Banking Crisis and Borrower Productivity

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In this paper, we propose a theoretical model in which a banking crisis (or bank distress) causes declines in aggregate productivity. When borrowing firms need additional bank loans to continue their businesses, a high probability of bank failure discourages ex ante investments (e.g., R&D investment) by firms that enhance their productivity. In a general equilibrium setting, we also show that there may be multiple equilibria: one in which bank distress continues and borrower productivity is low, and in the other, banks are healthy and borrower productivity is high. We show that the bank capital requirement may be effective in eliminating the bad equilibrium and may lead the economy to the good equilibrium in which the productivity of borrowing firms and the aggregate output are both high and the probability of bank failure is low.

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Abstract

In this paper, we propose a theoretical model in which a banking crisis (or bank distress) causes declines in aggregate productivity. When borrowing firms need additional bank loans to continue their businesses, a high probability of bank failure discourages ex ante investments (e.g., R&D investment) by firms that enhance their productivity. In a general equilibrium setting, we also show that there may be multiple equilibria: one in which bank distress continues and borrower productivity is low, and in the other, banks are healthy and borrower productivity is high. We show that the bank capital requirement may be effective in eliminating the bad equilibrium and may lead the economy to the good equilibrium in which the productivity of borrowing firms and the aggregate output are both high and the probability of bank failure is low.

1 Introduction

Historical episodes of banking crises apparently showed that bank distress causes deterioration of economic activities in the (very) short run mainly due to liquidity shortage. There may exist an additional effect of bank failures that changes the economic structure, possibly in the long run, and deters economic growth. This paper examines the causes and consequences of bank failure and checks how bank failures affect productivity

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growth. During the International Great Depression in the 1930s, many countries experienced banking crises and productivity declines. Cole, Ohanian, and Leung (2005) examined data on 17 countries during the 1930s, and pointed out that there may be a causal relationship between banking crises (or bank distress) and declines in aggregate productivity. In the 1990s, the Japanese economy experienced decade-long bank distress and a slowdown in productivity growth. The bank distress seemed to cause a persistent deterioration of the economy as a whole, though a well-functioning financial market had been developed in Japan.[1] What is the main mechanism which generates such a relationship?

The main purpose of this paper is to reexamine the mechanism of banking panics and to show how failures or panics affect productivity. We will show here that the ex ante investments (e.g., R&D investments) among borrowing firms may affect banking panics. In other words, a coordination failure among borrowers is a trigger for banking panics and productivity declines.

There are many papers which examine bank failures and panics. The causes of these crises have been debated. Some papers have shown that depositor panics are the main factors for the crises[2] and some papers have shown that external shocks generate bank failures. Diamond and Rajan (2005) focused on external shocks on borrowing firms. They showed that those shocks generate liquidity shortages and introduce banking panics. Although those papers implicitly assumed that banking panics affect economic conditions or macro performances, they have not explicitly examined the relation between banking panics and productivity.[3] Hence it is not so clear how bank failures affect economic conditions. Even when there is a bank run, for example, new banks might be able to offer alternative financial services.

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[1]For example, see Hayashi and Prescott (2002)

[2]The seminal paper is Diamond and Dybvig (1983). For example, Allen and Gale (2000), Bhattacharya and Gale (1987) are related papers.

[3]Levine and Zervos (1998) have shown that banks and stock markets provide different services and both contribute to economic growth. From this result we can infer that bank failures deter economic growth. They did not , however, explicitly examine this possibility.

In this paper, we will show another mechanism which generates bank panics. This paper focuses on the behaviors of borrowing firms. In this sense, this paper is related to Diamond and Rajan (2005). The crucial difference between this paper and Diamond and Rajan (2005) is that this paper assumes productivity conditions of firms are endogenously determined, although Diamond and Rajan (2005) have assumed there are exogenous random variables in the borrowing sector. By treating them endogenous, it becomes possible to acquire some important insights. First, we can derive another reason for banking panics. This paper stresses the coordination failure of borrowers. Of course, we do not contest the reasons those previous papers have explored. We will show there is another possibility. It might seem strange that borrowers affect the condition of a bank since they have already borrowed from the bank. If the borrowing firms may require additional investments or liquidities, however, it becomes natural that conditions of other borrowing firms affect the balance sheet of the lending bank and the incentive of a borrowing firm. We will stress this relation in this paper. Second, it becomes easier to explain the relation between banking panics and economic productivity. Since the productivity of each firm becomes endogenous, we can examine productivity and bank panics directly. The possibility of bank failure decreases the incentive of borrowing firms and decreases the productivity of borrowing firms.

To explain these points, we use a theoretical model in which bank distress causes a decline in the productivity of the borrowing firms, even though there exists a well-functioning financial market. The model is a modified version of the models of Diamond and Rajan (2005) and Holmstrom and Tirole (1998). We assume that a firm can enhance its own productivity through costly ex ante investment, which may be interpreted as investment in Research and Development activities. (Thus we call this ex ante investment "R&D investment.") The firm needs to borrow from a bank to start the business, and it also needs an additional investment at the interim period to continue its business if the firm is hit by a shock. From the aspect of specific skill as explored by Diamond and Rajan (2001, 2005), we assume the return of the project is not perfectly verifiable and seizable to lenders. Hence, as in Holmstrom and Tirole's model, firms cannot borrow

additional funds in the financial market when incumbent banks fail.

In this setting, if there is a positive probability of bank failures, a borrowing firm cannot get the necessary additional loan and must shut down its business with the positive probability. Hence when a high probability of bank failure is expected, a firm expects high probability of shutdown and low expected return on R&D investment. Since we assume R&D investment by individual firms enhances productivity, less R&D investment leads to a lower level of productivity. In other words, we can show that a banking crisis leads to less R&D investment in the borrowing firms, and the decline of aggregate productivity.

Next we embed this partial equilibrium model into a general equilibrium setting in which consumers, as depositors and bank shareholders, provide funds to firms through banks. Modeling the general equilibrium economy, we endogenize the probability of bank failure, and show that the economy may end up with two steady state equilibria: a good equilibrium and a bad equilibrium. The key point is an externality effect among borrowers. Less R&D investment not only decreases productivity but also increases the probability of bank failure. This means the level of R&D investment has the externality effect to other borrowing firms through the change of bank failure probability. Moreover, if those low productivities are anticipated by depositors, they will demand resources immediately and generate bank runs, as stressed by Diamond and Rajan (2005). Hence, in the good equilibrium, firms choose the highest level of R&D investment, which generates high aggregate productivity and low probability of bank failure. In the bad equilibrium, however, firms choose the lowest level of R&D investment, aggregate productivity is low, and the probability of bank failure is high.

