Intertemporal Distribution of Foreign Aid

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By

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Abstract

We analyze the dynamic effects of foreign aid on the economic growth and welfare of a recipient country. Based on an overlapping generations model with a productive capital, foreign aid is characterized by its uses: whether it takes a form of income compensation (income aid) or capital stock (capital aid), and by its recipients: which generation(s) or institution hold(s) its ownership. We found some different results from those of the traditional wisdom of foreign aid. First, foreign aid tends to be less efficient without any condition to the recipient. Second, capital aid can be more efficient than income aid, and its efficiency could be even higher when capital aid is loaned in stead of being granted. Third, a stable relationship between the donors and the recipient may harm the efficiency of foreign aid.

Keywords: Foreign Aid, Overlapping Generations, Modalities of Aid, Economic Growth, Conditionality, Fungibility

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1. Introduction

Foreign aid is defined as a transfer of resources from outside sources to a recipient country. The effectiveness of foreign aid on the economic growth of a recipient characterizes past literature. One-time foreign aid was sometimes referred to as a “big push” which means that a large one-time resource transfer will result in the self-sustained growth and welfare improvement of a recipient economy. According to Papanek (1972), one main issue in the literature was whether this big push of aid was recommended or not.

Chenery and others argued for foreign aid (cf. Chenery and Strout 1966, Chenery and Eckstein 1970), claiming that it increases the levels of investment, production, income and consumption, and that it will result in future economic growth. On the contrary, Griffin and Enos (1970) have argued against foreign aid, stating that it increases consumption and decreases domestic saving and investment, and so that it will result in no or slower economic growth. One of the most critical points was whether individuals in the recipient country will increase or decrease the amount of saving after receiving foreign aid (cf. Papanek 1973).

Whether the arguments were for or against aid, both the econometric and theoretical models which the above studies rely on were too simple. First, most results were obtained from OLS estimations based on cross country data because the econometric models were linear. Second, their theoretical models were so static that they cannot really explain the growth effects of aid. In other words, the decision-making in recipient countries and modalities of aid were too simplified (cf. Papanek 1973). The capacity level of each recipient country to implement foreign aid projects, sometimes represented as political stability, monetary and financial systems, market openness,

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1 The term, big push, is obviously different in meaning from the same term in the paper by Murphy, Shleifer, and Vishny (1989). Here, the dynamic saving and consumption decision on foreign aid is concerned while the demand spill-over effects among industries are focused on in their paper. In fact, both these problems plus some endogenous growth discussions can be found in the introduction of the paper by Rosenstein-Rodan (1961).

2 Hasan (2002) tried to reproduce the results of those studies adding new data and the possibility of non-linearity utilizing recent econometric techniques such as VAR and the Granger Test. Still, it was not made clear the direction of
varies widely. Besides, those studies seemed to treat the total amount of aid to each recipient country as one variable but obviously, there are many different uses of aid, such as building a dam, a highway or providing food and medicine. Consequently, each of them will affect the saving decisions of recipients and the future growth differently.

Burnside and Dollar (2000) analyzed the effectiveness of foreign aid in order to resolve the problems mentioned above. They adopted a policy variable to represent the capacity level of a government for receiving foreign aid. It is a weighted average of multiple factors, such as election and legal systems, corruption level of government, market openness, and financial and monetary systems. They made their econometric models non-linear by introducing additional terms: a multiple of foreign aid and policy, and a square of foreign aid. In particular, the square term shows a decreasing marginal effect of aid on growth. After their regression analysis based on a new panel data of foreign aid, they concluded that growth rate becomes higher under a better political environment, but that the marginal effect of aid is decreasing. This research revived this topic in the economic literature, and generated many studies to extend and verify their findings.

However, most structural models in these studies remained static without the dynamic optimization of individuals. We know that the notion of economic growth itself is dynamic because it means the difference in economic performances between two periods. So, in a situation where a saving rate endogenously adjusts to the presence of foreign aid, effects on growth explained by those models can make sense in a very short run. As Easterly (2003) clearly states that we need more theoretical analysis of foreign aid based on a dynamic macro model.

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In fact, there exists a small number of dynamic analysis of foreign aid under the (Neo-Classical) growth models. As a pioneering work among the representative agent (infinite horizon) models, Boone (1996) succeeded in constructing a dynamic model of alternative political (so-called, elitist or egalitarian) regimes. Galor and Polemarchakis (1987) invented a two-country dynamic model and discussed the effects of transferring resources between two countries. Kemp (1995) also analyzed the effects of foreign aid around a steady state in a two-country infinite horizon model, and Brock (1996) constructed a multiple (traded and non traded) goods model to analyze foreign aid. Chao and Yu E. S. H. (2001) consider the effects of import quotas in their model of multiple productive capital stocks. van Soest and Lensink (2000) applied an infinite horizon model into the analysis of deforestation. Naito and Ohdoi (2006) analyzed foreign aid in their two-country, multiple goods and multiple productive capital stocks model.


