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"Irrational Exuberance" in the Pigou Cycle under Collateral Constraints

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JEL Classifications:.

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(Incomplete and preliminary)

Keiichiro Kobayashi*and Masaru Inaba[†]

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Abstract

The boom-bust cycles such as the episode of the “Internet bubble” in the late 1990s may be described as the business cycle driven by changes in expectations, which is called the *Pigou cycle* by Beaudry and Portier (An exploration into Pigou’s theory of cycles, Journal of Monetary Economics, 2004). The key feature of the notion of the Pigou cycle is the comovements in the consumption, the labor, and the investment, in response to changes in expectations. We show that with the assumption that firms are subject to the collateral constraint in financing labor input (and investment), a fairly standard neoclassical model can generate the Pigou cycle. We also show that the collateral-constraint model with the private information can generate the “irrational exuberance,” i.e., a boom in which each firm correctly anticipates that *its own* productivity will not rise, while it also believes wrongly that the productivity of *the other* firms will rise dramatically.

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1 Introduction

The boom and bust of the "Internet bubble" in the late 1990s may have been driven by changes in expectations on future productivity of the economy; and substantial number of the boom-bust cycles in asset-prices and investment around the world may be explained by the same mechanism. In the literature of theoretical business cycle research, there is a growing number of papers that try to formalize this idea. Beaudry and Portier (2004a, b), Christiano, Motto, and Rostagno (2005), Christiano and Fujiwara (2005), Jaimovich and Rebelo (2005) and Den Haan and Kaltenbrunner (2004) among others explore the idea of the business cycles driven by changes in expectations. These research confirm that changes in expectations in the standard neoclassical models cannot replicate the boom-bust cycles in the real world, in which consumption, labor input and investment all increase in the boom period and all decrease in the bust period. In the standard models, for example, if consumers increase the consumption in the current period due to the welfare effect in response to the improved prospect of future productivity, the investment in the current period decreases, since labor input does not increase so much. Consumption and investment usually move in the opposite directions in response to changes in expectations on futuer productivity in the standard models.

In the business cycles in reality, however, three macroeconomic variables, i.e., consumption, labor, and investment, usually move in the same direction. These research, therefore, try to find a way to modify the standard model so that the comovements in consumption, labor, and investment are driven by changes in expectations on future technology. We call this comovements of the three macroeconomic variables driven by expectational changes the *Pigou cycle*, since, as Beaudry and Portier (2004a) point out, Pigou (1926) is one of the first economists who emphasizes this notion of business cycles.

Recent research show that in order to produce the Pigou cycle, we need to incorporate nonstandard twists into the standard Real Business Cycle model: Beaudry and Portier (2004a, b) introduce a certain type of complementarity between production technologies in a two-sector model; Christiano, Motto, and Rostagno (2005), Christiano and Fujiwara (2005), and Jaimovich and Rebelo (2005) introduce habit persistence in consumers' preference and the adjustment costs in investment; and Den Haan and Kaltenbrunner (2004) show that if the labor market is subject to matching frictions, changes in expectations induce the comovements in consumption, labor, and investment.

There are two objectives in this paper. The first objective is to propose a new set of twists that generates the Pigou cycle. We do not assume technological complementarity, habit persistence, adjustment costs for investment, nor matching frictions in the labor market. We introduce a (tangible or intangible) asset with fixed supply (e.g., land or new business ideas in the IT industry) and assume that firms are subject to collateral constraints in purchasing productive factors: The purchase must be financed by borrowing which is limited by the value of the firms' asset. We show that with the asset and the collateral constraints a standard neoclassical growth model can generate the Pigou cycle.

The second objective of this paper is to show that our collateral-constraint model can explain the continuation of the "irrational exuberance," i.e., a boom in which each firm correctly anticipates that *its own* productivity will not rise, while it also believes wrongly that the productivity of *the other* firms will rise dramatically. Our model shows that overinvestment and overemployment due to wrong expectations on future productivity cannot be stopped even in the case where each firm receives correct information on its own future productivity, if it is the private information. This may be an alternative to the rational bubble theory for explaining why (seemingly or apparently) irrational booms can continue for a long time. The heart of our story is that overinvestment and overemployment may become a signaling device that loosens the firms' collateral constraints during the time of euphoria. Suppose that the optimism prevails and people believe public expectation that the productivity of all firms will rise in the future. Our model shows that the collateral constraints of all firms are loosened and they increase their output in this case. Suppose also that each firm receives the private information that its own productivity will not rise.[1] This information is not known to the agents that lend money to the firm. If the firm revises its investment and employment based

[1]Note that it continues to believe that the other firms' productivity will rise in the future.

on the correct (private) information, the revised decision reveals to the lenders (i.e., the consumers) that the firm's productivity will not rise and makes them tighten the collateral constraint for the firm. In this case, the firm's decision on investment and employment becomes efficient, while it incurs the loss due to tightening of its collateral constraint. If the firm continues overinvestment and overemployment, the lenders continue to believe that the firm's productivity will rise and the collateral constraint remains loose. In this case, the firm's investment and employment are inefficient, while it enjoys the benefit of the loose collateral constraint. We show numerically for a range of parameters that the firms do better off in the latter case. Therefore, a firm that receives correct information privately hides it by continuing overinvestment and overemployment, and consumers continue to hold the false expectations that the productivity of all firms will rise.[2]

Organization of the paper is the following. In the next section, we describe the basic structure of the model and show that the basic model without the collateral constraint cannot generate the Pigou cycle. In Section 3, we show that the model with the collateral constraint can generate the Pigou cycle in response to changes in expectations. In Section 4, we show that the model with collateral constraint and the private information can generate the "irrational exuberance" in response to the wrong macroeconomic news and the correct private information. Section 5 provides concluding remarks.

2 Basic model without collateral constraints - No Pigou cycles

In this section we describe the basic structure of our model and show that the Pigou cycle does not occur if the collateral constraint is not imposed to the firms.

The economy is a variant of the discrete time neoclassical growth model, which is composed of consumers and firms. The economy is populated with a continuum of consumers with identical preferences, whose measure is normalized to one. There is also a continuum of firms with measure one.

[2]Each firm continues to believe that the productivity of all firms except for itself will rise.

Modeling strategy for changes in expectations Throughout this paper, we assume for simplicity of the analysis that the model is deterministic, i.e., there is no aggregate nor idiosyncratic risk. In order to analyze the dynamics in response to changes in the expectations on the future productivity, we formalize the changes as unexpected events in the sense that they are measure zero events. To be more specific, we analyze the following case in this paper: Until date -1, the economy is in the steady state where all agents believe that the productivity will not change forever; at date 0, an unexpected change in macroeconomic expectations hits the economy and all agents are suddenly made to believe that the productivity will rise permanently from date τ onward ($\tau > 0$); and at date τ, the economy is hit again by an unexpected event that the prospect of the productivity rise turns out to be wrong and all agents restore the belief that the productivity remains at the initial value forever. It seems unrealistic to assume that measure-zero events occur two times during a short period. But this extreme assumption suffices for our theoretical interest to judge whether our model can generate the Pigou cycle in response to exogenous changes in expectations. It is straightforward to generalize the model such that the expectations evolve following a certain stochastic process just like in Beaudry and Portier (2004) or Den Haan and Kaltenbrunner (2005).

