How Effective are Emission Taxes in an Open Economy?

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Abstract

This paper compares emission taxes with other taxes from the viewpoint of emission reduction in an open economy. Using a simple monopoly model, we show that emission taxes may not be very effective to protect environment because of the spillover effects between markets stemming from non-constant marginal costs and transboundary externalities. Other taxes such as production taxes and tariffs are more effective under certain conditions. Thus, an easy application of emission taxes should be discreet in the open economy framework.

\textit{JEL Classification Numbers:} F13, F18

\textit{Keywords:} emission taxes, tariffs, spillover effects, monopoly, transboundary pollution

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1 Introduction

Worldwide environmental destruction has been attracting considerable attention. Obviously, emissions from production and consumption (including disposal) activities are the major causes of the environmental problems. When firms (consumers) generate emissions, the conventional wisdom says that the government should intervene to restrict the production (consumption). Emission taxes are typical policy measures for this purpose. The taxes lower the level of production (consumption) and hence the negative externalities generated by emissions are internalized.

In this age of globalization, however, production and consumption may not be done at home. For example, by undertaking foreign direct investment (FDI), firms may not produce in their own countries. Foreign production does not imply that the domestic country is free from environmental damages. Transboundary pollution such as global warming, acid rain, and depletion of ozone layer has lately been a big problem. When the good is produced abroad, the domestic emission taxes are not effective. In this case, it is often claimed that trade policies such as tariffs should be used to affect foreign production. Moreover, goods are usually consumed not only at home but also abroad. That is, there usually exist multiple markets. Thus, the framework of an open economy is indispensable to examine emission regulations.

The purpose of this paper is to shed light on the effects of taxes on emissions with imperfect competition in the open economy. We focus on the case of monopoly in order to obtain clear-cut results and policy implications. We show that emission taxes may be less effective than other taxes. In particular, an emission tax raises environmental damages under certain circumstances. Thus, an easy application of the conventional wisdom in the open economy framework should be discreet.

The basic structure of our model is as follows. There are two segmented markets, domestic and foreign. A monopolist serves both markets but produces in either the domestic country or the foreign country. Emissions are generated in the process of either production or consumption and transboundary. To reduce emissions, the domestic government imposes various taxes including emission taxes and trade taxes.

Our counter-intuitive result stems from the spillover effects between the markets which are caused by non-constant marginal cost (MC). As long as the MC is constant, a tax in the domestic country affects only domestic market. Thus, the effects of the tax are basically the same as those in the closed economy. If the MC is not constant, on the other hand, the different markets are connected through the changes of the MC.

Intuition behind our results are somewhat similar to that under plant relocation. When the domestic government adopts tough environmental regulations, domestic firms may shift

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1 For recent studies on trade policies with spillover effects, see Ishikawa and Mukunoki (2004).
their plants abroad. If emission regulations are loose enough abroad, for example, environmental damages could become worse as a result of the plant relocation. Instead of such plant relocation, in our model, the single-plant monopolist adjusts its supplies between two markets.

The rest of the paper is organized as follows. Sections 2 and 3, respectively, examine the case of domestic production and the case of foreign production. Section 4 concludes the paper.

2 Production in the Domestic Country

We consider an industry in which a monopolist produces a good with a single domestic plant and serves both domestic and foreign markets. These two markets are segmented and hence the monopolist can set different prices across the markets. The inverse demand functions are given by

\[ p = P(x); \quad P' < 0 \quad \text{and} \quad p^* = P^*(x^*); \quad P'^* < 0 \]

(1)

where \( x \) and \( p \) are the demand and consumer price of the good in the domestic market. Foreign variables, parameters and functions are denoted by “\( * \)”. We define the elasticity of the slope of the inverse demand function \( \epsilon(x) \equiv -xP''(x)/P'(x) \) for the following analysis. The (inverse) demand curve is concave if \( \epsilon(x) \leq 0 \) and convex if \( \epsilon(x) \geq 0 \).