This paper tries to analyze foreign aid fully utilizing an overlapping two generations model with a productive capital, as a standardized tool of dynamic macro economy. We chose this model setting mainly because we can obtain direct answers on the fundamental but still unsolved question of foreign aid, whether the amount of savings will increase or decrease after receiving aid.

In this paper, foreign aid is a transfer from donors (one donor) to one recipient whose economy is represented by a well-known overlapping generations model. Intertemporal distribution of foreign aid must be characterized by its multiple modalities: multiple uses (how transferred resources to be spent) and multiple recipients (which generations or institutions are given the

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4 Other related studies are Svensson (1999), Mountford (1998), Brock (1996) and Casella and Eichengreen (1996).
ownership of aid). Foreign aid can be naturally utilized as capital stock (capital aid) and income compensation (income aid). Ownership of foreign aid is shared mostly among the young and old generations, but the recipient government and donors can hold part of the ownership of aid. It is an interesting question whether a recipient economy will observe the conditionalities on the modalities of foreign aid. Also, informational structure that the individuals predict or do not predict an occurrence of foreign aid, greatly affects the effectiveness of foreign aid. We will follow the directions of changes caused by foreign aid in a macro economy (output, wage, return of capital, domestic savings, and capital stock) and summarizes them into the levels of growth and welfare.

The structure of this paper after the introduction is as follows. Section 2 introduces an Autarky economy of overlapping generations and analyzes an intertemporal distribution of income aid. Section 3 analyzes the effects of capital aid, and explains differences between capital loans and capital grants. Section 4 analyzes mixed uses of aid and discusses the possibility of government intervention. Section 5 is the conclusions.

2. Model

2.1. Autarky Economy

Consider an overlapping two generations model with a constant population \( L > 0 \) where the young \( (L(t)) \) and old \( (L(t-1)) \) generations coexist in each period \( L(t) = L(t-1) = L, t > 1 \). The generation \( L(t) \) is distinguished as \( L_1(t) \) in period \( t \) and \( L_2(t) \) in period \( t+1 \), whose subscripts 1 and 2 express young and old respectively. A single good \( (Y) \) is produced as an output with inputs of

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5 In this paper, the term “conditionalities” is used in much broader sense than in tying conditions. Here, conditionalities means that donors decide the uses and owners of aid while tying usually means that donors decide the sources of procurement of aid. According to DAC annual report, Tied ODA is defined as loans or grants which are either in effect tied to procurement of goods and services from the donor country or are subject to procurement modalities implying limited geographic procurement eligibility.
capital stock ($K$) and labor ($L_i$) by a Neo-Classical production function ($Y = F(K, L_i)$). Defining per capita capital stock and production function\(^6\) as $k$ and $f$, per capita output ($y$) is represented as:

$$y(t) = \frac{Y(t)}{L_i(t)} = \frac{F(K(t), L_i(t))}{L_i(t)} = \frac{F\left(\frac{K(t)}{L_i(t)}\right)}{L_i(t)} = f\left(\frac{K(t)}{L_i(t)}\right)$$  (1)

Defining the return of capital stock and wage as $r$ and $w$, perfect inputs markets are shown as:

$$r(t) = f'(k(t))$$  (2)
$$w(t) = f'(k(t)) - kf''(k(t))$$  (3)

Each of the young ($L_1(t)$) consumes in its young ($c_1(t)$) and old periods ($c_2(t+1)$). Its welfare level is represented by a Neo-Classical utility function\(^7\) ($u(c_1(t), c_2(t+1))$) where both goods are normal ($u_{22}u_{12} > u_{12}u_{22} \text{ and } u_{11}u_{21} > u_{21}u_{11}$). Given $w(t)$ and an expected return of capital ($r^E(t+1)$), each young $L_1(t)$ chooses their saving ($s(t)$) to maximize their utility subject to the budget constraints below at the end of period $t$ when the output ($y(t)$) is ready:

$$c_1(t) + s(t) = w(t)$$  (4)
$$c_2(t + 1) = \left(r^E(t + 1) + 1\right)s(t)$$  (5)

Let’s confirm a following rule in this overlapping generations model for the later analysis of foreign aid before solving the maximization problem.