2.1 Consumer

A representative consumer maximizes the following utility:

$$\sum_{t=0}^{\infty} \beta^t U(c_t, n_t), \tag{1}$$

where β is the discount factor ($0 < \beta < 1$), c_t is the consumption at date t, and n_t is the labor input sold to firms for production. At each date t, the consumer is endowed with 1 unit of time, which can be divided into labor and leisure. Thus, $1 - n_t$ is the amount of leisure that the consumer can enjoy at date t. The flow utility $U(c_t, n_t)$ is concave, twice-differentiable, and increasing in both consumption (c_t) and leisure ($1 - n_t$). In order to simplify the analysis, the functional form for $U(c, n)$ is specified as

$$U(c, n) = \ln c + \gamma \ln(1 - n),$$

where γ (> 0) is a positive parameter. We assume that the consumers own all shares of the firms and receive dividends as a lump-sum transfer. As we describe in Section 2.2, firms make investment decisions and the consumers do not. Therefore, given that the market rate of wage is w_t, the consumer's income consists of wage $w_t n_t$ and dividends π_t. Thus, the consumer's problem is written as follows:

$$\max_{c_t, n_t} \sum_{t=0}^{\infty} \beta^t U(c_t, n_t)$$

subject to

$$c_t \leq w_t n_t + \pi_t. \tag{2}$$

The first-order conditions (FOCs) for the consumer's problem imply that

$$\lambda_t = \frac{\beta^t}{c_t}, \tag{3}$$

$$w_t = \frac{\gamma c_t}{1 - n_t}, \tag{4}$$

where λ_t is the Lagrange multiplier for (2).

2.2 Firm

There exist (potentially different) firms that compete in a perfectly competitive market. The firms are owned by the consumers. A firm owns one unit of a (potentially heterogenous) asset, which may be interpreted as a tangible asset, e.g., real estate or an intangible asset, e.g., a new business idea in the IT industry. In what follows we call this asset "land." We assume for a moment that the firm cannot sell its own land. (Assuming the firm-specificity in the usage of land, we show in Section 2.3 below that a firm has no incentive to sell its land or buy other firm's land in the competitive land market.) Since a firm can be represented by its land, we use firm i ($i \in [0, 1]$) and land i interchangeably when there is no possibility of confusion.

The firms make investment and accumulate capital stock. They have three options for the usage of its capital stock: first, they can produce the consumer goods using a Cobb-Douglas technology from capital and labor; second, they can produce the consumer goods from capital and land; and third, they can rent their capital to other firms at the market rate of rent r_t.

Therefore, firm i ($i \in [0,1]$) generates the following real dividend at date t:

$$\pi_{it} = A_t(k_t - k_{2t} - k_{3t})_t^{\alpha} n_t^{1-\alpha} + B_{it} k_{2t}^{\eta} a_{it}^{1-\eta} + r_t k_{3t} - [k_{t+1} - (1-\delta)k_t] - w_t n_t, \tag{5}$$

where A_t is the productivity of the Cobb-Douglas technology, which is common for all firms, k_t is the capital stock of firm i at date t, n_t is the labor input of firm i, B_{it} is the productivity of land i, k_{2t} is the capital input into the land, a_{it} is the amount of land i, k_{3t} is the capital that firm i rents to other firms, and δ is the depreciation rate. The consumers receive the total dividends of all firms, i.e., $\pi_t = \int_0^1 \pi_{it} di$.

The firms act in the interest of their owners, i.e., consumers, and maximize the present value of the dividend stream. Therefore, firm i's problem at date t is written as follows:

$$\max_{k_{s+1}, n_s, k_{2s}, k_{3s}} \sum_{s=t}^{\infty} \lambda_s \pi_{is}. \tag{6}$$

Our purpose in this section is to analyze the dynamics in the case where firms are not subject to the collateral constraints and to confirm that the model cannot generate the Pigou cycle without the collateral constraints. In the case without collateral constraints, the FOCs for firm i imply the following:

$$w_t = (1-\alpha) A_t \left(\frac{k_t - k_{2t} - k_{3t}}{n_t} \right)^{\alpha}, \tag{7}$$

$$r_t = \alpha A_t \left(\frac{n_t}{k_t - k_{2t} - k_{3t}} \right)^{1-\alpha} = \eta B_{it} \left(\frac{a_{it}}{k_{2t}} \right)^{1-\eta}, \tag{8}$$

$$\frac{c_{t+1}}{\beta c_t} = \alpha A_t \left(\frac{n_t}{k_t - k_{2t} - k_{3t}} \right)^{1-\alpha} + 1 - \delta. \tag{9}$$

2.3 Asset price

There exists a competitive land market. We assume that land i is a firm-specific asset for firm i in the sense that firm i can use land i most efficiently among all firms. To be more specific, we assume that the productivity of land i becomes $B_{it}\theta$ ($0 < \theta < 1$) if it is used by other firms. Thus the market price of land i is determined by

$$q_{it} = \max_{k_s} \sum_{s=t+1}^{\infty} \frac{\lambda_s}{\lambda_t} [B_{is} \theta k_s^{\eta} a_i^{1-\eta} - r_s k_s], \tag{10}$$

where $a_{it} = 1$. Solving this maximization problem, we obtain the law of motion for q_{it}:

$$q_{it} = \frac{\lambda_{t+1}}{\lambda_t} \left\{ \frac{(1-\eta)\eta^{\frac{\eta}{1-\eta}} B_{it+1}^{\frac{1}{1-\eta}} \theta^{\frac{1}{1-\eta}}}{r_{t+1}^{\frac{\eta}{1-\eta}}} + q_{it+1} \right\}. \tag{11}$$

Equations (10) and (11) imply that firm i has no incentive to sell land i in the market. More precisely, if $\theta = 1$, the firm is indifferent to sell or buy land, and if $\theta < 1$, the firm strongly prefers its own land to other firms' land. We assume that θ is sufficiently smaller than one. This assumption justifies our premise in Section 2.2 that firms do not trade their land.

2.4 Symmetric equilibrium

In this section we analyze the dynamics in which B_{it} is common for all firms. Heterogeneity of B_{it} becomes relevant in Section 4 where we analyze the case where private information exists.

In the symmetric equilibrium where all firms act identically, it must be the case that

$$k_{3t} = 0. \tag{12}$$

The resource constraints of the economy are

$$a_{it} = 1, \tag{13}$$

$$c_t + k_{t+1} = A_t(k_t - k_{2t})^\alpha n_t^{1-\alpha} + B_t k_{2t}^\eta + (1-\delta)k_t. \tag{14}$$

The FOCs for consumers and firms and the above equations imply that the dynamics of $\{c_t, n_t, k_t, k_{2t}\}_{t=0}^\infty$ in the symmetric equilibrium are described by the following system of equations:

$$\frac{\gamma c_t}{1-n_t} = (1-\alpha)A_t \left(\frac{k_t - k_{2t}}{n_t}\right)^\alpha, \tag{15}$$

$$\alpha A_t \left(\frac{n_t}{k_t - k_{2t}}\right)^{1-\alpha} = \eta B_{it} k_{2t}^{\eta-1}, \tag{16}$$

$$\frac{c_{t+1}}{\beta c_t} = \alpha A_t \left(\frac{n_t}{k_t - k_{2t}}\right)^{1-\alpha} + 1 - \delta, \tag{17}$$

and (14).

2.5 Dynamics in response to changes in expectations

We assume that A_t is time-invariant. We analyze the dynamics in response to the following change in expectations on future values of B_t: The economy is initially in the steady state where all agents believe that B_t remains a constant B forever; at date 0, a macroecnomic news hits the economy and all agents suddenly become to believe that the productivity of land will rise permanently at date τ ($\tau > 0$), i.e., $B_t = B + \Delta$ for $t \geq \tau$; and at date τ, the prospect of productivity rise turns out to be false and B_t remains at B for $t \geq \tau$, and all agents restore the correct expectation that $B_t = B$ forever.

If the consumption c_t, the labor n_t, and the investment $k_{t+1} - (1-\delta)k_t$ all rise during the priod between date 0 and date τ, we could say that the Pigou cycle is generated in our model without the collateral constraints. The numerical simulation below shows that it is not the case. The reason why it is so may be partly made clear by analysis on the steady state values of these macroeconomic variables. Therefore, before reporting the simulation results, we analytically describe the steady state.

