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Abstract

During some recent financial crises, the majority of domestic banks - or indeed the entire banking sector - became insolvent. We have analyzed the welfare effects of policy responses to bank insolvency by examining a modified version of the Diamond-Rajan model, introducing a fiat currency. The sources of inefficiency in our model are the "moral hazard in banking" and the "premature liquidation of bank assets". The model assumes that banking system insolvency is caused by an exogenous macroeconomic shock that destroys a portion of banks' assets.

If the government does not intervene in very severe cases of bank insolvency, a fire sale of all bank assets can occur along with dis-intermediation of the economy and falls in price levels (debt deflation).

We have analyzed the consequences of the following three different policy responses to bank insolvency: (1) providing deposit guarantees (without immediate recapitalization), (2) providing unlimited liquidity support, and (3) effecting bank recapitalization through either cash creation (monetary policy) or bond issuance (fiscal policy). In doing so, we showed that bank recapitalization by fiscal measures provides the optimal solution. Our findings imply that Japan's protracted recession and deflation problem may have been caused by an inappropriate policy response to bank insolvency.

JEL Classification: E5, E6, G2, H3.

Keywords: Financial crises, bank insolvency, debt deflation, recapitalization.

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1 Introduction

In this paper, we analyze a banking crisis, subsequent debt deflation, and the policy responses to the crisis in a model where fiat currency is introduced and contracts are made in nominal terms.

In the recent theoretical research on banking crises, many theories have been proposed concerning the mechanism by which financial crises occur (Diamond and Dybvig [1983], Postlewaite and Vives [1987], and Allen and Gale [1998, 2000, 2001]). But there are not so many theories that explain the difference of recovery paths from the crises in accordance with different policy responses. For example, Diamond and Dybvig (1983), Freixas, Parigi, and Rochet (1999), and Martin (2001) discuss the policies to prevent bank runs, but not policy responses to bank insolvency. Only a few authors like Diamond and Rajan (2002a, b), Bergoing, et al. (2002), and Boyd, Chang, and Smith (2000) consider *ex post* policy responses to financial crises.

The world has experienced a large number of banking crises in the last three decades. Caprio and Klingebiel (1999) identified 113 systemic banking crises that had occurred in 93 countries since the late 1970s, along with 50 borderline and smaller banking crises in 44 countries over the same period. In analyzing these experiences, researchers have come to pay more attention to bank insolvency than to bank runs. Recent crisis episodes suggest that bank insolvency is the central problem to be rectified and that a temporary shortage of liquidity is basically a symptom. Diamond and Rajan (2002a) also refer to the theoretical possibility of two-way linkage between bank insolvency and liquidity shortages.

One common observation concerning financial crises is the decline of the inflation rate after the onset of a banking crisis (Boyd et al. [2001]). The United States experienced severe deflation associated with a rash of bank failures in the 1930s, and Japan is now experiencing a bout of deflation following the onset of a banking crisis in the late 1990s. Boyd et al. (2001) show that the onset of a banking crisis decreases the growth rate of M2, which is a significant element contributing to inflation. The low inflation or deflation associated with a banking crisis may be modeled as debt deflation (Fisher [1933]). In

this paper we formalize the notion of debt deflation in a simple model.

Several facts about the recovery paths from banking crises have been found by case studies and empirical research on recent financial crises (see, for example, Claessens, Klingebiel, and Laeven [2001], Alexander et al. [1997], Caprio and Klingebiel [1996]). Honohan and Klingebiel (2000) find that open-ended liquidity support, regulatory forbearance, and an unlimited deposit guarantee are all significant contributors to the fiscal cost of resolving a banking crisis. They also find that liquidity support significantly increases the output loss or delays the economic recovery. (This result is confirmed by Bordo et al. [2001], who used a broader data set.) Boyd et al. (2001) also claim that incremental expenditures for banking system bailouts may increase output losses. On the other hand, Claessens, Klingebiel, and Laeven (2001) show that liquidity support and a deposit guarantee may be effective in promoting a recovery of profitability in the corporate sector if these measures are combined with the establishment of an asset management agency. Although empirical analyses have not produced a consensus concerning the effects of bank bailouts, it seems the case that a temporizing policy such as liquidity support or a deposit guarantee without the restoration of bank solvency may hinder economic recovery and magnify the fiscal cost.

In any case the banking system must be recapitalized in order for an economic system hit by a banking crisis to restore its normal functions. There are many important practical issues concerning bank recapitalization, such as the source of funds, the loss sharing by depositors, the incentive mechanism for banks' management, the design of financial instruments, and the exit strategy for the government (Honohan [2001]). Whether or not to monetize the cost of recapitalization is one big issue in the policy debate. For example, inflation targeting, or extraordinary monetary easing, has become the focus of the macroeconomic policy debate in Japan since the financial crisis of 1998. One of the objectives of inflation targeting in today's Japan would obviously be to monetize the cost of recapitalizing the banking system. Thus, the provision of a theoretical basis for judging the costs and effects of a monetization policy is very important as a practical matter for crisis-affected countries. The recent crisis episodes show that bank bailout

costs are typically not monetized (Boyd et al. [2001]). We can ask whether there was any economic ground for the policymakers' decision not to monetize the cost of bank recapitalization in those episodes.

Our aim in this paper is to formalize debt deflation, i.e., the decline of the inflation rate associated with a banking crisis, and to analyze the effects of different policy responses to bank insolvency. In order to model the debt deflation, we construct a variant of the Diamond-Rajan model (Diamond and Rajan [2001]) in which we introduce fiat currency using a cash-in-advance constraint. In this variant Diamond-Rajan model, banks are subject to *moral hazard*, i.e., inefficient use of capital, but this can be deterred by the threat of bank runs by depositors - a threat that is made credible by the use of demand deposit contracts. On the other hand, if bank runs occur, banks are forced to liquidate their assets prematurely. Thus, in this model the sources of inefficiency are moral hazard and the premature liquidation of bank assets.

In our model, we do not describe how financial crises occur; we merely posit that all banks become insolvent as a result of an unspecified macroeconomic shock (e.g., the bursting of an asset-price bubble or a fall in the currency exchange rate) that suddenly decreases the value of all bank assets. Taking this *banking system insolvency* as given, we focus our analysis on the welfare properties of the following policy responses: (1) a deposit guarantee, (2) unlimited liquidity support, (3) bank recapitalization through cash creation (monetary recapitalization), and (4) bank recapitalization through bond issuance (fiscal recapitalization).

If the government takes no action in response to bank insolvency, all households will withdraw their deposits immediately, since they know that the banks' assets are less than their liabilities. In this case, all the banks experience depositor runs, all their assets must be liquidated prematurely, price levels fall, and the welfare of households deteriorates.

If the government implements a deposit guarantee policy and/or an unlimited liquidity support policy, it is easily shown that, while bank runs are prevented, banks are inevitably subject to moral hazard. This is because under these policies the government gives the banks a commitment to provide them resources without limit.

In order to avoid generating moral hazard, the government must combine the provision of additional resources *ex ante* to banks (recapitalization) with a credible declaration that it will not provide any more *ex post*. The government has two options in financing the cost of recapitalization: monetization or taxation. In the case where the government implements the bank recapitalization by creating cash (monetary recapitalization), it is shown that moral hazard cannot be prevented, since the monetary recapitalization affects the banks' incentive to hold cash reserves through the cash-in-advance constraint. In the case where the government implements the bank recapitalization by issuing bonds (fiscal recapitalization), it is shown that the optimal outcome is achieved, avoiding both moral hazard and the premature liquidation of assets.

The rest of the paper is organized as follows: Section 2 describes the basic model. Section 3 analyzes debt deflation, which is triggered by bank insolvency resulting from an unexpected macroeconomic shock. Section 4 compares the welfare effects of several policy options. And Section 5 presents the conclusion.

2 Basic Model

The economy consists of continua of consumers and of banks. For simplicity of exposition, we shall normalize the measure of each continuum to 1. But we assume that each bank has infinitely many consumers as its depositors.¹ There is also the government. The economy continues for two consecutive periods bounded by three dates: $t = 0, 1, 2$. There is one type of good (*consumer good*) in this economy that can be consumed by consumers. Banks can transform the consumer good into productive capital, and only banks can use capital and produce the consumer goods at date 1 and date 2.

2.1 Consumers

A consumer maximizes his utility $u(c_0) + \beta u(c_1) + \beta^2 u(c_2)$, where $u(c)$ satisfies the usual neoclassical properties ($u'(c) > 0, u''(c) < 0, \lim_{c \rightarrow 0} u'(c) = \infty$), c_t is the consumption

¹This statement can be justified by assuming that a bank is indexed by α where $\alpha \in [0, 1]$, while a consumer is indexed by (α, γ) where $(\alpha, \gamma) \in [0, 1]^2$.

at date t and β is the time discount factor. For simplicity we assume $u(c) = \ln c$ in what follows. At date 0 each consumer is endowed with E units of consumer goods and M units of useless paper that is provided by the government and is called “cash.” The government levies a tax of M units of cash on each consumer at date 2. Although cash is intrinsically useless for consumers, we posit the following cash-in-advance constraint that makes cash a medium of exchange:

Assumption 1 (*Cash-In-Advance Constraint*) *A consumer must use cash to purchase the consumer good.*

There is no endowment for consumers at dates 1 and 2. There are three assets available to consumers as stores of value: the consumer good (stored rather than consumed), cash, and bank deposits.² If a consumer stores y units of the consumer good at date t , then he will still have y units of the goods at date $t + 1$. This is *storage technology*. Alternatively, a consumer can deposit the consumer goods and/or cash that he has at date t in a bank in exchange for a nominal claim on the bank (i.e., a demand deposit) D_t at date t . We make the following assumption for the deposit contract:

Assumption 2 *The deposit contract can be made only in nominal terms. A contract in real terms between consumers and banks is not allowed.*

This assumption can be justified as follows: Though it may be possible to observe a change of price levels, it is not easy for depositors or banks to verify the amplitude of the change; therefore, real-term contracts cannot readily be implemented in the private sector. For simplicity we prohibit real-term contracts in our model. We will see in Section 2.2 that the following characteristic of the demand deposit contract is necessary to prevent moral hazard for banks.

Assumption 3 *A demand deposit is a contract between a bank and a consumer such that the consumer (depositor) can withdraw any amount of cash up to his bank balance at any time.*

²In the equilibrium where the nominal interest rate is positive, consumers do not hold cash, but they hold bank deposits.

This assumption states that consumers can withdraw any amount of cash just before they buy the consumer good, implying that the cash-in-advance constraint does not enter in the consumer's optimization program as it usually does in ordinary cash-in-advance models (see, for example, Sargent [1987]). As we will state in Section 2.2 that banks' *production technology* is more efficient than storage technology, we consider for a moment the consumer's optimization problem on the premise that consumers choose bank deposits as their only assets.³ In this case, consumers solve the following problem taking prices (p_0, p_1, p_2) , nominal interest rates (i_0, i_1) , endowments (E, M) , and the tax $T_2 = M$ as given.

$$(PC) \quad \max_{c_0, c_1, c_2} u(c_0) + \beta u(c_1) + \beta^2 u(c_2)$$

subject to

$$\begin{cases} p_0 c_0 + D_0 \leq p_0 E + M, \\ p_1 c_1 + D_1 \leq (1 + i_0) D_0, \\ p_2 c_2 + T_2 \leq (1 + i_1) D_1, \end{cases} \quad (1)$$

where $u(c_t) = \ln c_t$, and D_t is the demand deposit remaining at date t .

