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Keiichiro Kobayashi†

March 30, 2005 (First draft: June 20, 2004)

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When a landowner can borrow money immediately using land as collateral, one can say that land essentially provides a transaction service. The transaction services that such an asset can provide increase as its price rises, since the asset owner can borrow more money against the asset's increased value. Thus an asset-price bubble can emerge due to the externality of *self-reference* wherein the asset price reflects the transaction services that it can provide, while the amount of the transaction services reflects the asset price. If the collateral ratio of the asset (θ) is not too high, there exists a steady state equilibrium where the asset price has a bubble component; if θ exceeds a certain value, there exists no stable monetary equilibrium.

The paper also analyzes the case where θ is determined as an equilibrium outcome. Finally, in the case where the equilibrium concept is relaxed to allow for sticky prices and a temporary supply-demand gap, the paper shows that there exists an equilibrium where a bubble develops temporarily and eventually bursts.

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Transaction services and asset-price bubbles*

(Incomplete and preliminary)

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that there exists an equilibrium where a bubble develops temporarily and eventually bursts.

1 Introduction

When an asset market overheats, the trading volume of the asset usually increases. Figure 1 shows the price and the trading volume of land in Japan during the "real estate bubble" period of 1987–91 and before and after that period.

Figure 1. Trading volume of land and urban land prices

On the one hand, the increase in demand for the asset caused by the expectation of a rising price results in vigorous trade. On the other hand, this vigorous trade enhances ease of sale, or liquidity, of the asset in the market. When liquidity increases, the asset owner can borrow more money from banks by putting up the asset as collateral. For example, during the "bubble" period in Japan, the collateral ratio of land in bank lending was said to be greater than 100% (i.e., some banks were alleged to have lent money in excess of 100% of the value of the collateralized land).

When an asset is easily exchanged for money, it can be said that it works as a de facto medium of exchange just like money itself. In other words, the asset can provide transaction services. This paper is a theoretical study of the deviation of an asset price from its fundamental value when the asset can provide transaction services as a medium of exchange. The basic idea can be roughly described as follows: Suppose that there exits a nondepletable asset (land) and that the landowner can obtain money immediately by borrowing from banks using the land as collateral. If the price of the asset is Q_t, it can be plausibly assumed that the amount of money the owner of one unit of land can borrow from a bank is weakly increasing in $Q_t\theta_t$, where θ_t ($0 \leq \theta_t < 1$) is a parameter representing the collateral ratio of the asset, which may be exogenously given or may be an equilibrium outcome determined by the inefficiencies of the real estate market. Therefore, the amount of transaction services (L_t) that the asset can provide can be expressed as

$$L_t = M(q_t \theta_t),$$

where $q_t \equiv \frac{Q_t}{P_t}$ is the real asset price, P_t is the general price level, and $M(\cdot)$ is a weakly increasing function. At the same time, the real price of the asset is determined as a discounted sum of the flow of dividends that the land yields and the flow of the value of transaction services that it provides. The land price can be expressed as

$$q_t = \sum_{s=t}^{\infty} \{y_s + g_s(L_s)\},$$

where y_s is the present value of the dividend at date s as of date t and $g_s(L_s)$ is the present value of liquidity L_s at date s as of date t. For simplicity, let us focus on the steady state where we can omit time subscripts. In the steady state, the transaction services L and the real asset price q are determined by

$$L = M(q\theta) \text{ and } q = Q(L), \tag{1}$$

where $Q(L)$ is an increasing function of L. As Figure 2 shows, L^* that solves (1) may be positive.

Figure 2. Land prices and liquidity

Thus, in the equilibrium, the asset may provide a positive amount of transaction services L^* and its price may become $q^* = Q(L^*)$, which is higher than the fundamental price of the asset $Q(0)$. The difference $Q(L^*) - Q(0)$ can be regarded as the "bubble" component of the asset price.[1] The bubble is generated by a particular type of externality, or a *self-reference* in transaction services that the asset can provide: An increase in the asset price results in an increase in transaction services that the asset can provide, since the asset is exchangeable for more money; and the increase in transaction services enhances

[1] To use the word "bubble" in this context may be somewhat misleading, since the difference $Q(L^*) - Q(0)$ reflects the fact that the asset provides transaction services in addition to the dividends. Thus we may be able to say that the fundamental price of an asset when it provides transaction services ($Q(L^*)$) is higher than the fundamental price of it when it does not provide transaction services ($Q(0)$). Nevertheless, I call the difference $Q(L^*) - Q(0)$ the bubble throughout in this paper, since the fundamental price of an asset usually refers to the value from the dividends, not from transaction services.

the value of the asset, causing a further increase in the asset price. Thus the amount of transaction services that the asset can provide reflects the asset price, which reflects, in turn, the transaction services.

There is a considerable amount of literature on asset-price bubbles (see Camerer [1989] for a survey of rational growing bubbles, fads, and information bubbles). Examples of recent theoretical developments are Allen and Gale (2000), in which information asymmetry and limited liability cause risk shifting from investors to banks, which leads to asset-price bubbles; and Allen, Morris, and Shin (2003), in which higher order beliefs under noisy public information generate distortions in asset pricing. But few authors have addressed the problem of the transaction services that the asset can provide. Among these few authors are Kiyotaki and Wright (1989) and Bansal and Coleman (1996).

Kiyotaki and Wright show that there exists a bubble equilibrium in which an intrinsically useless asset (cash) has positive value since it provides transaction services. The difference in their model from the present paper is that in their model, the amount of transaction services that the cash can provide is physically limited by the assumption that an exogenously fixed amount of cash is exchangeable for one unit of goods. Since I assume that the amount of transaction services that the asset can provide increases as the real price of the asset increases, the asset price can follow a complicated path as discussed in Section 3. Bansal and Coleman analyze a one-period bond as an asset that provides transaction services. Because their asset is a fixed-payment security with a short maturity, the bubble component generated by the transaction services is small, while in the present paper the asset is infinitely long-lived and allows the emergence of large bubbles. My model is quite similar to the model in Kiyotaki and Moore (2001) in which a borrowing constraint plays a crucial role in determining the asset price. The difference is that the collateral ratio θ in Section 3 in this paper is endogenously determined as an equilibrium outcome, while that in their model is exogenously given. Due to this difference, multiple monetary equilibria emerge in my model, while there is no such multiplicity in their model.

The organization of the paper is as follows: In the next section, I present the basic structure of the model, in which the collateral parameter θ is exogenous. It is shown that if θ is large, there exists no steady state equilibrium. This result may imply that when the asset market is too liquid, the economy becomes unstable. In Section 3, I argue for a mechanism that endogenously determines θ and show that there are multiple equilibria. In one equilibrium the asset price equals its fundamental price; in another it has a bubble component. Section 4 examines an equilibrium path for the asset price under the assumption of sticky prices. Under sticky prices, there exist equilibrium paths in which an asset-price bubble temporarily develops and eventually bursts. Section 5 provides some concluding remarks.

2 The basic model

The basic model is a variant of the Lucas tree economy with a cash-in-advance constraint, which is composed of an infinite number of consumers and banks, and one government. The economy is populated with a continuum of consumers with identical preferences, whose measure is normalized to one. There is also a continuum of banks with measure one. At date 0, a representative consumer maximizes the following utility:

$$\sum_{t=0}^{\infty} \beta^t u(c_t), \tag{2}$$

where β is the discount factor ($0 < \beta < 1$) and c_t is the consumption at date t.

At each date t, the consumer is endowed with z units of consumer goods, which are not durable. There is a nonperishable asset (land) in this economy, which has a fixed total supply of a. Initially each consumer owns a units of land at the beginning of date 0. I assume that one unit of land yields y units of consumer goods at each date without any cost. Thus the total supply of consumer goods is $c \equiv ya + z$ at each date. I assume that c is sufficiently larger than ya such that

$$(1 - \beta)c > \beta ya. \tag{3}$$

At each date t, the government provides M^s_{t+1} units of cash to this economy. The difference $X_t \equiv M^s_{t+1} - M^s_t$ is a lump-sum transfer to (from) the consumer from (to) the government at date t. (The initial amount M_0^s is given to the consumers at, say, date -1 as a lump-sum subsidy.)

