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Abstract

This paper models two-sided market platforms, which connect third-party suppliers (developers) of many different products and services to users who demand a variety of these products. From a positive perspective, our model provides a simple explanation for the stark differences in platform pricing structures observed across a range of industries, including software for computers and an increasing number of electronic devices, videogames, digital media, etc. We show that the optimal platform pricing structure shifts towards making a larger share of profits on developers when users have a stronger preference for variety and also when there is uncertainty with respect to the availability, or a limited supply, of third-party (high-quality) products. From a normative perspective, we show that the increasingly popular public policy presumption that open platforms are *inherently* more efficient than proprietary ones -in terms of induced product diversity, user adoption and total social welfare- is not justified in our framework. The key welfare tradeoff is between the extent to which a proprietary platform internalizes business-stealing, product diversity and indirect network effects and the two-sided deadweight loss it creates through monopoly pricing.

Keywords: Two-Sided Markets, Platforms, Indirect Network Effects, Product Variety, Social Efficiency.

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1 Introduction

An increasing number of industries in today's economy are organized around platforms, which enable consumers to purchase, access and use a great variety of products. These platforms and the markets in which they operate are said to be "multi-sided" because the vast majority of products is generally supplied by third-party (or independent) producers[1], so that in order to thrive platforms have to attract, through adequate pricing, both consumers and product suppliers.

A classic example is shopping malls: the mall developer has to attract retailers (with which he signs lease contracts) and shoppers. However, it is in industries at the core of the "new economy" that this form of market organization has become most important: to a certain extent one may think of them as "digital shopping malls". For example, in the computer industry, operating system vendors such as Microsoft, Apple, Sun, IBM, Novell, etc. control the software platform which allows computer users to access the large variety of applications supplied by independent developers, who also have to gain access to it. An ever-increasing number of consumer electronics products such as personal digital assistants, smart mobile phones, television sets and car navigation systems are also built around operating system platforms such as Palm OS, Symbian and Linux, which likewise allow consumers to acquire and use thousands of applications from many third-party developers. Internet sites such as Priceline.com allow users to select from a variety of products and services offered by companies having obtained the right to be listed on the site. In the videogame market, users have to purchase consoles such as Sony's Playstation, Microsoft's XBox and Nintendo's Gamecube in order to have access to hundreds of games supplied by independent publishers. Digital media platforms, from wireless networks such as Vodafone Live and NTT DoCoMo's i-mode, to software media players such as Real's Real Player, to on-demand and interactive cable television platforms such as TiVo and Sky Plus, enable users to access a variety of content (games, news, music, movies, etc.) from thousands of independent providers[2].

[1] By contrast, products supplied by the platforms themselves are "first-party".

[2] There are more than 70,000 applications developed for Windows; the Palm OS is supported by over 22,000 applications and its large community of developers is

This paper is the first to model two-sided platforms connecting buyers and sellers in markets in which product variety and competition between sellers are important, and to propose a formal explanation for the differences in platform pricing structures observed across some of the industries mentioned above. In particular, in an empirical survey of computer-based industries centered around software platforms, Evans Hagiu and Schmalensee (2004) document that platforms in this family of markets, economically very similar, have chosen strikingly different pricing structures in order to get the two sides -consumers and independent producers- "on board". At one end of the spectrum, all platforms in the markets for computers, handheld devices and mobile phones have chosen to subsidize or earn little if any profits on the developer side of the market and make virtually all of their profits on users, while on the other hand, in the videogame market, all console manufacturers make the bulk of their profits through royalties charged to third-party game publishers[3] and sell their consoles at or below cost to users. That paper also contains a comparative analysis of how these industries have evolved from an initially vertically integrated structure, in which customers bought fully integrated systems from one supplier, to the current multi-sided (or modular) one, in which platforms and complements are supplied by many different firms. However Evans Hagiu and Schmalensee (2004) does not provide clear conditions under which one should expect the pricing structure chosen by platforms to be tilted in favor of users or developers of complementary products. Accordingly, the first task of the present paper is positive: we seek to build a formal model of platforms operating in industries of the type described above and identify the main factors driving optimal pricing structures. The second task of the paper is normative. There are basically two types of two-sided platform governance that have emerged:

known as the "Palm Economy"; Symbian, the dominant operating system for smart mobile phones (phones with advanced capabilities such as multimedia, e-mail, etc.), offers users of Symbian-based phones a choice from over 2,500 software applications; Playstation is supported by over 800 games; and i-mode channels content from over 60,000 providers. Gawer and Cusumano (2002) and Evans Hagiu and Schmalensee (2004) survey the business and economics aspects of some of these plaftorms.

[3] They also make money through sales of first-party games. However, the proportion of first-party games has decreased significantly over time: it is less than 20% for Playstation and XBox today.

proprietary platforms (such as Windows, Playstation, PalmOS) and open platforms (such as Linux). It is therefore important for economists and policy-makers (industrial policy as well as competition policy) to understand the welfare tradeoffs between these two types of platforms and the model developed here is a first step in this direction as we explain below.

Our model predicts that a higher intensity of users' preference for product diversity shifts the optimal pricing structure towards making a higher share of profits on third-party producers relative to users. Since common intuition and empirical studies suggest that users care more about product variety in markets such as videogames than in more "productivity"-oriented markets such as computer software, this prediction constitutes a plausible explanation for the observed differences in pricing structures. We further show that the optimal pricing structure shifts in this same direction (i.e. in favor of users) when there is more uncertainty with respect to the availability of third-party products and consumers are more pessimistic regarding this uncertainty relative to producers, and when there is a limited supply of high-quality products. Once again, these predictions are consistent with empirical case studies which suggest that these factors are particularly important in the videogame market, relative to other software industries.

From a normative perspective, our model reveals a fundamental welfare tradeoff between two-sided profit-maximizing (proprietary) platforms and two-sided *open* platforms, which allow "free entry" on both sides of the market. On the one hand, a profit-maximizing platform creates two-sided deadweight loss through monopoly pricing, unlike an open platform which essentially prices at marginal cost on both sides. On the other hand however, precisely because it sets prices in order to maximize profits, a proprietary platform internalizes at least partially the positive indirect network externalities between users and third-party product suppliers and the direct negative externalities between producers, whereas an open platform does not. Therefore it is by no means obvious which platform will perform better in terms of induced product variety, user adoption and total social welfare. We show formally that the tradeoff hinges on the interplay between three factors: deadweight loss, the strength of the business-stealing effect versus the product diversity effect, and the extent to which a proprietary platform is able to internalize indirect network externalities.

This insight has important public policy implications. Indeed, the increasing popularity of the open-source software movement with open platforms such as the Linux operating system or the Apache web-server, has given rise to a heated debate among economists and policy-makers regarding the efficiency merits of open versus proprietary platforms[4]. In fact, an increasing number of governments around the world are considering or already enacting policies promoting open source software systems at the expense of proprietary systems[5]. Oftentimes these policies stem from the conviction that open software platforms are *inherently* more efficient than their proprietary counterparts. Although our model is highly stylized and does not incorporate many economic features specific to the open source form of organization for the software market, it is sufficient for exhibiting the welfare tradeoff described above. In fact, we even provide a specific example in which either form of platform governance (open or closed) may be the more efficient one. This implies that in our framework an *a priori* preference of open platforms over proprietary platforms (or the other way around) is not economically justified.

Related literature

Our paper belongs to very recent and quickly growing economics literature on two-sided markets, pioneered by Armstrong (2002), Caillaud and Jullien (2003) and Rochet and Tirole (2003) and (2004). A market is said to be two-sided if firms serve two distinct types of customers, who depend on each other in some important way, and whose joint participation makes platforms more valuable to each. In other words, there are indirect network externalities between the two different customer groups[6]. One of the main insights which has emerged from this literature is the importance of

[4]Hahn et al. (2002) contains a representative sample of the opposing views on this issue.

[5]For instance, Brazil has passed legislation mandating open source solutions be given preference in municipal governments and France has passed a parliamentary bill forbidding government-related institutions from using anything but open-source software. See Hahn et al. (2002) for a comprehensive overview of such policies.

[6]This is the definition offered by Evans (2003). Rochet and Tirole (2004) use a slightly different one: for them, a necessary and sufficient condition for a market to be two-sided is that the volume of transactions be sensitive to the distribution of total costs between the two sides.

platforms' choice of pricing structures in "getting the two sides on board". Similar to Armstrong (2002) and Rochet and Tirole (2003) and (2004), our model emphasizes the role of elasticities of demand for the platform on both sides of the market: the more elastic the demand on one side, the higher the price charged to the other side, and viceversa.

The innovation of our model is to introduce competition among members of one side of the market (developers)[7] and to show that the intensity of users' preferences for variety is a crucial determinant of the optimal platform pricing structure. This enables us to propose an explanation for the somewhat puzzling empirical finding of radically different pricing structures across a set of otherwise very similar industries[8]. By contrast, most of the two-sided markets literature up to now has either focused on individual industries such as credit cards (Rochet and Tirole (2002) and (2003)[9], Schmalensee (2002), Wright (2003)), intermediaries (Caillaud and Jullien (2003), Baye and Morgan (2001)), Yellow Page directories (Rysman (2003)) and broadcasting (Anderson and Coate (2003)), or has provided general and essentially symmetric models[10], inadequate for undertaking the type of cross-industry comparison we make here.

Second, our welfare comparison between open and proprietary platforms relates our paper to the literature on product variety, free entry and social

Both definitions imply that a platform can improve upon the market outcome through a pricing structure that rebalances costs between the two sides by internalizing (to a certain extent) the indirect externalities.

[7]Competition stems endogenously from consumers' concave preferences for product variety in our model. The only other models with two-sided platform pricing and explicit competition on one side we are aware of are Rochet and Tirole (2002) and Schmalensee (2002), in the credit card context. In those papers however competition is not between merchants (the equivalent of developers in our model), but between issuers and/or acquirers, i.e. the members of the credit-card association (the platform). Therefore product variety does not play the important role it does in our paper.

[8]Hagiu (2004b) also studies platforms of the type we are interested in here. However, that paper abstracts from the question of product diversity by assuming independent demand functions for applications and focuses instead on the issue of commitment and the use of variable fees (or royalties).

[9]The model contained in this paper is inspired by and primarily destined to credit cards but the authors show that some of the general insights they offer also apply to other industries.

[10]That is, demand functions on the two sides are symmetric and there is no competition within either side (Armstrong (2002), Rochet and Tirole (2004)).

efficiency, in particular Mankiw and Whinston (1986). Their paper is concerned with the inefficiencies associated with free-entry in product markets and shows that the sign of the inefficiency (i.e. whether there is excessive or insufficient entry) depends on the interplay between two opposite effects: the business-stealing effect and the product-diversity effect. Our paper can be viewed as an extension of their analysis in two important dimensions. First, Mankiw and Whinston's model is "one-sided" in the sense that the number of consumers participating in the market is fixed and only the number of producers is variable. This allows them to focus exclusively on *direct* externalities on the producer side and abstract from the positive *indirect* network externalities between consumer entry and producer entry, which are central to our paper. Thus, our two-sided open platforms are similar but more general than the free-entry regime studied by Mankiw and Whinston, Spence (1976), Dixit and Stiglitz (1977), Salop (1979), etc. since user participation in the market is endogenous in our model. Second and most important, our two-sided proprietary platforms controlling market access through prices charged to both users and independent product suppliers constitute a novel form of market organization, which has not been analyzed by the literature on product variety.