2.2 Banks

Our model of the banking sector is a simplified version of the Diamond-Rajan model (Diamond and Rajan [2001]).

Production Technology At date 0 a bank can transform K_0 units of the consumer good that are deposited by consumers into the same amount of capital, i.e., K_0 units of capital. Only banks, not consumers, can use the capital to produce the consumer good. Suppose that a bank (bank α , $0 \leq \alpha \leq 1$) forms capital of $K_{0\alpha}$ at date 0. Bank α can produce $AK_{0\alpha}$ units of the consumer good at date 1, while the capital depreciates

³Grossman and Weiss (1983) also construct a model in which they impose a restriction that only banks hold cash and the depositors do not, while in our model the banks' decisions on holding cash reserves are endogenous, as discussed in Section 2.4.

completely to zero.⁴ If bank α sells $y_{1\alpha}$ units of the consumer good to consumers, it can invest the rest and form capital of $K_{1\alpha}$, where $K_{1\alpha} = AK_{0\alpha} - y_{1\alpha}$. At date 2 bank α can produce $AK_{1\alpha}$ units of the consumer good, while its capital $K_{1\alpha}$ depreciates completely to zero. This is the standard AK-technology of the neoclassical growth model. We assume

$$1 < A,$$

which implies that the bank's production technology is more efficient than storage technology. We assume that bank α can also use its capital inefficiently. For simplicity of analysis, we assume the following:

Assumption 4 *Bank α can attain either high productivity (A) by efficient use of capital $K_{0\alpha}$ or low productivity (a) by inefficient use of capital, where*

$$1 \leq a < A. \tag{2}$$

Bank α 's choice variables are (1) the efficiency in use of capital (efficient or inefficient), and (2) the amounts of sale and investment (see the bank's problem (PB) below). Bank α obtains private utility at dates 1 and 2 if it continues to operate. The private utility at date t (U_t) is 0 if bank α stops its operation at date t , b if it chooses inefficient use of capital, and $b - \epsilon$ if it chooses efficient use of capital, where ϵ ($0 < \epsilon < b$) is the effort that is necessary to use capital efficiently. The effort ϵ is observable but not verifiable for the other agents so that contracts contingent on ϵ is infeasible. A bank's primary objective is to maximize the present value of its private utility

$$V \equiv \beta U_1 + \beta^2 U_2.$$

Like Diamond and Rajan, we assume the following "relation-specificity" in this production technology of banks: Only bank α can use $K_{0\alpha}$ most efficiently, since bank α has relation-specific knowledge that other banks do not have about $K_{0\alpha}$, such as the detailed structure of the business model and the efficient use of the equipment in question. If

⁴We assume for simplicity that the capital investment by a bank is reversible: the bank can transform the capital back into the consumer good at any time before the production takes place.

bank α' ($\alpha' \neq \alpha$, $0 \leq \alpha' \leq 1$) takes over $K_{0\alpha}$, it can produce $a'K_{0\alpha}$ units of the consumer good at date 1, while the capital $K_{0\alpha}$ depreciates to zero. We assume

$$1 \leq a' < a < A. \quad (3)$$

After selling y' units of the consumer good, bank α' can form $K'_1 = a'K_{0\alpha} - y'$, and can produce AK'_1 units of the consumer good at date 2, while K'_1 depreciates to zero.

Bank's Problem Competition for depositors among banks forces a bank to maximize their value, since otherwise it cannot collect deposits and is forced to stop operating, and its private utility is driven down to zero. When the nominal interest rate is strictly positive ($i_t > 0$), the competition among banks also makes them commit to efficient use of capital, and they obtain $U_1 = U_2 = b - \epsilon$; a bank chooses efficient use of capital if the other banks use capital efficiently, since otherwise it cannot collect deposit and is forced to stop operating. (See Lemma 1 for the case where $i_t = 0$.) Thus a bank (bank α) solves the following problem taking prices (p_0, p_1, p_2) , interest rates (i_0, i_1) where $i_0 > 0$ and $i_1 > 0$, and initially deposited consumer goods (E) as given.

$$(PB) \max_{y_0, y_1, y_2} p_0 y_0 + \frac{p_1}{1 + i_0} y_1 + \frac{p_2}{(1 + i_0)(1 + i_1)} y_2$$

subject to

$$\left\{ \begin{array}{l} y_0 \leq E, \\ K_{0\alpha} = E - y_0, \\ y_1 \leq AK_{0\alpha}, \\ K_{1\alpha} = AK_{0\alpha} - y_1, \\ y_2 \leq AK_{1\alpha}. \end{array} \right. \quad (4)$$

Cash Reserves and Fire Sales Consumers deposit units of the consumer good (E) and cash (M) in banks at date 0. In the equilibrium where the nominal interest rate is positive, value-maximizing banks will not hold cash unless cash enhances their efficiency. Instead they will use any cash they have to buy the consumer good, which they will then transform into capital. In order to make a non-trivial choice problem between cash and capital, we set the following (realistic) assumption for the cash reserve of banks:

Assumption 5 Let W_t be the withdrawal demand from a bank's depositors at date t . The bank can pay W_t in two ways. (1) It can use a cash reserve M_{t-1} that was set at date $t-1$. (2) It can sell the consumer good y_t at date t and use the cash income $p_t y_t$ to repay the depositors at date t . We will call the second method a "fire sale." If the bank uses the fire sale, it incurs a "deadweight loss": $\frac{1-x}{x} y_t$ units of the consumer goods where

$$0 < x < \beta.$$

The deadweight loss $\frac{1-x}{x} y_t$ is not incurred if the proceeds of a sale are kept as a cash reserve for date $t+1$.

This assumption states that a bank facing withdrawals by depositors must raise cash through an inefficient fire sale of goods if it does not have a sufficient cash reserve. The assumption implies that a bank must produce $\frac{1}{x} y_t > \beta^{-1} y_t$ in order to sell y_t in the fire sale. This loss represents the inefficiency involved in a fire sale of bank assets in reality. Note that the deadweight loss is not incurred if the proceeds of the sale are not paid out at the same date but are kept as a cash reserve for the next date. This feature of the deadweight loss can be justified as follows: The inefficiency of a fire sale occurs when a bank sells an asset to someone who cannot use it most efficiently. A bank cannot find the best buyer if it does not have enough time to search among the possible buyers. And banks that do not have enough reserves on hand must raise cash immediately to pay their depositors; they have no time to search for good buyers; they suffer the deadweight loss by selling the goods to suboptimal buyers. On the other hand, banks that have enough reserves have enough time to find the best buyer since they do not need to pay out the proceeds of the sale immediately; they do not incur the deadweight loss. This is the reason why a deadweight loss is incurred in our model only when the bank pays out the proceeds of the sale to the depositors on the same date.

Demand Deposit The above relation-specific technology justifies the aforesaid characteristic of demand deposits (Assumption 3). Suppose that D_0 is the demand deposit remaining at the end of date 0. As we will see later, in the equilibrium the date-0 withdrawal $p_0 c_0$ is equal to M , $y_0 = c_0$, and no fire sale occurs at date 0. In this case, if the

contract made at date 0 between the consumer and bank α is not renegotiated at a later date, then the value of the deposit must be $D_0 = \frac{p_1}{1+i_0}y_1 + \frac{p_2}{(1+i_0)(1+i_1)}y_2 + M$, where (y_1, y_2) is the solution to (PB). But the relation-specific technology prevents banks from committing not to renegotiate after a contract is established. We assume the following:

Assumption 6 *Bank α can walk away at any time, leaving the capital ($K_{0\alpha}$ between date 0 and date 1, or $K_{1\alpha} = AK_{0\alpha} - y_0$ between date 1 and date 2) for the depositors. If bank α walks away, the only thing the depositors can do is to deposit the capital that bank α left in another bank α' and let bank α' use the capital.*

After forming the relation-specific capital $K_{0\alpha}$, bank α , knowing that it can walk away at any time, has an incentive to offer $D'_0 = \max_{y'_1, y'_2} \frac{p_1}{1+i_0}y'_1 + \frac{p_2}{(1+i_0)(1+i_1)}y'_2 + M$, where $y'_1 \leq aK_{0\alpha}$ and $y'_2 \leq a(aK_{0\alpha} - y'_1)$, instead of D_0 to the consumer. Assumption 6 and equation (3) imply that the rational consumer will always accept the offer D'_0 .

Therefore, if the contract between consumers and banks is a renegotiable debt contract, banks will always renegotiate their obligation down to D'_0 , they will choose inefficient use of capital, and productivity will decline to a . Diamond and Rajan (2001) point out that *demand deposits* are a tool for banks to credibly commit not to renegotiate later. The intuition is as follows. Suppose that the demand deposit contract gives depositors the right to withdraw the full amount of their deposit from bank α at any time. If bank α tries to renegotiate at a time between date 0 and date 1, depositors will rationally exert their right to withdraw, and this will result in a run on the bank. The bank run results in the fire sale of all the bank's assets, and bank α is forced to stop operating at date 1. In this case, bank α cannot obtain the private utility U_t at dates 1 and 2. Anticipating this result, a bank will not try to renegotiate as long as the date 0 contract between depositors and banks is a demand deposit. The same argument implies that banks will not renegotiate at any time between dates 1 and 2. In Section 2.4 we will show more rigorously that demand deposits prevent renegotiation in the equilibrium.

In our model, depositors withdraw cash, not the consumer good as in the Diamond-Rajan model. This difference necessitates the following assumption for demand deposits to prevent renegotiation.

Assumption 7 *A consumer who withdraws his deposit from a bank can deposit the cash he has withdrawn in another bank at the same rate of return (i_0, i_1) .*

This assumption states that banks must accept deposits from consumers at any time on the same terms and conditions. Unless Assumption 7 holds, consumers who withdraw their deposits in a run on a bank must either hold the proceeds as cash or buy consumer goods and store them. Since holding cash or storing consumer goods provides less consumption to consumers, they may rationally accept the renegotiation by banks instead of withdrawing their deposits in bank runs. Therefore, both Assumptions 3 and 7 must hold for demand deposits to prevent renegotiation in our model, while Assumption 3 is sufficient in the Diamond-Rajan model, where the deposit contract is made in real terms.

2.3 Timing of Events

We summarize the timing of events in our model. At date 0, the consumers are endowed with E units of the consumer good and M units of cash. They deposit E and M in banks in exchange for the nominal claims D (demand deposits) on the banks. After receiving E and M , banks have a chance to buy and sell the consumer goods at price p_0 among themselves to adjust their asset portfolio and cash reserves. (In the Financial Intermediation Equilibrium [FIE] defined in Section 2.4 no trading occurs.) Then consumers withdraw $p_0 c_0$ units of cash from banks and buy c_0 units of the consumer good. It is shown in Section 2.4 that the withdrawal $p_0 c_0$ must be equal to the cash reserve M in the FIE. The remaining demand deposits become $D_0 = D - p_0 c_0$. At this stage, a bank has $E - c_0$ units of the consumer good and M units of cash as its assets and D_0 as its liabilities. Then banks have another chance to buy and sell the consumer good at price p_0 among themselves in order to achieve their optimal level of cash reserves M_0 . ($M_0 = M$ in the FIE.) The banks next transform the consumer good $E - c_0$ into capital $K_0 = E - c_0$. After forming their capital, banks have an incentive to renegotiate with the depositors, but they do not actually offer renegotiation in the equilibrium, since such an offer would induce a run on the bank that made it.

