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# **Business Cycle Accounting for the Japanese Economy Using the Parameterized Expectations Algorithm**

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Masaru Inaba\*

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Keywords: Business cycle accounting; parameterized expectations algorithm.

JEL Classification: E21, E22, E37

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# 1 Introduction

The idea of business cycle accounting (BCA hereafter) developed by Chari, Kehoe, and McGrattan (2002, 2004, 2007a) is to assess which wedge is important for the fluctuation of an economy which is assumed to be described as a prototype model with time-varying wedges. These wedges resemble productivity, labor and investment taxes, and government consumption. Since these wedges are measured using the production function and first order conditions to fit the actual macroeconomic data, this method can be interpreted as a generalization of growth accounting.

In this paper, we apply the parameterized expectations algorithm (PEA hereafter) to BCA. There are two contributions of this paper. The first one is application of the PEA to BCA. The PEA introduced by Marcet (1988) is one of the methods to solve the non-linear dynamic stochastic general equilibrium model. Marcet and Lorenzoni (1998) provide applications of PEA to some economic models. The basic idea of the PEA is to approximate the expectation function by a smooth function, a polynomial function in general. The PEA has an advantage<sup>1</sup> in that it is simpler and easier to understand and implement than the other non-linear solution methods.<sup>2</sup>

Secondly, we apply BCA to the Japanese economy using the PEA which relaxes the perfect foresight assumption and show that the result is similar to the main result in the deterministic BCA by Kobayashi and Inaba (2006). They assume perfect foresight in the prototype economy so that all wedges are given deterministically as in Chari et al. (2002). The assumption of perfect foresight enables us to avoid complicated calculations. As they point out, however, the effects of the investment wedge are sensitive to the assumption on the future values of wedges.<sup>3</sup> On the other hand, the stochastic model that wedges are assumed to be an exogenous stochastic process which is estimated from the data does not suffer from the arbitrary choices of the future values of wedges. Chakraborty (2004) also applies BCA to the Japanese economy using a log-linearized dynamic stochastic general equilibrium model. The simulation result on the investment wedge is somewhat different from Kobayashi and Inaba (2006). In this paper, we find that the result of BCA using PEA is similar to the result of the perfect foresight BCA. Therefore, we can conclude that the causes of the difference in the results between Chakraborty and Kobayashi-Inaba must be in data constructions, data sources, and log-linearization. In cases where the economy is far away from the steady state or highly non-linear, the approximation error may be large. Therefore, taking account of non-linearities may be important.

The organization of the paper is as follows. Section 2 describes the prototype model for BCA. Section 3 explains the accounting procedure using the parameterized expectations al-

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<sup>1</sup>There is also a disadvantage that the PEA needs a long simulation in order to obtain the fitted coefficients of the approximating function. Therefore the algorithm can be quite computationally demanding.

<sup>2</sup>Chari et al. (2004, 2007) implement BCA using the finite element method for the non-linear solution described by McGrattan (1996).

<sup>3</sup>In the perfect foresight assumption, the future value of wedges is arbitrarily given. Kobayashi and Inaba (2006) check the following four cases: (1) all wedges are assumed to remain constant at the last value of the target period; (2) the labor wedge and investment wedge are zero in the future and the efficiency wedge is the benchmark value, i.e., the 1984-1989 average; (3) the labor wedge and investment wedge are zero in the future and the efficiency wedge is the last value of the target period; (4) all wedges are the benchmark values in the future.

gorithm. Section 4 describes application of BCA based on the PEA to the Japanese economy in order to investigate the robustness of the result of Kobayashi and Inaba (2006). Section 5 concludes. The result in this paper is quite similar to the main result of Kobayashi and Inaba (2006). The effects of the investment wedge are not a significant cause of the persistent recession during the 1990s. The output due to the efficiency wedge roughly replicates actual output, while the discrepancy widened during the 1990s. The labor wedge had a large depressing effect on output during 1989-2005. The efficiency wedge explains the recent economic recovery.

## 2 The prototype model

This section describes the prototype model with time-varying wedges: the efficiency wedge  $A_t$ , the labor wedge  $1 - \tau_{l,t}$ , the investment wedge  $1/(1 + \tau_{x,t})$ , and the government wedge  $g_t$ .