Steady state Solving equations (14)–(17) analytically on the premise that c_t, n_t, k_t, k_{2t}, A_t, and B_t are all constant, we obtain the steady state values of the macroeconomic variables $\{c, n, k, k_2\}$ as follows:

$$c = (1-\alpha)A\left(\frac{\alpha A}{\beta^{-1} - 1 + \delta}\right)^{\frac{\alpha}{1-\alpha}} \frac{1-n}{\gamma}, \tag{18}$$

$$n = \frac{\frac{1-\alpha}{\gamma}A\left(\frac{\alpha A}{\beta^{-1}-1+\delta}\right)^{\frac{\alpha}{1-\alpha}} - \left(\frac{\beta^{-1}-1+(1-\eta)\delta}{\eta}\right)\left(\frac{\eta B}{\beta^{-1}-1+\delta}\right)^{\frac{1}{1-\eta}}}{\left(\frac{1-\alpha}{\gamma}+1\right)\left(\frac{\beta^{-1}-1+(1-\alpha)\delta}{\alpha}\right)\left(\frac{\alpha A}{\beta^{-1}-1+\delta}\right)^{\frac{1}{1-\alpha}}}, \tag{19}$$

$$k = k_2 + n\left(\frac{\alpha A}{\beta^{-1} - 1 + \delta}\right)^{\frac{1}{1-\alpha}}, \tag{20}$$

$$k_2 = \left(\frac{\eta B}{\beta^{-1} - 1 + \delta}\right)^{\frac{1}{1-\eta}}. \tag{21}$$

It is obvious from these values that

$$\frac{dn}{dB} < 0 \text{ and } \frac{dc}{dB} > 0, \tag{22}$$

and it is also easily shown that if $\eta \geq \alpha$,

$$\frac{dk}{dB} > 0. \tag{23}$$

These comparative statics imply that (the prospect of) a permanent rise in B may decrease the labor, while it would increase the consumption. Therefore, the analysis of the steady state indicates that the model without the collateral constraint may not generate the Pigou cycle in response to changes in expectations on future values of B_t. The numerical simulation shows that it is exactly the case.

Simulation Parameter values are set as follows: $A = 1$, $B = 0.1$, $\Delta = 0.01$, $\alpha = 0.3$, $\beta = 0.98$, $\gamma = 1.2$, $\delta = 0.06$, $\eta = 0.3$, $\theta = 0.1$, and $\tau = 10$. The variables are calculated by the backward shooting method. Since we assume that the economy is in the steady state at date 0, the capital stock at date 0 is given as its steady-state value k_{ss}. We calculate by the backward shooting method on k_{ss} the path from the initial steady state to the new steady state where $B_t = B + \Delta$, assuming that B_t changes at date τ. The economy follows this path from date 0 to date $\tau - 1$, since all agents in the economy believe that $B_t = B$ for $0 \leq t < \tau$ and $B_t = B + \Delta$ for $t \geq \tau$. The capital stock k_τ at date τ is given by this calculation. From date τ onward, the economy converges to the initial steady state, given the initial capital k_τ, since all agents change their expectations at date τ to that $B_t = B$ for $t \geq \tau$. We also calculate this path by the backward shooting on k_τ. Simulation result is shown in Figure 1.

Figure 1. Response to expectational change, Benchmark model without the collateral constraint

The initial response of the macroeconomic variables during the period between date 0 and date τ confirms our prediction that the Pigou cycle does not occur. In this period, the consumption rises, while the labor slightly decreases. Most noticeable is that the investment sharply declines in response to the improvement of the expectations on B_t. The investment decline is completely counter to the investment boom in the Pigou cycle. We also conduct the simulation for the case where η is larger than α. Initial responses during date 0 and date $\tau - 1$ are different in the case where $\eta = 0.5 (> \alpha)$:

The consumption sharply declines and the investment rises. These responses are again different from the Pigou cycle, in which the consumption, the labor, and the investment all rises.

3 The Pigou cycle in the economy with collateral constraints

We modify the basic model so that the firms are subject to the collateral constraint and show that the modified model generates the Pigou cycle in response to the changes in expectations.

3.1 Collateral constraint

We assume that at each date, productions of the consumer good take place before the firms pay wages to the workers (i.e., the consumers) in the form of the consumer good. Thus at each date, the workers must provide their labor before they are paid wages. We assume that the firms cannot fully commit to pay wages to the workers. We assume that the firms have a chance to abscond from the workers in a time after they produce the output using labor and before they pay the wages to the workers. But the firms can put up their own land as collateral to the workers. If the firms abscond without paying wages, they can bring the output with them but they cannot bring their land. The land would be left behind, and the workers would get the market value of the land. Under this environment, the workers are willing to provide their labor force only up to the amount, the market value of which is equal to that of the firm's land. Therefore, we can consider that firm i is subject to the following collateral constraint:

$$w_t n_t \leq q_{it} a_i. \tag{24}$$

In the modified model in this section, firm i solves (6) subject to (24). We set θ in equation (11) sufficiently small so that we can focus on the case where the collateral constraint is always binding. The FOCs for firm i are (8), (9), and

$$(1+c_t\mu_t)\frac{\gamma c_t}{1-n_t} = (1-\alpha)A_t\left(\frac{k_t-k_{2t}-k_{3t}}{n_t}\right)^{\alpha}, \tag{25}$$

instead of (7), where $\beta^t\mu_t$ is the Lagrange multiplier for (24). The FOCs for the consumers and the firms, the resource constraints, and the collateral constraint imply that the dynamics of $\{c_t, n_t, k_t, k_{2t}, \mu_t\}_{t=0}^{\infty}$ in the symmetric equilibrium where (24) is binding, $B_{it} = B_t$, and $k_{3t} = 0$ are described by equations (14), (16), (17), (25), and

$$\frac{\gamma c_t n_t}{1-n_t} = q_t, \tag{26}$$

where q_t is determined by by (11), where $r_t = \alpha A_t\left(\frac{n_t}{k_t-k_{2t}}\right)^{1-\alpha}$. We see in the numerical simulation below that this model geenrates the Pigou cycle, i.e., the comovements of consumption, labor, and investment in response to changes in the expectations on future values of B_t.

3.2 Steady state

Before proceeding to the simulation, we analyze the steady state where A_t and B_t are constant, and establish the responses of the steady-state values of the macroeconomic variables to the changes in B. Solving the system of equations (14), (16), (17), (25), and (26) on the premise that $A_t, B_t, c_t, n_t, k_t, k_{2t}$, μ_t, and q_t are all constant, we have the following:

$$k_2 = \left(\frac{\eta B}{\beta^{-1}-1+\delta}\right)^{\frac{1}{1-\eta}}, \tag{27}$$

$$k = \left(\frac{\alpha A}{\beta^{-1}-1+\delta}\right)^{\frac{1}{1-\alpha}} n + k_2, \tag{28}$$

$$c = \frac{\left(\beta^{-1}-1+(1-\alpha)\delta\right)A}{\beta^{-1}-1+\delta}n + Bk_2^{\eta} - \delta k_2, \tag{29}$$

$$\frac{\gamma c n}{1-n} = q, \tag{30}$$

$$q = \frac{\beta}{1-\beta}\frac{1-\eta}{\eta}\theta^{\frac{1}{1-\eta}}\left(\frac{\eta B}{\{(\beta^{-1}-1+\delta)\alpha A\}^{\eta}}\right)^{\frac{1}{1-\eta}}. \tag{31}$$

It is easily shown that $Bk_2^\eta - \delta k_2 > 0$ and $\frac{d}{dB}\{Bk_2^\eta - \delta k_2\} = \{Bk_2^\eta - \delta k_2\}\frac{B^{\frac{\eta}{1-\eta}}}{1-\eta} > 0$. Therefore, differentiation of (29) with respect to B implies

$$\frac{dc}{dB} = \frac{\left(\beta^{-1} - 1 + (1-\alpha)\delta\right) A}{\beta^{-1} - 1 + \delta}\frac{dn}{dB} + \{Bk_2^\eta - \delta k_2\}\frac{B^{\frac{\eta}{1-\eta}}}{1-\eta}. \tag{32}$$

It is also easily shown that $\frac{dq}{dB} = q\frac{B^{\frac{\eta}{1-\eta}}}{1-\eta}$. Therefore, differentiation of (30) with respect to B and (32) imply

$$\left\{\frac{\gamma n}{1-n}\frac{\left(\beta^{-1} - 1 + (1-\alpha)\delta\right) A}{\beta^{-1} - 1 + \delta} + \frac{\gamma c}{(1-n)^2}\right\}\frac{dn}{dB} = \left\{q - \frac{\gamma n}{1-n}(Bk_2^\eta - \delta k_2)\right\}\frac{B^{\frac{\eta}{1-\eta}}}{1-\eta}. \tag{33}$$

Since (29) and (30) imply that $\frac{\gamma n}{1-n}(Bk_2^\eta - \delta k_2) = q\frac{Bk_2^\eta - \delta k_2}{c} < q$, equation (33) implies that $\frac{dn}{dB} > 0$. Therefore, (32) and $Bk_2^\eta - \delta k_2 > 0$ imply that $\frac{dc}{dB} > 0$. It is also easily shown from (27) and (28) that $\frac{dk}{dB} > 0$ and $\frac{dk_2}{dB} > 0$. Therefore, the values of consumption, labor, and investment are all larger in the steady state where B is larger. This result indicates that in our model with the collateral constraint all three variables may rise in response to a prospect of rise in future value of B_t. The numerical simultion in the next subsection shows that it is exactly the case.