Finally, our paper is linked to the literature on indirect network effects, especially Church and Gandal (1992) and Church Gandal and Krause (2002). Both papers study two-sided technology (or platform) adoption, however in both models, the platform is assumed to be entirely passive, i.e. there is no strategic pricing on either side of the market. This is equivalent to an open platform in our model.

The remainder of the paper is organized as follows: the next section presents the model and sets up the optimization problem for a monopoly two-sided proprietary platform. Section 3 derives the optimal platform pricing structure and studies the effects of introducing uncertainty and limited supply of developers. Section 4 analyzes social efficiency, by comparing product variety, user adoption and social welfare under an open platform, a proprietary profit-maximizing platform and a benevolent social planner. Section 5 concludes.

2 Modelling framework

We are interested in modelling a two-sided platform whose value to users is increasing in the number of developers[11] it supports and whose value to developers is increasing in the number of users who adopt the platform. The platform controls the extent of adoption on both sides of the market through prices.

Net surplus for a user indexed by θ from buying a platform which charges her P^U and is supported by n applications is:

$$u(n) - P^U - \theta$$

where $u(n)$ is the surplus obtained from the n applications, *net of the prices charged by each application developer* and θ is a horizontal differentiation parameter distributed with c.d.f. F and continuously differentiable density f over an interval $[\theta_L, \theta_H]$. θ can be interpreted as the difference between the fixed (sunk) cost of learning how to use the system comprised by the platform and the applications and the sum of: a) the user's intrinsic "taste" for the system, b) the standalone value of the platform (in case it comes bundled with some applications). We denote by ε_F the elasticity of F, which is to be interpreted as the elasticity of user demand for the platform:

$$\varepsilon_F(\theta) = \frac{\theta f(\theta)}{F(\theta)}$$

Similarly, net profits for a developer indexed by ϕ from supporting a platform which charges P^D to developers and is adopted by all users $\theta \leq \theta_m$[12] are:

$$\pi(n) F(\theta_m) - P^D - \phi$$

where $\pi(n)$ is the profit *per platform user net of variable costs* and ϕ is the developer's fixed cost of writing an application. We assume ϕ is distributed

[11]Developers are third-party product suppliers: developers of software applications or games, content providers, etc. For simplicity and ease of interpretation throughout the paper we will use the blanket term "developers" instead of third-party producers and "applications" in order to refer to their products.

[12]Indeed, given the structure of user preferences assumed above, if user θ adopts the platform given n and P^U then all users $\theta' \leq \theta$ will also adopt.

on $[0, \phi_H]$ with c.d.f. $H(.)$ and continuously differentiable density $h(.)$. The elasticity of developer demand for the platform:

$$\varepsilon_H(\phi) = \frac{\phi h(\phi)}{H(\phi)} > 0$$

As suggested by this formulation we will ignore integer constraints and treat n as a continuous variable throughout the paper. The reason is that in the markets we have in mind there are hundreds or even thousands of applications as explained in the introduction. Continuity also renders the analysis very convenient by allowing us to reason in terms of demand elasticities.

There are three important assumptions embedded in the expressions of user surplus and developer profits above. First, all users are assumed to have the same marginal valuation for applications, i.e. there is no vertical differentiation among them. Second, all applications are assumed to be identical and fully interchangeable from the point of view of every user and developers are also solely horizontally differentiated by their fixed development cost. Third, platforms charge only fixed "access" fees and no variable fees. These assumptions greatly simplify the analysis, however our main insights hold for more general formulations[13].

Let:

$$V(n) = u(n) + n\pi(n)$$

denote the social surplus created by n applications per platform user, *gross of fixed development costs.*

We make the following assumption:

Assumption 1 *$u(n)$ is strictly increasing and concave, $\pi(n)$ is strictly decreasing and $V(n)$ is strictly increasing and concave.*

[13]For example, in Hagiu (2004c) we introduce vertical differentiation on both sides of the market. The formal analysis is slightly more complex but the main conclusions are unchanged, which is why we have chosen to focus on the simplest formulation.

Introducing variable *proportional* fees would not change anything here. In Hagiu (2004c) we do so, while at the same time introducing investment in product quality by developers. Nominal fees are more problematic, as they mey impact the price charged by developers to users: Hagiu (2004b) allows platforms to use royalties in a simpler model, by abstracting from the issue of product diversity.

This assumption is quite reasonable: it simply says that user surplus from applications is increasing at a decreasing rate (the 100th application is less valuable than the 10th), that each developer's profits per user are decreasing in n (crowding effect) and that total social surplus $V(n)$ is increasing at a decreasing rate.

Let us denote by ε_V the elasticity of V:

$$\varepsilon_V(n) = \frac{nV'(n)}{V(n)} \in]0,1[$$

The elasticity ε_V plays a central role in our model: it measures the intensity of users' preference for variety. The higher ε_V, the less concave $V(.)$ and therefore the higher the marginal contribution of an additional application to gross social surplus per platform user.

Also, it will prove useful to define:

$$\lambda(n) = \frac{\pi(n)}{V'(n)}$$

the ratio between developer profits and the marginal contribution of an additional developer to social surplus (per platform user). Intuitively, when $\lambda(n) > 1$, each developer is gaining more than his marginal contribution to social surplus, therefore one would expect a bias towards excessive entry under a free entry regime (open platform), and viceversa, when $\lambda(n) < 1$, free-entry contains a bias towards insufficient entry[14]. Of course, a two-sided proprietary platform may either correct or exacerbate this bias to a certain extent through its price P^D.

Let us clearly specify the timing of the pricing game we consider throughout the paper. There are 3 stages:

- Stage 1) The platform sets prices P^U and P^D for consumers and developers simultaneously
- Stage 2) Users and developers make their adoption decision simultaneously

[14]Note that this bias is on the developer side of the market and has to be compounded with a same or opposite sign bias on the user side, as we show in section 3.2.

- Stage 3) Developers set prices for consumers and those consumers who have acquired the platform in the second stage decide which applications to buy.

The slightly odd-sounding assumption that users decide whether or not to buy the platform *before* developers set their prices is made in order to simplify the analysis of the two-sided pricing game. It implies that when developers set their prices, they take consumer demand for the platform as given. In reality, users' decisions whether or not to adopt the platform and developers' pricing decisions overlap. However, it is quite reasonable to assume developers take user demand for the platform as given when they set their prices, i.e. each individual developer regards himself as being small enough so that his strategic decisions do not affect total user demand *for the platform* (they do of course affect his own demand). The reason is once again the large numbers of developers supporting the platforms we have set out to study. Still, our results do not hinge on this assumption: virtually all the analysis below carries over to the case when developers have positive mass and are allowed to take into account the effect of their individual prices on total user demand for the platform, on condition they are "small enough". Lastly, an additional benefit of assuming this timing in our model is that it allows us to introduce uncertainty with respect to the availability of applications in a simple and tractable way, as will become clear in section 3.1. below.

In order to illustrate how $u(n)$, $\pi(n)$ and $V(n)$ are obtained, we provide two specific examples, which we will use later in the paper.

***Example* 1** Suppose users' gross benefits have the Spence-Dixit-Stiglitz form $G\left(\sum_i v(q_i)\right)$, where q_i is the "quantity" of application i *consumed*, $v(0) = 0$, $v'(.) > 0$ and $v''(.) < 0$ and $G'(.) > 0$, $G''(.) < 0$.

User maximization implies that the quantity q_k demanded by each platform user[15] from developer k charging p_k satisfies:

$$p_k = v'(q_k)\, G'\left(\sum_i v(q_i)\right)$$

[15] This is because all users "agree" on the incremental benefits offered by applications.

Given our assumption of small developers each of them takes the market price $G'\left(\sum_i v\left(q_i\right)\right)$ as given when setting his price. Consequently, the stage 3 pricing equilibrium among developers is symmetric and defined by:

$$v'\left(q_n\right)G'\left(nv\left(q_n\right)\right)=p_n=\arg\max_p\left\{(p-c)\,v'^{-1}\left(\frac{p}{G'\left(nv\left(q_n\right)\right)}\right)\right\}$$

Then: $\pi\left(n\right)=\left(p_n-c\right)q_n$, $u\left(n\right)=G\left(nv\left(q_n\right)\right)-np_nq_n$ and $V\left(n\right)=G\left(nv\left(q_n\right)\right)-ncq_n$. Letting $v\left(q\right)=q^{\sigma}$ and $G\left(z\right)=z^{\frac{\alpha}{\sigma}}$, with $0<\alpha<\sigma<1$, we obtain:

$$\pi\left(n\right)=\left(1-\sigma\right)\alpha\left(\frac{\alpha\sigma}{c}\right)^{\frac{\alpha}{1-\alpha}}n^{-\frac{\sigma-\alpha}{\sigma(1-\alpha)}}$$

$$u\left(n\right)=\left(1-\alpha\right)\left(\frac{\alpha\sigma}{c}\right)^{\frac{\alpha}{1-\alpha}}n^{\frac{\alpha(1-\sigma)}{\sigma(1-\alpha)}}$$

$$V\left(n\right)=\left(1-\sigma\alpha\right)\left(\frac{\alpha\sigma}{c}\right)^{\frac{\alpha}{1-\alpha}}n^{\frac{\alpha(1-\sigma)}{\sigma(1-\alpha)}}$$

$$\varepsilon_V=\frac{\alpha\left(1-\sigma\right)}{\sigma\left(1-\alpha\right)}\in]0,1[$$

$$\lambda=\frac{\sigma\left(1-\alpha\right)}{1-\sigma\alpha}\in]0,1[$$

***Example* 2** Suppose users have unitary demand for applications (i.e. buy either 0 or one unit of each application) and gross benefits from using n applications are $V\left(n\right)$ with $V'\left(.\right)>0$, $V''\left(.\right)<0$. In this case the stage 3 price equilibrium is: $p_n=V'\left(n\right)$ leading to[16]: $\pi\left(n\right)=V'\left(n\right)$, $u\left(n\right)=V\left(n\right)-nV'\left(n\right)>0$ and $\lambda=1$. Letting $V\left(n\right)=An^{\beta}$, with $0<\beta<1$, we obtain[17]:

$$\pi\left(n\right)=\beta An^{\beta-1}$$

[16]Here we assume developers have 0 marginal costs: many of the real-life platforms we have in mind support digital applications whose marginal costs are virtually 0.

Note that this is not feasible in example 1 since it would lead to infinite prices and profits.

[17]This example is used by Church Gandal and Krause (2002).

$$u(n) = (1-\beta) A n^{\beta} > 0$$

$$\varepsilon_V = \beta$$

Let us now set up the optimization program for a two-sided profit-maximizing platform. Given the platform's prices P^U and P^D, it is indeed an (interior) equilibrium for n developers and all users $\theta \leq \theta_m$ to adopt the platform in stage 2 only if the following two conditions hold:

$$\pi(n) F(\theta_m) - P^D - H^{-1}(n) = 0 \tag{1}$$

$$u(n) - P^U = \theta_m \tag{2}$$

The first condition says that in equilibrium all profit opportunities are exhausted for developers (assuming an unlimited supply of developers[18]) and the second condition says that the marginal user θ_m must be indifferent between adopting and not adopting the platform.

Equation (1) determines developer demand n as a function $N\left(\theta_m, P^D\right)$ of user demand and the price charged to developers, whereas equation (2) determines user demand $F(\theta_m)$ or equivalently the marginal user θ_m as a function $\Theta\left(n, P^U\right)$ of developer demand and the price charged to users. Note that these two-way demand interdependencies or indirect network externalities are positive: $N\left(., P^D\right)$ and $\Theta\left(., P^U\right)$ are both increasing.

