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# **A Theory of Banking Crises (Part 1)**

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# A Theory of Banking Crises

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## Abstract

In order to protect the public's confidence in deposit money, governments usually guarantee bank deposits implicitly or through an explicit deposit insurance system. Thus bank insolvency does not induce immediate bank runs. In many episodes of banking crises, several years passed quietly after bank insolvency had occurred, with the insolvency continuing to develop under the surface, and the rash of bank failures broke out only when the bank insolvency exceeded a certain level.

In this paper I present a simple model that describes the dynamics of bank insolvency in a form that eventually results in banking system failure or bank recapitalization by the government. The main results are as follows: (1) The government cannot indefinitely postpone recognizing the fiscal loss associated with bank insolvency. (2) The consumption level is too high (low) before (after) bank recapitalization compared with the optimal level. Thus the price conditions become deflationary (inflationary) before (after) bank recapitalization. (3) Social welfare decreases as bank recapitalization is delayed.

JEL Classification: E31, E59, E63, G21.

Keywords: Bank insolvency, deposit money, price level determination, bank recapitalization

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# A Theory of Banking Crises (Part I)

## – Welfare Cost of Forbearance –

(Incomplete and Preliminary)

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### **Abstract**

In modern economies, bank deposits play an important role as a means of payment. On the one hand, deposits are private liabilities of banks, and on the other hand, they are public goods: deposit money. In order to protect the public's confidence in deposit money, governments usually guarantee bank deposits implicitly or through an explicit deposit insurance system. Thus bank insolvency does not induce immediate bank runs. In many episodes of banking crises, several years passed quietly after bank insolvency had occurred, with the insolvency continuing to develop under the surface, and the rash of bank failures broke out only when the bank insolvency exceeded a certain level.

In this paper I present a simple model that describes the dynamics of bank insolvency in a form that eventually results in banking system failure or bank recapitalization by the government. The main results are as follows: (1) Banking system failure or bank recapitalization by the government takes place in a finite period of time: i.e., the government cannot indefinitely postpone recognizing the fiscal loss associated with bank insolvency. (2) The consumption level is too high (low) before

(after) bank recapitalization compared with the optimal level. Thus the price conditions become deflationary (inflationary) before (after) bank recapitalization. (3) Social welfare decreases as bank recapitalization is delayed.

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## 1 Introduction

In the recent episodes of banking crises, the crises have developed through the following dynamics (Caprio and Klingebiel [1997]). First of all bank insolvency develops as a result of a collapse of an asset-price bubble or increase in inefficient lending.<sup>1</sup> Even though insolvency becomes severe, bank failures are prevented for several years by deposit guarantees from the government or liquidity support from the central bank. Under the surface, though, the bank insolvency continues to swell as time passes, and a system-wide rash of bank failures breaks out at the point when the bank insolvency becomes apparent to the general public or market participants.

This pattern of crisis development is consistent with many episodes of banking crises, such as in the United States (1980s) and Japan (1990s).

One major factor that produces this dynamic, it seems to me, is the special role of bank deposits. In modern economies, bank deposits play an important role as a means of payment. On the one hand, they are private liabilities of banks, and on the other hand, they are public goods: deposit money. In order to protect the public's confidence in deposit money, governments usually guarantee bank deposits implicitly or through an explicit deposit insurance system. If bank deposits were unprotected private liabilities just like ordinary corporate debts, the occurrence of bank insolvency would trigger bank runs immediately, and the losses associated with bank insolvency would be recognized

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<sup>1</sup>Caprio and Klingebiel (1997) define the onset of a systemic bank insolvency as a time when the ratio of nonperforming loans to total loans exceeds 5–10 percent.

and borne by the depositors immediately. Since, however, the government guarantees bank deposits and the central bank provides liquidity to support the banking system, bank runs do not necessarily happen when banks become insolvent. As a result, even after bank insolvency occurs, deposit money backed by no bank assets can continue to circulate for a considerable period of time.

Unbacked deposit money is an implicit liability of the government. If the government decides that it is unwilling to let depositors bear the losses resulting from bank insolvency, it has no other choice than to make up for the losses itself, ultimately meaning the use of taxpayers' money. In what follows, I use the term *bank recapitalization* to refer to this form of banking system recapitalization through the injection of taxpayers' money. To inject taxpayer's money into the banking system is an unpopular policy and politically difficult to implement. The government usually tries to put off recognizing bank insolvency and making up for the losses. Therefore, the government is tempted to continue an unsustainable policy: on the one hand, it declares deposit guarantees and provides liquidity support, and on the other hand, it postpones bank recapitalization.

The model in this paper describes the development of bank insolvency and the response of macroeconomic variables under this unsustainable policy. There is a literature in which the response of an economy to an unsustainable policy is theoretically examined, namely, the research on balance of payment crises (Krugman [1979], Calvo [1987]). In the models of BOP crises, a policy of fixing the exchange rate at unsustainable level causes the crisis. The intuition of my model is similar. In my model, the unsustainable policy (deposit guarantees without bank recapitalization) causes an inefficient outcome.<sup>2</sup> The paper which is closest to this paper is Dekle and Kletzer (2003). While I developed this model independently, the structure of my model is qualitatively very close to theirs, with some differences in the policy lessons: They stress that the stringent supervision that requires loan-loss reserving by banks is important to prevent the banking crisis, while I argue that quick recapitalization of banks are necessary once the banking system

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<sup>2</sup>The structure of the model is similar to Calvo (2003) in which an unexpected increase in the public debt affects the economic activities through the distortionary tax system.

falls into insolvency as a result of an exogenous shock.

