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- A Simple Theory of Great Depressions -

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Debt-Ridden Equilibria – A Simple Theory of Great Depressions –

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March 19, 2008 (First Draft: April 2007)

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Keywords: Great depressions; collateral constraint; indeterminacy.
JEL Classification: E22, E32, E37, G12.

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March 18, 2008 (First draft: April 2007)

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lower stock prices. There also exists indeterminacy in the equilibrium paths: since optimizations by agents alone cannot specify the path of the economy, the expectations which are exogenously given are necessary to uniquely pin down the equilibrium path. The model provides the policy implication that debt reduction in the corporate sector at the expense of consumers (or taxpayers) may be welfare-improving when firms are debt-ridden.

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I recognized this kind of paralysis from my Goldman Sachs days. The attitude of much of Japan's political establishment seemed to be that of a trader praying over his weakening positions, when what he needed to do was to reevaluate them unsentimentally and make whatever changes made sense.
(Robert E. Rubin, *In an Uncertain World* [New York: Random House, 2003], chap. 8)

1 Introduction

The 1930s in the United States and the 1990s in Japan are both characterized as persistent stagnations of economies with debt-ridden corporate and financial sectors subsequent to asset-price collapses.[1] This paper shows that a simple variant of a neoclassical growth model with collateral constraints can account for key features of these depression episodes. Pioneered by Cole and Ohanian (1999), there has been growing literature in which the neoclassical growth models are used to account for great depressions.[2] Liter-

[1]See Fisher (1933) for a description of debt-deflation in the US Great Depression.

[2]We use "great depression" to denote a large and decade-long recession such as the US Great Depression in the 1930s and the Lost Decade in Japan in the 1990s. Kehoe and Prescott (2002) define a great depression somewhat narrowly as a time period during which detrended output per working-age

ature includes, among others, Hayashi and Prescott (2002), Bergoeing, Kehoe, Kehoe, and Soto (2002), Fisher and Hornstein (2002), and Chari, Kehoe, and McGrattan (2004).

In these papers, it is shown that declines in total factor productivity (TFP) can explain observed declines in output and investment during the onset of great depressions. More challenging for neoclassical models are protracted slumps of a decade or more subsequent to economic collapses at the early stages. Mulligan (2002), Nakajima (2003), and Chari, Kehoe, and McGrattan (2004) show that during the US Great Depression inefficiencies in the factor markets, especially in the labor market, emerged in the early 1930s and continued for a few decades.[3] The persistent inefficiencies suggest that the steady state to which the US economy tended to converge had shifted during the Great Depression. Cole and Ohanian (2004) and Ebell and Ritschl (2007) try to explain the persistent labor inefficiency and naturally come up with models in which institutional changes in the labor market in favor of labor unions cause persistent inefficiency in wage bargaining.

In this paper, we propose a new explanation for persistent inefficiencies that gives us completely different policy implications. With two simple modifications, the standard neoclassical growth model exhibits indeterminacy, and it is shown that there exists a continuum of steady-state equilibria indexed by the amount of corporate debt. A steady state with a larger debt, which we call a *debt-ridden equilibrium*, has more inefficient factor markets, produces less output, and is characterized by lower stock prices. Our explanation is that a great depression is a shift of equilibrium to debt-ridden equilibrium from one with less debt. Two modifications in the model are that firms are subject to collateral constraint on borrowing their working capital (or liquidity) for financing the inputs, e.g., labor and intermediate goods; and that the firms can hold other firms' stocks as their assets and use them as the collateral.

The first modification is the same as that in Kobayashi, Nakajima, and Inaba's (2007) model. Firms must pay the costs for inputs, such as labor and intermediate goods, in

population falls at least 20% and a fall of at least 15% must occur within the first decade of the period.

[3]Persistent inefficiency in the labor market is also found in the 1990s in Japan. See Kobayashi and Inaba (2006b).

advance of production, and they need external funds to finance them. The amount that they can borrow is limited by the value of the collateral. It is easily shown that the financial inefficiency, i.e., the tightness of the collateral constraint, generates inefficiencies in the factor markets, e.g., wedges between marginal products of factors and their market prices. This setting does not necessarily imply that firms do not accumulate internal funds; it may be interpreted as depicting an aspect of the reality that a wide variety of working capital cannot be financed by internal funds in many cases, and external borrowing, which is constrained by collateral, is often necessary to finance the working capital. The idea that firms need external funds to finance working capital and are subject to collateral constraint in borrowing the funds is widely used in recent literature. See, for example, Chen and Song (2007), Jermann and Quadrini (2006), and Mendoza (2006).

The second modification is a novel feature of the present paper. We assume that firms can buy and hold corporate stocks issued by other firms as their financial assets and that they can use the stocks as collateral for input finance. These assumptions seem quite realistic but, to our knowledge, are excluded from standard growth and business cycle literature.

Firms issue risk-free debts to consumers and buy corporate stocks of other firms.[4] The firms do so in equilibrium where the collateral constraint binds because corporate stocks are more valuable than debts for firms, since the stocks can be used as collateral for finanicing the inputs. Corporate debts cannot be used as collateral because of the relation-specificity in lenders' monitoring technology: only the original lender of corporate debt can make the borrower repay, implying that corporate debt is not a collateralizable asset.

Due to these two modifications, our model shows the following interaction between stock prices and productive efficiency: A firm enjoys looser collateral constraint when the levels of stock prices of other firms are higher; the firm can then produce output

[4]Investment in corporate stocks financed by debt was allegedly widespread during the stock-price bubbles on the eves of depressions, e.g., 1929 in the US and 1990 in Japan.

more efficiently; and the higher efficiency raises the stock price of the firm; and thus the higher stock price of the firm loosens the collateral constraints of other firms in turn. This interaction between stock prices and productive efficiency gives corporate stocks an additional value as collateral. That a corporate stock has an additional value as collateral is an externality, since a stock-issuer firm that acts to enhance its own stock price also loosens (unintentionally) the collateral constraint of a stock-holder firm that own the stock. This externality then causes indeterminacy in equilibrium paths, which we call *Modigliani-Miller indeterminacy*: the number of equations, derived from the optimizations by consumers and firms, that describe the dynamics of the equilibrium path, becomes less than the number of macroeconomic variables. Since the optimizations cannot specify the equilibrium uniquely, it is necessary to add some exogenous expectation on the macroeconomic variables to pin down the path of the economy uniquely. The similar strategy to determine the equilibrium by imposing exogenous expectations on the economy, in which optimizations by agents generate a continuum of equilibria, is adopted by Hall (2005).

Due to Modigliani-Miller indeterminacy, steady-state equilibria also become indeterminate. There exists a continuum of steady-state equilibria which are indexed with the level of stock price or the amount of corporate debt. It is shown that in equilibrium where the amount of corporate debt is larger, factor markets are more inefficient, firms produce less output, and stock prices are lower.

We show numerically that this model can replicate the key features of great depressions; that the reduction of corporate debt by government policy at the expense of consumers (or taxpayers) may be welfare-improving when firms are debt-ridden; and that an optimistic change in exogenous expectations that corporate debt will decrease may bring about economic recovery and relief from the debt as a self-fulfilling prophecy.

In Appendix A, we show a modified version of our model in which the exogenous expectation is not necessary to pin down the equilibrium path. We assume that net repayment of corporate debt must be financed by working capital, which is subject to collateral constraint. In the modified model, there still exists a continuum of steady-state equilibria, while the path is uniquely determined for a given initial state without an exogenous expectation. The economy converges to a steady-state with a larger corporate debt if it has a larger initial debt. Most of the conclusions in this paper survive in the modified model without an exogenous expectation.

Organization of the paper is as follows. In the next section, we describe our model and its dynamics. Multiplicity of steady-state equilibria and indeterminacy of equilibrium paths are analyzed. In Section 3, we show the results of the numerical simulations. Section 4 provides policy implications and concluding remarks. In Appendix A, we show a modified model in which the equilibrium path is uniquely determined without imposing an exogenous expectation.

2 Model

In this section, we describe our model and analyze the dynamics. Indeterminacy in paths and a continuum of steady-state equilibria exist. Our model economy is a closed economy with discrete time, that consists of continua of identical consumers and firms, whose measures are both normalized to one. There are also identical banks with unit mass, which only play a role of passive liquidity suppliers. Firms are vehicles that issue stocks and risk-free debts, and maximize the market value of the discounted sum of the dividend flow. The total supply of corporate stocks issued by one firm is normalized to one. Stocks can be traded, and firms can own stocks issued by other firms as their financial assets. We assume without loss of generality that only consumers can hold corporate debts and that firms do not lend to other firms.[5] In this paper, we focus on the symmetric equilibrium where the amounts of capital stocks, corporate debts, and financial assets are identical among all firms. Heterogeneous distribution of these stock variables among firms will make the model analysis very complex. We are confident, however, that our qualitative results in this paper will still hold under heterogeneous distribution among firms (see footnote 8 for more on this).

[5] Allowing firms to hold corporate debts as their assets does not change our results qualitatively as long as we assume Assumption 1.