Plugging (2) into (1), we obtain n as an implicit function of the platform's prices P^D and P^U:

$$\pi(n) F\left(u(n) - P^U\right) = H^{-1}(n) + P^D \tag{3}$$

This expression makes clear that on the developer side of the market there are both positive indirect network effects contained in the term $F\left(u(n) - P^U\right)$ and *negative direct* network effects contained in the term $\pi(n)$.

Setting for simplicity platform marginal costs on both sides to 0, the expression of platform profits is:

$$\Pi^P = P^U F(\theta_m) + n P^D$$

[18]We relax this assumption in section 3.2.

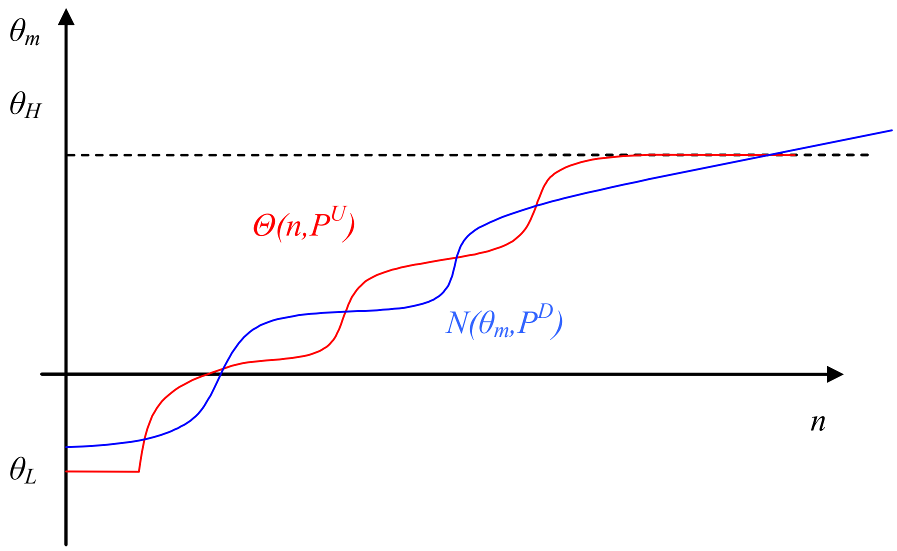

Figure 1:

Using (1) and (2) we obtain:

$$\Pi^P = (V(n) - \theta_m) F(\theta_m) - nH^{-1}(n) \tag{4}$$

which depends only on (θ_m, n). Therefore, rather than maximizing platform profits over (P^U, P^D) we will do so directly over (θ_m, n)[19].

The first-order conditions determining the optimal $(\theta_m^{2sp}, n^{2sp})$ are:

$$\frac{V(n) - \theta_m}{\theta_m} = \frac{1}{\varepsilon_F(\theta_m)} \tag{5}$$

$$V'(n) F(\theta_m) = nH^{-1\prime}(n) + H^{-1}(n) \tag{6}$$

Given the profit-maximizing $(n^{2sp}, \theta_m^{2sp})$, the corresponding profit maximizing prices (P_{2sp}^U, P_{2sp}^D) are then *uniquely* determined by (1) and (2).

The issue is that conversely, given $(P^U, P^D) = (P_{2sp}^U, P_{2sp}^D)$, (1) and (2) may have multiple solutions (θ_m, n) as can be seen in figure 1.

[19] A similar "trick" is used by Armstrong (2003) in a linear model. Below and in the appendix we discuss the question of when this transformation is legitimate in our model.

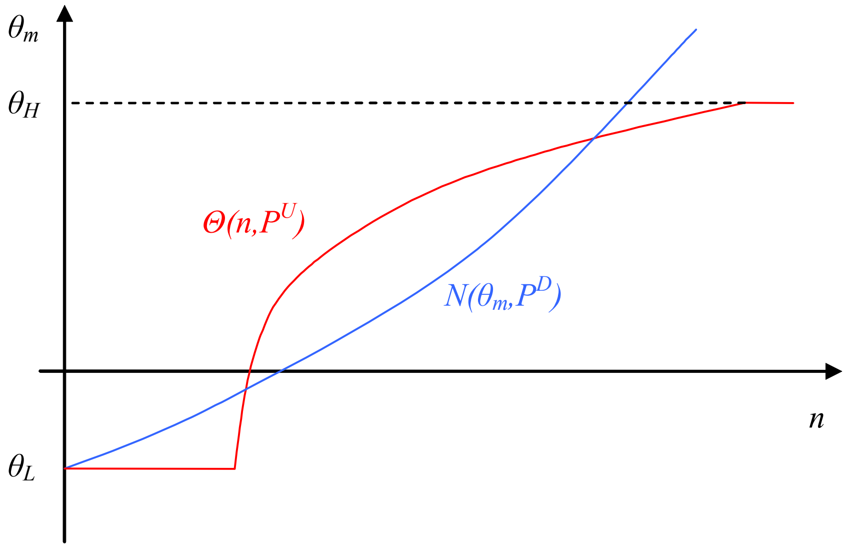

Figure 2:

This is a well-known feature in markets with indirect network effects[20]. In order to overcome this problem, we restrict attention to *stable* equilibria[21] and make the following assumption:

Assumption 2 *Given a set of prices* (P^U, P^D)*, the platform is able to coordinate users and developers on its most preferred, stable, adoption equilibrium to (1) and (2).*

This assumption is less restrictive than it might appear at first glance. First, under sufficient regularity conditions[22], (1) and (2) have at most two interior intersections (θ_m, n) given (P^U, P^D), only one of which is stable, as illustrated by figure 2.

Second, even when there are multiple stable equilibria, it is reasonable to expect users and developers will coordinate on the stable equilibrium with

[20]See for example Church and Gandal (1992).

[21]Given (P^U, P^D), a solution (θ_m, n) is stable if and only if the dynamic adjustment process starting at any nearby point (θ'_m, n') and following (1) and (2) converges to (θ_m, n). On our graphs, stable equilibria are the intersections of $N(\theta_m, P^D)$ and $\Theta(n, P^U)$ where $N(., P^D)$ crosses $\Theta(., P^U)$ from *below*.

[22]An example of such conditions are conditions 1, 2 and 3 in appendix A1.

the highest levels of entry on both sides of the market[23], otherwise, in the absence of any entry restrictions, there are strictly positive rents available to coalitions of users and developers which are left out of the market. Therefore the only potentially problematic case is when the platform's preferred stable equilibrium is not the one with the highest levels of entry. But then the platform can simply adopt a policy of entry restriction on either side, inducing both sides to coordinate on its most preferred equilibrium.

Consequently, given assumption 2, we simply need to impose conditions such that $\left(\theta_m^{2sp}, n^{2sp}\right)$ is a stable solution to (1) and (2) given $\left(P^U, P^D\right) = \left(P_{2sp}^U, P_{2sp}^D\right)$ and that Π_P is concave in (θ_m, n) (in order for (5) and (6) to define a maximum). In appendix A1 we provide an example of a simple set of conditions, which ensure the stability and concavity of all two-sided optimization problems we consider in this paper. It should be stressed that this example is merely an illustration of the technical issues arising in two-sided models, and that all the results and insights in this paper hold for significantly more general conditions.

3 Platform pricing structures

In an empirical survey of computer-based industries, Evans Hagiu and Schma lensee (2004) document that despite numerous economic similarities, software platforms operating in these markets have chosen radically different pricing structures. On the one hand, vendors of operating systems for computers and many other consumer electronics products (handheld digital assistants, smartphones, television sets) have chosen to subsidize or earn little if any profits on the developer side of the market, be it applications or hardware complements. Despite investing large amounts of money every year in "developer support", Microsoft, Apple, Symbian, Palm, Novell, Sun, all make virtually all of their profits by selling their platforms to users[24]. At the other end of the spectrum, in the videogame market, all console man-

[23] Note that because $N\left(., P^D\right)$ and $\Theta\left(., P^U\right)$ are increasing, if (θ_m, n) and $\left(\theta'_m, n'\right)$ are two equilibria given the same prices $\left(P^U, P^D\right)$ then $\theta_m < \theta'_m$ if and only if $n < n'$, so that it makes sense to talk about the highest level of entry on *both* sides.

[24] Either directly, in integrated hardware-software form, -like Apple and Sun-, or indirectly, by licensing it to OEMs -like Symbian and Microsoft-, or using both channels -like Palm.

ufacturers without exception since the introduction of the first Nintendo Entertainment System in the United States in 1988 make the bulk of their profits through per-game royalties charged to publishers-developers[25] and sell their consoles at or below cost to users[26]. Finally, pricing structures of digital media platforms seem to lie somewhere in-between these two extremes: for example, i-mode makes profits both on users and on content providers through variable fees based on the intensity of usage of the network and Real's revenues come from both subscription fees charged to users and access fees charged to "non-premium" content providers[27].

It should be stressed that there is absolutely nothing that prevents the first type of platforms above from charging developers, either fixed or variable fees, except of course business rationality. In fact, this pricing "puzzle" is all the more striking as it can be found within the same firm, Microsoft, which has two entirely opposite business models for Windows and XBox. More generally, this discussion can be extended to include other two-sided platforms: shopping malls charge nothing for access to consumers and recoup their initial layout by collecting rent from retailers[28]; Priceline.com allows Internet users to access a variety of services and product offerings for free, while charging sponsors of these services and products for the right to be listed[29]; Ticketmaster pays venues or promoters a small fee per ticket sold and recoups by charging users \$3 to \$6 in addition to the ticket's face value[30].

How can one make sense of these contrasting pricing structures? In this section we show that our model yields an explanation based on the intensity of users' preferences for variety. Of course, there are many other factors, specific to each industry, which have a significant influence on platforms'

[25]For example, Sony's Playstation 2, Nintendo's GameCube and Microsoft's XBox charge \$8-\$10 royalties per game to independent game publishers.

[26]Clements and Ohashi (2004).

[27]A few premium content providers are paid by Real. The company paid, for instance, the National Basketball Association \$20 million and a share of subscription revenues for the rights to stream NBA games for three seasons (Sloan (2003)).

[28]Pashigian and Gould (1998).

[29]Ideally, one should distinguish between pure advertisers and genuine product/service offerings (trips, hotels, flights, etc.), however, from the broad perspective we take here, these two types of "products" can be considered approximately similar.

[30]Bilodeau (1995).

pricing structures, however the intuitive explanation we propose has the merit of being applicable to a broad range of industries and, as we argue below, is quite plausible empirically, especially when one restricts attention to computer-based industries.