3.3 Simulation

The parameter values and simulation method are the same as those in Section 2. The simulation result is shown in Figure 2.

Figure 2. Response to expectational change, Model with the collateral constraint

This figure shows that the consumption, labor, and investment all jump up on impact when the expectations on future B improve at date 0; the consumption and labor continue to rise during the period between date 0 and date τ, while the investment remains at a high level in that period; and all three variables decline sharply when the expectations are corrected at date τ. Therefore, the model is quite successful in replicating the Pigou cycle in response to the emergence of the expectations that B_t will rise and subsequent disappointment. We also conduct the simulation of the case where $\eta = 0.5$ and confirm that the Pigou cycle is generated in this case, too.

Why is the collateral constraint crucial? Two features of the model are crucial in replicating the Pigou cycle: the collateral constraint and the production technology that land produces the consumer goods in combination with capital input, not labor input. When the expectations change at date 0 such that $B_t = B + \Delta$ for $t \geq \tau$, the land price rises and the collateral constraint is loosened. Therefore, labor input increases, and the output from the Cobb-Douglas technology (and the consumption) increases. Since k_2 is larger in the new steady state where $B_t = B + \Delta$, the investment must increase before date τ to smooth the path of capital accumulation. Therefore, the investment also increases in response to the expectational change, and the Pigou cycle is generated.

If we assume a different production technology, the model may not generate the Pigou cycle. For example, we can consider a variant of the momdel in which the production technology for land is $y_t = B_t n_{2t}^{\eta} a_t^{1-\eta}$, where n_{2t} is the labor input to land. The simulation shows that in this case, the investment decreases in response to the improvement of the expectations on future B_t. This variant of the model cannot generate the Pigou cycle.

What if the investment is subject to the collateral constraint? In the simulation shown in Figure 2, we set $B = 0.1$ and $\Delta = 0.01$. If we increase the values of B and Δ, the movement of investment becomes odd: For example, the investment jumps up at date 0 and then decreases gradually in the case where $B = 0.2$ and $\Delta = 0.05$. If the investment expenditures by firms are (partially) subject to the collateral constraint, the investment does not decrease during the period between date 0 and date τ. We posit the following collateral constraint:

$$w_t n_t + \chi\{k_{t+1} - (1-\delta)k_t\} \leq q_t, \tag{34}$$

instead of (24), where $0 < \chi < 1$, and conduct the simulation. We set $\chi = 0.5$, $B = 0.2$, $\Delta = 0.05$, and the other parameters at the same values as those in the previous simulations. The result is shown in Figure 3.

Figure 3. Response to expectational change, The collateral constraint on labor and investment

In this case where the investment expenditures are partially subject to the collateral constraint, the consumption, the labor and the investment all continue to increase during the period between date 0 and date τ. Thus it can be said that the plausible modification enables the model to replicate the Pigou cycle for a wide range of B and Δ.

4 "Irrational exuberance" in the Pigou cycle with private information

In this section, we modify the model such that each firm receives private information on the productivity of its own land. It may be plausible to assume that even during a boom period a firm has the correct prospect for its own productivity, while the firm may be influenced by the prevalent euphoria in judging the future productivity of the other firms. If these private information were aggregated quickly into the public informtion, the wrong macroeconomic expectations would be corrected soon and the boom caused by the wrong expectations would come to an end quickly. In this section, we show that when the firms are subject to the collateral constraints, they may have strong adverse incentive to hide the correct private information from others. Under such an environment, a wrong optimism on the macroeconomic productivity is not corrected for a long time once it spreads over the economy, even if each firm has the correct private information on its own future productivity; and the boom driven by the wrong macroeconomic expectations continue for considerable periods.

This story may be an alternative explanation to existing theories of bubbles (e.g., rational bubble discussed in, say, Blanchard and Fischer [1989] and risk-shifting in Allen and Gale [2000]) for episodes of "irrational exuberance." The interpretation of the irrational exuberance in our story is that it is a boom in which each firm correctly anticipates that *its own* productivity will not rise, while it also believes wrongly that the productivity of *the other* firms will rise dramatically.

4.1 Information structure

We assume that there are two kinds of expectations on the future values of B_{it}: Public expectation on the macroeconomic productivity, which is shared commonly by all consumers and firms, and private expectation on the productivity of each land i, which is given only to firm i. Before specifying the information structure for the expectations, we make clear that all past and present variables are public information:

Assumption 1 *At date t, all consumers and firms observe $\{A_s, B_{is}, c_s, n_{is}, k_{is+1}\}$ for all i and for all $s \leq t$, where n_{it} and $k_{it+1} - (1-\delta)k_{it}$ is the labor input and the investment for firm i at date t.*

Since our purpose is to analyze how the equilibrium path of the previous section changes if private information exists, we assume the following evolutions for the expectations.

Public expectation for future productivity The economy is initially in the steady state where $B_{it} = B$. Thus until date 0, people hold the public expectation that $B_{it} = B$ for all i and for all t. At date 0, the macroeconomic news hits the economy unexpectedly, and the public expectation changes to that $B_{it} = B$ for all i and for $0 \leq t \leq \tau - 1$, and $B_{it} = B + \Delta$ for all i and for $t \geq \tau$. People continue to hold this public expectation until date τ, unless the private expectations of firms are not revealed. Another news hits the economy unexpectedly at date τ and people changes their public expectation again to that $B_{it} = B$ for all i and for all $t \geq \tau$. In sum, a wrong public expectation comes at date 0, which is then corrected at date τ.

Private expectation for future productivity We assume that although a wrong news spreads during the period between date 0 and date τ, each firm always knows private information on its own land, which is not known to the other agents. Thus, we can assume that the firm always has correct prospect on the future productivity of its own land. Therefore, firm i has the private expectation that $B_{it} = B$ for all t. Note that what firm i knows about the future values of B_{jt} for all $j(\neq i)$ is the public expectation on B_{jt}.[3]

Revelation of private information Assumption 1 and the above assumptions on public and private expectations imply that discrepancy between private and public information can exist only in the period between date 0 and date τ. In this period, people infer the private information of firm i by observing the labor input n_{it} and the investment $k_{it+1} - (1-\delta)k_{it}$. We define the *public path* as the equilibrium path of the economy in which B_{it} changes as follows and all agents correctly anticipate the evolution of B_{it}: $B_{it} = B$ for all i and for $0 \leq t < \tau$ and $B_{it} = B + \Delta$ for all i and for all $t \geq \tau$. Therefore, through the public path the economy converges from the steady state where $B_{it} = B$ to the new steady state where $B_{it} = B + \Delta$. The public path is determined by solving the system of equations (14), (16), (17), (25), and (26), on the premise that $B_t = B$ for $0 < t < \tau$ and $B_t = B + \Delta$ for $t \geq \tau$. We denote the variables in the public path by putting overline on them: $\{\overline{c}_t, \overline{n}_t, \overline{k}_{t+1}, \overline{q}_t, \overline{r}_t, \overline{\lambda}_t\}$. If firm i chooses $n_{it} = \overline{n}_t$ and $k_{it+1} = \overline{k}_{t+1}$ at date t, firm i's private information on the future value of B_{it} is hidden from the public, and people continue to believe that $B_{it} = B + \Delta$ for $t \geq \tau$. Therefore, firm i can pretend that the productivity of its land will rise by mimicing the labor and the investment, which the other firms would choose under the belief that $B_{jt} = B + \Delta$ for $\forall j$ and $t \geq \tau$. On the other hand, if firm i chooses $n_{it} \neq \overline{n}_t$ and/or $k_{it+1} \neq \overline{k}_{t+1}$ at date t, consumers and the other firms infer that the public information that $B_{it} = B + \Delta$ for $t \geq \tau$ is wrong, and they infer the private information of firm i from the observed values of n_{it} and k_{it+1}. To simplify the process of information revelation, we assume the following assumption:

Assumption 2 *If firm i chooses $n_{it} \neq \overline{n}_t$ and/or $k_{it+1} \neq \overline{k}_{t+1}$ at date t $(0 \leq t < \tau)$, consumers and the other firms infer that $B_{it} = B$ for $t \geq \tau$.*

[3] For simplicity of the analysis, we assume that the correct expectation is that $B_{it} = B$ for all i and for all t. Alternative and more realistic assumption may be that $B_{it} = B + \Delta$ for $t \geq \tau$ for some firms, and $B_{it} = B$ for all t for the other firms. We leave this extension for future work.

We may be able to generalize the model by loosening this assumption so that the inferred value of B_{it} can be different from B. Although it may be worthwhile to generalize this model in this direction, we focus on the simple case with Assumption 2, since it suffices for our purpose to demonstrate the basic mechanism for firms to hide the private information.

Collateral constraint with private information When lenders (i.e., consumers) provide credit to firm i, they lend up to the value of the collateral: q_{it}. In the case where the discrepancy between private and public information on B_{it} exists, the collateral value q_{it} is determined in the market based on the observable variables $\{A_t, B_{it}, c_t, n_{it}, k_{it+1}\}$. Since firms must borrow money before it actually puts the labor input and makes the investment, q_{it} is determined based on the following information: $I_t = \{A_s, B_{is}, c_{s-1}, n_{js-1}, k_{js}(\text{ for } \forall j \in [0,1])\}_{s=-\infty}^{t}$. Therefore, equation (11) is replaced by

$$q_{it} = \frac{\overline{\lambda}_{t+1}}{\overline{\lambda}_t} \left\{ \frac{(1-\eta)\eta^{\frac{\eta}{1-\eta}} E\left[B_{it+1}^{\frac{1}{1-\eta}} \middle| I_t \right] \theta^{\frac{1}{1-\eta}}}{\overline{r}_{t+1}^{\frac{\eta}{1-\eta}}} + q_{it+1} \right\}, \tag{35}$$

where $E[\cdot|I_t]$ is the expectation conditional to the information set I_t. Note that deriving this equation, we assume that firm i believes that the equilibrium path of the economy is the public path. This equation and Assumption 2 imply the existence of the following trade-off for firm i in choosing n_{it} and k_{it+1} for $0 \leq t < \tau$. If firm i follows the path $\{\overline{n}_s, \overline{k}_{s+1}\}_{s=0}^{t}$, the market value of its collateral at the next date remains at $q_{it+1} = \overline{q}_{t+1}$, since its private information remains unrevealed when it borrows at date $t+1$. If firm i deviates and chooses $n_{it} \neq \overline{n}_t$ and/or $k_{it+1} \neq \overline{k}_{t+1}$ at date t, the market value of its collateral at the next date becomes the value q_{it+1}, which is calculated from (35) on the premise that $B_{it} = B$ for all t. This is because the deviation from the public path by firm i makes the lenders believe that the productivity of land i will not rise.

4.2 Firm's problem with private information

In this environment, a firm can choose at date t $(0 \leq t < \tau)$ whether or not it pretend that the productivity of its land will rise at date τ. To be more specific, we formulate the firm's problem as follows. Firm i chooses date T $(0 \leq T \leq \tau - 1)$, at which the firm stops pretending.[4] The firm it chooses $n_{it} = \overline{n}_t$ and $k_{it+1} = \overline{k}_{t+1}$ for $0 \leq t < T$; and that it solves the following maximization problem, given that $k_{iT} = \overline{k}_T$:

$$\max_{k_{t+1},k_{2t},k_{3t},n_t} \sum_{t=T}^{\infty} \overline{\lambda}_t[\overline{r}_t k_{3t} + A(k_t - k_{2t} - k_{3t})^{\alpha} n_t^{1-\alpha} + Bk_{2t}^{\eta} a_i^{1-\eta} - (k_{t+1} - (1-\delta)k_t) - \overline{w}_t n_t]$$

subject to

$$\begin{cases} \overline{w}_T n_T \leq \overline{q}_T a_i, \\ \overline{w}_t n_t \leq q_{it} a_i, \end{cases} \tag{36}$$

where q_{it} is given by (35) with $B_{it+1} = B$. Firm i chooses T such that

$$T = \arg\max_{T'} W_{T'}, \tag{37}$$

where

$$W_T = \sum_{t=0}^{\infty} \overline{\lambda}_t[\overline{r}_t \hat{k}_{3t} + A(\hat{k}_t - \hat{k}_{2t} - \hat{k}_{3t})^{\alpha} \hat{n}_t^{1-\alpha} + B\hat{k}_{2t}^{\eta} a_i^{1-\eta} - (\hat{k}_{t+1} - (1-\delta)\hat{k}_t) - \overline{w}_t \hat{n}_t], \tag{38}$$

where the variables with hat are the firm's choices, given a fixed value of T.

If T, the solution to the above problem, is close to 0, firms voluntarily reveal their private information, and the wrong public expectations are corrected quickly. If T equals or is close to $\tau - 1$, firms hide their private information for a long period, and the boom caused by the wrong public expectations continues for that period. The simulation below shows that the latter is the case for the plausible parameter values.

4.3 Equilibrium and simulation

We focus on the symmetric equilibrium where all firms act identically and choose the same value of T. Thus, in the symmetric equilibrium, the private information of all firms is revealed at the same time, i.e., date T. Since consumers and each firm take the revelation of the other firms' private information as an unexpected event, the (wrong) public information is unexpectedly corrected at date T and people become to believe at date T that $B_{it} = B$ for all i and for all t.

[4] Note that T cannot be τ, since the private information of firm i is revealed unconditionally at date τ. Thus there is no benefit for the firm of pretending at date $\tau - 1$. Therefore, T is at most $\tau - 1$.

Therefore, the equilibrium becomes as follows: There exists T such that the economy follows the public path until date $T-1$ and each firm chooses $n_{it} = \overline{n}_t$ and $k_{it+1} = \overline{k}_{t+1}$ for $t \leq T-1$; and that at date T the economy switches to the path, in which all agents believe that $B_{it} = B$ for all i and for all t, and converges to the initial steady state.

Finding T To find the value of T, we need to solve the firm's problem described in Section 4.2 on the premise that the economy follows the public path forever. Since $0 \leq T \leq \tau - 1$, we calculate W_T for all $T \in \{0, 1, \cdots, \tau - 1\}$ and find the optimal value of T, which maximizes W_T. For a given value of T, the solution to the firm's problem: $\{k_t, k_{2t}, k_{3t}, n_t, \mu_t\}$ for $t \geq T$ are determined by the following system of equations, where μ_t is the Lagrange multiplier for the collateral constraint. Note that the firm solves the problem holding the belief that the economy follows the public path forever, and that, therefore, these variables are not realized in the equilibrium, since the macroeconomic expectations unexpectedly change at date T in the symmetric equilibrium.

$$(\overline{\lambda}_t + \mu_t)\overline{w}_t = \overline{\lambda}_t(1-\alpha)A\left(\frac{k_t - k_{2t} - k_{3t}}{n_t}\right)^{\alpha}, \tag{39}$$

$$\overline{r}_t = \alpha A\left(\frac{n_t}{k_t - k_{2t} - k_{3t}}\right)^{1-\alpha}, \tag{40}$$

$$\overline{r}_t = \eta B\left(\frac{a_i}{k_{2t}}\right)^{1-\eta}, \tag{41}$$

$$\overline{\lambda}_t = \overline{\lambda}_{t+1}\left\{\overline{r}_{t+1} + 1 - \delta\right\}, \tag{42}$$

$$\overline{w}_T n_T \leq \overline{q}_T a_i, \text{ and} \tag{43}$$

$$\overline{w}_t n_t \leq q_t a_i, \text{ for } t \geq T+1. \tag{44}$$

Solving this system of equations, we get the discounted present value of the (expected) dividend stream W_T. The firm chooses the equilibrium value of T by solving (37).