At date 1, the banks produce AK_0 units of the consumer good. The demand deposit

becomes $(1 + i_0)D_0$. Consumers withdraw p_1c_1 units of cash and buy c_1 units of the consumer good. If the withdrawal $p_1c_1 > M_0$, a fire sale occurs. In the equilibrium $p_1c_1 = M_0 = M$. The remaining demand deposits become $D_1 = (1 + i_0)D_0 - p_1c_1$. Then banks have a chance to sell and buy the consumer good at price p_1 among themselves so as to achieve their desired level of cash reserves M_1 . (No trade occurs, and $M_1 = M$ in the FIE.) Then banks transform the remaining consumer goods $AK_0 - c_1$ into capital $K_1 = AK_0 - c_1$. After forming this new capital, banks again have an incentive to renegotiate, but again they do not actually offer renegotiation in the equilibrium.

At date 2, the banks produce AK_1 units of the consumer good. The demand deposits become $(1 + i_1)D_1$. Consumers withdraw p_2c_2 units of cash and buy c_2 units of the consumer good. If $p_2c_2 > M_1$, a fire sale occurs. In the equilibrium $p_2c_2 = M_1 = M$. The remaining demand deposits become $D_2 = (1 + i_1)D_1 - p_2c_2$. The government then requires consumers to pay a tax $T_2 = M$. Consumers withdraw T_2 from banks, they pay the tax, and the economy ends. In the FIE, $D_2 = T_2 = M$ must hold.

2.4 Optimal Equilibrium

We define the *Financial Intermediation Equilibrium (FIE)* as follows:

Definition 1 *The Financial Intermediation Equilibrium is the set of prices $(p_0, p_1, p_2, i_0, i_1)$, endowments (E, M) , tax (T_2) , and allocation (c_0, c_1, c_2) such that (1) the allocation (c_0, c_1, c_2) is the solution to (PC) given the prices, endowments, and tax; (2) $(y_0, y_1, y_2) = (c_0, c_1, c_2)$ is the solution to (PB) given the prices and the endowment E ; (3) Given the prices and the withdrawals (p_0c_0, p_1c_1, p_2c_2) , banks rationally choose to hold cash reserves of $M_t = p_{t+1}c_{t+1}$ at each date $t = 0, 1$; (4) The money market clears $M_t = M$ at each date t ; (5) Given the prices, banks rationally withhold offering renegotiation to depositors.*

We will show that the optimal allocation is attainable in the Financial Intermediation Equilibrium. The optimal allocation is given by the following social planner's problem:

$$(PO) \max_{c_0, c_1, c_2} u(c_0) + \beta u(c_1) + \beta^2 u(c_2)$$

subject to

$$\begin{cases} c_0 \leq E, \\ c_1 \leq A(E - c_0), \\ c_2 \leq A^2(E - c_0) - Ac_1. \end{cases} \quad (5)$$

The solution (c_0^*, c_1^*, c_2^*) is uniquely determined by the resource constraints and the first-order conditions (FOCs):

$$\frac{u'(c_0^*)}{\beta u'(c_1^*)} = \frac{u'(c_1^*)}{\beta u'(c_2^*)} = A. \quad (6)$$

Since we assume $u(c_t) = \ln c_t$, the solution is

$$c_0^* = \frac{E}{1 + \beta + \beta^2}, \quad c_1^* = \beta A c_0^*, \quad c_2^* = \beta^2 A^2 c_0^*. \quad (7)$$

We assume that the parameters satisfy the following:

Assumption 8 *The parameters β and x satisfy $(2-x)\beta^2 + (1-x)\beta > 1$ and $(2-x)\beta > 1$.*

For example, $\beta \geq .9$ and $x \leq .8$ satisfy this condition. This assumption guarantees that Assumptions 3 and 7 are sufficient to prevent renegotiation in the FIE. In a decentralized economy, the consumer's problem is (PC), the solution to which is characterized by the following FOCs:

$$\frac{u'(c_0^*)}{\beta u'(c_1^*)} = \frac{p_0}{p_1}(1 + i_0), \quad \frac{u'(c_1^*)}{\beta u'(c_2^*)} = \frac{p_1}{p_2}(1 + i_1). \quad (8)$$

The bank's problem is (PB), the solution to which is characterized by

$$\frac{p_0}{p_1}(1 + i_0) = \frac{p_1}{p_2}(1 + i_1) = A. \quad (9)$$

It is easily shown that the set of the prices $(p_0^*, p_1^*, p_2^*, i_0^*, i_1^*) \equiv (\frac{M}{c_0^*}, \frac{M}{c_1^*}, \frac{M}{c_2^*}, \beta^{-1} - 1, \beta^{-1} - 1)$ and the allocation (c_0^*, c_1^*, c_2^*) satisfies the FOCs (8) and (9). We can now demonstrate the following proposition:

Proposition 1 *The set of the prices $(p_0^*, p_1^*, p_2^*, i_0^*, i_1^*)$ and the allocation (c_0^*, c_1^*, c_2^*) is the Financial Intermediation Equilibrium.*

(Proof) It is sufficient to show that banks rationally set the cash reserve at $M_t = M$ for $t = 0, 1$, and that they rationally withhold offering renegotiation. Given the prices, the withdrawals at

each date are $p_0^*c_0^* = p_1^*c_1^* = p_2^*c_2^* = M$. At the beginning of date 0, each bank has M units of cash, as the consumers deposit all their cash. Before depositors withdraw $p_0^*c_0^*$, banks can change the level of their cash reserve (R) by buying and selling the consumer goods at price p_0^* . If $R < p_0^*c_0^* = M$, banks incur a deadweight loss (Assumption 5). Thus banks desire for R to be greater than or equal to M . Since banks have another chance to sell and buy the consumer goods at the same price p_0^* after the withdrawal of $p_0^*c_0^*$, they are indifferent whether $R = M$ or $R > M$. Therefore, banks set their reserves at the level of the money supply M .

When depositors withdraw $p_0^*c_0^*$, the cash M is withdrawn temporarily but is returned to the banks on the same date in the form of the proceeds from sales of the consumer good $y_0 = c_0^*$. At this point banks, anticipating that withdrawals at date 1 will be $p_1^*c_1^* = M$, have the chance to change the level of their cash reserves (M_0) by buying and selling the consumer good at p_0^* among themselves. If a bank sets M_0 to be greater than the expected withdrawals (M), it loses the opportunity to produce the consumer good, incurring the opportunity cost: $\{A\frac{p_1^*}{p_0^*} - 1\}(M_0 - M) = (\beta^{-1} - 1)(M_0 - M)$ units of cash. Thus M_0 must be no greater than M . If a bank sets $M_0 < M$, it can buy $\frac{1}{p_0^*}(M - M_0)$ units of the consumer good and produce $\frac{A}{p_0^*}(M - M_0)$ at date 1. But at date 1 the bank must sell part of its output in a fire sale in order to obtain cash $M - M_0$ to pay out M to the depositors. The payoff is $A\frac{p_1^*}{p_0^*}(M - M_0) - \frac{1}{x}(M - M_0) = (\beta^{-1} - x^{-1})(M - M_0)$, which is less than zero because of Assumption 5. Therefore banks set $M_0 = M$ given the prices (p_0^*, p_1^*) . It is shown that $M_1 = M$ by the same argument.

Next we show that banks rationally withhold renegotiation. In this equilibrium the budget constraint of (PC) implies that $(1 + i_1^*)D_1 = p_2^*c_2^* + T_2 = 2M$, $(1 + i_0^*)D_0 = p_1^*c_1^* + 2\beta M = (1 + 2\beta)M$, and $D_0 = (1 + 2\beta)\beta M$. Therefore the bank's liability at the end of date 0 is $D_0 = (1 + 2\beta)\beta M$, while the bank's assets are M units of cash and $E - c_0^*$ units of capital. The bank has the incentive to renegotiate with depositors at a time after it forms the capital $K_0 = E - c_0^*$ and before it produces the date 1 consumer good AK_0 . If at this time the bank offers renegotiation and experiences a run by the depositors, then it will be forced to transform $E - c_0^*$ back into units of the consumer good and to sell them in a fire sale. In this case the total cash the bank can pay out becomes $M + p_0^*x(E - c_0^*) = (1 + \beta x + \beta^2 x)M$. Assumption 8 guarantees that the bank will go bankrupt if there is a run on it by depositors at the end of date 0. Anticipating this result, banks rationally withhold renegotiation at the end of date 0.

Similarly, at the end of date 1 when the bank has the incentive to renegotiate, its liability is $D_1 = 2\beta M$, while the total cash the bank can pay out after the fire sale is $(1 + \beta x)M$. Thus Assumption 8 guarantees that the bank will go bankrupt if a bank run occurs, and that banks

rationally withhold renegotiation at the end of date 1. (End of Proof)

Thus in the FIE, the most efficient use of capital and the optimal consumption allocation are realized by demand deposit contracts between banks and consumers.

3 Banking Crisis

The recent episodes of banking crises have often involved the emergence and subsequent collapse of an asset-price bubble. In this paper we do not analyze how the bubble emerges or how it collapses. Our focus is on how the collapse affects the banking sector and on the overall economy. Thus in our model we describe the bubble's collapse as an unexpected macroeconomic shock that destroys a portion of the real output of the economy. The occurrence of this shock is assumed to be a measure-zero event, in the sense that the agents in the economy have no expectation of the shock beforehand. We can modify our model to treat the macro shock as a random variable the probability distribution of which is known *ex ante* to the agents. But this modification does not essentially change the results that we describe below, since our results mainly concern the responses of the economic agents *after* the shock hits. Thus for simplicity of exposition we assume that the *ex ante* probability of the shock (bubble collapse) equals zero⁵, and that all consumers and banks at date 0 make their contracts on the premise that prices will become $(p_0^*, p_1^*, p_2^*, i_0^*, i_1^*)$.

3.1 Bubble Collapse, Price Adjustment, and Zero Nominal Interest Rate

We formalize the bubble collapse at the beginning of date 1 as follows. At a time after $AK_0 = A(E - c_0^*)$ units of the consumer good are produced and the equilibrium price p_1^* is announced but before the goods are sold to consumers, a macro shock λ ($0 \leq \lambda < 1$) hits the economy unexpectedly and destroys $(1 - \lambda)AK_0$ of each bank's output. Since

⁵This modeling strategy is same as Loewy (1991). Burnside, Eichenbaum, and Rebelo (2001) also adopt the same modeling strategy to analyze currency crises.

the *ex post* output is λAK_0 after the bubble collapse at date 1, the optimal allocation after the shock becomes $(c_1, c_2) = (\lambda c_1^*, \lambda c_2^*)$. The optimal allocation is realized as an equilibrium outcome by the new price system $(p_1, p_2, i_1) = (\lambda^{-1} p_1^*, \lambda^{-1} p_2^*, i_1^*)$. We will show that the optimal equilibrium is realized by a standard price adjustment process if depositors cannot make a run on banks before the price system changes.