At each date t, the consumer chooses the amount of consumption c_t, cash holdings M_{t+1}, and land holdings a_{t+1}, given that he owns M_t units of cash and a_t units of land at the beginning of date t. Denoting the nominal price of consumer goods by P_t and the land price by Q_t, the budget constraint for the consumer at date t is written as

$$P_t c_t + M_{t+1} + Q_t a_{t+1} \leq P_t(y a_t + z) + Q_t a_t + M_t + X_t. \tag{4}$$

I assume as an ordinary cash-in-advance model that a consumer cannot consume his own endowment $y a_t + z$ and needs to buy c_t in the goods market from other consumers.

Consumers can buy the goods using cash and bank borrowing B_t. Thus, the consumers must choose c_t under the following liquidity constraint:

$$P_t c_t \leq M_t + B_t.$$

Banks lend B_t to consumers competitively at the beginning of date t, and consumers repay $R_t B_t$ to the banks at the end of date t. As a result of the competition among banks, the rate of return on bank borrowing within one date must be one: $R_t = 1$. I assume that B_t works as a medium of exchange exactly like cash. In other words, I assume that B_t is given in the form of a bank deposit and banks can create and provide transaction services to depositors without cost.

I assume, as in Kiyotaki and Moore (1997, 2001), that consumers can freely abscond, leaving their land, and that there is no way for banks to penalize such borrowers. Therefore, the consumers cannot precommit to repay B_t to banks, and only thing that banks can do when the borrowers abscond is to seize the land. Following the arguments by Kiyotaki and Moore, this assumption implies that a consumer is subject to the borrowing constraint:

$$B_t \leq Q_t \theta_t a_t,$$

where a_t is the land held by the consumer and θ_t $(0 \leq \theta_t < 1)$ is the collateral ratio. In this section, I simply assume that θ_t is an exogenously given parameter, while in the next section I argue an example of economic structure that determines θ_t as an equilibrium outcome. Under this borrowing constraint, a consumer who borrows B_t will never abscond and will repay B_t at the end of date t, since otherwise the bank will seize his land, the value of which is $Q_t a_t$ $(> B_t)$.

The above arguments imply that the reduced form of the liquidity constraint for the consumer is

$$P_t c_t \leq M_t + Q_t \theta_t a_t. \tag{5}$$

Therefore, the representative consumer's problem is to maximize (2) subject to (4) and (5). It is useful to clarify the timing of events. The representative consumer enters date t with cash holdings M_t and land holdings a_t. At the beginning of date t, he is given endowment (z) and yields on the land ($y a_t$), and he borrows $B_t (= Q_t \theta_t a_t)$ from a bank. The goods market opens first, and the consumer sells goods $y a_t + z$ and buys c_t under the constraint (5). After consumption takes place, the consumer repays B_t to the bank. After repayment, the asset market then opens, and the consumer buys M_{t+1} and a_{t+1} by selling the remaining assets, the nominal value of which is $P_t(y a_t + z) + M_t + Q_t a_t - P_t c_t$.

The equilibrium conditions for cash, land, and consumer goods are

$$M_t = M_t^s, \tag{6}$$

$$a_t = a, \tag{7}$$

$$c_t = c \; (\equiv y a + z). \tag{8}$$

The monetary competitive equilibrium is defined as follows:

Definition 1 *The monetary competitive equilibrium is a set of prices* $\{P_t, Q_t\}_{t=0}^{\infty}$ *and allocations* $\{c_t, a_t, M_t\}_{t=0}^{\infty}$ *that satisfies the following conditions: (a) The prices are positive and finite for all t:* $0 < P_t, Q_t < \infty$*; (b) given the prices, the allocations solve the consumer's problem (i.e., maximization of [2] subject to [4] and [5]); (c) the allocations satisfy the equilibrium conditions (6)–(8); and (d) the transversality conditions are satisfied:* $\lim_{t\to\infty} \lambda_t = 0$ *and* $\lim_{t\to\infty} \eta_t = 0$*, where* λ_t *and* η_t *are the Lagrange multiplier for (4) and (5), respectively.*

Note that, as in the model of Kiyotaki and Moore (2001), there always exists a nonmonetary equilibrium where cash has no value and only land provides transaction services.[2] In the nonmonetary equilibrium, P_t and Q_t are infinite, and $q_t = \frac{Q_t}{P_t}$ is finite.

In what follows in this section, I assume the liquidity parameter θ_t is constant over time, i.e.,

$$\theta_t = \theta,$$

and analyze whether there exists a steady state equilibrium where prices are constant over time. Denoting the Lagrange multipliers for (4) and (5) by λ_t and η_t, respectively, the first order conditions (FOCs) for the consumer's problem are

$$\beta^t u'(c_t) = (\lambda_t + \eta_t)P_t, \tag{9}$$

$$Q_t = \frac{\lambda_{t+1}}{\lambda_t}(P_{t+1}y + Q_{t+1}) + \frac{\eta_{t+1}}{\lambda_t}Q_{t+1}\theta, \tag{10}$$

$$\lambda_t = \lambda_{t+1} + \eta_{t+1}. \tag{11}$$

Note that since the fundamental price of the asset (Q_t^F) is defined as the asset price where the asset does not provide transaction services, Q_t^F satisfies

$$Q_t^F = \frac{\lambda_{t+1}}{\lambda_t}(P_{t+1}y + Q_{t+1}^F). \tag{12}$$

Since $1 = \frac{\lambda_{t+1}}{\lambda_t} + \frac{\eta_{t+1}}{\lambda_t}$ from (11), it can be interpreted that $\frac{\lambda_{t+1}}{\lambda_t}$ is the present value at t of one unit of cash at $t+1$ as a store of value, and $\frac{\eta_{t+1}}{\lambda_t}$ is the present value at t of transaction services that one unit of cash can provide at $t+1$. The second term of the right-hand side of (10) is the nominal present value at t of the transaction services that one unit of land can provide at date $t+1$. In this paper I mainly focus on the case where the supply of base money is constant: $M_t^s = M$ for all t. One justification for this is that since the monetary authorities in reality do not seem to target asset prices, it may be reasonable to assume that M_t^s is determined independently from changes in asset prices. When $M_t^s = M$ for all t, the steady state equilibrium exists, and the prices can be easily derived from the FOCs, the liquidity constraint (5), and the equilibrium

[2] I thank an anonymous referee for making this point.

conditions (6)–(8):

$$Q = \frac{\beta P y}{(1-\theta)(1-\beta)}, \tag{13}$$

$$P = \frac{M}{c - \frac{\beta\theta}{(1-\theta)(1-\beta)} ya}. \tag{14}$$

Note that the fundamental price of the asset in the steady state is $Q^F = \frac{\beta P y}{1-\beta}$. Therefore, the "bubble" component of Q is $Q - Q^F = \frac{\theta}{1-\theta} Q^F$ (> 0), which reflects the function of providing transaction services. That the price P in (14) must be positive and finite gives the following condition for the existence of the steady state:

$$0 \leq \theta < \overline{\theta} \equiv \frac{(1-\beta)c}{(1-\beta)c + \beta ya}. \tag{15}$$

If $\theta \geq \overline{\theta}$, there is no steady state monetary equilibrium for this economy. In fact, the following stronger results are obtained:

Lemma 1 *If $\theta \geq \overline{\theta}$ and $M_t^s = M$ for all t, a monetary competitive equilibrium does not exist.*

See Appendix A for the proof. In the case where the government can appropriately control money supply M_t^s, it seems likely that there exists a competitive equilibrium with a constant inflation rate. But this is not the case when θ is an exogenous constant.