Simulation We conduct the simulation in the case where the parameters are set at the same values as in Section 2. Since $\tau = 10$, we calculate W_T for $T = 0, 1, \cdots, 9$, and find that the value of T that maximizes W_T equals 9. Therefore, the equilibrium value of T equals 9. The equilibrium path is described as follows. During the period from date 0 to date 8, the economy follows the public path, which is determined by solving the system of equations (14), (16), (17), (25), and (26), on the premise that $B_t = 0.1$ for $0 \leq t \leq 9$ and $B_t = 0.11$ for $t \geq 10$. The correct private expectation is revealed at date 9, and the economy switches at date 9 to a new path that converges to the initial steady state where $B_t = 0.1$. This path is determined by solving the same system of equations (14), (16), (17), (25), and (26), on the premise that $B_t = 0.1$ for all t, given the initial capital stock k_9. The equilibrium dynamics are shown in Figure 4.

Figure 4. The Pigou cycle with private information

We also conduct the simulation in the case where the investment expenditure is subject to the collateral constraint, i.e., (34). We set the same parameter values as those in Figure 3. The numerical calculation shows that the equilibrium value of T in this case is also 9 $(= \tau - 1)$. The equilibrium dynamics are shown in Figure 5.

Figure 5. The Pigou cycle with private information, The collateral constraint on labor and investment

The result that firms choose $T = \tau - 1$ is robust for small changes in parameter values. This result means that in our model firms choose to hide their private information as long as possible. Therefore, the boom caused by wrong public expectation cannot be stopped quickly even if each firm has the correct private expectation on its own future productivity.

Private information in the model without collateral constraints Note that the adverse incentive for firms to hide the private information disappears in the case where they are not subject to the collateral constraint. In the model with no collateral constraint, each firm decides the labor input and the investment based only on its own private expectation, since there is no benefit from hiding its private expectation from the other agents. Therefore, the wrong public expectation has no impact on the equilibrium path of the economy, as long as the firms have correct private information on their own technology. The economy remains in the steady state where $B_t = B$ forever, although the wrong macroeconomic news hits the economy at date 0. The public expectation is corrected instantly at date 0, since all firms reveal their private information at date 0 through their investment and employment decisions.

5 Concluding remarks

The notion of the Pigou cycle, i.e., the business cycle driven by expectations, is attracting attention of increasing number of researchers as a theoretical tool for explaining the boom-bust cycles in the real world, such as the emergence and collapse of the "Internet bubble" in the late 1990s. While the comovements of the consumption, the labor, and the investment are the key feature of the Pigou cycle, the standard neoclassical models cannot generate the comovements in response to the changes in expectations. Existing literature proposes several twists for the standard models to generate the Pigou cycle: For example, technological complementarity, habit persistence, adjustment costs for investment, and labor frictions.

This paper has two contributions to the literature. First, we show that financial frictions may be crucial in generating the Pigou cyle. With an assumption that firms are subject to the collateral constraint in financing labor input or investment, a fairly standard model can reproduce the Pigou cycle in response to the changes in expectations.

Second, we propose an explanation for the "irrational exuberance," i.e., a boom in which each firm correctly anticipates that *its own* productivity will not rise, while it also believes wrongly that the productivity of *the other* firms will rise dramatically. We consider the case where a wrong macroeconomic news spreads over the economy, while each firm has the correct private information on its own future technology. If the correct private information were revealed quickly, the boom driven by the wrong expectation would be stopped early. We show that the collateral constraint may give firms an adverse incentive to hide the private information during the boom, since the revelation of the private information by a firm makes the other agents revise their expectations on the firm's future technology, leading to the tightening of the collateral constraint for the firm who reveals the private information. If the benefit of the loose collateral constraint is large for the firm, it has an adiverse incentive to hide the correct private information as long as the boom caused by the wrong news continues. The numerical simulation shows that it is the case for a plausible range of parameters.

Financial frictions represented by the collateral constraint are widely used in explaining the features of the (relatively large) business fluctuations.[5] It may be said that this paper confirms the usefulness of the collateral constraint as a building block in the theory of the business cycles by showing that it can explain the key features in the boom-bust cycles driven by changes in expectations.

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[5]For example, the seminal paper by Kiyotaki and Moore (1997) formalizes the interaction between fluctuations of the real variables and asset prices; Edison, Luangaram, and Miller (2000) adopt the Kiyotaki-Moore model to explain the East Asian crisis in 1997; Mendoza and Smith (2004) explain the Sudden-stops in small open economies; Kobayashi and Inaba (2005) use the collateral constraints to explain the labor deteriorations in the US Great Depression and the 1990s in Japan.

