RIETI Discussion Paper Series 06-E-036

An Optimal Rate of the National Burden in an Aging Japan

OKAMOTO Akira
Okayama University

Research Institute of Economy, Trade & Industry, IAA
The Research Institute of Economy, Trade and Industry
http://www.rieti.go.jp/en/

RIETI Discussion Paper Series 06-E -036

An Optimal Rate of the National Burden in an Aging Japan

Akira Okamoto*

Okayama University

November 19, 2006

Abstract

This paper examines an optimal rate of the national burden to establish guidelines for fiscal reform in Japan's graying society. The paper looks at Japanese tax and social security systems through an extended life-cycle general equilibrium simulation model. It explicitly considers the benefits that the government provides to households, which enables us to comprehensively evaluate the balance between benefits and burdens. Simulation results show that an optimal rate of the national burden is high when households put a great utility weight on the benefits coming from public services, and that it is low when the degree of publicness of government expenditure is high. The results also suggest that an optimal rate of the national burden would rise as Japan ages and may exceed 50% during the rapid aging of its population.

Keywords: Aging population; National burden rate; Government expenditure; Tax Reform; Life-cycle general equilibrium simulation model

JEL classification: H20; C68

RIETI Discussion Papers Series aims at widely disseminating research results in the form of professional papers, thereby stimulating lively discussion. The views expressed in the papers are solely those of the author(s), and do not present those of the Research Institute of Economy, Trade and Industry.

* *Correspondence to:* Associate Professor, Faculty of Economics, Okayama University, Tsushima, Okayama, 700-8530, Japan.
Phone & Fax: +81-86-251-7539 *E-mail:* okamoto@e.okayama-u.ac.jp

1 Introduction

With a population that is aging faster than any other in the world, Japan faces serious problems with its public finance, particularly when it comes to tax and social security issues.[1] Structural reforms are urgently needed to accommodate the impending demographic change. There has recently been increased discussion about the “rate of the national burden” in Japan. When reconstructions of public finance or fiscal structural reforms were implemented in past years, one of the government’s aims was to keep the rate of the national burden below 50%. One of the main reasons for this is that, as the rate of the national burden becomes high, economic vitality is diminished.

Several recent studies, however, have suggested that the size of the welfare state has no relation to the rate of economic growth. For example, Atkinson (1999) reported that the results of econometric studies of the relationship between social transfer spending and growth rates are mixed: some find that high spending on social transfers leads to lower growth, others find the reverse, and thus that the studies of the aggregate relationship between economic performance and the size of the welfare state reviewed in his book do not yield conclusive results.

This paper aims to establish guidelines for fiscal reform in Japan’s graying society by looking at Japanese tax and social security systems through an extended life-cycle general equilibrium simulation model. Many papers have studied tax reforms using this kind of model; for instance, Auerbach and Kotlikoff (1987), Altig *et al.* (2001), and Ihori *et al.* (2005). However, these papers incorporate taxes into their models but do not take account of the fact that government expenditure enhances households’ utility. Moreover, few studies have examined an optimal rate of the national burden, taking full account of the benefits that the government provides to households.

Therefore, we explicitly consider that public services improve households’ welfare, which is the main contribution that this paper aims to make. This enables us to comprehensively evaluate the balance between benefits and burdens, and thus to investigate an optimal rate of the national burden. We explore an optimal rate of the national burden during the rapid aging of the Japanese population.

When taking account of the fact that households receive benefits coming from government spending, there are two crucial aspects. One is to what degree households put a utility weight for the benefits coming from government expenditure, compared with their private consumption. The other is to what degree the goods and services that the government provides to households take on the character of public or private goods. If government expenditure takes on the character of public goods, its benefits to households are widespread. On the other hand, if it takes on the character of private goods, its benefits to households are restricted. Because simulation results depend substantially on the parameter values that determine the above degrees, we undertake sensitivity analyses for changes in these parameter values.

This paper is organized as follows. The next section identifies the basic model employed in the simulation analysis. Section 3 explains the method of simulation analysis and the assumptions adopted. Section 4 evaluates the simulation findings and discusses policy implications. Section 5 summarizes and concludes the paper.

2 Theoretical Framework

The life-cycle growth model employed in this paper is grounded in the microeconomics of intertemporal choice, and the macroeconomics of savings and growth. The simulation model has three features. First, aggregate assets of the economy in each period consist of the assets of different generations that maximize their lifetime utility. This allows us to rigorously analyze changes in the supply of assets caused by demographic changes. Second, assets in the capital market, where aggregate assets appear as real capital, affect the production level. Third, it is possible to estimate realistic consumption-savings profiles for the elderly, by incorporating life-length uncertainty and unintended bequests into the model.

We calibrate the simulation of the Japanese economy by employing population data estimated by the National Institute of Population and Social Security Research in 2002.

The model has 75 different overlapping generations. Three types of agents are considered: households, firms, and the government. The basic structure of households is as follows.

2.1 Household Behavior

Households are divided into three income classes: low, medium, and high.[2] A single household type represents each income class. Each household has the same mortality rate and the same utility function. Unequal labor endowments, however, create different income levels. Each household appears in the economy as a decision-making unit at the age of 21 and lives to a maximum of 95. Households face an age-dependent probability of death. Let $q_{j+1|j}$ be the conditional probability that a household of age j lives to $j+1$. Then the probability of a household of age 21 surviving until age s can be expressed by

$$p_s = \prod_{j=21}^{s-1} q_{j+1|j} \ . \tag{1}$$

The probability $q_{j+1|j}$ is calculated from data estimated by the National Institute of Population and Social Security Research (2002).

The utility of each household depends on the levels of consumption and the public services that the government provides to each household. There is no choice between leisure and labor supply. Each household works from age 21 to RE, the retirement age. The labor supply is inelastic before retirement and zero after retirement (see the first comment in Section 4.2 for the explanation of this assumption). Each household that maximizes the expected lifetime utility makes lifetime decisions at age 21 concerning the allocation of wealth between consumption and savings. The utility function of a representative household of income class i, the form of which is assumed to be time-separable, is

$$U_t^i = \frac{1}{1-\frac{1}{\gamma}} \sum_{s=21}^{95} p_s (1+\delta)^{-(s-21)} \left[\left\{ C_s^i \right\}^{1-\frac{1}{\gamma}} + \mu \left\{ \frac{G_t}{AN_t^{\rho}} \right\}^{1-\frac{1}{\gamma}} \right], \tag{2}$$

where C_s^i represents consumption (or expenditure) at age s, δ the adjustment coefficient for discounting the future, and γ is the intertemporal elasticity of substitution in different years. The superscript $i\,(=l,m,h)$ stands for low, medium, and high-income classes, respectively.

