RIETI Discussion Paper Series 03-E-014

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June 2003

Abstract

The purpose of this paper is to analyze the problem of optimally partitioning a design process of a complex product, and to derive several comparative statics results by utilizing the technique developed by Topkis (1998). By partitioning the product design and assigning each sub-design to a team, there are the benefit of having many smaller real options on the one hand, and the cost resulting from an increased incidence of across-team coordination on the other. Furthermore, by endogenizing the across-team coordination costs, our analysis shows that lower cost of within-team coordination induces coarser partitions and higher costs of across-team coordination, i.e. lower level of information and communication technology (ICT) investment. It is argued that these results may explain the reason for the retarded introduction of the ICT by Japanese firms in the 1970s and 1980s as well as the difference of performance between Route 128 and Silicon Valley in the 1990s. It is also argued that our results are consistent with the empirical finding by Brynjolfsson, Maline, Gurbaxani, and Kambil (1994) that ICT leads to decreases in firm size.

JEL Classification: D21;L23;O32

Keywords: Coordination; Modular design; Option value; Supermodularity; Monotone comparative statics

[1] I am grateful to Masahiko Aoki, Fukihiko Funaki, Nobuo Ikeda, Michihiro Kandori, Toshiji Kawagoe, Takuya Nakaizumi, Masahiro Okuno-Fujiwara, and Atsuomi Obayashi, Mitsuo Suzuki for their helpful comments and discussions.

If there are n workers on a project, there are $(n^2 - n)/2$ interfaces across which there may be communication, and there are potentially almost 2^n teams within which coordination must occur. The purpose of organization is to reduce the amount of communication and coordination necessary; hence organization is a radical attack on the communication problems treated above.

— Frederic P. Brooks, 1995, pp.78-79

1 Introduction

This paper analyzes the problem of optimally partitioning a design process of a complex product, and derives several comparative statics results by utilizing the technique developed by Topkis (1998). A designing organization comprises design tasks, each of which determines new design specification of a design parameter for a system product. Design tasks/parameters are usually intricately dependent upon one another. By partitioning the design and assigning sub-designs to design teams, there are the benefit of having many smaller real options on the one hand, and the cost resulting from an increased incidence of across-team coordination on the other.

The analysis in this paper is primarily relevant to organizations and/or quasi-organizations engaged in designing a complex system, such as software or computer system. However, it may also be applicable to organizations and/or quasi-organizations where containing coordination costs is of major importance. Dealing with the relationship between coordination costs and a design, this paper is also concerned with such design concepts as architecture, an interface and modularity, which have been highlighted by the outstanding development of information and communication technology (ICT) in the recent decades.

Our analysis shows that the optimal partition will be coarser if the cost of across-team coordination is higher, the cost of within-team coordination is lower, the uncertainty concerning R&D activities is lower, and the ICT investment is more expensive. These confirm the result obtained by Baldwin and Clark (2000) and Schaefer (1999). We also endogenize the cost of across-team coordination. Our findings are as follows; (1) lower cost of within-team coordination induces higher cost of across-team coordination (i.e. lower level of ICT investment) and coarser partitions to be chosen; and (2) lower cost of ICT investment induces lower cost of across-team coordination and finer partitions to be chosen. It is argued that the former result throws some light on the retarded introduction of new ICT by Japanese firms, while the latter result is consistent with the empirical finding by Brynjolfsson, Maline, Gurbaxani, and Kambil (1994) that the firm size becomes smaller as the firms invest in the ICT. We also discuss the relation of our analyses to the concept of modularization.

This paper is closely related to Baldwin and Clark (2000) and Schaefer (1999). To the best of our knowledge, Baldwin and Clark's is the most comprehensive work to elucidate the reason that the computer industry has dramatically increased its rate of product innovation since the 1970's and has been divided into many smaller sub-industries. They identified "modularity-in-design" as the major driving force behind the heightened pace of this change. By modularizing a system, one interdependent whole is transformed into many independent subsystems (i.e. modules). Then the system of one large option is turned into the sum of many smaller options, which creates more value (the value of splitting). Of course, they do note that modularization can be costly. They argue that modularization incurs the cost of creating and disseminating architecture, running experiments, and testing the compatibility of modules. However, they do not fully formalize these costs to analyze the determinants of optimal partition.

A first formal approach was taken by Schaefer, who combined Baldwin and Clark's concept of modularity-in-design with the economics of supermodular functions. In his paper, partitioning a designing organization creates more value because each design team can specialize in a smaller number of design tasks, while the cost aspect of his model is based on the fundamental insight by Milgrom and Roberts (1995). He assumes that partitioning a designing organization reduces the correlation between research shocks in different teams. Because the value function of the whole system is assumed to be supermodular in the values of component modules, finer partition lowers the value of the whole system.

The model in this paper may be regarded as an extention of the model by Baldwin and Clark in the sense that the benefit of partitioning a product design is derived from having many smaller options instead of one large option. However our model differs from theirs in that we explicitly model and focus upon coordination costs incurred in partitioning a product design, which enable us to conduct a formal comparative statics analysis regarding coordination costs. This paper is also in line with Schaefer's in that both Schaefer's and ours make a comparative statics analysis with respect to communication costs. However, he introduces communication cost as a casual parameter having effect on the cost of buying some level of correlation between research shocks in different components, while the coordination cost in the present paper is naturally derived from partitioning a design process into several sub-designs and assigning them to different teams. We also explicitly considers an optimization problem over the set of possible partitions, which is shown to be a lattice. Although the modeling approaches are different, Schaefer's paper and ours share the conclusion that a coarser partition is favored under higher cost of across-team coordination and lower cost of within-team coordination. This paper provides further insight into the relation between the cost of across-team and within-team coordination by endogenizing the cost of across-team coordination.

Although we would like to restrict our present focus to the design process of a complex product, some aspects of the results obtained in the paper seem to be relevant to a more general argument on the division of labor (Smith 1776/1991). Adam Smith argues that the division of labor leads to greater knowledge, while Becker and Murphy (1992) assert that the causation may also go from greater general knowledge to a more extensive division of labor and greater task-specific knowledge. Our results indicate that less expensive coordination costs, as realized by the development of transportational means and/or information and communication technologies, may cause a more extensive division of labor. It should be noted that our model focuses not on the efficiency of general production processes but on that of R&D activities. However, the our comparative statics result on the optimal partition is not so much dependent on the benefit from having a finer partition as on the coordination costs. So the same logic should be applicable to broader situations.

The next section sets up the framework for the analysis throughout the paper. Based upon the basic insight embodied in "Design Structure Matrix (DSM) Mapping," a grand design is defined as a binary relation on the set of design parameters. Design teams are introduced by partitioning design parameters (tasks) into groups. Section 3 focuses upon the cost of coordination arising from partitioning a designing organization, and the basic property of the cost function is derived. Section 4 turns to the analysis of the benefit of partitioning a designing organization. In this section we follow Baldwin and Clark and identifies the benefit of partitioning as having many smaller options instead of one large option. In Section 5, we integrate both cost and benefit aspects to give comparative statics results and relate those results to Baldwin and Clark's argument of modularization. Section 6 provides some discussion of the obtained results and concludes the paper.

