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Abstract

This paper studies the dynamic behavior of the general price level when the natural rate of interest declines substantially. Particular attention is paid to two constraints: the non-negativity constraint of nominal interest rates, and the government's intertemporal budget constraint. In a normal situation, nominal bond prices rise in response to the shock, which restores equilibrium. However, if the non-negativity constraint is binding, nominal bond prices cannot rise sufficiently. Equilibrium can then be restored only by a sufficient fall in the current price level. The required fall is greater when the maturity of government debt is shorter. To avoid deflation, the government must coordinate with the central bank by committing itself to reducing the current and future primary surplus.

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1 Introduction

Recent developments of the Japanese economy are characterized by the concurrence of two rare phenomena: zero interest rates and deflation. The uncollateralized overnight call rate has been practically zero since the Bank of Japan (BOJ) policy board made a decision in February 1999 to lower it to be "as low as possible." On the other hand, the year-on-year CPI inflation rate has been below zero for more than four years since the second quarter of 1998. The purpose of this paper is to study the connection between these two phenomena. More specifically, we investigate how and under what circumstances a central bank fails to stop deflation due to the presence of the zero bound on nominal interest rates.

At the center of our analysis are two constraints: the non-negativity constraint of nominal interest rates, and the government's intertemporal budget constraint. The importance of the first constraint in discussing deflation was first pointed out by Krugman (1998), who argued that the Japanese economic problem stems from a decline in the natural rate of interest to a negative level.[1] To illustrate his basic idea, we denote the current price level by P_0 and (the expectation of) the future price level by P_1. Given that the nominal interest rate cannot fall below zero, a decline of the natural rate of interest to a negative level implies that P_1/P_0 must rise to remain consistent with the Fisher equation. There are two ways of achieving this: lowering P_0 or raising P_1. The former adjustment, according to Krugman, has been happening in the Japanese economy since the mid-1990s.

It is unquestionable that P_1/P_0 must rise in response to a decline of the natural rate of interest to a negative level. However, it is not so clear how this required change in the relative price is divided between P_1 and P_0. In this respect, Krugman argues that (the expectation of) P_1 is sticky, so that almost all adjustment must take place in P_0 (Krugman (1998), pp.@@). Generally speaking, however, there is no a priori reason to believe that P_1 is sticky and P_0 is not. Rather, it is natural to think that both variables are sticky; moreover, the possibility that P_0 is stickier than P_1 cannot be denied.

An alternative way to determine how the required change in the relative price is divided between P_1 and P_0 is to use the government's intertemporal budget constraint, which is the second constraint that we will pay particular attention to in this paper. Sargent and

[1] The idea that the Japanese problem is rooted in a decline of the natural rate of interest below zero is shared by several researchers including Woodford (1999c). As shown later, however, a negative natural rate of interest is not a necessary condition for the non-negativity constraint on nominal interest rates to be binding.

Wallace's (1981) "unpleasant monetarist arithmetic" points out that the government's budget constraint implies an intertemporal restriction on the inflation rate in each period, and their idea is extended to the fiscal theory of the price level (FTPL). One of the most important assumptions in this line of research is that the government does *not* adjust expenditures and taxes so that its intertemporal budget constraint is satisfied. This is what Leeper (1991) calls "active" fiscal policy. We will see later that the division of the required change in the relative price between P_1 and P_0 is endogenously and uniquely determined under the assumption of active fiscal policy.

There are some criticisms against the fiscal theory, two of which are closely related to our analysis. First, several empirical researches report that actual data are not necessarily consistent with the fiscal theory, particularly with the assumption of active fiscal policy. For example, Canzoneri et al. (2001) find a regularity in postwar U.S. data, that a positive innovation in the primary surplus causes a rise in a future surpluses and a fall in future liabilities, arguing that the regime of passive fiscal policy offers a straightforward interpretation of this regularity, whereas the regime of active fiscal policy provides a rather implausible explanation. On the other hand, Cochrane (1998, 2001) and Woodford (2001) provide fiscal interpretations of the U.S. inflation, while Loyo (1999) argues that the Brazilian inflation of the 1980s can be explained by the fiscal theory. Given that the empirical evidence is mixed, it would not be appropriate to stick to the assumption of active fiscal policy. Our basic strategy is to extract as many implications as possible from the government's budget constraint, independently of whether fiscal policy is active or passive. More specifically, we will investigate what the government's budget constraint implies for the response of the price level to a substantial fall in the natural rate of interest under the assumption of active fiscal policy. At the same time, we will look at the other side of the coin by asking what type of fiscal adjustment is needed to maintain price stability even when the natural rate of interest declines substantially.[2]