The household maximizes:

$$\max_{c_t, k_{t+1}, l_t} E_0 \left[ \sum_{t=0}^{\infty} \beta^t U(c_t, l_t) N_t \right]$$

subject to

$$c_t + (1 + \tau_{x,t}) \left\{ \frac{N_{t+1}}{N_t} k_{t+1} - k_t \right\} = (1 - \tau_{l,t}) w_t l_t + r_t k_t + T_t, \quad 0 < \beta < 1,$$

where  $c_t$  denotes consumption,  $l_t$  employment,  $N_t$  population,  $k_t$  capital stock,  $w_t$  the wage rate,  $r_t$  the rental rate on capital,  $T_t$  the lump-sum taxes per capita. All quantities written in lower-case letters denote per-capita quantities except for  $T_t$ .

The firm maximizes

$$\max_{k_t, l_t} A_t F(k_t, \gamma^t l_t) - \{r_t + (1 + \tau_{x,t})\delta\} k_t - w_t l_t,$$

where  $\delta$  denotes the depreciation of capital stock and  $\gamma$  the balanced growth rate of technical progress. The resource constraint is

$$c_t + x_t + g_t = y_t, \tag{1}$$

where  $x_t$  is investment,  $g_t$  the government consumption and  $y_t$  the per-capita output. The law of motion for capital stock is

$$\frac{N_{t+1}}{N_t} k_{t+1} = (1 - \delta) k_t + x_t. \tag{2}$$

The equilibrium is summarized by the resource constraint (1), the law of motion for capital (2), the production function,

$$y_t = A_t F(k_t, \gamma^t l_t), \tag{3}$$

and the first-order conditions,

$$-\frac{U_{l,t}}{U_{c,t}} = (1 - \tau_{l,t}) A_t \gamma^t F_{l,t}, \tag{4}$$

$$U_{c,t}(1 + \tau_{x,t}) = \beta E_t U_{c,t+1} [A_{t+1} F_{k,t+1} + (1 - \delta)(1 + \tau_{x,t+1})], \quad (5)$$

where  $U_{ct}$ ,  $U_{lt}$ ,  $F_{lt}$ , and  $F_{kt}$  denote the derivatives of the utility function and the production function with respect to their arguments. The functional form of the utility function is given by  $U(c, l) = \ln c + \phi \ln(1 - l)$ , where  $\phi > 0$  is a parameter. Also, the functional form of the production function is given by  $F(k, l) = k^\alpha l^{1-\alpha}$ .

### 3 Accounting procedure

This section provides the accounting procedure to measure actual wedges using PEA.

#### 3.1 Measuring the wedges

We take the government wedge  $\tau_g$  directly from the data. To obtain the values of the other wedges, we use the data for  $y_t$ ,  $l_t$ ,  $x_t$ ,  $g_t$ , and  $N_t$ , together with a series on  $k_t$  constructed from  $x_t$  by (2). The efficiency wedge and the labor wedge are directly calculated from (3) and (4).

To find the investment wedge  $\tau_{x,t}$ , Kobayashi and Inaba (2006) assume the deterministic model and posit a strict assumption on the values of the wedges for the time period after the target period of business cycle accounting.<sup>4</sup> As they point out, however, the effects of the investment wedge depend on the assumption on the values of future wedges.

In this paper, to avoid this matter, we apply the PEA to find the investment wedge  $\tau_{x,t}$ . The algorithm is as follows.