In this general equilibrium model, we conduct numerical experiments in which we impose capital requirement for banks. The capital requirement policy has an effect, which eliminates the bad equilibrium for a certain range of parameter values. Therefore, the capital requirement policy may have an impact that increases the aggregate productivity of the economy and, through enhancing the R&D investment by firms, lowers the probability of a banking crisis. Our result implies that the bank capital requirement may

have a significant impact on social welfare as a whole by affecting aggregate productivity, while existing literature on this topic tends to stress moral hazard or adverse selection problems in the banking sector (see, for example, Morrison and White 2005).

The organization of this paper is as follows. In Section 2, we present a simple example to show our propositions intuitively, and we construct the partial equilibrium version of our model in Section 3. In Section 4, we embed the partial equilibrium model in the general equilibrium in which bank failure possibility is endogeously determined. We describe the model with and without bank capital requirements. In Section 5, we demonstrate numerical examples of the general equilibrium model. Section 6 provides concluding remarks.

2 Simplified Example

To clarify the basic structure of our model, we demonstrate a simplified model in this section. There are N firms which have a potential investment opportunity. This investmet requires 1 input at date 1 and will generate $R > 1$ at date 3. Only fR is seizable for banks, however, since R is not perfectly verifiable. In other words, $(1-f)R$ becomes the benefit of each firm under any type of contract. This investment opportunity may require additional investments by an idiosyncratic and independent shock at date 2. With probability q, the additional investment ρ becomes necessary. For simplicity, we assume that the firm generates 0 at date 3 if this additional investment is not implemented. The market interest rate is supposed to be zero for simplicity.

One crucial assumption is that R is dependent upon R&D investment, s, of each firm. Each firm can choose s^H or s^L and the private cost for choosing s^H (s^L) is C (0). Naturally $R(s^H)$ is higher than $R(s^L)$. R&D investment is observable and is chosen at date 1 before the loan contract is made. Since all firms choose their R&D investments simultaneously, they cannot coordinate their choice over s_H or s_L: Thus there is a possibility of coordination failure. It is assumed that

$$fR(s^H) > 1 + q\rho.$$

In other words, this lending is profitable for a bank as long as the firm chooses s^H, even though it can seize only $fR(s^H)$ and it has to pay the additional investment cost ρ. Here we assume, however,

$$fR(s^H) < \rho.$$

In this setting, as explored by Holmstrom and Tirole (1998), it is difficult to get ρ in the market after the shock at date 2. But a bank can offer a credit line contract at date 1 which guarantees to supply ρ when the additional investment is necessary.

Moreover, as long as there is no bank failure and

$$(1 - f)R(s^H) - C > (1 - f)R(s^L),$$

each firm has incentive to choose s^H. Hence it is a Nash equilibrium that all firms choose s^H.

If R&D investment affects a possibility of bank failure, however, there may exist another equilibrium. Suppose that a bank will fail at date 2 with probability ν_c and this probability is a decreasing function of R&D investment of the borrowing firms. When all firms choose s^H, ν_c becomes 0, but it becomes very high when all firms choose s^L. If a bank fails at date 2, the borrowing firm cannot get ρ after the shock and R becomes 0 even though the firm chooses s^H. In this situation, ν_c becomes high and it may become very difficult to get ρ if $N-1$ firms choose s^L. Hence another firm cannot have incentive to choose s^H. More rigorously, if

$$\{1 - q\nu_c(s_i^H, s_{-i}^L)\}(1 - f)R(s^H) - C < \{1 - q\nu_c(s_i^L, s_{-i}^L)\}(1 - f)R(s^L),$$

there is another equilibrium in which all firms choose s^L where $\nu_c(s_i^H, s_{-i}^L)$ is the probability of bank failure when firm i chooses s^H and the other firms choose s^L. In other words there are multiple equilibria.

This result is an intuitive explanation of our propositions. From the next section, we formulate a more rigorous model to examine this intuition.

3 Partial equilibrium model: Exogenous probability of bank run

In this section, we consider a partial equilibrium model in which the probability of a bank run is exogenously given. In the next section, we embed this model in a general equilibrium economy in which consumers provide funds to firms through banks and the probability of a bank run is endogenously determined.

The economy is a simplified version of Holmstrom and Tirole's (1998) model, in which there are continua of banks and firms. Measures of firms and banks are normalized to one. The economy continues for only one period, and agents can choose their actions two times: at the beginning of the period and at the middle of the period. Following Holmstrom and Tirole (1998), we assume that banks have all bargaining power over firms, and they maximize the expected return on loans to firms, making firms break-even.

3.1 Firm

Firms are indexed by i, where $i \in [0, 1]$. At the beginning of the period, firm i chooses its level of R&D investment, s_i, where $s_i \in [0, 1]$. We assume that s_i is observable. R&D investment incurs private cost ξs_i for the firm. After s_i is chosen, firm i borrows X_i units of consumer goods from a bank, and invest X_i in its production project. At the middle of the period, a macro shock $\nu \in [\underline{\nu}, 1]$ and an idiosyncratic shock $\rho_i \in \{0, \rho\}$ hit the economy. $\rho_i = 0$ with probability $1 - q$, and $\rho_i = \rho > 0$ with probability q. The macro shock ν indicates the success probability of a firm's project (see equations [1] and [2] below). If $\rho_i = \rho$, firm i needs to invest additional fund ρX_i at the middle of the period in order to continue the project. Otherwise, the project must be shut down leaving the liquidation value, $(1 - \delta)X_i$. If firm i successfully finances ρX_i or $\rho_i = 0$, it can continue the project.