There are some empirical findings on banking crises that are consistent with this model: (1) Output losses following a banking crisis are observed for a surprisingly long period of time (Boyd, Kwak, and Smith [2002]). (2) In a country that experienced a single crisis, the inflation rate typically falls after the onset of the bank insolvency (Boyd et al. [2001]). (3) Open-ended liquidity support and unlimited deposit guarantees are significant contributors to the fiscal cost of a banking crisis (Honohan and Klingebiel [2000]).

The organization of this paper is as follows: In the next section, I present the basic structure of the model. In Section 3, I introduce bank insolvency caused by an unexpected macroeconomic shock, and I describe the development of the banking crisis in an environment of deposit guarantees and postponed bank recapitalization. Section 4 provides concluding remarks.

## 2 The Basic Model

The economy continues for an infinite period from date 0. In this economy, consumption takes place at date  $t$  ( $t = 0, 1, 2, \dots$ ), and production takes place in a period between dates  $t$  and  $t + 1$ . I call the period between dates  $t$  and  $t + 1$  period  $t$ . This economy consists of one government and continua of consumers, firms, and banks. Each continuum (of consumers, firms, and banks) is of measure 1. The consumers are infinitely long-lived and maximize the following utility:

$$\sum_{t=0}^{\infty} \beta^t u(c_t), \quad (1)$$

where  $c_t$  is consumption at date  $t$ , and  $\beta$  is the discount factor, and  $u(c)$  is a concave and increasing function of  $c$ . I assume for simplicity that  $u(c) = \ln c$ . At the beginning of date 0, the consumers are endowed with  $k$  units of the consumer goods and  $B$  units of the nominal government bond.

I assume that consumers, firms, and banks act as price takers as a result of competition in each sector. I also make the following assumption about the medium of exchange

in this economy:

**Assumption 1** *Money consists of either bank deposits or government bonds. All transactions between a consumer and a firm must be mediated by money. A consumer and a firm cannot lend to or borrow from each other directly.*

**Firms and production technology** This economy is a one-good economy in which the consumer good is the only good. The production technology of a firm is as follows: An input of  $k_t$  units of the good at date  $t$  produces  $Ak_t$  units of the good ( $A > 1$ ) at date  $t + 1$ . Only firms have this production technology. The input  $k_t$  depreciates completely to zero during the process of production in period  $t$ .

I assume that firms continue for only one period. A firm of period  $t$  is established at date  $t$ , buys inputs  $k_t$  from consumers and produces the good during period  $t$ . At date  $t + 1$  it sells all of the output  $Ak_t$  to consumers, distributes any profit that it has made to all consumers equally as dividends, and is liquidated. A firm of period  $t$  solves the following maximization problem:

$$\max_k \Pi_t = P_{t+1}Ak - R_t^L P_t k, \quad (2)$$

where  $\Pi_t$  is the profit of the firm,  $P_t$  is the price level at date  $t$ , and  $R_t^L$  is the nominal rate of return on bank loans in period  $t$ . ( $R_t^L = 1 + I_t^L$ , where  $I_t^L$  is the nominal rate of interest of bank loans in period  $t$ .) In order to simplify the analysis of the equilibrium, I assume

$$A = \frac{1}{\beta}. \quad (3)$$

**Circulation of bank deposits** I assume that banks continue operating for infinite periods. In this model, bank failure is represented by bank recapitalization, i.e., a situation where the government is forced to make up for the losses from bank insolvency.

Banks make loans by creating deposits and giving them to firms in return for a promise of future repayment. I make the following assumption about contracts between banks and depositors and between firms and banks:

**Assumption 2** *Bank deposits and bank loans of period  $t$  are debt contracts in which the principal and the interest payment are fixed at date  $t$  and cannot be changed at date  $t + 1$ .*

Debt contracts are used for financing in this economy for some microeconomic reasons that are not specified in this model. For example, agency problems associated with information asymmetry can justify the use of debt contracts (see Gale and Helwig [1985]).<sup>3</sup>

At date  $t$ , a firm borrows  $P_t k_t$  units of deposit money from a bank in order to buy  $k_t$  units of the good as an input, promising to repay  $R_t^L P_t k_t$  to the bank at date  $t + 1$ , where  $R_t^L$  is the nominal rate of return on the bank loan. In making the loan, the bank creates a deposit with a nominal value of  $P_t k_t$ , and gives it to the firm: i.e., the firm becomes the holder of the bank deposit  $P_t k_t$ .<sup>4</sup> The firm buys  $k_t$  units of the good from a consumer in exchange for the bank deposit  $P_t k_t$ . The settlement of this transaction is made by changing the holder of the bank deposit  $P_t k_t$  from the firm to the consumer.

A bank's profit maximization can be expressed as

$$\max_{L_t, D_t} \sum_{t=0}^{\infty} \left( \prod_{s=0}^t \frac{1}{R_s} \right) \{R_t^L L_t - R_t D_t\}, \text{ subject to } L_t = D_t, \quad (4)$$

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<sup>3</sup>That the principal and interest are fixed ex ante in the bank deposit contract is closely related to the unique characteristic of the bank deposit that it is demandable. In order for depositors to have the right to withdraw any amount of their deposit at any time, the total amount of their deposit must be pre-fixed. The demandable nature of bank deposits must be the reason why they are used as a close substitute of cash and a means of payments in reality. Since I made an assumption that bank deposits are used as money (Assumption 1), the fact that deposits are demandable does not play any role in this model. Thus the reason why bank deposits are demandable is not specified in this model either: The need to prevent hold-up problems associated with relation-specificity in lending technology may drive banks to offer demand deposits to depositors (see Diamond and Rajan [2001]).