2 The model of a designing organization

According to Baldwin and Clark (2000), A *design* of an artifact is its complete description, which in turn can be broken down into smaller units called *design parameters*. For example, in order to design a mug cup, such design parameters as color, material, height, etc., have to be completely specified. Usually very intricate interdependencies among those parameters exist, which we call *design structure*. The design structure can be easily visualized by means of a technique called "Design Structure Matrix (DSM) mapping," which was invented by Steward (1981) and further developed by Eppinger (1991). In a DSM matrix, design parameters are listed on the rows and columns. If ith parameter is affected by jth parameter, then we put a mark "x" in the cell where ith row and jth column intersect. See Figure 1. The *design tasks* are to choose those parameters. It is easy to see that there is a one-to-one correspondence between a design parameter and a design task.

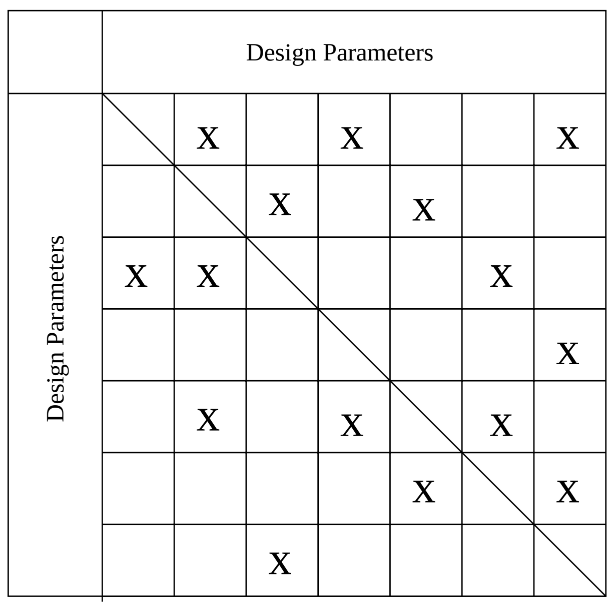

Figure 1: A generic design structure matrix (DSM)

Now consider an organization engaged in designing a complex artifact. We modify the above definition of a DSM matrix so that each design parameter/task requires design efforts by exactly one designer. This can be done by bundling the original design parameters/tasks that are closely related to one another into a new design parameter/task in the modified DSM matrix. Henceforth we suppose that one designer is engaged in each design task. Thus there is a one-to-one correspondence among a design parameter, a design task and a designer. Suppose that there are n design parameters/tasks. Let us denote the set of all design parameters/tasks by $N = \{1, \cdots, n\}$. A *grand design* or *architecture* is represented by a nonempty subset A of $N \times N$ with $(i, i) \notin A$ for each $i \in N$. We interpret $(i, j) \in A$ as expressing that design task i is dependent upon design task j so that some coordination is necessary between them. In terms of the DSM, this corresponds to the situation that the cell corresponding to (i, j) is marked by "x." In what follows, we investigate the optimal partition of N given a grand design A.

Mathematically, a partition $P = \{S_1, \cdots, S_{r(P)}\}$ of N is a family of subsets of N, such that

$$\forall j \quad S_j \neq \emptyset, \quad \forall j \neq k \quad S_j \cap S_k = \emptyset, \quad \cup_{j=1}^{r(P)} S_j = N,$$

where $r(P)$ is the number of elements in P. We introduce a binary relation on the set of all partitions on N, denoted $\mathcal{P}(N)$, as follows. Let P_1 and P_2 be two partitions of N. We say that P_2 is coarser than P_1 (alternatively P_1 is finer than P_2) if for each $S_i \in P_1$, there exist $S_j \in P_2$ such that $S_i \subset S_j$; we write this $P_1 \preceq P_2$. Thus defined binary relation on $\mathcal{P}(N)$ is easily shown to be a partial ordering, and the partially ordered set $(\mathcal{P}(N), \preceq)$ is shown to be a lattice, where $P_1 \vee P_2$ is the finest common coarsening of P_1 and P_2 and $P_1 \wedge P_2$ is the coarsest common refinement of P_1 and P_2. This enables us to treat the problem of optimally partitioning a product design as that of maximizing a function on a lattice and therefore to apply the results obtained by Topkis (1998). Given a partition, each design task belongs to one and the only one element of the partition. Let us denote the element of partition P to which design parameter i belongs by $S(i, P)$, whose cardinality we denote as $\#S(i, P)$. Since each design task requires input of one designer's effort, S_j is a group of design tasks as well as a group of people, which we call "design team." In what follows, we will use the partition of a product design and that of a design organization interchangeably.

Partitioning the set of design tasks and creating design teams have dual functions in the analyses that follow. First, we assume that the coordination across design teams is more costly than that within a design team. Thus fine partitions will be costly because of the increased incidence of across-team coordination. Second, each design team is a unit of decision-making regarding the adoption of new design parameters thereof. A new design will be adopted if and only if the result of R&D in the current period is judged to be better than the current value of the "component." Thus the second assumption implies that the size of design teams determine the size of real options. In sum, by partitioning the design, there are the benefit of having many smaller real options on the one hand, and the cost resulting from an increased incidence of across-team coordination on the other.

3 The Cost of Partitioning a Design Process

Let us now consider the coordination cost of partitioning a designing organization. Under a grand design A, suppose that $(i, j) \in A$. A specific partition P determines for each $(i, j) \in A$ whether $j \in S(i, P)$. Then let the cost of coordination between i and j be c_{ij} if both i and j belong to the same design team, and let the coordination cost between them be C_{ij} if they belong to different design teams. We assume that $0 < c_{ij} < C_{ij}$ for each $(i, j) \in A$. That is, we assume that the coordination across design teams is more costly than that within a design team. We write $C = (C_{ij})_{(i,j)\in A}$ and $c = (c_{ij})_{(i,j)\in A}$. Henceforth, we suppose that $\mathbb{R}^{\#A}$ is endowed with the product ordering relation based on the usual ordering relation on the real line $\mathbb{R}^1$. This makes $\mathbb{R}^{\#A}$ a lattice with $x' \vee x'' = (\max(x'_1, x''_1), \cdots, \max(x'_{\#A}, x''_{\#A}))$ and $x' \wedge x'' = (\min(x'_1, x''_1), \cdots, \min(x'_{\#A}, x''_{\#A}))$ for x' and x'' in $\mathbb{R}^{\#A}$.

Throughout the paper, we will use a broader term "coordination" rather than "communication," because resolving dependencies between design parameters can involve something more than just communication between them. Nevertheless, the cost of coordination will necessarily depend on that of communication.

Although some may think that the assumption that $C > c$ is not trivial, it seems to be classical. For example, Arrow (1975) presumes that integration yields superior auditing technology. In a context more similar to ours, Becker and Murphy (1992) also implicitly assume this condition, for their model supposes that each division incurs more coordination costs under finer division of labor. More recently, Wernerfelt (2003) provides a analytical model for explaining that coordination between divisions is harder than coordination inside divisions. Thus, our current position is that although this condition may needs further foundations, it is a stylized fact. As it appears in the opening quotation by Brooks (1995), some organizational arrangement needs to be contrived to mitigate this coordination problem. After all, the present paper aims at exploring the implication for organization of the assumption rather than its foundations.