Lemma 2 *Assume that $\theta_t = \theta$ (> 0) and the government can freely control M_t^s. Define a steady inflation equilibrium as a monetary competitive equilibrium in which $\pi_t = \frac{P_{t+1}}{P_t} = \pi(\neq 1)$ and constraint (5) is always binding. There is then no monetary policy $\{M_t^s\}_{t=0}^{\infty}$ that can realize a steady inflation equilibrium.*

See Appendix B for the proof. These lemmas imply that there is no stable equilibrium path for the economy if $\theta > \overline{\theta}$.[3] Although I cannot specify further the behavior of the model in the case where the government can freely control M_t^s, I conjecture that there is no monetary competitive equilibrium if $\theta > \overline{\theta}$ even in the case where the government can freely control money supply M_t^s.

[3] Note that there exists a nonmonetary equilibrium even if $\theta > \overline{\theta}$, which is not in our interest.

One way to understand why the economy becomes unstable when θ exceeds $\overline{\theta}$ is the following: If $\theta \geq \overline{\theta}$, it is shown from (13) that $\frac{Q\theta a}{Pc} \geq 1$, which implies that constraint (5) does not bind for any positive value of M. Therefore, it can be said that if $\theta \geq \overline{\theta}$, the asset provides more liquidity than needed. Since θ can be interpreted as the ease of borrowing from banks by putting up the asset as collateral, $Q_t\theta a$ in (5) can be interpreted as the amount of bank lending collateralized by land. As the left-hand side of (5) can be interpreted as the nominal output, the equivalent of $\frac{Q\theta a}{Pc}$ in reality may be the ratio of bank lending collateralized by land to nominal GDP. Figure 3 shows this ratio during and after the bubble period in Japan.

Figure 3. Ratio of loans covered by collateral to nominal GDP

The ratio increased markedly just before the bubble burst.

3 The model of endogenous liquidity

In the previous section I assumed that the collateral parameter θ is exogenously given. In this section, I explicitly posit a formal mechanism that determines θ_t and argue how the asset price behaves under changing θ_t.

I assume the following inefficiency in asset-seizure by banks. Suppose that a consumer borrows B_t from a bank at the beginning of date t. If the borrower absconds during date t, the bank seizes the borrower's land, a_t, and sells it at a price of Q_t at the end of date t. I assume that the bank needs to pay for maintenance of the seized land until it sells the land in the asset market, and the bank incurs the cost of maintenance, which is xa_t in terms of the consumer goods. Under these conditions, competition among profit-maximizing banks implies that in the equilibrium, $B_t = \max\{Q_t a_t - P_t x a_t, 0\}$. Since $B_t = Q_t\theta_t a_t$, the collateral ratio θ_t satisfies

$$\theta_t = \max\{1 - \frac{P_t x}{Q_t}, 0\}. \tag{16}$$

Therefore, if banks have the above inefficient technology for land maintenance, the collateral ratio θ_t evolves by (16), given Q_t and P_t. The definition of the monetary competitive equilibrium thus needs to be modified:

Definition 2 *The monetary competitive equilibrium is a set of prices and allocations* $\{P_t, Q_t, c_t, a_t, M_t, \theta_t\}_{t=0}^{\infty}$ *that satisfies all conditions in Definition 1 and (16).*

The representative consumer's problem is the same as the previous section, i.e., maximization of (2) subject to (4) and (5). The FOCs are (9), (11),

$$Q_t = \frac{\lambda_{t+1}}{\lambda_t}(P_{t+1}y + Q_{t+1}) + \frac{\eta_{t+1}}{\lambda_t}Q_{t+1}\theta_{t+1} \tag{17}$$

instead of (10). In order to make the price path interesting, I assume that the parameters satisfy

$$x > \frac{\beta y}{1-\beta}. \tag{18}$$

As shown below, there are infinitely many equilibria, even if the government fixes the money supply at a constant ($M_t^s = M$ for all t). But all equilibrium paths eventually jump to either of the following two equilibria:

Lemma 3 *If* $M_t^s = M$ *for all* t*, there exist two equilibria: the fundamental equilibrium and the steady inflation equilibrium. In the fundamental equilibrium,* $\{P_t, Q_t, c_t, a_t, \theta_t, M_t\} = \left\{\frac{M}{c}, \frac{M\beta y}{(1-\beta)c}, c, a, 0, M\right\}$*. In the steady inflation equilibrium, the inflation rate is constant* ($\frac{P_{t+1}}{P_t} = \pi^*$ (> 1) *for all* t*) and* θ_t *is always positive.*

(Proof) Given that parameters satisfy (18), it is obvious that the prices and allocations $\{P_t, Q_t, c_t, a_t, \theta_t, M_t\} = \left\{\frac{M}{c}, \frac{M\beta y}{(1-\beta)c}, c, a, 0, M\right\}$ satisfies all conditions for a competitive equilibrium.

Next I show that the equilibrium in which $\theta_t > 0$ for all t uniquely exists. Suppose $\theta_t > 0$ for all t. The equations (16) and (5) imply that in the equilibrium,

$$\theta_t = \frac{c - m_t}{c + ax - m_t}, \tag{19}$$

where $m_t = \frac{M}{P_t}$. Since $Q_t = \frac{P_t c - M}{\theta_t a}$, (19) implies

$$Q_t = \frac{P_t c - M}{a} + P_t x. \tag{20}$$

Equation (17) implies that in the equilibrium, the asset price follows $Q_t = Q_{t+1}\theta_{t+1} + \beta\frac{P_{t+1}}{P_{t+2}}\{P_{t+1}y + (1-\theta_{t+1})Q_{t+1}\}$. These conditions, taken together, give the difference equation for inflation rate $\{\pi_t\}_{t=0}^{\infty}$:

$$F(\pi_t) = G(\pi_{t+1}), \tag{21}$$

where $F(\pi) \equiv \frac{\frac{c}{a}+x}{\pi_t}$ and $G(\pi) \equiv \beta \frac{y+x}{\pi} + \frac{c}{a}$. It is shown as follows that the inflation rate in the equilibrium must be π^*, which is the solution to $F(\pi) = G(\pi)$. Since $F(\pi) < G(\pi)$ as $\pi \to \infty$ and (18) implies that $F(1) > G(1)$, the equilibrium inflation rate π^* must be larger than one. Suppose that $\exists \tau$ such that $\pi_\tau < \pi^*$. In this case, it is obvious from the functional forms of $F(\pi)$ and $G(\pi)$ that π_t $(t \geq \tau)$ monotonically decreases and becomes less than one in finite steps, implying that $P_t c$ becomes smaller than M eventually. This means that the financial constraint (5) becomes nonbinding from some time t' onward, implying that $P_{t+1} = \beta P_t$ for all $t > t'$. In this case $\lambda_t = \frac{\beta^{t+1} u'(c_{t+1})}{P_{t+1}} = \frac{\beta^{t'}}{P_{t'}} u'(c) > 0$ for all $t > t'$, implying that the transversality condition ($\lim_{t\to\infty} \lambda_t = 0$) is violated. This contradicts the assumption that P_t is in the equilibrium path. Thus π_t can never be less than π^*. Next suppose that $\exists \tau$ such that $\pi_\tau > \pi^*$. In this case, π_t monotonically increases. Since $\lim_{\pi\to\infty} F(\pi) = 0$ and $\lim_{\pi\to\infty} G(\pi) = \frac{c}{a}$, the difference equation (21) becomes impossible to solve at a finite t. Thus π_t can never exceed π^* in the equilibrium. Therefore, $\pi_t = \pi^*$ in the equilibrium. Given that $\pi_t = \pi^*$, the equilibrium values are determined by $P_t = (\pi^*)^t P_0$, and (20). (End of Proof of Lemma 3)

From (12), the fundamental asset-price in the equilibrium where $\pi_t = \pi^*$ is written as $Q_t^F = \frac{\beta P_t y}{1-\beta}$. The gap between $\frac{Q_t}{P_t}$ and $\frac{Q_t^F}{P_t}$ is the bubble component of the asset price:

$$\frac{Q_t}{P_t} - \frac{Q_t^F}{P_t} = \frac{c}{a} + x - \frac{\beta y}{1-\beta} - \frac{M}{P_t a}, \tag{22}$$

and it becomes larger as time passes and converges to $\frac{c}{a} + x - \frac{\beta y}{1-\beta}$.