Throughout the paper we will calculate the pricing structure as the ratio between the portion of total profits Π^P which is made on developers, Π^{PD}, and the portion which is made on users, Π^{PU}:

$$\Pi^P = \underbrace{P^U F(\theta_m)}_{\Pi^{PU}} + \underbrace{nP^D}_{\Pi^{PD}}$$

Using in (1) and (2), we can write:

$$\frac{\Pi^{PD}}{\Pi^{PU}} = \frac{n\pi(n) F(\theta_m) - nH^{-1}(n)}{(u(n) - \theta_m) F(\theta_m)}$$

Using (6), we obtain:

$$\begin{aligned}\frac{\Pi^{PD}}{\Pi^{PU}} &= \frac{n(\pi(n) - V'(n)) F(\theta_m) + n^2 H^{-1\prime}(n)}{(V(n) - n\pi(n) - \theta_m) F(\theta_m)} \\ &= \frac{nV'(n)\left(\frac{\pi(n)}{V'(n)} - 1 + \frac{nH^{-1\prime}(n)}{V'(n)F(\theta_m)}\right)}{V(n)\left(1 - \frac{\pi(n)}{V'(n)}\frac{nV'(n)}{V(n)} - \frac{\theta_m}{V(n)}\right)}\end{aligned}$$

But the first order conditions (5) and (6) imply[31]:

$$\theta_m = \frac{\varepsilon_F(\theta_m) V(n)}{1 + \varepsilon_F(\theta_m)}$$

$$\frac{V'(n) F(\theta_m)}{nH^{-1\prime}(n)} = 1 + \varepsilon_H\left(H^{-1}(n)\right)$$

Combining the last three expressions and using $\varepsilon_V(n) = \frac{nV'(n)}{V(n)}$ and $\lambda(n) = \frac{\pi(n)}{V'(n)}$, we obtain the following proposition.

Proposition 1 *The optimal platform pricing structure is given by*[32]*:*

$$\frac{\Pi^{PD}}{\Pi^{PU}} = \frac{\varepsilon_V(1 + \varepsilon_F)(1 - (1 - \lambda)(1 + \varepsilon_H))}{(1 + \varepsilon_H)(1 - \lambda\varepsilon_V(1 + \varepsilon_F))} \tag{7}$$

[31] We use the fact that: $\frac{nH^{-1\prime}(n)}{H^{-1}(n)} = \frac{1}{\varepsilon_H(H^{-1}(n))}$.

[32] We omit function arguments in order to avoid clutter.

If $\lambda \leq \frac{\varepsilon_H}{1+\varepsilon_H}$ *then the platform subsidizes developers (*$P^D < 0$*) and recoups on users.*

If $\lambda \geq \frac{1}{\varepsilon_V(1+\varepsilon_F)}$ *then the platform subsidizes users (*$P^U < 0$*) and recoups on developers.*

If $\frac{\varepsilon_H}{1+\varepsilon_H} < \lambda < \frac{1}{\varepsilon_V(1+\varepsilon_F)}$[33] *then the platform makes positive profits on both sides of the market and its optimal pricing structure is such that the share of profits made on developers relative to the share of profits made on the user side of the market is decreasing in the elasticity of developer demand* ε_H *and increasing in the elasticity of user demand for the platform* ε_F*, in the elasticity of user demand for applications* ε_V *and in* λ*, the ratio of developer profits per user over the marginal contribution of an additional developer to surplus per user .*

■

Part of the result contained in proposition 1 is consistent with a general pricing principle which has emerged from the early theoretical literature on two-sided markets (in particular Armstrong (2002) and Rochet and Tirole (2003)): the price charged to one side will be higher the *less* elastic the demand of that side for the platform and the *more* elastic the demand on the other side. In terms of indirect network effects: all other things equal, the side which "needs" the other side relatively more will pay more.

Our model however yields two new results. First, the platform makes relatively more profits on the developer side of the market when developers extract a larger share λ of their marginal contribution to social surplus (per platform user). In particular, if this share is large enough ($\lambda \geq \frac{1}{\varepsilon_V(1+\varepsilon_F)}$) then the platform may even find optimal to subsidize the participation of users and make all of its profits on developers. Conversely, if this share is too low ($\lambda \leq \frac{\varepsilon_H}{1+\varepsilon_H}$) - this happens for example when competition among developers is too strong - then the platform will subsidize developers and recoup on users. This result is quite intuitive and ressembles the pricing principle stated above.

Second and most important, developers pay relatively more when the "intensity" of users' preferences for variety ε_V is higher. To see this more

[33] Condition 3 in appendix A1 is necessary for stability and implies that $\varepsilon_V(1+\varepsilon_F) < 1$.

clearly, let us use the formulation of user preferences from example 2, with $V(n) = An^{\beta}$. Then the optimal pricing structure is:

$$\frac{\Pi^{PD}}{\Pi^{PU}} = \frac{\beta(1+\varepsilon_F)}{(1+\varepsilon_H)(1-\beta(1+\varepsilon_F))}$$

This expression contains an interesting and plausible partial explanation for the different pricing structures we observe, across software platforms in particular. Indeed, while we are aware of no empirical evidence that the elasticities of developer and user demand for platforms are significantly different across computer-based industries, there are good reasons to believe that user demand for application variety is higher for videogames than for productivity-oriented or professional software (for computers, PDAs, smartphones or other electronic devices). The most important such reason is durability: by definition, games get "played out"[34], whereas professional software is theoretically infinitely durable (technological obsolescence notwithstanding of course). Consequently, users of videogame consoles will demand a constant stream of games throughout a console's lifecycle, whereas computer users will generally stick to a few applications that they always use. As pointed out by Campbell-Kelly (2003):

"[...] Thus, while the personal computer market could bear no more than a few word processors or spreadsheet programs, the teenage videogame market could support an indefinite number of programs in any genre. In this respect, videogames were, again, more like recorded music or books than like corporate software..."[35]

Given this difference, our model predicts that the pricing structures should be such that videogame platforms make a larger relative share of profits on developers than the other software platforms, which is precisely what we observe[36].

More generally, our model implies that the platform pricing structure will "favor" users (i.e. developers will acount for a larger share of profits)

[34] Coughlan (2001).
[35] Campbell-Kelly (2003), p. 281.
[36] Evans Hagiu and Schmalensee (2004).

	Platform	Pricing structure	
Increasing user preference for variety →		Developers	Users
	Ticket Master	≤0	>0
	OS for computers	≤0	>0
	OS for handheld assistants	≤0	>0
	OS for smartphones	≤0	>0
	Priceline.com	>0	=0
	Shopping Malls	>0	=0
	Wireless networks (i-mode)	>0	>0
	Videogame consoles	>0	≤0

Figure 3:

in industries in which the intensity of user preferences for diversity is inherently high. The table in figure 3 contains most of the industries/platforms mentioned above with their corresponding pricing structures[37], organized by increasing order of user demand for variety[38]. With the exception of wireless networks, it appears that in a first-order approximation, the prediction of our model is consistent with what we observe in reality.

In the following two subsections we use our model to study the impact of two other factors on the optimal platform pricing structure: uncertainty with respect to the availability of applications and limited supply of high-quality applications. The empirical and case study literature suggests that these two factors are specific to the videogame industry, however it should be clear that the insights we draw apply more generally, to a larger variety of industries.

[37]Given the lack of precise data, we have contented ourselves with providing the sign of profits made on each side of the market.

[38]Although this ranking is quite intuitive, it should be noted that it is based solely on casual empiricism and not on rigorous econometric analysis, which is the next logical step of this research.

In order to keep the analysis clear and tractable, we will use the formulation of user preferences presented in example 2 throughout the remainder of this section, i.e. $V(n) = An^{\beta}$, $0 < \beta < 1$ and $\lambda = 1$. This specification does not lose any substance: it is straightforward to conduct the same elasticity-based analysis in a more general setting, however analytical complexity would obscure the main insights without yielding any additional ones.

3.1 Uncertainty and risk-aversion

Three studies of the videogame industry, Brandenburger (1995), Coughlan (2001) and Clements and Ohashi (2004), convincingly argue that videogame users are unwiling to pay too much for consoles because of uncertainty regarding future prices, availability and quality of videogames. Indeed, since games are less durable than "productivity-oriented" software, users acquire many of them in periods subsequent to the purchase of the platform (whereas most non-game applications in the case of computers or other electronic devices are acquired at the same time as the platform), so that there is significantly more scope for uncertainty. In other words, as Clements and Ohashi (2004) explain, the "user holdup" problem is much more severe in the market for videogames. Once a user has adopted a platform, he is locked in to a certain extent, therefore developers supporting that platform can extract a large part of his valuation for applications. Of course, there is no reason why users shouldn't be able to factor this into their *ex-ante* platform adoption decisions, however users might have a different (presumably more "conservative", given asymmetric information) view of the relevant uncertainty than developers.

We introduce uncertainty in our model in the following highly stylized way: we assume that between stages 2 and 3, with positive probability, *all* developers become unable to provide their applications to users of the platform due to some exogenous *common* shock[39]. We allow users and developers to have different perceptions of the probability of this "market

[39] To fix ideas, it could be for example that after users have purchased and developers have decided to adopt the platform, the latter turns out to be technologically deficient, so that it becomes impossible to write applications for it.

breakdown", namely developers believe it will happen with probability $u \in [0,1]$ whereas users believe it will happen with probability $u + \Delta u \in [0,1]$. Consistent with the case studies mentioned above, we assume $\Delta u \geq 0$, i.e. users are more "pessimistic" than developers: this could be because users are more "risk-averse" or because they discount the utility derived from future purchases of applications at a higher rate relative to developers.

As before, the marginal user θ_m is indifferent between adopting the platform and not adopting it, i.e. obtains 0 expected utility from adoption:

$$\theta_m = (1 - u - \Delta u)\left(V(n) - nV'(n)\right) - P^U \tag{8}$$

Similarly, developer demand n as a function of P^D and θ_m is such that the marginal developer $\phi = H^{-1}(n)$ obtains 0 expected profits:

$$(1-u)\, V'(n)\, F(\theta_m) - P^D - H^{-1}(n) = 0 \tag{9}$$

Platform profits are then:

$$\Pi^P = P^U F(\theta_m) + nP^D$$

$$= \left((1-u-\Delta u)\, V(n) + \Delta u n V'(n) - \theta_m\right) F(\theta_m) - nH^{-1}(n) \tag{10}$$

The first order conditions are now[40]:

$$\frac{(1-u-\Delta u)\, V(n) + \Delta u n V'(n) - \theta_m}{\theta_m} = \frac{1}{\varepsilon_F} \tag{11}$$

$$\left((1-u)\, V'(n) + \Delta u n V''(n)\right) F(\theta_m) = nH^{-1\prime}(n) + H^{-1}(n) \tag{12}$$

The following proposition characterizes the resulting optimal platform pricing structure.

Proposition 2 *Assume user preferences are as specified in example 2 and that with positive probability all developers become unable to supply applications for the platform in stage 3: users perceive the probability of*

[40] The three conditions in appendix A1 are also sufficient for ensuring concavity and stability for this case.

this happening as being $u + \Delta u$, while developers perceive it as being u. Then the optimal platform pricing structure is:

$$\frac{\Pi^{PD}}{\Pi^{PU}} = \frac{\beta \left(1 + \varepsilon_F\right) \left(1 - u + \Delta u \varepsilon_H \left(1 - \beta\right)\right)}{\left(1 + \varepsilon_H\right) \left(\left(1 - u\right) \left(1 - \beta \left(1 + \varepsilon_F\right)\right) - \Delta u \left(1 - \beta\right)\right)} \tag{13}$$

If $\frac{\Delta u}{1-u} \geq \frac{1-\beta(1+\varepsilon_F)}{1-\beta}$ then the optimal platform pricing structure is such that the platform subsidizes users, i.e. $\Pi^{PU} < 0$.

If $\frac{\Delta u}{1-u} < \frac{1-\beta(1+\varepsilon_F)}{1-\beta}$ then the platform makes positive profits on both sides of the market and the optimal pricing structure is such that the share of profits made on the developer side relative to the share of profits made on the user side of the market is increasing both in u and in Δu, decreasing in the elasticity of developer demand for the platform ε_H, increasing in the elasticity of user demand for the platform ε_F and increasing in the intensity of user preferences for application variety β.