**OG Rule 1:** Individuals’ decisions are made once at the end of the period when they are young.

\(^6\) Inada conditions are satisfied: $f(0) = 0, \frac{df}{dk} = f' > 0, \frac{df'}{dk} = f'' < 0, \lim_{k \to 0} f' = \infty \text{ and } \lim_{k \to \infty} f' = 0$.

\(^7\) Inada conditions are satisfied: $\frac{\partial u}{\partial c_1} = u_1 > 0, \frac{\partial u}{\partial c_2} = u_2 > 0, \lim_{c_1 \to 0} u_1 = \infty \text{ and } \lim_{c_2 \to 0} u_2 = \infty$. 
Since the first order condition is represented as: \( \frac{u_1}{u_2} = r^E(t+1) \) and the second order condition is always satisfied: \( d(\frac{u_1}{u_2})/d(s(t)) = (\frac{u_1u_2}{u_2} u_{11} + (1 + r^E(t+1))(u_2u_{12} - u_1u_{22}))/u_2 > 0 \), there exists a unique saving function: \( s(t) = s(w(t), r^E(t+1)) \). Now, let’s confirm a following rule which is suggested by this overlapping generations model.

**OG Rule 2:** Saved output becomes operational as capital stock in the next period. Putty becomes Clay when it passes a period line.

Then, capital market equilibrium \( (S(t) = K(t+1)) \) automatically means: \( k(t+1) = K(t+1)/L_1(t+1) = S(t)/L(t) = s(t) \). Since \( r^E(t+1) = r(t+1) = f'(k(t+1)) \) under the rational expectation, \( k(t+1) = s(f(k(t)) - k f'(k(t)), f'(k(t+1))) \) where \( s_1 > 0 \) because \( c_1(t) \) is normal goods. Assuming the substitution effects is not smaller than the income effects \( (s_2(t) \geq 0) \), an Autarky level of per capita capital stock \( (k_A) \) is represented by the following saving function \( (s_A) \):

\[
k_A(t+1) = s_A(t) = s_A(k_A(t)) \quad (6)
\]

Since \( s_A' > 0, \) and \( s_A'' < 0 \), there exists a unique non-trivial steady state \( (k^* > 0) \). Defining an Autarky levels of output or income, and first and second period consumptions as \( y_A, c_{1A} \) and \( c_{2A} \), growth rate from period \( t \) to \( t+1 \) \( (g_A(t+1)) \) and utility level of the generation \( L(t) \) under Autarky \( (u_A(t)) \) are shown as:

\[
g_A(t+1) = \left( \frac{y_A(t+1) - y_A(t)}{y_A(t)} \right) \quad (7)
\]

\[
u_A(t) = u(c_{1A}(t), c_{2A}(t+1)) \quad (8)
\]
Given steady state levels of income ($y^*$) and $k^*$, a recipient economy is assumed to be in an early development stage so that it is entitled to receive foreign aid: $k_A(t) << k^*$ and $y_A(t) << y^*$.

2.2. Income Aid

Donors send income aid ($M(t)$) at the end of period $t$ to the generations existing in period $t$: the young $L_1(t)$ and the old $L_2(t-1)$, where the ownership is shared between the young ($(1-\theta)M(t)$) and old ($\theta M(t)$): $0 \leq \theta \leq 1$. Normalizing the size of population as one, one budget constraint (equation (9)) for the young $L_1(t)$ and one (equation (10)) for the old $L_2(t-1)$ are altered by this income aid:

$$
c_1(t) + s(t) = w(t) + (1 - \theta)M(t) \quad (9)
$$
$$
c_2(t) = (r(t) + 1)s(t) + \theta M(t) \quad (10)
$$

2.3. Unpredicted Income Aid.

Both generations $L(t)$ and $L(t-1)$ do not predict this income aid just like an emergency aid. The young $L_1(t-1)$ maximize $u(w(t-1) - s(t-1), (1 + r_E(t))s(t-1))$. Their maximization problem is identical to the Autarky because they have already finished their optimizing saving and consumption decisions at the end of period $t-1$ (OG Rule 1). Then, their saving ($s_U(t-1)$) and capital stock ($k_U(t)$) are the same as those in the Autarky so are the capital return ($r_U(t)$), wage ($w_U(t)$) and output ($y_U(t)$) in period $t$: $s_U(t-1) = k_U(t) = s_A(t-1) = k_A(t), r_U(t) = r_A(t), w_U(t) = w_A(t), y_U(t) = y_A(t)$.