2.1 Consumer

A representative consumer maximizes her lifetime utility, U, defined over sequences of consumption, c_t, and leisure, $1 - n_t$, where n_t is labor supply. We assume the following class of utility functions:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\epsilon} [c_t (1-n_t)^{\gamma}]^{1-\epsilon}, \tag{1}$$

where E_0 denotes the expectation conditional on the information available in period 0. In period t, the consumer sells labor, n_t, at wage rate w_t, receives the gross returns of corporate debts, $(1 + r_t)b_t$, and of corporate stocks, $(\overline{\pi}_t + q_t)s_t$, where r_t is the interest rate, b_t the amount of debts lent in period $t - 1$, $\overline{\pi}_t$ the dividend of a corporate stock, q_t the stock price, and s_t the amount of stocks bought in period $t - 1$. The consumer purchases consumption goods, c_t, lends debts, b_{t+1}, and buys stocks, s_{t+1}, at the end of period t. Therefore, the consumer's problem is to maximize (1) subject to the following flow budget constraint:

$$c_t + b_{t+1} + q_t s_{t+1} \leq w_t n_t + (1 + r_t) b_t + (\overline{\pi}_t + q_t) s_t. \tag{2}$$

2.2 Firm

A representative firm maximizes the discounted sum of dividend flows, which is discounted by the market discount factor, λ'_t. It is shown later that $\lambda'_t = \lambda_t$ in equilibrium, where λ_t is the consumer's Lagrange multiplier for the budget cosntraint, (2). Therefore, the firm's objective is to maximize

$$V_0 \equiv \frac{1}{\lambda'_0} E_0 \sum_{t=0}^{\infty} \lambda'_t \pi_t, \tag{3}$$

where π_t is the dividend in period t. The firm's actions are as follows. In period t, it employs labor, n_t, buys intermediate inputs, m_t, and produce (gross) output, y_t, using the following production technology:

$$y_t = A_t^{(1-\eta)(1-\alpha)} m_t^{\eta} k_t^{(1-\eta)\alpha} n_t^{(1-\eta)(1-\alpha)}, \tag{4}$$

where A_t represents the level of productivity. The firm issues risk-free debts, b_t, and holds corporate stocks issued by other firms, s'_t, as its assets. We assume that s'_t is not an individual stock but is a share of a mutual fund that invests in stocks of all firms. Since investing in a mutual fund necessitates that the firm pay for financial intermediation, holding s'_t is costly for the stock-holder firm. The cost of holding s'_t is $q_t h(s'_t)$, where $h'(\cdot) > 0$ and $h''(\cdot) > 0$, which is paid to the stock-issuer firms through the mutual fund as a lump-sum transfer.[6] For the functional form, we assume

$$h(s) = \frac{1}{2}\xi s^2. \tag{5}$$

We assume that the firm must pay the costs for inputs, $w_t n_t + m_t$, in advance of production. We also assume that a bank can issue bank notes that can be circulated in the economy as payment instruments. The firm needs to borrow bank notes, d_t, in advance of production to pay input costs. Given d_t, the firm's choice of n_t and m_t is constrained by

$$w_t n_t + m_t \leq d_t. \tag{6}$$

Bank borrowing is intra-period; if R_t is the gross rate of bank loans, the firm is supposed to repay $R_t d_t$ after production. (As discussed below, $R_t = 1$ in equilibrium.) As in Kiyotaki and Moore (1997), however, the firm cannot fully commit itself to repaying the bank loan. It can abscond without repaying at the end of period t, and the bank cannot keep track of the absconder's identity from the next period on. Instead, an imperfect commitment technology is available for the firm and the bank: The firm can put up a part of the corporate stocks that it holds as collateral, and the bank can seize the collateral when the borrower absconds. Therefore, the value of collateral gives the upper limit of the bank loan:

$$d_t \leq \theta q_t s'_t, \tag{7}$$

[6] For simplicity of exposition, we assume that the representative consumer does not suffer from this agency problem and does not pay the stock-holding cost. Introducing the stock-holding cost to the representative consumer does not change our results qualitatively.

where θ $(0 \leq \theta \leq 1)$ is the ratio of corporate stocks that can be put up as collateral. The bank's problem is to maximize the return on the loan, $(R_t - 1)d_t$. Since the bank faces no risk of default if the intra-period loan d_t satisfies (7), competition among banks implies that the return on the loan should be zero $(R_t - 1 = 0)$ in equilibrium. Therefore, in equilibrium, the banks become indifferent to the amount of d_t, and work as passive liquidity suppliers to the firms. So we can neglect the banks' decision-making, since it has no effect on the equilibrium dynamics of this economy. Conditions (6) and (7) together imply the following collateral constraint for the firm:

$$w_t n_t + m_t \leq \theta q_t s'_t. \tag{8}$$

A similar type of collateral constraint is present in the models of Chen and Song (2007), Jermann and Quadrini (2006), and Mendoza (2006). At the end of period t, after production, the firm sells y_t, repays $R_t d_t$, determines dividend, π_t, makes investment in capital stocks, $k_{t+1} - (1-\delta)k_t$, receives gross return from corporate stocks, $(\overline{\pi}_t + q_t)s'_t$, buys new stocks, s'_{t+1}, pays the stock-holding cost, $q_t h(s'_t)$, receives the cost of stock-holding as a lump-sum transfer, T_t, from firms that hold its stocks, repays the old debts, $(1 + r_t)b_t$, and borrows new debts, b_{t+1}, subject to the flow budget constraint:

$$\pi_t + q_t s'_{t+1} + k_{t+1} - (1-\delta)k_t - b_{t+1} = (\overline{\pi}_t + q_t)s'_t + y_t - R_t d_t - (1 + r_t)b_t - q_t h(s'_t) + T_t, \tag{9}$$

where $R_t = 1$ in the equilibrium. The reduced form of the budget constraint is

$$\pi_t = y_t - m_t - k_{t+1} + (1-\delta)k_t - w_t n_t + b_{t+1} - q_t s'_{t+1} - (1 + r_t)b_t + (\overline{\pi}_t + q_t)s'_t - q_t h(s'_t) + T_t. \tag{10}$$

Therefore, the firm's problem is to maximize (3) subject to (4), (8), and (10).

Why is b_t not used as collateral? A key feature of this model is that the corporate stock, s'_t, is used as collateral, while the inter-period corporate debt, b_t, is *not* a financial asset that can be used as collateral for the intra-period borrowing of the working capital. If firms can buy and hold other firms' inter-period debt as a financial asset and can use it as collateral for borrowing intra-period working capital, the dynamics of the model will change completely. We assume, however, the following restriction on the lending technology for the inter-period corporate debt, b_t:

Assumption 1 *Only the original lender of* b_t *can build a relationship with the borrower-firm that enables the lender to monitor the borrower and to ensure repayment of the agreed amount,* $(1 + r_t)b_t$*. This monitoring technology is relation-specific and non-transferable: If the original lender sells* b_t *to another party, the new holder of* b_t *cannot make the borrower repay. (The new holder of* b_t *cannot impose any penalty on the repudiation.)*

This assumption ensures that the corporate debt, b_t, is worthless for anybody other than the original lender, and therefore b_t is not a transferable asset, implying that b_t cannot be put up as collateral for working capital borrowing. This assumption seems reasonable as a simplified description of lending technology during the 1920s in the United States or the 1980s in Japan. The market for corporate bonds has developed only recently; and corporate debt, which was usually in the form of bank lending, was quite illiquid and could not be used as collateral. Relation-specificity in monitoring technology seems a natural assumption for long-term corporate debt or bank loan under the existence of severe information asymmetry. It is a popular assumption in banking literature (see Diamond and Rajan [2000, 2005] for example).

Why is b_t not collateral constrained? Assumption 1 also explains why firms are not subject to collateral constraint when they borrow inter-period debt, b_t. Since the (original) lender has relation-specific technology that ensures the borrower-firm repay, the lender does not need collateral.

Why can a firm not use its own stocks as collateral? As we see in the end of the next subsection, if the firm can use its own stock as collateral in borrowing intra-temporal debt, the equilibrium dynamics are completely changed. We assume the following restriction:

Assumption 2 *A firm cannot use its own stocks as collateral in borrowing the intra-temporal debt, d_t.*

This assumption is justified by supposing that individual firms' stocks have idiosyncratic risk. Although we do not formally specify the risk in our model, it may be plausible to assume that the price of an individual stock is volatile even within a period due to an unspecified idiosyncratic shock to the firm. On the other hand, q_t, the price of s'_t, is stable, since s'_t is a share of the mutual fund that invests in an infinite number of firms and the Law of Large Numbers eliminates the idiosyncratic risk. Therefore, it is plausible to assume that banks do not accept individual stocks as collateral since they are risky, while they do accept shares of the mutual fund. With Assumption 2, we need not prohibit the firms from holding their own stocks as their financial assets. It is easily confirmed that as long as Assumption 2 holds, the equilibrium dynamics do not change qualitatively even if the firm holds its own stocks. In what follows, for simplicity of exposition we focus on the equilibrium where the firms do not hold their own stocks as their financial assets.