In this section, we assume that the monopolist produces in the domestic country. Production or consumption generates emissions which lead to negative externalities. Emissions measured in the domestic country are given by \( \omega \equiv ez + ke^*z^* \) where \( e, e^*, \) and \( k \) are parameters with \( 0 \leq k \leq 1 \). \( k \) is the degree of transboundary emissions. \( z \) is the domestic output level (i.e., \( z = x + x^* \) and \( z^* = 0 \)) if production generates emissions and the domestic consumption level (i.e., \( z = x \) and \( z^* = x^* \)) if consumption generates emissions. Domestic negative externalities are measured by \( B(\omega); \quad B' > 0 \) with \( B(0) = 0 \). The domestic government sets various specific taxes to control emissions: an emission tax, \( t \), a production tax, \( \nu \), a consumption tax, \( \theta \), and an export tax, \( \zeta \).

The profit of the monopolist is

\[ \Pi(x, x^*; t, \nu, \theta, \zeta) = [P(x) - \theta - \nu]x + [P^*(x^*) - \zeta - \nu]x^* - C(x + x^*) - tez, \]

(2)

where \( C(\cdot) \) is the cost function with \( C' > 0 \).

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2 See Markusen et al. (1993,1995) and Rauchscher (1995), for example.
2.1 Emission Taxes

We first examine emission taxes when production leads to negative externalities. Noting \( z = x + x^* \), the first-order conditions for the monopolist are

\[
\frac{\partial \Pi}{\partial x} = (P - \theta - \nu) + P'x - (C' + te) = 0, \quad (3)
\]

\[
\frac{\partial \Pi}{\partial x^*} = (P'^* - \zeta - \nu) + P'^*x^* - (C' + te) = 0. \quad (4)
\]

We assume that the second-order sufficient conditions are satisfied:

\[
2P' + P''x - C'' = P'(2 - \epsilon) - C'' < 0, \tag{5}
\]

\[
2P'^* + P'^{*}x^* - C'^* = P'^*(2 - \epsilon^*) - C'^* < 0, \tag{6}
\]

\[
\Omega \equiv (2P' + P''x - C'') (2P'^* + P'^{*}x^* - C'^*) - (C'')^2 = [P'(2 - \epsilon) - C''] P'^*(2 - \epsilon^*) - P'(2 - \epsilon) C'^* > 0 \tag{7}
\]

The following should be noted. First, in view of (5) and (6), \( \epsilon < 2 \) and \( \epsilon^* < 2 \) are necessary when \( C'' \leq 0 \). Second, the slope of the domestic MR curve is given by \( P'(2 - \epsilon) \). Third, it can be seen from the second-order conditions that if \( \epsilon \geq 2 \), then \( \epsilon^* < 2 \) is necessary and vice versa. Therefore, the following lemma is straightforward:

**Lemma 1.** (i) \( \epsilon > 2 \) or \( \epsilon^* > 2 \) holds only if \( C' + te \) is increasing. (ii) The domestic (foreign) MR curve is downward-sloping if and only if \( \epsilon < 2 \) (\( \epsilon^* < 2 \)). (iii) The MR curve can be upward-sloping or horizontal at most in one country.

To analyze the effects of the domestic emission tax on emissions, setting \( \nu = \theta = \zeta = 0 \), we totally differentiate (3) and (4) and obtain

\[
\begin{pmatrix}
2P' + P''x - C'' \\
- C''
\end{pmatrix}
\begin{pmatrix}
dx \\
- \frac{dx}{dx^*}
\end{pmatrix}
= \begin{pmatrix}
e \\
- e
\end{pmatrix} dt
\]

with the solution

\[
\begin{pmatrix}
dx \\
- \frac{dx}{dx^*}
\end{pmatrix} = \frac{1}{\Omega} \begin{pmatrix}
2P'^* + P'^{*}x^* - C'^* \\
C'^*
\end{pmatrix}
\begin{pmatrix}
e \\
- e
\end{pmatrix} dt.
\]

Thus, the effects of a change in \( t \) on the supplies are given by

\[
\frac{dx}{dt} = P'(2 - \epsilon)e, \quad \frac{dx^*}{dt} = P'^*(2 - \epsilon^*)e, \quad \frac{d(x + x^*)}{dt} = \frac{P'(2 - \epsilon) + P'^*(2 - \epsilon^*)}{\Omega} e. \tag{8}
\]
In view of Lemma 1, the following are straightforward. If $C'' \leq 0$, both $\epsilon < 2$ and $\epsilon^* < 2$ hold and hence an increase in $t$ necessarily decreases both $x$ and $x^*$. If $C'' > 0$, on the other hand, $x$ ($x^*$) increases if and only if $\epsilon^* > 2$ ($\epsilon > 2$). However, it can be shown with the aid of Figures 1 and 2 that the total supply decreases even if either $\epsilon > 2$ or $\epsilon^* > 2$ holds.