In order to consider the situation in which the return of the project is not perfectly seizable to investors, we consider the following moral hazard situation. After firm i chooses to continue the project, the firm faces an opportunity to shirk. If the firm shirks,

it enjoys private benefit, $b(s_i)$, but the output, y_i, at the end of the period becomes

$$y_i = \begin{cases} (r + s_i)X_i, & \text{with prob. } \nu - \underline{\nu}, \\ (1 - \delta)X_i, & \text{with prob. } 1 - \nu + \underline{\nu}, \end{cases} \tag{1}$$

where $r > 1$. If the firm does not shirk and works diligently, it does not obtain private benefit, but the output at the end of the period becomes

$$y_i = \begin{cases} (r + s_i)X_i, & \text{with prob. } \nu, \\ (1 - \delta)X_i, & \text{with prob. } 1 - \nu. \end{cases} \tag{2}$$

Therefore, if a firm shirks, the success probability of its project is lowered by $\underline{\nu}$. The private benefit for the firm of shirking, $b(s_i)$, may be increasing in the level of R&D investment, s_i. Note that R&D investment directly increases the output. Thus the average of s_i can be interpreted as "aggregate productivity" in this model. We assume a weakly convex benefit:

$$b(s_i) = \frac{b'}{2}s_i^2 + c's_i, \tag{3}$$

where $b' \geq 0$ and $c' > 0$. In order to give the incentive for not shirking, lenders have to abandon a part of the output as will be explained below.

3.2 Bank failure and debt contract

We assume that if the macro shock satisfies $\nu \leq \nu_c$, a bank run occurs and all banks are shut down at the middle of the period, where ν_c is an exogenous parameter in this section. (The bank run is endogenized in the general equilibrium setting in Section 4.)

A bank solves the following problem:

$$\max_{R_f(i)} \int_{\nu_c}^{1} \left[\nu \left\{ r + s_i - R_f(i) \right\} + (1 - \nu)(1 - \delta) \right] df(\nu) - 1 - q\rho, \tag{4}$$

subject to

$$\underline{\nu}R_f(i) \geq b(s_i), \tag{5}$$

where $f(\nu)$ is the p.d.f. for ν and $R_f(i)$ is the final payment to firm i. $R_f(i)$ must be determined such that firm i gets better expected payment when it works diligently than when it shirks. Binding (5) gives

$$R_f(i) = \frac{b(s_i)}{\underline{\nu}} \equiv \frac{b}{2} s_i^2 + c s_i, \tag{6}$$

where $b = b'/\underline{\nu}$ and $c = c'/\underline{\nu}$.

3.3 Firm's problem

Anticipating R_f in (6), firm i chooses s_i before it borrows from a bank. We assume that the loan contract between the bank and the firm survives even if the bank fails: The firm must repay retaining R_f as long as output is produced; and if firm i is hit by the idiosyncratic shock ρ_i after the bank failed, the firm cannot obtain the necessary funds for additional investment and its output becomes $(1 - \delta)X_i$, all of which is to be repaid to the creditor. (When the bank fails, the creditor of the loan contract is a group of bank depositors. See Section 4 for details.) Therefore, firm i solves

$$\max_s \int_{\nu_c}^1 (\frac{b}{2} s_i^2 + c s_i) \nu df(\nu) + \int_{\underline{\nu}}^{\nu_c} (1 - q)(\frac{b}{2} s_i^2 + c s_i) df(\nu) - \xi s_i. \tag{7}$$

We assume that

$$(r + s - \frac{b}{2} s^2 - cs)\nu_c + (1 - \nu_c)(1 - \delta) < \rho, \quad \text{for all } s \in [0, 1]. \tag{8}$$

This assumption ensures that a firm cannot finance ρ in the financial market when the bank fails. We also assume that

$$E(\nu)\left(r + s - \frac{b}{2} s^2 - cs\right) + (1 - E(\nu))(1 - \delta) > 1 + q\rho, \quad \text{for all } s \in [0, 1]. \tag{9}$$

This assumption ensures that a bank will commit to providing a credit line of ρ before ν is revealed in case of a liquidity shock. In the case where ν follows a uniform distribution, i.e., $f(\nu) = \frac{1}{1 - \underline{\nu}}$, the firm's problem can be rewritten as:

$$\max_{s_i} \frac{1 - q\nu_c^2 - (1 - q)\underline{\nu}^2}{2(1 - \underline{\nu})} \left[\frac{b}{2} s_i^2 + c s_i\right] - \xi s_i. \tag{10}$$

9

The derivative of the objective function in (10) with respect to s_i is

$$\frac{1 - q\nu_c^2 - (1 - q)\underline{\nu}^2}{2(1 - \underline{\nu})}\left[bs_i + c\right] - \xi. \tag{11}$$

Obviously, if ν_c, the probability of bank failure, is large, the equilibrium value of s_i may be 0, the lower bound, and that if ν_c is small, s_i may be 1, the upper bound.

3.4   Implication of the model

This partial equilibrium model implies that bank distress, i.e., a large ν_c, may lower the level of R&D investment of the borrowing firms and therefore may lead to a lower level of aggregate productivity for the economy. A higher probability of bank failure implies a higher probability that the borrowing firm fails to continue business, since the firm cannot obtain additional funds if the bank fails. Therefore, the expected return on the ex ante R&D investment for the firm becomes lower if ν_c is higher, and it chooses the lowest level of R&D investment. Since R&D investment enhances productivity, a bank distress causes productivity declines in our model. In this sense, our theory seems successful in explaining productivity declines observed during banking crises, such as the episodes during the US Great Depression (see Cole and Ohanian 1999 and Ohanian 2001) and the lost decade in Japan in the 1990s (see Hayashi and Prescott 2002). Our model may be regarded as one explanation for the conjecture by Cole, Ohanian, and Leung (2005) that the banking crises may have some causal linkage with productivity declines during the International Great Depression.

4   General equilibrium model

We can embed the model of the previous section in the general equilibrium setting and endogenously determine the probability of bank failure, ν_c. The summary of the structure of the model is as follows: firms choose the degree of R&D investment, s, taking ν_c as given; banks choose ν_c, taking s and the market rate of interest, R, as given; and R is determined as an outcome of the general equilibrium. Therefore, the equilibrium of this model can be regarded as a Nash equilibrium in a simultaneous game in which firms

choose s and banks choose ν_c, taking the other players' actions and R as given. (Although we used the term "simultaneous" game, the timing of the game is that banks choose ν_c after s is chosen by firms. Our theoretical and numerical results in this paper do not change even if the banks are the Stackelberg leaders, i.e., if the banks can precommit to ν_c before firms choose s, taking the best response of the firms into account. See footnote 7.) Similar to the previous section, we assume that the economy continues for only one period. There are continua of consumers, firms, and bank managers, whose measures are normalized to one. A consumer is given E units of the consumer goods as endowment at the beginning of the period. Consumers can either invest the endowment in bank capital, C, or put it into the banks as deposits, D:

$$C + D \leq E. \tag{12}$$

The relationship among bank managers, bank capital (consumers), and depositors (consumers) is similar to that in Diamond and Rajan's (2000) model. A bank manager has relation-specific technology to collect on loans from firms, but he can threaten the bank capital and the depositors that he will walk away without collecting the loans unless he is paid more (the hold-up problem). We assume for simplicity that renegotiation can take place only at the middle of the period. To prevent the hold-up problem by the bank managers, the bank capital and the depositors set the deposit contract as the demandable deposit. Therefore, the depositors can withdraw their deposits at any time they like during the period. Since bank deposit is demandable, depositors run on banks if the bank manager threaten the depositors by offering a renegotiation to lessen the payoff of the depositors, and the bank run destroys the bank manager's surplus. Anticipating this result, the bank manager cannot invoke renegotiation under demandable deposit. To make the bank deposit demandable is the optimal design to ensure that the rate of return to bank deposit is high and to increase the funds deposited in banks. (In the equilibrium, bank runs may not occur.)

This contractual arrangement may have a side-effect when a macro shock ν is introduced into the economy: Under demandable deposit contracts, bank runs occur when the macro shock ν is less than a certain threshold value, ν_c. The feature that a macro

shock triggers a bank run is the same as Allen and Gale's (1998) optimal finanical crisis model.

Bank runs: We assume the following for the payoffs of the agents in the event of a bank run. When the bank run occurs, the ownership of bank assets is transferred to the groups of depositors. Thus, the bank capital obtains zero. A borrowing firm produces $(r + s_i)X_i$ if it is not hit by the idiosyncratic shock ρ, while it can produce $(1 - \delta)X_i$ if it is hit by the shock ρ, since it cannot obtain the additional investment necessary to continue production. Therefore, a firm gets $R_f(i)$ with probability $(1 - q)$ and zero with probability q. We assume that the depositors get $(1 - \delta)X + q\rho X + L$, which is the sum of the liquidation value of bank lending, $(1 - \delta)X$, the remaining liquid asset, L, and $q\rho X$, which was kept for lending to the firms that will be hit by the shock ρ. Here we implicitly assumed that a firm's output that exceeds $(1 - \delta)X_i$ is simply vanished as a dead weight loss due to the resource-consuming negotiations among depositors (or rent-seeking activities). This inability of depositors is consistent with the assumption that only the bank managers have relation-specific technology to collect the full value of the bank loans.

4.1 Bank capital's problem

A bank capital, i.e., a coalition of consumers who invest C into a bank, takes the market rate of interest, R, and the level of the borrower's R&D investment, s_i, as given. The bank capital chooses the deposit, D, that it borrows, the investment in a safe asset, L, the investment in the (risky) firms, X, the deposit rate, R_d, the rate of final payment to firm i, $R_f(i)$, and the threshold value of the macro shock that triggers a bank run, ν_c. Safe asset L is just storage of the consumer goods. Thus one unit of L can be converted to one unit of consumer goods at any time. X is the loan to firms, which is invested in the production projects by the borrowers. We assume that a bank lends to an infinite number of firms so that the idiosyncratic risk, ρ_i is perfectly diversified for the bank. Therefore, a bank that lends X to firms must lend $q\rho X$ additionally to the firms at the

interim period when the idiosyncratic liquidity shocks are revealed. (We assume that (9) is satisfied so that the commitment to the credit line ρX is ex ante optimal for a bank.) A bank run occurs if $\nu < \nu_c$. If $\nu = \nu_c$, it must be the case that depositors are indifferent on whether or not to run on the bank. This condition is equivalent to $R_d D = [\nu_c \{r + s_i - R_f(i)\} + (1 - \nu_c)(1 - \delta)]X + L$. We assume that the bank capital can obtain only the fraction θ (< 1) of the total surplus without the help of the bank manager who has the relation-specific technology of collecting loans. This assumption implies that the total surplus of the bank is divided by bargaining such that θ goes to the bank capital and $1 - \theta$ to the bank manager. Finally, in order to prevent the hold-up problem, the bank capital needs to set the assets and liabilities such that the liquidation value of total assets is less than the deposit liabilities, i.e., $(1 - \delta)X + q\rho X + L \leq R_d D$.

Therefore, the bank capital solves

$$\max_{D, L, X, \nu_c, R_d, R_f} \theta \int_{\nu_c}^{1} \left([\nu\{r + s_i - R_f(i)\} + (1 - \nu)(1 - \delta)]X + L - R_d D\right) f(\nu)d\nu, \quad (13)$$

subject to

$$D + C = L + (1 + q\rho)X, \tag{14}$$

$$\{(1 - \delta)X + q\rho X + L\} \cdot \text{Prob}(\nu < \nu_c) + R_d D \cdot \text{Prob}(\nu \geq \nu_c) \geq RD, \tag{15}$$

$$R_d D = [\nu_c\{r + s_i - R_f(i)\} + (1 - \nu_c)(1 - \delta)]X + L, \tag{16}$$

$$\underline{\nu} R_f(i) \geq b(s_i), \tag{17}$$

$$(1 - \delta)X + q\rho X + L \leq R_d D, \tag{18}$$

where (14) is the balance sheet identity for the bank, and (15) is the participation constraint for depositors. Incentive compatibility for firm i implies

$$R_f = \frac{b}{2}s_i^2 + cs_i. \tag{19}$$

4.2 Solution to the bank's problem

We focus on the case where the macro shock, ν, follows uniform distribution over $[\underline{\nu}, 1]$, i.e., $f(\nu) = \frac{1}{1 - \underline{\nu}}$. We define s as the average level of R&D investment of the bank's

borrowers. We focus on the symmetric equilibrium in which all firms choose the same R&D investment: $s_i = s$. We assume that conditions (14)–(17) are binding, while (18) is not binding in the equilibrium. We justify that (18) is satisfied with strict inequality for the parameter values in the numerical examples in Section 5. The reduced form of the bank capital's problem is

$$\max_{X,L,\nu_c} \frac{\theta}{2(1-\underline{\nu})}[1-\nu_c]^2 \left[r + s - \frac{b}{2}s^2 - cs - (1-\delta)\right] X, \quad (20)$$

subject to

$$g(\nu_c, R)X + (R-1)L \leq RC, \quad (21)$$

where

$$g(\nu_c, R) \equiv (1+q\rho)R - (1-\delta) - \frac{(\nu_c - \nu)}{1-\underline{\nu}}q\rho - \frac{1-\nu_c}{1-\underline{\nu}}\nu_c \left[r + s - \frac{b}{2}s^2 - cs - (1-\delta)\right]. \quad (22)$$