Next I will show that there are infinitely many equilibria in which $\theta_t = 0$ initially, and at some time τ, θ_τ becomes a positive value and the prices follow the steady inflation equilibrium path ($\pi_t = \pi^*$) from date τ onward.

Lemma 4 *Assume that $M_t^s = M$ for all t. For any date τ ($\tau > 0$), there exist (multiple) equilibria in which $\theta_t = 0$ for $t < \tau$, $\theta_t > 0$ for $t \geq \tau$, and the prices are determined by $P_t = (\pi^*)^{t-\tau} P_\tau$ and (20) from date τ onward.*

The following claim is useful to prove the lemma:

Claim 1 *In the case where $M^s = M$ for all t, the liquidity constraint (5) is binding if $\theta_t = 0$.*

See Appendix C for the proof of this claim.

(Proof of Lemma 4) Proof is by construction. Suppose such an equilibrium exists for a given τ. The prices from τ onward are determined by $P_t = (\pi^*)^{t-\tau} P_\tau$ and (20). Since $\theta_t = 0$ for $t < \tau$ and (5) is binding from Claim 1, the price before τ satisfies $P_t = \frac{M}{c}$ for $t < \tau$. Since the economy is in the steady inflation equilibrium from τ onward, the asset price at $\tau - 1$ satisfies

$$Q_{\tau-1} = Q_\tau \theta_\tau + \frac{\beta}{\pi^*}\{P_\tau y + (1-\theta_\tau)Q_\tau\} = \frac{P_\tau c - M}{a} + \frac{\beta}{\pi^*} P_\tau y + \frac{\beta}{\pi^*} P_\tau x. \qquad (23)$$

Since $\theta_{\tau-1} = 0$ and $\theta_\tau > 0$, (16) implies that

$$\frac{Q_{\tau-1}}{P_{\tau-1}} \leq x < \frac{Q_\tau}{P_\tau}. \qquad (24)$$

Equation (23) and (20) for $t = \tau - 1$ imply that the first inequality of (24) is rewritten as

$$\pi_{\tau-1}\left\{\frac{c}{a} + \frac{\beta}{\pi^*}(y+x)\right\} \leq x + \frac{c}{a}, \qquad (25)$$

which holds if $\pi_{\tau-1} \leq \pi^*$, since π^* is the solution to $F(\pi) = G(\pi)$. The second inequality of (24) is equivalent to $P_\tau > \frac{M}{c}$, which also holds if $\pi_{\tau-1} > 1$, since $P_{\tau-1} = \frac{M}{c}$. Therefore, there exists a continuum of P_τ (or $\pi_{\tau-1}$) that satisfies (24): $1 < \pi_{\tau-1} \leq \pi^*$. Once P_τ is given, P_t is given by $P_t = (\pi^*)^{t-\tau} P_\tau$, and Q_t is determined by (20), for $t \geq \tau$. The asset prices before τ are determined recursively by (23) and

$$Q_{t-1} = \beta \frac{M}{c} y + \beta Q_t, \text{ for } t \leq \tau - 1. \qquad (26)$$

(End of Proof of Lemma 4)

The reverse of Lemma 4 does not hold:

Lemma 5 *If $M_t^s = M$ for all t, there exists no equilibrium in which $\exists \tau$ such that $\theta_t > 0$ for $t < \tau$ and $\theta_t = 0$ for all $t \geq \tau$.*

(Proof) Suppose such an equilibrium exists. Since Claim 1 holds and $\theta_t = 0$ for all $t(\geq \tau)$, $P_t = \frac{M}{c}$ and $Q_t = \frac{M\beta y}{(1-\beta)c}$ for $t \geq \tau$. Since $Q_{\tau-1}$ satisfies (20) and $Q_{\tau-1} = \beta\frac{P_\tau}{P_{\tau+1}}(P_\tau y + Q_\tau)$, $P_{\tau-1}$ must satisfy

$$\frac{P_{\tau-1}c - M}{a} + P_{\tau-1}x = \frac{M\beta y}{(1-\beta)c}. \tag{27}$$

Denoting the left-hand side of (27) by $K(P_{\tau-1})$, the condition for $P_{\tau-1} > \frac{M}{c}$ is written as $K(\frac{M}{c}) < \frac{M\beta y}{(1-\beta)c}$. This condition is rewritten as $x < \frac{\beta y}{1-\beta}$, which cannot hold, since (18) is assumed. Therefore, there is no $P_{\tau-1}$ that realizes the equilibrium in which $\theta_t > 0$ for $t < \tau$ and $\theta_t = 0$ for all $t \geq \tau$. (End of Proof)

The above Lemmas 3, 4, and 5 imply there are at most four types of competitive equilibria in this economy if $M_t^s = M$ for all t: (a) The fundamental equilibrium; (b) the steady inflation equilibrium in which $\pi_t = \pi^*$; (c) the equilibrium where $\theta_t = 0$ initially and the economy jumps to the steady inflation equilibrium at some date τ; and (d) the equilibrium where $\exists\tau$ (≥ 1) and $\exists s$ (≥ 1) such that $\theta > 0$ for $t < \tau$, $\theta_t = 0$ for $t = \tau, \tau+1, \cdots, \tau+s-1$, and the economy jumps to the steady inflation equilibrium at date $\tau + s$. The fourth type may or may not exist depending on the parameter values. If it exists, π_t $(t < \tau)$ decreases following $F(\pi_t) = G(\pi_{t+1})$ and Q_t is determined by (20).

The welfare implication is trivial. In this endowment economy, the consumption is the same constant c in any equilibrium. Therefore, the utility of the representative consumer is the same for all equilibria.

In the case where the government can control M_t^s freely, there may be a more complicated equilibrium path for the asset price, but I will not fully specify the equilibrium behavior of the model under flexible M_t^s. I assumed the constant M, since decision-making by the monetary authorities in reality seems independent from asset prices.

4 Equilibrium with sticky prices and bursting bubbles

The arguments in the previous section imply that there is no equilibrium path in which the asset-price bubble bursts. The reason why the asset-price bubble never bursts in the previous model can be explained intuitively as follows. In the previous model, the consumption c_t is constant in any equilibrium, implying that the real interest rate is always $\beta^{-1}-1$. If the bubble bursts, the liquidity constraint (5) implies that the general price level P_t must plummet, leading to a spike in the real interest rate. Since the real interest rate cannot rise in the equilibrium, there is no equilibrium in which the bubble bursts.

In this section it is shown that if the equilibrium condition $c_t = c$ is relaxed to $c_t \leq c$ and prices are sticky in the sense that P_t is predetermined at date $t-2$ and P_0 and P_1 are exogenously given, then there exist equilibria in which the bubble bursts at some date. In what follows I assume $u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}$, where $0 < \sigma < 1$.