#### Algorithm <sup>5</sup>

- **Initialization:** Apply the deterministic method of business cycle accounting as described in Kobayashi and Inaba (2006), and regard the derived investment wedge as the initial value of  $\tau_{x,t}^{(0)}$ , and set a stopping parameter  $\epsilon > 0$
- **Step 1:** Specify a vector AR1 process for the four wedges  $s_t = (\log(A_t), \tau_{l,t}, \tau_{x,t}^{(j)}, \log(g_t))$  of the form

$$s_{t+1} = P_0 + P s_t + \eta_{t+1}, \quad (6)$$

where  $\eta_t \sim i.i.d. N(0, \Omega)$ .<sup>6</sup>

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<sup>4</sup>Their procedure is as follows. Denoting the target period of BCA by  $t = 0, 1, 2, \dots, T$ , we assume that  $A_t = A^* = A_T$ ,  $\tau_{l,t} = \tau_{l,T}^* = \tau_{l,T}$  and  $g_t = g^* = g_T$  for  $t \geq T + 1$ . They also assume that  $\tau_{x,t}$  is an unknown constant  $\tau_x^*$  for  $t \geq T$ , and use the shooting method to find  $\tau_x^*$  so that  $\tau_{x,T} = \tau_{x,T+1} = \tau_x^*$ . Once  $\tau_x^* = \tau_{x,T}$  is determined by this method,  $\tau_{x,t}$  for  $t = 0, 1, 2, \dots, T - 1$ , are obtained by solving (5) backward.

<sup>5</sup>For details, see technical appendices Inaba (2007).

<sup>6</sup>The OLS estimation of this stochastic process can be non-stationary. We then use the maximum likelihood procedure described in McGrattan (1994) to estimate the parameters  $P_0, P$  of the vector AR1 process for the wedges. To ensure stationarity, we add to the likelihood function a penalty term proportional to  $\{\max(|\lambda_{\max}| - 0.99, 0)\}^2$ , where  $\lambda_{\max}$  is the maximal eigenvalue of  $P$ . If  $\lambda_{\max} < 0.99$ , we use the OLS estimation.

- **Step 2:** Apply the parameterized expectation algorithm to get the non-linear solution of the model. Then we get an approximation function  $\Phi(\cdot)$  for the expectation function<sup>7</sup>:

$$\beta E_t U_{c,t+1} \left\{ A_{t+1} F_{k,t+1} + (1 - \delta)(1 + \tau_{x,t+1}^{(j)}) \right\}.$$

$\Phi(\cdot)$  is a polynomial function of  $k_t$ ,  $A_t$ ,  $\tau_{l,t}$ ,  $\tau_{x,t}^{(j)}$ , and  $g_t$ .

- **Step 3:** To find the value of  $\hat{\tau}_{x,t}$  that realizes the actual data,  $c_t$  and  $l_t$ , solve the following equation for  $\hat{\tau}_{x,t}$ ,

$$U_{c,t}(1 + \hat{\tau}_{x,t}) = \Phi(k_t, A_t, \tau_{l,t}, \hat{\tau}_{x,t}, g_t) \quad (7)$$

- **Step 4:**  $\tau_{x,t}^{(j+1)} = \nu \hat{\tau}_{x,t} + (1 - \nu) \tau_{x,t}^{(j)}$ ,  $0 < \nu < 1$ .
- **Step 5:** if  $\| \tau_{x,t}^{(j+1)} - \tau_{x,t}^{(j)} \| < \epsilon$ , STOP; else go to step 1.

## 3.2 Decomposition

To see the effect of the measured wedges on movements in macroeconomic variables from the initial date  $t = 0$ , we decompose the movements as Chari, Kehoe, and McGrattan (2007a)<sup>8</sup>. For example, to evaluate the effects of the efficiency wedge, we compute the decision rules for the economy with only the efficiency wedge, denoted  $y^e(s_t, k_t)$ ,  $c^e(s_t, k_t)$ ,  $l^e(s_t, k_t)$ , and  $x^e(s_t, k_t)$  under an exogenous stochastic process which is assumed to be the combination of (6) and a one-to-one mapping function;

$$\log A(s_t) = \log A_t, \quad \tau_l(s_t) = \bar{\tau}_l, \quad \tau_x(s_t) = \bar{\tau}_x, \quad \text{and} \quad \log g(s_t) = \log \bar{g}. \quad (8)$$

Starting from initial value of capital stock,  $k_0$ , we then use the actual value of wedges,  $s_t$ , the decision rules for the economy with only the efficiency wedge, and the capital accumulation law to compute the realized sequence of output, consumption, labor, and investment,  $y_t^e$ ,  $c_t^e$ ,  $l_t^e$ , and  $x_t^e$  which we call the *efficiency wedge components* of output, consumption, labor, and investment. In a similar manner, we define the *labor wedge components*, *investment wedge components*, *government wedge components*, and *benchmark components*.