Proof Using the first-order conditions, we can write the pricing structure as:

$$\frac{\Pi^{PD}}{\Pi^{PU}} = \frac{n^2 H^{-1\prime}\left(n\right) - \Delta u n^2 V''\left(n\right) F\left(\theta_m\right)}{\left(\left(1 - u - \Delta u\right) \left(V\left(n\right) - nV'\left(n\right)\right) - \theta_m\right) F\left(\theta_m\right)}$$

The first order conditions (11) and (12) also imply (using $V\left(n\right) = An^{\beta}$):

$$\frac{\theta_m}{V\left(n\right)} = \frac{\varepsilon_F \left(1 - u - \left(1 - \beta\right) \Delta u\right)}{1 + \varepsilon_F}$$

and:

$$\frac{V'\left(n\right) F\left(\theta_m\right)}{nH^{-1\prime}\left(n\right)} \left(1 - u + \left(\beta - 1\right) \Delta u\right) = 1 + \varepsilon_H$$

where we have used $\frac{nV''(n)}{V'(n)} = \beta - 1$.

We obtain:

$$\begin{aligned}
\frac{\Pi^{PD}}{\Pi^{PU}} &= \frac{nV'\left(n\right)}{V\left(n\right)} \frac{\frac{1-u-(1-\beta)\Delta u}{1+\varepsilon_H} - \Delta u \left(\beta - 1\right)}{\left(1 - u - \Delta u\right)\left(1 - \beta\right) - \frac{\varepsilon_F (1-u-(1-\beta)\Delta u)}{1+\varepsilon_F}} \\
&= \frac{\beta \left(1 + \varepsilon_F\right) \left(1 - u + \Delta u \varepsilon_H \left(1 - \beta\right)\right)}{\left(1 + \varepsilon_H\right) \left(\left(1 - u\right) \left(1 - \beta \left(1 + \varepsilon_F\right)\right) - \Delta u \left(1 - \beta\right)\right)}
\end{aligned}$$

Finally, the only comparative static which is not obvious is the influence of β. To prove that $\frac{\Pi^{PD}}{\Pi^{PU}}$ is increasing in β when it is positive, it suffices to show that $\frac{1-u+\Delta u\varepsilon_H(1-\beta)}{(1-u)(1-\beta(1+\varepsilon_F))-\Delta u(1-\beta)}$ is increasing in β, and this fraction can be re-written as:

$$\frac{\Delta u\varepsilon_H}{(1-u)(1+\varepsilon_F)-\Delta u}+\frac{1-u+\Delta u\varepsilon_H\left(1-\frac{1-u-\Delta u}{(1-u)(1+\varepsilon_F)-\Delta u}\right)}{1-u-\Delta u-\beta\left((1-u)(1+\varepsilon_F)-\Delta u\right)}$$

Since the denominator of the last term on the right is positive and decreasing in β, it suffices to show that the numerator is positive, which is equivalent to:

$$1-u+\Delta u\varepsilon_H\frac{(1-u)\varepsilon_F}{(1-u)(1+\varepsilon_F)-\Delta u}>0$$

and this inequality holds because $(1-u)(1+\varepsilon_F)-\Delta u>0$.

■

Note that when there is no uncertainty, i.e. $u=\Delta u=0$ or even when there is uncertainty but users' and developers' expectations are consistent, i.e. $\Delta u=0<u$, (13) is identical to (7) with $\lambda=1$ and $\varepsilon_V=\beta$.

Despite the highly simplistic way in which we have modeled uncertainty, the result contained in Proposition 2 yields a prediction which is consistent with the empirical and case studies of the videogame market mentioned above and reveals a rather interesting insight. If at the time they adopt the platform users discount the surplus they derive from purchasing developers' products at a higher rate (true in the videogame market because a larger relative share of that surplus is more distant in the future), then we have shown that the optimal pricing structure involves making relatively more money on developers. Also, keeping the difference between the uncertainty perceived by users relative to developers Δu constant, when there is more uncertainty, i.e. u increases, the optimal pricing structure shifts again towards making more profits on developers. Note however that for this to be true it is necessary that $\Delta u>0$, i.e. user and developer expectations have to be inconsistent.

Also, if there is sufficient uncertainty, i.e. for u and Δu high enough, the optimal pricing structure is such that the platform subsidizes users, which is exactly what we observe in the videogame market.

3.2 Limited supply of high-quality developers

Up to now we have assumed that the platform benefitted from an unlimited supply of third-party applications, or at least greater than it needed in order to satisfy users' demand for variety and maximize profits. This may of course not be realistic, precisely in markets such as videogames, in which, as argued above, users have a particularly strong preference for variety. More specifically, even if there are a lot of willing independent suppliers of videogames, many of them may be of doubtful quality. It seems indeed that the quality of third-party games is an important issue: since Nintendo entered the US videogame market in 1986, all console manufactorers have used a security chip designed to lock out unauthorized third-party game publishers and have enforced strict policies with respect to developer access in order to avoid an overflow of poor quality games[41]. This suggests that although videogame users demand great variety, console manufacturers have to restrict the supply of games to a certain extent. We leave the interesting antitrust implications of this form of exclusion for future research and focus here on its effects on the optimal platform pricing structure.

For simplicity, in this subsection we also assume that all developers (high and low quality) have the same fixed development cost ϕ and that the elasticity of user demand for the platform ε_F is constant and satisfies $\varepsilon_F \leq 1$ and $\beta (1 + \varepsilon_F) < 1$[42].

If all applications are of high quality ($q_H = 1$) then the optimal levels of user and developer adoption, θ_m^{2sp} and n^{2sp}, are defined by the two first-order conditions:

$$\frac{V(n^{2sp}) - \theta_m^{2sp}}{\theta_m^{2sp}} = \frac{1}{\varepsilon_F}$$

[41] See Kent (2003) and Campbell-Kelly (2003). Playstation's and XBox's current access policies can be found at http://www.scea.sony.com/2b_ldev.php for Sony and http://www.xbox.com/en-us/dev/NARegDev.htm for Microsoft.

The security chip and, most famously, the Seal of Quality, were introduced by Nintendo in the wake of the videogame market crash of 1982-3. The crash is attributed by most industry analysts to a flood of poor-quality games, which console manufacturers did not control and which led to the collapse of hardware and software prices, forcing many firms out of business, Atari being the most prominent victim (see Campbell-Kelly (2003), p. 279-86).

[42] This is condition 3 in appendix A1 for the case $\lambda = 1$, $\varepsilon_V = \beta$: it ensures stability.

$$V'\left(n^{2sp}\right)F\left(\theta_m^{2sp}\right)=\phi$$

Assume however that the supply of high-quality applications is limited to $N<n^{2sp}$ and that the rest are of quality $q_L<1$. This means that gross user surplus from using N high-quality applications and $n\geq 0$ low-quality applications is:

$$V\left(N+q_L n\right)$$

If users can distinguish between high and low quality applications[43], the price equilibrium among developers determined in example 2 is easily extended to this case:

$$p_H=V'\left(N+nq_L\right)$$

$$p_L=q_L V'\left(N+nq_L\right)$$

where p_i is the price of a quality i application, $i=L,H$. In other words, each developer extracts his marginal contribution to user surplus, which is the same across users.

Assuming the platform can discriminate between high and low quality applications (i.e. charge two different prices P_L^D and P_H^D), its profits are:

$$\Pi^P=\left(V\left(N+nq_L\right)-\theta_m\right)F\left(\theta_m\right)-\left(N+n\right)\phi$$

The first order condition in θ_m is:

$$\theta_m=\frac{V\left(N+q_L n\right)\varepsilon_F}{1+\varepsilon_F}$$

However, the platform may find it profitable *not* to allow any low-quality applications in the market. Using the envelope theorem, this happens whenever:

$$\frac{\partial\Pi^P}{\partial n}\left(\theta_m=\frac{V\left(N+q_L n\right)\varepsilon_F}{1+\varepsilon_F},n=0\right)<0$$

[43] This is a reasonable assumption nowadays, given that there are hundreds of specialized magazines and websites which review videogames, so that users can form a fairly precise idea of the "quality" of a game prior to purchasing it (modulo some inherent intangible "value" dimensions of course).

which is equivalent to $q_L V'(N) F\left(\frac{\varepsilon_F V(N)}{1+\varepsilon_F}\right) < \phi$[44]. In this case, the platform restricts access to the N high-quality developers by charging

$$P^D = V'(N) F\left(\frac{\varepsilon_F V(N)}{1+\varepsilon_F}\right) - \phi$$

Let $\alpha(N) = \frac{\phi}{V'(N)F\left(\frac{\varepsilon_F V(N)}{1+\varepsilon_F}\right)} < 1$, where $\alpha(.)$ is increasing[45] and $\alpha(n_{2sp}) = 1$. The resulting platform pricing structure is:

$$\frac{\Pi^{PD}}{\Pi^{PU}} = \frac{\beta(1+\varepsilon_F)(1-\alpha(N))}{1-\beta(1+\varepsilon_F)}$$

Thus, when the available supply of complements becomes smaller (N decreases), the pricing structure shifts in favor of users, i.e. $\frac{\Pi^{PD}}{\Pi^{PU}}$ *increases*. This result is understood in terms of relative scarcity: the more scarce developers become relative to what users demand, the higher their marginal value to users and therefore the larger the revenues they are able to extract from users, so that it is optimal for the platform to charge developers relatively more.

If on the other hand $\frac{\partial \Pi^P}{\partial n}\left(\theta_m = \frac{V(N+q_L n)\varepsilon_F}{1+\varepsilon_F}, n=0\right) \geq 0$ then the platform will allow n_L low-quality applications, where n_L is defined by:

$$q_L V'(N+n_L q_L) F\left(\frac{\varepsilon_F V(N+n_L q_L)}{1+\varepsilon_F}\right) = \phi \qquad (14)$$

The pricing structure in this case is:

$$\frac{\Pi^{PD}}{\Pi^{PU}} = \frac{N(1-q_L)}{N+n_L q_L}\frac{\beta(1+\varepsilon_F)}{1-\beta(1+\varepsilon_F)}$$

The comparative statics obtained from the last two equations are quite interesting: the pricing structure shifts in favor of developers (i.e. $\frac{\Pi^{PD}}{\Pi^{PU}}$ decreases) when q_L *increases* and N *decreases*[46]. The effect of q_L is intuitive: users pay more when the quality of applications increases. The effect of N

[44]Note that the condition for there to be a shortage of high-quality developers $N < n^{2sp}$ is equivalent to $V'(N) F\left(\frac{\varepsilon_F V(N)}{1+\varepsilon_F}\right) > \phi$ and therefore does not preclude the profitability of excluding low quality developers.

[45]Indeed: $\frac{d\left(V'(N)F\left(\frac{\varepsilon_F V(N)}{1+\varepsilon_F}\right)\right)}{dN} = V''(N) F\left(\frac{\varepsilon_F V(N)}{1+\varepsilon_F}\right)\left(1-\frac{\beta\varepsilon_F}{1-\beta}\right) < 0$

[46]To see this, note that, as long as $q_L V'(N) F\left(\frac{\varepsilon_F V(N)}{1+\varepsilon_F}\right) \geq \phi$, (14) implies that $x = N + nq_L$ does not depend on N and is an increasing function of q_L.

however is opposite to the one when low-quality developers are excluded: the lost high-quality applications are replaced by low-quality ones so that total developer revenues are in effect decreasing when N decreases and therefore the platform finds it optimal to extract relatively more revenues from users.

