RIETI Discussion Paper Series 04-E-025

A Key Currency and a Local Currency
- A simple theoretical model and its welfare implications

KOBAYASHI Keiichiro
RIETI

RIETI
Research Institute of Economy, Trade & Industry, IAA

The Research Institute of Economy, Trade and Industry
http://www.rieti.go.jp/en/

RIETI Discussion Paper Series 04-E-025

A key currency and a local currency – A simple theoretical model and its welfare implications

Keiichiro Kobayashi*

July 15, 2004

Abstract

Is there any welfare distortion by the asymmetry between a key currency and a local currency in international trade? Interpreting this asymmetry as an international liquidity constraint for countries that cannot issue the key currency, I show that the answer may be yes. It is shown that the country that issues the key currency can raise its level of consumption by increasing the amount of its currency held by foreigners, or in other words, by increasing its external debt.

* Research Institute of Economy, Trade and Industry,
e-mail: kobayashi-keiichiro@rieti.go.jp

A key currency and a local currency – A simple theoretical model and its welfare implications.

(Incomplete and preliminary)

Keiichiro Kobayashi

Fellow, The Research Institute of Economy, Trade and Industry.

1-3-1 Kasumigaseki, Chiyoda-ku, Tokyo 100-8901, JAPAN

+81-3-3501-8308 (phone), +81-3-55101-3927 (fax)

E-mail: kobayashi-keiichiro@rieti.go.jp

July 15, 2004

Abstract

Is there any welfare distortion by the asymmetry between a key currency and a local currency in international trade? Interpreting this asymmetry as an international liquidity constraint for countries that cannot issue the key currency, I show that the answer may be yes. It is shown that the country that issues the key currency can raise its level of consumption by increasing the amount of its currency held by foreigners, or in other words, by increasing its external debt.

1 Introduction

This short paper is a simple theoretical exercise on the welfare implications of asymmetry between currencies: a key currency and local currencies. A key currency is the currency that is accepted as a (only one) medium of exchange in international trade, while local currencies are accepted only within the borders of the issuer countries. The key currency

in reality is of course the US dollar.[1]

In a two-country model, assuming that Country A issues the key currency and Country B the local currency, I show that Country A can increase its level of consumption permanently by increasing the amount of its currency held by foreigners, or in other words, by increasing its external debt. This is because both countries have the liquidity constraint that they need to have cash of the key currency beforehand when they buy tradable goods, while the government of Country A can exploit seigniorage revenue for its domestic consumers by issuing additional amounts of the key currency. This result gives one potential mechanism that may explain the reason why the US economy has been so strong in the last decade while its external debt has increased enormously.

The asymmetry between currencies is modeled as a variant of the cash-in-advance constraint that people must have cash of the key currency beforehand when they buy tradable goods, and that only Country A can issue the key currency. This liquidity constraint has a significant welfare effect in the model, leading to the above seemingly paradoxical result.

2 Model

There exist two countries: Country A and Country B. Each country is populated with a continuum of identical consumers whose measure is normalized to one. Each country has a government that can freely issue any amount of the national currency within the country and give it to domestic consumers as a lump-sum subsidy: Government A (B) issues Currency A (B). In this economy, time is discrete and continues from zero to infinity: $t = 0, 1, 2, \cdots, \infty$. Consumers in both countries live indefinitely and are endowed with y units of the domestic goods and z units of tradable goods at each date t. The consumer in Country A (B), henceforth Consumer A (B), have identical preferences:$\sum_{t=0}^{\infty} \beta^t (\ln d_t + \ln c_t)$, where β $(0 < \beta < 1)$ is the time discount factor, d_t is

[1]McKinnon (2002) points out that the US dollar's facilitating roles as international money are not only a medium of exchange, but also a store of value, a unit of account, and a standard of deferred payment.

consumption of the domestic goods at date t, and c_t is consumption of tradable goods at date t. The domestic goods are traded only within each country, and the total supply of the domestic goods in each country at each date is y. The tradable goods are traded internationally, and the total world supply of tradable goods is $2z$, which is (unequally) divided between Consumers A and B. I assume that Currency A is the key currency that can be used for payments in international trade.

I will denote a variable for Consumer B by putting an asterisk on it. So I denote the amount of Currency A that Consumer B set aside at date $t-1$ by M_t^*. I assume that Government A can freely set the sequence $\{X_t\}_{t=0}^{\infty}$, where X_t is the cash injection of Currency A to Consumer A. I also assume that Government B can freely set the sequence $\{Y_t^*\}_{t=0}^{\infty}$, where Y_t^* is the cash injection of Currency B to Consumer B. Thus the representative consumer's problems for both countries are written as follows. The problem for Consumer A is

$$\max_{d_t, c_t, M_{t+1}} \sum_{t=0}^{\infty} \beta^t (\ln d_t + \ln c_t)$$

subject to

$$P_t d_t + Q_t c_t + M_{t+1} \leq P_t y + Q_t z + M_t + X_t, \tag{1}$$

$$\text{and} \quad P_t d_t + Q_t c_t \leq M_t + X_t, \tag{2}$$

while the problem for Consumer B is

$$\max_{d_t^*, c_t^*, M_{t+1}^*, N_{t+1}^*} \sum_{t=0}^{\infty} \beta^t (\ln d_t^* + \ln c_t^*)$$

subject to

$$e_t P_t^* d_t^* + Q_t c_t^* + M_{t+1}^* + e_t N_{t+1}^* \leq e_t P_t^* y + Q_t z + M_t^* + e_t (N_t^* + Y_t^*), \tag{3}$$

$$Q_t c_t^* \leq M_t^*, \tag{4}$$

$$\text{and} \quad P_t^* d_t^* \leq N_t^* + Y_t^*, \tag{5}$$

where P_t (P_t^*) is the price of domestic goods in Country A (B) in terms of Currency A (B), d_t (d_t^*) is consumption of domestic goods by Consumer A (B), Q_t is the (international)

price of tradable goods in terms of Currency A, c_t (c_t^*) is consumption of the tradable goods by Consumer A (B), M_{t+1} (M_{t+1}^*) is cash of Currency A held by Consumer A (B), N_{t+1}^* is cash of Currency B held by Consumer B, and e_t is the exchange rate of Currency B in terms of Currency A.

