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# Payment Uncertainty, the Division of Labor, and Productivity Declines in Great Depressions

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## Abstract

This paper proposes a simple model that formalizes a variant of Ohanian's (2001) conjecture explaining the productivity declines observed in the Great Depression.

If a large payment shock like an asset-price collapse renders many firms insolvent, other economic agents become exposed to a higher risk of not being paid (*payment uncertainty*). The payment uncertainty causes endogenous disruptions of the division of labor among firms, thereby lowering macroeconomic productivity. The prediction of the model is that productivity correlates negatively with bankruptcies and positively with the cost share of intermediate inputs, which is consistent with the data from depression episodes. The model implies that the so-called failure of macroeconomic policy in the United States during the early 1930s, when a rash of bankruptcies occurred, could actually have been welfare enhancing, since the quick exit of insolvent agents can resolve payment uncertainty quickly.

JEL Classification: D24, E13, E32, E65.

Key words: Payment uncertainty, productivity, division of labor, great depression.

# Payment uncertainty, the division of labor, and productivity declines in great depressions*

(Incomplete and preliminary)

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**Abstract**

This paper proposes a simple model that formalizes a variant of Ohanian's (2001) conjecture explaining the productivity declines observed in the Great Depression. If a large payment shock like an asset-price collapse renders many firms insolvent, other economic agents become exposed to a higher risk of not being paid (*payment uncertainty*). The payment uncertainty causes endogenous disruptions of the division of labor among firms, thereby lowering macroeconomic productivity.

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# 1 Introduction

The recently growing literature on great depressions,[1] in which the general equilibrium growth model is normally used as the paradigm of the analyses, shows that productivity declines were the primary contributor to the depressions in many cases.

It is shown that the declines in total factor productivity (TFP) explain almost all declines in output and investment during the 1929–33 period in the United States (Cole and Ohanian [1999]; Chari, Kehoe, and McGrattan [2002]). Hayashi and Prescott (2002) show that the protracted recession in Japan during the 1990s is consistent with a standard growth model, given the persistent slowdown of TFP growth. Bergoeing, Kehoe, Kehoe, and Soto (2002) find that the difference between the spectacular recovery of Chile and the long stagnation of Mexico subsequent to the external debt crises that hit both countries in the early 1980s is explained by the difference of recoveries of the productivity in both countries: The detrended TFP began to grow again quickly in Chile, while it continued to decline for a long period in Mexico. In Germany, Fisher and Hornstein (2002) find falling productivity was one of the most important contributors to the severe decline in the 1928–1932 period. Thus the literature has shown that general equilibrium growth theory can account for several depression episodes very well, taking productivity changes as given. The next question is what were the sources of the productivity declines in those depressions.

[1]The January 2002 issue of *The Review of Economic Dynamics*, edited by Timothy J. Kehoe and Edward C. Prescott, examines nine depression episodes from the perspective of growth theory. Kehoe and Prescott (2002) defines a great depression as a time period during which detrended output per working-age population falls at least 20% and the fall at least 15% must occur within the first decade of the depression. According to this definition, the current Japanese depression is not a great depression, but they argue that it will become a great one soon if the Japanese economy continues to stagnate.

This paper presents a simple theoretical model that possibly explains the productivity changes in those depression episodes. The model may be regarded as a formalized variant of a conjecture made by Ohanian (2001). Some empirical evidence is provided using data from the Great Depression in the United States and the 1990s in Japan.

The model focuses on the payment process in the economy, in which a firm buys an intermediate input, transforms it into the next-stage intermediate good, and sells it to another firm. The intermediate goods are passed down from firm to firm in the market and are finally transformed into consumer goods. At a certain time, an economy operating under this production technology is hit by an exogenous macroeconomic shock that disturbs the payment process and renders many firms insolvent. The shock can be interpreted as the emergence and subsequent collapse of asset-price bubbles or an abrupt change in exchange rates.

On the one hand, I postulate an assumption that seems fairly orthodox in economics (Smith [1776]; Becker and Murphy [1992]) but does not generally receive much attention in recent business cycle literature. This is that productivity is enhanced by the division of labor. In other words, even when the total amount of inputs does not change, output increases if the number of specialized firms that are involved in production increases. On the other, I assume that the increase of insolvent firms continuing to operate on the verge of bankruptcy makes a persistent "payment uncertainty," that there remains a positive probability that an insolvent firm will go bankrupt and will fail to pay its suppliers. An increase in the number of firms involved in the production process results in an increase in productivity through the division of labor, which enhances the profit of a firm, while it also causes a rise in payment uncertainty, which depresses the expected profit of the firm. Thus payment uncertainty causes an endogenous decline in productivity through firms' decision making over the division of labor.

It is assumed that government policies determine the bankruptcy rate, at which insolvent firms go bankrupt and are restructured to be healthy firms. Taking the bankruptcy rate as an exogenously determined policy variable, it is shown that the path of the model productivity with a high bankruptcy rate replicates that in the Great Depression, and that the productivity with a low bankruptcy rate replicates that of the 1990s in Japan. Since fast recovery of productivity enhances social welfare unless there are some labor or investment frictions, a government policy that allows quick bankruptcies of insolvent firms may be welfare enhancing. A numerical example shows that the presumed failure of US macroeconomic policy in the early 1930s, which led to a rash of bankruptcies, could have been welfare enhancing, and that the extraordinary fiscal and monetary expansion during the 1990s in Japan, which kept many insolvent firms afloat for a long period, could have been welfare reducing.

The decline in productivity in this paper is ultimately driven by disruption of the division of labor among firms. This mechanism is similar to Blanchard and Kremer (1997) and Kobayashi (2004a). The novelty of the present paper is that the endogenous disruption of the division of labor occurs through perfect competition in a frictionless market, in which firms trade intermediate goods as atomic sellers or buyers. The other papers assume that firms form a *team* for production explicitly, and the results in these papers therefore crucially depend on the specific assumptions on relationships among firms in a team. The results in the present paper do not depend on any strategic relationships among firms, and thus they hold in a more general environment.

Payment uncertainty associated with trade credits plays a central role in the disruption of the division of labor in this model. Kiyotaki and Moore (1997) and Calvo (2000) address the problem of trade credits, and they propose theoretical models in which a disruption of a chain of trade credits amplifies a recession. The basic structure of their models is that a liquidity shortage is amplified through disruption of the chain of credits, and it seems to explain investment friction in a temporary recession, although not productivity declines. In my model, payment uncertainty faces new creditors and suppliers, not incumbent creditors that roll over bad loans to insolvent firms. Lamont (1995) argues that investments and outputs may inefficiently shrink if new creditors have a risk of not being paid in full and incumbent creditors take most of the outputs. Although the Lamont model shares the thinking of my model in some respects, it does not show a decrease in productivity, while it does show that a decrease in investments can be caused by a demand shortage in an economy of monopolistic competition.

The paper is organized as follows. Section 2 briefly describes productivity changes in depressions. Section 3 develops the basic structure of the model. Section 4 introduces payment uncertainty subsequent to a macroeconomic shock and describes productivity declines under the payment uncertainty. Section 5 provides some empirical evidence, and Section 6 presents a summary and conclusions.

## 2 Productivity and payment uncertainty in depressions

Figures 1 and 2 show output, investment, labor, and TFP in the Great Depression in the United States and in the 1990s in Japan. The variables are detrended by the growth rate of TFP.

Figure 1. Macroeconomic variables in the Great Depression

Figure 2. Macroeconomic variables in the 1990s in Japan

The recent research on the Great Depression shows that a main cause of the severe output decline during the 1929–33 period was a productivity fall, and that financial constraints on investments might be insignificant (Chari, Kehoe, and McGrattan [2002]). Ohanian (2001) assesses five common explanations for TFP declines: changes in capacity utilization, labor quality, and production composition; labor hoarding; and increasing returns to scale. He points out that only 5 percentage points of the 18 percent TFP decline during 1929–33 is explained by them, and the other 13 points are left unexplained. He conjectures that declines in organization capital may be the explanation for the rest: The breakdowns in supplier-customer relationships could have required firms to search for new suppliers and customers or to adopt new technologies; the search activities could have lowered efficiency by reducing managers' labor inputs dedicated to organizing and planning production; and new technologies could have been used only inefficiently, since firms were inexperienced with these technologies. Thus, the primary cause of inefficiencies in Ohanian's conjecture is an increase in bankruptcies possibly associated with the asset-price collapse in 1929. Disruption of the division of labor due to payment uncertainty can be another potential mechanism for declines in organization capital. The rash of commercial and bank failures in the early 1930s indicates that economic agents then might have felt a very high risk of not being paid by their customers. The rise of payment uncertainty could have lead to endogenous shrinkage of the division of labor among economic agents through the mechanism described in the following sections.

To check the relevancy of the model, it is necessary to measure payment uncertainty during the Great Depression. Although there may be other economic variables that represent payment uncertainty, I chose the liabilities of failed businesses and suspended banks as the most straightforward proxy. Figure 3 shows the sum of liabilities of failed businesses and suspended bank deposits. This figure shows a surge of liabilities of failed businesses and banks in the 1930–33 period. There is a caveat for this figure. If bank liabilities are excluded, the surge in the early 1930s disappears. The level of liabilities of commercial failures in the Great Depression was similar to that in the 1921–22 depression. Thus the surge in Figure 3 mainly reflects the surge of bank failures in the Great Depression. Yet I believe that the sum of commercial and bank liabilities is a good proxy of payment uncertainty, since the banking sector was the core of the payment process and suspended bank liabilities were not protected then by the government or deposit insurance.

Figure 3. Sum of commercial failures and bank suspensions (United States)

As for the 1990s in Japan, Hayashi and Prescott (2002) show that the annual growth rate of TFP in the 1991–2000 period was 0.3% while that in the 1983–91 period was 3.7%, and stress that this sharp and persistent decline of TFP growth was the main cause of Japan's lost decade.[2] They also assert that financial frictions may not have been an important factor in Japan's recession, since they find that Japanese corporations were

[2]The TFP slowdown during the 1990s in Japan is confirmed by several authors. Jorgenson and Motohashi (2003) report that Japan's TFP growth was 1.01% in the 1975–90 period and 0.74% in the 1990–95 period. Miyagawa (2003) reports that TFP growth was 1.63% in the 1981–90 period and 0.84% in the 1991–99 period. The differences seem mainly due to differences in the definitions of capital inputs and the TFP factor.

able to find financing for investments in the 1990s.[3] The explanation in the present paper is consistent with their view that productivity slowed down even though investments were not constrained. Figure 4 shows the liabilities of failed firms in the Japanese economy.[4] After the asset-price bubbles burst, the level of liabilities rose to about 10 trillion yen on the average in the 1990s from about 3 trillion yen in the 1980s. This increase in bankruptcies indicates that economic agents began to feel more risk of not being paid by their customers in the 1990s.