We assume and justify later that $g(\nu_c, R) \geq 0$ and $R > 1$. Then we get the solution:

$$L = 0, \quad (23)$$

$$X = \frac{RC}{g(\nu_c, R)}, \quad (24)$$

$$\nu_c = \frac{r + s - \frac{b}{2}s^2 - cs + q\rho - (1-\delta) - 2(1-\underline{\nu})(1+q\rho)R + 2(1-\underline{\nu})(1-\delta) - 2\underline{\nu}q\rho}{r + s - \frac{b}{2}s^2 - cs - q\rho - (1-\delta)}. \quad (25)$$

4.3 General equilibrium

In the general equilibrium, the arbitrage condition between the return rate of bank capital and the market rate of interest determines the value of R:

$$\frac{\theta}{2(1-\underline{\nu})}[1-\nu_c]^2 \left[r + s - \frac{b}{2}s^2 - cs - (1-\delta)\right] X = RC, \quad (26)$$

where $X = \frac{RC}{g(\nu_c,R)}$. This condition determines R (for given s). Finally, given $R(s)$ by (26) and $\nu_c(s)$ by (25), firm's problem (10) determines the value of s in the general equilibrium. Note that firm i chooses s_i to solve (10), taking $\nu_c(s)$ as given, where s is the average level of R&D investment for the bank's borrowers. Since the objective

function of firms is quadratic, the solution must be a corner solution: $s_i = 1$ if $\nu_c(s)$ is small and $s_i = 0$ if $\nu_c(s)$ is large. Therefore, either $s = 1$ or $s = 0$ in the equilibrium.[4] Note that s and ν_c can be regarded as the outcome of a simultaneous game between firms and banks: Firms choose s, taking ν_c as given; and banks choose ν_c, taking s and R as given. In Section 5 we will show numerical examples.

4.4 A model with bank capital requirements

In this subsection, we consider the economy where a capital requirement is imposed by the government. Capital requirements have recently become a major part of the banking regulation. We will show in the numerical experiments in Section 5 that the capital requirements may be effective in improving social welfare by eliminating the bad equilibrium or by leading the economy to the good equilibrium.

The banks in this economy are subject to the following constraint: $X \leq \lambda C$.

Bank: The bank's problem is reduced to

$$\max_{\nu_c} \frac{\theta}{2(1-\underline{\nu})}(1 - \nu_c)^2 \left[r + \left(1 - \frac{b}{2}s - c \right) s - (1 - \delta) \right] X, \qquad (27)$$

subject to

$$X \quad \leq \quad \lambda C, \qquad (28)$$

$$-g(\nu_c)X - (R - 1)L + RC \quad \geq \quad 0. \qquad (29)$$

We define $\tilde{R}$ and $g(\tilde{\nu}_c)$ as the solutions in the case where there are no capital requirements, that is, the equilibrium values in the previous subsection. If it holds that

$$\lambda < \frac{\tilde{R}}{g(\tilde{\nu}_c)}, \qquad (30)$$

$X = \lambda C$ should hold in the equilibrium. We assume $\lambda C < \frac{\tilde{R}C}{g(\tilde{\nu})}$ here.[5] The problem is reduced to[6]

$$\max_{X, \nu_c} \frac{\theta}{2} \frac{(1 - \nu_c)^2}{1 - \underline{\nu}} \lambda \left[r + (1 - \frac{b}{2} s - c)s - (1 - \delta) \right], \tag{31}$$

subject to

$$g(\nu_c) \leq \frac{R}{\lambda}. \tag{32}$$

Thus, under our assumption, since the constraint should be binding,

$$g(\nu_c) = \frac{R}{\lambda}. \tag{33}$$

Solving (33) gives us $\nu_c(R, s)$.

General equilibrium: In a general equilibrium, by the arbitrage condition,

$$\frac{\theta}{2} \frac{(1 - \nu_c)^2}{1 - \underline{\nu}} \left[r + (1 - \frac{b}{2} s - c)s - (1 - \delta) \right] \lambda = R. \tag{34}$$

This condition gives $R(s)$. Given $R(s)$ and $\nu_c(R, s)$, the first-order condition for the firm's problem, (11), gives the equilibrium value of s. There may be unique equilibrium or multiple equilibria, depending on the parameter values.

5 Numerical example

In this section, we show some numerical examples in order to see the workings of the model. We employ the benchmark values of parameters as in Table 1. We mention this set of variables as a baseline. In Sections 5.1–5.4, we change the values of b and λ and see the effects of their changes on the equilibrium outcome. In Section 5.5, we exemplify the welfare effect of the bank capital requirement by showing the ranges of ξ and b that generate the good equilibrium, multiple equilibria, and bad equilibrium, with

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[5]In our numerical examples in Section 5, we checked that this assumption holds.

[6]Here we also assume that (18) is satisfied with strict inequality. This assumption is justified for the numerical examples in Section 5.

r	δ	C	θ	$\bar{s}$	ξ	ρ	$\underline{\nu}$	b	c	q	λ
5	.5	2	.5	1	.32	3	.3	.1	.5	.3	1.5

Table 1: Parameter Values (1)

and without the bank capital requirement for a range of λ. First of all, we justify that (18) is satisfied with strict inequality for these parameter values in this section. From (16), (18) can be rewritten as

$$\frac{q\rho}{r + s - R_f - 1 + \delta} \leq \nu_c. \tag{35}$$

Since $\nu \geq \underline{\nu}$, ν_c must be no less than $\underline{\nu}$. The above condition is satisfied with strict inequality because $\frac{q\rho}{r+s-R_f-1+\delta} < \underline{\nu}$. Therefore, we can simply take it for granted that (18) is satisfied in the numerical examples in this section.