In order to characterize such an equilibrium, it is necessary to relax the definition of competitive equilibrium to allow for a temporary supply-demand gap:

Definition 3 *A sticky price equilibrium is same as a monetary competitive equilibrium defined by Definition 2, except for that (a) the price level at date t is predetermined at date $t-2$; (b) instead of (8), $c_t \leq c$ is satisfied; and (c) if $c_t < c$, the supply-demand gap $(c - c_t)$ perishes without being consumed by anyone at date t and is borne as a lump-sum cost by the consumer (= seller), and the budget constraint for the consumer at date t becomes*

$$P_t c_t + M_{t+1} + Q_t a_{t+1} \leq P_t(y a_t + z) + Q_t a_t + M_t + X_t - \Delta_t, \tag{28}$$

where Δ_t is a lump-sum cost, which is exogenous for the consumer, and $\Delta_t = P_t \cdot (c - c_t)$ holds in the equilibrium.

The price at date t is predetermined at $t-2$, and the consumer (=seller) cannot reduce the price P_t at date t even though he cannot sell all of his goods at P_t. Note that even under sticky prices, the FOCs: (9), (11), and (17) must be satisfied, since the consumers solve their optimization problem, taking the entire price path as given. Therefore, the FOCs are always satisfied in a sticky price equilibrium, while the market clearing conditions may not. This implies that if $\theta_t > 0$ and the market clearing conditions are satisfied for some t in the sticky price equilibrium, the equation (21) must be satisfied for t.

The concept of sticky price equilibrium is useful to analyze the situation where the initial value of inflation rate $\pi_0 (= \frac{P_1}{P_0})$ exceeds π^*. (If prices are not sticky, the (expected) price P_1 adjusts instantaneously at date 0 so that π_0 never exceeds π^*.[4]) If $\pi_0 > \pi^*$ and prices are sticky, the price path, or π_t, is determined by the difference equation (21) for the time being, but the equation becomes impossible to solve eventually. In this case, there exists a sticky price equilibrium where the bubble bursts at some date τ and $c_\tau < c$.

Lemma 6 *Assume that $M_t^s = M$ for all t, π_0 exceeds π^*, and $P_0 c > M$. There exists a sticky price equilibrium where $a_t = a$ for all t, and $\exists\tau$ such that $\theta_t > 0$ for $t < \tau$, $\theta_t = 0$ for $t \geq \tau$, $c_\tau < c$, and $c_t = c$ for $t \neq \tau$.*

(Proof) Proof is by construction. I construct an equilibrium where the economy jumps to the fundamental equilibrium at date τ. Thus I assume that $P_t = \frac{M}{c}$ for $t \geq \tau + 1$ and $Q_t = \frac{M\beta y}{(1-\beta)c}$ for $t \geq \tau$. Since $c_\tau < c$ in this equilibrium, $P_\tau = \frac{M}{c_\tau}$. Given π_0, generate a (finite) sequence of π_t by $F(\pi_t) = G(\pi_{t+1})$. Define τ by $F(\pi_{\tau-4}) > \frac{c}{a} > F(\pi_{\tau-3})$. Note that $\pi_{\tau-3}$ is the last element of the above sequence. In this equilibrium, P_t $(t \leq \tau - 2)$ is determined by $P_t = \pi_{t-1}P_{t-1}$, and Q_t $(t \leq \tau - 2)$ is determined by (20).

From the FOCs, $Q_{\tau-1}$ must satisfy

$$Q_{\tau-1} = \beta \frac{c^{1-\sigma}}{c_\tau^{1-\sigma}} \left\{ \frac{M}{c_\tau} y + \frac{M\beta y}{(1-\beta)c} \right\}. \tag{29}$$

Since $Q_{\tau-1}$ satisfies (20),

$$Q_{\tau-1} = \frac{P_{\tau-1}c - M}{a} + P_{\tau-1}x. \tag{30}$$

Equation (17) implies that $Q_{\tau-2} = \beta \frac{c^\sigma}{c_\tau^\sigma} \frac{P_{\tau-1}}{P_\tau} \{P_{\tau-1}y + (1-\theta_{\tau-1})Q_{\tau-1}\} + Q_{\tau-1}\theta_{\tau-1}$. This

[4] The initial condition is determined by exogenous or historical factors that are not specified in this paper. As for Japan's real estate bubble of the late 1980s, the most commonly accepted view is that it developed through a combination of: (1) the expansion of the business cycle in the 1980s, which tightened the real estate market and caused banks to rationally begin providing loans to real estate development projects; (2) deregulation of the financal industry, which made the banking industry more competitive and banks inclined to take greater risk; and (3) economic growth and appreciation of the yen, which allowed large firms to accumulate huge retained earnings and reduced the need for bank loans in traditional industries. This historical coincidence encouraged Japanese banks to pour money in real estate backed loans recklessly. I thank Masaru Yoshitomi for reminding about this history. Hoshi and Kashyap (1999) empirically support this interpretation.

condition and (20) at $\tau - 2$ imply

$$P_{\tau-2}\left(\frac{c}{a}+x\right)=\frac{\beta}{M}c^{\sigma}c_{\tau}^{1-\sigma}P_{\tau-1}^{2}\left\{y+x\right\}+P_{\tau-1}\frac{c}{a}. \tag{31}$$

It is easily shown that the system of three equations (29), (30), and (31) for three unknowns $Q_{\tau-1}$, $P_{\tau-1}$, and c_τ always has a unique solution that satisfies $c_\tau < c$. Therefore, prices and allocations of a sticky price equilibrium are fully specified. (End of Proof)

Figure 4 shows the above sticky price equilibrium for specific parameter values.

Figure 4. Sticky price equilibrium with a bursting bubble

Note that this is not the unique sticky price equilibrium for given initial conditions (P_0, M, π_0). The bubble can burst at any time that is less than or equal to τ defined above. There may be a sticky price equilibrium that corresponds to each timing of the bubble's puncturing. The timing τ in the above lemma is the upper limit for continuation of the bubble when the initial inflation rate is given as $\pi_0 > \pi^*$.

The welfare implications are straightforward. Welfare in a sticky price equilibrium with a bursting bubble is lower than in the fundamental equilibrium, since $c_\tau < c$.

5 Concluding Remarks

This paper has examined the emergence of asset-price bubbles in the case where the asset can provide transaction services, using a variant of the cash-in-advance model. The transaction services the asset can provide increase as its (real) price becomes higher, since the owner of the asset can borrow more money by putting it up as collateral. Thus the asset price may exceed its fundamental price, since the transaction services that it can provide are an increasing function of the asset price, which reflects the value of the transaction services that it can provide.

Introducing a parameter that represents the collateral ratio of the asset (θ), I showed that the asset price can exceed the fundamental price in the steady state equilibrium, that it is increasing in θ, and that if θ exceeds a threshold value, no stable equilibrium can exist. In the case where θ is endogenously determined, there exist multiple equilibria, where, in one equilibrium, the asset price equals its fundamental price, and it has a bubble component in another. It was also shown that if the equilibrium concept is relaxed to allow for sticky prices and a temporary supply-demand gap, there exists an equilibrium in which a bubble develops temporarily and eventually bursts.

These theoretical results of this simplified model imply that if a bubble is generated by the mechanism examined in this paper, an economy may become unstable or the bubble may burst when θ becomes too large. If θ is measured by the collateral ratio in bank lending, we may say that an asset-price bubble may emerge when the collateral ratio of the asset exceeds its historical average.

Further research to measure the real equivalent of θ for land, stocks, and other assets may be useful to measure asset-price bubbles and to predict their collapse.