<sup>4</sup>For readers who consider the notion of deposit *creation* awkward, here is an alternative explanation for bank lending. Suppose that the consumer deposits a portion of their holdings of the government bond  $B_{t-1}$  in the bank. The bank lends  $L_t$  units of the bonds to a firm, and makes the firm deposit back the bonds in the bank immediately. At this point in time, the bank has the bonds ( $B_{t-1}$ ) and the loan ( $L_t$ ) in the asset side, and the deposits from the consumers ( $B_{t-1}$ ) and from the firm ( $L_t$ ) in the liability side. Repeating this process, the bank can make loans larger than its bond holdings. Suppose that the consumers, who are indifferent between the bonds and the bank deposits, withdraw the bonds ( $B_{t-1}$ ) after the bank made loans ( $P_t k_t$ ) to firms. At this point in time, the bank has the loans ( $P_t k_t$ ) in the asset side, and the deposits ( $P_t k_t$ ) in the liability side. Therefore, the deposits  $P_t k_t$  are created.



where  $R_t$  is the nominal rate of return on bank deposits. In the competitive equilibrium, (2) and (4) imply that

$$R_t^L = R_t = A \frac{P_{t+1}}{P_t}. \quad (5)$$

We can show that the bank deposit ( $P_t k_t$ ) is cleared at date  $t + 1$  in this competitive equilibrium: At date  $t + 1$ , the consumer's bank deposit has grown to  $R_t P_t k_t$ . The firm sells the output  $A k_t$  to the consumer and receives the proceeds  $P_{t+1} A k_t$ . In the competitive equilibrium where  $P_{t+1} A = R_t P_t$ , the proceeds are equal to the consumer's bank deposit. Thus the consumer buys the good  $A k_t$  from the firm by paying the entire bank deposit  $R_t P_t k_t$  to the firm. At this point in time, the bank has a loan  $R_t^L P_t k_t$  to the firm, and the firm has a bank deposit  $R_t P_t k_t$ . In the competitive equilibrium where  $R_t^L = R_t$ , the repayment from the firm to the bank is made by offsetting the loan with the deposit. Therefore, in the competitive equilibrium, the deposit created at date  $t$  is cleared at date  $t + 1$ .

**Government bonds and the price level** The government issues bonds at each date  $t$  and redeems them using revenue from a consumption tax. The government bonds are a liquid asset (see Assumption 1). Thus a consumer (or a firm) holding bonds can freely exchange them for a bank deposit of the same nominal value. Bank recapitalization in Section 3 is a policy under which the government gives the bonds to depositors in exchange for bank deposits that are not backed by bank assets.

The government budget constraint is as follows:

$$\begin{aligned} \tau_0 P_0 c_0 &= B - B_0, \\ \tau_t P_t c_t &= R_{t-1} B_{t-1} - B_t, \quad \text{for } t \geq 1, \end{aligned} \quad (6)$$

where  $B$  is the initial amount of the bonds held by consumers and  $\tau_t$  is the rate of consumption tax at date  $t$ . The left-hand side of (6) is the tax revenue. The government can choose the values of  $\tau_t$  and  $B_t$  under the constraint that (6) is satisfied.

We can interpret that the equations (6) determine the price level  $P_t$ , given the government's policy choice of values  $(\tau_t, B_t)$ . This interpretation is the same as the logic of

price determination in the fiscal theory of price levels (see Woodford [2001], Cochrane [2000]).

I have assumed that there is only one kind of taxation: a consumption tax. This assumption is for simplicity of analysis, and the results do not change qualitatively even if the government can tax income ( $P_t k_t$ ). But the following assumption is crucial for the results of this paper:

**Assumption 3** *The government cannot impose a lump-sum tax on consumers.*

If the government uses a lump-sum tax to finance the cost of bank recapitalization, the consumption level becomes optimal. Thus a delay in bank recapitalization does not have any effect on social welfare in this case. But in this economy bank recapitalization using a lump-sum tax on consumers is nearly equivalent to defaults on bank deposits that destroy the credibility of bank deposits as money. Thus I assume that the government cannot impose a lump-sum tax on consumers since it wants to maintain the public's confidence in deposit money.

**Competitive equilibrium** The consumer's optimization problem is

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t)$$

subject to

$$\begin{cases} (1 + \tau_0)P_0 c_0 + P_0 k_0 + D_0 + B_0 \leq P_0 k + B + P_0 k_0, \\ (1 + \tau_t)P_t c_t + P_t k_t + D_t + B_t \leq R_{t-1}(D_{t-1} + B_{t-1}) + P_t k_t + \Pi_t, \text{ for } t \geq 1, \end{cases} \quad (7)$$

where  $k$  is the consumer's initial holdings of the good at date 0 and  $D_t$  is the consumer's bank deposit remaining at date  $t$ . The term  $P_t k_t$  in the left-hand side of the consumer's budget constraint is the payment from the consumer to a firm of period  $t - 1$  for the purchase of  $k_t$ , while the term  $P_t k_t$  in the right-hand side is the proceeds from selling  $k_t$  to a firm of period  $t$ .

The government's budget constraint (6), the consumer's budget constraint (7), and the resource constraints

$$\begin{aligned} c_0 + k_0 &= k, \\ c_t + k_t &= A k_{t-1} \text{ for } t \geq 1, \end{aligned} \quad (8)$$

imply

$$D_t = P_t k_t \text{ for } t \geq 0. \quad (9)$$

Since I have assumed that  $u(c_t) = \ln c_t$ , the first-order conditions (FOCs) of the consumer's problem and (5) imply

$$\frac{c_{t+1}}{c_t} = \beta \frac{(1 + \tau_t) P_t}{(1 + \tau_{t+1}) P_{t+1}} R_t = \beta A \frac{1 + \tau_t}{1 + \tau_{t+1}} = \frac{1 + \tau_t}{1 + \tau_{t+1}}. \quad (10)$$

The last equality follows from  $A = \beta^{-1}$ .