2.3 Dynamics

The equilibrium is the set of prices and allocations such that the allocations are solutions to the consumer's and the firm's problems, given the prices, and the following market clearing conditions are satisfied:

$$y_t = c_t + k_{t+1} - (1-\delta)k_t + m_t, \qquad s_t + s'_t = 1. \tag{11}$$

Sinec we focus on symmetric equilibria throughout this paper, the following equilibrium conditions are also satisfied:

$$\pi_t = \overline{\pi}_t, \qquad q_t h(s'_t) = T_t. \tag{12}$$

If the collateral constraint, (8), does not bind, our model would virtually reduce to the standard business cycle model. Throughout this paper, we focus on the case where the collateral constraint always binds. The first order conditions (FOCs) for the consumer are

$$\lambda_t = E_t[(1 + r_{t+1})\lambda_{t+1}], \tag{13}$$

$$\lambda_t q_t = E_t[\lambda_{t+1}(\overline{\pi}_{t+1} + q_{t+1})], \tag{14}$$

$$w_t = \frac{\gamma c_t}{1 - n_t}, \tag{15}$$

where λ_t is the Lagrange multiplier for (2). The FOCs for the firm are

$$\lambda'_t = E_t[(1 + r_{t+1})\lambda'_{t+1}], \tag{16}$$

$$\lambda'_t q_t = E_t[\lambda'_{t+1}(\overline{\pi}_{t+1} + q_{t+1} - q_{t+1}h'(s'_t)) + \mu_{t+1}\theta q_{t+1}], \tag{17}$$

$$(\lambda'_t + \mu_t)w_t = (1 - \eta)(1 - \alpha)\frac{y_t}{n_t}\lambda'_t, \tag{18}$$

$$\lambda'_t = E_t[\lambda'_{t+1}\{(1 - \eta)\alpha y_{t+1}/k_{t+1} + 1 - \delta\}], \tag{19}$$

$$(\lambda'_t + \mu_t)m_t = \eta y_t \lambda'_t, \tag{20}$$

where μ_t is the Lagrange multiplier for (8) in the firm's problem. Since the stock price, q_t, should be equal to the market value of the firm, V_t, equations (3) and (17) together with (5) imply that in equilibrium,

$$E_t[\theta q_{t+1} x_{t+1}] = \xi s'_{t+1} E_t[q_{t+1}],$$

where $x_t \equiv \mu_t/\lambda'_t$. Equations (14) and (17) then imply

$$\lambda'_t = \lambda_t, \tag{21}$$

in equilibrium. Therefore, the FOC with respect to b_{t+1} for the consumer, (13), and that for the firm, (16), are identical and redundant. Since (13) and (16) are redundant, the system of equations that describes the dynamics reduces to 11 equations for 12 unknowns (y_t, c_t, n_t, k_t, m_t, λ_t, x_t, q_t, r_{t+1}, $(1 + r_{t+1})b_{t+1}$, $\overline{\pi}_{t+1}$, s'_t),[7] where $x_t = \mu_t/\lambda_t$ measures

[7] We solve the system of equations by backward shooting.

the tightness of the collateral constraint:

$$\lambda_t = \beta^t \frac{(1-n_t)^{\gamma(1-\epsilon)}}{c_t^{\epsilon}}, \tag{22}$$

$$\lambda_t = E_t\left[\lambda_{t+1}(1+r_{t+1})\right], \tag{23}$$

$$\lambda_t q_t = E_t\left[\lambda_{t+1}\left\{\overline{\pi}_{t+1} + q_{t+1}\right\}\right], \tag{24}$$

$$\frac{\gamma c_t}{1-n_t} = \frac{(1-\eta)(1-\alpha)}{1+x_t}\frac{y_t}{n_t}, \tag{25}$$

$$\lambda_t = E_t\left[\lambda_{t+1}\left\{(1-\eta)\alpha\frac{y_{t+1}}{k_{t+1}} + 1 - \delta\right\}\right], \tag{26}$$

$$m_t = \frac{\eta}{1+x_t} y_t, \tag{27}$$

$$\frac{\gamma c_t}{1-n_t} n_t + m_t = \theta q_t s'_t, \tag{28}$$

$$(1+r_t)b_t - b_{t+1} = c_t - \frac{\gamma c_t}{1-n_t} n_t - q_t(s'_{t+1} - s'_t) - \overline{\pi}_t(1-s'_t), \tag{29}$$

$$c_t + m_t + k_{t+1} - (1-\delta)k_t = y_t, \tag{30}$$

$$y_t = A_t^{(1-\eta)(1-\alpha)} m_t^{\eta} k_t^{(1-\eta)\alpha} n_t^{(1-\eta)(1-\alpha)}, \tag{31}$$

$$E_t[\theta q_{t+1} x_{t+1}] = \xi s'_{t+1} E_t[q_{t+1}]. \tag{32}$$

This system of equations cannot specify the equilibrium path uniquely. If this system consisted of 12 equations, the equilibrium path would have been determined uniquely for the initial values of the two state variables, k_0 and $(1+r_0)b_0$, by choosing the initial values of the two control variables, c_0 and x_0.

Modigliani-Miller Indeterminacy: Note that in the case where the collateral constraint does not bind, the variables b_{t+1} and q_t are also indeterminate because of the redundancy of (13) and (16). In this case, however, the equilibrium allocation of goods, labor, and capital is uniquely determined. Therefore, the indeterminacy between b_{t+1} and q_t is innocuous if the collateral constraint does not bind. This is exactly what Modigliani and Miller's theorem states, i.e., that the means of finance is irrelevant to the real allocations. Therefore , we may call this indeterminacy between b_{t+1} and q_t due to redundancy between (13) and (16) the Modigliani-Miller indeterminacy. On one hand, the Modigliani-Miller indeterminacy is innocuous when the collateral constraint binds.

On the other hand, in the case where the collateral constraint binds, the Modigliani-Miller indeterminacy is not innocuous, since the equilibrium allocation of goods, labor, and capital becomes indeterminate. We analyze this case in this paper.[8] The Modigliani-Miller indeterminacy due to redundancy between (13) and (16) causes indeterminacy in real variables in our model when the collateral constraint binds: The Modigliani-Miller indeterminacy makes b_{t+1} indeterminate, which in turn makes π_{t+1} and thus q_t indeterminate through equation (10); a different value of q_t corresponds to a different value of x_t, which corresponds to a different inefficiency in the labor market through (25) and in the intermediate goods market through (27); therefore, the Modigliani-Miller indeterminacy in b_{t+1} and q_t causes indeterminacy in real variables such as labor and output. Note that the setting of our model wherein other firms' stocks are used as collateral is crucial in generating indeterminacy. The indeterminacy is caused by redundancy of the FOCs with respect to b_{t+1} for consumers and firms; the redundancy arises from that the firm's choice on b_{t+1} does not affect the value of its collateral; and this is because the collateral is other firms' stocks. It is easily shown that the FOCs with respect to b_{t+1} are not redundant if the collateral is the borrower's own stock and that in this case the equilibrium is unique if it exists at all.[9]

[8] Note that the Modigliani-Miller indeterminacy is present even under heterogeneous distribution of capital, k_{it}, corporate debt, b_{it}, and corporate stock, s'_{ijt}, among firms, though in this paper we focus on the symmetric equilibrium where these variables are identical among firms. Suppose that there are N firms with heterogeneous initial values of $\{k_{it}, b_{it}, \{s'_{ijt}\}_{j\neq i}^{N}\}$ for $i = 1, 2, \cdots, N$, where s'_{ijt} is the amount of stock of firm j held by firm i at t. They solve the firm's problem, given these initial values and the market prices: $\{w_t, r_t, \{q_{jt}, \overline{\pi}_{jt}\}_{j\neq i}^{N}, \{\lambda_t\}_{t=0}^{\infty}\}$. The arbitrage on corporate stocks implies that in equilibrium the tightness of collateral constraint is equal among firms: $x_{it} = x_t \ \forall i$, where $x_{it} = \mu_{it}/\lambda_t$ and μ_{it} is the Lagrange multiplier associated with (8) for firm i. The FOCs with respect to b_{it} for $i = 1, 2, \cdots, N$ and for the consumer are identical and redundant. Therefore, the Modigliani-Miller indeterminacy is present due to the redundancy of $N + 1$ equations. This example indicates that we have a higher degree of indeterminacy with heterogeneous distribution than with identical firms.

[9] We illustrate this argument by a modified model, in which we discard Assumption 2 and a firm can use its own stock as collateral. In this case, s'_t is the firm's own stock, and the firm chooses s'_{t+1} regarding that its dividend π_{t+1} and price q_{t+1} are functions of s'_{t+1}. With the Implicit Function Theorem, we can easily show that the FOC with respect to s'_{t+1} is identical to (24), while the FOC for the firm with

2.4 Collateral constraint and productive inefficiencies

A key variable that measures the effect of the collateral constraint is $x_t = \mu_t/\lambda_t$, which represents the tightness of the collateral constraint: If (8) does not bind, $x_t = 0$, and if it binds, $x_t > 0$; and the larger the value of x_t, the tighter the collateral constraint. Therefore, x_t can be viewed as a measure of finanical market inefficiency. At the same time, (25) implies that x_t works as a wedge between the marginal rate of substitution between consumption and leisure and the marginal product of labor. In other words, the financial market inefficiency generates inefficiency in the labor market. Therefore, if x_t is lowered for some reason, the economy experiences a boom, since a reduction in x_t causes an increase in the labor demand (see Kobayashi, Nakajima, and Inaba [2007]). Introduction of intermediate inputs, m_t, in the production technology (4) amplifies the business cycles by generating procyclical movements in the "observed" TFP in the production of value added, $y_t - m_t$. Using (27), the production function for value added can be written as

$$y_t - m_t = \left(1 - \frac{\eta}{1+x_t}\right)\left(\frac{\eta}{1+x_t}\right)^{\frac{\eta}{1-\eta}} A_t^{1-\alpha} k_t^{\alpha} n_t^{1-\alpha}. \tag{33}$$