Figure 4. Total liabilities and number of failed businesses (Japan)

Bergoeing et al. (2002) show that difference of productivity growth is the most important factor behind the sharp contrast between economic recovery in Chile and long stagnation in Mexico during the 1980s and the 1990s. They demonstrate that productivity growth was faster in Chile than in Mexico, and they hypothesize that Chile's earlier policy reforms in banking and bankruptcy procedures generated this difference. They report that the number of business bankruptcies in Chile surged in 1982–3, when TFP plummeted, and then quickly went back to normal in 1984, when TFP started to grow again. The path of the number of bankruptcy indicates the quick resolution of payment uncertainty in Chile, which may explain the recovery of TFP through the mechanism described in this paper.

[3] Hosono and Watanabe (2002) also confirm empirically that the liquidity constraint for Japanese firms did not become severer in the 1990s. Andolfatto (2003) also argues that monetary and financial problems in Japan's lost decade may be irrelevant to the output decline.

[4] The liabilities of failed banks are not included in Figure 4, since the Japanese government guaranteed all bank liabilities during the 1990s. I assume that liabilities of failed banks do not affect payment uncertainty, since the creditors are protected. (The rise in bankruptcies in the 1990s would be simply emphasized by the inclusion of bank failures in Figure 4, since there were very few bank failures until 1995.) Japan has had deposit insurance since 1971, and while the protection had a formal upper limit of 10 million yen per depositor, the public came to believe that the government would never let any bank fail and would guarantee all bank liabilities. In 1995, amid a deepening financial crisis, the government in fact made just such a guarantee of liabilities explicit.

# 3 Basic model

In this section, I describe the basic structure of the model and the competitive equilibrium in the case where the economy is not subject to payment uncertainty. In Section 4, I introduce payment uncertainty caused by an exogenous macroeconomic disturbance in the payment process.

## 3.1 Environment

The economy is comprised of consumers, firms, and a government. In this economy, time is discrete and continues from zero to infinity: $t = 0, 1, 2, \cdots, \infty$. There are infinitely many consumers who have identical preferences and maximize

$$\sum_{t=0}^{\infty} \beta u(c_t),$$

where $\beta$ $(0 < \beta < 1)$ is a discount factor, $u(c)$ is an increasing and concave function, and $c_t$ is the consumption in period $t$. The measure of the consumers is normalized to one. There are also infinitely many firms with measure one, who are risk-neutral and maximize profits. Only firms, not consumers, have access to the production technology described below.

## 3.2 Production technology

Consumers are endowed with a nondepletable asset (land), the total supply of which is $K$, at the beginning of period 0. They are also endowed with $L$ units of labor at the beginning of each period $t$ $(t = 0, 1, 2, \cdots)$. I assume that there are $\overline{n}$ different goods indexed by $i \in \{1, 2, \cdots, \overline{n}\}$, where $\overline{n}$ is a large integer, and that consumers can consume good-1 only, while good-$i$ $(i = 2, 3, \cdots, \overline{n})$ is an intermediate good that can be used for production of good-$(i-1)$ but cannot be consumed by consumers. If good-$i$ $(i = 2, 3, \cdots, \overline{n})$ is produced during period $t$ and is not used as an input for producing good-$(i-1)$ by the following technology (2), then good-$i$ perishes at the end of period $t$ without being consumed or stored.

In each period $t$, firms produce goods using one of the following two production technologies. Good-$i$ can be produced from capital (i.e., land) and labor by

$$y_i = \phi(i) k_{i+1}^{\theta} l_{i+1}^{1-\theta}, \tag{1}$$

where $\phi(i)$ is the productivity for good-$i$ production, $\theta$ $(0 < \theta < 1)$ is a parameter, $y_i$ is the output of good-$i$, $l_{i+1}$ is the labor input, $k_{i+1}$ is the capital input of land. I assume that $\phi'(i) > 0$ and $\phi''(i) < 0$. Good-$i$ $(i \in \{1, 2, \cdots, \overline{n} - 1\})$ can also be produced from the intermediate input of good-$(i + 1)$, and labor and capital inputs by

$$y_i = A y_{i+1}^{\alpha} k_{i+1}^{(1-\alpha)\theta} l_{i+1}^{(1-\alpha)(1-\theta)}, \tag{2}$$

where $A$ is a productivity parameter, and $y_{i+1}$ is the intermediate input of good-$(i + 1)$.

As shown in Lemma 1 below, in the equilibrium, there exists $n \geq 1$ such that good-$n$ is produced by technology (1) and good-$i$ $(1 \leq i < n)$ is produced by technology (2). From the assumption that $\phi(n)$ is increasing in $n$ and good-$(i + 1)$ is used only for production of good-$i$, it is interpreted that $n$ is the degree of the division of labor, and that the division of labor enhances productivity $\phi(n)$.

I assume a technological constraint that if a firm is to produce good-$i$ by technology (2), it must buy the intermediate input (good-$(i + 1)$) from other firms; it cannot conduct production of the next-stage good using its own output. This constraint can be interpreted as saying that a firm specializes in the production of only one kind of good during period $t$ and cannot use its output for the next-stage production during the same period. This constraint makes the payment process relevant to aggregate productivity.

### 3.3 Firm's problem

Since this paper focuses on a decentralized market economy, it is assumed that all firms are price-takers. At the beginning of period $t$, a firm chooses the good to produce (good-$i$), production technology ([1] or [2]), and the amounts of inputs, in order to maximize its profit, given period-$t$ prices:$\{R, P_1, P_2, \cdots, P_{\overline{n}}\}$, where $P_i$ is the price of good-$i$ in period $t$ and $R$ is the rent of capital (i.e., land) in period $t$. Labor input is taken as the numeraire, and thus the wage rate is set at one. To simplify notation, I omit time subscript $t$ when there is no possibility of confusion. Given prices, a firm that produces good-$i$ from land and labor solves the profit maximization and obtains the following profit:

$$\pi_\phi(i) \equiv \max_{k_{i+1},l_{i+1}} P_i\phi(i)k_{i+1}^\theta l_{i+1}^{1-\theta} - Rk_{i+1} - l_{i+1}. \tag{3}$$

The firm buys labor $l_{i+1}$ from consumers and rents land $k_{i+1}$ from (other) consumers too. Given prices, a firm that produces good-$i$ using technology (2) obtains the following profit:

$$\pi(i) \equiv \max_{y_{i+1},k_{i+1},l_{i+1}} P_i A y_{i+1}^\alpha k_{i+1}^{(1-\alpha)\theta} l_{i+1}^{(1-\alpha)(1-\theta)} - P_{i+1}y_{i+1} - Rk_{i+1} - l_{i+1}. \tag{4}$$

Since the firm cannot use its own output in the next-stage production, it buys good-$(i+1)$ from other firms and rents land $k_{i+1}$ and buys labor $l_{i+1}$ from consumers. In any case, a firm must buy inputs from other firms and consumers, and sell the output to other firms and other consumers. Since there are continuously and indefinitely many consumers and firms of measure one in this economy, I can assume the following for trading in the market:

**Assumption 1** *In each period $t$, firms buy inputs from and sell outputs to economic agents whom they randomly encounter in the market. The random matching is efficient in the sense that all markets clear every period without any friction.*

This assumption is crucial to generate payment uncertainty under an environment where many firms operate on the verge of bankruptcy (see Section 4). In order to simplify the analysis, I assume the following for the firm's choice of technology:

**Assumption 2** *If $\pi_\phi(i) = \pi(i)$, a firm that wants to produce good-i will choose technology (2).*

The following lemma holds in the equilibrium where firms are price-takers and earn zero profits.

**Lemma 1** *Given prices $\{R, P_1, P_2, \cdots, P_{\overline{n}}\}$, define $n$ by*

$$n = Max[\arg\max_i P_i\phi(i)], \tag{5}$$

*where $Max[\Omega]$ is the maximum element of the set of integers $\Omega$. In the equilibrium where prices are given as $\{R, P_1, P_2, \cdots, P_{\overline{n}}\}$ and price-taking firms earn zero profits, good-$n$ is produced from labor and land using technology (1), good-$i$ ($1 \leq i < n$) is produced from good-$(i+1)$ using technology (2), and good-$i$ ($i > n$) is not produced.*

(Proof) It is shown as follows that $\pi_\phi(n) \geq \pi_\phi(i)$, for all $i \neq n$. Suppose $(k^*_{i+1}, l^*_{i+1})$ satisfies $\pi_\phi(i) = P_i\phi(i)(k^*_{i+1})^\theta(l^*_{i+1})^{1-\theta} - Rk^*_{i+1} - l^*_{i+1}$. Since $P_n\phi(n) \geq P_i\phi(i)$ by definition, it follows that $\pi_\phi(n) \geq P_n\phi(n)(k^*_{i+1})^\theta(l^*_{i+1})^{1-\theta} - Rk^*_{i+1} - l^*_{i+1} \geq P_i\phi(i)(k^*_{i+1})^\theta(l^*_{i+1})^{1-\theta} - Rk^*_{i+1} - l^*_{i+1} = \pi_\phi(i)$.

The competition among firms implies that $\pi_\phi(n) = 0 \geq \pi_\phi(i)$ for all $i \neq n$ in the equilibrium. This fact and Assumption 2 imply that only good-$n$ is produced from labor and land using technology (1). The other goods must be produced from intermediate goods using technology (2), if they are produced at all.

It is easily shown that good-$i$ ($n < i \leq \overline{n}$) is not produced by induction: Since good-$\overline{n}$ is not produced from labor and land, it is not produced at all. For $i > n$, if good-$i$ is produced, it must be produced from good-$(i+1)$; But since good-$\overline{n}$ is not produced, backward induction implies that good-$i$ is not produced for all $i > n$.

It is also easily shown that good-$i$ for $1 \leq i < n$ is produced: Suppose that good-$i$ is not produced in the equilibrium. Since good-$(i+1)$ cannot be consumed, it perishes at the end of period $t$, implying that good-$(i+1)$ is not produced if good-$i$ is not produced. The contraposition of this statement implies that if good-$(i+1)$ is produced in the equilibrium, good-$i$ is also produced. Since good-$n$ is produced, it is shown by induction that good-$i$ ($1 \leq i < n$) is also produced.

Therefore, since it is shown that good-$i$ ($1 \leq i < n$) is produced in the equilibrium and it is not produced using technology (1), it must be produced from good-$(i+1)$ using technology (2). (End of proof)

### 3.4 Consumer's problem

In the competitive equilibrium, consumers solve the following utility maximization problem:

$$\max_{c_t, k_{t+1}} \sum_{t=1}^{\infty} \beta^t u(c_t) \tag{6}$$

subject to

$$P_{1,t}c_t + Q_t k_{t+1} \leq R_t k_t + Q_t k_t + L,$$

and $k_0 = K$, given prices $\{R_t, P_{1,t}, Q_t\}_{t=0}^{\infty}$, where $c_t$ is the consumption in period $t$, $k_t$ is the land holding at the beginning of period $t$, $P_{1,t}$ is the price of good-1, $Q_t$ is the price of land, and $R_t$ is the rent of capital during period $t$.