Proposition 3 summarizes the preceding analysis.

Proposition 3 *If the supply of high-quality developers is limited to N then:*

- *If $V'(N)\,F\left(\frac{\varepsilon_F V(N)}{1+\varepsilon_F}\right) \leq \phi$ then $N \leq n^{2sp}$, the constraint is not binding and the optimal platform pricing structure is $\frac{\Pi^{PD}}{\Pi^{PU}} = 0$*
- *If $\phi < V'(N)\,F\left(\frac{\varepsilon_F V(N)}{1+\varepsilon_F}\right) \leq \frac{\phi}{q_L}$ then the constraint is binding, the platform restricts access to the N high-quality applications and $\frac{\Pi^{PD}}{\Pi^{PU}}$ is decreasing in N*
- *If $V'(N)\,F\left(\frac{\varepsilon_F V(N)}{1+\varepsilon_F}\right) > \frac{\phi}{q_L}$ then the constraint is binding but the platform allows some low-quality applications to enter and $\frac{\Pi^{PD}}{\Pi^{PU}}$ is increasing in N*

■

Corollary *When there are no low-quality applications (i.e. $q_L = 0$) $\frac{\Pi^{PD}}{\Pi^{PU}}$ is decreasing in N (weakly).*■

Figure 4 illustrates the results contained in Proposition 3.

4 Two-sided proprietary platforms, open platforms and social efficiency

Up to here we have focused exclusively on two-sided proprietary platforms. However, given the increasing popularity of open platforms such as Linux, Apache and other open source software systems, it is interesting from an economic theory perspective and important from an economic policy perspective to compare the efficiency of proprietary, profit-maximizing platforms to that of open platforms, in terms of induced product variety, user adoption and total social welfare.

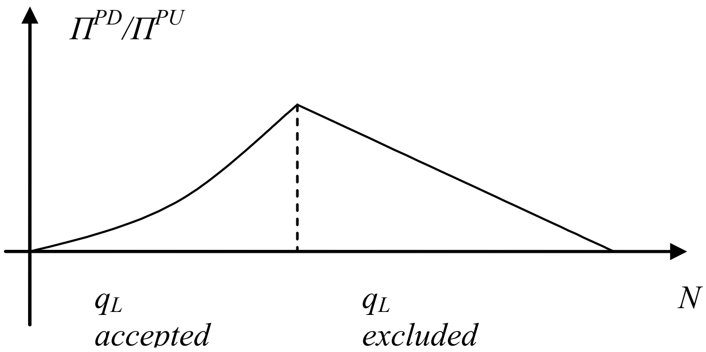

Figure 4:

In our framework, an open platform is simply a platform allowing free-entry of both users and developers, i.e. charging prices equal to marginal costs (0) on both sides of the market. Although this may be a very simplified conception of, say, the open source software form of market organization[47], we believe it is sufficient for revealing a fundamental welfare tradeoff between the two types of platform. An open platform avoids the two-sided deadweight loss due to monopoly pricing but at the same time does not internalize any of the indirect network effects between users and developers, whereas a profit-maximizing platform does so to a certain extent through its prices.

Note that in a one-sided market the comparison is straightforward: a firm pricing at marginal cost does always better in terms of social welfare than a profit-maximizing monopolist who cannot price-discriminate and therefore inefficiently restricts output. By contrast, in a two-sided context, things are more complex: as we show below, a proprietary platform need not necessarily induce less developer entry and user adoption than an open platform, nor need it even result in socially insufficient entry.

A benevolent social planner maximizes total welfare, which in our framework is the difference between total surplus from indirect network effects and

[47]In particular, "free entry" of users and developers is certainly not a perfect representation of the licensing agreements characteristic of open source software (BSD or GPL).

the costs of entry on the two sides of the market:

$$W\left(\theta_m, n\right) = V\left(n\right) F\left(\theta_m\right) - \int_0^{\theta_m} \theta f\left(\theta\right) d\theta - \int_0^{H^{-1}(n)} \phi h\left(\phi\right) d\phi \quad (15)$$

By contrast, a two-sided proprietary platform maximizes profits:

$$\Pi^P\left(\theta_m, n\right) = \left(V\left(n\right) - \theta_m\right) F\left(\theta_m\right) - nH^{-1}\left(n\right)$$

In the case of an open platform (or free entry regime) n and θ_m are determined as follows:

$$\pi_m^D\left(n\right) = \pi\left(n\right) F\left(\theta_m\right) - H^{-1}\left(n\right) = 0 \quad (16)$$

$$\theta_m = u\left(n\right) = V\left(n\right) - n\pi\left(n\right) \quad (17)$$

where $\pi_m^D\left(n\right)$ are net profits of the marginal developer when n developers have entered.

Consider first the developer side of the market. The derivative of total social welfare with respect to n is:

$$\frac{\partial W}{\partial n} = V'\left(n\right) F\left(\theta_m\right) - H^{-1}\left(n\right) = \pi_m^D\left(n\right) + \left(V'\left(n\right) - \pi\left(n\right)\right) F\left(\theta_m\right) \quad (18)$$

Thus, if one looks only at the developer side of the market, what drives a wedge between the levels of product diversity under an open platform relative to the socially optimal level is the term $\left(V'\left(n\right) - \pi\left(n\right)\right) F\left(\theta_m\right)$. If developer profits per platform user $\pi\left(n\right)$ exceed the marginal contribution of an additional developer to social welfare per platform user $V'\left(n\right)$ (i.e. $\lambda > 1$), then $\frac{\partial W}{\partial n} < \pi_m^D\left(n\right)$ and therefore an open platform tends to induce excessive entry of developers *all other things equal*. And viceversa. This is precisely the insight of Mankiw and Whinston (1986). To see this more clearly, consider example 1:

$$V'\left(n\right) - \pi\left(n\right) = \underbrace{n\left(G'v' - c\right)\frac{\partial q_n}{\partial n}}_{\text{business-stealing}} + \underbrace{G' \times \left(v - v'q_n\right)}_{\text{product diversity}}$$

The first term represents the business stealing effect and is negative as long as $\frac{\partial q_n}{\partial n} < 0$ and the price $G'v'$ is above marginal cost, whereas the second term is the product diversity effect and is positive since v is concave. The inefficiency of an open platform on the developer side depends on which of these two effects dominates. In example 2 we have $\pi(n) = V'(n)$, so that the open platform introduces no bias with respect to developer entry *all other things equal.*

But of course, all other things are *not* equal in our model, since developer entry depends on user entry and viceversa. As we show below, the open platform induces too little user entry, which in turn leads to too little developer entry, an indirect effect which does not exist in Mankiw and Whinston (1986).

Consider now the derivative of a two-sided platform's profits with respect to n:

$$\frac{\partial \Pi^P}{\partial n} = V'(n) F(\theta_m) - H^{-1}(n) - nH^{-1\prime}(n) \tag{19}$$

$$= \pi_m^D(n) + (V'(n) - \pi(n)) F(\theta_m) - nH^{-1\prime}(n) \tag{20}$$

Comparing (20) with (18), the proprietary platform introduces no inefficiency through the business stealing and the product diversity effects. This is of course due to our simplifying assumption that both users and developers are only horizontally differentiated, so that the platform fully internalizes developer revenues $n\pi(n)$ and user gross surplus $V(n) - n\pi(n)$[48]. What does induce a bias however is the proprietary platform's inability to perfectly price discriminate among developers: it consequently discounts the total social value created by an additional developer by $nH^{-1\prime}(n)$, the revenues lost on existing developers by reducing the price P^D in order to accomodate the additional developer. Since this bias is negative, the proprietary platform tends to induce too little entry on the developer side, keeping everything else constant.

Turning now to the user side of the market, the first order condition with respect to θ_m are:

$$\theta_m = V(n) \tag{21}$$

[48] In Hagiu (2004c) we introduce user and developer vertical differentiation and show that the main results continue to hold.

for the social planner and:

$$u(n) - P^U = \theta_m = \frac{\varepsilon_F V(n)}{1+\varepsilon_F} \tag{22}$$

for the proprietary platform.

Comparing (21) to (17), the open platform induces too little user adoption all other things equal, because each developer who enters does not take into account the effect of his price on *total* user demand for the platform. Comparing (22) to (21), the proprietary platform also induces too little user entry: it perceives the benefits of an additional user as the difference between the extra revenues $P^U + n\pi(n) = V(n) - \theta_m$, which are also equal to the total social value created by the additional user[49], and $\frac{F(\theta_m)}{f(\theta_m)}$, the revenues lost on existing users by reducing the price P^U in order to accomodate the additional user.

Comparing (17) and (22), it is not possible to say in general which of the open platform or the proprietary platform restricts user adoption more. It all depends on the sign of P^U: the proprietary platform induces less restriction of user entry if and only if it subsidizes users, i.e. sets $P^U < 0$. This stresses the importance of the choice of pricing structure for overall efficiency: by being able to balance the interests of the two sides, a proprietary platform may come closer to the socially optimal level of adoption than a platform simply pricing at marginal cost on both sides.

To sum up this discussion, given that a proprietary platform induces a bias towards insufficient entry on both sides of the market, the combination of the two leads unambiguously to insufficient product diversity and user adoption relative to the socially optimal level. This of course is not a robust result: in Hagiu (2004c) we provide an example in which developers are vertically differentiated and the platform is unable to extract all of their revenues, so that it *overestimates* the magnitude of positive indirect effects and therefore induces excessive entry on both sides.

The robust result is that the comparison between open platforms, proprietary platforms and social planner in terms of the induced levels of product diversity and user adoption is indeterminate. Indeed, although in our framework an open platform is biased towards socially insufficient entry on the

[49]This is because users are horizontally differentiated.

user side, its bias on the developer side can go both ways and, through the mechanism of indirect network effects, may or may not outweigh the direct bias. To see this clearly, let us write the equations[50] which implicitly determine n^{2sp}, n^{fe}, n^{so}:

$$V'\left(n^{2sp}\right)F\left(\frac{\varepsilon_F V\left(n^{2sp}\right)}{1+\varepsilon_F}\right)=n^{2sp}H^{-1\prime}\left(n^{2sp}\right)+H^{-1}\left(n^{2sp}\right) \qquad (23)$$

for the proprietary platform;

$$\pi\left(n^{fe}\right)F\left(u\left(n^{fe}\right)\right)=H^{-1}\left(n^{fe}\right) \qquad (24)$$

for the open platform and

$$V'\left(n^{so}\right)F\left(V\left(n^{so}\right)\right)=H^{-1}\left(n^{so}\right) \qquad (25)$$

for the social planner.

Since $V(n) > \frac{\varepsilon_F V(n)}{1+\varepsilon_F}$, it is clear that $n^{2sp} < n^{so}$. However, since $\frac{\pi(n)}{V'(n)} = \lambda(n)$ can be either larger or smaller than 1, it is not possible to say whether $n^{fe} \lesseqgtr n^{so}$. When $\lambda(n) \leq 1$ we have $n^{fe} < n^{so}$, but if $\lambda(n) > 1$ it may well be that the business-stealing effect prevails so that $n^{fe} > n^{so}$.