Maximizing (1) subject to (8), we obtain the socially optimal allocation of consumption:

$$c_t = c^* = (1 - \beta)k, \text{ and } k_t = k^* = \beta k, \text{ for all } t \geq 0. \quad (11)$$

The condition (10) implies that this optimal allocation is attained in the competitive equilibrium if the tax rate  $\tau_t$  is set at a constant rate. (The price level  $P_t$  and the nominal return  $R_t$  can vary under a constant tax rate if  $B_t$  varies.)

For simplicity I assume that the tax rate and the volume of bond issuance is time invariant:  $\tau_t = \tau^*$  and  $B_t = B^* \equiv \beta B$  for all  $t \geq 0$ . In this case the equilibrium price becomes  $(P_t, R_t) = (P^*, R^*) \equiv (\frac{(A-1)B^*}{\tau^* c^*}, A)$ , and the equilibrium allocation becomes  $(c_t, k_t) = (c^*, k^*)$ . This equilibrium is the social optimum.

### 3 Banking Crisis

I assume that, at date 0, all agents in the economy expected that the economy would follow the optimal path  $(P_t, R_t, c_t, k_t) = (P^*, R^*, c^*, k^*)$ . The banks created  $P^* k^*$  units of bank deposits and lent them to firms. The nominal rate of return for period 0 (from date 0 to date 1) was fixed at  $R^*$ . At date 0, firms made a promise to repay  $R^* P^* k^*$  to banks at date 1, and banks made a promise to repay  $R^* P^* k^*$  to depositors at date 1.

I assume that an unexpected macroeconomic shock ( $\lambda$ ) hit the economy at a time between date 0 and date 1, where

$$0 < \lambda < 1, \quad (12)$$

and  $\lambda$  is close to 1. This shock completely destroyed the output of  $1 - \lambda$  firms. Thus at date 1,  $\lambda$  firms produce the output  $Ak^*$ , while the other  $1 - \lambda$  firms produce nothing. The total production at date 1 becomes  $\lambda Ak^*$ . In this case, the optimal allocation from date 1 onward becomes  $c_t = c^o \equiv \lambda c^*$ , and  $k_t = k^o \equiv \lambda k^*$ .

**Bank insolvency** Based on the above assumptions, let us now examine the process of payments after the unexpected shock  $\lambda$  hits the economy.

At date 1, banks have assets (loans) of  $P^* Ak^*$ . But since  $1 - \lambda$  firms failed to produce any output, the banks cannot recover their loans to them. These firms go bankrupt at date 1. Thus the banks have irrecoverable loans with a face value of  $(1 - \lambda)P^* Ak^*$ .

Next we assume that the new equilibrium price at date 1 becomes  $P$ . The price  $P$  may be either greater or less than  $P^*$ . The amount the bank can collect from the remaining  $\lambda$  firms is  $\lambda \min\{P, P^*\} Ak^*$ . Therefore, the total amount of irrecoverable loans at date 1 is  $N_1$ , where

$$N_1 = \left[ (1 - \lambda) + \lambda \max\left\{1 - \frac{P}{P^*}, 0\right\} \right] P^* Ak^*. \quad (13)$$

Meanwhile, at date 1, the consumers have bank deposits of  $R^* P^* k^* = P^* Ak^*$ , out of which they use  $\lambda P Ak^*$  to purchase the outputs. (We can take it that the consumption tax  $\tau^* P c_1$  is paid with a part of consumers' holdings of government bonds  $[R^* B^*]$ , since the equation  $\tau^* P c_1 = R^* B^* - B_1$  must hold ex post in the new equilibrium for nonnegative  $B_1$ .) The total profits of firms at date 1 are  $\lambda \max\{P - P^*, 0\} Ak^*$ , which are paid out to the consumers in equal dividends. Therefore, after all the transactions on period 0 output are over, the remaining bank deposits of consumers are  $P^* Ak^* - \lambda P Ak^* + \lambda \max\{P - P^*, 0\} Ak^* = (1 - \lambda)P^* Ak^* + \lambda \max\{P^* - P, 0\} Ak^* = N_1$ .

The firms of period 0 are all liquidated at date 1. Therefore, at date 1, the banks have irrecoverable loans of  $N_1$  on the asset side and uncleared deposits of  $N_1$  from consumers on the liability side.

If bank deposits were perceived as private liabilities like ordinary debts of firms, and if the government did not guarantee them, the irrecoverable loans  $N_1$  and the unbacked deposits  $N_1$  would be immediately written off at date 1 with the entire cost  $N_1$  being

borne by the consumers as a lump sum. After this immediate adjustment, the new equilibrium prices ( $P_t = \frac{1}{\lambda}P^*$ ,  $R_t = A$  for  $t \geq 1$ ) and the optimal allocation ( $c_t = c^o$ ,  $k_t = k^o$ ) would be realized.

But in this economy, where the bank deposits circulate as money, the government does not allow the occurrence of defaults on bank deposits, nor do the economic agents expect the government to allow such defaults. Thus the expectation prevails that the government will make up for the losses from bank insolvency by paying the cost of guaranteeing the deposits of failed banks or by injecting taxpayers' money into the insolvent banks.

But this does not necessarily mean that the government must recapitalize banks at date 1. As long as people believe that the government will make up for the losses from bank insolvency sometime in the future, the public's confidence in deposit money is maintained.<sup>5</sup> The unbacked deposits  $N_1$  will grow to  $N_t = \{\prod_{s=1}^t R_s\}N_1$  at date  $t$ . Since the banks consider that the government will make up for the loss ( $N_t$ ) in the future, the unbacked deposit  $N_t$  does not appear in the bank's profit maximization; the banks have the unbacked deposit  $N_t$  in the liability side, while they have the asset  $N_t$ , i.e., the (implicit) deposit guarantee from the government. Therefore, the bank's problem is still (4).

