this proposition is readily understood by referring to the threshold match quality \(z^*(\varphi)\) in (12). Equation (12) shows that as firms’ intrinsic productivity goes up, the threshold match quality falls through weakening the relative bargaining-power effect.

### 3.2 FDI Sales

The total mass of home FDI firms, \(K_{IH}\), is given by

\[
K_{IH} = K_H \left[ (1 - G(\bar{\varphi}_H)) + \int_{\underline{\varphi}_H}^{\bar{\varphi}_H} \frac{1 - z^*_H(\varphi)}{1 - \lambda} g(\varphi) d\varphi \right].
\]

(20)

Then, the average productivity of home FDI firms, \(\bar{\varphi}_{IH}^{\sigma - 1}\), is

\[
\bar{\varphi}_{IH}^{\sigma - 1} = \frac{K_H}{K_{IH}} \left[ \int_{\underline{\varphi}_H}^{\bar{\varphi}_H} \bar{z}_H(\varphi) \varphi^{\sigma - 1} g(\varphi) d\varphi + \int_{\bar{\varphi}_H}^{\infty} \left[ \frac{1 + \lambda}{2} \right] \varphi^{\sigma - 1} g(\varphi) d\varphi \right].
\]

(21)

Thus, FDI average sales per firm are given by \(\sigma M_F(A_H \bar{\varphi}_{IH})^{\sigma - 1}\), and the total FDI sales, \(R_{IH}\), are given by

\[
R_{IH} = \sigma K_H M_F(A_H)^{\sigma - 1} \left[ \int_{\underline{\varphi}_H}^{\bar{\varphi}_H} \bar{z}_H(\varphi) \varphi^{\sigma - 1} g(\varphi) d\varphi + \int_{\bar{\varphi}_H}^{\infty} \left[ \frac{1 + \lambda}{2} \right] \varphi^{\sigma - 1} g(\varphi) d\varphi \right].
\]

(22)
Table 1: Parameter values and some key variables

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elasticity of substitution between varieties</td>
<td>$\sigma = 5$</td>
</tr>
<tr>
<td>Shape parameter</td>
<td>$k = 6$</td>
</tr>
<tr>
<td>Trade cost</td>
<td>$T_l = 0.0625$ ($\tau_l = 2$)</td>
</tr>
<tr>
<td>Match efficiency (informational frictions)</td>
<td>$\lambda = 0.1, 0.6, 1$</td>
</tr>
<tr>
<td>Average productivity for domestic production</td>
<td>$\bar{\varphi} = 1.32$</td>
</tr>
<tr>
<td>Upper threshold productivity</td>
<td>$\bar{\varphi} = 1.29(\lambda = 0.4), 2.19(\lambda = 0.9)$</td>
</tr>
<tr>
<td>Lower threshold productivity</td>
<td>$\varphi = 1.13$</td>
</tr>
</tbody>
</table>

Recall that $M_F = \gamma E_F/(P_F^{1-\sigma} \alpha^{1-\sigma})$. As shown in the Appendix, the price index is given by

$$P_F^{1-\sigma} = K_H(\alpha A_H \bar{\varphi}_{HF})^{\sigma-1} + K_F(\alpha A_F \bar{\varphi})^{\sigma-1}$$

(23)

where $\bar{\varphi}_{HF}^{\sigma-1}$ is the average productivity of home firms when accessing foreign. The Appendix gives the exact expression of $\bar{\varphi}_{HF}^{\sigma-1}$.

4 Numerical Examples

This subsection illustrates how the model predicts the distribution of FDI firms with numerical examples. In so doing, we need to set several parameter values. The baseline parameter values used in the examples are reported in Table 1. The elasticity of substitution between differentiated goods is set to $\sigma = 5$. The degree of firm heterogeneity is $k = 6$. Mayer and Ottaviano (2007) report that it is 3.03 and 2.55 for Italy and France. Wakasugi, Toda, Sato, Nishioka, Matsuura, Ito, and Tanaka (2008) estimate that $k$ is about 1.7 for Japanese firms.\textsuperscript{12} However, in order for the size distribution of firms to have a finite mean, we need $k > \sigma - 1$. Thus, if $\sigma = 5$ is used, appropriate $k$s are greater than 4. Here I set $k = 6$ as a baseline parameter.\textsuperscript{13} Trade cost $\tau_i$ is set to 2 for both home and foreign, which implies that $T_l = 0.0625$.

First, we need to derive the conditional pdf for FDI firms. For simplicity, it is assumed that the two countries are symmetric. Also, the effectiveness of one unit of labor is set at 1 ($A_H = A_F = 1$). When the two countries are symmetric, the derivation of total expenditure

\textsuperscript{12}This estimate appears too small. There is a possibility that the data set used in their study might suffer from the lack of data especially for small firms. In fact, after abandoning samples from very small firms, re-estimation generates $k$ greater than 2.