The TFP for production of value added, $\tilde{A}_t$, is defined by $y_t - m_t = \tilde{A}_t^{1-\alpha} k_t^{\alpha} n_t^{1-\alpha}$. Therefore,

$$\tilde{A}_t \equiv \left(1 - \frac{\eta}{1+x_t}\right)^{\frac{1}{1-\alpha}}\left(\frac{\eta}{1+x_t}\right)^{\frac{\eta}{(1-\eta)(1-\alpha)}} A_t, \tag{34}$$

where $\partial \tilde{A}/\partial x < 0$ if $\eta, x_t > 0$. Thus, a fall in financial market inefficiency increases TFP in the production of value added. Chari, Kehoe, and McGrattan (2004) also describe a similar mechanism of amplification due to frictions in financing intermediate inputs. The result that the observed TFP, $\tilde{A}_t$, decreases as the financial market inefficiency,

respect to b_{t+1} becomes

$$\lambda_t = \lambda_{t+1}(1 + r_{t+1})(1 + \theta x_{t+1} s'_{t+1}).$$

Since the collateral constraint implies that $s'_{t+1} > 0$ in equilibrium, the above condition and the consumer's FOC with respect to b_{t+1} imply that $b_{t+1} = 0$ in equilibrium. (We implicitly assumed that b_{t+1} cannot be negative.) Therefore, the equilibrium path is uniquely determined such that $b_t = 0$.

x_t, increases may support our thesis that great depressions may have a causal linkage with financial frictions, since the literature repeatedly reports that declines in (observed) TFP were the main cause of great depressions in many historical episodes (see Kehoe and Prescott [2002], Hayashi and Prescott [2002], and Chari, Kehoe, and McGrattan [2004]). This quantitative research on great depressions raises the causes of TFP declines as a puzzle (see also Ohanian [2001] for the productivity puzzle of the US Great Depression and Kobayashi [2006] for a theory for the puzzle). Our model may provide a potential explanation. If finance for intermediate input is constrained by collateral and the collateral constraint becomes tighter (because of, for example, a collapse in the prices of collateralized assets) at the onset of depressions, the observed TFP declines.

2.5 Indeterminacy and exogenous expectations

Since the dynamics of the economy are described by equations (22)–(32), 11 equations for 12 unknowns, the equilibrium path is indeterminate. The state variables in period $t + 1$ and the control variables in period t are indeterminate, given the state variables in period t; and the steady state to which the economy converges eventually is also indeterminate. In this subsection we first analyze the continuum of steady states and then argue the role of exogenous expectations in determining the equilibrium path.

See Appendix A for a modification of the model which determines a unique equilibrium path for a given initial state, while the continuum of the steady-state equilibria is preserved. In the modified model, there is no need to add exogenous expectations to specify the equilibrium path.

2.5.1 Continuum of Steady-State Equilibria

Solving equations (22)–(32) analytically for a steady state where the variables are invariant over time, we obtain the equilibrium values of variables, indexed with x:

$$
\begin{aligned}
n(x) &= \frac{1}{1+\gamma\Phi(x)},\\
k(x) &= \left[\frac{(1-\eta)\eta^{\frac{\eta}{1-\eta}}\alpha}{(1+x)^{\frac{\eta}{1-\eta}}r_k}\right]^{\frac{1}{1-\alpha}} An(x),\\
c(x) &= \left[\left(1-\frac{\eta}{1+x}\right)\frac{r_k}{(1-\eta)\alpha}-\delta\right]k(x),\\
y(x) &= \frac{r_k}{(1-\eta)\alpha}k(x),\\
m(x) &= \frac{\eta}{(1+x)(1-\eta)\alpha}r_k k(x),\\
q(x) &= \frac{(1-\eta)(1-\alpha)+\eta}{(1+x)x\theta^2}\xi y(x),\\
\pi(x) &= \frac{1-\beta}{\beta}q(x),\\
rb(x) &= \left[1-\frac{(1-\eta)(1-\alpha)+\eta}{1+x}\left\{1+\frac{1}{\theta}\left(\frac{\xi}{\theta x}-1\right)\right\}-\frac{(1-\eta)\alpha\delta}{r_k}\right]y(x)
\end{aligned}
$$

where $r=\beta^{-1}-1$, $r_k=\beta^{-1}-1+\delta$, and

$$\Phi(x)=\frac{1+x}{(1-\eta)(1-\alpha)}\left[1-\frac{\eta}{1+x}-\frac{(1-\eta)\alpha\delta}{r_k}\right]. \tag{35}$$

It is easily confirmed from the above solutions that gross output, $y(x)$, consumption, $c(x)$, capital, $k(x)$, labor, $n(x)$, intermediate inputs, $m(x)$, and stock price, $q(x)$, are all decreasing in x.[10] All these variables are smaller in a steady-state equilibrium where the financial market inefficiency, x, is larger. Whether corporate debt, $b(x)$, is increasing or

[10]Only that $c(x)$ is decreasing in x is not straightforward. It is shown that $c'(x)=f(x)y(x)-g(x)\gamma\Phi'(x)y/(1+\gamma\Phi(x))$, where

$$
\begin{aligned}
f(x) &= \frac{\eta}{1+x}-\frac{\eta\left(1-\frac{\eta}{1+x}-\frac{(1-\eta)\alpha\delta}{r_k}\right)}{(1-\eta)(1-\alpha)},\\
g(x) &= 1-\frac{\eta}{1+x}-\frac{(1-\eta)\alpha\delta}{r_k}.
\end{aligned}
$$

Since $r_k>\delta$, $g(x)>0$ for $x>0$ and $f(0)<0$. Since $f'(x)<0$ for $x>0$, $f(0)<0$ implies $f(x)<0$. Therefore, $c'(x)<0$ for $x>0$.

decreasing in x is ambiguous. It can be shown, however, that if ξ is sufficiently small, $b(x)$ is increasing in x in the feasible region: $0 \leq x \leq \xi/\theta$. A steady-state equilibrium with a large b can be called a *debt-ridden equilibrium* in this case: A large debt induces a large financial inefficiency, and lowers output, labor, investment, consumption, and stock prices. Figure 1 shows the steady-state allocations and prices as functions of x. We show $\tilde{A}_t$ defined in (34) as the TFP and the value added, $y_t - m_t$, as the output. The parameter values are given as follows: $\beta = .99$; $\gamma = 1.6$; $\epsilon = 1$; $\delta = .02$; $\eta = .5$; $\alpha = .33$; $\theta = .3$; $\xi = .03$; and $A = 1$. Most of these values seem standard. We set the values of β and δ so that the unit of time is a quarter; the value of γ is chosen so that the steady-state value of n is in the neighborhood of 0.3. The vaule of θ is chosen so that there exists a sufficiently large difference between the real variables (e.g., output) in the initial steady state and those in the final steady state in our numerical experiments in Section 3. All our results in this paper are replicated with smaller magnitudes even if we set $\theta = 1$.

2.5.2 The role of exogenous expectations in resolving indeterminacy

To determine the equilibrium path uniquely, we need to add one exogenous condition for each t to the system of the 11 equations. We give three examples (or candidates) for the exogenous condition. Agents in this economy may believe that the tightness of collateral constraint will be constant over time; this exogenous expectation corresponds to the condition that $x_t = x^*$ for all t. Alternatively, agents may believe that the level of corporate debt will be constant over time; this correspnds to that $b_t = b^*$ for all t. Or agents may believe that the wage rate will be constant over time; this sticky wage expectation corresponds to that $w_t = w^*$ for all t. The system of equations (22)–(32), together with one of the above three conditions, can determine the equilibrium path uniquely. The additional condition can be interpreted as the *exogenous expectation* on the values of macroeconomic variables in the future. If the exogenous expectation changes for some reason, the same optimizations by consumers and firms generate a different equilibrium path. Note that the exogenous expectation does not work as a constraint in the optimization problems by consumers or firms, but it works as the equilibrium condition that the aggregate variables, i.e., the solutions to the optimizations, must obey in the equilibrium. Therefore, it can be said that the exogenous expectation is compatible with optimizations by agents. Adding an exogenous expectation to pick a unique equilibrium from a continuum of equilibria is the strategy adopted by Hall (2005). Hall uses the sticky wage expectation to pin down the equilibrium outcome of the wage bargaining economy, which has a continuum of bargaining outcomes.

The exogenous expectations on future values of x_t and/or b_t may be translated into various expectations in reality on wealth distribution in the future between the household and corporate sectors. If agents believe that corporate debt, b_t, will become large and market capitalization, q_t, small in the future, then agents have the exogenous expectation that the tightness of the collateral constraint, x_t, will eventually be large. It can be said that in our model the exogenous expectations (on, for example, wealth distribution in the future) drive the business fluctuations and significantly affect productive efficiencies and the resource allocations both in the short- and long-run. This feature of our model that the exogenous expectation affects the equilibrium path may be regarded as one way to formalize Keynes' view that long-term expectations affect today's economic activities (see Keynes [1936], ch.12).

3 Numerical Experiments for Great Depressions

In this section we report the results of our numerical experiments. The parameter values are the same as those in Figure 1 (see Section 2.5.1). Each figure in this section is divided into upper and lower panels, and the variables shown in the upper panel are normalized such that the initial value at $t = 0$ is set at one. (The variables in the lower panel are not normalized.) In all experiments, the dynamics are assumed to be deterministic. In other words, the respective shocks to which the economy responds in the experiments are treated as totally unexpected events (or measure-zero events). We are confident that the nature of the dynamics of our model will be invariant in stochastic cases. Confirming this conjecture is a topic of our future research.