In the figures, $RR$ and $R^*R^*$, respectively, show the locus of the first-order condition (3) and (4) on the quantity plane. The initial equilibrium, $E$, is given by the intersection of $RR$ and $R^*R^*$. Using the implicit function theorem, the slopes of $RR$ and $R^*R^*$ are, respectively, given by

\[
\begin{align*}
\frac{dx^*}{dx}|_{RR} & = \frac{P'(2 - \epsilon) - C''}{C''}, \\
\frac{dx^*}{dx}|_{R^*R^*} & = \frac{P^*(2 - \epsilon^*) - C''}{P^*P''(2 - \epsilon^*) - C''}.
\end{align*}
\]

Since the numerator of (9) and the denominator of (10) are negative from the second-order conditions, the signs of the slopes depend only on the sign of $C''$. While $RR$ is vertical and $R^*R^*$ is horizontal with $C'' = 0$, both loci are downward-sloping (upward-sloping) with $C'' > 0$ ($C'' < 0$). The second-order conditions imply that $RR$ is steeper than $R^*R^*$. The absolute value of the slope of $RR$ ($R^*R^*$) is greater (less) than 1 if and only if $\epsilon < 2$ ($\epsilon^* < 2$).

Figure 1 illustrates the case where both $\epsilon < 2$ and $\epsilon^* < 2$ hold, while Figure 2 shows the case where $\epsilon < 2$ and $\epsilon^* > 2$ hold.

An increase in $t$ shifts both $RR$ and $R^*R^*$ downward. Suppose that $RR$ ($R^*R^*$) shifts downward to $R'R'$ ($R^*R^*$) in Figure 1. Then the equilibrium moves from $E$ to $E'$ which must be located in the southwest of $E$, because both $x$ and $x^*$ fall. In Figure 2, the new equilibrium $E'$ must be located to the southeast of $E$, because $x$ increases while $x^*$ decreases. Noting $R^*R^*$ and hence $RR$ are steeper than the 45 degree line, the total output at $E'$ is less than that at $E$.

It is somewhat puzzling that an increase in $t$ could raise $x$ ($x^*$) and its condition depends on the foreign (domestic) demand (i.e., $\epsilon^*(\epsilon)$). The intuition is as follows. Suppose that $C'' > 0$ and $\epsilon^* > 2$. An increase in $t$ decreases $x^*$. The decrease in $x^*$ is much larger under the upward-sloping MR curve in the foreign country, i.e., $\epsilon^* > 2$, than under the downward-sloping MR curve, i.e., $\epsilon^* < 2$. This effect is large enough to make $C' + te$ lower, which actually dominates the original increase in $t$. As a result, $x$ rises, but since $x^*$ falls by a large amount, $x + x^*$ lowers.

The above analysis establishes the following proposition.

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3 This can easily be verified in Figure 2.
Proposition 1. Suppose that a monopolist produces its product in the domestic country and serves both domestic and foreign markets and that production generates emissions. Then an increase in the domestic emission tax necessarily reduces the total emissions in the domestic country. If one of the MR curves is upward-sloping, however, the effect is mitigated.