Baseline Case If consumers cannot make a run on banks quickly, they solve the following problem under the new price system (p_1, p_2, i_1)

$$(PC1) \max u(c_1) + \beta u(c_2)$$

subject to

$$\begin{cases} p_1 c_1 + D_1 \leq (1 + i_1^*) D_0^* = (1 + 2\beta)M, \\ p_2 c_2 + T_2 \leq (1 + i_1) D_1. \end{cases} \quad (10)$$

The banks solve

$$(PB1) \max_y p_1 y + \frac{p_2}{1 + i_1} A \{ \lambda AK_0 - y \}.$$

We assume that the new interest rate $i_1 > 0$ (we will examine the case where $i_1 = 0$ later). In this case, consumers never hold cash; they hold only bank deposits. We also assume that the government does not transfer goods or cash from consumers to banks. Under these assumptions, when banks sell units of the consumer good at date 2, their remaining liability after withdrawals $((1 + i_1) D_1 - p_2 c_2)$ must be equal to their income $(p_2 c_2)$. Thus we have the equilibrium condition: $p_2 c_2 = \frac{1}{2}(1 + i_1) D_1 = T_2$. Since the government does not change its tax policy, $T_2 = M$. Thus, in the equilibrium $p_2 c_2 = M$ and $(1 + i_1) D_1 = 2M$. The FOCs for consumers and banks $(\frac{c_2}{\beta c_1} = \frac{p_1}{p_2}(1 + i_1) = A)$ imply $p_1 c_1 = \frac{p_2 c_2}{\beta(1 + i_1)} = \frac{M}{\beta(1 + i_1)}$. This condition and the budget constraint (10) imply $1 + i_1 = \beta^{-1}$. Therefore $p_1 c_1 = M$. The FOC $(\frac{c_2}{c_1} = \beta A)$ and the resource constraint $(c_2 = A\{\lambda AK_0 - c_0\})$ imply that $c_1 = \frac{\lambda AK_0}{1 + \beta} = \lambda c_1^*$ and $c_2 = \beta A c_1 = \lambda c_2^*$. Thus, the new equilibrium price system $(p_1, p_2, i_1) = (\lambda^{-1} p_1^*, \lambda^{-1} p_2^*, \beta^{-1} - 1)$ is uniquely determined in the case where $i_1 > 0$.

If the government sets the interest rate at zero: $i_1 = 0$, the consumers hold both cash

and bank deposits. We can show that banks will choose the inefficient use of capital if the nominal interest rate is zero:

Lemma 1 *Suppose that the economy is in an equilibrium where the nominal interest rate is zero and banks continue to operate at dates 1 and 2. Each bank chooses the inefficient use of capital and the aggregate productivity of the economy becomes a .*

(Proof) Since the banks continue to operate at dates 1 and 2 under a zero nominal interest rate, the prices satisfy the FOC for the banks:

$$\frac{p_1}{p_2} = \tilde{A}, \quad (11)$$

where \tilde{A} is the productivity of capital, which is A if the bank chooses the efficient use of capital, and is a if it chooses the inefficient use. We will prove that a bank holds only cash rather than capital if the other banks choose efficient use of capital. Suppose that all banks choose the efficient use of capital. Then $\frac{p_1}{p_2} = A$. This condition and Assumption 4 imply that a bank can obtain the same return A by holding cash without exerting effort ϵ . Therefore when a bank enters into deposit contracts with depositors at date 1, it promises them the same return as the other banks, and it holds only cash, because by doing so the bank can fulfill the promise made to the depositors and can save the effort ϵ that must be exerted to fulfill the promise by using capital. Therefore no banks will hold capital in the equilibrium where $i_1 = 0$ and $\frac{p_1}{p_2} = A$. This is a contradiction, since if no banks hold capital then $\frac{p_1}{p_2} = A$ does not hold. Thus, in the equilibrium where $i_1 = 0$ it must be the case that $\frac{p_1}{p_2} = a$, and all banks choose the inefficient use of capital. (End of Proof)

This lemma says that if $i_1 = 0$ the banks never promise the efficient use of capital in the first place. Therefore the demand deposit contract, which does not allow subsequent renegotiation, cannot prevent the inefficiency of moral hazard when $i_1 = 0$. The implication of this lemma is similar to Smith's view that the Friedman rule is not the optimal policy in the economy where the financial intermediaries play a significant role (Smith [2002]).

It is easily shown that if $i_1 = 0$ and the depositors cannot make runs on banks, there exists a continuum of equilibria in which moral hazard exists for the banks. When

$i_1 = 0$, consumers holds cash (M_1) and bank deposit (D_1). The FOC ($\frac{c_2}{\beta c_1} = \frac{p_1}{p_2}$) and the budget constraint imply $p_1 c_1 = \frac{2\beta M}{1+\beta}$ and $p_2 c_2 = \frac{2\beta^2 M}{1+\beta}$. The FOCs and the resource constraint imply that $(c_1, c_2) = (\lambda c_1^*, \lambda \beta a c_1^*)$. The equilibrium condition for banks is $D_1 - W = p_2 c_2$, and the equilibrium conditions for consumers are $W + M_1 - R = p_2 c_2$ and $D_1 - W + R = T_2$, where W is the amount of withdrawals at date 2 and R is cash held by consumers that is not used for the purchase of the goods but used for tax payment. The equilibrium price system is uniquely determined as $(p_1, p_2, i_1) = (\frac{2\beta}{\lambda(1+\beta)}p_1^*, \frac{2\beta^2}{\lambda\beta a(1+\beta)}p_1^*, 0)$. The deposits and cash holdings are indeterminate: $D_1 = \frac{2\beta^2}{1+\beta}M + W$ and $M_1 = M - W$ for $0 \leq W \leq M$. It is easily shown that banks have incentive to hold cash reserves W given that withdrawals at date 2 are W .

The above baseline case for $i_1 > 0$ is realized if consumers cannot make runs on banks before the price system changes. Although it is quite plausible to assume that prices adjust instantaneously in a neoclassical growth model, the assumption of perfectly flexible prices is not realistic in our model, where depositors who anticipate the price change can make runs on banks and can buy consumer goods very quickly. In order to formalize bank runs that occur as quickly as price changes, we assume that when the price changes in the wake of the macroeconomic shock, a small group of the fastest withdrawers can buy consumer goods at the old price, while the other withdrawers can buy the goods only at the new equilibrium price:

Assumption 9 (*Bank Runs and Price Adjustment*) *Suppose that the equilibrium price changes from p_1^* to the new price p_1 after the shock λ hits the economy. If a consumer withdraws deposit and has cash in hand before he faces the price p_1^* or p_1 at date 1, there is a probability of π ($0 < \pi < 1$) that he will be able to buy the goods at p_1^* using his cash in hand. In this case the consumer obtains an arbitrage opportunity in which he can buy the goods at p_1^* and sell them at the new equilibrium price p_1 . If he waits to withdraw his deposit until he faces the date-1 price, he loses the chance to buy the goods at p_1^* , and he can buy the goods only at the new equilibrium price p_1 .*

This assumption says that if the depositors withdraw fast, they have a chance to buy the

goods at the original price p_1^* , but they lose the chance and have to buy the goods at the new equilibrium price p_1 if they wait to withdraw. Therefore, when the shock λ hits the economy, a depositor must decide whether to make a withdrawal and, if so, how much to withdraw on the premise that he can buy the goods at the old price p_1^* with probability π and at the new price with probability $1 - \pi$. Assumption 7 guarantees that as long as banks continue to operate, a depositor will decide about his withdrawal on the premise that he can redeposit the cash he withdraws if it turns out after his withdrawal that he cannot buy the consumer goods at a favorable price.

3.2 Bank Runs and Debt Deflation

When the unexpected shock hits the economy at the beginning of date 1, depositors may or may not make runs on banks anticipating that the equilibrium price system will change from the original equilibrium price (p_1^*, p_2^*, i_1^*) to the new one. Note that bank runs can occur even in the FIE described in Proposition 1 if each depositor believes that the other depositors will make a run on his bank, since a depositor who waits while all the other depositors make a run gets nothing. This can be called a bank run due to self-fulfilling prophecy. Since our focus is on whether or not the shock λ (the bubble collapse) causes bank runs, we simply exclude the possibility of self-fulfilling prophecy, as do Allen and Gale (2001).

Assumption 10 *Bank runs due to self-fulfilling prophecy do not occur. Bank runs occur only if withdrawing the entire deposit is a strictly better strategy for a depositor than waiting, even if the other depositors do not make a run on the bank.*

We also make the following assumption for simplicity.

Assumption 11 *In the bank runs where all depositors seek to withdraw their entire deposits, each depositor obtains an equal share of the total cash that the bank can pay.*

We can show that if there is no government intervention, the bubble collapse (λ) causes all depositors to make runs on their banks, and the economy is thus disintermediated.

The following proposition shares the same intuition and implication with the results of Loewy (1991).

Proposition 2 (*Disintermediation*) *Suppose that the government does not intervene after the shock λ hits the economy. For λ that satisfies $0 \leq \lambda < \frac{2\beta}{1+\beta}$, all depositors withdraw their entire deposits, and all banks go bankrupt at date 1. For λ that satisfies $\frac{2\beta}{1+\beta} \leq \lambda < 1$, there exist equilibria where $i_1 = 0$, depositors do not make runs on banks, banks continue to operate at dates 1 and 2, and the productivity becomes a as a result of banks' moral hazard.*

(Proof) We examine the following two cases: (1) the new equilibrium price p_1^R is larger than p_1^* ; and (2) $p_1^R \leq p_1^*$.

In the case where $p_1^R > p_1^*$, it is obvious that the optimal choice for a depositor is to withdraw his entire deposit in order to make use of the arbitrage opportunity that he can hope for with a probability of π . Thus all depositors make runs on banks. In this case, banks' liabilities are $(1+i_0^*)D_0 = (1+2\beta)M$, against which they have cash reserves M and the consumer goods λAK_0 . In order to meet the depositors' demand for cash, banks are first of all forced to pay out all their cash reserves. After they have done so, the remaining demand for cash is $2\beta M$, while the total cash in the economy is M , which banks can obtain by selling their goods. In the end, Assumption 11 implies that each depositor obtains $2M$ units of cash. The price p_1^R is determined as follows. Since $p_1^R > p_1^*$, the lucky withdrawers (measure π) spend $2M$ to buy goods at p_1^* . Thus

$$p_1^R x \left\{ \lambda AK_0 - \frac{\pi 2M}{x p_1^*} \right\} = M - \pi 2M,$$

which implies⁶

$$p_1^R = \frac{1 - 2\pi}{x \left\{ \lambda(1 + \beta) - \frac{2\pi}{x} \right\}} p_1^*. \quad (12)$$

Therefore, if $\lambda < \frac{1}{x(1+\beta)}$, the new equilibrium price p_1^R satisfies $p_1^R > p_1^*$, and all banks go bankrupt at date 1.

In the second case, where $p_1^R \leq p_1^*$, we prove by contradiction that all banks go bankrupt if $\frac{1}{x(1+\beta)} \leq \lambda < \frac{2\beta}{1+\beta}$. Suppose that banks continue to operate at dates 1 and 2. In this case, the date 2 price p_2^R and the interest rate i_1^R must satisfy

$$\frac{p_2^R}{1 + i_1^R} = \frac{p_1^R}{A}, \quad (13)$$

⁶We assume that the lucky withdrawers obtain the cash $2M$ from the reserves of banks.

because the banks maximize the present value of their assets. We assume that there is no fire sale in this equilibrium⁷. First we consider the case where $i_1^R > 0$. In this case banks' cash reserves are M . At date 2, a bank's liabilities must be equal to its assets. Thus $(1 + i_1^R)\{(2\beta + 1)M - p_1^R c_1^R\} = p_2^R A(\lambda AK_0 - c_1^R) + M$, which can be rewritten as

$$(2\beta + 1)M = \frac{p_1^R}{p_1^*} \lambda(1 + \beta)M + \frac{M}{1 + i_1^R}. \quad (14)$$

In this case if $\lambda < \frac{2\beta}{1+\beta}$ the condition (14) cannot hold for any values of $p_1^R (\leq p_1^*)$ and $i_1^R \geq 0$. This is a contradiction.