### 3.5 Payment process

During period $t$, firms buy and sell intermediate goods, and consumers sell labor and rent land to firms and buy consumer good (good-1) from firms. I assume that all transactions during a period are done by trade credits, which are settled at the end of the period. In Section 4, I describe bankruptcies and defaults on trade credits after an exogenous payment shock hits the economy.

### 3.6 Equilibrium

The competitive equilibrium is defined as follows:

**Definition 1** *The competitive equilibrium is a set of prices $\{P_{1,t}, P_{2,t}, \cdots, P_{\overline{n},t}, Q_t, R_t\}_{t=0}^{\infty}$ and allocations $\{n_t; y_{1,t}, y_{2,t}, \cdots, y_{n_t,t}; k_t, k_{2,t}, k_{3,t}, \cdots, k_{n_t+1,t}; l_{2,t}, l_{3,t} \cdots, l_{n_t+1,t}\}_{t=0}^{\infty}$ that satisfies the following conditions (a) – (h):*
*(a) Given prices, $n_t$ is the solution to (5); (b) Given prices, $c_t = y_{1,t}$ and $k_{t+1}$ solve the consumer's problem (6); (c) Given prices, $l_{n_t+1,t}$ and $k_{n_t+1,t}$ solve the firm's problem (3) for $i = n_t$, and $\pi_\phi(n_t) = 0$; (d) Given prices, $\{y_{i+1,t}, k_{i+1,t}, l_{i+1,t}\}$ $(1 \le i < n_t)$ solve the firm's problem (4), and $\pi(i) = 0$; (e) $k_t = \sum_{i=1}^{n_t} k_{i+1,t} = K$; (f) $\sum_{i=1}^{n_t} l_{i+1,t} = L$; (g) $y_{n_t,t} = \phi(n_t) k_{n_t+1,t}^{\theta} l_{n_t+1,t}^{1-\theta}$; and (h) for $1 \le i < n_t$, $y_{i,t} = A y_{i+1,t}^{\alpha} k_{i+1,t}^{(1-\alpha)\theta} l_{i+1,t}^{(1-\alpha)(1-\theta)}$.*

In what follows in this subsection, I omit the time subscript for simplicity, since the competitive equilibrium is a steady state equilibrium as shown below.

The equilibrium prices and allocations are characterized as follows. The first-order conditions (FOCs) of the firm's problem (4) are

$$\alpha P_{i-1} A \left(\frac{k_i}{y_i}\right)^{(1-\alpha)\theta} \left(\frac{l_i}{y_i}\right)^{(1-\alpha)(1-\theta)} = P_i, \tag{7}$$

$$(1-\alpha)(1-\theta) P_{i-1} A \left(\frac{y_i}{l_i}\right)^{\alpha} \left(\frac{k_i}{l_i}\right)^{(1-\alpha)\theta} = 1, \tag{8}$$

$$(1-\alpha)\theta P_{i-1}A\left(\frac{y_i}{k_i}\right)^{\alpha}\left(\frac{l_i}{k_i}\right)^{(1-\alpha)(1-\theta)} = R. \tag{9}$$

The FOCs for (3) are

$$(1-\theta)P_n\phi(n)\left(\frac{k_{n+1}}{l_{n+1}}\right)^{\theta} = 1, \tag{10}$$

$$\theta P_n\phi(n)\left(\frac{l_{n+1}}{k_{n+1}}\right)^{1-\theta} = R. \tag{11}$$

These FOCs imply $\frac{l_i}{k_i} = \frac{(1-\theta)R}{\theta} = \frac{L}{K}$ for all $i \in \{2,3,\cdots,n+1\}$, $\frac{k_{i+1}}{k_i} = \frac{l_{i+1}}{l_i} = \alpha$ for $i \in \{2,3,\cdots,n-1\}$, and $\frac{k_{n+1}}{k_n} = \frac{l_{n+1}}{l_n} = \frac{\alpha}{1-\alpha}$. Thus, since $\sum_{i=1}^n k_{i+1} = K$ and $\sum_{i=1}^n l_{i+1} = L$, it is easily shown that, given $n$,

$$\begin{aligned} &\tfrac{k_i}{K} = \tfrac{l_i}{L} = (1-\alpha)\alpha^{i-2}, \quad \text{for } 2 \le i \le n, \\ &\tfrac{k_{n+1}}{K} = \tfrac{l_{n+1}}{L} = \alpha^{n-1}. \end{aligned} \tag{12}$$

Equations (7)–(9) also imply that

$$P_i^{\alpha}\left(\frac{L}{K}\right)^{(1-\alpha)\theta} = \alpha^{\alpha}(1-\alpha)^{1-\alpha}AP_{i-1}.$$

For simplicity of exposition, I assume the following for the parameter values:

$$(1-\alpha)^{1-\alpha}\alpha^{\alpha}A = 1. \tag{13}$$

Under this assumption, the equilibrium price of good-$i$ must satisfy

$$\ln P_i = \frac{\alpha^{-i+1}-1}{1-\alpha}\ln\gamma + \alpha^{-i+1}\ln P_1, \tag{14}$$

where $\gamma = \left(\frac{K}{L}\right)^{(1-\alpha)\theta}$, as long as good-$i$ is produced. Thus the problem (5) that determines $n$ is written as

$$n = \arg\max_i \frac{\alpha^{-i+1}-1}{1-\alpha}\ln\gamma + \alpha^{-i+1}\ln P_1 + \ln\phi(i).$$

The solution $n$ satisfies the FOC for the above problem:

$$-\frac{\alpha^{-n+1}}{1-\alpha}\ln\gamma\ln\alpha - \alpha^{-n+1}\ln\alpha\ln P_1 + \frac{\phi'(n)}{\phi(n)} = 0. \tag{15}$$

Given $n$, $P_n$, $k_{n+1}$, and $l_{n+1}$, the equilibrium rent $R$ is determined by (11). The equilibrium values of $n$ and $P_1$ are determined by (15) and (10). From these equations, it is shown that $n$ is uniquely determined by

$$\frac{\phi'(n)}{\phi(n)} = -\{\ln(1-\theta) + \ln \phi(n)\} \ln \alpha. \tag{16}$$

In this equation and in what follows, I treat $n$ as a real number, in order to simplify the exposition. The restriction that $n$ is an integer can be easily incorporated with minor adjustments in the analysis.

The land price $Q_t$ is determined by the FOC for the consumer's problem: $Q_t = \frac{\beta u'(c_{t+1})}{u'(c_t)} \frac{P_{1,t}}{P_{1,t+1}} \{R_t + Q_{t+1}\}$. In the equilibrium, $Q_t$ is given by $Q_t = \frac{\beta}{1-\beta} R$. Thus, the competitive equilibrium is completely characterized.

The output of consumer goods in each period in the competitive equilibrium is

$$y_1 = A(n) K^{\theta} L^{1-\theta},$$

where

$$\ln A(n) \equiv \alpha^{n-1} \ln \phi(n) - (1 - \alpha^{n-1}) \ln(1-\theta).$$

The number $n$ can be seen as the degree of the division of labor in this economy, and $A(n)$ the aggregate productivity. It is easily shown that $A(n)$ is maximized by the equilibrium value of $n$ that satisfies (16). Figure 5 plots $A(n)$ as a function of $n$, given that $\phi(n) = n^a$, $\alpha = 0.9$, $a = 0.1$, and $\overline{n} = 1000$. The equilibrium value of $n$ is 44.

Figure 5: Aggregate productivity

It is also easily confirmed that the allocation of the competitive equilibrium is the social optimum. The social planner's problem is

$$\max_{n, k_i} y_1 \tag{17}$$

subject to

$$\begin{cases} y_n = \phi(n) k_{n+1}^{\theta} l_{n+1}^{1-\theta}, \\ y_i = A y_{i+1}^{\alpha} k_{i+1}^{(1-\alpha)\theta} l_{i+1}^{(1-\alpha)(1-\theta)}, \quad \text{for } 1 \leq i < n, \\ k_2 + k_3 + \cdots + k_{n+1} = K, \\ l_2 + l_3 + \cdots + l_{n+1} = L. \end{cases} \tag{18}$$

It is easily shown that the solution to this problem ($k_i$, $l_i$, and $n$) satisfy (12) and (16).

I make a digression before ending this section. The result in this section that the division of labor is endogenously limited in a perfectly competitive economy is interesting in itself. This is because Adam Smith's famous theorem, "the division of labor is limited by the extent of the market," has been long regarded as holding not in perfect competition but only in a setting with imperfect competition or market frictions (see, for example, Stigler [1951], Baumgardner [1988], Kim [1989], and Kobayashi [1998]). Since $\phi(n)$ is increasing in $n$, one may expect that perfect competition in a frictionless economy will realize the maximum value of $n$ (i.e., $\overline{n}$), which is not the case in the competitive equilibrium of this model. Thus it can be said that this model suggests one way of reconciliation between competitive price theory and Smith's theorem. As the degree of the division of labor ($n$) increases, the amount of factor inputs to production of one intermediate goods decrease as a result of the frictionless market competition. And thus, aggregate productivity $A(n)$ becomes quite different from $\phi(n)$, and it decreases if $n$ becomes too large.

# 4 Productivity declines under payment uncertainty

This section describes the productivity changes subsequent to a large disruption in the payment process, such as the emergence and collapse of asset-price bubbles.

## 4.1 Payment shock

In this paper, it is assumed that there are no real shocks on productivity or preferences. I focus on an exogenous payment shock that renders some firms insolvent. Prior to the shock, the model assumes that while there are trade credits between firms and consumers, there are no nonperforming loans from consumers to firms. After the shock, as a result of disruption of the payment process, nonperforming loans owed by firms to consumers are generated instantaneously, and they are far larger than ordinary trade credits. I assume that these nonperforming loans emerge at a portion of firms with measure $z$ $(0 < z < 1)$

at the end of period 0, after payoffs concerning the activities in period 0 are over. Since the firms have no assets after the payoffs, they are clearly insolvent.

**Assumption 3** *An insolvent firm owes a nonperforming loan to only one consumer. The nonperforming loan to the firm is observable only to the firm itself and the creditor. An insolvent firm has the same production technology as other firms; therefore, other consumers or other firms cannot distinguish it from a healthy firm.*

Note that an insolvent firm is not inefficient *per se* in terms of production technology; the only difference is that the creditor (consumer) has too large a claim on the prospective assets of the insolvent firm. The reason why nonperforming loans are generated is not specified in this paper. I simply assume that some exogenous shock (e.g., the emergence and bursting of asset-price bubbles) made some firms insolvent. I denote the amount (measured in terms of labor units) of the nonperforming loan to each insolvent firm at the beginning of period 1 by $N$. I assume that $N$ is very large (see Section 4.3). An insolvent firm has no assets corresponding to the liability $N$. Consumers have nonperforming loans $Nz$ to $z$ firms as their assets in addition to landholdings at the beginning of period 1.