Next I will describe the competitive equilibrium in which the date of bank recapitalization is not date 1 and  $N_t$  circulates in the economy as money.<sup>6</sup>

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<sup>5</sup> Otherwise the confidence will collapse. If this happens, the write-off of  $N_1$  occurs immediately, and transactions among economic agents become impossible (for a while) because bank deposits are money in this cashless economy. Thus we assume that the loss of confidence in deposit money generates prohibitively high social costs.

<sup>6</sup>Note that a positive productivity shock at another date cannot help banks to recover the loans  $N_t$ , since in the loan contracts the repayments are fixed competitively before the shock hits the economy. For example, suppose that an unexpected macroeconomic shock at date  $t$  ( $t \geq 1$ ) changes the productivity from  $A$  to  $A + \epsilon$ . In this case, banks cannot recover  $N_t$  from firms since the repayments to the banks are fixed at date  $t - 1$ . Firms' profits from the productivity increase are distributed to consumers, and thus the unbacked deposits ( $N_t$ ) of the consumers do not decrease. Therefore, the problem of bank insolvency persists even if the economy is hit by both negative and positive shocks at different dates.

**Equilibrium in which banks are recapitalized at date  $T$**  In reality, bank recapitalization is not a popular policy. Therefore, the government of a country hit by a banking crisis usually puts off recognizing the crisis and paying the costs of bank recapitalization.

In order to formalize this postponement of bank recapitalization in my model, I assume that immediately after the shock  $\lambda$  hits the economy, the following expectation prevails:

**Assumption 4** *People expect that the government will give  $N_T$  units of bonds to the consumers in exchange for the unbacked deposit  $N_T$  at date  $T$  ( $T > 1$ ), and that the government will change the tax policy ( $\tau_t$ ) only at date  $T$ .*

Under this assumption, equation (10) implies that the consumption is invariant from date 1 to date  $T - 1$ , and from date  $T$  onward. Therefore, to simplify the analysis I assume that the economy stays in a steady state until date  $T - 1$  and moves to another steady state at date  $T$ . The economic variables can be written as follows:<sup>7</sup>

$$P_t = P, c_t = c, \tau_t = \tau^*, B_t = B^*, \text{ for } t \leq T - 1, \quad (14)$$

$$P_t = P', c_t = c', k_t = k', \tau_t = \tau', B_t = B', \text{ for } t \geq T. \quad (15)$$

At date  $T$ , the government pays  $N_T$  units of the bonds to the depositors in order to make up for the losses from bank insolvency. Thus the constraint at date  $T$  becomes  $\tau' P' c' = R_{T-1} B^* + N_T - B'$ .

Profit maximization by firms implies  $\frac{P_t}{P_{t+1}} R_t = R_t = A$ . Therefore,  $N_t = A^{t-1} N_1$  for  $t \leq T - 1$ . Since the nominal rate of return from date  $T - 1$  to date  $T$  is  $R_{T-1} = A \frac{P'}{P}$ , the amount of the bonds that the government must give to the banks is

$$N_T = R_{T-1} N_{T-1} = \frac{P'}{P} A^{T-1} N_1. \quad (16)$$

The government's budget constraint is

$$\tau^* P c = A B^* - B^*, \text{ for } t \leq T - 1, \quad (17)$$

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<sup>7</sup>I assumed that the economy jumps to the latter steady state at date  $T$ . The results do not change qualitatively even if I assume that the values of the variables at date  $T$  are allowed to be different from those of the period from date  $T + 1$  onward.

$$\tau' P' c' = R_{T-1} B^* + N_T - B', \quad \text{for } t = T, \quad (18)$$

$$\tau' P' c' = AB' - B', \quad \text{for } t \geq T + 1. \quad (19)$$

These conditions imply

$$N_{T-1} = \left( \frac{\tau' c'}{\tau^* c} - 1 \right) B^*. \quad (20)$$

The FOCs for the consumers from date  $T - 1$  to date  $T$  imply

$$\frac{c'}{c} = \frac{1 + \tau^*}{1 + \tau'}, \quad (21)$$

since  $\frac{P}{P'} R_{T-1} = A$  and  $\beta A = 1$ . The equations (20) and (21) imply

$$N_{T-1} = \left( \frac{1 + \frac{1}{\tau^*}}{1 + \frac{1}{\tau'}} - 1 \right) B^*. \quad (22)$$

Given that  $(\tau_t, B_t) = (\tau^*, B^*)$  for  $t \leq T - 1$ , the condition (22) implies that the government must raise the tax rate  $\tau'$  if it delays bank recapitalization.

Since the economy is in a steady state from date  $T$  onward, the resource constraints ( $c_1 + k_1 = \lambda A k^*$ ,  $c_t + k_t = A k_{t-1}$  for  $2 \leq t \leq T - 1$ ) and (21) imply

$$c = \frac{(A - 1) \lambda k^*}{1 - \frac{1}{A^{T-1}} \left( 1 - \frac{1 + \tau^*}{1 + \tau'} \right)}. \quad (23)$$

The conditions (13), (22), and  $P^* = \frac{(A-1)B^*}{\tau^* c^*}$  imply

$$\frac{1 + \tau^*}{1 + \tau'} = 1 - \left[ (1 - \lambda) + \lambda \max \left\{ 1 - \frac{P}{P^*}, 0 \right\} \right] A^{T-1}. \quad (24)$$

If  $P \geq P^*$ , this condition reduces to  $\frac{1 + \tau^*}{1 + \tau'} = 1 - (1 - \lambda) A^{T-1}$ , which determines the relationship between  $T$  and  $\tau'$  uniquely. If the government chooses a larger  $\tau'$  for a given  $T$ , the condition (24) determines the price level  $P (< P^*)$ . But I assume that the government benevolently chooses the smallest value of  $\tau'$  for a given  $T$ . In this case, (24) implies that the equilibrium price  $P$  will be greater than or equal to  $P^*$ . (It is shown that  $P = P^*$  in the equilibrium. See equation (29).)