Next, in the case where $i_0^R = 0$ the condition that the bank's liabilities equal its assets at date 2 is

$$(2\beta + 1)M = \frac{p_1^R}{p_1^*} \lambda(1 + \beta)M + R_1, \quad (15)$$

where R_1 is the cash reserve of the bank that satisfies $R_1 \leq M$. If $\lambda < \frac{2\beta}{1+\beta}$, the condition (15) cannot hold for any $p_1^R (\leq p_1^*)$. Thus, for $\lambda < \frac{2\beta}{1+\beta}$, all banks go bankrupt from bank runs for any $i_1^R \geq 0$. Therefore, for $\frac{1}{x(1+\beta)} \leq \lambda < \frac{2\beta}{1+\beta}$, banks are forced to sell all their goods in a fire sale, since the cash in the economy is M , while the depositors' demand for cash is $2\beta M$. Since the expectation is that $p_1^R \leq p_1^*$, the lucky withdrawers who have a chance to buy the goods at p_1^* wait to buy the goods at p_1^R . Thus

$$p_1^R = \frac{M}{x\lambda AK_0} = \frac{p_1^*}{\lambda x(1 + \beta)}.$$

Thus for $\frac{1}{x(1+\beta)} \leq \lambda < \frac{2\beta}{1+\beta}$ the new price p_1^R satisfies $p_1^R \leq p_1^*$, and all banks go bankrupt at date 1.

If $\frac{2\beta}{1+\beta} \leq \lambda < 1$, there exist sets of (p_1^R, i_1^R) that satisfy (14). Suppose that $i_1^R > 0$. In this case, an argument similar to that of the baseline case (page 17) holds, implying $p_1^R c_1^R = M$. Since we assume that there is no fire sale, the FOCs for banks and consumers imply $\frac{c_2^R}{\beta c_1^R} = A$, and the resource constraint ($c_2^R = A\{\lambda AK_0 - c_1^R\}$) holds. Thus $c_1^R = \frac{\lambda AK_0}{1+\beta}$ and $c_2^R = \beta A c_1^R$. Therefore it must be the case that $c_1^R < c_1^*$, implying that $p_1^R > p_1^*$, which is a contradiction. Therefore, $i_1^R = 0$ in the equilibrium where $\lambda \geq \frac{2\beta}{1+\beta}$ and banks continue to operate. Since consumers hold both cash and bank deposits when $i_1^R = 0$, the condition (14) becomes (15). Lemma 1 implies that the banks operate the capital inefficiently when $i_1^R = 0$ so that the productivity becomes a . Therefore, for $\frac{2\beta}{1+\beta} \leq \lambda < 1$, there is a continuum of equilibria in which $c_1^R = \lambda c_1^*$, $c_2^R = \beta a c_1^R$, $\frac{2\beta}{(1+\beta)\lambda} p_1^* \leq p_1^R \leq p_1^*$, $\{2\beta + 1 - \lambda(1 + \beta)\}M \leq R_1 \leq M$, $i_1^R = 0$, and $p_2^R = a^{-1} p_1^R$. Note that the bank-run equilibrium does not exist for $\frac{2\beta}{1+\beta} < \lambda < 1$, since we make Assumption 10.

⁷It is easily shown that the following argument still holds for the case where the fire sale occurs.

In sum, we have shown the following: if $0 < \lambda < \frac{1}{x(1+\beta)}$, bank runs occur and all banks go bankrupt, resulting the equilibrium price $p_1^R > p_1^*$; if $\frac{1}{x(1+\beta)} \leq \lambda < \frac{2\beta}{1+\beta}$, bank runs occur and all banks go bankrupt, while the equilibrium price $p_1^R \leq p_1^*$; if $\frac{2\beta}{1+\beta} \leq \lambda < 1$, $i_1^R = 0$, there are no bank runs, but moral hazard occurs. (End of Proof)

When $\frac{1}{x(1+\beta)} \leq \lambda < \frac{2\beta}{1+\beta}$, the new price p_1^R becomes less than p_1^* . We can interpret this price change from p_1^* to p_1^R as the *debt deflation* caused by bank runs and fire sales. In this case, after all banks go bankrupt and the economy is disintermediated, each consumer holds λxAK_0 units of the consumer good and M units of cash.

Equilibrium after Debt Deflation In order to specify the equilibrium after debt deflation, we make the following assumption:

Assumption 12 *After the disintermediation, no production technology is available, and consumers must store the consumer goods for the date-2 consumption.*

Since banks cease to exist, consumers solve the following problem:

$$(PRC) \max_{c_1, M_1, s, c_2} u(c_1) + \beta u(c_2)$$

subject to

$$\begin{cases} p_1^R c_1 + p_1^R s + M_1 \leq p_1^R \lambda xAK_0 + M, \\ p_2^R c_2 + T_2 \leq p_2^R s + M_1, \\ p_2^R c_2 \leq M_1 \quad (\text{CIA constraint}), \end{cases} \quad (16)$$

where $T_2 = M$. The solution (c_1, c_2) to this consumer's program is feasible if $c_1 \geq 0$, $c_2 \geq 0$, and $c_1 + c_2 \leq \lambda xAK_0$. It is easily shown that (PRC) has a feasible solution only if $p_2^R = \infty$, and the unique feasible solution is $(c_1, c_2, s, M_1) = (\lambda xAK_0, 0, 0, M)$.

4 Policy Responses to Bank Insolvency

After the shock λ hits the economy, the government has several policy options to cope with bank runs and subsequent disintermediation. In this section we compare the welfare effects of three policies: a deposit guarantee, unlimited liquidity support, and bank

recapitalization. Note that these policy responses are necessitated since the shock λ hits all banks.⁸

A deposit guarantee is a policy under which the government guarantees that all deposits will be repaid in full, but it does not supply cash or goods to banks unless they run out of assets. In order to fulfill the commitment to guarantee deposits, the government must impose a tax on consumers and transfer the goods to banks after the banks' assets are exhausted.

Unlimited liquidity support is a policy under which the government supplies as much cash to banks as they need. The cash is created by the government, and it is redeemed by imposing taxes on consumers at date 2. This policy involves a transfer of value from consumers to banks by seigniorage.

Bank recapitalization is a policy under which the government transfers goods or cash to banks as a subsidy in order to restore their solvency. The cost of recapitalization is financed by seigniorage or taxation. Under a recapitalization policy, the government transfers a fixed amount of resources *ex ante*, but it does not transfer any resources *ex post*, unlike in the case of a deposit guarantee or unlimited liquidity support.

4.1 Deposit Guarantee

We define the deposit guarantee policy as follows. The government declares at date 1 that all deposits will be repaid in full at any time. But the government does not supply cash or transfer the goods before the withdrawals occur at date 1. If the banks exhaust their assets during the withdrawals at date 1 or date 2, the government uses a portion of its tax revenue to pay back the depositors. Thus, the government redistributes the goods from consumers to withdrawers through taxation. We can also assume that when the government implements a deposit guarantee policy, consumers rationally expect that the government will collect the cost of the deposit guarantee policy at date 1 or date 2 as a lump-sum tax on consumers.

⁸If the shock λ is idiosyncratic and observable, deposit insurance among the banks can prevent bank runs.

We examine the equilibrium $(p_1^D, p_2^D, i_1^D, c_1^D, c_2^D)$ under a deposit guarantee policy. If $p_1^D > p_1^*$, all depositors withdraw their entire deposits in order to make use of the arbitrage opportunity in which they buy goods at p_1^* and sell them at p_1^D . Therefore the same argument as in the proof of Proposition 2 implies that in the case where $0 \leq \lambda < \frac{1}{x(1+\beta)}$, banks go bankrupt and stop to operate at date 1 in spite of the deposit guarantee by the government. The cost of the deposit guarantee is collected through a lump-sum tax on consumers. Thus we have the following lemma.

Lemma 2 *If $0 \leq \lambda < \frac{1}{x(1+\beta)}$ and the macroeconomic expectation is that $p_1^D > p_1^*$, all banks go bankrupt and the economy is disintermediated at date 1 even with a deposit guarantee by the government.*

If $\frac{1}{x(1+\beta)} \leq \lambda < 1$, the equilibrium is different from the disintermediation. We can show the following:

Lemma 3 *If $\frac{1}{x(1+\beta)} \leq \lambda < 1$, the new equilibrium price satisfies $p_1^D \leq p_1^*$.*

(Proof) Suppose that $p_1^D > p_1^*$. Then all depositors withdraw their entire deposits and the banks go bankrupt. The fire sale drives the price down to $\frac{M}{x\lambda AK_0}$, which is no greater than p_1^* because $\lambda \geq \frac{1}{x(1+\beta)}$. This is a contradiction. Thus $p_1^D \leq p_1^*$. (End of Proof)

In the equilibrium under a deposit guarantee policy where the macroeconomic expectation is $p_1^D \leq p_1^*$, banks continue to operate at dates 1 and 2. In this situation, the deposit guarantee policy has a serious side effect. Since the government protects bank deposits, the depositors have no incentive to make a run on banks even when the latter are using capital inefficiently. Therefore, moral hazard for banks inevitably results from the deposit guarantee policy in our model. The bank's problem under a deposit guarantee policy is

$$(PBD) \max_y p_1^D y + \frac{p_2^D}{1+i_1^D} a\{\lambda AK_0 - y\}.$$

The consumer's problem is

$$(PCD) \max u(c_1^D) + \beta u(c_2^D)$$

subject to

$$\begin{cases} p_1^D c_1^D + D_1 \leq (1 + 2\beta)M, \\ p_2^D c_2^D + T_2 \leq (1 + i_1^D)D_1, \end{cases} \quad (17)$$

where $T_2(\geq M)$ is the expected amount of tax, including the additional cost of the deposit guarantee policy. The FOCs imply

$$(1 + i_1^D) \frac{p_1^D}{p_2^D} = a = \frac{c_2^D}{\beta c_1^D}.$$

We can show the following proposition. As we show in the proof of the proposition, there is a range of values of T_2 that supports the equilibrium. If the government credibly announces an inappropriate value of T_2 at date 1, there is no equilibrium. We assume that the government declares the deposit guarantee without announcing T_2 , and the consumers and banks form a rational expectation of T_2 , given the government's declaration.