6 Appendix A

Proof of Lemma 1

First I show that there is no equilibrium where liqiudity constraint (5) is always binding. Proof is by contradiction. Suppose that such an equilibrium exists. The FOCs and the equilibrium conditions imply $Q_t = \frac{\beta}{\pi_{t+1}} P_{t+1} y + \left\{\theta + \frac{\beta}{\pi_{t+1}}(1-\theta)\right\} Q_{t+1}$, for all t, and $Q_t = \frac{P_t c - M}{\theta a}$, for all t, where $\pi_t = \frac{P_{t+1}}{P_t}$. Therefore, the inflation rate π_t follows

$$\pi_t = f(\pi_{t+1}, \frac{M}{P_t}) \equiv \frac{c\pi_{t+1} - (1-\theta)(\pi_{t+1} - \beta)\frac{M}{P_t}}{\theta c \pi_{t+1} + \theta\beta y a + (1-\theta)\beta c}. \tag{32}$$

Note that $f(\beta, \frac{M}{P}) = \frac{c}{c+\theta y a} > \beta$. The last inequality is from (3). This inequality implies that there is a solution to $\pi = f(\pi, \frac{M}{P})$ that is larger than β, since $f(\pi, \frac{M}{P})$ is increasing in π and bounded from above. Define $\overline{\pi}$ as the (unique) solution to $\pi = f(\pi, 0)$. $\overline{\pi} = \frac{c-(1-\theta)\beta c - \beta\theta y a}{\theta c}$. Note that $\overline{\pi} \leq 1$ because $\theta \geq \overline{\theta}$ by the assumption of this lemma. Any path of $\{\pi_t\}_{t=0}^{\infty}$ that is determined by (32) satisfies either that $\pi_t \leq \overline{\pi}$ for all t or that $\exists t$ such that $\pi_t > \overline{\pi}$. I will show the nonexistence of $\{\pi_t\}_{t=0}^{\infty}$ by showing $\{\pi_t\}_{t=0}^{\infty}$ cannot satisfy either condition.

Suppose that $\pi_t \leq \overline{\pi}$ for $\forall t$. Then $\frac{M}{P_t} \geq \frac{M}{P_0}$. Since $f(\pi, \frac{M}{P})$ is decreasing in $\frac{M}{P}$ for

$\pi > \beta$, $f(\pi, \frac{M}{P_t}) < f(\pi, \frac{M}{P_0}) < f(\pi, 0)$ for all π ($> \beta$) and t. See Figure 5.

Figure 5

Define $\hat{\pi}$ as the solution to $\pi = f(\pi, \frac{M}{P_0})$. Obviously $\beta < \hat{\pi} < \overline{\pi} \leq 1$. Since $f(\pi, \frac{M}{P_t}) < f(\pi, \frac{M}{P_0})$, it is easily shown from Figure 5 that if $\exists\tau$ such that $\pi_\tau > \hat{\pi}$, then π_t $(t > \tau)$ monotonically increases and becomes $+\infty$ in finite steps. Therefore, if the equilibrium in which $\pi_t \leq \overline{\pi}$ for all t exists, it must be the case that $\pi_t \leq \hat{\pi} < \overline{\pi}$ for all t. But in this case, since $P_t c \leq (\hat{\pi})^t P_0 c$ and $\hat{\pi} < 1$, $\lim_{t\to\infty} P_t c = 0 < M$. Therefore constraint (5) becomes nonbinding eventually. This fact contradicts the assumption that (5) is always binding in the equilibrium. Therefore $\{\pi_t\}$ cannot satisfy $\pi_t \leq \overline{\pi}$ for all t. Suppose that $\exists\tau$ such that $\pi_\tau > \overline{\pi}$. In this case, it is also easily shown from Figure 5 that π_t $(t > \tau)$ is monotonically increasing and becomes $+\infty$ in finite steps. Thus π_t cannot exceed $\overline{\pi}$ in the equilibrium. The above arguments imply that there exists no equilibrium in which constraint (5) is binding for all t.

Next, suppose that there exists an equilibrium in which $\exists\tau$ such that $\eta_\tau = 0$, i.e., constraint (5) becomes nonbinding for some date τ. I will show that if $\eta_\tau = 0$, then $\eta_{\tau+1} = 0$. Since $\eta_\tau = 0$, $\lambda_{\tau-1} = \lambda_\tau$, which implies $P_{\tau+1} = \beta P_\tau$. Since $\lambda_\tau = \lambda_{\tau+1} + \eta_{\tau+1} \geq \lambda_{\tau+1}$, the asset price satisfies $Q_\tau = \frac{\lambda_{\tau+1}}{\lambda_\tau}(P_{\tau+1}y + Q_{\tau+1}) + \frac{\eta_{\tau+1}}{\lambda_\tau}Q_{\tau+1}\theta \leq P_{\tau+1}y + (1-\theta)Q_{\tau+1} + \theta\frac{\lambda_{\tau+1}+\eta_{\tau+1}}{\lambda_\tau}Q_{\tau+1} = P_{\tau+1}y + Q_{\tau+1}$. Suppose that $\eta_{\tau+1} > 0$. In this case (5) is binding, and $P_{\tau+1}c = M + Q_{\tau+1}\theta a$. Then $M + Q_{\tau+1}\theta a - \beta(M + Q_\tau\theta a) \geq (1-\beta)(M + Q_{\tau+1}\theta a) - \beta P_{\tau+1}y\theta a = P_{\tau+1}\{(1-\beta)c - \beta y\theta a\} > 0$. The last inequality is from (3). This inequality implies that $P_{\tau+1}c = \beta P_\tau c \leq \beta M + \beta Q_\tau \theta a < M + Q_{\tau+1}\theta a$, which contradicts the assumption that (5) is binding at date $\tau + 1$. Therefore, it has been shown that $\eta_{\tau+1} = 0$ if $\eta_\tau = 0$.

By induction, if $\eta_\tau = 0$, then $\eta_t = 0$ for all t $(\geq \tau)$. In this case, $P_{t+1} = \beta P_t$ for all t $(\geq \tau)$, which implies that $\lim_{t\to\infty} \lambda_t = \frac{\beta^\tau}{P_\tau}u'(c) > 0$. This violates the transversality condition. It thus has been shown that there is no competitive equilibrium in which $\exists\tau$ such that (5) is nonbinding at date τ.

The above arguments imply that there is no competitive equilibrium if $\theta \geq \overline{\theta}$ and M_t^s is constant over time.

7 Appendix B

Proof of Lemma 2

The asset price Q_t satisfies $Q_t = \frac{\beta}{\pi_{t+1}} P_{t+1} y + \left\{ \theta + (1-\theta) \frac{\beta}{\pi_{t+1}} \right\} Q_{t+1}$, and liquidity constraint (5) implies $Q_t = \frac{P_t c - M_t}{\theta a}$. Therefore, in the equilibrium where $\pi_t = \pi (\neq 1)$, the following equation must be satisfied:

$$c = \beta y a \theta + \omega \pi c + \frac{1}{\pi^t P_0} \left\{ M_t - \omega M_{t+1} \right\}, \tag{33}$$

where $\omega = \theta + (1-\theta)\frac{\beta}{\pi}$. This equation must hold for all t given that $\pi \neq 1$. Therefore, the monetary policy must satisfy the constraint that $M_t^s - \omega M_{t+1}^s = \pi^t P_0 x$, where x is a constant. Therefore, M_t^s must follow

$$M_t = \omega^{-t} \left\{ M_0 + \frac{1-(\omega\pi)^t}{1-\omega\pi} P_0 x \right\}. \tag{34}$$

In this case, (33) implies that $\pi = \frac{1-\beta}{\theta} + \beta + \frac{1}{\theta c}(x - \beta\theta y a)$. In a steady inflation equilibrium, $\frac{M_t}{P_t} = (\omega\pi)^{-t} \left\{ \frac{M_0}{P_0} + \frac{1-(\omega\pi)^t}{1-\omega\pi} x \right\}$ must be positive and finite. If $\omega\pi < 1$, then $\lim_{t\to\infty} \frac{M_t}{P_t} = \infty$; if $\omega\pi = 1$, then $\frac{M_t}{P_t} = \frac{M_0}{P_0} + tx$, which goes to infinity as time passes; if $\omega\pi > 1$, then $\lim_{t\to\infty} \frac{M_t}{P_t} = \frac{x}{\omega\pi - 1}$, which is finite and positive. Therefore, in a steady inflation equilibrium, the government must set x such that $\omega\pi > 1$, i.e., $x > \beta\theta y a$. But in this equilibrium, constraint (5) must be binding. Therefore, $c \geq \frac{M_t}{P_t}$ for all t, implying that $c \geq \frac{x}{\omega\pi - 1} = \frac{x}{x - \beta\theta y a} c > c$, which is a contradiction. Therefore, there is no steady inflation equilibrium in this economy.