Intuitively speaking in the present context, members in the same design team will resort to face-to-face communication very frequently, perhaps because they are located closely. On the other hand, coordination across design teams will require some communication devices like a facsimile, a telephone, the Internet, etc., which may limit the use of subtle and complicated coordination. We may realize lower cost of across-team coordination by installing such devices with some costs, the implication of which we will explore later by making C an endogenous variable.

Suppose that the cost of coordination incurred by the whole designing organization, under the partition P and coordination costs (C, c), is the sum of coordination costs over (i, j)'s in A. Thus we have

$$\begin{aligned} K(P, C, c) &= \sum_{(i,j):(i,j)\in A, j\in S(i,P)} c_{ij} + \sum_{(i,j):(i,j)\in A, j\notin S(i,P)} C_{ij} \\ &= \sum_{(i,j):(i,j)\in A} c_{ij} + \sum_{(i,j):(i,j)\in A, j\notin S(i,P)} (C_{ij} - c_{ij}) \end{aligned} \tag{1}$$

Next lemma is the key to the main theorem.

Lemma 1 *$K(P, C, c)$ is decreasing and submodular in P on $\mathcal{P}(N)$, and has increasing differences in $(P, (-C, c))$.*

Proof We first show that $K(P, C, c)$ is submodular in P on $\mathcal{P}(N)$. Pick any P_1 and P_2 from $\mathcal{P}(N)$. For each $(i, j) \in A$, either $j \in S(i, P_1)$ or not, and similarly either $j \in S(i, P_2)$ or not. Therefore we can partition A into 4 subsets. The subset of A, denoted G, consists of all (i, j)'s such that $j \in S(i, P_1)$ and $j \in S(i, P_2)$. The second subset, denoted by H, comprises all (i, j)'s such that $j \in S(i, P_1)$ but $j \notin S(i, P_2)$. Similarly the third subset I is composed of all (i, j)'s in A with $j \in S(i, P_2)$ but $j \notin S(i, P_1)$. Finally the fourth subset J contains all (i, j)'s with $j \notin S(i, P_1)$ and $j \notin S(i, P_2)$. Obviously they are disjoint and $G \cup H \cup I \cup J = A$.

$$\begin{aligned} K(P_1, C, c) &= \sum_{(i,j):(i,j)\in G\cup H} c_{ij} + \sum_{(i,j):(i,j)\in I\cup J} C_{ij} \\ K(P_2, C, c) &= \sum_{(i,j):(i,j)\in G\cup I} c_{ij} + \sum_{(i,j):(i,j)\in H\cup J} C_{ij} \end{aligned}$$

Since $P_1 \wedge P_2 = \{S_i \cap S_j : S_i \in P_1 \text{ and } S_j \in P_2, S_i \cap S_j \neq \emptyset\}$, we have

$$K(P_1 \wedge P_2, C, c) = \sum_{(i,j):(i,j)\in G} c_{ij} + \sum_{(i,j):(i,j)\in H\cup I\cup J} C_{ij}$$

Observing that $S(i, P_1) \cup S(i, P_2) \subset S(i, P_1 \vee P_2)$,

$$K(P_1 \vee P_2, C, c) \leq \sum_{(i,j):(i,j)\in G\cup H\cup I} c_{ij} + \sum_{(i,j):(i,j)\in J} C_{ij}$$

Thus we have

$$
\begin{aligned}
&K(P_1)+K(P_2)-K(P_1\wedge P_2)-K(P_1\vee P_2)\\
&\geq\left[\sum_{(i,j):(i,j)\in G\cup H} c_{ij}+\sum_{(i,j):(i,j)\in G\cup I} c_{ij}-\sum_{(i,j):(i,j)\in G} c_{ij}-\sum_{(i,j):(i,j)\in G\cup H\cup I} c_{ij}\right]\\
&+\left[\sum_{(i,j):(i,j)\in I\cup J} C_{ij}+\sum_{(i,j):(i,j)\in H\cup J} C_{ij}-\sum_{(i,j):(i,j)\in H\cup I\cup J} C_{ij}-\sum_{(i,j):(i,j)\in J} C_{ij}\right]\\
&=0
\end{aligned}
$$

That $K(P,C,c)$ is decreasing in P should be obvious. Next we show $K(P,C,c)$ has increasing differences in $(K,(-C,c))$. Pick any P' and P'' with $P'\prec P''$.

$$
\begin{aligned}
&K(P'',C,c)-K(P',C,c)\\
&=\sum_{(i,j)\in A,j\notin S(i,P'')}(C_{ij}-c_{ij})-\sum_{(i,j)\in A,j\notin S(i,P')}(C_{ij}-c_{ij})\\
&=-\sum_{(i,j)\in A,j\in S(i,P''),j\notin S(i,P')}(C_{ij}-c_{ij})
\end{aligned}
$$

which obviously is increasing in $-C$ and c. □

Given the assumption that coordination cost is higher *across* design teams than *within* a design team, it is obviously the least costly to have the largest partition $\{N\}$. However, there are also benefits of partitioning a organization, to which we now turn.

4 The Benefit of Partitioning a Design Process

There can be several reasons why partitioning a design process can be beneficial. For example, we may attribute it to the benefit of specialization as in Schaefer (1999) or in Becker and Murphy (1992). Alternatively, as pointed out by Baldwin and Clark (2000), it may be because we have more number of smaller real options under a finer partition of the whole design process. We first formulate our benefit function by keeping loyal to the original formulation by Baldwin and Clark (2000), and then deviate to a more general benefit function.

4.1 Baldwin and Clark's Option Value

In a designing organization, each design task i has its own R&D activity, which yields a potential value of a new design specification. Let us denote by X_i the potential value created at design task i in the current period. As in Baldwin and Clark (2000), we assume that $X_i \sim N(0, \sigma_i^2)$ and X_i's are independently distributed. Let $\sigma = (\sigma_1, \cdots, \sigma_n)$. In this formulation, each design parameter already has a default design specification, whose value is normalized as zero.

Consider a partition P and a design team $S_j \in P$ thereof. S_j will possibly have several design tasks. We assume that the potential value created in S_j in the current period is the sum of the potential value created by all the design tasks belonging to S_j. Thus $X_{S_j} = \sum_{i \in S_j} X_i \sim N(0, \sum_{i \in S_j} \sigma_i^2)$. Denoting the potential value of the whole system product by X, we thus have $X = \sum_{i=1}^n X_i \sim N(0, \sum_{i=1}^n \sigma_i^2)$. So if the results of R&D activities in the current period are all adopted, there is no sense in partitioning the product design.

However, recall that each design team functions as a unit of decision-making as to the adoption of new designs in it. Also recall that the organization already has default designs for respective design parameters whose values are normalized as zero. Then new designs are adopted by the team S_j if and only if the sum of the potential values of new designs in S_j turns out to be greater than the default value *ex post*. Thus the realized value in S_j is a random variable $X_{S_j}^+ = \max(0, X_{S_j})$. This is the reason why Baldwin and Clark (2000) compared the value resulting from R&D activities to "real options." The realized value of the whole designing organization is the sum of those realized values of S_j's in P, that is $\sum_{S_j \in P} X_{S_j}^+$. Since the realized results of R&D activities are real options, it makes sense to partition the designing process. As we shall see, it is more profitable to have many smaller options than to have one large option.

In the previous paragraph, we have made two assumptions concerning the value created in a design organization. First, we assumed X_i's are independently distributed. Second, the value created in a larger unit is simply assumed to be the sum of values of smaller units contained in it. These certainly require some justification.