3.1 Impulse response to productivity shocks

In this subsection we show the impulse responses to temporary and permanent productivity shocks, respectively. Our objective here is to show that our model can replicate the ordinary business cycles in response to (small) productivity shocks. As we argued in Section 2.5.2, we need to add an exogenous expectation to pin down the equilibrium path. Since our interest is on the role of collateral constraint, we put a condition on x_t, the tightness of the collateral constraint, as the exogenous expectation.

Figure 2 shows the impulse response to a temporary productivity shock: The economy was in a steady state initially; and A_t increases by 5% at $t = 1$ unexpectedly, and then decreases by 0.5% each period for $t = 2, \cdots, 10$, and returns to the original level at $t = 11$. We assume that the evolution of A_t for $t \geq 2$ is perfectly foreseen on impact at $t = 1$. We assume as the exogenous expectation that x_t jumps to a certain value, x^n, on impact at $t = 1$, and $x_t = x^n$ for all $t \geq 2$. Therefore, x^n is the value of x_t in the new steady state to which the economy converges after the shock.[11] The response of our model is similar to that of the standard business cycle models. The shift of the steady state to which the economy converges is negligibly small for the temporary shock.

Figure 3 shows the impulse response to a permanent productivity shock: A_t increases by 5% permanently at $t = 1$ unexpectedly. We assume the same exogenous expectation as the experiment for a temporary shock: $x_t = x^{nn}$ for $t \geq 1$. (Note that x^{nn} may not be equal to x^n.) The response of our model seems quite plausible.

3.2 Emergence and collapse of stock-price bubble

Figure 4 shows the response of the model to an emergence and collapse of a stock-price bubble. We assume that corporate debt drastically increases during the bubble period, which lingers after the stock-price bubble collapses.

[11]The value of x^n and the initial value of consumption, c_1, are determined such that capital k_1 and debt b_1 at $t = 1$ are equal to their respective values in the initial steady state, where k_1 and b_1 are given by solving the dynamics, (22)–(32), backward.

3.2.1 Equilibrium path with stock-price bubble

The economy was initially in a steady state equilibrium in which the collateral constraint binds. At $t = 1$ stock-price bubble, S_t, emerges and the stock price becomes $q_t^b = q_t + S_t$, where q_t is the fundamental value of the stock given by (24). The bubble evolves by

$$\lambda_t S_t = E_t[\lambda_{t+1} S_{t+1}], \tag{36}$$

while the initial value is given by $S_1 = 5.1409$ in our experiment. The dynamics of the economy are described by the following system of equations: (22)—(27), (30)—(32), (36), and

$$\frac{\gamma c_t}{1 - n_t} n_t + m_t \leq \theta q_t^b s_t', \tag{37}$$

$$(1 + r_t) b_t - b_{t+1} = c_t - \frac{\gamma c_t}{1 - n_t} n_t - q_t^b (s_{t+1}' - s_t') - \overline{\pi}_t (1 - s_t'). \tag{38}$$

We assume for simplicity that S_t is large enough such that the collateral constraint does not bind once the bubble emerges. Therefore, (37) does not bind and $x_t = 0$. We also assume that all economic agents believe that S_t grows deterministically forever and that there is no possibility of the stock-price bubble collapsing. Under this setting, the real allocations $\{y_t, c_t, n_t, k_t, m_t, \lambda_t\}_{t=1}^{\infty}$ of the economy with stock-price bubble are determined uniquely: This is because the economy follows the path of the standard neoclassical growth model, since the collateral constraint does not bind. On the other hand, the finanical varilables, $\{(1 + r_t) b_t, \overline{\pi}_t, s_t'\}_{t=1}^{\infty}$, are indeterminate due to the Modgliani-Miller indeterminacy. Therefore we need to set one additional condition for the financial variables to pin down the equilibrium path with a bubble. As a build-up of corporate debt is usually observed in an asset-price bubble episode, we assume that $(1 + r_{t+1}) b_{t+1}$ is fixed at a large constant for $t = 1, 2, \cdots, 9$; and that $\pi_{t+1}/[(1 + r_{t+1}) b_{t+1}]$ is constant for $t \geq 10$.[12] We set $(1 + r_{t+1}) b_{t+1} = 2.6828$ for $1 \leq t \leq 9$.

[12] We assume that $\pi_t = \pi_{10}$ for $1 \leq t \leq 9$. The constant value of $\pi_{t+1}/[(1 + r_{t+1}) b_{t+1}]$ is determined endogenously such that s_1', which is calculated by the backward shooting method, equals s' in the initial steady state. We also assume $r_1 = E_0[r_1]$, which is the value in the initial steady state.

3.2.2 Equilibrium path after the collapse of the bubble

Although the agents believe that the bubble grows deterministically forever, it unexpectedly collapses at $t = 6$: In our experiment displayed in Figure 4, we set $S_t = 0$ for $t \geq 6$. As a result, the collateral constraint binds and the dynamics are determined by (22)–(32) for $t \geq 6$. As argued in Section 2.5.2, we need to set an exogenous expectation to pin down the equilibrium path uniquely. We assume as the exogenous expectation that x_t jumps to a certain value, x^d, from zero at $t = 6$, and $x_t = x^d$ for all $t \geq 6$. In other words, we assume that in this economy, people believe that tightness of the collateral constraint for firms is invariant over time after the bubble collapse.[13]

A large amount of corporate debt lingers as a result of the collapse of the stock-price bubble. This increase in corporate debt may be a plausible description of the economic turmoil caused by the emergence and collapse of the asset-price bubble at the onset of the US Great Depression and the Lost Decade in Japan in the 1990s. It is shown in Figure 4 that after the bubble collapse the economy stagnates persistently and converges to a steady state where output, labor, investment, consumption, and stock prices are all lower and corporate debt larger than their respective values in the initial state.[14]

3.3 Debt reduction policy

How can we model policy responses to great depressions such as the Bank Holiday[15] in March 1933 during the US Great Depression and the (gradual) disposal of nonperforming loans in the 1990s in Japan? In our model, these policy responses may be modeled as

[13] The values of x^d, c_6, and q_6 are uniquely determined by the backward shooting method such that k_6, b_6, and s'_6 are equal to their respective values in the bubble path. We assume that the realized values of r_6 and π_6 are those expected in the bubble path, i.e., $r_6 = E_5[r_6]$ and $\pi_6 = E_5[\pi_6]$.

[14] The value of π_7 becomes a large negative number. We interpret the negative dividend as a voluntary capital augmentation by the stock holders in response to the bubble collapse and the unexpected tightening of collateral constraint.

[15] Operations of all banks in the United States were suspended for one week and more than 5,000 banks were finally liquidated. Since banks are financial conduits from households to the corporate sector, the bank closures can be regarded as a reduction of debts in the corporate sector at the expense of the household sector.

an exogenous decrease in corporate debt, b_t, by a lump-sum transfer from consumers to firms. Figure 5 shows the response of the economy to an exogenous and unexpected debt reduction: The evolution of the economy is the same as the previous experiment for $1 \leq t \leq 15$; and at $t = 16$ the corporate debt changes unexpectedly to $(1 + r_{16})b_{16} - \Delta$, where $\Delta = 0.7095$. We assume as the exogenous expectation that x_t jumps to a new value x^{dd} at $t = 16$ and $x_t = x^{dd}$ for $t \geq 16$.[16] The economy picks up when debt-reduction policy is implemented and converges to another steady state, which is more inefficient than the initial steady state but more efficient than the steady state where the economy converges in the case of no debt reduction, shown in Figure 4. Figure 5 shows the behaviors of the macroeconomic variables that seem similar to those in the US Great Depression (see, for example, Chari, Kehoe, and McGrattan [2004]).

3.4 Optimistic expectations

In the experiment shown in Figure 4 we assumed as the exogenous expectation that x_t is constant at a large value after the bubble collapse. This expectation may be interpreted as pessimism over the future of the debt-ridden corporate sector, or a lack of confidence. In the historical episodes of depressions, economic recoveries seemed to be associated with the recovery of confidence. We illustrate the remarkable effect of a change in expectations on the equilibrium path: Figure 6 shows the result of an experiment which is the same as that in Section 3.2 except for the exogenous expectation. We assume as the exogenous expectation that x_t jumps up to x^o at $t = 6$ and $x_t = x^* + 0.5^{t-6}(x^o - x^*)$ for $t \geq 6$, where x^* is the value of x_t in the initial steady state. In other words, we assume that in this experiment people are optimistic about the future and believe that the level of corporate debt will recede toward the initial level rapidly after the bubble collapse.

Figure 6 shows that a change in the exogenous expectation changes the equilibrium path drastically. Once fallen into the depression, the economy recovers toward the initial steady state. Note that this change in expectation is not a change in constraints in

[16] We assume that the realized values of r_{16} and π_{16} are those expected at $t = 15$, i.e., $r_{16} = E_{15}[r_{16}]$ and $\pi_{16} = E_{15}[\pi_{16}]$.

the individual optimization problems for consumers or firms. A difference only in the expectation makes a difference in the equilibrium path between a permanent depression (Figure 4) and a quick economic recovery (Figure 6).

4 Conclusion

The US Great Depression in the 1930s and Japan's Lost Decade in the 1990s are both characterized as persistent recessions with debt-ridden corporate sectors subsequent to asset-price booms and their collapses. Recent literature shows that the persistent stagnations were associated with persistent inefficiencies in the factor markets, especially in the labor market. In this paper, we propose a simple theory of depressions in which two modifications of the standard growth model generate indeterminacy in the dynamics and a continuum of equilibria; and a persistent depression is modeled as a shift of the equilibrium path due to an emergence of large corporate debt resulting from asset price collapse.