Next we investigate emission taxes when consumption generates emissions. Since $z = x$ and $z^* = x^*$, the first-order conditions are modified as follows:

$$\frac{\partial \Pi}{\partial x} = (P - \theta - \nu) + P'x - (C' + te) = 0,$$

$$\frac{\partial \Pi}{\partial x^*} = (P^* - \zeta - \nu) + P'^*x^* - C'^* = 0.$$ (11) (12)

Then we obtain

$$\left(\begin{array}{c}
dx \\
dx^*\end{array}\right) = \frac{1}{\Omega} \left(\begin{array}{cc}2P'^* + P''x^* - C'' & C'' \\ C'' & 2P' + P''x - C''\end{array}\right) \left(\begin{array}{c}e \\
0\end{array}\right) dt.$$ (13)

Thus,

$$\frac{dx}{dt} = \frac{2P'^* + P''x^* - C''}{\Omega} e < 0, \quad \frac{dx^*}{dt} = \frac{C''}{\Omega} e, \quad \frac{d(x + x^*)}{dt} = \frac{P'^*(2 - e^*)}{\Omega} e.$$ (13)

An increase in $t$ lowers $x$. Whether $x^*$ decreases or not depends on the sign of $C''$. When $C'' > 0$, $x^*$ increases. This is because a decrease in the supply to the domestic market caused by an emission tax reduces the MC of production and hence the monopolist has an incentive to increase the supply to the other market, i.e., the foreign market. If $e^* > 2$ in addition to $C'' > 0$, the total output increases.\(^4\) However, if $e^* < 2$, the total output falls.

We can confirm the above result in Figures 1 and 2. When $t$ rises, only $RR$ shifts downward. In Figure 1, the new equilibrium is at $E'$. Whereas $x$ falls, $x^*$ rises. Since $R^*R^*$ is less steep than 45 degree line, the total output necessarily lowers. In Figure 2, on the other hand, the new equilibrium is given by $a$. Since $R^*R^*$ is steeper than the 45 degree line, the total output necessarily rises.

We thus have the following proposition.

Proposition 2. Suppose that a monopolist produces its product in the domestic country and serves both domestic and foreign markets and that consumption generates emissions. If the MC curve is upward-sloping, an increase in the domestic emission tax lowers the domestic emissions but raises the foreign ones, which may increase negative externalities measured in the domestic country.

Next we examine the effects of emission taxes on domestic welfare. Domestic welfare is
measured by sum of the consumer surplus (CS), profits, tax revenue, and negative externalities measured in the domestic country:\footnote{If the monopolistic firm is foreign owned, we need to take into account the rent-shifting effect from the foreign monopolist to the domestic country. For details, see Ishikawa (2000).}

\[ W = CS(x) + \Pi(x, x^*; t, \nu, \theta, \zeta) + tez + \nu(x + x^*) + \theta x + \zeta x^* - B(ez + ke^* z^*). \]

We first consider the case where production generates emissions. Setting \( \nu = \theta = \zeta = 0, z = x + x^* \) and \( z^* = 0 \), we differentiate domestic welfare (14) with respect to \( t \) and obtain\footnote{An emission tax is harmful to the monopolist.}

\[ \frac{dW}{dt} = -xP' \frac{dx}{dt} + (t - B' e) \frac{d(x + x^*)}{dt}. \] (15)

Evaluating (15) at \( t = 0 \), we can obtain the effect of introducing an emission tax:

\[ \frac{dW}{dt} \bigg|_{t=0} = -xP' \frac{dx}{dt} - B' e \frac{d(x + x^*)}{dt}. \] (16)

Since \( d(x + x^*)/dt < 0 \), an emission tax enhances domestic welfare if \( dx/dt > 0 \) (i.e., \( e^* > 2 \)).\footnote{\( (dW/dt)\big|_{t=0} > 0 \) implies that the optimal emission tax is positive.} Intuitively, under monopoly, the domestic market is undersupplied. Thus, if an increase in the supply as well as a reduction of the total emissions are realized, domestic welfare improves. If \( dx/dt < 0 \) (i.e., \( e^* < 2 \)), however, the welfare loss caused by the reduction of the domestic supply may outweigh the welfare gain from the reduction of the emissions.

In the case where consumption generates emissions, setting \( \nu = \theta = \zeta = 0, z = x \) and \( z^* = x^* \), we differentiate domestic welfare (14) with respect to \( t \) and evaluate it at \( t = 0 \):

\[ \frac{dW}{dt} \bigg|_{t=0} = -xP' \frac{dx}{dt} - B' e \frac{dx}{dt} + ke \frac{dx^*}{dt}. \] (17)

the sign of which is generally ambiguous. If \( e(dx/dt) + e^*(dx^*/dt) > 0 \), which holds only when \( C'' > 0 \), then an emission tax deteriorates domestic welfare. An emission tax improves domestic welfare only if the welfare gain from the reduction of negative externalities exceeds the welfare loss caused by the reduction of the domestic consumption.