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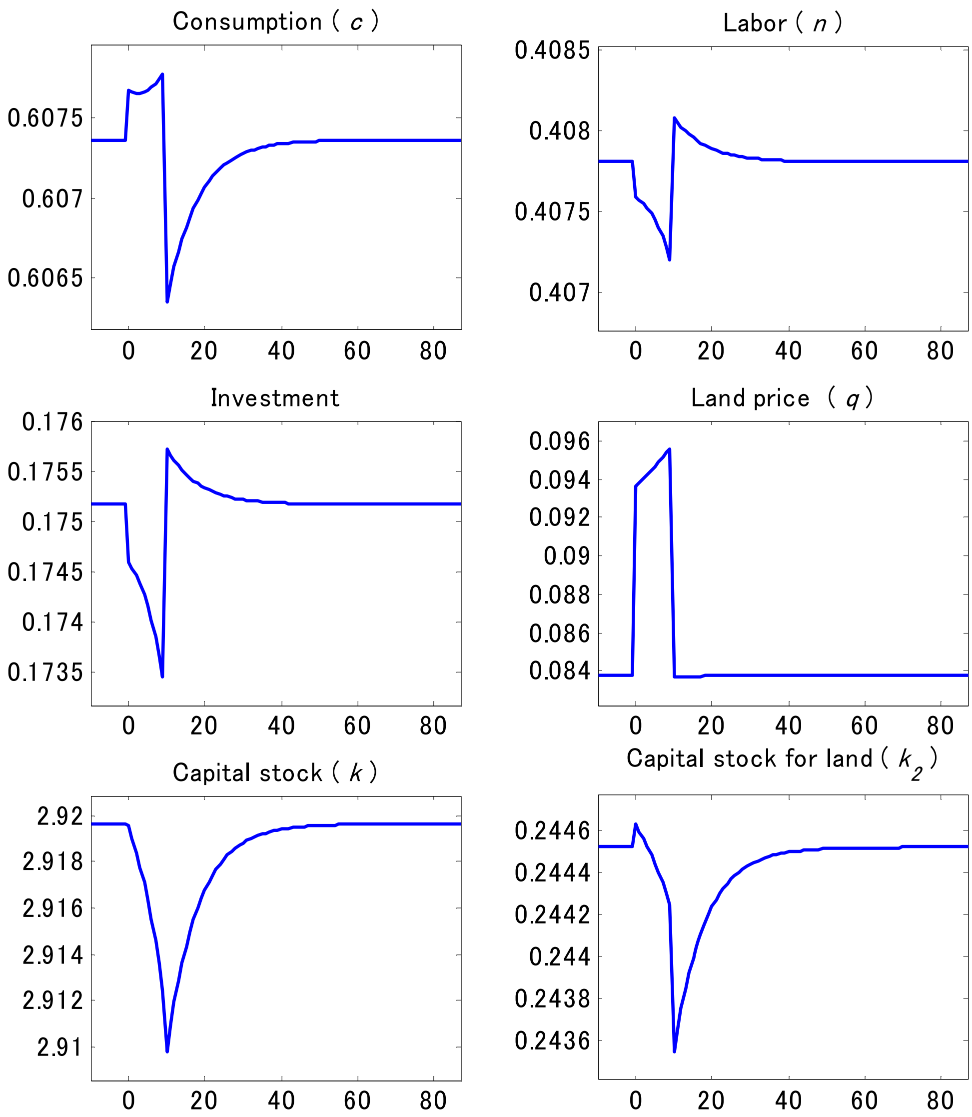
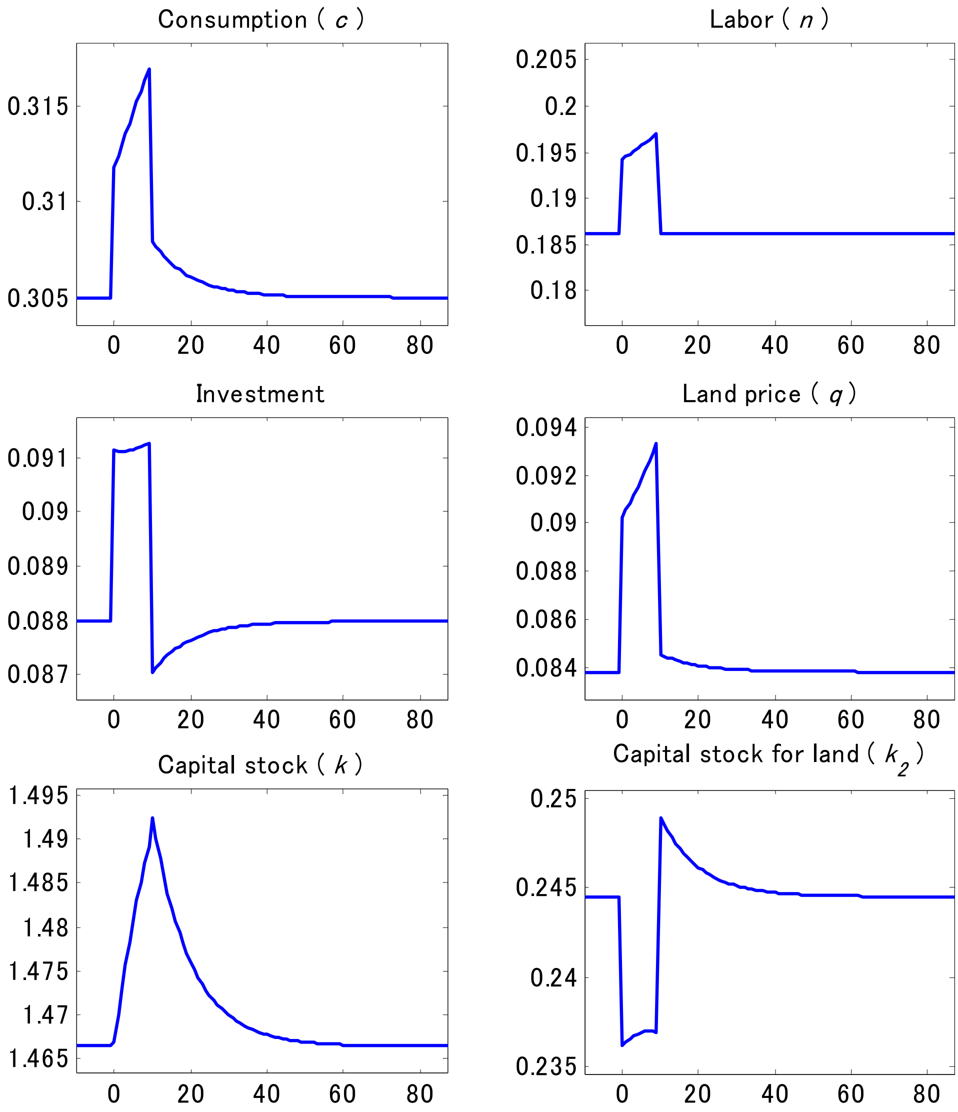
Figure 1. Response to expectational change; Benchmark model without the collateral constraint



Parameters in this graph are following:

α=0.3, β=0.98, γ=1.2, δ=0.06, η=0.3, χ=0, A=1, B=0.1, Δ=0.01, τ=10.

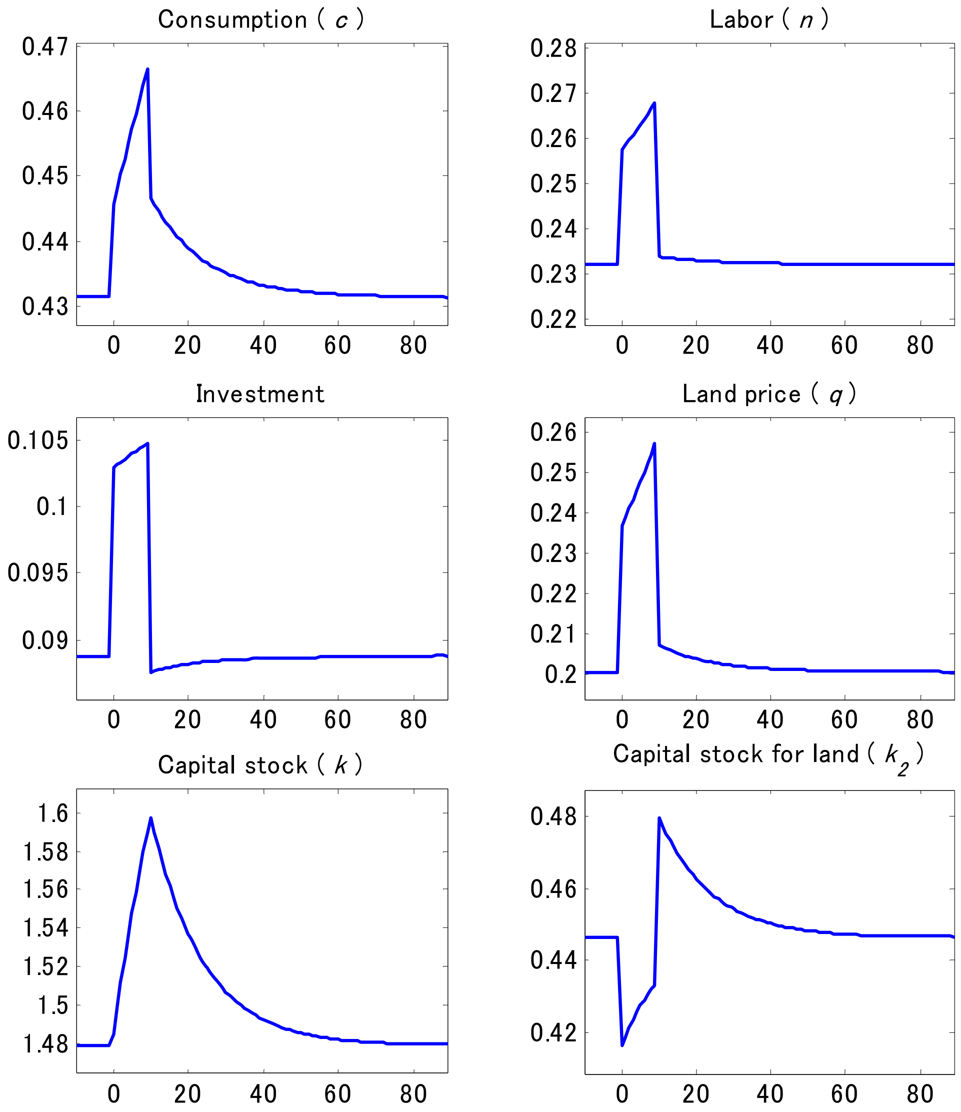
Figure 2. Response to expectational change; Model with the collateral constraint



Parameters in this graph are following:

α=0.3, β=0.98, γ=1.2, δ=0.06, η=0.3, χ=0, A=1, B=0.1, Δ=0.01, τ=10.