The two modifications are the assumptions that firms need to borrow working capital for input cost and the borrowing is subject to collateral constraint, and that firms can buy and hold other firms' stocks as financial assets and can use the stocks as collateral. It was easily shown that the equilibrium path is indeterminate and there also exists a continuum of steady-state equilibria which are indexed with the amount of debt: In a steady state with a larger debt, the factor markets are more inefficient, stock prices are lower, and output is smaller. To pin down the equilibrium dynamics we need to add the exogenous expectation, which implies that a change in the expectations changes the equilibrium path of the economy.

This model provides us with straightforward but surprising implications for economic policy: Debt reduction in the corporate sector at the expense of consumers (or taxpayers) may improve efficiency and social welfare when firms are debt-ridden. That debt reduction is welfare improving is easily confirmed by reducing the value of b_t by a lump-sum transfer from consumers to firms (Figure 5). If our model is a precise description of the decade-long stagnation associated with the persistent nonperforming loans problem in the 1990s in Japan, the policy implications above may be a theoretical translation of what Robert E. Rubin, the 70th US Secretary of the Treasury, said about Japan (see the epigraph). In fact, the Japanese economy has been picking up since 2002, when the government changed its policy stance toward aggressive disposal of nonperforming loans. The Bank Holiday in March 1933 in the United States may also be an example of debt reduction policy in a debt-ridden equilibrium: As is well known, the US economy picked up from March 1933 (see, for example, Cole and Ohanian [1999] and Chari, Kehoe, and McGrattan [2004]).

The experiments in Sections 3.2 and 3.4 sharply illustrate the importance of changes in exogenous expectations or public confidence. If confidence is lost and people believe that corporate debt will remain high for a long time, the economy stays in a persistent recession (Figure 4) and the pessimism becomes a self-fulfilling prophecy. If confidence is not lost, or is recovered after once lost, and people believe that corporate debt will soon return to the original level, the economy also recovers quickly and the optimism is justified (Figure 6). These examples show that exogenous expectations that do not constrain the agents' optimizations may crucially affect the dynamics of the aggregate economy, implying that economic policies or political events that affect public confidence may drastically change the path of the economy.

This model may also provide a new interpretation of the Keynesian prescriptions for recessions. Keynesian fiscal and monetary policies may be effective in our model only if these policies reduce corporate debt, b_t: For example, an expansionary fiscal policy is interpreted as reduction of b_t by a lump-sum transfer from taxpayers (consumers) to firms. Therefore, the Keynesian notion of "demand stimulus" may be interpreted as a policy to reduce corporate debt or change exogenous expectations in our model.

Note that our model of debt-ridden equilibria is purely real. Nominal factors, such as deflation in nominal prices, may be relevant only if they affect the amount of debt by redistributing wealth between consumers and firms. This implication seems consistent with Fisher's (1933) debt-deflation theory. Our model also suggests that the decade-long deflation in Japan since the late-1990s was not a direct cause of the persistent recession:

On the contrary, the deflation in Japan, which still continues into 2008, may be a natural response of nominal prices to the zero-nominal-interest-rate policy adopted by the Bank of Japan in a debt-ridden equilibrium.[17] In our model, liquidity injection by the central bank to lenders of working capital, i.e., banks, is not effective for the economy to escape from the debt-ridden equilibrium, since the borrowing constraint due to borrowers' lack of commitment is not relaxed. What is necessary is to reduce corporate debt to an appropriate level, that is, to make whatever changes make sense.

Appendix A

We propose a modified version of our model in which we can preserve the continuum of the steady-state equilibria and uniquely specify the equilibrium path from a given initial state without appealing to the exogenous expectation. The modified model is the same as the original model except for the collateral constraint: We assume that net repayment of the inter-period debt, $(1 + r_t)b_t - b_{t+1}$, must be made before production and the firm needs liquidity (or intra-period bank lending) to finance the net repayment. This seems a plausible assumption for corporate finance. The modified collateral constraint is

$$w_t n_t + m_t + (1 + r_t)b_t - b_{t+1} \leq \theta q_t s'_t, \tag{39}$$

instead of (8). In this model, the FOCs with respect to b_{t+1} for the consumers and the firms under perfect foresight imply that

$$x_t = x_{t+1}, \qquad \forall t, \tag{40}$$

which makes the equilibrium path unique, given the initial value of the debt, b_0. Therefore, the dynamics of the modified model are quite similar to those of the original model with the exogenous expectation that x_t is constant over time. Thus the effect of an unexpected increase (decrease) in corporate debt is qualitatively equal to that in the original

[17] Since the real interest rate takes on a positive value in the equilibrium, the credible commitment of the Bank to set the nominal interest rate at zero for a long period may generate the expectation that price deflation continues. This mechanism is similar to the one that generates a deflationary equilibrium in Benhabib, Schmitt-Grohé, and Uribe (2002).

model. Most of the results in Section 3 and policy implications survive in the modified model except for that in Section 3.4, where the dynamics drastically change in response to an arbitrary change in the exogenous expectation.

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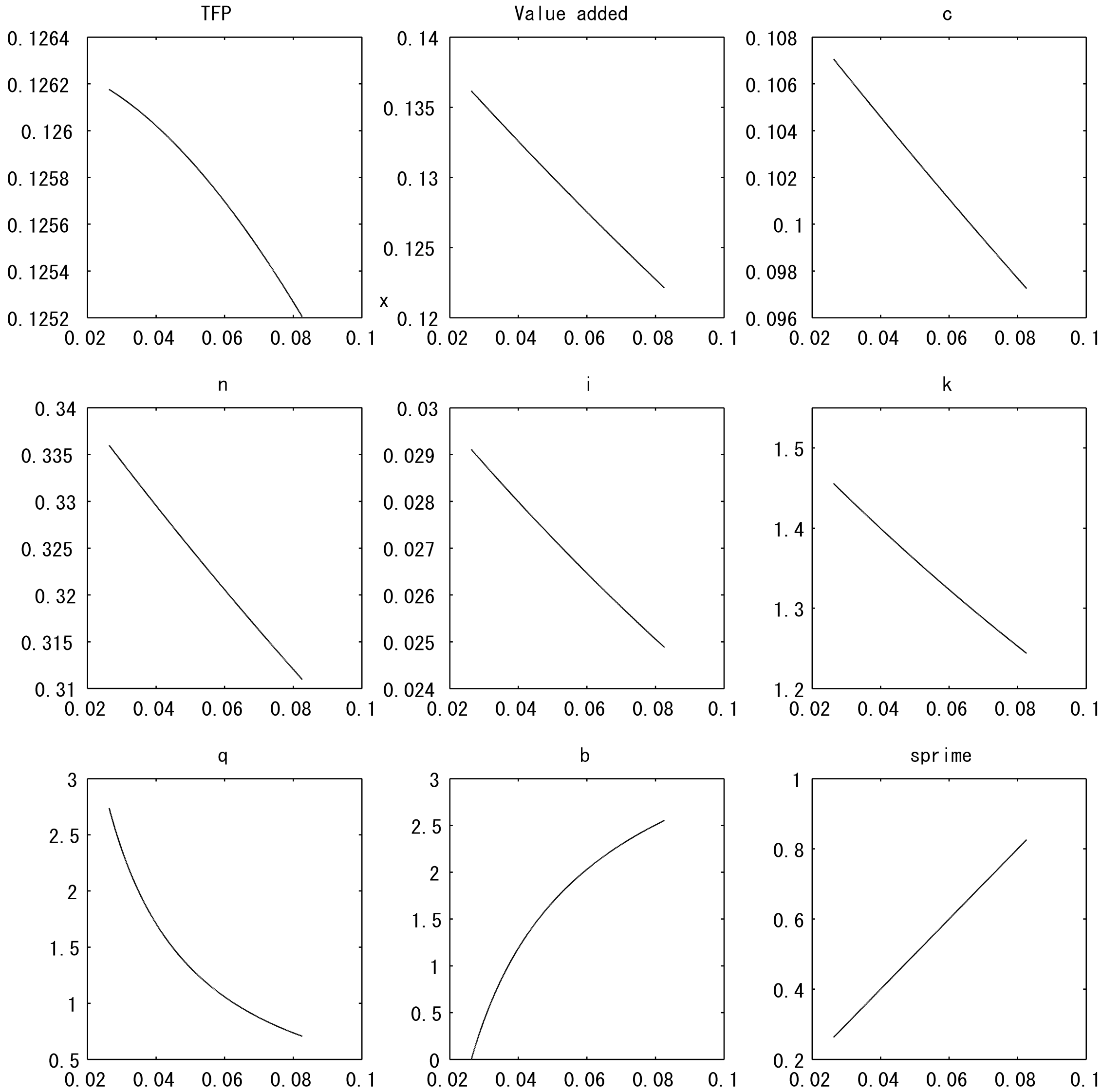