The young $L_1(t)$ maximize $u(w(t) + (1-\theta)M(t) - s(t), (1 + r_E(t)+1)s(t))$. Since both consumptions are normal, their saving ($s_U(t)$) and capital stock ($k_U(t+1)$), and wage ($w_U(t+1)$) and output ($y_U(t+1)$) in period $t+1$ increase while the capital return ($r_U(t+1)$) decreases: $s_U(t) = k_U(t+1) \geq s_A(t) = k_A(t+1), r_U(t+1) \leq r_A(t+1), w_U(t+1) \geq w_A(t+1), y_U(t+1) \geq y_A(t+1)$. Then, the next period growth rate becomes higher: $g_U(t) = g_A(t), g_U(t+1) \geq g_A(t+1)$ and welfare levels of all generations
after period $t$ also becomes higher: $u_U(t+j) \geq u_A(t+1+j), j \geq -1$. Therefore, unpredicted income aid is Pareto improving.

2.4. Predicted Income Aid.

As Svensson (2000) and Azam and Laffont (2003) succeeded in formulating their principal agent models of foreign aid distribution, information structure is a key element also in this overlapping generations model. Now, both generations predict the occurrence of this income aid in period $t$. The young $L_1(t-1)$ maximize $u(w(t-1) - s(t-1), (1 + r^E(t))s(t-1) + \theta M(t))$ in period $t-1$ anticipating to receive income aid in period $t$. Then, they decrease the amounts of saving and capital stock so that wage and output in the next period decrease (expressing the predicted aid case as a subscript $P$): $s_P(t-1) = k_P(t) \leq s_A(t-1) = k_A(t), r_P(t) \geq r_A(t), w_P(t) \leq w_A(t), y_P(t) \leq y_A(t)$.

The young $L_1(t)$ maximize $u(w(t) + (1-\theta)M_1(t) - s(t), (1 + r^E(t+1))s(t))$ as in the unpredicted case. But, their income $(w_P(t) + (1-\theta)M(t))$ may not be larger than the Autarky one $(w_A(t))$ because their wage is lower owing to the decreased amount of capital. Their saving $(s_P(t))$ and capital $(k_P(t+1))$ may not be larger than the Autarky so are the next period wage $(w_P(t+1))$ and output $(y_P(t+1))$. Then, growth rate in period $t$ is sure to decrease $(g_U(t) = g_A(t) \geq g_P(t))$, but that in period $t+1$ $(g_P(t+1))$ is unclear compared with $g_A(t+1)$ or could be even less than zero. A bizarre result can happen that $g_P(t+1)$ is higher than $g_U(t+1)$. Since the old $L_2(t-1)$ consumes more, amounts of saving and output in period $t$ decrease. If $y(t)$ goes down too much, the next period growth $(g(t+1))$ might jump to the even higher level.

The welfare of generation $L(t-1)$ becomes higher than the unpredicted aid case $(u_P(t-1) \geq u_U(t-1) \geq u_A(t-1))$ while those of generations $L(t)$ and after become less than the unpredicted case $(u_U(t+j) \geq u_P(t+j), j \geq 0)$ and could be less than the Autarky levels. These results are summarized by the following proposition:
PROPOSITION 1

*Both unpredicted and predicted income aid might be Pareto improving. But, as an aid program is more prepared, it tends to be less efficient.*

2.5. Governance and Conditionality.

As more aid is transferred to the young and the less is to the old (i.e. less $\theta$), all the proceeding amounts of saving, capital, and output ($s(t-1+j), k(t+j)$), and $y(t+j), j \geq 0$) will be higher in both unpredicted and predicted cases. So, growth rate in period $t$ ($g(t)$) and all the welfare levels of generations $L(t)$ and after ($u(t+j), j \geq 0$) will be higher although $u(t-1)$ becomes lower. $\theta$ means a share of rent for generation $L(t-1)$ because it is not only a waste (in a unpredicted aid) but has an adverse effect on the future growth (in a predicted aid). Therefore, $\theta$ can be seen as a governance parameter of the recipient so that the less $\theta$ is, the better the governance is.

When donors’ sole objective is to enhance the economic growth of a recipient, their conditionality on aid should be the best governance ($\theta = 0$). Then, since the young $L_1(t-1)$ know that they are not given any aid, they will not change their saving decision in their young period. So, the highest effectiveness of income aid is attained: $g_U(t) = g_P(t) = g_A(t), g_U(t+1) = g_P(t+1) > g_A(t+1), u_U(t-1) = u_P(t-1) = u_A(t-1), u_U(t+j) = u_P(t+j) > u_A(t+j), j \geq 0$.