Denoting the Lagrange multipliers for (1), (2), (3), (4), and (5) by λ_t, η_t, λ_t^*, η_t^*, and ξ_t^* respectively, the first order conditions (FOCs) are

$$\frac{\beta^t}{d_t} = (\lambda_t + \eta_t)P_t, \tag{6}$$

$$\frac{\beta^t}{c_t} = (\lambda_t + \eta_t)Q_t, \tag{7}$$

$$\lambda_t = \lambda_{t+1} + \eta_{t+1}, \tag{8}$$

$$\frac{\beta^t}{d_t^*} = (\lambda_t^* + \xi_t^*)e_t P_t^*, \tag{9}$$

$$\frac{\beta^t}{c_t^*} = (\lambda_t^* + \eta_t^*)Q_t, \tag{10}$$

$$\lambda_t^* = \lambda_{t+1}^* + \eta_{t+1}^*, \tag{11}$$

$$e_t\lambda_t^* = e_{t+1}(\lambda_{t+1}^* + \xi_{t+1}^*). \tag{12}$$

The equilibrium conditions are

$$d_t = d_t^* = y, \tag{13}$$

$$c_t + c_t^* = 2z, \tag{14}$$

$$M_{t+1} + M_{t+1}^* = M_t + M_t^* + X_t, \tag{15}$$

$$N_{t+1}^* = N_t^* + Y_t^*. \tag{16}$$

Note that since Currency B is relevant only to the domestic goods in Country B, P_t^* is determined independently from international trade and e_t is determined so that P_t^* and other variables satisfy the FOCs for Consumer B.

Thus I focus on the conditions on Currency A. Equations (6), (7), and (2) imply

$$P_t d_t = Q_t c_t = \frac{1}{2}(M_t + X_t). \tag{17}$$

For simplicity of exposition, I assume that Government A chooses X_t such that

$$M_{t+1} + M_{t+1}^* = \pi(M_t + M_t^*), \tag{18}$$

where π (> 1) is the growth rate of the outstanding amount of Currency A. In this case, $\frac{1}{2}(M_t + X_t) = \frac{\pi}{2}M_t + \frac{\pi-1}{2}M_t^*$. Equation (3), (4), (5), and (17) in the equilibrium imply $M_{t+1}^* = Q_t z = \frac{1}{2}\{M_t^* + \frac{1}{2}(M_t + X_t)\} = \frac{\pi}{4}M_t + \frac{\pi+1}{4}M_t^*$. This equation and (18) imply that $M_{t+1} = \frac{3\pi}{4}M_t + \frac{3\pi-1}{4}M_t^*$. Assuming the initial values:$M_0 = M$ and $M_0^* = M^*$, the sequence of $\{M_t, M_t^*\}$ can be solved as

$$M_t = \frac{\pi^{t-1}(3\pi - 1)(M + M^*) + 4^{-t}\{M + (\pi^{-1} - 3)M^*\}}{4 - \frac{1}{\pi}}, \tag{19}$$

$$M_t^* = \frac{\pi^t(M + M^*) - 4^{-t}\{M + (\pi^{-1} - 3)M^*\}}{4 - \frac{1}{\pi}}. \tag{20}$$

Since $X_t = (\pi - 1)\pi^t(M + M^*)$, equations (17) and (4) imply that

$$\frac{c_t}{c_t^*} = \frac{4\pi - 2 + (4\pi)^{-t}\left\{\frac{M+(\pi^{-1}-3)M^*}{M+M^*}\right\}}{2 - (4\pi)^{-t}\left\{\frac{M+(\pi^{-1}-3)M^*}{M+M^*}\right\}}, \tag{21}$$

which converges to $2\pi - 1$ as time passes. P_t is determined by $P_t y = \frac{1}{2}(M_t + X_t)$. The FOCs for Consumer B imply that the exchange rate e_t is determined by $e_{t-1} = \frac{M_t^*}{N_t^* + Y_t^*}$, and P_t^* is determined by $P_t^* y = N_t^* + Y_t^*$. This result shows that the growth of money supply of Currency A does not necessarily imply the depreciation of Currency A compared to Currency B, because e_{t-1} can decrease if Y_t^* grows at a sufficiently high rate.

Assuming that Government A can choose π arbitrarily large, equation (21) implies that Government A can make the level of consumption, and thus welfare, in Country A higher by setting a higher rate of growth of the key currency outstanding. This result can be also interpreted as showing that Country A enjoys a higher level of consumption by making its external debt (M_t^*) larger.

3 Conclusion

This paper showed that the difference between a key currency and local currencies may have significant welfare effects on countries. There is an asymmetry between the issuer country of the key currency and the other countries, i.e., the issuer country earns the seigniorage revenue, while the other countries are liquidity-constrained. The issuer

country of the key currency can exploit the advantage of this asymmetry and make its consumption level higher by increasing the amount of its currency held by foreigners (or by increasing its external debt).