## 4.2 Government policy on insolvent firms

Since the consumers' nonperforming loans $Nz$ are not backed by firms' assets, the insolvent firms would go bankrupt immediately. But distortion-producing government policies or inefficient economic institutions do not allow all insolvent firms to go bankrupt immediately. I assume that there is a parameter $x$ ($0 < x < 1$), which is determined by government policy or institutional factors, that represents the rate of bankruptcy. Denote the measure of the remaining insolvent firms at the beginning of period $t$ by $z_t$. I assume the following.

**Assumption 4** *During period $t$, the remaining insolvent firms of measure $z_t$ conduct production activities and provide and receive trade credits, just like other healthy firms. Among $z_t$ insolvent firms, $xz_t$ of them go bankrupt at the end of period $t$, after all trade credits of period $t$ are made but before any of them are settled.*

Therefore, $z_t$ evolves by

$$z_{t+1} = (1 - x)z_t,$$

where $z_1 = z$. I assume that the government cannot set $x$ at zero, but there is a lower bound $\underline{x}$ $(> 0)$ such that $\underline{x} \leq x < 1$.

We can interpret the paths of the bankruptcies in the United States (Figure 3) and Chile as corresponding to the case of a high $x$, while that in Japan (Figure 4) may correspond to the case of a low $x$.

In order to simplify the calculation, I assume that at the beginning of period $t + 1$, bankrupt firms of measure $xz_t$ are replaced by newly established firms of the same number, and thus the total measure of the firms remains constant as one.

### 4.3 Bankruptcies

When an insolvent firm goes bankrupt, the incumbent creditor (consumer) of the nonperforming loan can seize all the assets of the firm, and all payments of trade credits from the firm to other creditors are cancelled. To make clear the meaning of this assumption, let us consider the case where an insolvent firm continues to operate and conducts production activities during period $t$ and goes bankrupt at the end of period $t$. We have the following from Assumption 1 and the fact that the insolvency of a firm is not observable. At the settlement time, the firm has as its assets the trade credits to its customers, while it has as its liabilities the nonperforming loan $N$ from a consumer and the account payable to its suppliers of the intermediate good and to other consumers for the rent of land and the wage. If this firm goes bankrupt at the end of period $t$, the creditor that provided $N$ seizes all the assets and cancels payment to the debtor's other creditors. Caveats for this assumption on bankruptcy follow. One may consider that the assumption that the incumbent creditor takes everything is too strong as a model of bankruptcy. But in practice, it usually happens that suppliers fail to collect on their claims. In this paper, I made the above extreme assumption in order to focus on the uncertainty that suppliers feel when they sell goods to (possibly insolvent) firms. Another caveat is that I implicitly assume that $N$ is always larger than the amount of assets that the firm holds when it goes bankrupt. This assumption is just for simplicity of calculation. Otherwise the incumbent creditor seizes only $N$ if the assets of the firm exceed it, and the other creditors of the trade credits get paid partially, making the analysis more complicated without making substantial changes in the results.

### 4.4 Firm's problem

If insolvent firms of measure $xz_t$ go bankrupt at the end of period $t$, there emerges a risk that a payment will not be settled. Since Assumption 3 implies that a seller cannot tell whether the buyer is an insolvent firm or not, sellers become constantly exposed to a positive probability of not being paid. Therefore, prices of intermediate goods are distorted by this payment uncertainty. Consider a (healthy) firm that produces and sells good-$i$ ($2 \leq i \leq n_t - 1$) in period $t$, where only good-$n_t$ is produced from labor and capital. A buyer will go bankrupt and fail to pay with probability $xz_t$. In order to simplify the analysis, I assume the following:

**Assumption 5** *A firm that produces good-i divides its output into infinitesimally small fractions and sells them to infinitely many firms. The law of large numbers implies that the firm obtains the revenue* $(1 - xz_t)P_{i,t}y_{i,t}$ *with probability one, where* $y_{i,t}$ *is the total output.*

Under this assumption, the profit of the firm is

$$\pi(i; xz_t) \equiv \max_{y_{i+1}, k_{i+1}, l_{i+1}} (1 - xz_t)P_{i,t}Ay_{i+1}^{\alpha}k_{i+1}^{(1-\alpha)\theta}l_{i+1}^{(1-\alpha)(1-\theta)} - P_{i+1,t}y_{i+1} - R_t k_{i+1} - l_{i+1}. \tag{19}$$

And since consumers do not go bankrupt, the profit of the firm that sells good-1 to consumers is $\pi(1; 0)$.

Firms choose $n$, $k_{n+1}$, and $l_{n+1}$ to maximize the profit $\pi_\phi(n; xz_t)$, where

$$\pi_\phi(n; xz_t) \equiv \max_{k_{n+1}, l_{n+1}} (1 - xz_t)P_{n,t}\phi(n)k_{n+1}^{\theta}l_{n+1}^{1-\theta} - R_t k_{n+1} - l_{n+1}. \tag{20}$$

By the similar arguments as the proof of Lemma 1, it is easily shown that $n = \arg\max_i q_t P_i \phi(i)$, where $q_t = 1 - xz_t$. The resource constraints ($\sum_{i=1}^{n} k_{i+1} = K$ and $\sum_{i=1}^{n} l_{i+1} = L$) and the FOCs of (19) and (20) imply

$$\begin{aligned} \tfrac{k_i}{K} &= \tfrac{l_i}{L} = \tfrac{(1-q_t\alpha)q_t^{i-2}\alpha^{i-2}}{1+\frac{1-q_t}{1-\alpha}q_t^{n-1}\alpha^n}, \quad \text{for } 2 \leq i \leq n, \\ \tfrac{k_{n+1}}{K} &= \tfrac{l_{n+1}}{L} = \tfrac{(1-q_t\alpha)q_t^{n-1}\alpha^{n-1}}{1-\alpha+(1-q_t)q_t^{n-1}\alpha^n}, \end{aligned} \tag{21}$$

where $q_t = (1 - xz_t)$. Using (13), the FOCs also imply

$$P_{i+1,t} = (\gamma q_t)^{\alpha^{-1}} P_{i,t}^{\alpha^{-1}}, \quad \text{for } 2 \leq i < n_t,$$

and $P_{2,t} = \gamma^{\alpha^{-1}} P_{1,t}^{\alpha^{-1}}$. Therefore, in the equilibrium where the profit-maximizing firms earn zero profits, prices of intermediate goods are determined by

$$P_{i,t} = \gamma^{\frac{\alpha^{1-i}-1}{1-\alpha}} q_t^{\frac{\alpha^{2-i}-1}{1-\alpha}} P_{1,t}^{\alpha^{-i+1}}, \quad \text{for } 2 \leq i < n_t.$$

Since $n = \arg\max_i q_t P_i \phi(i)$, the above equation implies

$$n = \arg\max_i \frac{\alpha^{1-i}-1}{1-\alpha} \ln\gamma + \frac{\alpha^{2-i}-\alpha}{1-\alpha} \ln q_t + \alpha^{-i+1} \ln P_{1,t} + \ln\phi(i).$$

The FOC implies

$$-\frac{\alpha^{1-n}}{1-\alpha} \ln\alpha \ln\gamma - \frac{\alpha^{2-n}}{1-\alpha} \ln\alpha \ln q_t - \alpha^{-n+1} \ln\alpha \ln P_{1,t} + \frac{\phi'(n)}{\phi(n)} = 0. \tag{22}$$

The FOC with respect to $l_{n+1}$ of (20) implies

$$\ln(1-\theta) + \frac{\alpha^{2-n}-\alpha}{1-\alpha} \ln q_t + \frac{\alpha^{1-n}-1}{1-\alpha} \ln\gamma + \alpha^{1-n} \ln P_{1,t} + \ln\phi(i) + \theta \ln\left(\frac{K}{L}\right) = 0. \tag{23}$$

Since $n$ must be no less than one, its equilibrium value is determined as follows: $n_t = \max\{n'_t, 1\}$, where $n'_t$ is the solution to the following equation that is derived from (22) and (23):[5]

$$\frac{\phi'(n)}{\phi(n)} = -\left\{\ln(1-\theta) - \frac{\alpha}{1-\alpha} \ln(1-xz_t) + \ln\phi(n)\right\} \ln\alpha. \tag{24}$$

Since $\ln(1 - xz_t) < 0$ and $\phi'(n)$ is decreasing in $n$, it is easily shown that

$$n_t < n^*,$$

[5]Here I treat $n$ as a real number to simplify the exposition, as in the previous section. The restriction that $n$ is an integer can be incorporated with some trivial adjustments.

where $n^*$ is the solution to (16). Therefore, when economic agents face payment uncertainty, the degree of the division of labor declines. Given $n_t$, the FOC with respect to $k_{n+1}$ of (20) determines the rent $R_t$ by

$$R_t = \theta q_t P_n \phi(n) \left( \frac{l_{n+1}}{k_{n+1}} \right)^\theta .$$

The conditions (21) and resource constraints (18) imply that the final output of good-1 (i.e., consumption $c_t$) in the equilibrium is

$$y_{1,t} = c_t = \Phi(q_t) K^\theta L^{1-\theta}, \tag{25}$$

where

$$\ln \Phi(q_t) = \frac{\alpha - \alpha^{n_t}}{1-\alpha} \ln q_t + \ln \frac{1 - q_t \alpha}{1 - \alpha + (1-q_t) q_t^{n_t - 1} \alpha^{n_t}} + \ln A(n_t).$$

Since $A(n_t)K^\theta L^{1-\theta}$ is the value of output that is maximized by the social planner's problem (17) with $n$ fixed at $n_t$, the equilibrium productivity $\Phi(q_t)$ under the payment uncertainty must satisfy $\Phi(q_t) \leq A(n_t)$.

### 4.5 Consumer's problem

The consumer's problem after the payment shock is as follows:

$$\max_{c_t, k_{t+1}} \sum_{t=1}^{\infty} \beta^t u(c_t)$$

subject to

$$P_{1,t} c_t + Q_t k_{t+1} \leq (1 - x z_t) R_t k_t + (1 - x z_t) L + Q_t k_t + T_t,$$

given prices ($P_{1,t}$, $Q_t$, $R_t$), the lump-sum gain ($T_t$), and the initial value $k_0 = K$. The lump-sum gain $T_t$, the amount of which is specified below, is the gains that the consumers (i.e., the incumbent creditors of nonperforming loans to insolvent firms) obtain by asset-seizures in bankruptcies of $x z_t$ insolvent firms. Moreover, the consumers have as their assets the nonperforming loans $N z_t$, which are nontradable and decrease over time.[6]

[6] We can posit that nonperforming loans are tradable and let them enter into the consumer's budget constraint by assuming that for some political reasons the government guarantees growth of the nonperforming loans at the market rate of interest. I did not choose this modification, since it would complicate the analysis without changing the results essentially. See Kobayashi (2004b) for the model with this setting.