Comparing (23) and (24), it is even harder to say whether product diversity (and user adoption) is higher under an open or a proprietary platform. Figure 5 illustrates the case when $n^{2sp} < n^{fe} < n^{so}$.

Finally, we have to compare the levels of total social welfare under a proprietary platform and an open platform. Given the variety of possibilities we have obtained above regarding the relative levels of product variety and user adoption, one would expect the same indeterminacy with respect to total social welfare.

Proposition 4 confirms this indeterminacy and illustrates some of the possibilities described above through a specific example.

Proposition 4 *Assume that user preferences are as described in example 1 above, with* $0 < \alpha < \sigma < 1$, $F(\theta) = \frac{\theta}{\theta_H}$; *that all developers have*

[50] We assume that n^{2sp}, n^{so} and n^{fe} are well-defined, i.e. the left-hand sides of (23), (24) and (25) are decreasing in n, while the right-hand sides are increasing. This is true for example under the three conditions provided in appendix A1.

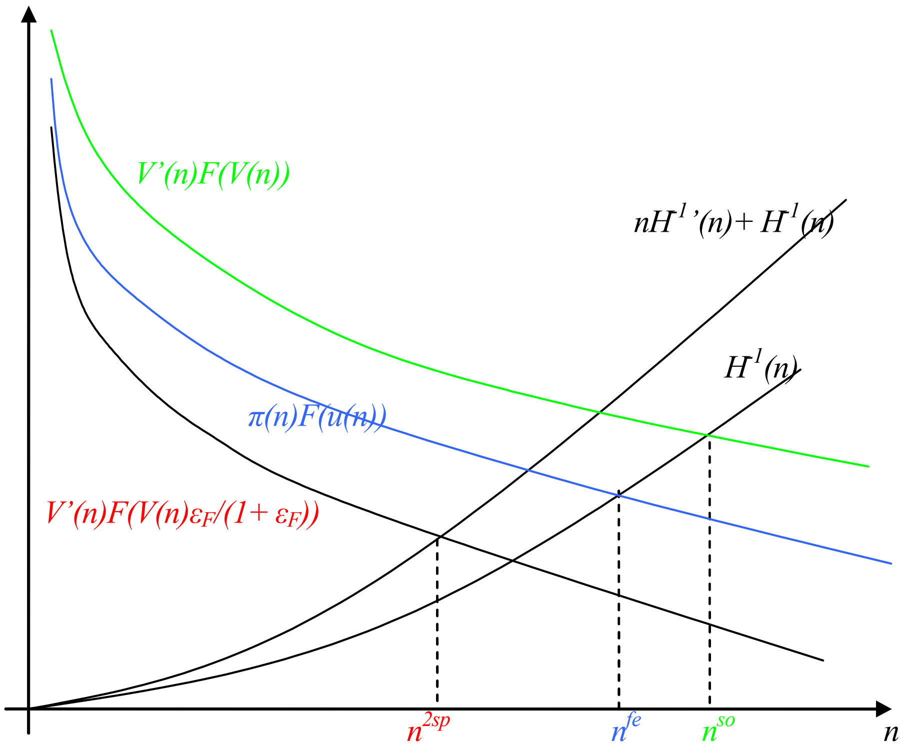

Figure 5:

the same fixed cost $\phi > 0$ and that the stability-concavity conditions 1, 2 and 3 in appendix A1 hold.

Then both the open platform and the two-sided proprietary platform induce insufficient product diversity and user adoption: $n^{2sp}, n^{fe} < n^{so}$ and $\theta_m^{2sp}, \theta_m^{fe} < \theta_m^{so}$. Furthermore, $n^{2sp} > n^{fe}$ if and only if $(1-\sigma\alpha)^2 > 2\sigma(1-\alpha)^2$.

Total social welfare can also be higher with either type of platform:

- *If* $\alpha \to 0$, $\sigma \to 0$ *and* $\frac{\alpha}{\sigma} \to k < 1$, *then* $\frac{W_{2sp}}{W_{fe}} \to +\infty$, *so that social welfare becomes infinitely higher under a profit-maximizing platform*
- *If* $\sigma \to 1$ *and* $\alpha < 1$ *is fixed, then* $\frac{W_{2sp}}{W_{fe}} \to \frac{3}{4}$, *so that social welfare is higher under an open platform.*

Proof See appendix.■

To sum up the preceding analysis, the efficiency of a proprietary platform relative to an open platform depends on the interaction of three factors: two-sided deadweight loss from monopoly pricing, the strength of the business-stealing effect relative to the product diversity effect and the extent to which the proprietary platform internalizes indirect network externalities on both sides of the market. In general, either of these three factors may predominate, implying that in some cases proprietary platforms are more efficient than open platforms and viceversa in other cases.

5 Conclusion

This paper has presented a model of two-sided platforms which -we hope- contributes to throwing some light on the economic factors driving business choices made by firms operating in an increasing number of industries central to the new economy, such as the Internet, software for computers and other electronic devices, videogames, digital media and others.

From a positive perspective we have identified the intensity of user preferences for variety as a key factor driving optimal platform pricing structures. We have shown that when users care more about diversity the optimal pricing structure shifts towards making more profits on developers. This prediction of our model constitutes a plausible explanation for the contrasting choices of pricing structures observed in the industries mentioned above, in particular videogame consoles relative to software plaforms in most other computer-based industries. Indeed, it is consistent with the observation that videogames are intrinsically less durable goods than other types of software (so that game users have a stronger preference for variety) and with the empirical finding that videogame console manufacturers earn most of their profits from game publishers while operating system vendors for computers and other consumer electronics make the largest share of their revenues on users. Our model also predicts that the pricing structure shifts in the same direction (i.e. in favor of users) when there is more uncertainty with respect to the availability of applications and users have a more pessimistic view of this uncertainty than developers, and when the supply of high-quality applications is limited. This is again consistent with empirical studies of the videogame market.

From a normative perspective, we have compared the levels of product variety, user adoption and social welfare under two-sided proprietary (profit-maximizing) platforms and two-sided open platforms, and we have shown that either of these two forms of platform governance can be the more socially efficient one. This result implies that, in our framework, there is little economic justification for an a priori industrial policy preference for open platforms over proprietary ones. It therefore questions the validity of a presumption which has become increasingly popular with governments around the world and according to which open source software platforms such as Linux are *inherently* more efficient for the development of software industries than proprietary platforms such as Windows[51].

Clearly, this paper constitutes only an initial formal exploration of the

[51]Of course, there might be strategic reasons for such a preference which are not captured by our model. For example, one of the main reasons behind the Japanese Ministry of Economy Trade and Industry (METI)'s decision to participate in a recent joint government-industry alliance with China and Korea in order to develop open-source software is the perceived strength of Japan's consumer electronics sector but the relative weakness of its software industry. In this context, a hypothetical cross-device open source software platform would "commoditize" the operating system and allow strong hardware brands such as Sony, Matsushita, NEC, Fujitsu and Hitachi to extract most of the economic value of the hardware-software systems their products are part of. This is the exactly opposite scenario to the current situation in the personal computer industry, which is dominated by Microsoft through Windows.

economic issues raised by the category of two-sided market platforms on which we have focused and which we believe should be the topic of promising future research. On the theoretical side, our model can be extended to tackle the important issues of platform competition, developer multi-homing and exclusivity and the efficiency of alternative forms of platform governance[52]. On the empirical side, our model can provide the starting point for a more rigorous cross-industry analysis of pricing structures and other important business decisions such as the extent of vertical (dis)integration defining "platform scope".

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6 Appendix A1

In this appendix we prove that under the following three conditions all maximization problems we have considered in this paper are well-defined and the profit-maximizing solutions (θ_m, n) are stable given the coresponding prices

$\left(P^U, P^D\right)$.

Condition 1 *The elasticities* ε_H *and* ε_F *are constant, i.e.* $F(\theta_m) = \left(\frac{\theta_m}{\theta_H}\right)^{\varepsilon_F}$ *and* $H(\phi) = \left(\frac{\phi}{\phi_H}\right)^{\varepsilon_H}$.

Condition 2 *The elasticity* ε_V *is constant*[53], *i.e.* $V(n) = An^{\varepsilon_V}$, *the ratio* $\lambda = \frac{\pi(n)}{V'(n)}$ *is constant and* $\lambda\varepsilon_V < 1$.

Condition 3 $\varepsilon_F \leq 1$ and $0 < \varepsilon_V (1+\varepsilon_F) \max\left(\frac{1-\lambda\varepsilon_V}{1-\varepsilon_V}, 1\right) < 1$.

Note that condition 2 is satisfied by the explicit functional forms given in examples 1 and 2.

We start by proving stability. Consider the following two equations:

$$\theta_m = (1-u-\Delta u)(1-\lambda\varepsilon_V) V(n) - P^U \tag{26}$$

$$(1-u)\lambda V'(n) F(\theta_m) - P^D - H^{-1}(n) = 0 \tag{27}$$

with Δu, $u + \Delta u \in [0,1]$.

Lemma 1 *Assume* $\left(n^{2sp}, \theta_m^{2sp}\right)$ *is the unique*[54] *solution to:*

$$\frac{(1-u-(1-\lambda\varepsilon_V)\Delta u) V(n) - \theta_m}{\theta_m} = \frac{1}{\varepsilon_F} \tag{28}$$

$$(1-u-(1-\lambda\varepsilon_V)\Delta u) V'(n) F(\theta_m) = nH^{-1\prime}(n) + H^{-1}(n) \tag{29}$$

and let $\left(P^U_{2sp}, P^D_{2sp}\right)$ *be the unique solution to (26) and (27) given* $(\theta_m, n) = \left(\theta_m^{2sp}, n^{2sp}\right)$. *Then, conversely,* $\left(\theta_m^{2sp}, n^{2sp}\right)$ *is a stable solution to (26) and (27) given* $\left(P^U, P^D\right) = \left(P^U_{2sp}, P^D_{2sp}\right)$.

Proof Given $\left(P^U_{2sp}, P^D_{2sp}\right)$, (26) is equivalent to $\theta_m = \Theta_1(n)$ and (27) is equivalent to $\theta_m = \Theta_2(n)$. Stability simply means that at the intersection point $\left(\theta_m^{2sp}, n^{2sp}\right)$, $\Theta_2(n)$ cuts $\Theta_1(n)$ from below, or equivalently that the slope of $\Theta_2(n)$ is steeper than the slope of $\Theta_1(n)$, both evaluated at $n = n^{2sp}$, i.e.:

$$(1-u-\Delta u)(1-\lambda\varepsilon_V) V'\left(n^{2sp}\right) < \frac{d}{dn}\left[F^{-1}\left(\frac{P^D + H^{-1}(n)}{(1-u)\lambda V'(n)}\right)\right]\left(n^{2sp}\right)$$

[53] Recall also that $\varepsilon_V < 1$ by assumption 1.