The goods and services that the government provides to households are incorporated, with reference to Borcherding (1985). G_t is government spending on goods and services in year t, AN_t the size of the households, namely, the total population at period t, μ a weight placed on the benefits coming from government expenditure (i.e., public services), and the exponent ρ is the degree of publicness. If ρ is zero, then government expenditure, G_t, is purely public. On the other hand, if ρ is unity, then G_t is purely private. It needs to be noted that, when the value of ρ is large, the degree of publicness is low.

The flow budget constraint equation for each household at age s is

$$A_{s+1}^i = \{1 + r(1-\tau_r)\}\,A_s^i + \{1 - \tau_w(wx^i e_s) - \tau_p\}\,wx^i e_s + b_s^i + a_s^i - (1+\tau_c)C_s^i, \qquad (3)$$

where A_s^i represents the amount of assets held by the household at the beginning of age s, r the interest rate, w the wage rate per efficiency unit of labor, and e_s is the age profile of earnings ability for the household that belongs to the medium-income class.[3] b_s^i is the amount of public pension benefit, and a_s^i is the amount of bequest to be inherited at age s. $\tau_w(wx^i e_s)$ is the tax rate on labor income, τ_c that on consumption, τ_r that on interest income, and τ_p is the contribution rate to the public pension scheme. x^i is the weight coefficient corresponding to the different levels of labor endowments among the three income classes. The medium-income class is used as a yardstick, and thus $x^m = 1$. x^l and x^h reflect the realistic differences in earnings ability across the three income classes.

The tax system consists of labor income, interest income, consumption, and inheritance taxes. Labor income is progressively taxed. The symbol $\tau_w(wx^i e_s)$ in equation (3) signifies that τ_w is a function of $wx^i e_s$. The progressive tax schedule is incorporated in the same manner as in Auerbach and Kotlikoff (1987): by choosing two parameters labeled α and β, we set the average tax rate $\tau_w(wx^ie_s)$ equal to $\alpha + 0.5\beta wx^ie_s$ for all values of wx^ie_s. The corresponding marginal tax rate is $\alpha + \beta wx^ie_s$. Setting $\beta = 0$ amounts to proportional taxation. One may make the tax system more progressive, holding revenue constant, by increasing β and decreasing α simultaneously. On the other hand, interest income, consumption, and inheritances are proportionally taxed.

Variables related to the public pension program in a pay-as-you-go system are represented by

$$\begin{cases} b_s^i = \theta H^i & (s \geq ST) \\ b_s^i = 0 & (s < ST) \end{cases}, \tag{4}$$

where the age at which each household starts to receive public pension benefits is ST, the average annual remuneration is $H^i\left(= \frac{1}{RE - 20}\sum_{s=21}^{RE} wx^ie_s\right)$, and the replacement ratio is θ. Thus b_s^i rigorously reflects different earnings abilities across the three income classes.

There are unintended bequests caused by uncertainty over the length of life. The bequests, which were held as assets by deceased households, are handed to surviving 50-year-old households. Therefore a_s^i is positive if and only if $s = 50$, and otherwise is zero. The inheritance is transferred within the households that belong to the same income class. When BQ_t^i is the sum of bequests inherited by 50-year-old households at period t, a_{50}^i is defined by

$$a_{50}^i = \frac{(1-\tau_h)BQ_t^i}{N_t p_{50}(1+n)^{-29}}, \tag{5}$$

where

$$BQ_t^i = N_t \sum_{s=21}^{95} (p_s - p_{s+1})(1+n)^{-(s-21)} A_{s+1}^i, \tag{6}$$

N_t is the number of new households entering the economy as decision-making units at period t, n is the common growth rate of successive cohorts, and τ_h is the tax rate on inheritances of bequests. In the steady state of a life-cycle growth model, the amount of inheritances received is linked to the age profile of assets chosen by individuals.

Let us consider the case in which each household maximizes lifetime utility under a constraint. Each household maximizes equation (2) subject to equation (3) (see Appendix A). From the utility maximization problem, the equation expressing evolution of consumption over time for each household is characterized by

$$C_{s+1}^{i} = \left[\left(\frac{p_{s+1}}{p_s} \right) \left\{ \frac{1 + r(1 - \tau_r)}{1 + \delta} \right\} \right]^{\gamma} C_s^i . \tag{7}$$

If the initial consumption level, C_{21}^{i}, is specified, optimal consumption behavior of all ages can be derived from equation (7). The amount of assets held by each household at each age can be obtained from equation (3). The expected lifetime utility of each household is derived from equation (2).

The social welfare function, which takes account of different earnings abilities and thus provides different levels of consumption, is given by

$$SW = U^{l} + U^{m} + U^{h} . \tag{8}$$

This function is derived from a summation of the expected lifetime utilities at age 21 for the three income classes. When comparing steady states, it is not necessary to take account of the utilities of all overlapping generations existing at period t . A comparison of the lifetime utility of a single cohort is sufficient, because our aim is to compare the welfare level among simulation cases. The social welfare function is of the "Benthamite type," but depends mainly on the utility of the low-income class, like the "Rawlsian type."[4] It is maximized if all income classes have the same level of consumption.

With regard to the basic structure of firms, a single production sector is assumed to behave competitively using capital and labor, subject to a constant-returns-to-scale production function. See Appendix B for the basic structures of firms and the government, and market equilibrium conditions.

3 Simulation Analysis

The method of simulation employed in our analysis represents the economy as a general equilibrium system. We investigate an optimal tax rate on consumption, by comparing the equilibrium solutions that are calculated under given and counterfactual conditions. This method permits us to numerically calculate various economic variables and to recognize concrete policy implications. It is especially useful when examining policy packages, or when there are multiple policies that work in a differing direction.

3.1 Method of Simulation

The simulation model presented in the previous section is solved under the hypothesis of perfect foresight by households that correctly anticipate the interest, wage, tax, and contribution rates. If the tax and public pension systems are determined, the model can be solved using the Gauss-Seidel method (see Auerbach and Kotlikoff (1987) for the computation process). The parameter values α and β, which determine tax progressivity on labor income, and the tax rates on interest income (τ_r) and inheritances (τ_h), are exogenously given. Thus, tax revenue neutrality makes the tax rate on consumption (τ_c) endogenous. With regard to the public pension system, the replacement ratio (θ) is exogenously given. Therefore, the contribution rate (τ_p) is made endogenous.