Proposition 3 *Assume that the government adopts a deposit guarantee policy after a shock λ hits the economy. If $\frac{1}{x(1+\beta)} \leq \lambda < \frac{x}{\beta}$, the nominal interest rate becomes zero (i.e., $i_1^D = 0$) and the inflation rate is $\frac{p_2^D}{p_1^D} = a^{-1}$ in the equilibrium. If $\frac{x}{\beta} < \lambda < 1$, there exist two continua of equilibria: In one continuum, $i_1^D = 0$ and $\frac{p_2^D}{p_1^D} = a^{-1}$. In the other, $i^D = (\beta\lambda)^{-1} - 1 > i_1^*$ and $\frac{p_2^D}{p_1^D} = (\beta a \lambda)^{-1} > \frac{p_2^*}{p_1^*}$.*

(Proof) We consider the condition for $i_1^D > 0$ in the equilibrium. In this case the consumers solve (PCD), and the banks solve (PBD). The FOCs for (PCD) and (PBD) and the resource constraint $c_2 = a(\lambda AK_0 - c_1)$ imply that

$$c_1^D = \lambda c_1^*, \text{ and } c_2^D = a\beta c_1^D.$$

Lemma 3 guarantees that p_1^D satisfies $p_1^D = z p_1^*$ for $z \in [0, 1]$. Thus $p_1^D c_1^D = z \lambda p_1^* c_1^* = z \lambda M < M$. Therefore, in the equilibrium, consumers withdraw $z \lambda M$ from banks and buy λc_1^* . Since banks already hold M units of cash reserve and they can do nothing to equate the date-1 withdrawal with M , the cash $(1 - z \lambda)M$ remains in banks after consumers' withdrawals at date 1. Since $i_1^D > 0$, consumers choose to hold no cash at the end of date 1, and the banks must hold all the cash (M) as the reserve for date-2 withdrawals. And when banks hold M as the reserve, it must be the case that M is no greater than the expected amount of withdrawals ($p_2^D c_2^D$) at date 2, because the opportunity cost of holding one unit of cash reserve in excess of $p_2^D c_2^D$ is

$\frac{1}{p_1^D}ap_2^D - 1 = i_1^D > 0$. On the other hand, the opportunity cost of holding one unit of cash reserve less than $p_2^D c_2^D$ is $-\frac{1}{p_1^D}ap_1^D + \frac{1}{x} = \frac{1}{x} - (1 + i_1^D)$. If $\frac{1}{x} - (1 + i_1^D) < 0$, then banks choose to have no cash reserve. Therefore, in the equilibrium where banks have a cash reserve M , it must be the case that $\frac{1}{x} - (1 + i_1^D) \geq 0$. In this case, $M = p_2^D c_2^D$. Thus $1 + i_1^D = \frac{1}{z\beta\lambda}$, $p_2^D = \frac{1}{\beta a\lambda}p_1^*$. Then the condition $\frac{1}{x} - (1 + i_1^D) \geq 0$ is rewritten as $\lambda \geq \frac{x}{z\beta}$. Therefore, we have shown that if $\frac{x}{\beta} \leq \lambda < 1$, there exists an interval of z : $[\frac{x}{\beta\lambda}, 1]$ such that for $z \in [\frac{x}{\beta\lambda}, 1]$ there exists an equilibrium where $p_1^D = zp_1^*$, $p_2^D = \frac{p_1^*}{a\beta\lambda}$, $1 + i_1^D = \frac{1}{z\beta\lambda}$, $c_1^D = \lambda c_1^*$, and $c_2^D = a\beta c_1^D$. In this equilibrium, $i^D = (z\beta\lambda)^{-1} - 1 > i_1^*$, $\frac{p_2^D}{p_1^D} = (z\beta a\lambda)^{-1} > \frac{p_2^*}{p_1^*}$, and $T_2 = \frac{1+2\beta-z\lambda-z\beta\lambda}{z\beta\lambda}M$. (Note that if the government sets the tax at $T_2 \neq \frac{1+2\beta-z\lambda-z\beta\lambda}{z\beta\lambda}M$, then there is no equilibrium in which $i_1^D > 0$.) At date 2 the government fulfills its deposit-guarantee commitment by transferring a portion of the goods collected as the tax (T_2) from consumers to the withdrawers, since the banks run out of assets at date 2.

Next we show that there exists another equilibrium in which $i_1^D = 0$ for all λ that satisfies $\frac{1}{x(1+\beta)} < \lambda < 1$. Suppose that $i_1^D = 0$ and $p_1^D = zp_1^*$ ($0 \leq z \leq 1$) in the equilibrium. The FOC for (PBD) implies that $p_2^D = \frac{1}{a}p_1^D$. The consumption allocation is $(c_1^D, c_2^D) = (\lambda c_1^*, \beta a c_1^D)$. In this case, the cash demand by consumers at date 2 is $p_2^D c_2^D = z\beta\lambda M$. Let W be the withdrawals at date 2, and R_1 the cash reserves of the banks. For a bank, the opportunity cost of holding cash in excess of W is zero, and the opportunity cost of holding cash less than W is $\frac{1}{x} - 1 > 0$. Thus banks set their cash reserves at $R_1 \geq W$ at the end of date 1. Since consumers are indifferent between bank deposits and cash when the nominal interest rate is zero, they hold $M - R_1$ units of cash in hand at the end of date 1. The budget constraint implies that $M - R_1 + D_1 = (1 + 2\beta - z\lambda)M$ and $T_2 = (1 + 2\beta - z\lambda - z\lambda\beta)M$. The values of W and R_1 are indeterminate but satisfy the cash-in-advance constraint: $W + M - R_1 \geq z\beta\lambda M$. This equilibrium exists for all $z \in [0, 1]$. (End of Proof)

This proposition states that if the macro shock is large (i.e., $\lambda \leq \frac{x}{\beta}$), a deposit guarantee policy leads the economy into an equilibrium where the nominal interest rate is zero and moral hazard occurs. In this equilibrium, the inflation rate is $\frac{1}{a}$, while it is $\frac{\lambda^{-1}p_2^*}{\lambda^{-1}p_1^*} = \frac{1}{\beta A}$ in the baseline case. If the parameter a is close to A such that $a > \beta A$, then we have mild deflation under a zero nominal interest rate in the equilibrium, just as we have had in the Japanese economy since the late 1990s. Low interest rates during

deflation were also observed in the U.S. economy during the Great Depression.

The deposit guarantee prevents the inefficiency associated with a fire sale (or liquidation) of bank assets, but at the same time this policy inevitably induces moral hazard for banks.

4.2 Unlimited Liquidity Support

We can show that the unlimited liquidity support policy brings about welfare effects similar to those of a deposit guarantee policy. We define the unlimited liquidity support policy as follows. At date 1, the government declares that it will supply an unlimited amount of liquidity on demand to banks. The government supplies cash ΔM on demand to banks for payment to withdrawers. At the end of date 1 the banks choose efficient or inefficient use of capital, and the consumers redeposit their cash after observing the banks' choice. The total cash that exists in the economy between dates 1 and 2 becomes $M + \Delta M$. At date 2, the government collects all cash as the tax: $T_2 = M + \Delta M$. Note that this policy is not the same as the ordinary provision of liquidity by the central bank in normal circumstances. Unlimited liquidity support involves the transfer of value from consumers to banks by seigniorage.⁹ We can prove the following proposition:

Proposition 4 *Suppose that π is close to zero: $\pi \approx 0$. If*

$$x > \frac{\beta}{2\beta + 1}, \quad (18)$$

there is no equilibrium with a positive interest rate under the unlimited liquidity support policy.

(Proof) We assume that $i_1^L > 0$. In this case we can show that $p_1^L > p_1^*$ by contradiction. Under the liquidity support policy, the banks continue to operate at dates 1 and 2. Suppose that $p_1^L \leq p_1^*$ in the equilibrium where the interest rate is positive. Arguments similar to those of the baseline case hold, and we have $p_1^L c_1^L = M$. The FOCs for consumers and banks under $p_1^L \leq p_1^*$ imply that $\frac{c_2^L}{\beta c_1^L} = \tilde{A}$ where $\tilde{A} = A$ or a is the equilibrium productivity, and the resource constraint says

⁹The ordinary liquidity lending at the market rate of interest does not help the banks hit by the shock λ since the difficulty they face is not only the liquidity shortage but also insolvency.

$c_2^L = \tilde{A}\{\lambda AK_0 - c_1^L\}$. Therefore the consumption allocation is uniquely determined as $c_1^L = \lambda c_1^*$ and $c_2^L = \tilde{A}\beta c_1^L$, which implies $p_1^L = \lambda^{-1}p_1^* > p_1^*$. This is a contradiction. Therefore, the equilibrium price must satisfy $p_1^L > p_1^*$.

Since $p_1^L > p_1^*$, Assumption 9 implies that all depositors withdraw $(1 + 2\beta)M$ at date 1. Therefore the cash injection at date 1 is $\Delta M = 2\beta M$. The lucky depositors (measure π) buy $(2\beta + 1)c_1^*$ units of the goods at p_1^* , while the other can buy the goods only at p_1^L . The lucky consumers solve

$$(PC\pi) \quad \max u(c'_1) + \beta u(c'_2)$$

subject to

$$\begin{cases} p_1^L c'_1 + D'_1 \leq (2\beta + 1)p_1^L c_1^*, \\ p_2^L c'_2 + T_2 \leq (1 + i_1^L)D'_1. \end{cases} \quad (19)$$

The unlucky consumers solve

$$(PC1 - \pi) \quad \max u(c''_1) + \beta u(c''_2)$$

subject to

$$\begin{cases} p_1^L c''_1 + D''_1 \leq (2\beta + 1)M, \\ p_2^L c''_2 + T_2 \leq (1 + i_1^L)D''_1. \end{cases} \quad (20)$$

Define $c_1^L \equiv \pi c'_1 + (1 - \pi)c''_1$, $c_2^L \equiv \pi c'_2 + (1 - \pi)c''_2$, and $D_1^L \equiv \pi D'_1 + (1 - \pi)D''_1$. The aggregate budget constraint is

$$\begin{aligned} p_1^L c_1^L + D_1 &\leq (2\beta + 1)(\pi p_1^L c_1^* + (1 - \pi)M), \\ p_2^L c_2^L + T_2 &\leq (1 + i_1^L)D_1. \end{aligned} \quad (21)$$

Banks solve $\max p_1^L c_1^L + \frac{p_2^L}{1 + i_1^L} \tilde{A}\{\lambda AK_0 - c_1^L\}$. The FOCs are

$$\frac{c_2^L}{\beta c_1^L} = \frac{p_1^L}{p_2^L}(1 + i_1^L) = \tilde{A}.$$

The FOCs and the budget constraint (21) imply $p^L(c_1^L - \pi c_1^*) = (1 - \pi)M$. Thus $p_1^L = \frac{1 - \pi}{\lambda - \pi} p_1^*$. The FOCs and the budget constraint (21) imply that $D_1 = \frac{(1 - \pi)\lambda}{\lambda - \pi} 2\beta M$, and $1 + i_1^L = \frac{1 + 2\beta}{\beta} \frac{\lambda - \pi}{(1 - \pi)\lambda}$. The conditions for the banks to set the cash reserve at $p_2^L c_2^L$ are $1 + i_1^L - 1 = i_1^L > 0$ and $\frac{1}{x} - (1 + i_1^L) > 0$. When $\pi \approx 0$, the latter is equal to

$$x < \frac{\beta}{2\beta + 1},$$

which is violated if (18) holds. (The condition (18) holds for $x > .33$ if $\beta = .9$.) Therefore, an equilibrium with a positive interest rate does not exist in this case. (End of Proof)

This proposition states that under the unlimited liquidity support policy, the government provides too much cash so that no banks would hold cash in the case where $i_1 > 0$.