As we have seen, design tasks are usually intertwined with one another in a very complicated manner in a grand design. However, such dependencies may be considered as resolved each time by paying appropriate coordination costs as already formulated. This means that the interdependent design tasks decide upon a specific interface and on such a basis they work out their part independently. Given such coordination process, the value they create may be assumed to be independent and also summable.

Realizing that $E(X^+) = \frac{\hat{\sigma}}{\sqrt{2\pi}}$ for $X \sim N(0, \hat{\sigma}^2)$, $E(X_{S_j}^+) = \sqrt{\frac{\sum_{i \in S_j} \sigma_i^2}{2\pi}}$. Thus, the value created in the whole designing organization is

$$V(P,\sigma) = \frac{1}{\sqrt{2\pi}} \sum_{j=1}^{r(P)} \sum_{i \in S_j} \sigma_i^2 \tag{2}$$

This value function has the following property.

Lemma 2 *$V(P,\sigma)$ is strictly decreasing in P and has strictly increasing differences in $(P,-\sigma)$.*

Proof Pick any P' and P'' from $\mathcal{P}(N)$ with $P' \prec P''$. Then

$$V(P'',\sigma) - V(P',\sigma) = \frac{1}{\sqrt{2\pi}} \left[\sum_{j=1}^{r(P'')} \sqrt{\sum_{i \in S_j} \sigma_i^2} - \sum_{j=1}^{r(P')} \sqrt{\sum_{i \in S_j} \sigma_i^2} \right].$$

Since $P' \prec P''$, there exists $S_j \in P''$ that is a union of at least two elements in P'. Thus it suffices to show that

$$\sqrt{\sigma_{i1}^2 + \cdots + \sigma_{ik}^2 + \sigma_{ik+1}^2 + \cdots + \sigma_{im}^2} - \sqrt{\sigma_{i1}^2 + \cdots + \sigma_{ik}^2} + \sqrt{\sigma_{ik+1}^2 + \cdots + \sigma_{im}^2} < 0.$$

This is shown to be true by a simple calculation. Thus the first part of the statement holds. Pick any l with $1 \le l \le k$ and differentiating the left hand side of the above inequality with respect to any σ_{il}. This yields

$$\frac{\sigma_{il}}{\sqrt{\sigma_{i1}^2 + \cdots + \sigma_{im}^2}} - \frac{\sigma_{il}}{2\sqrt{\sigma_{i1}^2 + \cdots + \sigma_{ik}^2}} < 0.$$

It is easy to see the same is true for l with $k+1 \le l \le m$. Thus $V(P'',\sigma) - V(P',\sigma)$ is strictly decreasing in σ, which complete the proof. □

$V(P,\sigma)$ is decreasing in P, because finer partitions create more number of smaller real options and the sum of their value is greater than the value of one large real option. This is what Baldwin and Clark call the "value of splitting." Thus partitioning a designing organization is benefitial. At this stage, some reader may realize that $V(P,\sigma) - K(P,C,c)$ is supermodular in P if $V(P,\sigma)$ is supermodular in P. Unfortunately, however, $V(P,\sigma)$ is not necessarily supermodular in P. However, this lack of supermodularity does not prevent us from conducting comparative statics analysis if we restrict our attention to a chain on $\mathcal{P}(N)$.

4.2 A More General Benefit Function

As we already stated, the benefit of partitioning a design process can stem from another factor: specialization. Following Schaefer (1999) and Becker and Murphy (1992), we will not delve into the foundations here, but just choose to formulate a real-valued benefit function in a reduced form $B(P, \beta)$, where $\beta \in \mathbb{R}$. We assume that $B(P, \beta)$ is strictly decreasing in P and supermodular in P. Some may wonder why we assume supermodularity, although it turns out that $V(P, \sigma)$ is not necessarily supermodular in P. This assumption is rather innocuous however. Even when $B(P, \beta)$ is not supermodular in P, the same comparative statics results obtain, if we restrict our attention to an optimization problem over a chain on $\mathcal{P}(N)$ and we think this is a plausible situation.

We also assume that $B(P, \beta)$ has strictly increasing differences. This means that the marginal benefit from increase in β is higher with coarser partition. In what follows, we interpret β as the degree of generality in overall skill investment. A higer value of β corresponds to the situation where members of the design organization invest in skills that are more productive when they are engaged in a wider range of design tasks. Likewise, a lower β corresponds to the situation in which specialized skill formation is prevalent.

5 Comparative Statics of Optimal Partitions

We are now in a position to derive comparative statics results by using the properties of cost and benefit function. First some comparative statics results regarding the optimal partition are provided. We then go on to further analysis by endogenizing across-team coordination costs.

5.1 Analysis of the Optimal Partition

Our objective function will be as follows, when we adopt the benefit function à la Baldwin and Clark, $V(P, \sigma)$.

$$\Pi^{BC}(P, \sigma, C, c) = V(P, \sigma) - K(P, C, c) \tag{3}$$

As already suggested, this benefit function $V(P, \sigma)$ is not generally supermodular in P. Thus, this objective function is not necessarily supermodular in P. So we restrict the constraint set, from which P is chosen, to a subset that is a chain rather than the set of all possible partition of N. Let a chain on $\mathcal{P}(N)$ be denoted $\mathcal{P}_C(N)$. It is easy to see that a chain in $\mathcal{P}(N)$ exists. Any chain is trivially a lattice. Any function is trivially supermodular on a chain. Thus $V(P, \sigma)$ is supermodular in P.

Restricting the domain to a chain might seem to be a major setback. However, doing so makes sense. It is widely observed that organizations usually divide further or integrate the currently existing sections when environmental parameters change. This means that organizations adjust their partition on some chain more frequently than on a general subset of $\mathcal{P}(N)$. In other words, organization cannot escape from historical path dependence. In this setup, the next proposition obtains.

Proposition 1 *Let $\mathcal{P}_C(N) \subset \mathcal{P}(N)$ be a chain. Consider the objective function $\Pi^{BC}(P, \sigma, C, c)$ as defined in (3). Then,*

(a). $\Pi^{BC}(P, \sigma, C, c)$ is supermodular in $(P, C, -c)$ on $\mathcal{P}_C(N)$ and has strictly increasing differences in $(P, -\sigma)$.

(b). $\arg\max_{P \in \mathcal{P}_C(N)} \Pi^{BC}(P, \sigma, C, c)$ is increasing in $(C, -c, -\sigma)$.

Proof By Lemma 1, $-K(P, C, c)$ has increasing differences in $(P, (C, -c))$. Obviously $-K(P, C, c)$ is supermodular in C and in $-c$ as well as in P. Furthermore it has increasing differences in $(C, -c)$. Thus, by Fact 2, $-K(P, C, c)$ is supermodular in $(P, C, -c)$. Note that this inference is correct regardless of the domain of P. Since $V(P, \sigma)$ is supermodular in P on $\mathcal{P}(N)$, $\Pi^{BC}(P, \sigma, C, c)$ is supermodular in $(P, C, -c)$. Obviously, $\Pi^{BC}(P, \sigma, C, c)$ has strictly increasing differences in $(P, -\sigma)$. This proves part (a).