We compare the effect of each wedge as follows. First, we construct the benchmark components by solving the prototype model with constant wedges. The values of the benchmark wedges are determined as the initial values at  $t = 0$ , or the averages of the values of the wedges for some period prior to the target period. Therefore, we solve the model assuming that  $s_t$  is a constant vector consisting of the benchmark wedges. The derived sequences  $y_t^b$ ,  $c_t^b$ ,  $x_t^b$ , and  $l_t^b$  are taken as the benchmark case. We then compare the each components of output with the benchmark components. If the derived output is below the benchmark, we say that the efficiency wedge has a depressing effect compared to the benchmark case.

<sup>7</sup>It is well known that the main drawback of the PEA is that it is not a contraction mapping technique and does not guarantee that a solution will be found. To avoid this, we modified the PEA following Maliar and Maliar (2003). They discuss a moving bounds method of imposing stability on the PEA to avoid the explosive case due to poor initial parameter values and achieve enhancement of the convergence property of the PEA.

<sup>8</sup>We also implement the alternative decomposition in Chari, Kehoe, and McGrattan (2004) and find similar result. For details, see technical appendices (Inaba [2007]). The differences between the two decomposition are explained in Chari, Kehoe, and McGrattan (2007b).

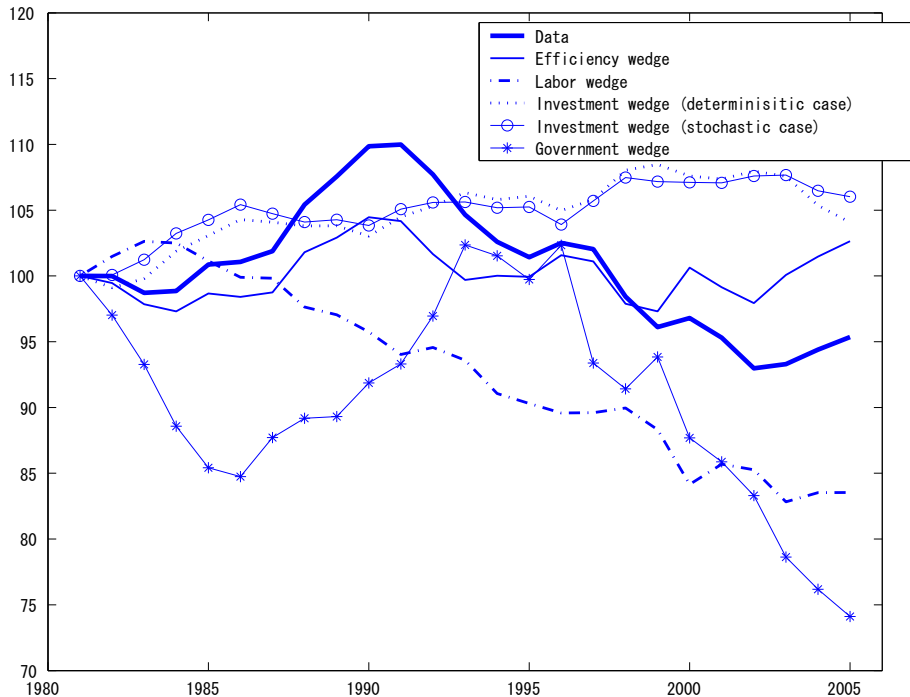


Figure 1: Output and the four measured wedges (100 in 1981)

## 4 BCA for Japan

The target period of our accounting exercise is 1981-2005. We update<sup>9</sup> the Kobayashi and Inaba (2005) data set and assume the same assumption as Kobayashi and Inaba (2006) except for the accounting algorithm. We set  $\beta = 0.98$ ,  $\alpha = 0.372$  and  $\delta = 0.0892$ , which are the averages during 1984-1989 except for  $\beta$ . We also set  $g_n = 0$ , and  $g_z = 0.0214$ , where  $g_n$  is the population growth rate, and  $(1 + g_z)^{1-\alpha} = \gamma$  for the simulation. The trend rate of technical progress  $(1 + g_z)$  is set as the average during 1981-2005.