2.6. Multi-period Aid

Donors send income aid ($M_U(t)$ and $M_P(t+1)$) to a recipient for two consecutive periods. As before, ownership is shared between the young ($1-\theta(j)$) and the old ($\theta(j)$), $0 \leq \theta(j) \leq 1, j = t, t+1$. Three generations $L(t-1), L(t)$ and $L(t+1)$ receive aid, and total of four budget constraints are altered as:
It is clear that both generations $L_1(t)$ and $L_1(t+1)$ will increase their savings in the unpredicted aid case, which leads to even higher growth rate and utility levels of generations $L(t-1)$ and after. On the other hand, the young $L_1(t-1)$ will decrease their saving in the predicted aid case. Furthermore, the young $L_1(t)$ may not increase their saving, partly because their second period income becomes higher with the income aid, and partly because the amount of their first period income is ambiguous to increase even with an income aid owing to the lower wage.

If a strict conditionality of best governance cannot be implemented, the effects of multiple-period aid on future growth and utility could be even worse than one-time aid. Thus, we can conclude that the stable relationship between donors and a recipient may harm the efficiency of aid.

3. Capital Aid


Donors send capital aid ($M_k(t)$) at the end of period $t$ only to the young $L_1(t)$ under the conditionality of best governance ($\theta = 0$). Ownership is shared between the donors ($\phi - 1$) and the young ($\phi$) where $0 \leq \phi \leq 1$ so that capital loans and capital grants are shown as $\phi = 0$ and $\phi = 1$.

Then, two budget constraints of generation $L(t)$ are affected:

$$c_1(t) + s(t) = w(t) \quad (4')$$
$$c_2(t + 1) = (r(t + 1) + 1)(s(t) + \phi M_k(t)) \quad (5')$$

3.2. Unpredicted Capital Aid
When the young $L_1(t)$ receive the unpredicted capital aid, they maximize $u(w(t) - s(t), (1 + r^E(t+1))s(t))$. Since capital aid can be seen as (saved) output produced in the donors, it is natural to think that capital aid is Putty which becomes Clay (operational) in the next period (OG Rule 2).

Then, their saving does not change but the amount of capital stock increases, so do wage and output in the next period: $s_U(t) = s_A(t)$, $k_U(t+1) = s_U(t) + M_K(t) \geq k_A(t+1)$, $r_U(t+1) \leq r_A(t+1)$, $w_U(t+1) \geq w_A(t+1)$, $y_U(t+1) \geq y_A(t+1)$.

Growth rate in the next period becomes higher: $g_U(t) = g_A(t)$, $g_U(t+1) \geq g_A(t+1)$, and welfare levels of generations $L(t)$ and after also become higher: $u_U(t+j) \geq u_A(t+j)$, $j \geq 0$. Thus, unpredicted capital aid is Pareto improving, noting that donors can not only get back their share of principal but also earn their return of capital: $(1 + r(t+1))(I - \phi)MK(t)$.

3.3. Predicted Capital Aid

Above results are seriously affected when the young $L_1(t)$ predict to receive capital aid. They maximize $u(w(t) - s(t), (1 + r^E(t+1))(s(t) + \phi M_K(t)))$ knowing that they will get a return and principal of their saving plus their share of capital aid in their old period. Saving of $L_1(t)$ turns to:

$$s_P(t) = s\left(w(t), (r^E(t+1) + 1)(s(t) + \phi M_K(t))\right) = s\left(w(t), r^E(t+1), (r^E(t+1) + 1)(s(t) + \phi M_K(t))\right)$$ (11)

Thus, their saving is a function of three terms: income in their young period (wage), expected return of capital, and income in their old period. When the young predict to receive higher old income because of this capital aid, they will consume more and save less in their young period. Even so, they will not consume a whole increased portion of income of capital aid: $-1 < s_3 < 0$ since both young and old consumptions are normal.

Capital stock in period $t+1$ is a sum of their saving and capital aid: $k_P(t+1) = s_P(t) + M_K(t)$.

Under the rational expectation: $r^E(t+1) = r_P(t+1) = f'(k_P(t+1))$, their saving ($s_P(t)$) and the next
period capital stock \((k_{P(t+1)})\) are considered to be a function of the present period capital stock \((k(t))\) and capital aid \((M_{K(t)})\), assuming \(s_2 \geq 0\). The next equation explains whether or not capital aid will increase the amount of capital stock or not:

\[
\left(1 - (s_2 + s_3 \phi M_{K(t)}(t))f''(k_{P(t+1)})\right) \frac{dk_{P(t+1)}}{dM_K(t)} = \phi((f'(k_{P(t+1)})+1))s_3 + 1 \quad (12)
\]

Since \(s_2\) is non negative and \(s_3\) is negative, a sign of the biggest parenthesis in the left-hand side of the equation (12) is indeterminate. Even when we simplify the value of that parenthesis as one by neglecting the second order effects, we cannot be sure to determine a direction of change in capital stock. The first term in the right-hand side represents a change of capital stock caused by a lower saving (i.e. negative). The second term shows a direct change of capital stock installed as a capital aid (i.e. one).