[54] Uniqueness follows from the concavity proof below.

which, using (27) and $F^{-1\prime} = \frac{1}{f}$, is equivalent to (we omit function arguments in order to avoid clutter):

$$(1-u-\Delta u)(1-\lambda\varepsilon_V)V' < \frac{H^{-1\prime} - (1-u)\lambda V''F}{(1-u)\lambda V'f}$$

Given that $H^{-1\prime} > 0$, it is sufficient that:

$$(1-u-\Delta u)(1-\lambda\varepsilon_V)V'^2 f + V''F < 0 \Leftrightarrow \frac{(1-u-\Delta u)(1-\lambda\varepsilon_V)V'^2}{(-V'')V}\frac{Vf}{F} < 1$$

But (28) implies $V(n^{2sp}) = \frac{(1+\varepsilon_F)\theta_m}{\varepsilon_F(1-u-(1-\lambda\varepsilon_V)\Delta u)}$ so that the last inequality above is equivalent to:

$$\frac{(1-u-\Delta u)(1-\lambda\varepsilon_V)(1+\varepsilon_F)V'^2}{(1-u-(1-\lambda\varepsilon_V)\Delta u)(-V''V)} < 1$$

or:

$$\varepsilon_V(1+\varepsilon_F)\frac{(1-\lambda\varepsilon_V)(1-u-\Delta u)}{(1-\varepsilon_V)(1-u-\Delta u+\lambda\varepsilon_V\Delta u)} < 1$$

which is implied by condition 3 since $\frac{1-u-\Delta u}{1-u-\Delta u+\lambda\varepsilon_V\Delta u} \leq 1$.$\blacksquare$

Lemma 1 implies stability of the solutions defined by (1), (2), (5) and (6) setting $u = \Delta u = 0$ and of the solutions defined by (8), (9), (11) and (12) setting $\lambda = 1$ and noting that, under conditions 1, 2 and 3:

$$(1-u-\Delta u)V(n) + \Delta u n V'(n) = (1-u-(1-\varepsilon_V)\Delta u)V(n)$$

and

$$(1-u)V'(n) + \Delta u n V''(n) = (1-u-(1-\varepsilon_V)\Delta u)V'(n)$$

Let us turn now to concavity.

Lemma 2 *Let $\psi(\theta_m, n, K) = (KV(n) - \theta_m)F(\theta_m)$. Given K, ψ is concave in (θ_m, n) for all (θ_m, n) such that $KV(n) \geq \theta_m$.*

Proof $\frac{\partial\psi^2}{\partial n^2} = KV''(n)F(\theta_m) < 0$; $\frac{\partial\psi^2}{\partial\theta_m^2} = (KV(n) - \theta_m)f'(\theta_m) - 2f(\theta_m) < 0$, when $KV(n) \geq \theta_m$ and F has constant elasticity $\varepsilon_F \leq 1$. $\frac{\partial\psi^2}{\partial\theta_m\partial n} = KV(n)f(\theta_m)$. Thus, all we have left to verify is that the determinant of the hessian of ψ is non-negative, i.e. $\frac{\partial\psi^2}{\partial\theta_m^2}\frac{\partial\psi^2}{\partial\theta_m^2} \geq \left(\frac{\partial\psi^2}{\partial\theta_m\partial n}\right)^2$, or:

$$KV'(n)^2 f(\theta_m)^2 < V''(n) F(\theta_m) [(KV(n) - \theta_m) f'(\theta_m) - 2f(\theta_m)]$$

Using conditions 1, 2 and 3, after several straightforward simplifications, this is equivalent to:

$$KA(1-\varepsilon_F-\varepsilon_V) n^{\varepsilon_V} + (1-\varepsilon_V)(1+\varepsilon_F)\theta_m > 0$$

Since $KAn^{\varepsilon_V} \geq \theta_m$, for the last inequality to be satisfied it is sufficient that $(1-\varepsilon_F-\varepsilon_V)+(1-\varepsilon_V)(1+\varepsilon_F) > 0$, or $1-\varepsilon_V+1-\varepsilon_V(1+\varepsilon_F) > 0$, which is implied by the concavity of V and condition 3.■

The expression of platform profits (4) in section 1 can be written as $\Pi^P(\theta_m, n) = \psi(\theta_m, n, 1) - nH^{-1}(n)$. Applying lemma 2 and noting that $nH^{-1}(n)$ is convex in n under condition 1, Π^P is concave in (θ_m, n), so that the solution to the first order conditions (5) and (6) maximizes Π^P. Similarly, the expression of platform profits (10) in section 3.1. can be written as $\Pi^P(\theta_m, n) = \psi(\theta_m, n, 1-u-(1-\varepsilon_V)\Delta u) - nH^{-1}(n)$ and, by the same argument, it is also concave in (θ_m, n) and maximized by the solution to the first order conditions (11) and (12). Finally, under conditions 1, 2 and 3, the expression of social welfare (15) in section 4 can be written as:

$$\begin{aligned}
W &= V(n) F(\theta_m) - \frac{\varepsilon_F \theta_m^{\varepsilon_F+1}}{(1+\varepsilon_F)\theta_H^{\varepsilon_F}} - \frac{n^{\frac{1}{\varepsilon_H}+1}}{\frac{1}{\varepsilon_H}+1} \\
&= \left(V(n) - \frac{\varepsilon_F\theta_m}{1+\varepsilon_F}\right) F(\theta_m) - \frac{n^{\frac{1}{\varepsilon_H}+1}}{\frac{1}{\varepsilon_H}+1} \\
&= \frac{\varepsilon_F}{1+\varepsilon_F}\psi\left(\theta_m, n, \frac{1+\varepsilon_F}{\varepsilon_F}\right) - \frac{n^{\frac{1}{\varepsilon_H}+1}}{\frac{1}{\varepsilon_H}+1}
\end{aligned}$$

From expression (15) it is clear that W cannot be maximized by any $\theta_m > V(n)$ because a slight decrease in θ_m would increase W, therefore in the relevant domain $\frac{1+\varepsilon_F}{\varepsilon_F}V(n) > V(n) \geq \theta_m$. Then lemma 2 and the convexity of $n^{\frac{1}{\varepsilon_H}+1}$ imply that W is concave in (θ_m, n) on the relevant domain and is therefore maximized by the solution to the first-order conditions (18) and (21).

Lastly, it remains to show that the right-hand sides of (23), (24) and (25) are increasing and the left-hand sides are decreasing in n. Under condition 1, $H^{-1}(n)$ and $nH^{-1\prime}(n) + H^{-1}(n)$ are both increasing in n and under conditions 1,2 and 3:

$$\frac{d\left(V'(n)\,F\left(\frac{\varepsilon_F V(n)}{1+\varepsilon_F}\right)\right)}{dn} = -V''(n)\,F\left(\frac{\varepsilon_F V(n)}{1+\varepsilon_F}\right)\left(\frac{\varepsilon_V \varepsilon_F}{1-\varepsilon_V} - 1\right) < 0$$

$$\frac{d\left(\pi(n)\,F(u(n))\right)}{dn} = -\lambda V''(n)\,F(u(n))\left(\frac{\varepsilon_V \varepsilon_F}{1-\varepsilon_V} - 1\right) < 0$$

$$\frac{d\left(V'(n)\,F(V(n))\right)}{dn} = -V''(n)\,F(V(n))\left(\frac{\varepsilon_V \varepsilon_F}{1-\varepsilon_V} - 1\right) < 0$$

7 Appendix A2

Proof of Proposition 4 Let $\beta = \frac{\sigma-\alpha}{\sigma(1-\alpha)} = 1-\varepsilon_V$ and recall $\lambda = \frac{\sigma(1-\alpha)}{1-\sigma\alpha} \in]0,1[$. Then the second part of condition 3 is equivalent to:

$$2(1-\beta) < \frac{\beta}{1-\lambda(1-\beta)} \tag{30}$$

Note that (30) and $\lambda < 1$ imply that $\beta > \frac{1}{2}$. (23), (24) and (25) become:

$$\frac{(1-\beta)(1-\sigma\alpha)^2}{2} n_{2sp}^{1-2\beta} = \frac{\theta_H \phi}{B^2} \tag{31}$$

$$\alpha(1-\sigma)(1-\alpha)\, n_{fe}^{1-2\beta} = \frac{\theta_H \phi}{B^2} \tag{32}$$

$$(1-\beta)(1-\sigma\alpha)^2\, n_{so}^{1-2\beta} = \frac{\theta_H \phi}{B^2} \tag{33}$$

where $B = \left(\frac{\alpha\sigma}{c}\right)^{\frac{\alpha}{1-\alpha}}$. Then, since $1-2\beta < 0$ and $\frac{(1-\beta)(1-\sigma\alpha)^2}{\alpha(1-\sigma)(1-\alpha)} = \left(\frac{1-\sigma\alpha}{1-\alpha}\right)^2 \frac{1}{\sigma} > 1$, we have $n^{2sp}, n^{fe} < n^{so}$. Moreover $n^{2sp} > n^{fe}$ if and only if $\frac{(1-\beta)(1-\sigma\alpha)^2}{2} >$

$\alpha(1-\sigma)(1-\alpha)$, which is equivalent to $(1-\sigma\alpha)^2 > 2\sigma(1-\alpha)^2$. It remains to be verified that this inequality may hold or not, while still satisfying (30). If $\alpha \to 0$ then $\beta \to 1$ and $\lambda \to \sigma$, so that (30) is satisfied, and in the limit $n^{2sp} > n^{fe}$ if and only if $\sigma < \frac{1}{2}$, so that both cases are possible.

Social welfare has the following expression:

$$W = \frac{\theta_m}{\theta_H} V(n) - \frac{\theta_m^2}{2\theta_H} - n\phi$$

Using (31), (32) and $\theta_{fe} = u(n_{fe}) = (1-\lambda(1-\beta)) V(n_{fe})$, $\theta_{2sp} = \frac{V(n_{2sp})}{2}$ and $V(n) = (1-\sigma\alpha) Bn^{1-\beta}$ we obtain:

$$\begin{aligned} W_{2sp} &= \frac{B^2(1-\sigma\alpha)^2}{\theta_H}\left(\frac{1}{2}-\frac{1}{8}\right) n_{2sp}^{2-2\beta} - \frac{B^2(1-\beta)(1-\sigma\alpha)^2}{2\theta_H} n_{2sp}^{2-2\beta} \\ &= \frac{B^2}{2\theta_H}\left(\beta - \frac{1}{4}\right)\left(\frac{1-\beta}{2\phi}\right)^{\frac{2-2\beta}{2\beta-1}} (1-\sigma\alpha)^{\frac{2}{2\beta-1}} \end{aligned}$$

and:

$$\begin{aligned} W_{fe} &= \frac{B^2}{\theta_H}\left[(1-\sigma\alpha)^2\left(1-\lambda(1-\beta) - \frac{(1-\lambda(1-\beta))^2}{2}\right) - \alpha(1-\sigma)(1-\alpha)\right] n_{fe}^{2-2\beta} \\ &= \frac{B^2(1-\alpha)^2}{2\theta_H}\left(\frac{\alpha(1-\sigma)(1-\alpha)}{\phi}\right)^{\frac{2-2\beta}{2\beta-1}} \end{aligned}$$

Finally:

$$\frac{W_{2sp}}{W_{fe}} = \left(\beta - \frac{1}{4}\right)\left(\frac{\sigma}{\lambda}\right)^{\frac{2}{2\beta-1}} \frac{1}{(2\sigma)^{\frac{2-2\beta}{2\beta-1}}}$$

Let $\sigma = x$, $\alpha = kx$ with $0 < k < \frac{1}{3}$ and $x \to 0$. Then $\lambda \to 0$ and $\beta \to 1-k$ so that (30) is satisfied in the limit and at the same time $\frac{\sigma}{\lambda} \to 1$ and therefore $\frac{W_{2sp}}{W_{fe}} \to +\infty$.

Now let $\sigma \to 1$ keeping α fixed: $\lambda, \beta \to 1$ so that (30) is satisfied and $\frac{W_{2sp}}{W_{fe}} \to \frac{3}{4}$.

■