Part (b) follows from the standard result about maximizing a supermodular function on a lattice, i.e., Fact 4 in the Appendix. □

Next we go for a more general benefit function. The objective function in this case will be

$$\Pi^G(P, \beta, C, c) = B(P, \beta) - K(P, C, c). \tag{4}$$

For this objective function, we have the next proposition:

Proposition 2 *Consider the objective as defined in (4). Then,*

(a). $\Pi^G(P, \beta, C, c)$ is supermodular in $(P, \beta, C, -c)$.

(b). $\arg\max_{P \in \mathcal{P}(N)} \Pi^G(P, \beta, C, c)$ is increasing in $(\beta, C, -c)$.

Proof As in the proof of Proposition 1, $-K(P, C, c)$ is supermodular in $(P, C, -c)$. Since $B(P, \beta)$ is also supermodular in $(P, C, -c)$, $\Pi^G(P, \beta, C, c)$ is supermodular in $(P, C, -c)$. Since $B(P, \beta)$ has increasing differences in (P, β), so does $\Pi^G(P, \beta, C, c)$. Furthermore $\Pi^G(P, \beta, C, c)$ has increasing differences in $(\beta, C, -c)$ and β is a value on a chain. Thus by Fact 2 in the appendix, $\Pi^G(P, \beta, C, c)$ is supermodular in $(P, \beta, C, -c)$, which complete the proof of part (a).

Part (b) follows from part (a) by Fact 4 in the Appendix. □

It is common to both Proposition 1 and 2 that the optimal partition is increasing in $(C, -c)$. This means the followings: (1) as the cost of across-team coordination increases, the optimal partition becomes coarser; (2) as the cost of within-team coordination increases, the optimal partition becomes finer. Specific to Proposition 1 is the prediction about the effect of uncertainty on optimal partition: (3) as the uncertainty regarding the R&D activities conducted in each design task increases, the optimal partition becomes finer. Proposition 2 contains the following prediction: (4) as the investment in skill formation become more general, the optimal partition becomes coarser.

These results should be intuitive. When the cost of across-team coordination is high, it is better to decrease the incidence of across-team coordination, namely to have a coarser partition. If the cost of within-team coordination is low, it is better to have the coordination conducted within a team, which also means having a coarser partition. The result on the effect of uncertainty confirms the result obtained by Baldwin and Clark (2000). Finally, more general skills favor a large team size.

5.2 The Cost of Maintaining Coordination Devices

As has been already argued, across-team coordination is usually accomplished by means of such coordination devices as a facsimile, telephone and the Internet etc. More generally we may even suppose within-team coordination can be facilitated by installing some device. However members within a team will mainly resort to face-to-face communication. Then it would be rather natural to think that the design organization buy and install coordination devices to reduce the cost of across-team coordination, with the cost of within-team coordination fixed exogenously.

We introduce such a cost as the cost of maintaining some level of difference between across-team and within-team coordination costs. Let the differences in costs between across-team and within-team coordination be denoted by $z = C - c \in \mathbb{R}_+^{\#A}$. We denote a generic component of z by z_{ij}, where $(i, j) \in A$. The cost of maintaining a coordination device that realizes cost differences z be denoted by $\kappa(z, \alpha)$, where $\alpha \in \mathbb{R}^1$ is a parameter. In what follows, we assume the following: (1) $\kappa(z, \alpha)$ is submodular in $(z, -\alpha)$ and ; (2) decreasing and concave in z_{ij} for each $(i, j) \in A$.

The first assumption of submodularity in $(z, -\alpha)$ means both submodularity in z and submodularity in $(z_{ij}, -\alpha)$ for each $(i, j) \in A$. The submodularity in z implies that reduction in the cost difference at $(i, j) \in A$ makes it less costly to reduce the cost difference in $(k, l) \neq (i, j)$ in A. Such a complementarity property would be easy to imagine, because the communication device across design teams usually have a network effect. On the other hand, the assumption of submodularity in (z_{ij}, α) implies that the marginal cost incurred in decreasing the cost difference at (i, j) decreases as α increases. In this sense, α may be regarded as measuring the inexpensiveness of ICT investment. The second assumption that $\kappa(z, \alpha)$ is concave in z_{ij} for each $(i, j) \in A$ means that allowing larger cost differences in each $(i, j) \in A$ saves increasingly more costs of maintaining communication devices.

The following lemma gives a property of $\kappa(C - c, \alpha)$, the proof of which utilizes a result on convex transformation of increasing supermodular functions (Topkis 1998, p.56, Lemma 2.6.4).

Lemma 3 *Suppose that $\kappa(z, \alpha)$ is submodular in $(z, -\alpha)$, and decreasing and concave in z_{ij} for each $(i, j) \in A$. Then $k(C - c, \alpha)$ is submodular in $(C, -c, -\alpha)$ and decreasing in $(C, -c)$.*

Proof Obviously, for each (i, j), $z_{ij} = C_{ij} - c_{ij}$ is increasing and supermodular in $(C, -c)$. Since $\kappa(z, \alpha)$ is submodular in z and decreasing and concave in z_{ij} for each $(i, j) \in A$, $-\kappa(z, \alpha)$ is supermodular in z and increasing and convex in z_{ij}. Because $-\kappa(C - c, \alpha)$ is a composite function of $-\kappa(z, \alpha)$ and $z_{ij} = C_{ij} - c_{ij}$, it is supermodular and increasing in $(C, -c)$ by Fact 3 in the Appendix. Thus $\kappa(C - c, \alpha)$ is submodular and decreasing in $(C, -c)$.

By Fact 2 in the Appendix, increasing differences in variables and supermodularity in each variable together imply supermodularity in those variables. Thus it suffices to show that $\kappa(C - c, \alpha)$ has decreasing differences in $(C, -\alpha)$ and $(-c, -\alpha)$. Since $-\kappa(z, \alpha)$ is supermodular in $(z, -\alpha)$, it has increasing differences in $(z, -\alpha)$ by Fact 1 in the Appendix. Then $-\kappa(C - c, \alpha)$ has increasing differences in $(C, -\alpha)$ and in $(-c, -\alpha)$. This completes the proof. □

Some caveats are in order here about the specification of cost function $\kappa(C - c, \alpha)$. $\kappa(C - c, \alpha)$ may depend upon both C_{ij} and c_{ij} in the present formulation. Some may think that, given a partition, only c_{ij} or C_{ij} should be included in $\kappa(C - c, \alpha)$ according to whether (i, j) belongs to the same design team or not. However, in reality, organizations, such as firms, do not set up communication devices specifically for each (i, j) each time, but always have some communication devices available and apply them when coordination is necessary. For example, the arrangement may be such that face-to-face communication is utilized within a team and the Internet is used for across-team communication. Then

i and j will use either face-to-face communication or the Internet, according to whether $j \in S(i, P)$ or not. Thus choosing the level of (C, c) is choosing a specific kind of communication devices for within-team and across-team coordination. Even so, different (i, j)'s in A may incur different coordination costs, because the difficulty of coordination may be different among (i, j)'s.