3.2 Simulation Cases

We consider three demographic regimes, namely, the 2005 current steady state and the 2025 and 2050 aged steady states. Case A is the current benchmark of 2005. Cases B and C are the benchmarks of an aged Japan projected for 2025 and 2050, respectively. Cases A, B, and C differ in population structure.[5] Different survival probabilities (p_s) and different growth rates of successive cohorts (n) create different age structures of the population among the three demographic regimes (see Table 1 for the parameter values characterizing the three steady states).

Under conditions of aged steady states, Japan will face a decline in the proportion of its working population, with a subsequent decrease in aggregate output (Y_t); hence, the ratio of the total tax revenue (T_t) to aggregate output (Y_t) is raised, representing a substantial increase in the tax burden on an aging society. Under tax revenue neutrality, there is an extra burden caused by the transition from the current state (Case A) to the aged states (Cases B and C). In an aging society, tax policies that stimulate capital accumulation can be required; and for this a consumption tax promotes capital formation more than other tax regimes (see Okamoto (2004) for the details). Furthermore, there is the political possibility that the tax rate on consumption can be raised in present-day Japan. Therefore, in our simulation, a consumption tax covers the extra tax burden caused by transition to an aging society. Additionally, when maximizing social welfare, we consider cases in which the tax rate on consumption is adjusted.

The simulation results depend on the weight parameter, μ, placed on public services in the utility function. The parameter value is assigned to 0.1 in the benchmark cases. Because the setting of the weight parameter is crucial for the aim of our analysis, a sensitivity analysis is undertaken. We consider simulation cases in which it is decreased to 0.01 and increased to 0.3. Also, the results depend on the parameter, ρ, which determines the degree of publicness on government expenditure. The parameter value is assigned to 0.88 in the benchmark cases. We consider cases in which it is decreased to 0.75 and increased to unity. The following simulation cases are now considered.

1 Three benchmarks for 2005, 2025, and 2050

Table 2 shows the simulation results for the three benchmarks for 2005, 2025, and 2050.

Case A (Benchmark of the 2005 current state)

The utility weight, μ, is set to 0.1. The tax system on labor income has a realistic progressiveness, with an average tax rate of 6.77%. Tax rates on consumption, interest income, and inheritances are 5%, 20%, and 10%, respectively.

Cases B (Benchmark of the 2025 aged state) ***and C*** *(Benchmark of the 2050 aged state)*

The utility weight, μ, is the same as in Case A. The extra tax burden, caused by a transition to an aged society, is covered by a consumption tax. Other tax regimes remain the same as in Case A.

2 Cases that maximize social welfare for 2005, 2025, and 2050

Table 3 presents the simulation results in cases that maximize social welfare for the three benchmarks.

*Case A**

In Case A, the tax rate on consumption is adjusted to maximize social welfare. A rise in the tax rate leads to an increase in government expenditure.

Cases B and C**

In Cases B and C, the tax rate on consumption is adjusted to maximize social welfare, which gives Cases B* and C*, respectively.

3 Changes in the utility weight parameter given for public services for 2005, 2025, and 2050

In contrast to Cases A, B, and C, the weight parameter, μ, given for the benefits coming from government expenditure is changed to 0.01 and 0.3. Table 4 presents the simulation results of the transitions of an optimal tax rate on consumption, an optimal rate of the tax burden, and an optimal rate of the national burden for $\mu = 0.01$, 0.1, and 0.3.

Cases A-1 and A-2

In contrast to Case A, the utility weight, μ, is changed to 0.01 and 0.3, which yields Cases A-1 and A-2, respectively.

Cases A-1 and A-2**

In Cases A-1 and A-2, the tax rate on consumption is adjusted to maximize social welfare, which gives Cases A-1* and A-2*, respectively.

Cases B-1, B-2, B-1, and B-2**

In the 2025 aged steady state, simulations similar to Cases-A are conducted. Cases B-1, B-2, B-1*, and B-2* correspond to Cases A-1, A-2, A-1*, and A-2*, respectively.

Cases C-1, C-2, C-1, and C-2**

In the 2050 aged steady state, simulations similar to Cases-A are undertaken. Cases C-1, C-2, C-1*, and C-2* correspond to Cases A-1, A-2, A-1*, and A-2*, respectively.

4 Changes in the degree of publicness on government expenditure

Table 5 shows the simulation results for different degrees of publicness on government expenditure in the 2005 current benchmark case.

Case A-a

In contrast to Case A, the exponent, ρ, that determines the degree of publicness is decreased to 0.75, which yields Case A-a. In this case, government expenditure takes on more of the character of public goods.

Case A-b

In contrast to Case A, the exponent, ρ, is increased to unity, which gives Case A-b. In this case, government expenditure takes on the character of purely private goods.

Cases A-a and A-b**

In Cases A-a and A-b, the tax rate on consumption is adjusted to maximize social welfare, which gives Cases A-a* and A-b*, respectively.

3.3 Specification of Parameters

This paper examines an optimal rate of the national burden for the economy in an aging Japanese society through comparing steady states. We choose parameter values that are realistic for the economy. Thus, the economic variables in the benchmark simulation (i.e., Case A), such as the ratio of capital to income (K/Y) or that of capital to labor (K/L), are close to the values that are suggested by the Economic and Social Research

Institute (2005). The parameter values used in the simulation are given in Table 6.

First, the weight parameter, μ, in the utility function in equation (2) is assigned to 0.1 in the benchmark cases. Because no studies have estimated explicitly this utility weight parameter, we made reference to Aschauer (1985) as a second-best means. In that study, effective consumption C^* is characterized by

$$C^* = C + \phi G,$$

where C is private consumption and ϕ denotes the degree of substitution between C and G (i.e., government spending). The parameter ϕ signifies the marginal benefits of government expenditure. Aschauer's study estimated ϕ to lie between 0.2 and 0.4.

Second, survival probabilities (p_s) are calculated from the National Institute of Population and Social Security Research (2002). Our model makes no distinction by sex, and thus this study uses male–female average values for 2005, 2025, and 2050. Based on the above data, the percentages of the aged population (65 or over) in the total population (21 or over) in 2005, 2025, and 2050 are 24.90%, 34.63%, and 44.47%, respectively. Common growth rates of successive cohorts (n) are chosen so that the percentages in the simulation equal the estimated values (see Table 1).