Second, Gordon and Leeper (2002) argue that an important necessary condition for the fiscal theory to hold is that nominal bond prices are pegged by a central bank. Otherwise,

[2] With respect to this, Krugman (2000) states, "We assume ... that any implications of the [open market] operation for the government's budget constraint are taken care of via lump-sum taxes and transfers" (Krugman (2000), p.@@@). Here the government is assumed to adjust the primary surplus so that the intertemporal budget constraint is satisfied by *any* sequences for non-fiscal endogenous variables including the price level. This is the kind of fiscal policy termed "passive" by Leeper (1991).

it would be possible that nominal bond prices, rather than the general price level, make a necessary adjustment in response to shocks.[3] Based on this understanding, Gordon and Leeper maintain that the fiscal theory should play a larger role in determining the price level during "periods when the central bank supports bond prices (as it might during wars)".[4] Turning to the Japanese current situation, the BOJ does not officially adopt bond-price support policy, but instead has been committing itself to continuing a zero interest rate policy (ZIRP) until "the consumer price index registers a stable year-on-year increase of zero percent or more." The policy intention behind this commitment is to spread the zero overnight call rate to longer-term nominal interest rates, including Japanese Government Bond rates (see, for example, Ueda (2000)).[5] In this sense, the BOJ's ZIRP could be seen as a weaker form of bond-price support policy, which would make it more likely that fluctuations in the price level are accounted for by the fiscal theory.

Our main findings are as follows. First, the non-negativity constraint on nominal interest rates becomes binding (or equivalently, nominal bond prices hit their upper bounds) when the natural rate of interest exhibits a substantial decline in terms of magnitude and persistence. In that case, equilibrium can be restored only by a sufficient fall in the current price level. Note that a negative natural rate of interest is not a necessary condition for the non-negativity constraint to be binding, which sharply contrasts with previous studies on liquidity traps. Second, a central bank's commitment to continuing a ZIRP for a long time is effective in weakening the downward pressure on the current price level, although it cannot eliminate the pressure completely. A side effect of this commitment is that it creates another downward pressure on the price level in future periods. In this sense, the best a central bank can do through this commitment is to postpone deflation to future periods. This implies that targeting a higher future price level creates an additional downward pressure on the current price level. Third, deflation in the current period can be avoided if the government coordinates

[3]This point cannot be see in typical FTPL models in which government bonds mature in one period. It was first observed by Cochrane (2001), who extended the fiscal theory to an economy with a rich maturity of government bonds.

[4]A famous episode of bond-price pegging is U.S. monetary policy from 1942 up until the Treasury-Fed Accord of 1951. Prior to that agreement, the Fed maintained a ceiling of 2 and 1/2 percent on 25-year Treasury bonds for nearly a decade. Woodford (2001) gives a fiscal explanation for fluctuations in the price level during this period.

[5]In the discussion of what the Fed should do if captured by a liquidity trap, Bernanke (2002) states that the BOJ's commitment is an "indirect" method, and shows his preference for announcing explicit ceilings for yields on longer-maturity Treasury debt.

with the central bank by committing itself to reducing the current and future primary surplus.

The rest of the paper is organized as follows. Section 2 derives a price level equation. Section 3 conducts a comparative statics analysis: we ask how and under what circumstances does the non-negativity constraint on nominal interest rates becomes binding, and investigate the roles of monetary and fiscal policies. Section 4 discusses the optimal monetary and fiscal policies in a liquidity trap by solving an intertemporal loss minimization problem. Section 5 concludes the paper.