Definite directions of changes in saving, capital, return of capital, wage and output are shown as: \(s_P(t) \leq s_A(t) \leq s_U(t), k_U(t+1) \geq k_P(t+1), r_P(t+1) \geq r_U(t+1), w_P(t+1) \leq w_U(t+1), y_P(t+1) \leq y_U(t+1)\). Then, growth rates and welfare becomes to: \(g_U(t+1) \geq g_P(t+1), u_P(t) \geq u_U(t) \geq u_A(t), u_U(t+j) \geq u_P(t+j), j \geq 1\). Therefore, we obtained the following proposition in this capital aid, similar to Proposition 1:

**PROPOSITION 2**

*Both unpredicted and predicted capital aid might be Parato improving. But, as an aid program is more prepared, it tends to be less efficient.*

3.4. Capital Loans.
As more aid is transferred to individuals in a recipient country (i.e. higher $\phi$), capital stock and output will not be altered in period $t$ and after in the unpredicted aid case while both will be lower in the predicted aid. Then, $u(t)$ will be higher, but $g(t+1)$ and $u(t+j), j \geq 1$ will be identical in the unpredicted aid while $u(t)$ will be higher, and $g(t+1)$ and $u(t+j), j \geq 1$ will be lower in the predicted aid. So, we can conclude that capital loans tend to be more efficient on growth and welfare of a recipient country. This seems to be a counter intuitive result that loans are more efficient than grants. Still, the reason is quite simple; the young save less when they expect more income in their old period.

Besides, capital loans have an attractive characteristic for donors. When donors send more capital loans (i.e. lower $\phi$), donors become better off without harming the welfare of the individuals in a recipient country. Donors can earn larger share of their principal and return of capital, and enjoy probably much higher return of capital than their own because the amount of capital stock is assumed to be quite low in the early stage of development.$^8$

4. Mix of Uses of Aid

4.1. Income and Capital Aid

Donors send foreign aid ($M(t)$) with conditionalities of $\theta = 1$ and $0 \leq \phi \leq 1$ at the end of period $t$. This time, they mixed the uses as income ($\psi$) and capital ($1-\psi$) only to the young $L_1(t)$: $0 \leq \psi \leq 1$.

Then, budget constraints of generation $L(t)$ are shown as:

$$c_1(t) + s(t) = w(t) + \psi M(t) \quad (4^*)$$

$$c_2(t+1) = (r(t+1) + 1)(s(t) + (1-\psi)\phi M(t)) \quad (5^*)$$

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$^8$ Capital loans here could be substituted by a foreign direct investment (FDI) because a high return is expected. As Kimura and Todo (2007) shows in their empirical study, an ODA loans for capital investment can cause a new FDI flow.
With the unpredicted aid, the young $L_1(t)$ maximize $u(w(t) + \psi M(t) - s(t), (1 + r^E(t+1))s(t))$. The young $L_1(t)$ increase their savings with more first period income, thus the amount of capital stock increases with larger savings plus direct investment through capital aid: $s(t) > s(t), k(t+1) = s(t) + (1-\psi)M(t) \geq k_A(t+1), r_A(t+1) \leq r_A(t+1)$. Since wage and output becomes larger: $w_A(t+1) \geq w_A(t+1)$, $y_A(t+1) \geq y_A(t+1)$, growth rate and welfare becomes higher: $g_A(t+1) \geq g_A(t+1), u(t+j) \geq u(t+j), j \geq 0$, which means Pareto improving.

As more income aid is transferred to the young (i.e. high $\psi$), capital and output ($k(t+j), y(t+j), j \geq 1$) become lower. Then, $g(t+1)$ is lower and $u(t+j), j \geq 1$ will be lower while $u(t)$ is higher. So, we can conclude that capital aid is more efficient than income aid. This is quite different from a well-known lesson from the past foreign aid literature that an income compensation without any condition is the most beneficial to a recipient. The young $L_1(t)$ must consume a positive portion of income aid ($\psi M(t)$) in their first period while they are not allowed to consume any portion of $(1-\psi) \phi M(t)$ as a capital aid.

The other side of this result enables us to see $\psi$ as a fungibility parameter. It measures a tendency of recipient individuals to consume foreign aid resources in stead of investing them.

Even when donors send only capital aid, the young with high $\psi$ in a recipient country may consume a large portion of aid, and the efficiency of aid on growth and welfare is greatly diminished.