5.3 Comparative Statics Results with Endogenous Across-Team Coordination Costs

Now we consider extended maximization problems with endogenous across-team coordination costs. According as we adopt the benefit function à la Baldwin and Clark or a more general benefit function, we have the following objective functions.

$$\Pi_E^{BC}(P, \sigma, C, c, \alpha) = V(P, \sigma) - K(P, C, c) - \kappa(C - c, \alpha) \tag{5}$$

$$\Pi_E^{G}(P, \beta, C, c, \alpha) = B(P, \beta) - K(P, C, c) - \kappa(C - c, \alpha) \tag{6}$$

As a choice variable, C has to be chosen from a set $\{x : x \in \mathbb{R}^{\#A}, x > c\}$. Rather than having this constraint set, we assume that C is chosen from a fixed set $D \subset \mathbb{R}^{\#A}$ such that D is a sublattice of $\mathbb{R}^{\#A}$ and each member of D is greater than any possible value of c. When the objective function (5) is adopted, the following proposition obtains.

Proposition 3 *Let $\mathcal{P}_C(N) \subset \mathcal{P}(N)$ be a chain. Suppose that, when C is a choice variable, it is chosen from a sublattice D of $\mathbb{R}_+^{\#A}$ such that each $C \in D$ is greater than any possible value of c. Consider the objective function $\Pi_E^{BC}(P, \sigma, C, c, \alpha)$ as defined in (5). Then,*

- *(a).* $\Pi(P, \sigma, C, c, \alpha)$ *is supermodular in* $(P, C, -c, -\alpha)$ *and has increasing differences in* $(P, -\sigma)$.
- *(b).* $\arg\max_{P \in \mathcal{P}_C(N)} \Pi(P, \sigma, C, c, \alpha)$ *is increasing in* $(-\sigma, C, -c, -\alpha)$.
- *(c).* $\arg\max_{(C,P): C \in D, P \in \mathcal{P}_C(N)} \Pi(P, \sigma, C, c)$ *is increasing in* $(-c, -\alpha)$.

Proof As in the proofs of the previous propositions, $-K(P, C, c)$ is supermodular in $(P, C, -c)$ and thus in $(P, C, -c, \alpha)$. Since $V(P, \sigma)$ is supermodular in P on $\mathcal{P}(N)$, it is supermodular in $(P, C, -c)$. By Lemma 3, $-\kappa(C - c, \alpha)$ is supermodular in $(C, -c, -\alpha)$, and thus in $(P, C, -c, -\alpha)$. These together imply that $\Pi_E^{BC}(P, \sigma, C, c)$ is supermodular in $(P, C, -c, -\alpha)$.

By lemma 2, $V(P,\sigma)$ has increasing difference in $(P,-\sigma)$. Thus $\Pi_E^{BC}(P,\sigma,C,c)$ has increasing difference in $(P,-\sigma)$. This proves part (a).

Part (b) follows from part (a) and Fact 4 in the Appendix. Part (c) follows from application of Fact 4 and Fact 5 in the Appendix. □

In the case with a general benefit function, we have the following proposition.

Proposition 4 *Suppose that, when C is a choice variable, it is chosen from a sublattice D of $\mathbb{R}_+^{\#A}$ such that each $C \in D$ is greater than any possible value of c. Consider the objective function $\Pi_E^G(P,\beta,C,c,\alpha)$ as defined in (6). Then,*

(a). $\Pi_E^G(P,\beta,C,c,\alpha)$ *is supermodular in* $(P,\beta,C,-c,-\alpha)$.

(b). $\arg\max_{P\in\mathcal{P}(N)} \Pi_E^G(P,\beta,C,c,\alpha)$ *is increasing in* $(\beta,C,-c,-\alpha)$.

(c). $\arg\max_{(C,P):C\in D,P\in\mathcal{P}(N)} \Pi_E^G(P,\beta,C,c,\alpha)$ *is increasing in* $(\beta,-c,-\alpha)$.

Proof Since $B(P,\beta)$ is supermodular in P, has increasing difference in (P,β), and β is a value on a chain, it is supermodular in (P,β). Thus it is supermodular in $(P,\beta,C,-c,-\alpha)$. $-K(P,C,c)$ is supermodular in $(P,C,-c)$ and thus in $(P,\beta,C,-c,-\alpha)$. $\kappa(C-c,\alpha)$ is supermodular in $(C,-c,-\alpha)$ and thus in $(P,\beta,C,-c,-\alpha)$. All these together imply that $\Pi_E^G(P,\beta,C,c,\alpha)$ is supermodular in $(P,\beta,C,-c,-\alpha)$.

Part (b) follows from part (a) and Fact 4. Part (c) follows from application of Fact 4 and Fact 5 in the Appendix. □

Here we have the following additional results: (1) as the cost of within-team coordination decreases, the chosen level of across-team coordination costs becomes higher and the partition becomes coarser; (2) as the ICT investment becomes inexpensive, the chosen level of across-team coordination costs becomes lower and the partition becomes finer; (3) as the skill formation becomes more general, the chosen level of across-team coordination costs becomes higher and the partition becomes coarser.

5.4 Relation to Modularization

Baldwin and Clark (2000) defines "modularization in design" as a process of design rationalization. Suppose there are interdependencies among several design parameters, which might involve a cycling and require complex coordination. Such intricate interdependencies, however, can be eliminated by setting a "design rule" that each relevant designer must obey. Carrying through this process results in a "modular structure" of the DSM as shown in Figure 5.4. Design rules are inserted at the top row and the leftmost column of the original DSM. We now have modular blocks of design parameters. Within each block there are interdependencies as before, while there are no interdependencies across blocks.

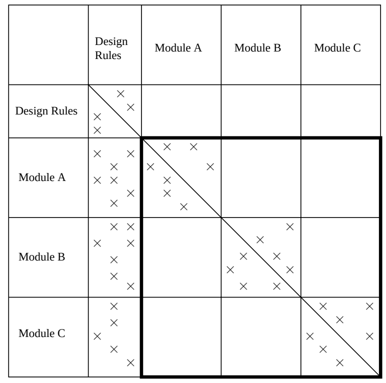

Figure 2: An example of modular structure (adapted from Figure 3.4 of Baldwin and Clark (2000)).

One of the major contributions of Baldwin and Clark (2000) is that they identified the value-enhancing aspects of a modular design in the following points:

- Modularity creates options;
- Modular designs evolve as the options are pursued and exercised.

Section 4.1 of the present paper analyzed the first point. However, we submit that the modularization also works to reduce the coordination costs among designers.

The process of modularization can be very costly as experienced in the course of modular design of IBM System/360, because finding all the potential interdependencies is often difficult and takes time. However, the cost of modularizing a design will be sunk once it has been done. At a first sight, it might appear that new costs are now to result from the dependence of design parameters upon design rules. However they can be regarded as negligible, because design rules are always visible to relevant designers and fixed for a relatively long period of time. After all, design rules determine the architecture and the interfaces among several design parameters. Therefore we can safely abstract from the cost arising from dependencies of design parameters upon design rules, and will do so henceforth. Then, by modularization, interdependencies among design tasks are reduced so that the coordination costs among them may also be substantially saved in the process of designing new products consecutively.

Our analysis of optimal partition should be relevant to the idea of "modularization" described above. Indeed, it can be shown that the optimal partition cannot be strictly coarser than the partition naturally induced by modularization. This implies that modularization works to set an upper bound for the optimal partition. Let us now turn to the formalization of this idea.