5.1 From multiple equilibria to good equilibrium

In the first example, there are multiple equilibria ($s = 0$ and $s = 1$) if the bank capital requirement is not imposed; and imposition of the capital requirement can eliminate the bad equilibrium in which $s = 0$, and the good equilibrium in which $s = 1$ becomes the unique equilibrium. Parameter values are given in Table 1. In this case, the equilibrium of the basic model in which no capital requirement is imposed is as in Table 2. There are multiple equilibria. We find that these two equilibria are stable.[7] If we introduce the bank capital requirement, however, there is unique equilibrium with $s = \bar{s} = 1$ as in Table 3. This unique equilibrium is also stable. Therefore, we can make the good equilibrium the unique equilibrium using the capital requirement.[8] The capital

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[7] We checked whether bank payoff can be improved if the banks choose $\nu_c(\hat{s})$ and the firms choose $\hat{s}$, where $\hat{s} = 1 - s^*$ and s^* ($= 0$ or 1) is the value in the equilibrium, while the market rate of interest is fixed at the equilibrium value, R^*. If there is such a possibility and the banks are the Stackelberg leader, they have an incentive to deviate from the equilibrium, and therefore the equilibrium is unstable. (Note that banks want to deviate, taking $R = R^*$ as given, while R will change from R^* if they actually deviate.) If there is no such $\hat{s}$ that improves bank payoffs, we call the equilibrium stable.

[8] This result is dependent on the value of $\xi = .32$. If $.3113 < \xi < .3750$, there are multiple equilibria in the case without capital requirement; the equilibrium with $s = 1$ is unique equilibrium for ξ below this

s	R	ν_c	C/X
0	1.4586	.4667	.3134
1	1.5673	.4545	.3356

Table 2: Result (1) - Without BIS Restriction

s	R	ν_c	C/X
1	1.2994	.3000	.6667

Table 3: Result (1) - With Capital Requirement

requirement is effective in eliminating the bad equilibrium is simply because a bank with more capital is less susceptible to a bank run. Suppose that X and L are fixed and that C increases, i.e., D decreases; condition (16) implies that ν_c decreases in this case; and therefore, the derivative of the objective function of the firm's problem, (11), implies that the equilibrium value of s is more likely to be one, the upper bound.

5.2   From bad equilibrium to good equilibrium

In the second example, we change b to .01 from .1 in the baseline case as in Table 4. Other variables are the same as those in Table 1. If b is large, the difference of firm revenue when $s = 1$ and when $s = 0$ becomes large, implying that (11) tend to be positive for $s = 1$ and negative for $s = 0$ and that there may exist two equilibria. Therefore, if b is small, multiplicity may disappear. This is actually confirmed in the numerical experiment. In this case, there is unique equilibrium with $s = 0$ in the case without capital requirement, and it is stable. However, if we introduce the capital requirement,

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region; and that with $s = 0$ is unique equilibrium for ξ above this region. If $.3250 < \xi < .3900$, there are multiple equilibria in the case with capital requirement. Therefore, the multiplicity of equilibria seems a "knife-edge" result which is crucially dependent on the value of ξ. We can easily confirm, however, that the range of values of ξ that give multiplicity becomes wide if we set b and q large. For example, if $b = 2$ and $q = .5$, the range of multiplicity without and with capital requirement are $.286 < \xi < 1.4$ and $.311 < \xi < 1.56$, respectively.

r	δ	C	θ	$\bar{s}$	ξ	ρ	$\underline{\nu}$	b	c	q	λ
5	.5	2	.5	1	.32	3	.3	**.01**	.5	.3	1.5

Table 4: Parameter Values (2)

s	R	ν_c	C/X
0	1.4586	.4667	.3134

Table 5: Result (2) - Without Capital Requirement

there is unique equilibrium with $s = \bar{s}$ as in Table 6. This unique equilibrium is also stable. Bank capital requirement switches equilibrium from bad to good.

s	R	ν_c	C/X
1	1.3112	.3000	.6667

Table 6: Result (2) - With Capital Requirement

5.3 A case where capital requirements do not matter

We change λ to 2.3 from 1.5 in the baseline as in Table 7. This means that capital requirement is loosened. As expected, the loose capital requirement does not have a significant effect toward eliminating the bad equilibrium. Other variables are the same as those in Table 1. In this case, there are multiple equilibria in the case without capital requirement. Only the equilibrium with $s = 1$ is stable. Even if we introduce the capital requirement, there are multiple equilibria as in Table 9. Only equilibrium with $s = 1$ is stable. In this case, the bank capital requirement does not matter in equilibrium selection.[9]

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[9] In Table 9, the probability of a bank run, ν_c, is larger in the equilibrium with $s = 1$ than with $s = 0$. This apparent counterintuitive result can be explained as follows: since in the equilibrium with $s = 1$, the market rate R is larger, and the depositors demand higher returns. Therefore, they run on banks at a higher ν_c. In a sense, the depositors are more impatient in the equilibrium with $s = 1$ than in that

r	δ	C	θ	$\bar{s}$	ξ	ρ	$\underline{\nu}$	b	c	q	λ
5	.5	2	.5	1	.32	3	.3	.1	.5	.3	**2.3**

Table 7: Parameter Values (3)

s	R	ν_c	C/X
0	1.4586	.4667	.3134
1	1.5673	.4545	.3356

Table 8: Result (3) - Without Capital Requirement

5.4   From bad equilibrium to multiple equilibria

We change both λ and b from the baseline as in Table 10: b is set small and λ large. It is confirmed that even the loose capital requiement has a subtle effect of generating the good equilibrium. Other variables are the same in Table 1. In this case, there is unique equilibrium with $s = 0$ in the case without capital requirement, and this is stable. If we introduce a capital requirement, there are multiple equilibria as in Table 12. Only the equilibriuim with $s = 1$ is stable.[10] Therefore, the introduction of a capital requirement generates the possibility that the economy shifts from bad equilibrium to good equilibrium.

s	R	ν_c	C/X
0	1.4379	.3763	.4348
1	1.5527	.3821	.4348

Table 9: Result (3) - With Capital Requirement

with $s = 0$.