In figure 1 we display the actual data for output (detrended by  $1 + g_z$ ) and the four measured wedges for 1981-2005: the efficiency wedge  $A_t$ , the labor wedge  $(1 - \tau_{l,t})$ , the investment wedge  $1/(1 + \tau_{x,t})$ , and the government wedge  $g_t$ . All variables are plotted as indices set at 100 in 1981. The fluctuations of the investment wedge derived by PEA are quite similar to those of the deterministic case.

The decomposition results for output are shown in figure 2. In our decomposition exercise, we assumed the values of the benchmark wedges as follows:  $A$ ,  $\tau_l$ ,  $\tau_x$ , and  $g$  are the averages for the 1984-1989 period. In figure 2, we display the separate contributions of each wedge. We plot the actual output, benchmark case, and simulated outputs due to each of the four wedges. We plot the benchmark as a horizontal line at 100 and the other outputs as deviations from the benchmark. If output due to a wedge is below (above) the benchmark case, we judge that the wedge had a depressing (expanding) effect on output. The result is quite similar to Kobayashi and Inaba (2006). The effects of the investment wedge are

<sup>9</sup>While Kobayashi and Inaba use "Private final consumption expenditure" as consumption, we use "Actual final consumption of households".



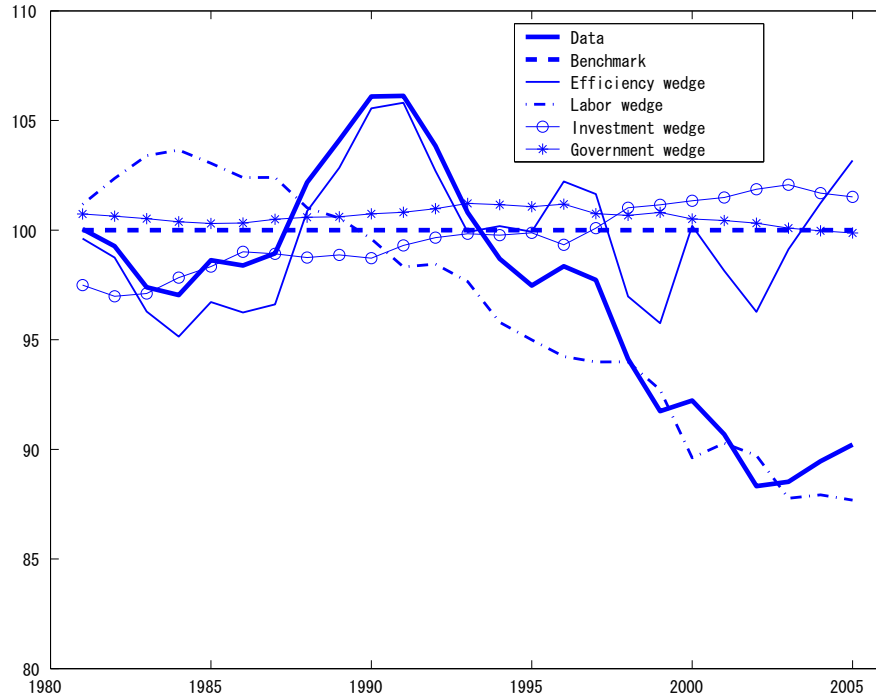


Figure 2: Decomposition of output with just one wedge

not a significant cause of the persistent recession during the 1990s. The output due to the efficiency wedge roughly replicates actual output, while the discrepancy widened during the 1990s. The labor wedge had a large depressing effect on output during 1989-2005. The efficiency wedge explains the recent economic recovery.

## 5 Concluding remarks

This paper proposes an application of the parameterized expectation algorithm to business cycle accounting. The PEA is a simple algorithm and easier to understand and implement than the other non-linear solution methods. Moreover, under a less arbitrary assumption about the process of wedges than the perfect foresight BCA, we show that the result of BCA using the PEA is similar to the main result in the deterministic BCA by Kobayashi and Inaba (2006).

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