Generally a unique partition of N is associated to each grand design A in the following natural manner. Let $\hat{P}$ be a partition of N such that

$$(i, j) \in A \text{ implies } j \in S(i, \hat{P})$$

Namely any two dependent design parameters are contained in the same element of $\hat{P}$. It is easy to see that such a partition necessarily exists, because the largest partition $\{N\}$ satisfies the condition. Let the set of all partitions of N that satisfy the above condition be denoted by $\mathcal{P}_A(N)$, which is a subset of $\mathcal{P}(N)$ and thus finite. Now pick any P' and P'' from $\mathcal{P}_A(N)$ and consider $P' \wedge P''$. Pick any $(i, j) \in A$. Since $P', P'' \in \mathcal{P}_A(N)$, $j \in S(i, P')$ and $j \in S(i, P'')$. Thus $j \in S(i, P') \cap S(i, P'') = S(i, P' \wedge P'')$. So $P' \wedge P'' \in \mathcal{P}_A(N)$. Since $\mathcal{P}_A(N)$ is finite, there exists the smallest (thus unique) element P^A in $\mathcal{P}_A(N)$, which we call partition induced by A. It is obvious that $K(P, C, c) = K(P^A, C, c)$ for each P with $P^A \preceq P$.

Proposition 5 *Let P^A be the partition induced by a grand design A. Let $\Pi_E^{BC}(P, \sigma, C, c, \alpha)$ be as defined in (5) and suppose $P^* \in \arg\max_{P \in \mathcal{P}(N)} \Pi_E^{BC}(P, \sigma, C, c, \alpha)$. Then $\neg P^A \prec P^*$.*

Proof Suppose $P^A \prec P^*$. Then

$$K(P^A, C, c) = K(P^*, C, c)$$

However, since $V(P, \sigma)$ is strictly increasing in P, $V(P^A, \sigma) > V(P^*, \sigma)$. Thus we have $\Pi(P^A, \sigma, C, c, \alpha) > \Pi(P^*, \sigma, C, c, \alpha)$, which is a contradiction. □

A modular design induces a partition of N that is strictly finer than $\{N\}$. Proposition 5 states that the optimal partition is not strictly coarser than the partition induced by a modular design. Note that the choice set is not restricted to a chain in the hypotheses of the above proposition. By restricting the choice set to a chain $\mathcal{P}_C(N)$, the next corollary holds.

Corollary 1 *Let P^A be the partition induced by A and let $\mathcal{P}_C(N)$ be a chain containing P^A. Suppose that C is chosen from a sublattice D of $\mathbb{R}_+^{\#A}$ such that each $C \in D$ is greater than any possible value of c.*

(a). Then for each $P^* \in \arg\max_{P \in \mathcal{P}_C(N)} \Pi_E^{BC}(P, \sigma, C, c, \alpha)$, $P^* \preceq P^A$.

(b). $\arg\max_{C \in D} \Pi_E^{BC}(P^*, \sigma, C, c, \alpha) \preceq \arg\max_{C \in D} \Pi_E^{BC}(P^A, \sigma, C, c, \alpha)$.

Proof Part (a) follows from Proposition 5 and the assumption that $\mathcal{P}_C(N)$ is a chain. Part (b) follows from Proposition 1 (a). □

Thus a modular design makes the size of each team smaller. Since that in turn increases the incidence of across-team coordination, a modular design induces more ICT investment in order to reduce the cost of across-team coordination.

6 Discussion and Conclusion

This paper analyzes the problem of optimally partitioning a product design or a design organization, and derives several comparative statics results. The most natural and rigorous interpretation of our model is that there is a single agent who faces and solve optimization problems. In this interpretation, the results obtained are concerned with the characteristics observed in an integrated firm such as IBM faces. We believe that there can be another interpretation of the model. Our comparative statics results may be considered as approximating the outcome arising from the joint arrangement among multiple firms. Although there are plenty of factors that lead to coordination failure, it is expected that the most efficient outcome will emerge through contractual and other organizational arrangements.

When the coordination costs are assumed to be exogenous, our analysis shows the followings: the optimal partition will be coarser, (1) if the cost of across-team coordination is higher, (2) the cost of within-team coordination is lower, (3) the uncertainty concerning R&D activities is lower, and (4) the degree of generality of skill is higher. The first result is a confirmation of the result obtained in Schaefer (1999), and the third result coincide with the insight by Baldwin and Clark (2000). Note however that our modeling approach is different from theirs.

In a setting where across-team coordination costs are endogenous, the following results are obtained: a higher level of across-team coordination costs (lower level of ICT investment) and coarser partitions are induced by (1) lower cost of within-team coordination, (2) higher cost of ICT investment, and (3) higher degree of generality of skill formation. In our objective functions, higher across-team coordination costs and coarser partition are complementary, and thus a change in some factor moves them in the same direction. For example, if the cost of within-team coordination is low enough, then a firm will tend to rely more on within-team coordination than across-team coordination. Then the firm will not want to have a finer partition. Accordingly the firm will not invest much for reducing the cost of across-team coordination.

These results seem to be consistent with some evidences, both casual and empirical. It is well known that Japanese firms forged an efficiently working within-firm network based upon their workers' cultural homogeneity by the 1970s, which often used to be regarded as one of the sources of their strength in the 1980s. However, it has also been argued that the very same factor worked to prevent Japanese firms from making full use of the emerging ICT such as the Internet in the 1990s (Ikeda 1997). One possible interpretation of lower costs of within-team coordination in our model may be cultural homogeneity among members of the team. Thus our result may explain the Japanese firms' retarded introduction of ICT in the 1970s and 1980s. Furthermore, Japanese workers are known to invest more in general communication skills than in specialized skills, which can be interpreted as a higher degree of generality in skill formation in our model. Thus Japanese workers' tendency to investment in general skills might have been another cause of less ICT investment of Japanese firms.

The same result is also instrumental to understanding the interesting comparison of industrial regions between Silicon Valley and Route 128 by Saxenian (1994). She observes that the Silicon Valley firms are marked by high mobility of workers, frequent communications and substantial degrees of information sharing among different firms, quite in contrast to the Route 128 firms. Thus it would be natural to think that the cost of across-firm coordination is substantially lower in Silicon Valley than in Route 128. The above result may explain why there are a lot of small independent firms in Silicon Valley, while large integrated firms are dominant in Route 128.

Our model exhibits the property that the chosen level of across-team coordination cost becomes lower and the corresponding partition becomes finer, as inexpensive ICT becomes available. If we interpret this results in terms of a joint arrangement of many firms and regard each team as an independent firm, it seems to suggest that the size of firms decreases, as new ICT is developed and deployed by those firms. Such prediction is in accordance with the empirical findings by Brynjolfsson, Maline, Gurbaxani, and Kambil (1994) that ICT investment has lead to smaller firm size in the US.[2]

[2]Strictly speaking, interpreting each team in our model as an independent firm implies that the boundary of a firm is determined by the ease of coordination. Brynjolfsson, Maline, Gurbaxani, and

Furthermore, an interesting property of our model is that less expensive ICT, higher cost of across-team coordination, lower cost of within-team coordination and finer partitions are all complementary in the objective function. As Milgrom and Roberts (1995) demonstrated for the emerging paradigm of modern manufacturing firms, this means necessity for a systematic response. Namely, higher cost of within-team coordination and/or lower cost of the ICT investment lead to a systematic response: more investment in ICT; and finer partitions of organization. Introducing the one without the other will not be profitable, and the one will induce the other. Actually the above two factors seem to go together in the new paradigm of ICT.