### 2.2 Other Taxes

Propositions 1 and 2 imply that emission taxes may not be very effective to reduce negative externalities. In the following, we examine the effects of the other taxes. First of all, the following should be noted. The effects of an emission tax are qualitatively the same with those of a production or consumption tax. In particular, an emission tax becomes equivalent
to a production or consumption tax if \( e = 1 \). That is,

\[
\frac{dx}{d\nu} = \frac{P'(2 - \epsilon)}{\Omega}, \quad \frac{dx^*}{d\nu} = \frac{P'(2 - \epsilon)}{\Omega}, \quad \frac{d(x + x^*)}{d\nu} = \frac{P'(2 - \epsilon) + P'(2 - \epsilon^*)}{\Omega}, \quad (18)
\]

The effects of an export tax can be obtained analogously:

\[
\frac{dx}{d\zeta} = \frac{C''}{\Omega}, \quad \frac{dx^*}{d\zeta} = \frac{2P' + P''x - C''}{\Omega} < 0, \quad \frac{d(x + x^*)}{d\zeta} = \frac{P'(2 - \epsilon)}{\Omega}, \quad (19)
\]

A rise of \( \zeta \) decreases \( x^* \), increases \( x \) if and only if \( C'' > 0 \), and reduces \( x + x^* \) if and only if \( \epsilon < 2 \). Noting that only \( R^*R^* \) shifts downward with an increase in \( \zeta \), it is straightforward to confirm the result in Figures 1 and 2. The effects of these taxes are summarized in Tables 1, 2 and 3.

[Tables 1, 2 and 3 around here]

If domestic consumption and exports are jointly taxed at the same level (i.e., \( \theta = \zeta \)), this gives rise to the same effects as an equal tax on domestic production. Thus, in the case of production externalities, a production tax is more effective than a consumption or export tax as long as both \( \epsilon < 2 \) and \( \epsilon^* < 2 \) hold. However, if this is not the case, a consumption tax or an export tax is more effective. Therefore, a consumption (an export) tax may be more effective than an emission tax if \( \epsilon > 2 \) (\( \epsilon^* > 2 \)).

In the case of consumption externalities, a production tax or an export tax may be more effective than an emission tax. If both \( \epsilon < 2 \) and \( \epsilon^* < 2 \) hold, a production tax lowers consumption in both countries and hence necessarily decreases negative externalities. If \( C'' < 0 \), an export tax decreases consumption in both countries. If \( C'' > 0 \), an export tax reduces foreign emissions but raises the domestic ones. If the foreign reduction is large enough, the total negative externalities in the domestic country become smaller.

We thus have the following proposition.

**Proposition 3.** When production generates emissions, an increase in the consumption (export) tax reduces the total emissions if the foreign (domestic) MR curve is downward-sloping and hence may be more effective than that of an emission tax. When consumption generates emissions, an increase in the production tax decreases the total emissions if both domestic and foreign MR curves are downward-sloping and hence may be more effective than that of the emission tax.
3 Production in the Foreign Country

In this section, we examine the case where the monopolist is producing in the foreign country. We specifically analyze the effects of an import tariff, $\tau$, imposed by the domestic government. The profit of the monopolist is modified as follows

$$\Pi(x, x^*; \tau) = [P(x) - \tau]x + P^*(x^*)x^* - C^*(x + x^*),$$

(21)

where $C^*(\cdot)$ is the cost function when producing abroad. The first-order conditions become

$$\frac{\partial \Pi}{\partial x} = (P - \tau) + P'x - C'' = 0,$$

(22)

$$\frac{\partial \Pi}{\partial x^*} = P^* + P'^*x^* - C'' = 0.$$

(23)

The second-order sufficient conditions are given by

$$2P' + P''x - C''' = P'(2 - \epsilon) - C'' < 0,$$

(24)

$$2P'^* + P''^*x^* - C''' = P'^*(2 - \epsilon^*) - C'' < 0,$$

(25)

$$(2P' + P''x - C''')(2P'^* + P''^*x^* - C'') - (C''')^2 > 0.$$

(26)