Third, the method of assigning the weight given to labor endowments for the three income classes is explained. Table 7 shows the data from the Ministry of Finance (2005). This table presents the effective tax rates of wageworkers on a national income tax and a residence tax, with regard to a couple with two children. In our model, the three representative households, namely, low, medium, and high-income classes, have different earnings abilities. Table 7 suggests that each income class, which accounts for one third of the total population, corresponds to the representative household earning 5, 7, or 10 million yen, respectively, on an annual base. The weight on labor endowments for each income class corresponds to the ratio of its amount of earned income. The medium-income class is used as a yardstick, that is, $x^m = 1$; x^l and x^h are assigned to reflect different earnings abilities across the three income classes.

Fourth, the method of assigning the parameter values that determine tax progressivity on labor income, namely, α and β, is described. Table 7 presents the effective tax rate calculated from a national income tax and a residence tax for each income class. The parameters on labor income are chosen so that the effective tax rate for each income class in the simulation is close to the estimated value.

Finally, with regard to the public pension system, the replacement ratio of pension payments (θ) in Case A is adjusted so that the contribution rate (τ_p) equals the actual value of 13.58% in employee pension plans *(Kosei Nenkin)* in 2005.

4 Simulation Results

According to the Ministry of Finance (2005), the rate of the tax burden (= (National taxes + Local taxes) / National income) is 21.5%, the rate of social security (= Social security / National income) is 14.4%, and thus the rate of the national burden, which is the sum of them, was 35.9% in 2005. In our benchmark simulation (i.e., Case A), the rate of the tax burden is 12.94%, the rate of social security is 12.06%, and the rate of the national burden is 25.00%, as shown in Table 2. The rate of the tax burden in our simulation is substantially lower than the actual rate, because our model does not introduce such taxes as corporate and property ones. On the other hand, the rate of social security is close to the actual rate, although our simulation deals only with a public pension scheme. Therefore, in the setting of our simulation, the rate of the national burden is lower than that in reality in Japan.

This paper investigates an optimal tax rate on consumption to maximize the social welfare function. A rise in the tax rate on consumption increases the total tax revenue, which means an increase in government expenditure. This increase enhances households' utility, because our model takes account of the benefits that the government provides to households. With regard to the social security system, our model only considers the current Japanese public pension program operated in a manner that is similar to a pay-as-you-go style. The replacement ratio in the current public pension system is kept constant in all simulations.[6] Therefore, it should be noted that an optimal rate of the national burden in this paper signifies the sum of an optimal rate of the tax burden (which is led by examining an optimal rate on consumption) and a constant rate of the public pension (= Public pension / National income).

In this paper, the influence on capital accumulation is regarded as an indicator of efficiency. The reason is that, under the assumption of an inelastic labor supply, the level of the total output depends solely on the level of the capital stock, as indicated by equation (14). The social welfare function represented by equation (8) is dependent on the aspects of both efficiency and equity.

4.1 Findings and Policy Implications

1 Three benchmarks for 2005, 2025, and 2050

Table 2 shows the simulation results for the three benchmarks for 2005, 2025, and 2050. Cases A, B, and C are the benchmarks for 2005, 2025, and 2050, respectively. Under revenue neutrality, a consumption tax covers an extra tax burden caused by the transition from the current state to the aged states. Therefore, in Cases B and C, the revenue-neutral tax rate on consumption is increased to 6.54% and 9.57%, respectively. As the population ages, the contribution rate to the public pension scheme (i.e., payroll tax) sharply rises under conditions of a constant replacement ratio. Table 1 suggests that the rate rises from 13.58% (Case A) to 21.57% (Case B) and 32.61% (Case C).

The capital-labor (K/L) ratio increases from 2.78 (Case A) to 3.10 (Case B), and then decreases slightly to 2.93 (Case C). The K/L ratio is determined by the amounts of capital accumulation and labor supply. Both would decline as the population ages. With regard to capital formation, in an aging society, there are many cohorts that dissave their assets based on their life-cycle motive. With respect to the labor supply, it decreases drastically from 349.6 to 306.2 and 260.0 under a constant population, as shown in Table 1.

The rate of the tax burden rises from 12.94% (Case A) to 14.61% (Case B) and

17.31% (Case C), because there is the extra tax burden caused by transition to the aged states. The rate of social security dramatically rises from 12.06% (Case A) to 19.30% (Case B) and 29.06% (Case C), because the current public pension program is fundamentally a pay-as-you-go style. Thus, the rate of the national burden rises substantially from 25.00% (Case A) to 33.90% (Case B) and 46.37% (Case C).

2 Cases that maximize social welfare for 2005, 2025, and 2050

Table 3 presents the simulation results in cases that maximize social welfare for the three benchmarks for 2005, 2025, and 2050. In the current benchmark, the optimal tax rate on consumption rises drastically from 5% (Case A) to 20.17% (Case A*).[7] As the population ages, it rises slightly to 20.38% (Case B*) and 20.45% (Case C*). The rate of the tax burden rises from 12.94% (Case A) to 23.71% (Case A*). As the population ages, it also rises slightly to 24.57% (Case B*) and 25.14% (Case C*). Thus, the rate of the national burden rises from 25.00% (Case A) to 35.77% (Case A*). As the population ages, it rises dramatically to 43.86% (Case B*) and 54.20% (Case C*), because of a substantial increase in the rate of social security.

Thus, the simulation results suggest that an optimal rate of the national burden would rise as the population ages and may exceed 50% during the rapid aging of the Japanese population. When evaluating this simulation result, the fact that the rate of the national burden in the setting of our simulation is lower than the actual rate needs to be noted.

3 Changes in the utility weight parameter given for public services

With regard to the weight parameter, μ, given for the benefits coming from government expenditure in equation (2), we assign 0.1 in the benchmark simulation. To ascertain the effects of different weight parameters on the simulation results, we consider the cases in which μ is changed to 0.01 and 0.3.

Table 4 shows that, when the weight is lowered to 0.01, the optimal tax rate on consumption decreases dramatically from 20.17% (Case A*) to 7.48% (Case A-1*). As the population ages, it rises slightly to 7.60 % (Case B-1*) and 7.71 % (Case C-1*). When the weight is raised to 0.3, the optimal tax rate on consumption increases to 29.34% (Case A-2*). As the population ages, it rises slightly to 29.61 % (Case B-2*) and 29.65 % (Case C-2*).

Table 4 also presents the simulation results for optimal rates of the tax burden and the national burden for 2005, 2025, and 2050. The results are similar to those for an optimal tax rate on consumption. When the utility weight parameter is high (i.e., $\mu = 0.3$) in 2050, the optimal rate of the national burden is approximately 60% (Case C-2*). The results suggest that, if households think highly of the benefits coming from government expenditure for their utility, the optimal tax rate on consumption (and thus the optimal rate of the national burden) is high.