The gap $Nz_{t+1} - Nz_t = -Nxz_t$ is recognized as a lump-sum loss of bankruptcies of $xz_t$ insolvent firms incurred by the consumers (i.e., the creditors of the nonperforming loans).

I assume for simplicity of calculation that consumers obtain the rent $(1-xz_t)R_t k_t$ and the wage $(1-xz_t)L$ deterministically by, say, forming fair insurance among themselves. As equation (25) shows, the equilibrium allocations are $k_t = K$ and $c_t = \Phi(1-xz_t)K^\theta L^{1-\theta}$. The asset price $Q_t$ is determined by

$$Q_t = \frac{\beta u'(c_{t+1})P_{1,t}}{u'(c_t)P_{1,t+1}}\{(1-xz_{t+1})R_{t+1} + Q_{t+1}\}.$$

The consumers' gain of asset-seizure $T_t$ is given by

$$T_t = xz_t \left[ P_{1,t}y_{1,t} + \sum_{i=2}^{n}(1-xz_t)P_{i,t}y_{i,t} \right]. \tag{26}$$

Firms' profit maximization implies that $P_{1,t}y_{1,t} = P_{2,t}y_{2,t} + R_t k_{2,t} + l_{2,t}$, $(1-xz_t)P_{i,t}y_{i,t} = P_{i+1,t}y_{i+1,t} + R_t k_{i+1,t} + l_{i+1,t}$ for $2 \le i < n$, and $(1-xz_t)P_{n,t}y_{n,t} = R_t k_{n+1,t} + l_{n+1,t}$. These conditions and (26) imply

$$T_t = \sum_{i=2}^{n} xz_t P_{i,t}y_{i,t} + xz_t R_t k_t + xz_t L,$$

where $k_t = k_{2,t} + k_{3,t} + \cdots + k_{n+1,t}$. Using $P_{2,t}y_{2,t} = P_{1,t}y_{1,t} - R_t k_{2,t} - l_{2,t}$, $xz_t P_{i,t}y_{i,t} = P_{i,t}y_{i,t} - P_{i+1,t}y_{i+1,t} - R_t k_{i+1,t} - l_{i+1,t}$ for $2 \le i < n$, and $xz_t P_{n,t}y_{n,t} = P_{n,t}y_{n,t} - R_t k_{n+1,t} - l_{n+1,t}$, we obtain

$$T_t = P_{1,t}y_{1,t} - (1-xz_t)R_t k_t - (1-xz_t)L.$$

Therefore, the equilibrium allocations $k_t = K$ and $c_t = y_{1,t} (= \Phi(1-xz_t)K^\theta L^{1-\theta})$ obviously satisfy the consumer's budget constraint, given the above $T_t$.

### 4.6 Share of intermediate inputs

In order to conduct an empirical test later, it is useful to clarify the prediction of the model concerning the share of intermediate inputs in total cost. The share ($s_M$) is defined by

$$s_M = \frac{\sum_{i=2}^{n} P_i y_i}{\sum_{i=1}^{n} P_i y_i}.$$

In the equilibrium, $P_1y_1 = P_2y_2 + Rk_2 + l_2$, $qP_iy_i = P_{i+1}y_{i+1} + Rk_{i+1} + l_{i+1}$ for $2 \leq i < n$, and $qP_ny_n = Rk_{n+1} + l_{n+1}$, where $q = 1 - xz_t$. These conditions imply

$$\sum_{i=1}^{n} P_iy_i = \sum_{i=2}^{n+1} \frac{2 - q - q^{i-2}}{(1-q)q^{i-2}}(Rk_i + l_i), \text{ and } \sum_{i=2}^{n} P_iy_i = \sum_{i=3}^{n+1} \frac{1 - q^{i-2}}{(1-q)q^{i-2}}(Rk_i + l_i).$$

Using (21), they are rewritten as follows:

$$\sum_{i=1}^{n} P_iy_i = \frac{1 + (1-q)\alpha - q^{n-1}\alpha^n}{1 - \alpha + (1-q)q^{n-1}\alpha^n}(RK+L), \text{ and } \sum_{i=2}^{n} P_iy_i = \frac{\alpha - q^{n-1}\alpha^n}{1 - \alpha + (1-q)q^{n-1}\alpha^n}(RK+L).$$

Thus, the share of intermediate inputs is written as

$$s_M = \frac{\alpha - q^{n-1}\alpha^n}{1 + (1-q)\alpha - q^{n-1}\alpha^n}. \tag{27}$$

Since $n = \max\{n', 1\}$, where $n'$ is the solution to (24), is a function of $q = 1 - xz_t$, it is difficult to show whether $s_M$ is increasing or decreasing in $q$. It is numerically shown, however, that $s_M$ is an increasing function of $q$, given that $\phi(n) = n^a$, $a = 0.1$, and $\alpha = 0.9$. See Figure 6. The plausibility of the value of $\alpha$ is argued in the next subsection.

Figure 6. Cost share of intermediate inputs

This figure implies that in the equilibrium with payment uncertainty, aggregate productivity is positively correlated with the share of intermediate inputs. I will check this prediction in Section 5 using data from the 1990s in Japan.

## 4.7 Productivity and welfare

This model implies that when a macroeconomic shock disturbs the payment process and renders many firms insolvent unexpectedly, the government policy on the insolvent firms affects the path of TFP growth subsequent to the shock. In Figure 7, the paths of TFP, i.e., $\Phi(1 - xz_t)$, corresponding to $x = 0.9$ and $x = 0.3$ are plotted, given that $\phi(n) = n^a$, $z = 0.2$, $a = 0.1$, and $\alpha = 0.9$.

The value of $\alpha$ is taken from a calibration by Basu (1995). As shown in equation (27), $\alpha$ approximates the share of intermediate inputs in total cost in this model economy. The data from United States and Japan show that the share of intermediate inputs is approximately 0.5, implying that $\alpha = 0.9$ is not reasonable. Basu concludes, however, that the counterpart of $\alpha$ in his model must be in the range of $0.8-0.9$ from an estimation of markups. He justifies his calibration as follows: It is natural to assume that there are large fixed costs of production; it is easy to amend the production function to allow for fixed costs, without changing the results of the analysis; and with fixed costs, $\alpha$ is no longer the share of intermediate inputs in total cost, but the share in total *variable* cost. Assuming that the fixed inputs consist of labor and capital, we can have a very high value of $\alpha$, while the share of intermediate inputs in total cost stays at roughly 0.5. In this paper I follow Basu's argument, and reinterpret $s_M$ in (27), which is approximated by $\alpha$ for a large $n$, as the share of intermediate inputs in total variable cost.

Figure 7. Productivity and welfare

In the case of $x = 0.9$, where the government allows quick bankruptcies of insolvent firms, productivity plummets at first and recovers very quickly. It can be said that this pattern replicates the path of productivity in the United States and Chile. In the case of $x = 0.3$, where the government postpones reforms and keeps many insolvent firms afloat, productivity stays relatively high at first but stagnates at a low level for a longer period. It can be said that this pattern replicates the path of productivity in the 1990s in Japan.

This figure also shows social welfare $\sum_{t=0}^{\infty} \beta^t u(c_t)$ as a function of government policy $x$. The functional form and parameters are given by $u(c) = \ln c$, $\beta = 0.99$, $L = 1$, and $K = 1$. The consumption is given by $c_t = \Phi(q_t)$ in this case. The figure implies that immediate bankruptcies of insolvent firms bring about the highest welfare for consumers in the case where the government policy $x$ is constrained by $0.3 \leq x < 1$. This result means that the "failure" of macroeconomic policy in the early 1930s in the United States, which brought about a rash of bankruptcies, could actually have been welfare enhancing, as could the policy reforms in Chile, and that the "successful" macroeconomic stabilization by extraordinary fiscal and monetary expansion in the 1990s in Japan, which kept many insolvent firms afloat, could have been welfare reducing. Since TFP in the United States recovered quickly starting in 1934, Cole and Ohanian (2002, 2004) and Chari, Kehoe, and McGrattan (2002) argue that the reduced welfare triggered by the Great Depression persisted during 1934–39 not because of low-level productivity but because of labor friction, possibly due to New Deal policies. Their view may support the implication of this paper that the rash of bankruptcies and bank failures in 1930–33 could have been welfare enhancing in the sense that it could have brought about a rapid recovery of aggregate productivity through a quick resolution of payment uncertainty.

There is an important caveat for the above policy implication. The model in this paper, like other neoclassical models, does not take account of the disutility caused by unemployment, which must have been very significant during great depressions. If we take account of this disutility, we may not be able to say that the quick exit of insolvent firms always enhances social welfare. Since, historically, the primary objective of macroeconomic policy has been reduction of unemployment, the above policy implication may be quite odd for policy practitioners. This paper claims that only under a special assumption that social welfare is not a function of unemployment, welfare becomes larger as insolvent firms go bankrupt quicker.

There are two related policy implications. First, suppose that the government overlooks the causal link between its policy $x$ and payment uncertainty, and mistakenly regards productivity $\{\Phi(q_t)\}_{t=1}^{\infty}$ to be an exogenous process independent of $x$. In this case, if there is some political demand to lower $x$, a benevolent government, wanting simply to maximize social welfare, may set $x$ at such a small value that it unintentionally causes productivity to stagnate and social welfare to decline.[7]

Second, if the government is not constrained by $0.3 \leq x$ and can set $x$ at any nonnegative value, it can maximize productivity and social welfare by setting the value of $x$ close to zero. In other words, if the government gives large subsidies to insolvent

[7] This implication exhibits a contrast to that of Bergoeing et al. (2002). They provide the explanation for the growth difference that before policy reform, the government favors one sector by allocating larger resources to it. This explanation means that the government intentionally lowers productivity in order to give favors to a specific sector, and that the government is not maximizing social welfare as a whole. In the present paper, the forbearance policy (i.e., a large $x$) does not involve direct costs, since insolvent firms are not inefficient per se (see Assumption 3). Thus a welfare-maximizing government may lower $x$, if it overlooks the effect of payment uncertainty on productivity.

firms, thereby enabling them to continue operating for a sufficiently long period, social welfare will become larger than in the case without such subsidies. This means that sufficiently aggressive fiscal (and monetary) expansion may effectively mitigate declines in productivity and welfare even in a great depression. An aggressive subsidy policy might, however, be politically infeasible, since taxpayers might not consent to bailing insolvent firms out using the government expenditures. Moreover, if people can confidently expect the government to give huge subsidies to failed firms, firm managers will encounter grave moral hazards, and serious agency problems may result, leading to other types of productivity declines.

## 5 Some empirical evidence

The model implies that in the equilibrium under payment uncertainty, $n_t$ and productivity decrease as $xz_t$ increases. Using the total liabilities of failed businesses as a proxy for $xz_t$, this relationship can be checked by estimating the correlation between productivity and bankruptcies.