Then (13) implies  $N_{T-1} = A^{T-1} (1 - \lambda) P^* k^*$ . From (24) we have

$$\tau' = \frac{A^{T-1} (1 - \lambda) + \tau^*}{1 - (1 - \lambda) A^{T-1}}. \quad (25)$$

If a finite and positive value of  $\tau'$  does not satisfy (25) for a given  $T$ , there exists no equilibrium. So we have the following lemma:

**Lemma 1** *The government must choose the date of bank recapitalization ( $T$ ) so that*

$$T \leq \bar{T}, \quad (26)$$

where  $\bar{T}$  is the largest integer less than or equal to  $1 - \frac{\ln(1-\lambda)}{\ln A}$ . The tax rate  $\tau'$  is determined by (25), and the amount of the government bonds  $B'$  is determined by  $B' = \frac{1}{A}(R_{T-1}B^* + N_T)$ .

This lemma says that the government can delay the date of bank recapitalization only up to  $\bar{T}$ .<sup>8</sup>

Since  $\bar{T}$  is an increasing function of  $\lambda$ , the maximum time that the government can postpone bank recapitalization becomes shorter as  $N_1 = (1 - \lambda)AP^*k^*$  becomes larger.

What will happen if the government declares that it will postpone bank recapitalization beyond date  $\bar{T}$ ? The above lemma implies that there is no equilibrium in which the expectation of recapitalization at  $T > \bar{T}$  prevails. Therefore, no one will believe the government's declaration. But on the other hand, it is feasible for the government to postpone bank recapitalization until  $\bar{T}$ . Thus if the government declares its intention to recapitalize at  $T > \bar{T}$ , people will believe that the government will be forced to make up for the losses from bank insolvency at date  $\bar{T}$ , since otherwise the confidence in deposit money will collapse at that point and the government will incur prohibitively high social costs (see footnote 5).

For  $T \leq \bar{T}$ , the equilibrium consumption becomes

$$c = c^* \quad \text{for } t \leq T - 1, \quad (27)$$

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<sup>8</sup>This result appears to depend crucially on the fact that the real rate of return on assets is  $A$ . But we can show the existence of an upper bound for  $T$  in a more general setting. The necessary assumptions are (1) the real rate of return has a lower bound  $r$ , which is greater than 1; (2) the output  $y_t$  is finite for all  $t$ . Since the real value of  $N_t$  is  $\frac{N_t}{P_t} \geq r^{t-1} \frac{P^*}{P_1} (1 - \lambda) Ak^*$ , it increases unboundedly as time passes. On the other hand, the discounted present value of consumption tax revenue (in real terms) for the government ( $G_t < \sum_{s=t}^{\infty} r^{t-s} y_s$ ) is finite. Therefore, there exists  $T$  such that  $\frac{N_t}{P_t} < G_t$  for  $t \leq T$  and  $\frac{N_t}{P_t} > G_t$  for  $t > T$ . Since for  $t > T$  the government cannot make up for the loss  $N_t$  with any fiscal policy under the no-Ponzi condition,  $T$  is the time limit for bank recapitalization. For example, this result holds in the model where the production technology is the Cobb-Douglas ( $y_{t+1} = Ak_t^\alpha n_t^{1-\alpha}$ ) and the consumer is endowed with a constant amount of labor  $n_t = 1$  at each date.



$$c' = \{1 - (1 - \lambda)A^{T-1}\}c^* \text{ for } t \geq T. \quad (28)$$

Therefore,  $c$  is larger than the optimal level of consumption  $c^o = \lambda c^*$ . For  $T \geq 2$ ,  $c'$  is smaller than  $c^o$  and decreases as  $T$  increases. Also the consumption from date  $\bar{T}$  onward becomes almost zero if the government delays the recapitalization until date  $\bar{T}$ .

If the government uses the  $\tau^*$  and  $B^*$  fixed before the shock  $\lambda$  hits the economy, (27) implies

$$P = \frac{(A-1)B^*}{\tau^*c} = P^* < \frac{1}{\lambda}P^*. \quad (29)$$

If the irrecoverable loans  $N_1$  are written off at date 1, the optimal consumption  $c^o = \lambda c^*$  will be realized along with the equilibrium price  $\frac{1}{\lambda}P^*$  under the policy  $(\tau_t, B_t) = (\tau^*, B^*)$ .

Therefore, in the equilibrium with bank insolvency, the price level before bank recapitalization is lower than the optimal price ( $\frac{1}{\lambda}P^*$ ), and the consumption level before recapitalization is larger than the optimal level ( $c^o$ ). The result that the price level after the shock stays at the original ( $P^*$ ) indicates that one of the sources of nominal price rigidity in business cycles may be the government policy or the social norm that prevents the banks from defaulting on their deposits immediately after their assets are impaired.

The price  $P'$  and the bonds  $B'$  from date  $T$  onward cannot be determined uniquely, but must satisfy

$$\frac{B'}{P'} = \frac{c^*}{A-1} \{\tau^* + A^{T-1}(1-\lambda)\}. \quad (30)$$

In this representative consumer economy, the social welfare is  $W = \sum_{t=1}^{T-1} \beta^t u(c) + \sum_{t=T}^{\infty} \beta^t u(c')$ . Therefore,

$$W = u(c^*) \frac{\beta}{1-\beta} + \frac{\beta^T}{1-\beta} \ln\{1 - (1-\lambda)A^{T-1}\}. \quad (31)$$

This equation implies that social welfare decreases as bank recapitalization is delayed (see Figure 1).