\textsuperscript{13}Eaton, Kortum, and Kramarz (2008) find that $k/(\sigma - 1)$ is about 1.5 for French firms.
$E$ is straightforward. Since trade is balanced, the total expenditure is

$$E = L + \frac{\gamma E}{\sigma}.$$  
Solving this equation, we obtain $E = \sigma L/(\sigma - \gamma)$.

The conditional pdf for FDI firms, $h(\varphi)$, is given by

$$h(\varphi) = \begin{cases} \Gamma^{-1} \left[ \frac{1-z^*(\varphi)}{1-\lambda} \right] k\varphi^k \varphi^{-k-1} & \text{if } \varphi \in [\bar{\varphi}, \bar{\varphi}], \\ \Gamma^{-1} k\varphi^k \varphi^{-k-1} & \text{if } \varphi \in [\bar{\varphi}, \infty), \end{cases}$$

(24)

where $\Gamma = A_1 k^{\bar{\varphi} - k} + A_2 k^{\bar{\varphi} - k} - k^{\bar{\varphi} - k}$.

Figure 3 illustrates conditional probability density functions of FDI firms for three different cases: $\lambda = 1, 0.6, \text{ and } 0.1$. In the case of $\lambda = 1$, there is no match frictions for FDI so that the pdf is of a Pareto distribution (the dotted curve in the figure). Existing firms’ productivity level starts at $\varphi = 1$ and the model gives the cutoff productivity level of $\varphi = 1.32$, above which firms can always undertake FDI.

Once we introduce matching frictions into the model, the pdfs change dramatically. The curve expressed by a solid line is the case of $\lambda = 0.6$. The shape is much more akin to those of empirically obtained from the data of Japanese firms. With uncertainty about foreign managers’ quality, even relatively productive firms may fail FDI. In this case, firms with productivities between 1.13 and 1.29 may export or undertake FDI. Here two elements govern the FDI firm distribution: firms’ intrinsic productivity $\varphi$ and match quality $z$. As is shown in the total cost function, FDI firms’ efficiency is determined by these two elements. In particular, the extent to which firms match with appropriate managers is crucial for firms with low $\varphi$. However, obtaining high match quality is difficult. Thus, even though there are many firms who might undertake FDI near the threshold productivity level $\varphi$, only a limited number of firms can do so. In contrast, highly productive firms do not need to be concerned about match quality. Thus, in high productivity regions in the figure, the effect of the distribution of firms’ productivity $\varphi$ becomes dominant.

5 Concluding Remarks and Extensions

Firm-level data often suggest that firms with very similar productivities select different modes of internationalization although the most productive firms still tend to choose FDI.
for entering foreign markets. This paper examines the foreign direct investment decisions of individual firms with a simple framework where firms and managers have to make matches for production. We find that predicted firm distributions are much more akin to those suggested by real data, namely, there exists a range of firm productivities in which more productive firms may export while less productive firms may undertake FDI. Such a range of firm productivities becomes wider when either matching frictions increase or trade costs decline. Furthermore, matching frictions hurt production efficiency more for productive firms than for less productive firms.

This study also addresses the extent to which informational frictions (the lack of information about foreign skilled labor market) hurt industry efficiency through disturbing productive firms becoming multinationals. Several issues should be considered further but are left for future research. Two of them are as follows.
Structure of matching and bargaining  The model is static and a one-shot game. In particular, the sequence of events is important for solving the model. In particular, firms and managers use status-quo payoffs from domestic matching at the stage of international matching. However, at the domestic matching stage, it is impossible for players to use international matching as a threat point. If we allow the repetition of matching, it is necessary to extend the model to the direction of the standard search-matching model where unemployed managers and firms without managers continuously seek match. In this case, the status-quo payoffs will be more generalized.

FDI sales  The model highlights two distinct elements that affect firms’ FDI decision making: trade costs and matching frictions (the lack of information about the foreign skilled labor market). The interaction between these two elements should be deeply considered. In particular, effects on FDI sales are important. For example, the gravity estimation of FDI sales in Wakasugi, Toda, Sato, Nishioka, Matsuura, Ito, and Tanaka (2008)) reveals that the variation of the extensive margin of FDI sales is largely explained by the distance between two countries. Geographical distances between two countries can be broadly interpreted such as a proxy of transportation costs as well as a proxy of informational frictions in skilled labor (managers). Thus, it is interesting to examine the extent to which the informational frictions highlighted in the paper influences FDI sales.
A Appendix

A.1 Proof of Lemma 1

Both the payoff schedules $\pi_{IH}(\varphi,z)$ and $\pi_{XH}(\varphi)$ are monotonically increasing in $\varphi^{\sigma-1}$. Thus, the slope of $\pi_{IH}$ is steeper than that of $\pi_{XH}$ only when $(2z + T_{F}\omega)/4 > T_{F}\omega/2$. The worst match quality is given by $z = \lambda$. Thus, we can establish the sufficient condition for the FDI viability such that $\lambda > T_{F}\omega/2$.