Proposition 5 *Suppose that $\pi \approx 0$. If the government sets $i_1^L = 0$ under the unlimited liquidity support policy, it can attain the equilibrium where there are no bank runs, but moral hazard occurs for banks.*

(Proof) First we show that if $i_1^L = 0$, it must be the case that $p_1^L \leq p_1^*$. Suppose that $p_1^L > p_1^*$ while $i_0^L = 0$. The similar arguments as in the proof of Proposition 4 imply that ΔM must be $2\beta M$, and the aggregate budget constraint is (21). Setting $i_1^L = 0$ and $T_2 = M + \Delta M = (2\beta + 1)M$, we have

$$p_1^L c_1^* \{(1 + \beta)\lambda - (2\beta + 1)\pi\} = -(2\beta + 1)\pi M,$$

which implies $p_1^L < 0$ for a small π . This is a contradiction. Therefore, $p_1^L \leq p_1^*$ if $i_1^L = 0$.

Next we show that there exists an equilibrium where $p_1^L = zp_1^*$ ($0 < z < 1$) under a liquidity support policy that satisfies $i_1^L = 0$. Since the government sets $i_1^L = 0$, the price must satisfy $p_1^L = zp_1^*$ for some z ($0 < z \leq 1$). Since the government declares that it supplies an unlimited amount of liquidity to banks, consumers and banks have the expectations that banks continue to operate at dates 1 and 2. Thus the FOCs for the bank's problem and Lemma 1 imply $\frac{c_2^L}{\beta c_1^L} = a$, and the resource constraint is $c_2^L = a(\lambda AK_0 - c_1^L)$. Therefore $c_1^L = \lambda c_1^*$ and $c_2^L = \beta a c_1^L$, implying $p_1^L c_1^L = z\lambda M$ and $p_2^L c_2^L = \beta z\lambda M$. The budget constraint for consumers and $T_2 = M + \Delta M$ imply

$$\Delta M = \{2\beta - (1 + \beta)z\lambda\}M. \quad (22)$$

In the equilibrium where $i_1 = 0$, the values of withdrawal W and cash reserves R_1 are indeterminate, but satisfies $R_1 > W$ and $W + M - R_1 \geq z\beta\lambda M$. We have shown that for each z there exists an equilibrium in which $i_1^L = 0$, and moral hazard occurs. (End of Proof)

The above propositions state that in the equilibrium under the unlimited liquidity provision policy, moral hazard is induced for banks, and the aggregate productivity declines from A to a . Thus the welfare effect of the liquidity provision policy is same as that of a deposit guarantee policy: the policy can prevent the inefficiency of a fire sale (or liquidation) of bank assets, but it inevitably induces moral hazard for banks.

4.3 Bank Recapitalization

Our interest is in determining whether there exists an optimal policy that can prevent both fire sales and moral hazard. Since moral hazard is caused by the commitment to provide an unlimited supply of goods or liquidity when banks run short, we should consider the type of policies in which the government supplies a fixed amount of resources to the banks before withdrawals occur and declares that it will not supply additional resources *ex post*.

In this section we examine bank recapitalization through the infusion of public funds from this point of view. In our model, we can consider two types of bank recapitalization policy. One is an approach that transfers value from consumers to banks by monetary policy or seigniorage. In this case the government creates cash ΔM and gives it to the banks at date 1, and collects all the cash in the economy ($M + \Delta M$) by taxation on consumers ($T_2 = M + \Delta M$) at date 2. The other type of recapitalization policy is an approach that transfers value by fiscal policy. There are various different fiscal measures that can be used to transfer value, but it can be easily understood that they result in the same welfare effects within our simple model. We examine the case in which the government issues bonds (B), gives B to the banks at date 1, and redeems the bonds by taxation on consumers ($T_2 = M + (1 + i_1)B$).

We denote the variables in the equilibrium under the bank recapitalization policy with superscript C : prices (p_1^C, p_2^C, i_1^C) and the allocation (c_1^C, c_2^C) .

4.3.1 Case 1: Recapitalization by Monetary Policy

We examine the first type of recapitalization policy. At date 1, the government creates cash ΔM and gives it to the banks before withdrawals occur. The government declares that it will not provide any additional resources to the banks during or after the withdrawals of date 1. The government levies the tax $T_2 = M + \Delta M$ on consumers at date 2. We call this policy “monetary recapitalization” in the following. We prove the following proposition.

Proposition 6 *Suppose that $\pi \approx 0$ and $x > \frac{2}{3}$. If the government implements monetary recapitalization that satisfies $\Delta M \geq \max\{(2\beta - (1 + \beta)\lambda)M, (\beta - \frac{1}{2})M\}$, the equilibrium interest rate becomes zero, and moral hazard inevitably occurs for banks.*

(Proof) There are four cases¹⁰ for the equilibrium interest rate and price: (Case 1) $i_1^C = 0$ and $p_1^C \leq p_1^*$, (Case 2) $i_1^C = 0$ and $p_1^C > p_1^*$, (Case 3) $i_1^C > 0$ and $p_1^C \leq p_1^*$, and (Case 4) $i_1^C > 0$ and $p_1^C > p_1^*$.

Case 1. If $i_1^C = 0$ and $p_1^C \leq p_1^*$, the equilibrium outcome is same as that described in Proposition 5. Lemma 1 implies that moral hazard occurs in this case.

Case 2. Lemma 1 also implies that moral hazard occurs in an equilibrium where $i_1^C = 0$ and $p_1^C > p_1^*$, if it exists.

Case 3. We show that if $i_1^C > 0$ then p_1^C cannot be less than or equal to p_1^* . Suppose that $i_1^C > 0$. Consumers deposit their all assets in banks, and they do not hold cash. The banks hold $M + \Delta M$ units of cash as the reserves. Suppose that $p_1^C \leq p_1^*$ in this case. If $p_1^C \leq p_1^*$, no depositors withdraw their entire deposits at date 1. Therefore the budget constraint for consumers becomes $p_1^C c_1^C + D_1 \leq (1 + 2\beta)M$, and $p_2^C c_2^C + T_2 = (1 + i_1^C)D_1$. Since $i_1^C > 0$ the opportunity cost for banks to hold cash in excess of date-2 withdrawals is positive. The opportunity cost for banks to hold cash less than date-2 withdrawals is $\frac{1}{x} - (1 + i_1^C)$. Suppose that $\frac{1}{x} - (1 + i_1^C) < 0$. In this case no banks hold cash and so there is no equilibrium where $i_1^C > 0$. Thus in the equilibrium where $i_1^C > 0$, it must be the case that $\frac{1}{x} - (1 + i_1^C) \geq 0$, which implies $M + \Delta M = p_2^C c_2^C$. Since a bank's liability must be equal to its assets at date 2, $(1 + i_1^C)D_1 = p_2^C c_2^C + M + \Delta M$. The FOCs for the consumer is $p_2^C c_2^C = (1 + i_1^C)\beta p_1^C c_1^C$. All these equations imply $p_1^C c_1^C = M$. Since $c_1^C = \lambda c_1^*$ in the equilibrium where bank runs do not occur at date 1, the price $p_1^C = \frac{M}{\lambda c_1^*} > \frac{M}{c_1^*} = p_1^*$. This is a contradiction. Thus, in the equilibrium where $i_1^C > 0$, it must be the case that $p_1^C > p_1^*$.

Case 4. We show that there is no equilibrium in which $i_1^C > 0$ and $p_1^C > p_1^*$. Suppose that $i_1^C > 0$ and $p_1^C > p_1^*$ in the equilibrium. In this case, all depositors try to withdraw their entire deposits $(1 + 2\beta)M$. If $\Delta M \geq 2\beta M$, the outcome is the same as that described in Proposition 4. Thus in the case where $\Delta M \geq 2\beta M$, there is no equilibrium. (Note that Proposition 5 implies that the equilibrium with $i_1^C = 0$ exists only if $\Delta M < 2\beta M$.) If $(\beta - \frac{1}{2})M \leq \Delta M < 2\beta M$, the banks can pay $(2\beta + 1)M$ to the depositors at date 1 by selling a portion of their assets in a fire sale. In this case, the aggregate budget constraint for the consumers is similar to (21) since

¹⁰Note that there is no disintermediation, since $M + \Delta M \geq (\beta + \frac{1}{2})M$, which implies that banks can repay all depositors in full if they sell (a portion of) their assets in a fire sale.

$i_1^C > 0$:

$$\begin{aligned} p_1^C c_1^C + D_1 &\leq (2\beta + 1)(\pi p_1^C c_1^* + (1 - \pi)M), \\ p_2^C c_2^C + T_2 &\leq (1 + i_1^C)D_1. \end{aligned} \quad (23)$$

In this equilibrium it must be the case that

$$\frac{1}{x} - (1 + i_1^C) \geq 0, \quad (24)$$

since otherwise no banks hold cash. We will clarify the condition for (24) later. Assuming that (24) is satisfied, we have $p_2^C c_2^C = M + \Delta M$. The FOC for the consumer is $p_2^C c_2^C = \beta(1 + i_1^C)p_1^C c_1^C$.

Therefore

$$p_1^C c_1^C = \left\{ \frac{p_1^C}{p_1^*} \pi + 1 - \pi \right\} M. \quad (25)$$

Since ΔM is predetermined, the withdrawal of $(1 + 2\beta)M$ causes a partial fire sale of the bank assets, which produces a deadweight loss of $\frac{(1-x)(2\beta M - \Delta M)}{xp_1^C}$ units of the consumer good. Therefore in this equilibrium

$$c_1^C = \frac{1}{1 + \beta} \left\{ \lambda AK_0 - \frac{(1-x)(2\beta M - \Delta M)}{xp_1^C} \right\}. \quad (26)$$

The equations (25) and (26) imply

$$\frac{p_1^C}{p_1^*} = \frac{1 - \pi}{\lambda - \pi} + \frac{1 - x}{x} \frac{2\beta M - \Delta M}{(1 + \beta)M(\lambda - \pi)}. \quad (27)$$

Now we examine the condition for (24). Since equation (27) and $p_2^C c_2^C = \beta(1 + i_1^C)p_1^C c_1^C$, (24) is rewritten as

$$\frac{\Delta M}{M} \leq \frac{\frac{1-\pi}{\lambda-\pi}\lambda + \frac{1-x}{x} \frac{2\beta\pi}{(1+\beta)(\lambda-\pi)} - \frac{x}{\beta}}{\frac{x}{\beta} + \frac{1-x}{x} \frac{\pi}{(1+\beta)(\lambda-\pi)}}. \quad (28)$$

Since the condition (28) does not hold for ΔM that satisfies $(\beta - \frac{1}{2})M \leq \Delta M < 2\beta M$ if $\pi \approx 0$ and $x > \frac{2}{3}$, there exists no equilibrium in which $i_1^C > 0$ and $p_1^C > p_1^*$.

The above analysis on Cases 1–4 implies that the equilibrium interest rate under monetary recapitalization must be zero, and thus moral hazard occurs in the equilibrium. (End of Proof)

The analysis on Case 4 implies that monetary recapitalization provides too much cash so that $\frac{1}{x} - (1 + i_1) < 0$ holds and banks are unwilling to hold cash reserves. To have $\frac{1}{x} - (1 + i_1) > 0$, the government must supply insolvent banks non-cash assets.

4.3.2 Case 2: Recapitalization by Fiscal Policy

The second type of recapitalization is the following: At date 1, after the shock λ hits the economy, the government issues bonds amounting to B , and gives this amount to

the banks before withdrawals occur. The government declares that it will not provide any additional resources to the banks during or after the withdrawals of date 1. The government levies the tax $T_2 = M + (1 + i_1^C)B$ on consumers at date 2. We call this policy “fiscal recapitalization.”¹¹ We assume that the government bond B has the same return profile as a bank deposit, and that the sale of the bonds does not result in a deadweight loss, unlike a fire sale of consumer goods.