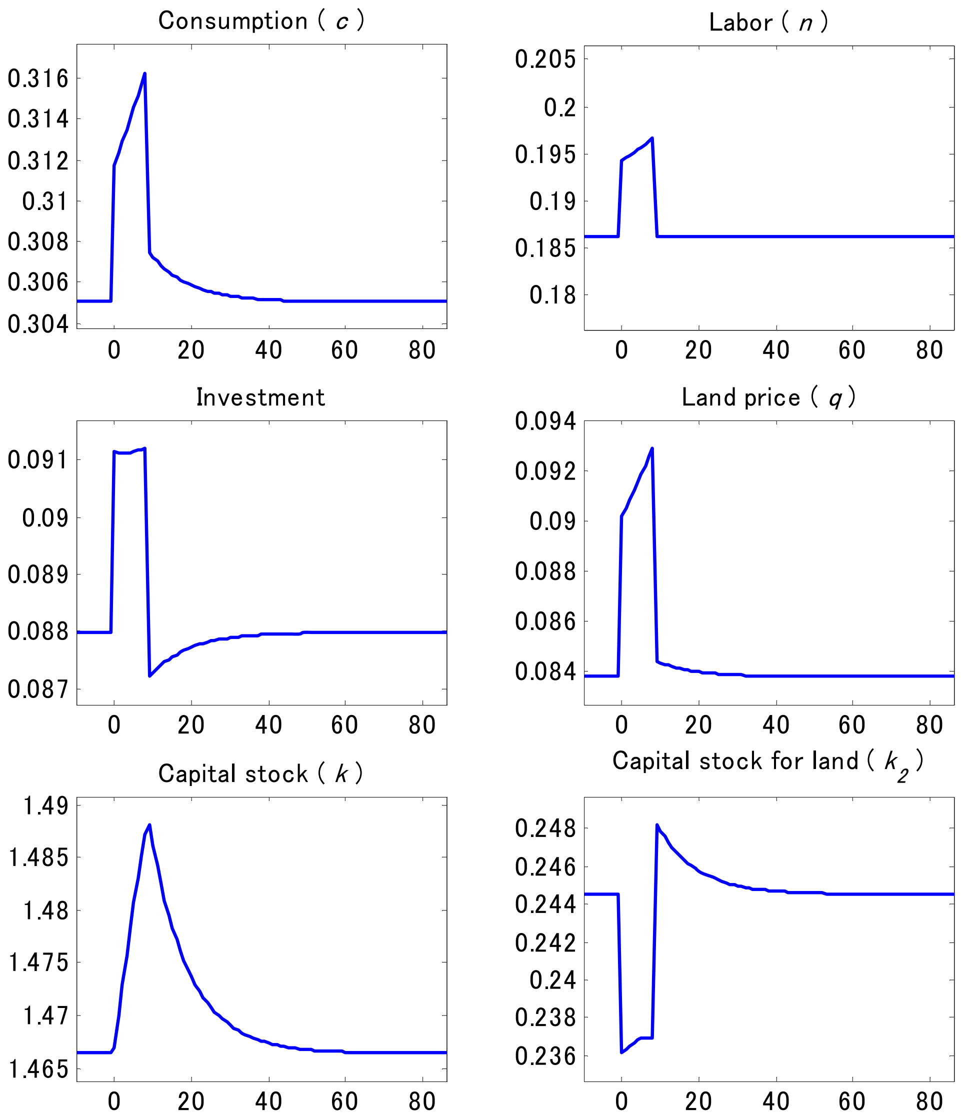
Figure 3. Response to expectational change; The collateral constraint on labor and investment



Parameters in this graph are following:

α=0.3, β=0.98, γ=1.2, δ=0.06, η=0.3, χ=0.5, A=1, B=0.2, Δ=0.05, τ=10.

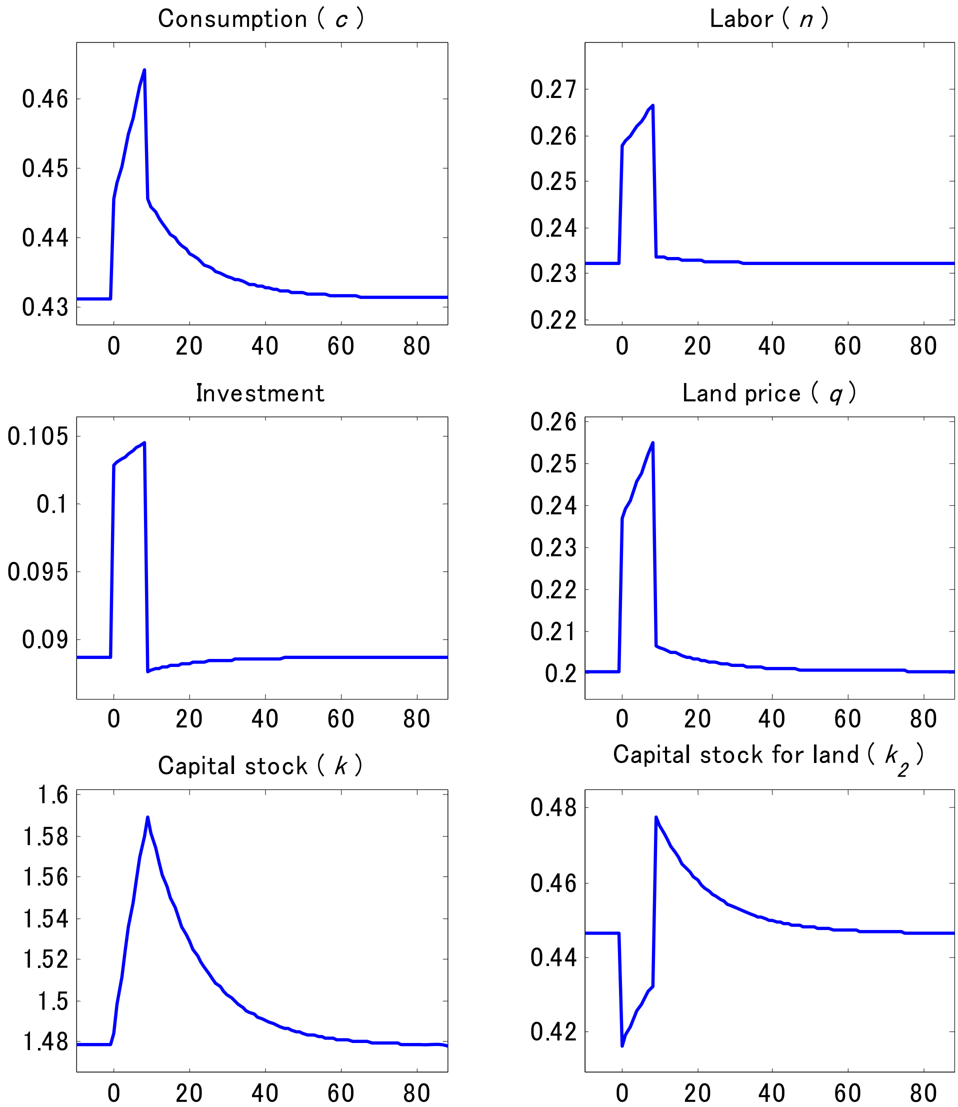
Figure 4. The Pigou cycle with private information



Parameters in this graph are following:

α=0.3, β=0.98, γ=1.2, δ=0.06, η=0.3, χ=0, A=1, B=0.1, Δ=0.01, τ=10.

Figure 5. The Pigou cycle with private information; The collateral constraint on labor and investment



Parameters in this graph are following:

α=0.3, β=0.98, γ=1.2, δ=0.06, η=0.3, χ=0.5, A=1, B=0.2, Δ=0.05, τ=10.