8 Appendix C

Proof of Claim 1

Suppose that $\theta_\tau = 0$ and (5) becomes nonbinding at date τ in the equilibrium. In this case, the Lagrange multiplier for (5) becomes zero at τ: $\eta_\tau = 0$. The FOCs and (5) imply that $P_{\tau+1} = \beta P_\tau$, and $P_\tau c \leq M$. Therefore, $P_{\tau+1} c = \beta P_\tau c < P_\tau c \leq M$, which implies that (5) is nonbinding at date $\tau+1$. Thus, by induction, it is shown that $\eta_t = 0$ for $\forall t (\geq \tau)$ and therefore $P_{t+1} = \beta P_t$ for $\forall t(\geq \tau)$. In this case $\lambda_t = \frac{\beta^{t+1} u'(c_{t+1})}{P_{t+1}} = \frac{\beta^\tau}{P_\tau} u'(c) > 0$ for

$t > \tau$, implying that the transversality condition ($\lim_{t\to\infty} \lambda_t = 0$) is violated. Therefore, $\eta_t \neq 0$ if $\theta_t = 0$ in the equilibrium.

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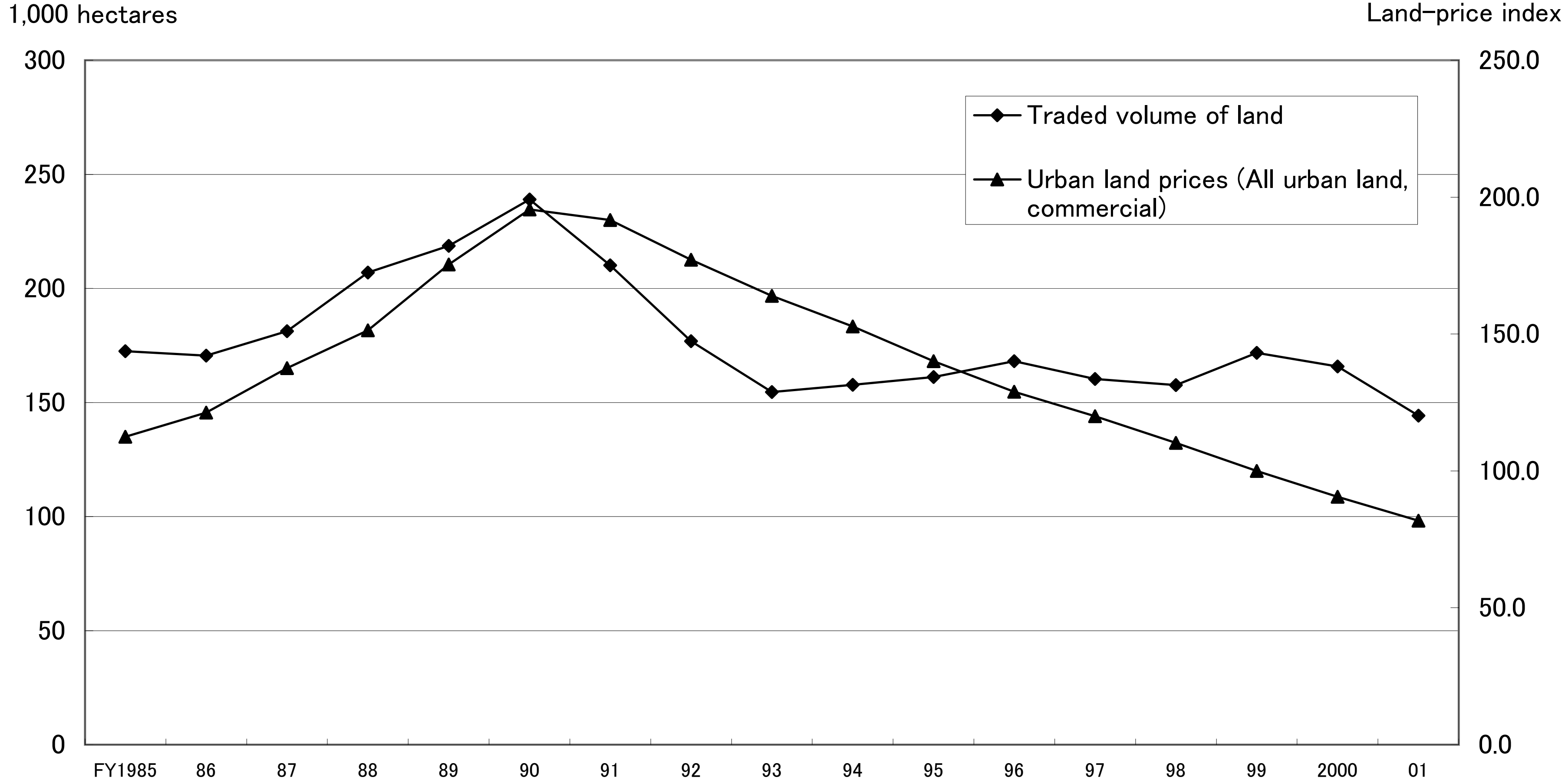
Figure 1. Traded volume of land and urban land prices



Sources: White paper on land, infrastructure, and transport in Japan, Ministry of Land, Infrastructure, and Transport.
Urban price index, Japan Real Estate Institute.
Note: Data for the index of urban land prices are as of the end of the fiscal year. End of FY1999 = 100.
The right axis is the index of urban land prices.