**Recapitalization at a date different from the expected date  $T$**  The above argument is based on the assumption of perfect foresight on the date of bank recapitalization ( $T$ ). But since the recapitalization takes place only once, the government can and will recapitalize the banks at different date from the expected date  $T$ .

Suppose that the government recapitalizes the banks at date  $T'$ , which is different from the expected date  $T$ . ( $T'$  may be larger or smaller than  $T$ .) Recapitalization at date  $T'$  is a surprise attack for all economic agents. I formalize the surprise recapitalization as follows. (1) Since the recapitalization is a surprise, the policy is invariant until date  $T'$ :  $\tau_t = \tau^*, B_t = B^*$ , for  $t \leq T' - 1$ . Thus the FOCs for consumers and firms imply  $P_t = P^*, c_t = c^*$  for  $t \leq T'$ . (2) Since  $R_{T'-1} = A$ , the amount of unbacked bank deposits at date  $T'$  is  $N_{T'} = R_{T'-1} N_{T'-1} = A^{T'-1} N_1$ , which is equal to the amount of the bonds that the government injects into the banking system at date  $T'$ . (3) Since the price level and consumption at date  $T'$  are both fixed *before* the government injects  $N_{T'}$  into the banks in this case of surprise recapitalization, all of the injected bonds  $N_{T'}$  must be newly issued at date  $T'$ . (5) Thus the government's liabilities at the beginning of date  $T' + 1$  are  $R_{T'}\{B^* + N_{T'}\}$ , and the budget constraint for the government at date  $T' + 1$  is  $\tau'' P'' c'' = R_{T'}\{B^* + N_{T'}\} - B''$ , where  $(\tau'', P'', c'', B'')$  are the variables in the steady state from date  $T' + 1$  onward.

Therefore, the economic variables can be written as follows:

$$P_t = P^*, c_t = c^*, \tau_t = \tau^*, B_t = B^*, \text{ for } t \leq T', \quad (32)$$

$$P_t = P'', c_t = c'', k_t = k'', \tau_t = \tau'', B_t = B'', \text{ for } t \geq T' + 1. \quad (33)$$

These conditions imply that the surprise recapitalization at date  $T'$  is equivalent to the scheduled recapitalization at date  $T' + 1$ . Since  $N_{T'} = \left(\frac{1+\frac{1}{\tau^*}}{1+\frac{1}{\tau''}} - 1\right) B^*$  and  $N_{T'} = A^{T'-T+1} N_{T-1}$ , the tax rate after bank recapitalization ( $\tau''$ ) must satisfy the following:

$$A^{T'-T+1} = \frac{\frac{1}{\tau^*} - \frac{1}{\tau''}}{\frac{1}{\tau^*} - \frac{1}{\tau'}} \frac{1 + \frac{1}{\tau'}}{1 + \frac{1}{\tau''}}, \quad (34)$$

where  $\tau'$  is determined by (25). The government must choose  $T'$  so that  $\tau''$  is finite. Therefore,

$$T' \leq \bar{T}' \equiv T - 1 + \frac{\ln \left( \frac{1+\frac{1}{\tau'}}{1-\frac{1}{\tau^*}} \right)}{\ln A} = \bar{T} - 1. \quad (35)$$

This condition says that in spite of the people's expectations concerning the date of bank recapitalization ( $T$ ), the government can recapitalize the banks earlier than this date, and it can also delay the recapitalization up to the time limit  $\bar{T}$ .

## 4 Concluding remarks

If banks can default on their deposit liabilities when their assets are impaired as a result of an asset-price bubble collapse or some other macroeconomic shock, the optimal allocation  $(c_t, k_t) = (c^o, k^o)$  can be realized in a competitive equilibrium. But since bank deposits circulate in the economy as money, the government generally does not allow the occurrence of defaults on bank deposits, and people generally expect that it will not. The government will guarantee the deposits and provide liquidity to banks with the result that bank deposits in excess of bank assets continue circulating in the economy. The government has no other choice than to make up for the gap between liabilities and assets of banks at some point by using taxpayer's money in order to restore the solvency of the banking system. But since bank recapitalization policy is politically unpopular, the government tries to delay recapitalization. This paper examined the consequences of this unsustainable policy, namely, deposit guarantees (and liquidity support) without recapitalization of the banking sector.

One of our main findings is that the government cannot postpone bank recapitalization forever. The degree of the impairment of bank assets and other technological constraints determine the time limit for postponement. If the time limit comes without recapitalization, the banking system will collapse, and the government will be forced to make up for the losses from bank insolvency by using taxpayers' money.

Social welfare decreases as bank recapitalization is delayed. The consumption level before (after) recapitalization is too high (low) compared with the optimal level. Thus the price conditions before (after) recapitalization become deflationary (inflationary).

The reason that the outcome becomes inefficient is the distortion in financing the cost of bank recapitalization: Consumers anticipating a tax increase after recapitalization set their level of consumption too high for the period before recapitalization. This inefficiency can be avoided if the government allows the occurrence of defaults on bank deposits, i.e., bank recapitalization through a lump-sum transfer from consumers. But this is not an option for a government that wants to maintain the public's confidence in deposit money.