When foreign aid is predicted, the above results are greatly altered. The young $L_1(t)$ maximizes $u(w(t) + \psi M(t) - s(t), (1 + r^E(t+1))(s(t) + \phi(1-\psi)M(t)))$. Similarly to the subsection 3.3, saving function is represented as:

$$s_p(t) = \left( w(t) + \psi M(t), (r^E(t + 1) + 1)}(s(t) + \phi(1-\psi)M(t)) \right)$$

$$= s(w(t) + \psi M(t), r^E(t + 1) + 1)}(s(t) + \phi(1-\psi)M(t)) \right) \quad (11')$$
Saving is a function of the young and old incomes, and expected capital return where \(0 < s_1 < 1\) and \(-1 < s_3 < 0\). Under the rational expectation, both saving \((s_P(t))\) and next period capital stock \((k_P(t+1) = s_P(t) + (1-\psi)M(t))\) are considered to be a function of present period capital \((k_P(t))\) and foreign aid \((M(t))\). The next equation explains whether or not foreign aid with a given mix of income and capital will increase the amount of capital stock or not:

\[
(1 - (s_2 + s_3(1-\psi)M(t)))f"(k_p(t+1))\frac{dk_p(t+1)}{dM(t)} = s_5(1-\psi)\left(f'(k_p(t+1))+1\right)s_3 + 1 - \psi \quad (13)
\]

Even neglecting second order effects, capital stock is not sure to increase because a sign of right-hand side of equation (13) is indeterminate. Let’s summarize definite changes of this macro economy in both unpredicted and predicted cases as follows:

\(s_{U(t)} \geq s_{A(t)}, s_{U(t)} \geq s_{P(t)}, k_{U(t+1)} \geq k_{A(t+1)}, k_{U(t+1)} \geq k_{P(t+1)}, r_{U(t+1)} \geq r_{A(t+1)}, r_{U(t+1)} \geq r_{P(t+1)}, w_{U(t+1)} \geq w_{A(t+1)}, w_{U(t+1)} \geq w_{P(t+1)}, y_{U(t+1)} \geq y_{A(t+1)}, y_{U(t+1)} \geq y_{P(t+1)}\).

Then, growth rate and welfare are shown as:

\(g_{U(t+1)} \geq g_{P(t+1)}, u_{P(t)} \geq u_{U(t)} \geq u_{A(t)}, u_{U(t+j)} \geq u_{A(t+j)}, u_{U(t+j)} \geq u_{P(t+j)}, j \geq 1\).

When more predicted income aid is transferred to the young (high \(\psi\)), the results are not clear compared with unpredicted aid. Capital stock could increase even when more income aid is given. The reason is shown by the next equation:

\[
(1 - (s_2 + s_3(1-\psi)M(t)))f"(k_p(t+1))\frac{dk_p(t+1)}{d\psi} = M(t)(s_5(1-\psi)\left(f'(k_p(t+1))+1\right)s_3 + 1 - \psi - 1) \quad (14)
\]

When the young receive more income aid (and less capital aid), they save more (both consumptions are normal goods), but the direct capital increase through the aid is lower. Income aid becomes more attractive in the predicted aid than in the unpredicted aid. If the relationship between the donors and the recipient is stable, income aid could be a better choice. For this reason,
we had better not treat $\psi$ as a fungibility parameter in the predicted aid case. Next proposition summarizes the results as follows:

**PROPOSITION 3**

*Capital aid is more efficient than income aid when aid is unpredicted. Income aid becomes more attractive when aid is predicted.*

4.2. Recipient Government.

Donors transfer full ownership of capital aid with no income aid ($\psi = 0$). The recipient government and the young $L_{1}(t)$ share the ownership of capital aid ($0 \leq \phi < 1$). The recipient government’s revenue at the end of period $t$ is shown as: $(1 + r(t+1))(1-\phi)M(t)$. The government transfer its revenue to the young $L_{1}(t+1)$ at the end of period $t+1$ just like a sellout of state-owned enterprises.

With this income transfer, the young $L_{1}(t+1)$ maximize $u(w(t+1) + (1 + r(t+1))(1-\phi)M(t) - s(t+1), (1 + r^E(t+2))s(t+1))$. Since they receive more first period income, they are sure to increase their saving. Expressing a subscript of UGT as unpredicted government transfer and PGT as predicted government transfer, the directions of the macroeconomy after this income transfer are as follows:

$s_{U}(t+1) = s_{P}(t+1) = k_{U}(t+2) = k_{P}(t+2) \leq s_{UGT}(t+1) = s_{PGT}(t+1) = k_{UGT}(t+2) = k_{PGT}(t+2)$

$r_{U}(t+2) = r_{P}(t+2) \geq r_{PGT}(t+2) = r_{UGT}(t+2)$,

$w_{U}(t+2) = w_{P}(t+2) \leq w_{PGT}(t+2) = w_{UGT}(t+2)$,

$y_{U}(t+2) = y_{P}(t+2) \leq y_{PGT}(t+2) = y_{UGT}(t+2)$. Then, growth rate and welfare turns to: $g_{UGT}(t+2) = g_{PGT}(t+2) \geq g_{U}(t+2) = g_{P}(t+2)$,

$u_{UGT}(t+j) = u_{PGT}(t+j) \geq u_{U}(t+j) = u_{P}(t+j), j \geq 1$.