Figure 1: Continuum of steady-state equilibria indexed with x

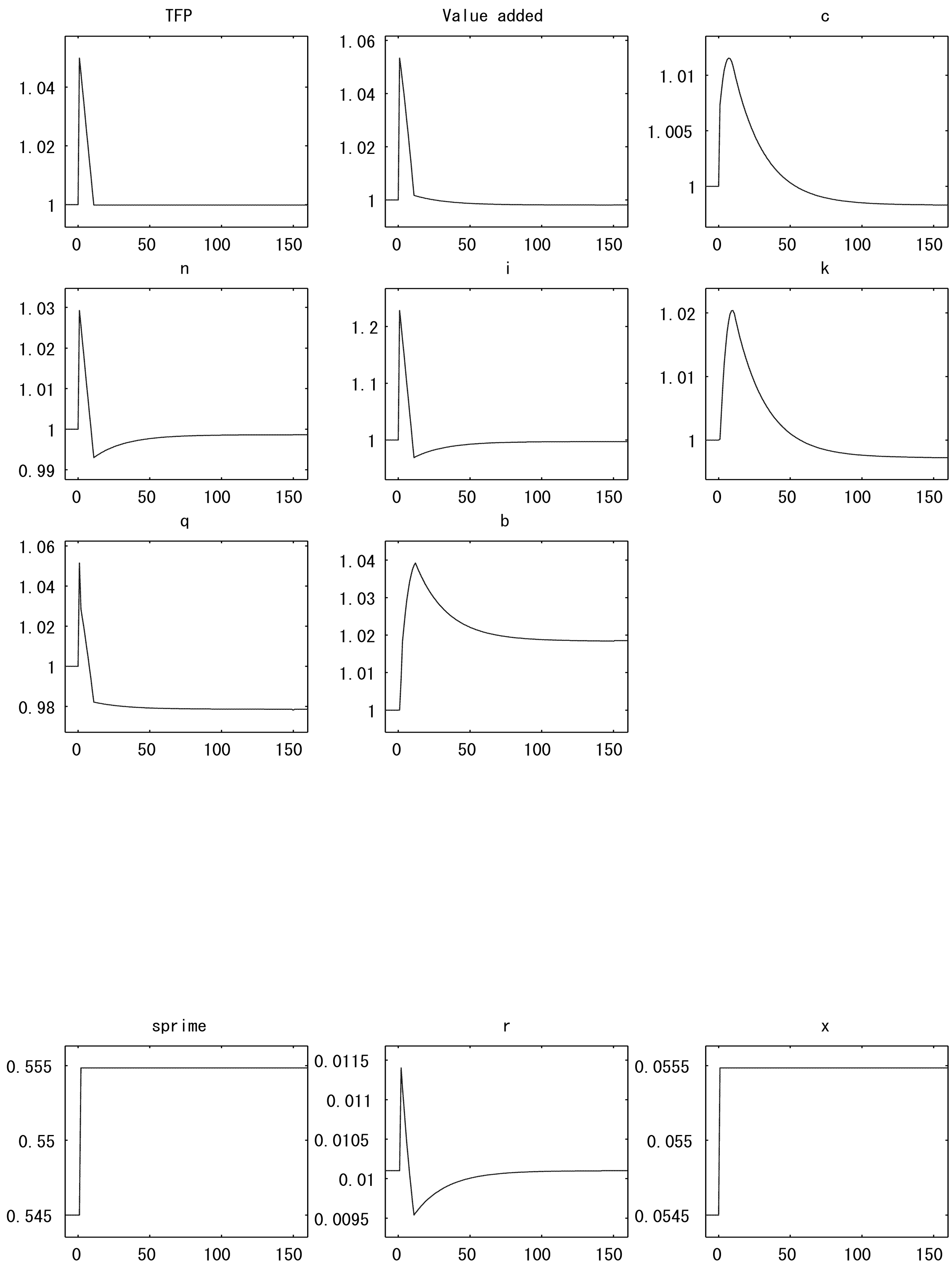

Figure 2: Impulse response to a temporary productivity shock

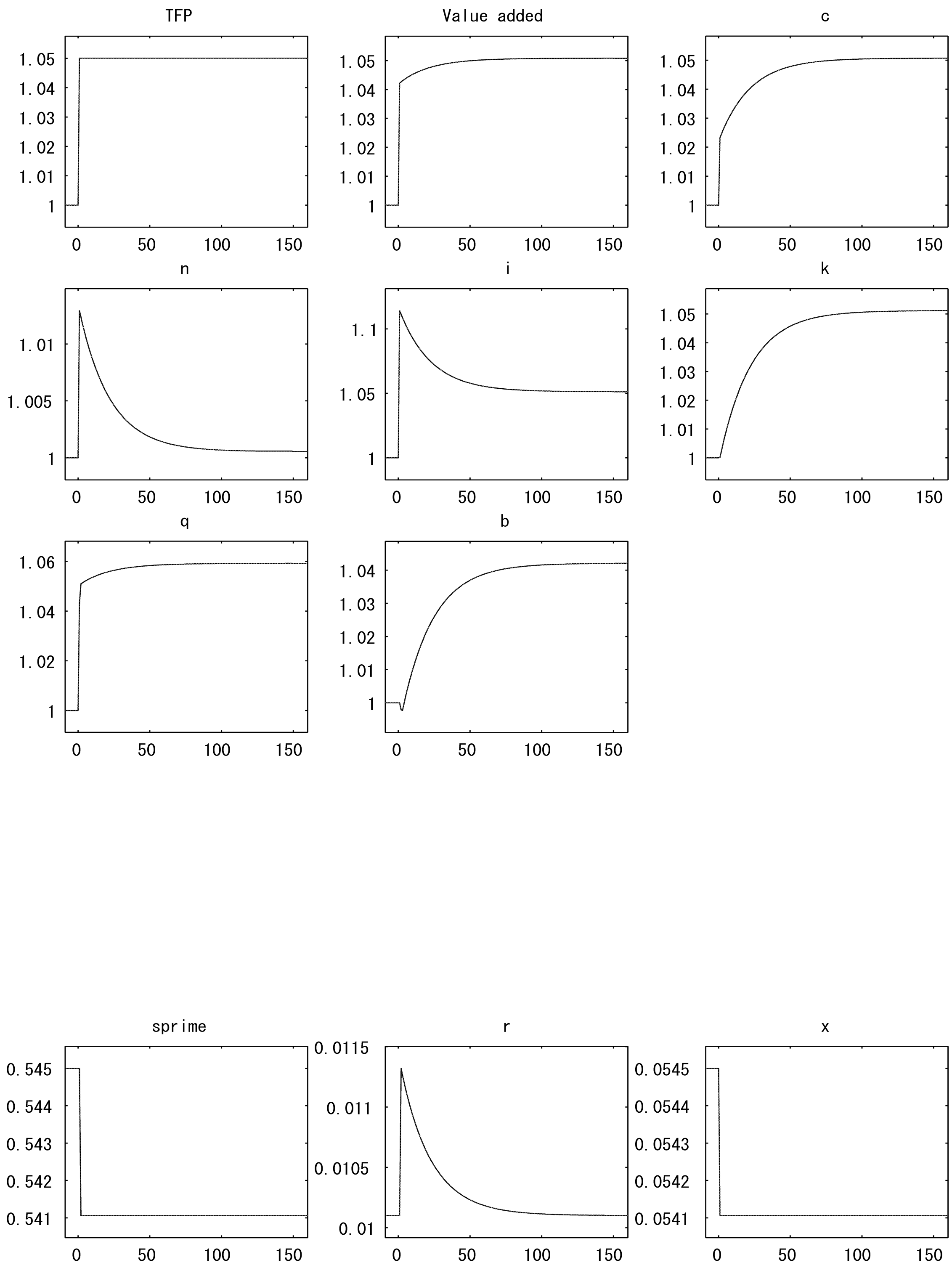

Figure 3: Impulse response to a permanent productivity shock

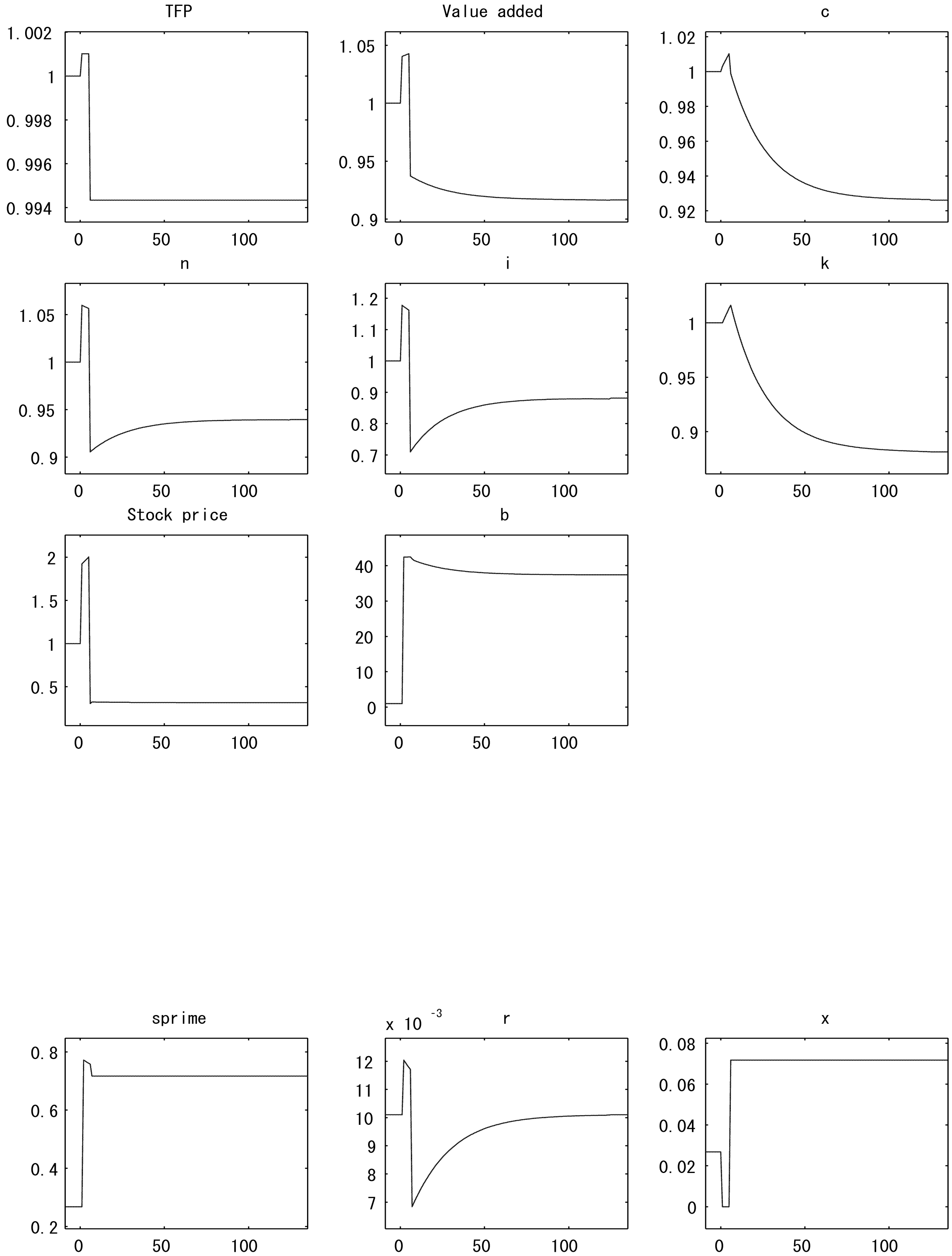