The estimation results for the Great Depression (1930–39) and the 1990s in Japan are shown in Table 1.[8] In the estimation, detrended TFP is regressed on the liabilities of failed businesses. In the case of the United States, TFP is detrended by the growth rate of 1.83%, which is the trend growth rate of TFP in the 1921–28 period, and in the case of Japan, it is detrended by 1.22%, the trend growth in the 1975–87 period.[9] The total liabilities of failed businesses include bank failures for the United States but not for Japan. The results imply that productivity is negatively correlated with bankruptcies in

[8] As for the Japanese macroeconomic data in this section, Fumio Hayashi kindly permitted me to use the database for Hayashi and Prescott (2002).

[9] I assume that the 1921–28 period was representative of the balanced growth path, in which TFP grows at the rate of technological progress, which can be regarded as a constant. I assumed so because the distorting effects of the World War I seem to have disappeared by 1921, and the distortion of asset-price bubbles seem to have started growing in 1929. For Japan, I posit that the economy was on the balanced growth path in the 1975–87 period, since the distortion caused by the oil crisis of 1973–74 seems to have disappeared by 1975, and that by asset-price bubbles became significant in 1988.

both depression cases.

Table 1. Correlation between productivity and business failures

One can argue that these results may simply reflect seemingly apparent countercyclicality of bankruptcies and procyclicality of TFP. But the negative correlation between these variables is not apparent from standard growth theory, since bankruptcies may mostly represent reductions of inputs rather than declines of TFP. To check whether these results have relevancy to payment uncertainty, I also conducted the same estimation for the 1921–28 period in the United States and for the 1975–87 period in Japan, using both detrended and original TFPs.[10] The results show that there was no significant correlation between bankruptcies and productivity in either case, implying that significant payment uncertainty could have emerged only after the asset-price collapses of 1929 in the United States and of 1990–91 in Japan.

As shown in Section 4.6, the model also implies that the share of intermediate inputs in total cost positively correlates with productivity in the equilibrium with payment uncertainty.[11] Because of the data availability on the cost share of intermediate inputs, the estimation can be conducted only for the 1990s in Japan. The result is shown in Table 2. Note that TFP is also detrended at the growth rate of 1.22%. The result is consistent with the model, showing that the detrended TFP correlates positively with the cost share of intermediate inputs.

Table 2. Productivity and cost share of intermediate inputs

# 6 Conclusion

In this paper, I presented a possible mechanism of productivity declines in depressions. If a macroeconomic shock, such as a bursting of asset-price bubbles, renders many firms

[10]The independent variables for the regression of the original TFP are liabilities of failed businesses, the trend, and a constant.

[11]Since according to the model, productivity correlates with the share of intermediate inputs in total variable cost ($s_M$), the share in total cost must correlate with the productivity, too.

insolvent, other firms become exposed to payment uncertainty, i.e., a higher risk of not being paid by their customers. The payment uncertainty distorts relative prices of intermediate goods and causes an endogenous disruption of the division of labor among firms, leading to shrinkage of chains of productions and lower aggregate productivity.

The model implies that the presumed failure of macroeconomic stabilization during the Great Depression, which lead to a rash of bankruptcies, could have been welfare enhancing, since the quick exit of insolvent firms could have resolved payment uncertainty.

The model predicts that productivity negatively correlates with bankruptcies and positively correlates with the cost share of intermediate inputs. The data from the Great Depression and the 1990s in Japan are consistent with these predictions, giving (weak) support for the model.

This paper may be regarded as a preliminary step in an inquiry into the causes of productivity declines during great depressions. In the future, various models should be proposed and examined empirically using more complete data sets for depression episodes.

# 7 References


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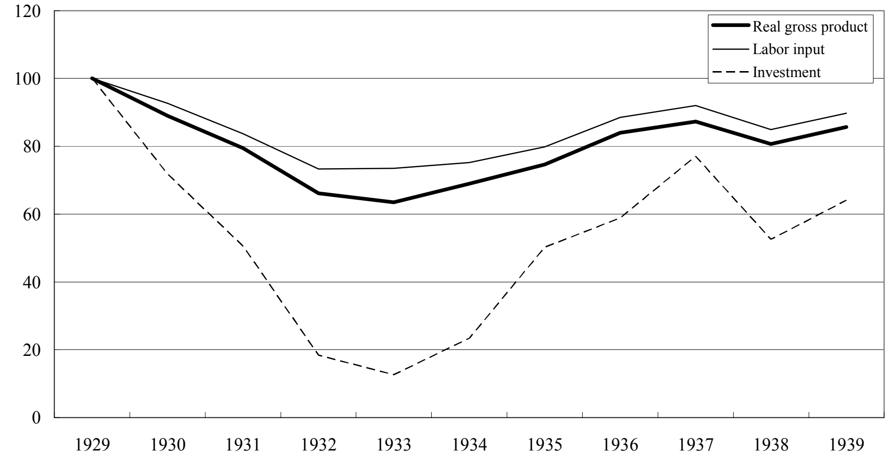
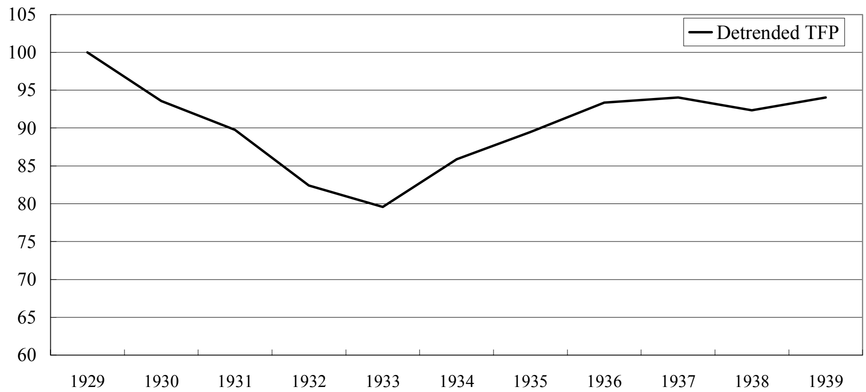
**Figure 1. Macroeconomic variables in the 1930s in the United States**



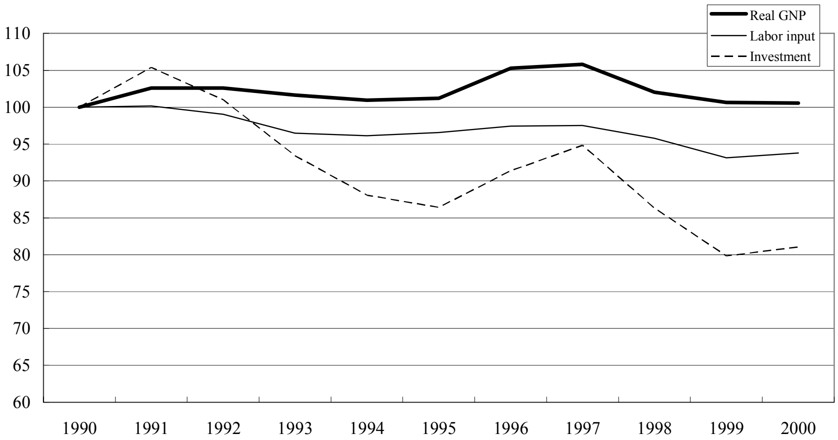



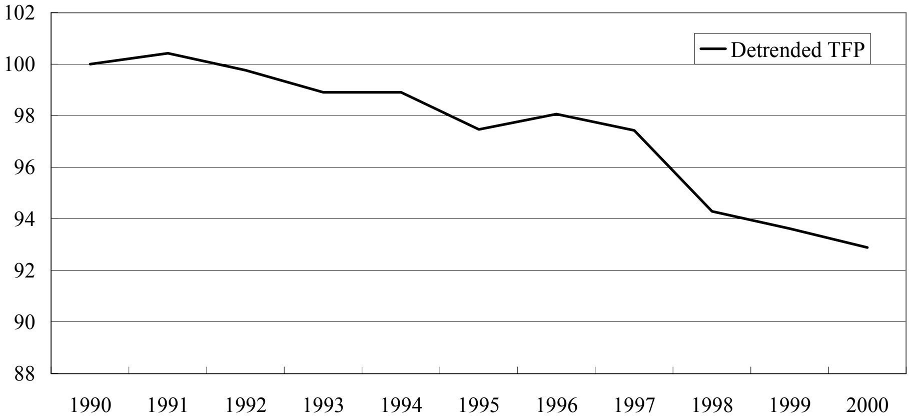

Source: Kendric (1961), Table A-IIb and Table A-XIX.
Note: Investment = New construction and equipment + Change in business inventries + Net foreign investment.
TFP = 100*(Real gross product/Total factor input).
All data are detrended by 1.83%, the average growth rate of TFP from 1921 to 1928.
1929 = 100.

## Growth accounting for the United States

| Year | Growth rate of real GNP | Factors | | |
|---|---|---|---|---|
| | | TFP | Capital | Labor |
| 1929 | 6.3% | 3.8% | 1.0% | 1.5% |
| 1930 | -9.4% | -4.5% | 0.6% | -5.5% |
| 1931 | -9.1% | -2.1% | 0.2% | -7.1% |
| 1932 | -15.3% | -5.9% | -0.5% | -8.9% |
| 1933 | -2.3% | -1.7% | -0.8% | 0.2% |
| 1934 | 10.7% | 10.0% | -0.8% | 1.6% |
| 1935 | 10.2% | 6.3% | -0.4% | 4.3% |
| 1936 | 14.5% | 6.7% | -0.1% | 7.9% |
| 1937 | 5.9% | 2.7% | 0.3% | 2.9% |
| 1938 | -5.8% | 0.0% | 0.1% | -6.0% |
| 1939 | 8.1% | 3.9% | 0.0% | 4.3% |

**Figure 2. Macroeconomic variables in the 1990s in Japan**




Source: Hayashi and Prescott database, http://www.e.u-tokyo.ac.jp/~hayashi/Hayashi-Prescott1.htm.

Note: TFP = Real GNP / Total factor input, where total factor input equals labor input raised to the labor share times capital input raised to the capital share.