[10]If the banks are the Stackelberg leaders, the good equilibrium is the unique equilibrium in this economy.

r	δ	C	θ	$\bar{s}$	ξ	ρ	$\underline{\nu}$	b	c	q	λ
5	.5	2	.5	1	.32	3	.3	**.01**	.5	.3	**2.3**

Table 10: Parameter Values (4)

s	R	ν_c	C/X
0	1.4586	.4667	.3134

Table 11: Result (4) - Without Capital Requirement

5.5 Sensitivity to parameter values

We checked the sensitivity of the equilibrium outcome to parameter values and found that changes in values of ξ, b, and λ have almost dominant effects on whether the outcome of the model is the good equilibrium, bad equilibrium, or multiple equilibria. ξ is the parameter for the cost of R&D investment, b is a parameter for the private benefit for the firm of shirking, and λ is the inverse of the capital ratio in the bank capital requirement. We show three figures in this section. The values other than specified in the figues are those given in Table 1. All figures are divided into the upper and lower panels. The upper panel shows the outcome in the case without the bank capital requirement, and the lower panel shows that in the case with the bank capital requirement. Figure 1 shows the equilibrium outcome in the ξ-b space. With the bank capital requirement, the regions of the good equilibrium and the multiple equilibria expand, while that of the bad equilibrium shrinks. The similar results are obtained in Figures 2 and 3. All these figures exemplify that changes in ξ and b change the equilibrium outcome drastically and that

s	R	ν_c	C/X
0	1.4379	.3763	.4348
1	1.5641	.3826	.4348

Table 12: Result (4) - With Capital Requirement

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the effects of ξ and b are affected by λ significantly when the capital requirement is in place. We can conclude that the bank capital requirement generally has a positive effect on productivity and social welfare.

5.6 Bank-induced instability and capital requirements

Our general equilibrium model has interesting implications. The banks in this economy provide insurance for the idiosyncratic liquidity shocks to firms, as those in Holmstrom and Tirole's (1998) model. This insurance function enables firms to undertake production projects and ex ante R&D investment, and thus increases the aggregate productivity of the economy. If the economy is not subject to the macro shock, ν, the existence of banks leads the economy to the good equilibrium. This is consistent with well-known results in the literature that financial deepening is relevant to or even crucial for economic growth (see Levine [1997]). Our model implies that if a macro shock exists, the existence of banks may generate multiple equilibria on the premise that the banks are subject to bank runs à la Diamond and Rajan (2000). In this case, the economy may become instable and fluctuate between the good and bad equilibria. Therefore, the existence of banks is good in that they provide insurance and may generate the good equilibrium, but is not sufficiently good in that they cannot necessarily eliminate the bad equilibrium if the economy is subject to the macro shock, causing large instability in the economy.

The possibility of bank runs decreases the expected return on R&D investment for firms, and thus increases instability. The capital requirement policy in this model can be regarded as a complement to the financial sector, which eliminates or reduces instability in the economy. The capital requirement changes the equilibrium composition of C and D, so that bank runs are less likely to occur. Therefore, the bank capital requirement may raise aggregate productivity through reducing the probability of bank runs, ν_c, and enhancing R&D investment by firms.[11]

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[11]It is often pointed out that introduction of the stringent bank capital requirement in Japan in 1998 led to significant reduction in bank lending and caused the banking crisis. This episode appears contradictory to the prediction of our model. We can interpret the introduction of a capital requirement in 1998 as that in the interim period in our model: if a tight capital requirement is suddenly introduced in the interim

6   Conclusion

We introduced borrowers' choice for R&D investment into Holmstrom and Tirole's (1998) model, and showed that R&D investment is negatively affected by bank distress. A high probability of bank failure discourages borrowers' R&D investment ex ante, and lowers the aggregate productivity of the economy. Our theory seems successful in explaining productivity declines observed during banking crises, such as the episodes during the Great Depression in the 1930s and the "lost decade" of Japan in the 1990s.

The general equilibrium version of our model also provides a potential motive for bank capital requirements. The model implies that bank capital requirements may be able to lead the economy to the good equilibrium where firms choose a higher level of R&D investment. Bank regulation may be effective for enhancing aggregate productivity through reducing bank-induced instability or eliminating the bad equilibrium. Multiplicity of equilibria or bank-induced instability may be important in explaining large business fluctuations associated with banking crises, especially in emerging markets. The effectiveness of capital requirements in reducing bank-induced instability may be worth studying further to deepen our understanding of the necessity of bank regulations.

7   References

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period (i.e., after ν is revealed), the bank lending of ρ to firms is hindered and the liquidations of firms increase. Our model implies that if a capital requirement is imposed before firm activities start, it has a good effect on the productive efficiency of the economy, while if it is imposed in the interim period after firm activities have already started, it has a bad effect on efficiency. Therefore we regard that the model is consistent with the episode.

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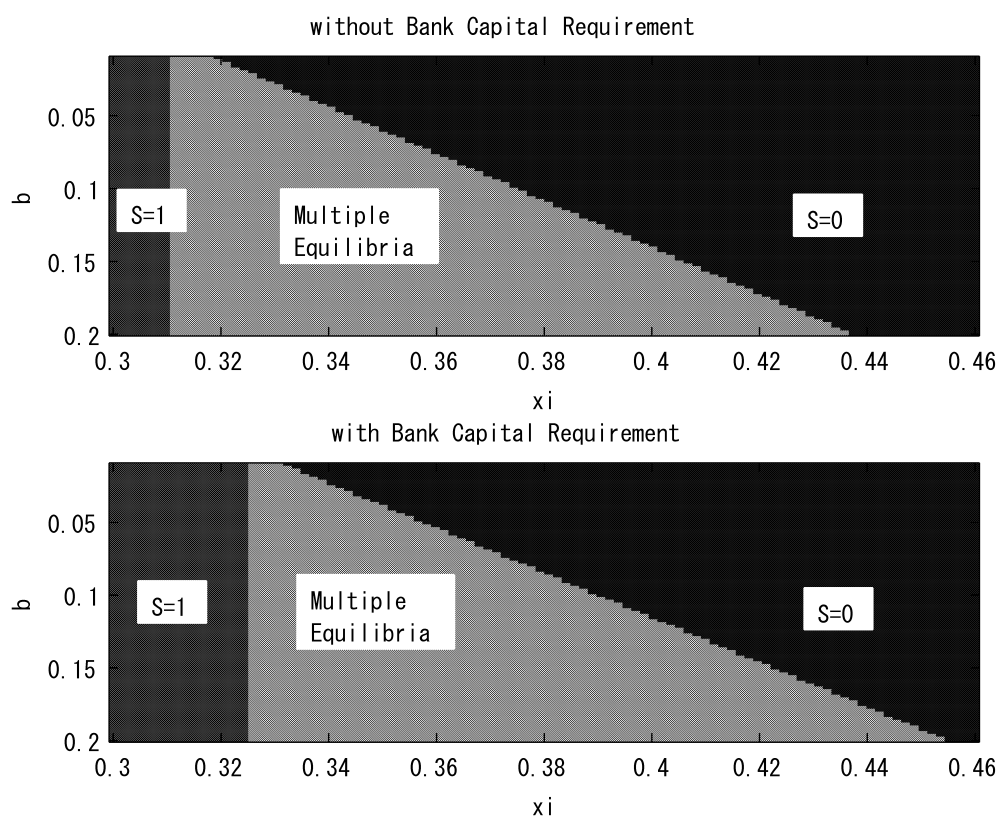
without Bank Capital Requirement