Note that under a conditionality of best governance, the results become identical between the unpredicted and predicted cases. If the young $L_{1}(t)$ receives less capital aid (i.e. less $\phi$), the government earns more revenue and the young $L_{1}(t+1)$ will save more. Then, the growth rate in
period $t+1$ ($g(t+1)$) becomes lower while future growth after that becomes higher, so is the welfare of generations $L(t+1)$ and after.

4.3. Government Reinvestment

The recipient government with a full ownership of capital aid ($\phi = 0$) does not transfer its revenue, but reinvests it in the recipient economy. Then, the young $L_y(t+1)$ maximizes $u(w(t+1) - s(t+1), (1 + r(t+1))s(t+1))$ without expecting any additional income transfer from the government.

Expressing a subscript of GR as government reinvestment, directions of macroeconomy after this income transfer are as follows: $s_{U(t+1)} = s_{GR(t+1)}$, $k_{U(t+2)} \leq k_{GR(t+2)} = s_{U(t+1)} + (1 + r(t+1))(1-\phi)M(t)$, $r_{U(t+2)} \geq r_{GR(t+2)}$, $w_{U(t+2)} \leq w_{GR(t+2)}$, $y_{U(t+2)} \leq y_{GR(t+2)}$. Please note that unpredicted and predicted cases are identical to each other.

Comparing with the government transfer, growth rate in period $t+2$ and the future welfare levels are shown as: $g_{GR(t+2)} > g_{GT(t+2)}$, $u_{GR(t+j)} > u_{GT(t+j)}$, $j \geq 1$. If the recipient government continues to reinvest its share of output, the future growth and welfare will be maximized utilizing the full potential of its economy. This result stands even in the predicted aid case. Even if the young predict the capital aid, they know that they cannot get any income transfer from the government when they become old.

If the recipient government consumes any portion of its share of output, the effects will be much smaller. As the recipient government is more corrupt, we cannot expect such a high improvement in growth and welfare. In the worst case, the economy goes back to the Autarky level. If it happened, donors had better give a whole ownership to the individuals, relying on a decentralized economy ($\phi = 1$). Still, let’s note that the role of local government to reinvest can be played by the donors. If benevolent donors send capital aid once and continue to reinvest their returns and principal, recipient economy will be better off to perform the highest growth rate.
5. Conclusions

We analyze an intertemporal distribution of foreign aid based on an overlapping generations model with a productive capital. The recipient economy is in the early development stage so that the levels of its capital stock and output are quite low compared with their steady state levels. Foreign aid is defined as a transfer from donors to a recipient, which is distinguished according to whether it takes the form of income compensation (income aid) or of capital stock (capital aid). Ownership of foreign aid is shared among the young and old generations, and donors and recipient governments.

Results of this paper are summarized on a table below. Compared with the Autarky levels, the effects of aid are marked as lower (-), none (0), higher (+), and indeterminate (?).

Table 1

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<th>s(t-1)</th>
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<td>Predicted</td>
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Generally speaking, both income and capital aid can be effective for the growth and welfare improvement of the recipient economy. However, as we see clearly in the above table, the amounts of domestic saving and capital stock do not always increase after the aid is sent. They tend to increase when aid is unpredicted but can decrease when aid is predicted.
Besides, we found some interesting results which are totally different from the traditional wisdom of foreign aid literature. First, both income and capital aid lose efficiency without any condition to the individuals in a recipient country. Second, if the individuals in a recipient country predict an occurrence of aid, economic growth and future welfare will be damaged. So, a stable relationship between the donors and the recipient may harm the efficiency of foreign aid. Third, if the individuals in a recipient country do not predict aid, capital aid is more efficient than income aid. Furthermore, its efficiency could be even higher when capital aid is loaned in stead of being granted.

In order to make the results much sharper, we should immediately start constructing a numerical example of this model using Cobb-Douglas production and utility functions case. For future research, we had better analyze foreign aid in a small open economy version of this model, assuming the perfect capital mobility. Then, we may be able to explain the distribution of concessional loans characterized by grant elements. Finally, it will be desirable and fruitful to apply this research into econometric studies such that we can find solutions for better distribution of foreign aid.

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