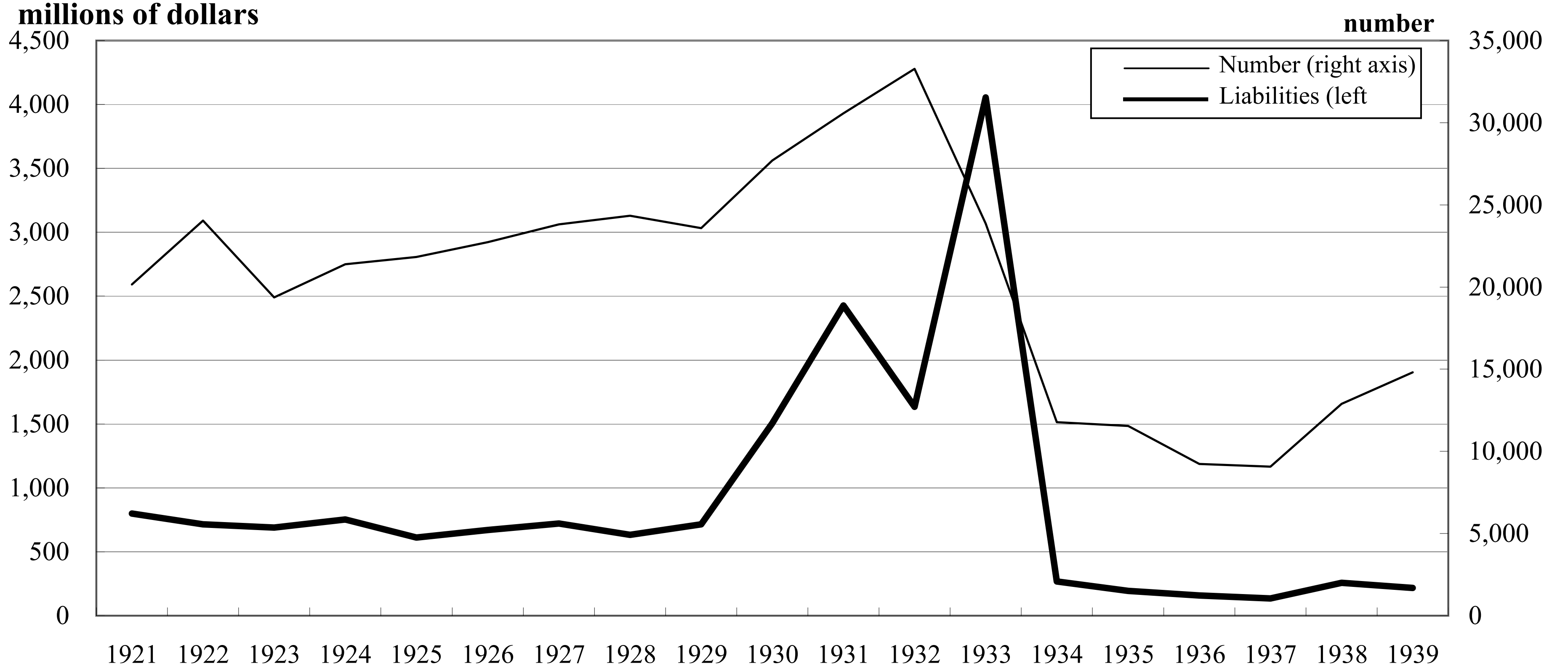
All data are detrended by 1.22%, the average growth rate of TFP from 1975 to 1987.

1990 = 100.

**Growth accounting for Japan**

| Year | Growth rate of real GNP | Factors | | |
|---|---|---|---|---|
| | | TFP | Capital | Labor |
| 1990 | 5.1% | 2.1% | 3.0% | 0.0% |
| 1991 | 3.8% | 1.4% | 2.3% | 0.1% |
| 1992 | 1.2% | -0.2% | 2.1% | -0.7% |
| 1993 | 0.3% | 0.3% | 1.7% | -1.6% |
| 1994 | 0.5% | -0.4% | 1.2% | -0.2% |
| 1995 | 1.4% | 0.1% | 1.0% | 0.3% |
| 1996 | 5.3% | 3.5% | 1.3% | 0.6% |
| 1997 | 1.7% | 0.6% | 1.1% | 0.1% |
| 1998 | -2.4% | -2.9% | 1.6% | -1.1% |
| 1999 | -0.2% | 1.1% | 0.5% | -1.8% |
| 2000 | 1.1% | -0.5% | 1.2% | 0.4% |

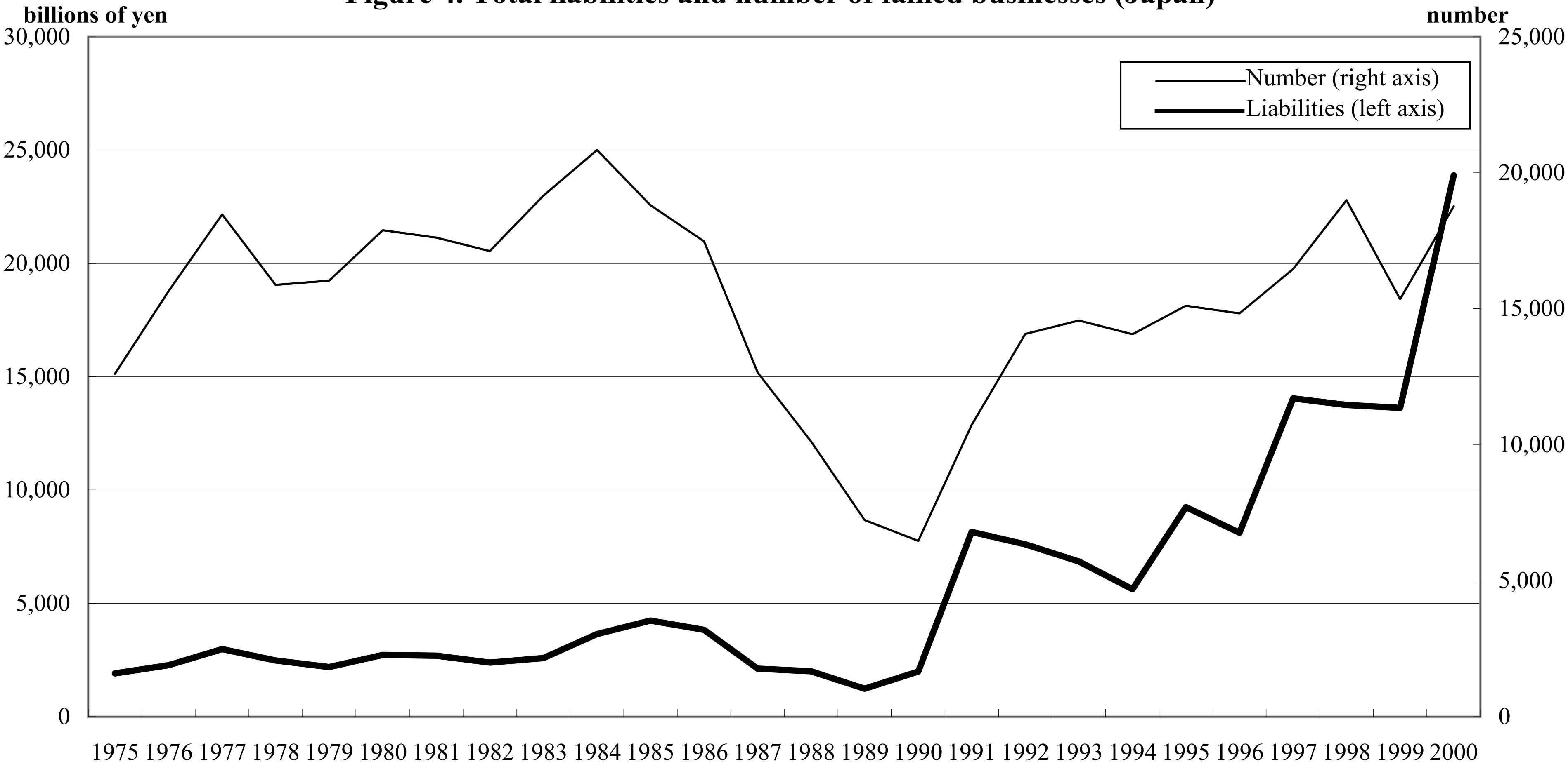
**Figure 3. Sum of commercial failures and bank suspensions (United States)**




Sources: NBER Macrohistory, IX. Financial Status of Business, U.S. Liabilities of Business Failures, All Commercial, http://www.nber.org/databases/macrohistory/contents/chapter09.html (accession number m09033a).
Board of Governors of the Federal Reserve System (1943), "Banking and Monetary Statistics 1914--1941."
George Thomas Kurian, "Datapedia of the United States, 1790--2000: America year by year," 1994.

Note: Liabilies = suspended deposit liablities + liablities of failed businesses.
The number and liabilities of business failures in 1938 and 1939 are based on George Thomas Kurian (1994).

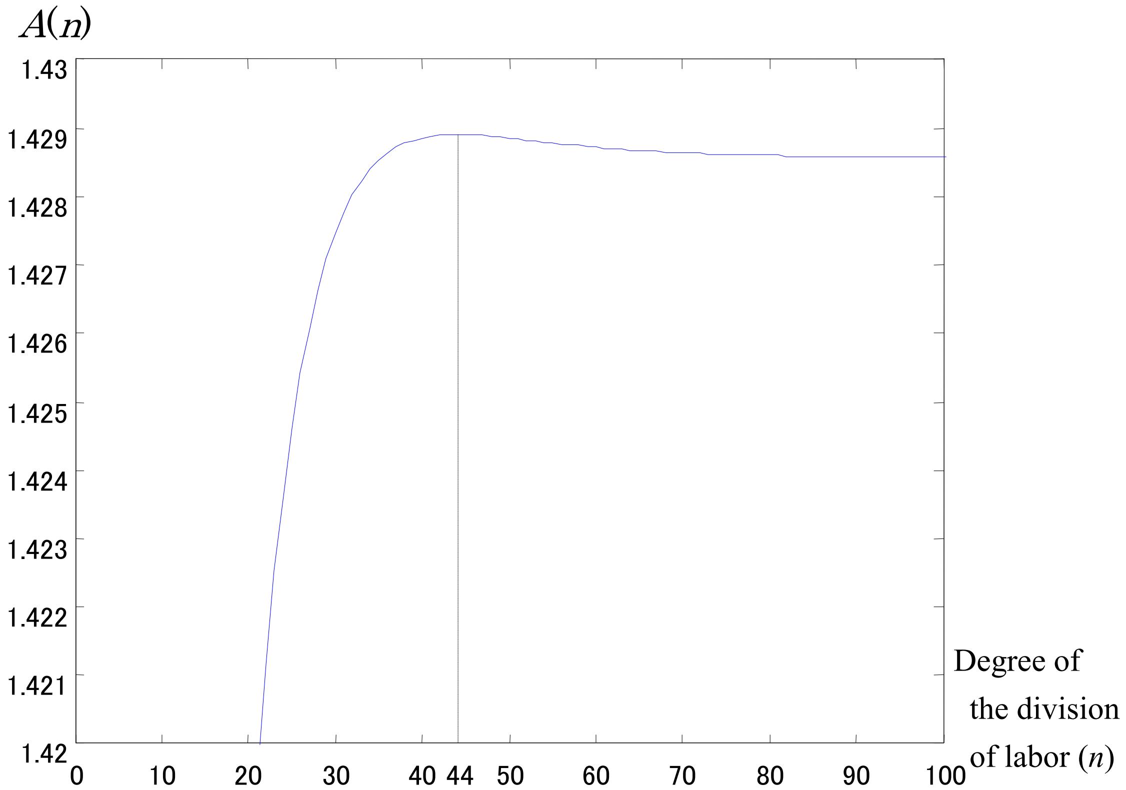
**Figure 4. Total liabilities and number of failied businesses (Japan)**




Source: Tokyo Shoko Research, Annual Business Failure [in Japanese], http://www.tsr-net.co.jp/topics/zenkoku/level_4/tousan_suii.html.