To establish the relationship between the assignment of the utility weight parameter μ and the reality of the Japanese economy, an additional simulation was conducted. The result shows that the current Japanese tax rate on consumption of 5% is optimal when the weight parameter μ equals 0.0052. This means that, only when the utility weight for public services is this low, is the current tax rate on consumption justified.

4 Changes in the degree of publicness on government expenditure

With regard to the exponent, ρ, that determines the degree of publicness on government expenditure in equation (2), we assign 0.88 in the benchmark simulation, with reference to Borcherding (1985). To ascertain the effects of different degrees of publicness on the simulation results, we consider cases in which ρ is changed to 0.75 and unity. In Case A-a where ρ is 0.75, government expenditure takes on more of the character of public goods. On the other hand, in Case A-b where ρ is unity, government expenditure is purely private goods.

Table 5 shows the simulation results for different degrees of publicness on government expenditure. When government expenditure takes on more of the character of public goods ($\rho = 0.75$), the optimal tax rate on consumption decreases dramatically from 20.17% (Case A*) to 8.64% (Case A-a*). Hence, in Case A-a*, the optimal rates of the tax burden and the national burden diminish to 15.80% and 27.86%, respectively. On the other hand, when government expenditure takes entirely on the character of purely private goods ($\rho = 1$), the optimal tax rate on consumption increases substantially to 37.47% (Case A-b*). Thus, in Case A-b*, the optimal rates of the tax burden and the national burden increase to 33.09% and 45.15%, respectively.

Therefore, the simulation results suggest that, when the degree of publicness of government spending is high, that is, as government expenditure takes on more of the character of public goods, the optimal rate of the national burden is low. These results are generated by the concavity of the underlying utility function employed in our model.

4.2 Comments

For interpreting the simulation results, the following four comments need to be taken into account. First, our model assumes an *inelastic* labor supply. Therefore, there is only a partial impact of progressive taxation and no excess burden. An *elastic* labor supply needs to be introduced to comprehensively analyze the effects of tax reforms. Several investigations, however, showed that labor supply is fairly inelastic for the after-tax wage rate *in Japan*. For instance, Asano and Fukushima (1994) reported that the estimated value of compensated elasticity of labor supply is 0.27 in Japan. It should be noted that their study estimated only the size of the substitution effect. If the income effect were also estimated, a still smaller elasticity of labor supply for the after-tax wage rate would be obtained.

An elastic labor supply can be assumed by incorporating leisure into the utility function in addition to consumption, such as in Auerbach and Kotlikoff (1987), Altig *et al.* (2001), and Okamoto (forthcoming):

$$U^i = \frac{1}{1-\frac{1}{\gamma}} \sum_{s=21}^{95} p_s (1+\delta)^{-(s-21)} \left\{ \left(C_s^i\right)^{1-\frac{1}{\xi}} + \phi \left(l_s^i\right)^{1-\frac{1}{\xi}} \right\}^{\frac{1-\frac{1}{\gamma}}{1-\frac{1}{\xi}}},$$

where l_s^i represents leisure at age s, ϕ the utility weight on leisure, and ξ is the intratemporal elasticity of substitution between consumption and leisure. The weight on leisure ϕ is assigned to 1.5 in Auerbach and Kotlikoff (1987) and 1.0 in Altig *et al.* (2001), respectively. On the other hand, Homma *et al.* (1987b) estimate the weight parameter to be 0.1 using Japanese data, which is much smaller than that in the United States. Thus, the parameter ϕ is assigned to 0.1 in studies on the Japanese economy, such as in Homma *et al.* (1987a), Iwamoto (1990), and Uemura (2001). Such a large difference may suggest that, even if elastic labor supply were introduced, the simulation results would not change so much as long as we conduct the analysis on the Japanese economy.

Second, our simulations are limited to the steady states. The model takes account of the effects of demographics, by comparing the three steady states for 2005, 2025, and 2050. Thus, our study lacks a consideration of the transitional path. Tax and social security reforms have different effects on different generations. Specifically, current and future generations would experience different impacts of these reforms.[8] Therefore, it is necessary to take account of not only steady states but also the transitional process in an aging society.

Third, the simulation model is solved under the hypothesis of perfect foresight. Households correctly anticipate the interest, wage, tax, and contribution rates. Without the assumption of perfect foresight, households that maximize their lifetime utility would have more assets, resulting in more capital accumulation. As households are more risk averse, this effect is greater.

Finally, because the simulation results are dependent on the given parameters, we must be careful about the effects of any parameter changes. As shown in Tables 4 and 5, a slight change in the parameters of μ (the weight on public services) and ρ (the degree of publicness) in the utility function has a substantial effect on the results. Also, a slight change in the parameter of intertemporal elasticity of substitution, γ, has a great affect on capital formation.

5 Conclusions

This paper examined an optimal rate of the national burden to establish guidelines for fiscal reform in Japan's graying society. The paper looked at Japanese tax and social security systems through an extended life-cycle general equilibrium simulation model. It considers the benefits that the government provides to households, which enabled us to comprehensively evaluate the balance between benefits and burdens.

The simulation results showed that, when households put a great utility weight on the benefits coming from public services, an optimal rate of the national burden is high. This implies that, if the government appropriately provides public services that meet the demands of households, the optimal rate is high. The results also suggested that, when the degree of publicness of government expenditure is high, an optimal rate of the national burden is low. In other words, if government expenditure takes on more of the character of public goods, the optimal rate is low.

Moreover, the simulation results indicated that, as the population ages, an optimal tax rate on consumption rises gradually and the contribution rate to the public pension program with a pay-as-you-go scheme dramatically rises under conditions of a constant replacement ratio. Therefore, an optimal rate of the national burden would rise as the population ages, and may exceed 50% during the rapid aging of the Japanese population.

Acknowledgements

This paper was presented at the *61st Congress of the International Institute of Public Finance (IIPF)* in Korea in 2005, and at the Spring Seminar of the Department of

Economics at the University of Wisconsin-Milwaukee in 2006. The author is very grateful for insightful comments and suggestions by Professors Masaru Yoshitomi, Yuji Hosoya, Nobuhiko Yamazaki, Masayuki Morikawa, Toshiaki Tachibanaki, Toshiya Hatano, Masumi Kawade, Naomi Miyazato, Sunwoong Kim, Mohsen Bahmani-Oskooee, Yoshio Niho, and Syun-ichiro Bessyo. The author acknowledges the financial supports from the Research Institute of Economy, Trade and Industry (RIETI) in Japan, and from the Ministry of Education, Culture, Sports, Science and Technology in Japan (Grant-in-Aid for Scientific Research (C) No.15530215).