Then, the difference between a firm’s FDI payoff and export payoff, $\pi_{IH}(\varphi,z) - \pi_{XH}(\varphi)$, is given by

$$
\pi_{IH}(\varphi,z) - \pi_{XH}(\varphi) = \left[\frac{2z - T_{F}\omega}{4}\right] M_{F}\varphi^{\sigma-1} - \left[\frac{M_{H}T_{H} + M_{F}}{4}\right]\tilde{\varphi}^{\sigma-1},
$$

(A.1)

which is increasing in $\varphi^{\sigma-1}$.

A.2 Price Index

The derivation of the foreign price index $P_{F}$ is as follows. The average relative productivity of home firms in foreign, $\tilde{\varphi}_{HF}^{\sigma-1}$, is given by

$$
\tilde{\varphi}_{HF}^{\sigma-1} = \int_{1}^{\infty} T_{F}\omega \varphi^{\sigma-1} g(\varphi) d\varphi + \int_{\tilde{\varphi}_{H}}^{\tilde{\varphi}_{H}} \left[\frac{1 - z_{H}^{*}(\varphi)}{1 - \lambda}\right] \left[\tilde{z}_{H}(\varphi) - T_{F}\omega\right] \varphi^{\sigma-1} g(\varphi) d\varphi
$$

$$
+ \int_{\tilde{\varphi}_{H}}^{\infty} \left[\frac{1 + \lambda}{2} - T_{F}\omega\right] \varphi^{\sigma-1} g(\varphi) d\varphi.
$$

(A.2)

This expression on $\tilde{\varphi}_{HF}$ is easy to interpret. The first term of the right-hand side (RHS) simply means average productivity when all home firms would export to country $F$. FDI brings about productivity gains as shown in the second and third terms. For those belonging to $(\tilde{\varphi}_{H}, \tilde{\varphi}_{H})$, firms obtain the productivity gain $\tilde{z}_{H}(\varphi) - T_{F}\omega$ with probability $[1 - z^{*}(\varphi)]/(1 - \lambda)$. For firms with $\varphi \geq \tilde{\varphi}_{H}$, the productivity gains from FDI is $(1 + \lambda)/2 - T_{F}\omega$.

The computation is straightforward but tedious. The sum of the first term and the last term is given by

$$
\frac{k}{k + 1 - \sigma} \left[T_{F}\omega + \tilde{z}_{H}(\varphi)^{\sigma-k-1} \left(\frac{1 + \lambda}{2} - T_{F}\omega\right)\right].
$$

(A.3)

The second term is much more complicated but given by

$$
\frac{A_{1}k}{k + 1 - \sigma} \left[(\tilde{\varphi}_{H})^{\sigma-k-1} - (\tilde{\varphi}_{H})^{\sigma-k-1}\right] + \frac{A_{2}k}{-\sigma - k + 1} \left[(\tilde{\varphi}_{H})^{\sigma-k+1} - (\tilde{\varphi}_{H})^{\sigma-k+1}\right]
$$

$$
+ A_{3} \left[(\tilde{\varphi}_{H})^{\sigma-k} - (\tilde{\varphi}_{H})^{\sigma-k}\right],
$$

(A.4)
where

\[ A_1 = \frac{1}{8(1 - \lambda)} [4 - T_F \omega - 5(T_F \omega)^2] \quad (A.5) \]

\[ A_2 = \frac{1}{2(1 - \lambda)} \left[ 1 + \frac{T_H m_H}{2} \left( \frac{A_H}{A_F} \right)^{1-\sigma} \right]^{2} \quad (A.6) \]

\[ A_3 = \frac{3T_F \omega}{2(1 - \lambda)} \left[ 1 + \frac{T_H m_H}{2} \left( \frac{A_H}{A_F} \right)^{1-\sigma} \right] \quad (A.7) \]

Using this \( \tilde{\varphi}_{HF}^{\sigma-1} \), the average price of home firms, \( \bar{p}_{HF}^{1-\sigma} \) is given by \( \bar{p}_{HF}^{1-\sigma} = (\alpha A_H \tilde{\varphi}_{HF})^{\sigma-1} \).

Therefore, the ideal price index \( P_F \) is expressed by

\[ P_F^{1-\sigma} = K_H \bar{p}_{HF}^{1-\sigma} + K_F \bar{p}_{F}^{1-\sigma} = K_H (\alpha A_H \tilde{\varphi}_{HF})^{\sigma-1} + K_F (\alpha A_F \tilde{\varphi})^{\sigma-1} \quad (A.8) \]

Furthermore, letting \( \tilde{\varphi}_{tF} \) be the weighted productivity average for the ideal price index \( P_F \), \( \tilde{\varphi}_{tF} \) is expressed by

\[ \tilde{\varphi}_{tF}^{\sigma-1} = \frac{1}{K_H + K_F} \left[ K_F \tilde{\varphi}_{tF}^{\sigma-1} + A_H \frac{A_H}{A_F} K_H \tilde{\varphi}_{HF}^{\sigma-1} \right]. \quad (A.9) \]
References