The enormous impact of the recent development of ICT on economy has highlighted the importance of such concepts as architecture, an interface, compatibility, standardization, information encapsulation, modularization and so forth. All of these concepts concern the process of designing a complex system. Today there seems to be a widespread belief that they are indispensable for understanding the way that we conduct economic transactions. The literature on this subject has begun to proliferate. The present paper can be regarded as one of such contributions.

With respect to the concept of modularization, our model indicates that modularization makes finer partitions more favorable in the sense that it sets an upper bound for the optimal partitions, and thus induces higher ICT investment to reduce the cost of across-team coordination. In this loose sense, modularization, ICT investment and smaller size of firms are all complementary.

The analysis in this paper sheds some light on the relationship between modularization, ICT investment and smaller size of firms by considering the coordination costs in a designing organization. However, our analysis is a static one. Probably more interesting and important theme is to explore the dynamics of how a complex system evolves. That will be a subject of another paper however.

Kambil (1994) explains this assertion by broadly interpreting coordination costs as "transaction cost" in general. However this is the converse of the assertion that coordination becomes easier within a firm, for which, to the best of our knowledge, there is no established theory. Our point is that the result is still suggestive of the current tendency for the size of firms to decrease.

Appendix

This appendix presents basic properties of supermodular functions and their maximization problems used in the paper for those who are not familiar with these analytical tools. For more detailed exposition and the proofs, see Topkis (1998).

A **binary relation** $\preceq$ on a set X specifies for all x' and x'' in X either $x' \preceq x''$ is true or false. We usually write $x' \prec x''$ if $x' \preceq x''$ and $x' \neq x''$. A binary relation $\preceq$ is **reflexive** if $x \preceq x$ for each $x \in X$, **antisymmetric** if $x' \preceq x''$ and $x'' \preceq x'$ imply $x' = x''$, and **transitive** if $x' \preceq x''$ and $x'' \preceq x'''$ imply $x' \preceq x'''$. A binary relation is called a **partial ordering** if it is reflexive, antisymmetric and transitive. A **partially ordered set** is a set X on which there is a partial ordering $\preceq$. A partially ordered set is a **chain** if it does not contain an unordered pair of elements.

Let X be a partially ordered set and X' be its subset. If $x' \in X$ and $x \preceq x'$ ($x' \preceq x$) for each $x \in X'$, then x' is an **upper (lower) bound** for X'. If x' in X' is an upper (lower) bound for X', then x' is the **greatest (least)** element of X'. If two elements, x' and x'', of a partially ordered set X have a least upper bound (greatest lower bound) in X, it is their **join (meet)** and is denoted $x' \vee x''$ ($x' \wedge x''$). A partially ordered set that contains the join (least upper bound) and the meat (greatest lower bound) of each pair of its elements is called a **lattice**. If X' is a subset of a lattice X and X' contains the join and meet with respect to X of each pair of elements of X', then X' is a **sublattice** of X.

Let X and T be partially ordered sets and $f(x,t)$ be a real-valued function on $X \times T$. $f(x,t)$ has **increasing differences, strictly increasing differences** if $f(x,t'') - f(x,t')$ is increasing, strictly increasing in x for all $t' \prec t''$. Suppose X_α is a partially ordered set for each $\alpha \in A$ and $f(x)$ is a real-valued function on $\times_{\alpha \in A} X_\alpha$. $f(x)$ has **increasing differences, strictly increasing differences** on $\times_{\alpha \in A} X_\alpha$ if for all distinct α' and α'' in A, $f(x)$ has increasing differences, strictly increasing differences in $(x_{\alpha'}, x_{\alpha''})$. Now suppose X is a lattice and $f(x)$ is a real-valued function on X. $f(x)$ is **supermodular** if

$$f(x') + f(x'') \leq f(x' \vee x'') + f(x' \wedge x'')$$

for all x' and x'' in X. $f(x)$ is **submodular** if $-f(x)$ is supermodular. If $f(x)$ and $g(x)$ are supermodular on X, then $f(x) + g(x)$ is supermodular on X.

Fact 1 shows that supermodularity implies increasing differences, while the converse hold under certain conditions.

Fact 1 *If X_α is a lattice for each $\alpha \in A$, X is a sublattice of $\times_{\alpha \in A} X_\alpha$, and $f(x)$ is supermodular on X, then $f(x)$ has increasing differences on X.*

Fact 2 *If X_i is a lattice for $i = 1, \cdots, n$, $f(x)$ has (strictly) increasing differences on $\times_{i=1}^n X_i$ and $f(x)$ is (strictly) supermodular in x_i on X_i for $i = 1, \cdots, n$, then $f(x)$ is (strictly) supermodular on $\times_{i=1}^n X_i$.*

Fact 3 shows that increasing and convex transformation of increasing supermodular functions results in a supermodular function.

Fact 3 *If X is a lattice, $f_i(x)$ is increasing and supermodular on X for $i = 1, \cdots, k$, Z_i is a convex subset of $\mathbb{R}^1$ containing the range of $f_i(x)$ on X for $i = 1, \cdots, k$, Z_i, and $g(z_1, \cdots, z_k, x)$ is supermodular in $(z_1, \cdots, z_k, x)$ on $(\times_{i=1}^k Z_i) \times X$ and is increasing and convex in z_i on Z_i, then $g(f_1(x), \cdots, f_k(x), x)$ is supermodular on X.*

In order to conduct comparative statics, we have to compare the set of maximizers. Suppose X is a lattice with ordering relation $\preceq$. The **induced set ordering** $\sqsubseteq$ is defined on the collection of nonempty members of the power set of X such that $X' \sqsubseteq X''$ if $x' \in X'$ and $x'' \in X''$ imply that $x' \wedge x'' \in X'$ and $x' \vee x'' \in X''$. Let $\mathcal{L}(X)$ be the collection of all nonempty sublattices of a lattice X. It is easy to see that if X is a lattice, then $\mathcal{L}(X)$ is a partially ordered set with the ordering relation $\sqsubseteq$. A function whose range is included in the collection of all subsets of some set is a correspondence. A correspondence S_t is **increasing** in t on T if the domain T is a partially ordered set, the range $\{S_t : t \in T\}$ is in $\mathcal{L}(X)$, and $t' \prec t''$ implies $S_{t'} \sqsubseteq S_{t''}$ in $\mathcal{L}(X)$. The next fact is the main tool for conducting comparative statics for the maximization problem on a lattice.

Fact 4 *If X and T are lattices, S is a sublattice of $X \times T$, S_t is a section of S at t in T, and $f(x,t)$ is supermodular in (x,t) on S, then* $\arg\max_{x \in S_t} f(x,t)$ *is increasing in t on* $\{t : t \in T, \arg\max_{x \in S_t} f(x,t)$ *is not empty*$\}$

The next fact implies that if one optimizes a system of complementary variables with respect to any subset of the variables then the remaining variables would still be complementary.

Fact 5 *If X and T are lattices, S is a sublattice of $X \times T$, S_t is a section of S at t in T, and $f(x,t)$ is supermodular in (x,t) on S, and $g(t) = \sup_{x \in S_t} f(x,t)$ is finite on the projection $\Pi_T S$ of S on T, then $g(t)$ is supermodular on $\Pi_T S$.*

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