References

Altig, D., Auerbach, A. J., Kotlikoff, L. J., Smetters, K. A., and Walliser, J. (2001) "Simulating Fundamental Tax Reform in the United States," *American Economic Review* 91 (3), 574-95.

Asano, S. and Fukushima, T. (1994) "Some Empirical Evidence on Demand System and Optimal Commodity Taxation," *Discussion Papers in Economics and Business,* 94-01, Faculty of Economics, Osaka University.

Aschauer, D. A. (1985) "Fiscal Policy and Aggregate Demand," *American Economic Review* 75 (1), 117-127.

Atkinson, A. B. (1999) *The Economic Consequences of Rolling Back the Welfare State*, The MIT Press.

Auerbach, A. J. and Kotlikoff, L. J. (1987) *Dynamic Fiscal Policy,* Cambridge: Cambridge University Press.

Borcherding, T. E. (1985) "The Causes of Government Expenditure Growth: A Survey of the U.S. Evidence," *Journal of Public Economics* 28 (3): 359-82.

Homma, M., Atoda, N., Iwamoto, Y., and Ohtake, F., (1987a) "Pension: Aging Society and Pension Plan," in Hamada, K., Horiuchi, A., and Kuroda, M. eds., *Macroeconomic Analysis of the Japanese Economy*, Tokyo University Shuppankai, 149-175 (in Japanese).

Homma, M., Atoda, N., Iwamoto, Y., and Ohtake, F., (1987b) "Life-Cycle Growth Model and Sensitivity Analysis," Osaka Economic Papers 36, Osaka University, 99-108 (in Japanese).

Ihori, T., Kato, R. R., Kawade, M., and Bessho, S., (2005) "Public Debt and Economic Growth in an Aging Japan," COE Discussion Papers COE-F-100, Faculty of Economics, The University of Tokyo.

Iwamoto, Y., (1990) "Pension Plan and Bequest Behavior," *Quarterly Journal of Social Security Research 25,* 388-411 (in Japanese).

Okamoto, A. (2004) *Tax Policy for Aging Societies: Lessons from Japan*, Springer.

Okamoto, A. (2005a) "Simulating Progressive Expenditure Taxation in an Aging Japan," *Journal of Policy Modeling* 27 (3): 309-325.

Okamoto, A. (2005b) "Simulating Fundamental Tax Reforms in an Aging Japan," *Economic Systems Research* 17 (2): 163-185.

Okamoto, A. "Optimal Tax Combination in an Aging Japan," *International Economic Journal*, forthcoming.

Okamoto, A. and Tachibanaki, T. (2002) "Integration of Tax and Social Security Systems," in *Social Security Reform in Advanced Countries* edited by Ihori, T. and Tachibanaki, T., Routledge, London and New York: 132-160.

Uemura, T. (2001) *Economic Analysis of the Financial Burden: Evaluation of Tax Reform and Pension Policy,* Kwansei Gakuin University Press (in Japanese).

Data

Economic and Social Research Institute (2005) *Annual Report on National Accounts (Kokumin Keizai Keisan Nenpo) 2005*, Cabinet Office, Japan.

Ministry of Finance (2005) *Let's Talk about Taxes*, May, Japan.

Ministry of Health, Labor and Welfare (2004) *Basic Survey on Wage Structure (Chingin Sensasu) 2003*, Japan.

National Institute of Population and Social Security Research (2002) *Population Projections for Japan: 2001-2100 (Estimation in January, 2002)*, Japan.

Notes

1 Japan's population has the highest old-age dependency ratio. However, the Republic of Korea is currently the fastest aging country in the world.

2 Okamoto (2005a) also incorporated three representative households with different earnings abilities. Altig *et al.* (2001) dealt with differences of lifetime earnings ability by incorporating 12 lifetime-income groups into a life-cycle model. Furthermore, Okamoto (2005b) introduced numerous representative households with continuous income distribution in each cohort.

3 To estimate the age profile of earnings ability, e_s, the following equation is employed:

$$Q = a_0 + a_1 A + a_2 A^2 + a_3 L + a_4 L^2 ,$$

where Q denotes average monthly cash earnings, A age, and L the length of one's service for all workers. Table 8 presents the parameter values estimated using data in the Ministry of Health, Labor and Welfare (2004). Because bonuses account for a large part of earnings in Japan, monthly cash earnings used here contain bonuses.

4 The model employs a simple utilitarian social welfare function based on the steady state utility for a single generation. The function is most sensitive to income changes for the low-income class due to the concavity of the underlying utility function. As the parameter of intertemporal elasticity of substitution, γ, is lower, it depends more on the welfare of the low-income class. It becomes Rawlsian if $SW = U^l$.

5 Population is the same across Cases A, B, and C. The population AN_t is assigned to 180.98 in all simulation cases.

6 Under a constant replacement ratio of the current Japanese public pension system, we maximize the social welfare. This is because, due to the rapid aging of the Japanese population, the benefits to be paid through the system are not profitable especially for future generations. Therefore, we decided that the current replacement ratio is also maintained in aged simulation cases.

7 The model in this paper does not introduce such taxes as corporate and property ones, while it takes full account of government expenditure. Therefore, in our simulation, the optimal tax rate on consumption inevitably becomes high. It should be noted that we do not claim that, in the current Japanese system, the optimal tax rate on consumption is approximately 20%.

8 For example, a change from a labor income tax to a consumption tax creates income transfers among generations during the transition. At the onset of policy reform, the elderly who had already paid their labor income tax will have to pay an additional consumption tax. Because this generation would suffer from a double burden, the transition to a consumption tax is not Pareto improving.

9 Okamoto and Tachibanaki (2002) incorporate the basic pension (i.e., the flat part) into the public pension program. In that study, the general tax revenue covers one third of the basic pension, along the lines of the current Japanese tax and public pension systems.