Thus, in the situation where the government cannot use a lump-sum transfer to

finance bank recapitalization, bank insolvency causes distortions of resource allocation in any case.<sup>9</sup>

There may be another source of distortion associated with bank insolvency that is not modeled in this paper. It is uncertain how the cost of making up for the gap between bank liabilities and assets will be financed in a real-life banking crisis. The government may finance the cost by, for example, inflation (see footnote 9), or increases in labor taxes or capital taxes. This uncertainty about cost distribution can be an additional source of inefficiency in the model where consumers are risk averse.<sup>10</sup>

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<sup>9</sup>A noteworthy policy is to set the nominal interest rate on bank deposits ( $R_t$ ) at a lower level than that on bank loans ( $R_t^L = A\pi_t$ , where  $\pi_t = \frac{P_{t+1}}{P_t}$ ): a zero nominal interest rate policy. This policy aims to make up for the irrecoverable loans by inducing inflation and thereby generating positive bank profits  $(R_t^L - R_t)P_t k_t$ . Assume that (1)  $u(c) = c^{1-\theta}$  ( $0 < \theta < 1$ ); (2) the government can monopolize the banking sector and can set  $R_t < R_t^L = A\pi_t$ ; (3) consumers can hold  $B_t$  or  $D_t$ , the rate of return on which is  $R_t$ , but cannot hold real assets ( $k_t$ ). In this case, the government can choose  $R_t = R$ ,  $g$  ( $0 < g < 1$ ), and  $T$ , and set  $B_t$  such that the banks will be able to make up for their losses on irrecoverable loans in a period from date 1 to date  $T$ , and that the equilibrium price and consumption levels satisfy  $\pi_t = \frac{\beta}{g^\theta} R$ , and  $\frac{c_{t+1}}{c_t} = g$  for  $1 \leq t \leq T-1$ . This is realized by setting  $B_t = R^{t-1} B_1 + \frac{\gamma^{t-1} - \gamma}{1-\gamma} R^t B_0$  for  $1 \leq t < T$ , where  $\gamma = g^{1-\theta} \beta$ . It can be shown that

$$\frac{N_{t+1}}{P_{t+1}} = \frac{g^\theta}{\beta} \left[ \frac{N_t}{P_t} - \left( \frac{1}{g^\theta} - 1 \right) A^{t-1} \left\{ \lambda A k_0 - c_1 \frac{1 - g^t \beta^t}{1 - g\beta} \right\} \right].$$

Therefore, the government can wipe out  $N_t$  in a finite period by setting  $g$  at a small value. But obviously the consumption allocation is not optimal in this case, and the inflation rate becomes high. Moreover, the assumptions (2) and (3) that are necessary to have a successful elimination of  $N_t$  seem too restrictive. Thus in reality, a zero nominal interest rate policy may not be able to eliminate the losses from irrecoverable loans; for example, if the assumption (2) does not hold and the rates of returns are set  $R_t = R_t^L = 1$ , then the inflation rate becomes negative ( $\pi_t = A^{-1} < 1$ ), and  $\frac{N_t}{P_t}$  grows explosively as time passes.

<sup>10</sup>Bad loans that are rolled over may be another source of uncertainty about cost distribution. Since bank deposits are guaranteed and bank runs are prevented, the banks can rollover bad loans and can allow the borrower firms to continue operating. The rolled-over bad loans constitute corporate debt overhang for the borrowers. This corporate debt overhang brings about uncertainty about cost distribution if there exists legal uncertainty about the order of seniority of the claims on the borrower firm held by banks, new creditors, suppliers, and laborers, since the cost of bank insolvency is borne by other claimants if the banks successfully recover the bad loans from the borrowers.

There seem to exist several mechanisms by which the circulation of deposit money on which banks cannot easily default causes a serious distortion in a case of bank insolvency; this paper demonstrates one such mechanism.

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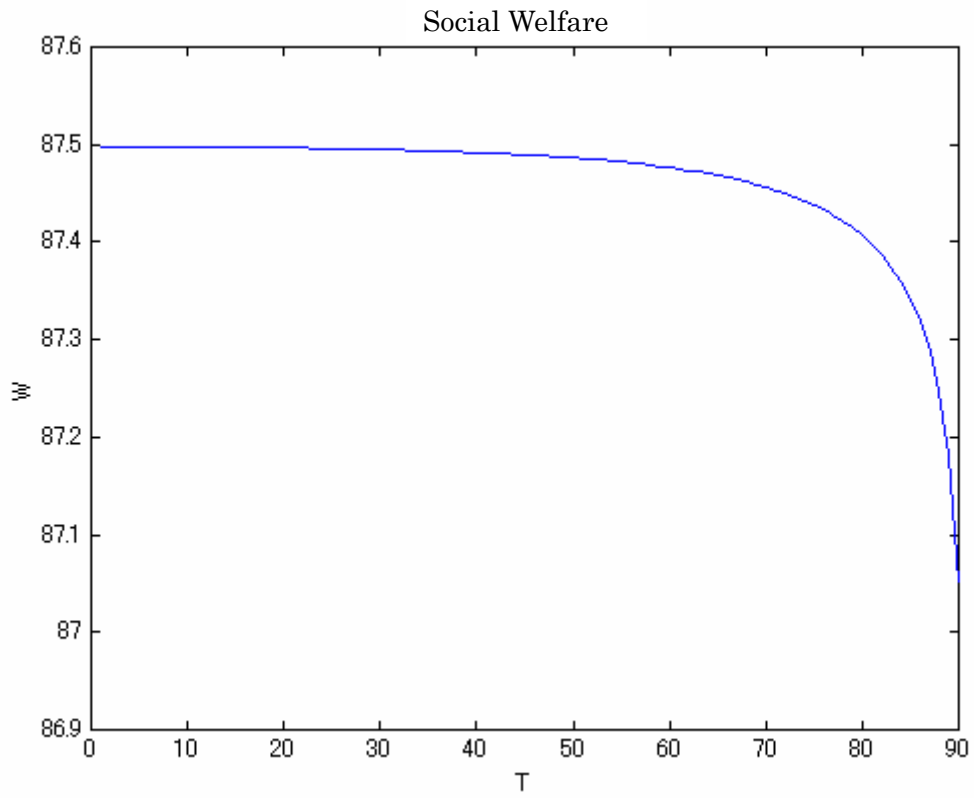
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Parameter:  $A = \frac{1}{\beta} = \frac{1}{0.95}$ ,  $\lambda = 0.99$ ,  $c^0 = 100$ ,