S=1

Multiple
Equilibria

S=0
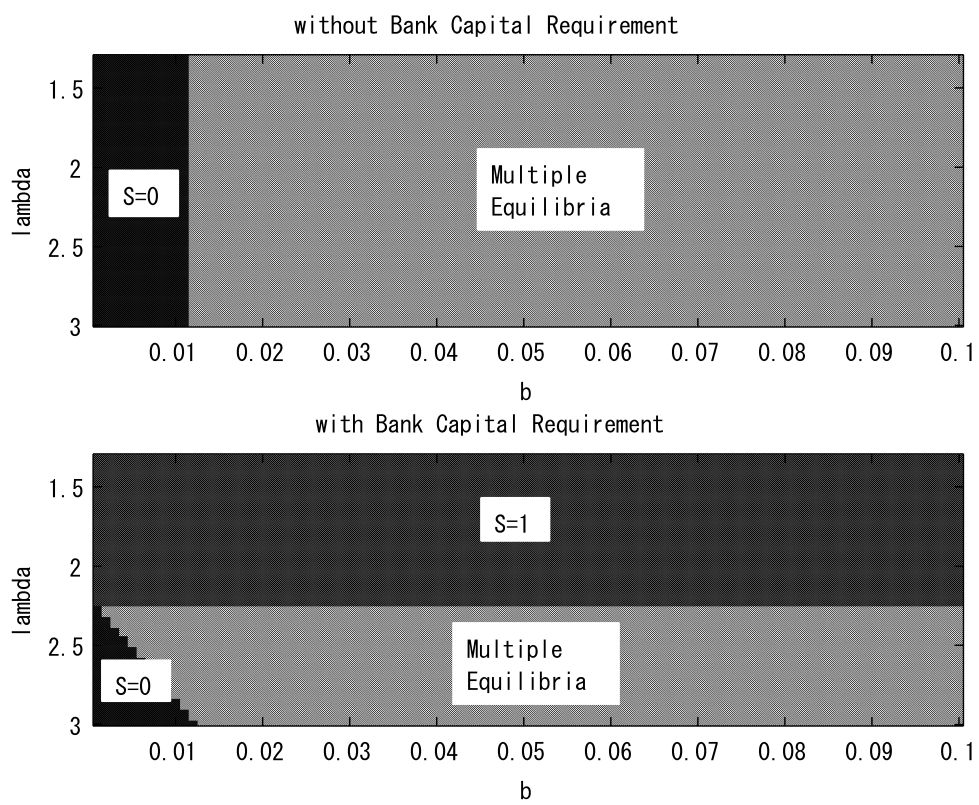
with Bank Capital Requirement

S=1

Multiple
Equilibria

S=0

Figure 1: Equilibrium Outcome in the ξ-b Space

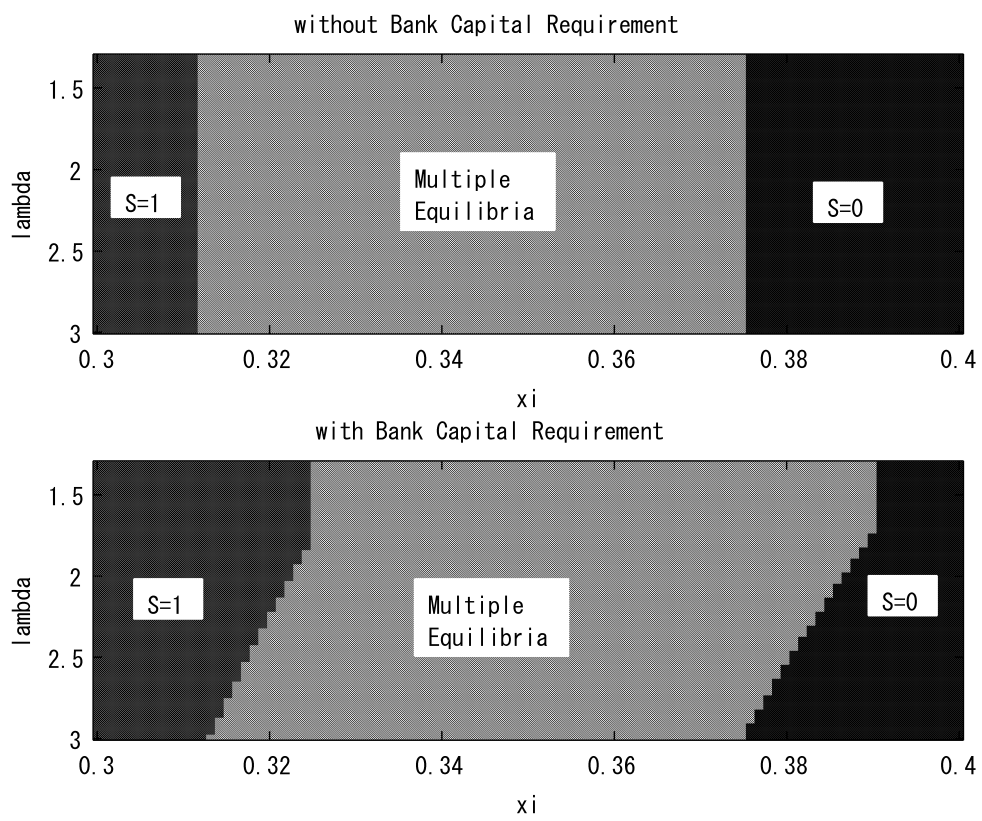Figure 2: Equilibrium Outcome in the b-λ Space

without Bank Capital Requirement

with Bank Capital Requirement

Figure 3: Equilibrium Outcome in the ξ-λ Space