Appendix A

To consider the utility maximization problem over time for each income class, namely the maximization of equation (2) subject to equation (3), let the Lagrange function be

$$L^i = U^i + \sum_{s=21}^{95} \lambda_s^i \left[- A_{s+1}^i + \{1 + r(1-\tau_r)\} A_s^i + \{1 - \tau_w(wx^i e_s) - \tau_p\} wx^i e_s + b_s^i + a_s^i - (1+\tau_c) C_s^i \right] \tag{9}$$

where λ_s^i represents the Lagrange multiplier for equation (3) and superscripts $i\,(=l, m, h)$ denote low, medium, and high-income classes, respectively. The first-order conditions for $s = 21, 22, \cdots, 95$ can be expressed by

$$\frac{\partial L^i}{\partial C_s^i} = p_s (1+\delta)^{-(s-21)} \left\{ C_s^i \right\}^{-\frac{1}{\gamma}} - \lambda_s^i (1+\tau_c) = 0\,, \tag{10}$$

$$\frac{\partial L^i}{\partial A_{s+1}^i} = -\lambda_s^i + \lambda_{s+1}^i \{1 + r(1-\tau_r)\} = 0\,. \tag{11}$$

The combination of equations (10) and (11) yields equation (7) that determines the slope of the age-consumption profile over the life cycle. For a given C_{21}^i, equation (7) solves the path for consumption. Transformation of equation (7) leads to the following expression:

$$C_s^i = \left(\frac{p_s}{p_{21}}\right)^{\gamma} \left\{\frac{1+r(1-\tau_r)}{1+\delta}\right\}^{\gamma(s-21)} C_{21}^i . \tag{12}$$

Integrating equation (3) and using the initial and terminal conditions $A_{21}^i = A_{96}^i = 0$, caused by no intended bequests, produces the following equation:

$$\sum_{s=21}^{95}\{1+r(1-\tau_r)\}^{-(s-21)}(1+\tau_c)C_s^i$$
$$=\sum_{s=21}^{RE}\{1+r(1-\tau_r)\}^{-(s-21)}[1-\tau_w(wx^ie_s)-\tau_p]wx^ie_s+\sum_{s=ST}^{95}\{1+r(1-\tau_r)\}^{-(s-21)}b_s^i+\{1+r(1-\tau_r)\}^{-29}a_{50}^i \tag{13}$$

To derive C_{21}^i, equation (12) is substituted into this lifetime budget constraint. Thus, we can find an optimum solution for C_{21}^i.

Appendix B

Section 2 describes the basic structure of households in the simulation model. This appendix presents the basic structures of firms and the government, and market equilibrium conditions.

Firm Behavior

The model has a single production sector that is assumed to behave competitively using capital and labor, subject to a constant-returns-to-scale production function. Capital is homogeneous and non-depreciating, while labor differs only in its efficiency. All forms of labor are perfect substitutes. Households in different income classes or of different ages, however, supply different amounts of some standard measure per unit of labor input.

The production function is assumed to be of the constant elasticity of substitution form:

$$Y_t = B\left[\varepsilon K_t^{1-\frac{1}{\sigma}} + (1-\varepsilon)L_t^{1-\frac{1}{\sigma}}\right]^{\frac{1}{1-\frac{1}{\sigma}}}, \tag{14}$$

where Y_t represents the total output, K_t the total capital, L_t the total labor supply measured by the efficiency units, B a scaling constant, ε a parameter measuring the intensity of use of capital in production, and σ is the elasticity of substitution between K_t and L_t. Using the property subject to a constant-returns-to-scale production function, we can obtain the following equation:

$$Y_t = rK_t + wL_t. \tag{15}$$

Government Behavior

The government sector consists of a narrower government sector and a pension sector. The narrower government sector collects taxes and spends them on general governmental expenditure. There is no outstanding debt, and thus balanced budget policies are assumed.

The budget constraint of the narrower government sector at time t is given by

$$G_t = T_t, \tag{16}$$

where T_t is the total tax revenue from labor income, interest income, consumption, and inheritances. The public pension system is assumed to be a simple pay-as-you-go scheme. The budget constraint of the pension sector at time t is given by

$$R_t = B_t, \tag{17}$$

where R_t is the total contribution to the pension schedule, and B_t is the total pension benefit to generations of age ST or over.

Both of these sectors are financed independently and separately. No transfer is made between the sectors.[9] G_t, T_t, R_t and B_t are defined by

$$G_t = N_t \sum_{s=21}^{95} p_s (1+n)^{-(s-21)} g, \tag{18}$$

$$T_t = LX_t + \tau_r rAS_t + \tau_c AC_t + \tau_h BQ_t, \tag{19}$$

$$R_t = \tau_p wL_t, \tag{20}$$

$$B_t = N_t \sum_{s=ST}^{95} p_s (1+n)^{-(s-21)} \{b_s^l + b_s^m + b_s^h\}, \tag{21}$$

where g is annual government expenditure for each cohort, and

$$BQ_t = BQ_t^l + BQ_t^m + BQ_t^h . \qquad (22)$$

LX_t is the tax revenue from labor income, which is led by a summation of the three income classes with the same weight:

$$LX_t = N_t \sum_{s=21}^{RE} p_s (1+n)^{-(s-21)} \left[\alpha w x^l e_s + \frac{1}{2}\beta (w x^l e_s)^2 + \alpha w x^m e_s + \frac{1}{2}\beta (w x^m e_s)^2 + \alpha w x^h e_s + \frac{1}{2}\beta (w x^h e_s)^2 \right]$$

(23)

Similarly, aggregate assets supplied by households, AS_t, and aggregate consumption, AC_t, are obtained by a simple summation of the three income groups:

$$AS_t = N_t \sum_{s=21}^{95} p_s (1+n)^{-(s-21)} \{A_s^l + A_s^m + A_s^h\}, \qquad (24)$$

$$AC_t = N_t \sum_{s=21}^{95} p_s (1+n)^{-(s-21)} \{C_s^l + C_s^m + C_s^h\}. \qquad (25)$$

Market Equilibrium

Finally, equilibrium conditions for the capital, labor, and goods markets are described.

1 Equilibrium condition for the capital market

Because aggregate assets supplied by households are equal to real capital, we get

$$AS_t = K_t . \qquad (26)$$

2 Equilibrium condition for the labor market

Measured in efficiency units, because aggregate labor demand by firms is equal to aggregate labor supply by households, we get

$$L_t = N_t \sum_{s=21}^{RE} p_s (1+n)^{-(s-21)} \{x^l + x^m + x^h\} e_s . \qquad (27)$$

3 Equilibrium condition for the goods market

Because aggregate production is equal to the sum of consumption, investment, and government expenditures, we get

$$Y_t = AC_t + (K_{t+1} - K_t) + G_t . \qquad (28)$$

An iterative program is performed to obtain the equilibrium values of the above equations.