Figure 4: Emergence and collapse of stock-price bubble (with expectations that debt lingers for $t \geq 6$)

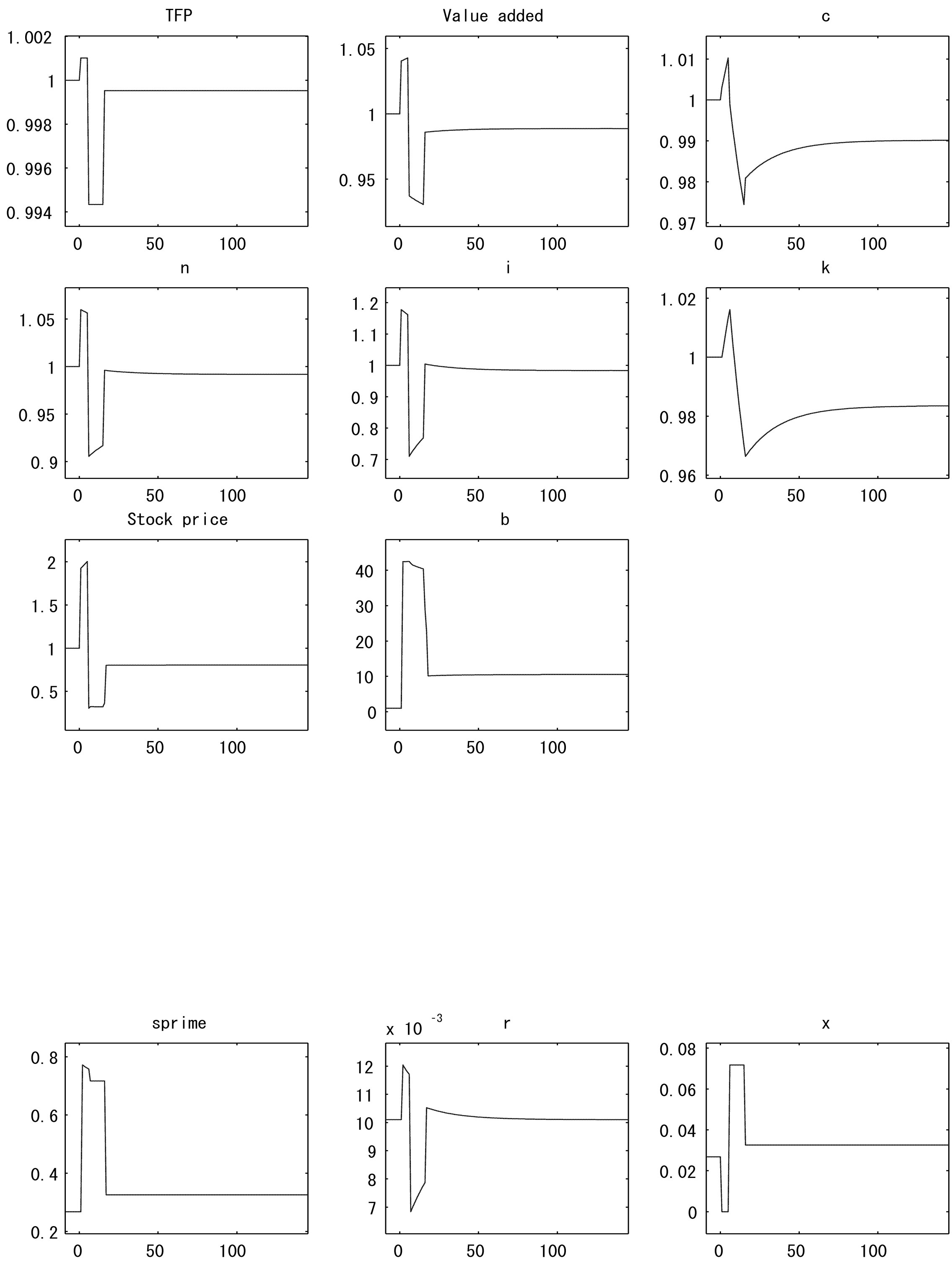

Figure 5: Debt-reduction policy: Debt is unexpectedly reduced at $t = 16$

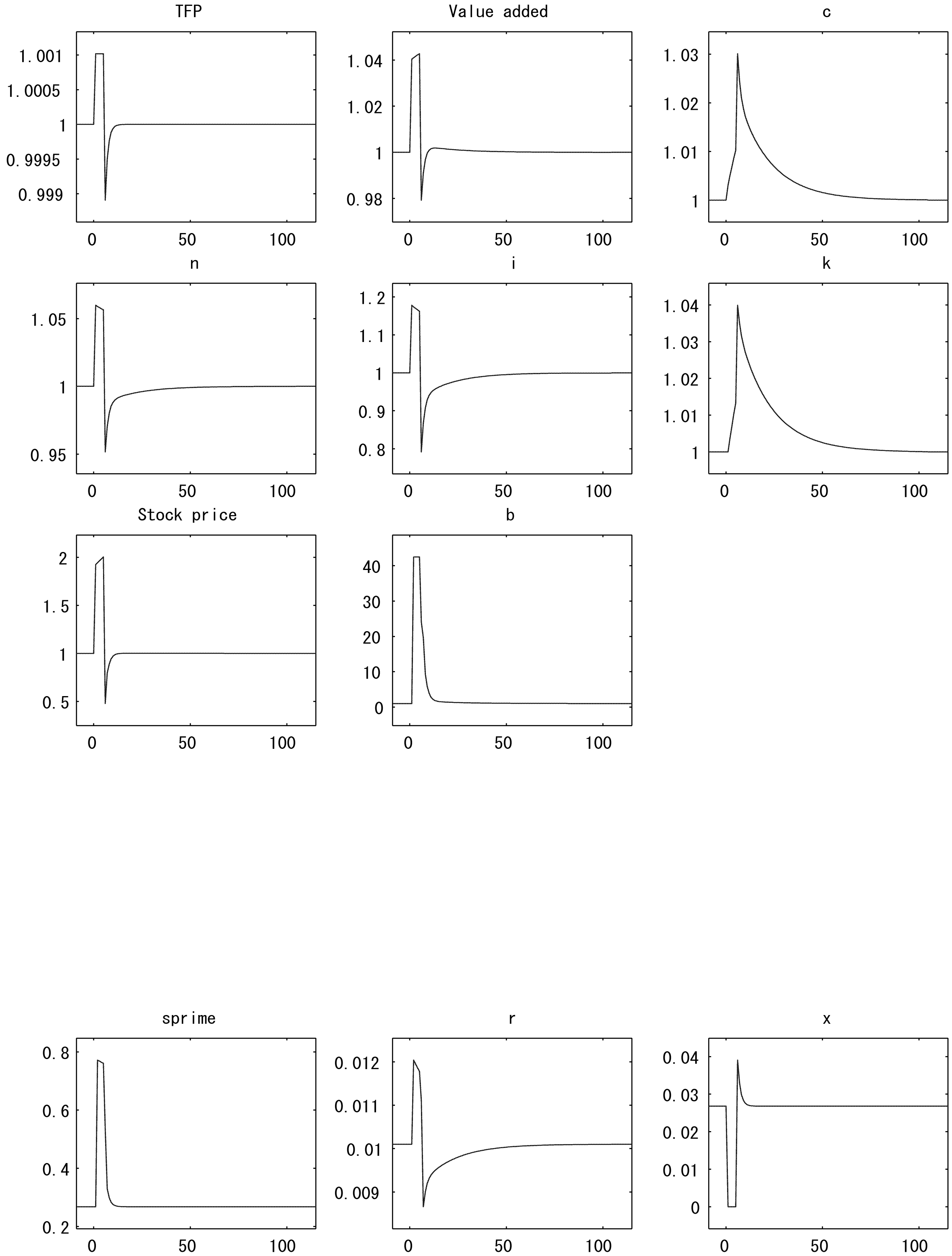

Figure 6: Optimistic expectations that debt goes down to the initial level