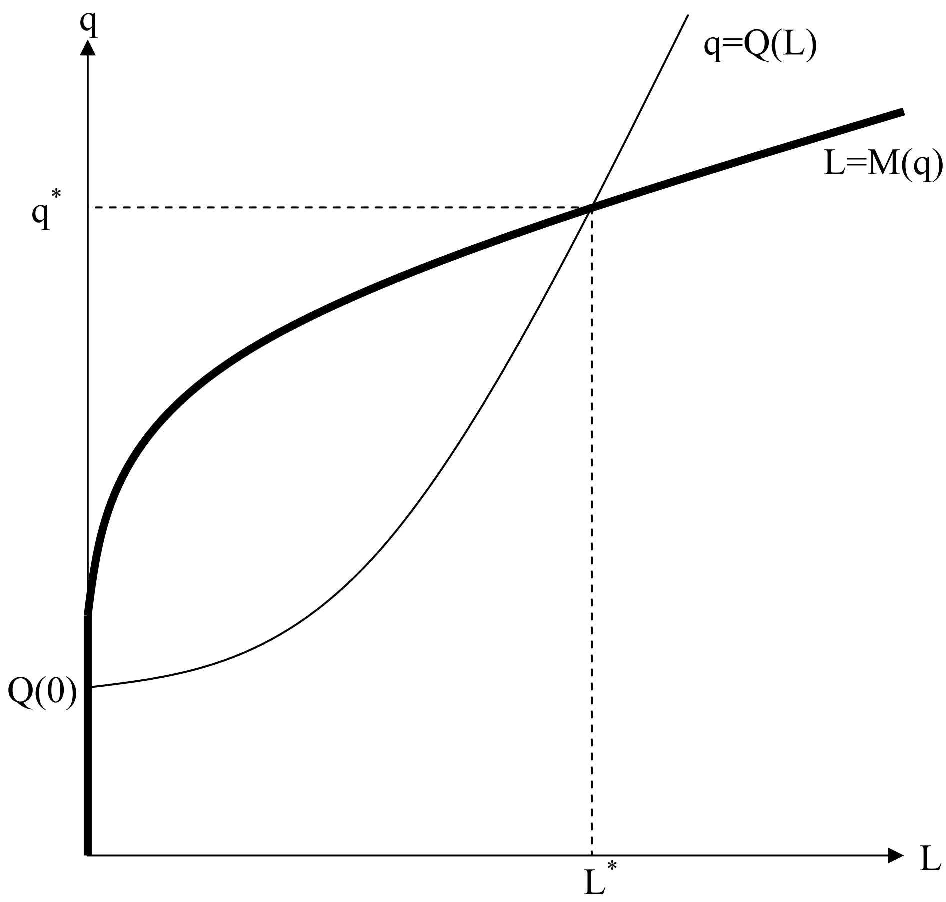
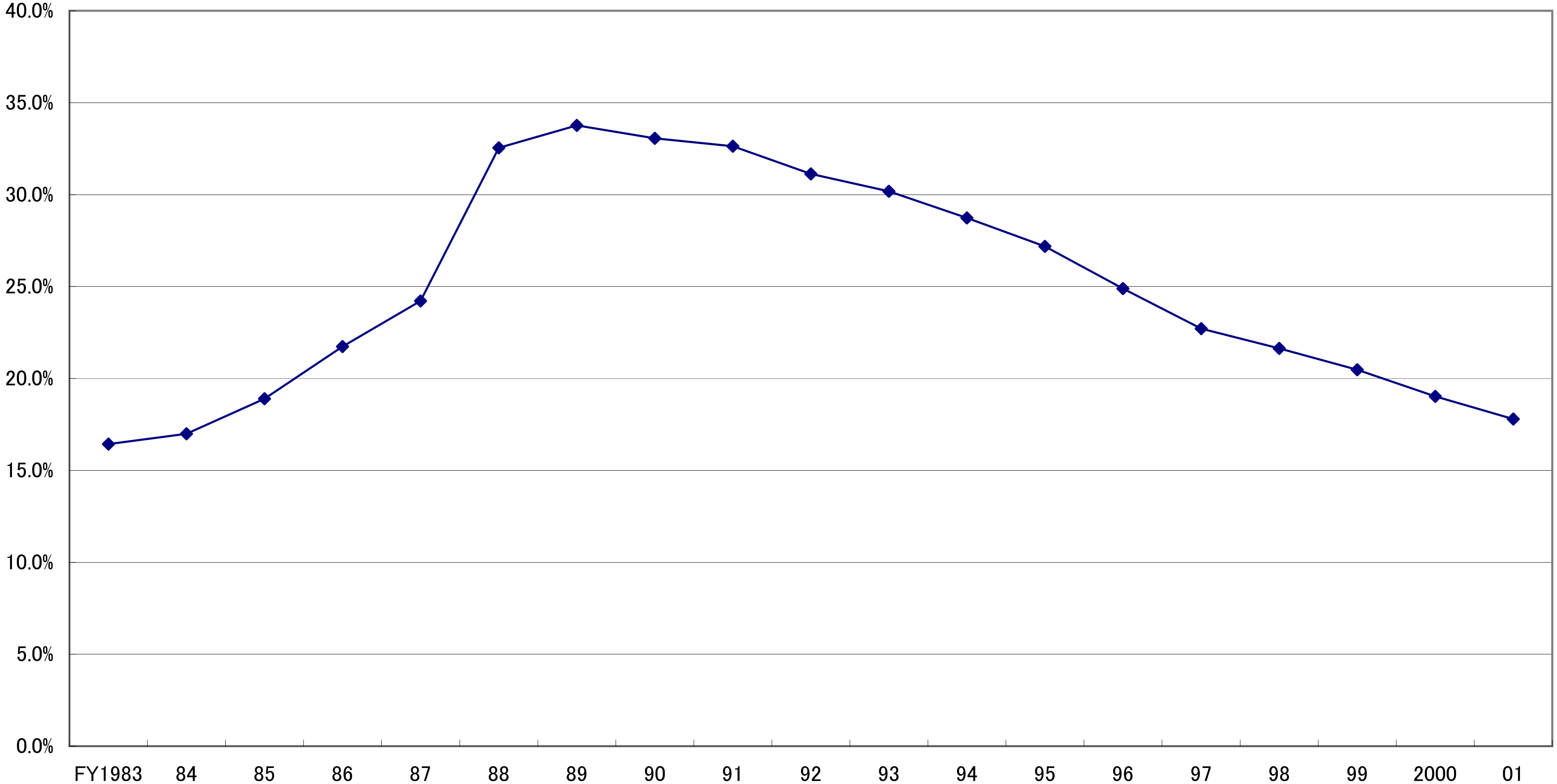


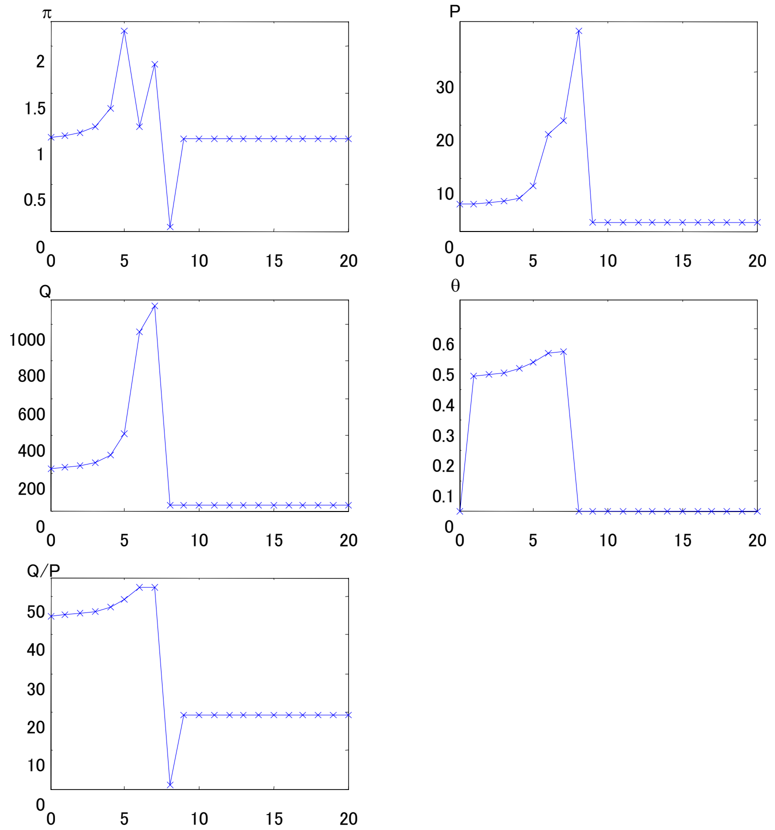
Figure 2. Land prices and liquidity

Figure 3. Ratio of loans covered by collateral (real estate and floating mortgage) to nominal GDP



Sources: Bank of Japan; Economic and Social Research Institute, Cabinet Office, Government of Japan.
Note: Data for loans covered by collateral are as of January of each fiscal year.

Figure 4. Sticky price equilibrium with a bursting bubble



Time	P	π	Q	θ	c
0	5.00	1.02	225.00	0	30.00
1	5.10	1.03	230.50	0.45	30.00
2	5.27	1.06	239.63	0.45	30.00
3	5.59	1.13	257.47	0.46	30.00
4	6.33	1.33	298.24	0.47	30.00
5	8.43	2.18	413.42	0.49	30.00
6	18.37	1.13	960.25	0.52	30.00
7	20.80	1.82	1093.70	0.52	30.00
8	37.76	0.04	31.67	0	1.32
9	1.67	1.00	31.67	0	30.00
10	1.67	1.00	31.67	0	30.00

$\pi^* \approx 1.01$

Parameters: c=30, x=25, a=1, y=1, M=50, P_0=5, β =0.95, σ =0.1, $\pi_0 = \pi^* + 0.01$

Figure 5.