Table 1 Parameter values characterizing the three steady states

Parameter	Current steady state *(Cases A)*	Aged steady state *(Cases B)*	More aged steady state *(Cases C)*
Survival probabilities, p_s (year)	2005	2025	2050
Growth rate of successive cohorts, n	0.0069	-0.0047	-0.0153
New entrants in period t, N_t	1.2179	0.8280	0.5485
Labor supply, L_t	349.6	306.2	260.0
Contribution rate, τ_p	13.58%	21.57%	32.61%

Table 2 Simulation results for the three benchmarks for 2005, 2025, and 2050

	Case A (Benchmark for 2005)	Case B (Benchmark for 2025)	Case C (Benchmark for 2050)
Weight parameter in utility, μ	[a]0.1	[a]0.1	[a]0.1
Government expenditure, G	50.96	[a]50.96	[a]50.96
Tax rate on labor income,[b] $\tau_w(wx^i e_s)$	[a]6.768%	[a]7.167%	[a]7.228%
Tax rate on consumption, τ_c	[a]5%	6.54%	9.57%
Tax rate on interest income, τ_r	[a]20%	[a]20%	[a]20%
Tax rate on inheritances, τ_h	[a]10%	[a]10%	[a]10%
Capital-labor ratio, K/L	2.78	3.10	2.93
Interest rate, r	4.54%	3.87%	4.21 %
Wage rate, w	1.000	1.020	1.010
Rate of tax burden, T/Y	12.94%	14.61%	17.31%
Rate of social security, B/Y	12.06%	19.30 %	29.06%
National burden rate, $(T+B)/Y$	25.00%	33.90%	46.37%
Social welfare, SW	-32.541	-35.816	-44.008

[a] The symbol ([a]) before the rate indicates that the variable is exogenous.

[b] The parameter values that determine the tax progressivity on labor income are $\alpha = -0.0325$, $\beta = 0.0628$ in all simulation cases. The rate presented is an average tax rate on labor income.

Table 3 Cases that maximize social welfare for 2005, 2025, and 2050

	Case A* *(2005)*	Case B* *(2025)*	Case C* *(2050)*
Weight parameter in utility, μ	[a]0.1	[a]0.1	[a]0.1
Government expenditure, G	[a]93.38	[a]85.71	[a]74.03
Tax rate on labor income,[b] $\tau_w(wx^i e_s)$	[a]6.768%	[a]7.167%	[a]7.228%
Optimal tax rate on consumption, τ_c^*	*20.17%*	*20.38%*	*20.45%*
Tax rate on interest income, τ_r	[a]20%	[a]20%	[a]20%
Tax rate on inheritances, τ_h	[a]10%	[a]10%	[a]10%
Capital-labor ratio, K/L	2.78	3.10	2.93
Interest rate, r	4.54%	3.87%	4.21 %
Wage rate, w	1.000	1.020	1.010
Rate of tax burden, T/Y	23.71%	24.57%	25.14%
Rate of social security, B/Y	12.06%	19.30 %	29.06%
National burden rate, $(T+B)/Y$	35.77%	43.86%	54.20%
Maximized social welfare, SW^*	*-16.346*	*-21.326*	*-33.552*

[a] The symbol ([a]) before the rate indicates that the variable is exogenous.

[b] The parameter values that determine the tax progressivity on labor income are $\alpha = -0.0325$, $\beta = 0.0628$ in all simulation cases. The rate presented is an average tax rate on labor income.

Table 4 Simulation results for cases with different utility weights on public services for 2005, 2025, and 2050

1 Transition of an optimal tax rate on consumption

The utility weight	2005	2025	2050
$\mu = 0.01$	7.48% (Case A-1*)	7.60 % (Case B-1*)	7.71 % (Case C-1*)
$\mu = 0.1$	20.17% (Case A*)	20.38% (Case B*)	20.45% (Case C*)
$\mu = 0.3$	29.34% (Case A-2*)	29.61 % (Case B-2*)	29.65 % (Case C-2*)

2 Transition of an optimal rate of the tax burden

The utility weight	2005	2025	2050
$\mu = 0.01$	14.91% (Case A-1*)	15.46% (Case B-1*)	15.81 % (Case C-1*)
$\mu = 0.1$	23.71% (Case A*)	24.57% (Case B*)	25.14% (Case C*)
$\mu = 0.3$	29.00% (Case A-2*)	30.03% (Case B-2*)	30.74% (Case C-2*)

3 Transition of an optimal rate of the national burden

The utility weight	2005	2025	2050
$\mu = 0.01$	26.97% (Case A-1*)	34.75% (Case B-1*)	44.87% (Case C-1*)
$\mu = 0.1$	35.77% (Case A*)	43.86% (Case B*)	54.20% (Case C*)
$\mu = 0.3$	41.05% (Case A-2*)	49.33% (Case B-2*)	59.80 % (Case C-2*)

Table 5 Simulation results for different degrees of publicness on government expenditure

	$\rho = 0.75$ *(Case A-a*)*	$\rho = 0.88$ *(Case A*)*	$\rho = 1.00$ *(Case A-b*)*
Optimal tax rate on consumption, τ_c^*	8.64%	20.17%	37.47%
Rate of tax burden, T/Y	15.80%	23.71%	33.09%
National burden rate, $(T+B)/Y$	27.86%	35.77%	45.15%
Maximized social welfare, SW^*	-10.919	-16.346	-27.995

Table 6 Parameter values used in the simulation analysis

Parameter description	Parameter values
Adjustment coefficient for discounting the future	$\delta = 0.02$
Intertemporal elasticity of substitution	$\gamma = 0.25$
The degree of publicness of government spending	$\rho = 0.88$
Elasticity of substitution in production	$\sigma = 0.6$
Weight parameter in production	$\varepsilon = 0.2$
Scale parameter in production	$B = 0.9634$
Retirement age	$RE = 60$
Starting age for receiving public pension benefits	$ST = 65$
Replacement ratio for public pension	$\theta = 0.3718$

Table 7 Effective tax rates for national income tax and residence tax of wageworkers

Income class	Total amount of annual income (million yen)	Weight on labor endowments	Total amount of annual taxes: national income tax and residence tax (thousand yen)	Effective tax rates (%)
Low	5	$x^l = 0.7143$	177	3.54
Medium	7	$x^m = 1$	418	5.97
High	10	$x^h = 1.4286$	1,041	10.41

Data given are for a couple with two children.

Source: Ministry of Finance (2005).

Table 8 Estimation of the age profile of earnings ability

a_0	a_1	a_2	a_3	a_4
-0.31291 (-0.53353)	0.08333 (2.04195)	-0.00112 (-3.04843)	0.12496 (2.22492)	-0.00168 (-1.12888)

$$\begin{cases} S.E. = 0.0847 \\ R^2 = 0.9903 \end{cases}$$