We totally differentiate (22) and (23) and obtain:

$$\left( \frac{dx}{dx^*} \right) = \frac{1}{\Omega^*} \left( \begin{array}{c} 2P'^* + P''^*x^* - C''' \\ C'''' \\ 2P' + P''x - C'''' \end{array} \right) \left( \begin{array}{c} 1 \\ C'''' \\ 0 \end{array} \right) d\tau,$$

where $\Omega^* \equiv (2P' + P''x - C''')(2P'^* + P''^*x^* - C'') - (C''')^2 > 0$. Thus, the effects of a change in $\tau$ on the supplies are given by

$$\frac{dx}{d\tau} = \frac{2P'^* + P''^*x^* - C'''}{\Omega^*} < 0, \quad \frac{dx^*}{d\tau} = \frac{C'''}{\Omega^*}, \quad \frac{d(x + x^*)}{d\tau} = \frac{P''(2 - \epsilon^*)}{\Omega^*}.$$

(27)

A tariff necessarily decreases the domestic imports, but decreases the total output if and only if $\epsilon^* < 2$.\(^8\) Thus, as long as $\epsilon^* > 2$, tariffs are ineffective to reduce negative externalities generated by production. Furthermore, in the case of consumption externalities, a tariff raises the foreign consumption and hence foreign emissions when $C''' > 0$. If this effect is

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\(^8\) As one may expect, the effects are similar to those of consumption taxes in the last section. The only difference is the cost functions. As long as the good is produced only abroad, tariffs are equivalent to consumption taxes if their rates are the same, i.e., $\tau = \theta$. Moreover, the effects of an emission tax in the case of consumption externalities are qualitatively the same with those of a tariff.
strong enough, a tariff raises negative externalities in the domestic country. We thus have the following proposition.

**Proposition 4.** When emissions are generated by production, an increase in the import tariff lowers the total emissions if and only if the foreign MR curve is downward-sloping. When emissions are generated by consumption, an increase in the tariff always lowers the domestic emissions but raises the foreign emissions if and only if the foreign MC curve is upward-sloping.

### 4 Concluding Remarks

We have examined whether various taxes such as emission taxes reduce emissions in an open economy. It has been shown that emission taxes may not be very effective to reduce negative externalities. The shapes of the MR and MC curves are the key to our results. The upward-sloping MR curve particularly plays a crucial role.

Although one may think that the increasing MR is peculiar, it has been recognized by a number of studies as an important possibility. These works show that the conditions giving rise to the upward-sloping MR curve are not stringent: any convex demand function that is consistent with the law of demand can have upward-sloping MR curve (see Formby et al., 1982). In addition, Walters (1980) shows the evidence of positively sloping marginal revenue in the pricing of the Port of Singapore. These suggest that there should be no theoretical and empirical reasons that rule out the possibility of upward MR curve, and, in fact, new results have been obtained by incorporating the possibility in various topics such as dual equilibria in monopolistic competition (Ireland, 1984), cartel and anti-trust (Smith et al., 1987), and third-degree price discrimination (Nahata et al., 1990).

To make our point as clearly as possible, we have presented a monopoly model. It is also possible to extend our analysis to an oligopoly framework. For example, introducing local firms into our model, we can still verify the existence of a Cournot equilibrium without declining MR and obtain similar results as long as the local firms serve only their own markets.

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9 A classical work by Robinson (1933) pointed out the possibility and its importance. However, it was in 1980's when the analysis of upward-sloping MR curve was actually developed. See Formby et al. (1982), Coughlin (1984), Beckman and Smith (1993), for example. In particular, using the elasticity of the slope of the demand curve, Coughlin (1984) shows that the MR function is increasing with respect to quantity if and only if the value of the elasticity is greater than two.
References


### Table 1
Effects of production tax

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### Table 2
Effects of consumption tax

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Effects of export tax

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Figure 1: $\Gamma'' > 0, \epsilon < 2$ and $\epsilon^* < 2$

Figure 2: $\Gamma'' > 0, \epsilon < 2$ and $\epsilon^* > 2$