Note: Summary of major business failures with more than 10 million yen in liabilities.

Figure 5. Aggregate productivity



# Figure 6. Cost share of intermediate inputs

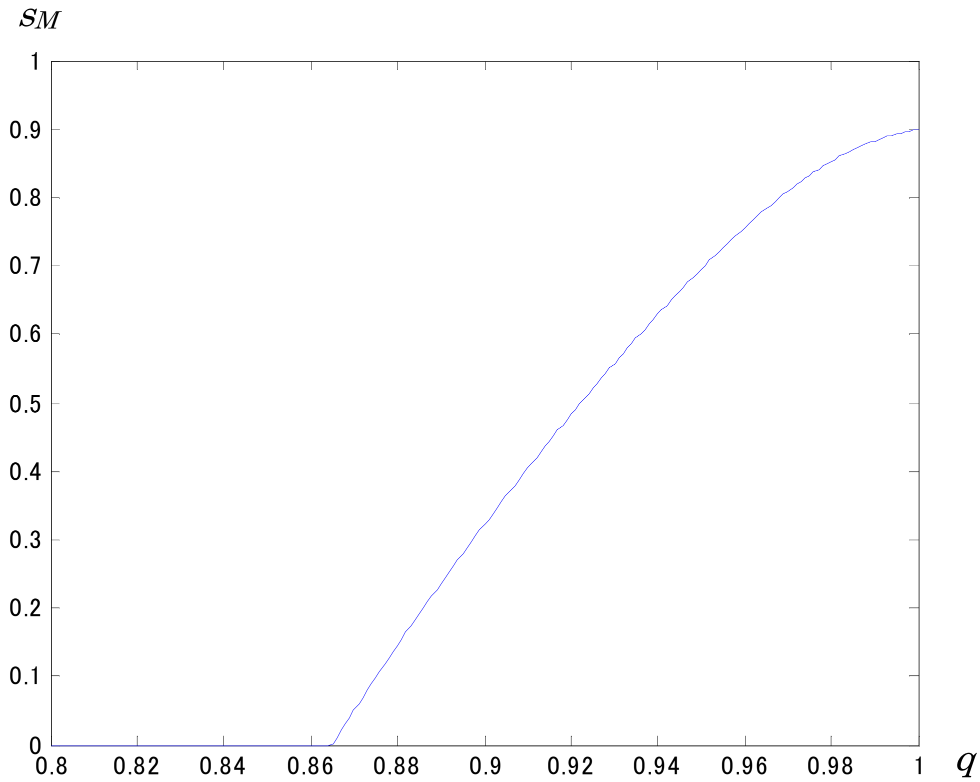
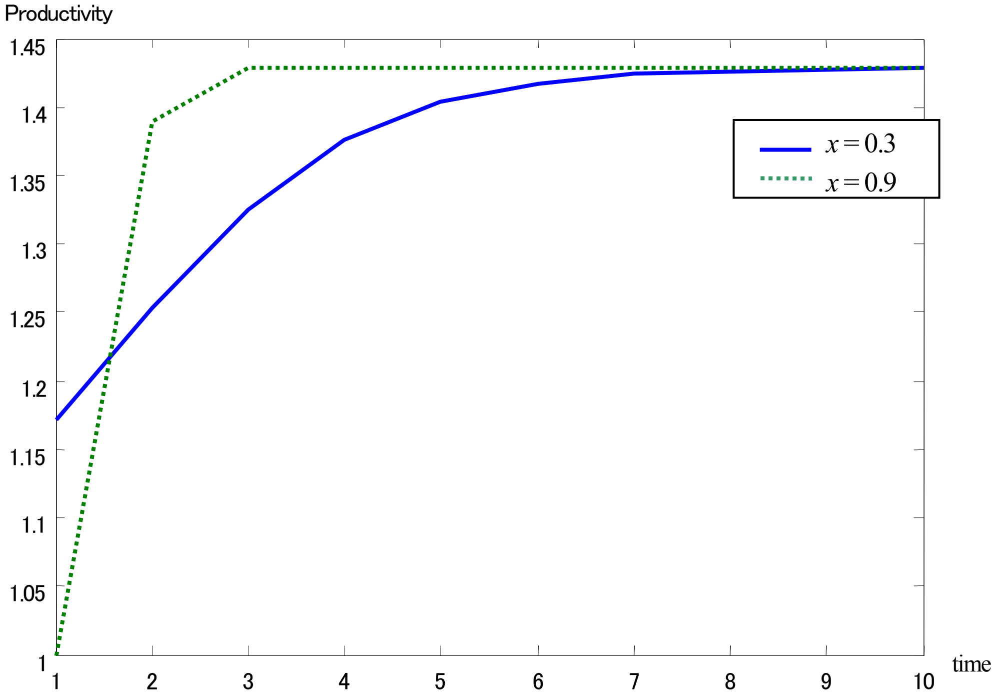
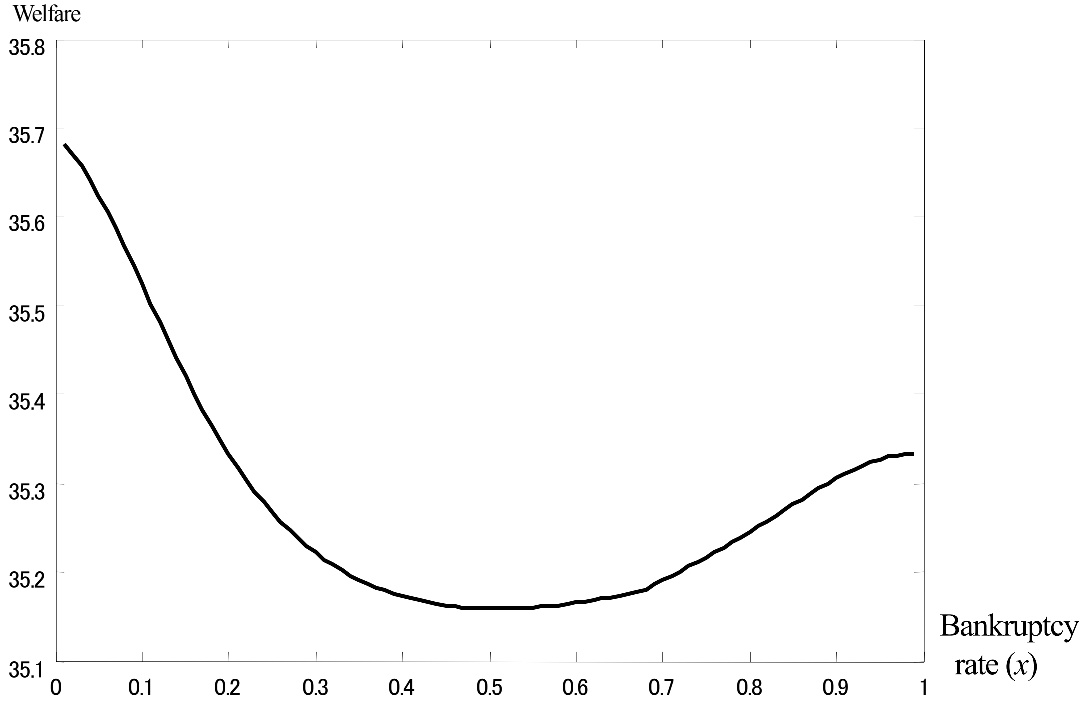


Figure 7. Productivity and welfare




**Parameters**: $z_0 = 0.2$, $a = 0.1$, $\alpha = 0.9$, $\beta = 0.99$, $\theta = 0.3$, $K = 1$, $L = 1$



## Table 1. Correlation between productivity and business failures

**United States**
Method: Least squares
Sample: 1930--39
Included observations: 10

log(detrended TFP)= c(1)+c(2) log(L)

| | Coefficient | Std. error | t-statistic | Prob. |
|---|---|---|---|---|
| **Constant, c(1)** | 4.68 | 0.08 | 58.03 | 0.00 |
| **log(L), c(2)** | -0.03 | 0.01 | -2.32 | 0.05 |

| | |
|---|---|
| R-squared | 0.40 |
| Adjusted R-squared | 0.33 |

Note: Detrended TFP is detrended by 1.83%, the average growth rate of TFP from 1921 to 1928.
L is the sum of the deposits of suspended banks and the liabilities of business failures.
Sources: Kendrick (1961), Table A-XIX.
NBER Macrohistory database, U.S. Number of Business Failures.
Board of Governors of the Federal Reserve System (1943), "Banking and Monetary Statistics 1914--1941."
The number and liabilities of business failures in 1938 and 1939 are based on
George Thomas Kurian, "Datapedia of the United States, 1790-2000: America year by year," 1994.

**Japan**
Method: Least squares
Sample: 1991--2000
Included observations: 10

log(detrended TFP)= c(1)+c(2) log(L)

| | Coefficient | Std. error | t-statistic | Prob. |
|---|---|---|---|---|
| **Constant, c(1)** | 5.20 | 0.13 | 39.02 | 0.00 |
| **log(L), c(2)** | -0.05 | 0.01 | -4.68 | 0.00 |

| | |
|---|---|
| R-squared | 0.73 |
| Adjusted R-squared | 0.70 |

Note: L is the liabilities of business failures.
Detrended TFP is detrended by 1.22%, the average growth rate of TFP from 1975 to 1987.
Sources: Hayashi and Prescott database, http://www.e.u-tokyo.ac.jp/~hayashi/Hayashi-Prescott1.htm.
Tokyo Shoko Research, Annual Business Failure [in Japanese],
http://www.tsr-net.co.jp/topics/zenkoku/level_4/tousan_suii.html.

## Table 2. Productivity and cost share of intermediate inputs

**Japan**

Method: Least squares

Sample: 1990--2000

Included observations: 11

log(detrended TFP) = C(1)+C(2)*(cost share of intermediate goods)

| | Coefficient | Std. error | t-statistic | Prob. |
|---|---|---|---|---|
| **Constant** | **3.71** | **0.27** | **13.83** | **0.00** |
| **Cost share of intermediate goods** | **1.85** | **0.57** | **3.23** | **0.01** |

R-squared 0.518922

Adjusted R-squared 0.465469

Note: Detrended TFP is detrended by 1.22%, the average growth rate of TFP from 1975 to 1987.

Sources: Hayashi and Prescott database, http://www.e.u-tokyo.ac.jp/~hayashi/Hayashi-Prescott1.htm.
Economic and Social Research Institute, Cabinet office, Government of Japan; "Annual Report on National Accounts of 2004."
(Supporting Table 2. Gross Domestic Product and Factor Income Classified by Economic Activities [at current prices])