Assumption 13 *The government bond B issued at date 1 yields $(1 + i_1^C)B$ units of cash at date 2. The bond B is exchangeable for B units of cash at date 1. When the banks sell the bond at date 1 and pay the proceeds of the bond sale to the depositors on the same date, they do not incur the deadweight loss as associated with a fire sale of consumer goods.*

This assumption states that government bonds and bank deposits are equivalent assets for consumers. The latter part of the assumption is just for simplicity of exposition: we can derive qualitatively identical results even if a fire sale of the bonds generates a deadweight loss.

Proposition 7 *There exists a fiscal recapitalization policy that supports the optimal equilibrium in which $p_1^C \leq p_1^*$, $i_1^C > 0$ and both fire sales of the goods and moral hazard for the banks are prevented.*

(Proof) It is sufficient to examine the case where $i_1^C > 0$ (Lemma 1). We derive the conditions for B that induce the optimal equilibrium, where fire sales and moral hazard are prevented.

We derive the condition for $p_1^C \leq p_1^*$ in the equilibrium where $i_1^C > 0$. Suppose that p_1^C exceeds p_1^* . Then all depositors try to withdraw $(2\beta + 1)M$. Since the banks’ cash reserve is M , and the banks can obtain cash M by selling all their assets, the total cash that the banks can pay to the depositors is $2M$, which is less than $(2\beta + 1)M$. Therefore if $p_1^C > p_1^*$, the banks go

¹¹There are several other fiscal measures to recapitalize the banks. For example, the government can tax consumers at date 1 and transfer the tax revenue (i.e., cash or goods) to the banks on the same date, or it can issue bonds, sell them to consumers, and give the proceeds of the bond sale (i.e., cash) to the banks on date 1. It is easily confirmed that these policies have the same welfare effect as the above fiscal recapitalization policy.

bankrupt, and p_1^C is determined by

$$p_1^C = \frac{M}{\lambda x A K_0 + \frac{B}{p_1^C}}. \quad (29)$$

Since no production technology is available in this case (see Assumption 12), the equilibrium after disintermediation would be the solution to

$$\max_{c_1, s, M_1, B_1, c_2} u(c_1) + \beta u(c_2)$$

subject to

$$\begin{cases} p_1^C c_1 + p_1^C s + M_1 + B_1 \leq p_1^C \lambda x A K_0 + B + M, \\ p_2^C c_2 + T_2 \leq p_2^C s + M_1 + (1 + i_1^C) B_1, \\ p_2^C c_2 \leq M_1, \end{cases} \quad (30)$$

where $T_2 = M + (1 + i_1^C)B$. In the disintermediated economy, the nominal interest rate i_1^C is set to satisfy $1 + i_1^C = \frac{p_2^C}{p_1^C}$ since the only asset is the consumer good that is stored; one unit of cash is transformed into $\frac{1}{p_1^C}$ units of the consumer goods at date 1, whose return is $\frac{p_2^C}{p_1^C}$ units of cash at date 2. As in the case of equilibrium after debt deflation (page 23), it is easily shown that $p_2^C = \infty$ and $(c_1, s, c_2, M_1, B_1) = (\lambda x A K_0, 0, 0, M, B)$ in the equilibrium after disintermediation; each consumer stores cash (M) and bonds (B), and pays them to the government as the tax $T_2 = M + (1 + i_1^C)B = \infty$.

If the government sets B such that

$$B \geq \{1 - \lambda x(1 + \beta)\}M, \quad (31)$$

then the equation (29) implies that $p_1^C = \frac{M-B}{\lambda x A K_0} \leq p_1^*$. Therefore, if the government sets B to satisfy (31), the price in disintermediation (p_1^C) must be no greater than p_1^* . So we assume that the government sets B in such a way. Since $p_1^C \leq p_1^*$ in this case, there are no depositors who make runs on banks in the equilibrium. Thus the aggregate budget constraint is

$$\begin{aligned} p_1^C c_1^C + D_1 &\leq (1 + 2\beta)M, \\ p_2^C c_2^C + T_2 &\leq (1 + i_1^C)D_1. \end{aligned} \quad (32)$$

Since $i_1^C > 0$, the opportunity cost for the banks to hold cash reserves in excess of the date-2 withdrawal ($p_2^C c_2^C$) is $i_1^C > 0$, while the opportunity cost for the banks to hold cash reserve less than $p_2^C c_2^C$ is $\frac{1}{x} - (1 + i_1^C)$. We will verify later that

$$\frac{1}{x} - (1 + i_1^C) > 0. \quad (33)$$

Assuming that (33) holds, we have $p_2^C c_2^C = M$. The FOC for the consumers implies $p_2^C c_2^C = \beta(1 + i_1^C) p_1^C c_1^C$. Since $T_2 = M + (1 + i_1^C)B$, we have

$$p_1^C c_1^C = M - \frac{B}{2\beta + 1}. \quad (34)$$

Since there is no fire sale when $p_1^C \leq p_1^*$, the FOCs and the resource constraint imply $c_1^C = \frac{1}{1+\beta} \lambda A K_0 = \lambda c_1^*$. If the government sets

$$B \equiv (2\beta + 1)(1 - z)M \geq (2\beta + 1)(1 - \lambda)M, \quad (35)$$

the equilibrium price $p_1^C = \frac{z}{\lambda} \frac{M}{c_1^*} = \frac{z}{\lambda} p_1^* \leq p_1^*$. We have shown that by setting $z \leq \lambda$, the government can induce a situation where $p_1^C \leq p_1^*$ and prevent fire sales.

We will now examine the condition for z (or B) to prevent moral hazard. Suppose that a bank offers its depositors to renegotiate at date 1 after it forms its capital of $\lambda A K_0 - c_1^C$. The bank's liability is $D_1 = (1 + 2\beta)M - p_1^C c_1^C = (1 + 2\beta)M - zM$, while its assets are M units of cash, B units of the bond, and $\lambda A K_0 - c_1^C = \beta c_1^C$ units of capital. The amount of cash that the bank can collect by selling the bonds and the goods in a fire sale is $M + B + x p_1^C \beta c_1^C = M + (1 + 2\beta)(1 - z)M + \beta x z M$. This amount is less than the bank's liability if

$$1 \leq z\beta(2 - x). \quad (36)$$

If (36) holds, the bank's offer of renegotiation results in the bankruptcy of the bank at date 1. Therefore the banks never try to renegotiate if (36) holds. In this case $c_2^C = \beta A c_1^C = \beta A \lambda c_1^*$, $p_2^C = \frac{M}{c_2^C}$, and $1 + i_1^C = \frac{1}{\beta z} \leq 2 - x$. Now we verify (33): $\frac{1}{x} - (1 + i_1^C) \geq \frac{1}{x} + x - 2 > 0$ for $0 < x < 1$. Thus if B satisfies (31), (35), and (36), then the equilibrium in which $i_1^C > 0$ is uniquely determined for a fixed value of B . The sufficient condition for the existence of such B is

$$0 < Z \equiv \min \left\{ \frac{\lambda x(1 + \beta) + 2\beta - 1}{1 + \beta x}, \frac{3\beta - \frac{1}{2} + \lambda x(1 + \beta)}{1 + 2\beta} \right\}. \quad (37)$$

Obviously this condition (37) is satisfied, since the discount factor β is larger than $\frac{1}{6}$. Therefore the government can pick z that satisfies $0 \leq z < Z$, and set $B = (1 + 2\beta)(1 - z)M$. In this equilibrium, both the fire sale and moral hazard are prevented, and the optimal consumption allocation is realized. (End of Proof)

This result appears to be quite different from that of Diamond and Rajan (2002b), who claim that recapitalization of failing banks during a financial crisis may worsen the crisis. Diamond and Rajan (2002b) seem to use the term "recapitalization" to represent

a policy that transfers liquidity to an insolvent bank from other banks. Thus in their model, the amount of aggregate liquidity does not increase at date 1, and therefore bank bailouts result in more inefficient liquidation of bank assets. This outcome seems similar to that of a deposit guarantee in our model (Lemma 2).

In the fiscal recapitalization policy of our model, however, the government can create aggregate liquidity when it recapitalizes insolvent banks by issuing government bonds, which can be sold in the market at date 1: The government bond in our model is accepted as a liquid asset by consumers. Thus in our model, fiscal recapitalization solves the problems of both insolvency and illiquidity. Diamond and Rajan (2002a) also state that capital injections can provide better outcomes when combined with injections of liquidity.

Propositions 5 and 6 imply, however, that cash injections alone are not sufficient to attain the optimal allocation. This is because the increase in the money supply distorts the decision making of banks through the cash-in-advance constraint, while fiscal recapitalization can avoid this distortion.¹²

5 Conclusion

When the economy is hit by a macroeconomic shock that makes the banking system insolvent and thus raises the risk of debt deflation, different policy responses have quite different welfare effects.

A deposit guarantee, unlimited liquidity support, and monetary recapitalization generate inefficient outcomes. Fiscal recapitalization is the optimal policy that can prevent both moral hazard (inefficient use of capital) at banks and the premature liquidation of bank assets. In crisis-hit countries, policies of temporization, such as a deposit guarantee and unlimited liquidity support, are often adopted. Our results imply that a deposit

¹²Boyd, Chang, and Smith (2000) argue that to monetize the cost of bank bailouts is better than taxation. In their model, cash is introduced by the reserve requirement imposed by the government, and it does not induce moral hazard for banks as it does under a zero nominal interest rate in our model (See Lemma 1). Therefore, monetization is not inefficient in their model, while it is in ours.

guarantee and unlimited liquidity support may give rise to a combination of mild deflation and low interest rates (Propositions 3, 5, and 6). Our results also imply that temporizing without recapitalizing insolvent banks causes a larger welfare loss. These implications are consistent with the observations on recent financial crises that we refer to in Section 1.

Let us examine the Japanese economy of the 1990s in our theoretical framework.¹³ Although Japan experienced a full-fledged crash of its asset-price bubble at the beginning of the 1990s, the government did not begin to recapitalize major banks until 1998; during the 1990s the government insisted that it would never allow the occurrence of a bank closure; meanwhile, the central bank has kept the short-term interest rate at zero since 1995. This attitude of the Japanese government can be interpreted as employing the policy of a deposit guarantee¹⁴ and liquidity support.¹⁵ The decade-long stagnation of the Japanese economy seems consistent with the predictions of our model. The deflation since the late 1990s can also be interpreted as a result of the implementation of a deposit guarantee without sufficient recapitalization.

In short, Japan's prolonged recession and current deflation may have been caused by an inappropriate policy response to bank insolvency: temporization with "too little and too late" recapitalization.

¹³We can easily generalize this two-period model into a multi-period model preserving the intuition so that the basic implication of the model can be applied to the decade-long recession of the Japanese economy.

¹⁴Although Japan established a deposit insurance system in the 1970s that guarantees up to 10 million yen for any single depositor, there was a tacit understanding that the government would never let any bank close, meaning in practice that deposits were guaranteed without limit. In 1995, the government declared explicitly that it would guarantee all deposits without limit for the time being. This unlimited deposit guarantee has still not been eliminated as of 2003.

¹⁵The monetary easing by the Bank of Japan was very aggressive, but it turned out that the speed and the scale of the monetary policy were insufficient. The BOJ's intention was to provide temporary liquidity support but not to subsidize the insolvent banks by seigniorage. Therefore, the monetary policy in the 1990s was insufficient for the restoration of bank solvency.

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