2 Determination of the Price Level

This section will derive a price level equation, with the equilibrium price level on the left, and other exogenous variables, including the nominal interest rate and the natural rate of interest, on the right.

We will adopt the basic framework of FTPL developed by Leeper (1991), Sims (1994) and Woodford (1994, 1995, 1996, 1998a, 1998b), including the assumption of perfectly flexible prices,[6] but deviate from it in the following respects. First, our main interest is in deflation, particularly deflation under a liquidity trap, whereas most of the existing FTPL studies deal with inflation.[7] Second, we are interested in changes in the natural rate of interest as an exogenous shock to the economy, whereas existing FTPL studies typically analyze the effects of fiscal shocks such as changes in the government primary surplus.[8] Third, we allow for the existence of long-term government debt. The existing FTPL studies typically assume that there is only short-term (one-period) government debt and that the government rolls it over. In this simplified maturity structure of government debt, the price level is determined by the ratio between two variables: the value of nominal debt, which is issued in the previous period and matures at the end of the current period, and the present value of primary surpluses. Because neither of these variables is directly affected by any changes in the future path of

[6] The companion paper, Iwamura, Jung and Watanabe (2003), attempts to extend the discussion to an economy with sticky prices.

[7] An exception is Woodford (2001), who gave a fiscal explanation for deflation in the U.S. over the 1948-50 period. Woodford attributed deflation in this period to an improvement of U.S. government budget resulting from the end of wartime deficits. Also, Woodford argued that the abandonment of the bond price-support regime after 1951 had a downward pressure on the price level.

[8] Some of the FTPL studies discuss the impacts of interest rate shocks on the price level (for example, Canzoneri and Diba (2000) and Woodford (2001)); however, somewhat surprisingly, little has been said about the effects of changes in the natural rate of interest.

short-term nominal interest rates, a central bank's commitment regarding the future course of monetary policy has no role in determining the price level. On the other hand, in an economy with long-term government bonds, a change in the future path of short-term nominal interest rates alters the market values of the existing long-term bonds, thereby affecting the current price level. Cochrane (2001) extends the fiscal theory to an economy with a rich maturity structure of government debt, which will be used in this paper with some modifications.

2.1 The Natural Rate of Interest

Consider an exchange economy, in which each household receives a single non-storable endowment each period. We denote equilibrium marginal utility by $u'(c)$, and define real discount factor $D_{t,t+j}$ as

$$D_{t,t+j} \equiv \beta^j \frac{u'(c_{t+j})}{u'(c_t)}, \tag{2.1}$$

where β is the consumer's subjective discount factor. Denote the endowment in period t by e_t, and autonomous variation in spending not motivated by intertemporal optimization by ν_t.[9] The market clearing condition is expressed by $c_t = e_t - \nu_t$.[10] Substituting the market clearing condition into equation (2.1) yields

$$D_{t,t+j} = \beta^j \frac{u'(e_{t+j} - \nu_{t+j})}{u'(e_t - \nu_t)}, \tag{2.2}$$

which shows how the real discount factor is determined by the two exogenous disturbances to the economy, e_t and ν_t. Changes in e_t could be interpreted as a supply shock, while changes in ν_t could be labeled as a demand shock.

By defining $1 + r_t \equiv 1/D_{t,t+1}$, we have a convenient expression

$$D_{t,t+j} = \frac{1}{(1+r_t)(1+r_{t+1}) \times \cdots \times (1+r_{t+j-1})},$$

[9] For example, a temporary and autonomous decline in aggregate expenditure that is triggered by, say, debt-overhang in the corporate sector, is interpreted as ν_t. A similar assumption about autonomous changes in spending not motivated by intertemporal substitution is adopted by Woodford (1999) and Jung et al. (2001) among others.

[10] Here we assume that the public sector (the government plus the central bank) does not "eat" goods, so that the market clearing condition is not affected by their behavior. To be more specific, the public sector's activity in our model is to collect lump-sum taxes and transfer them to the private sector. Woodford (2001) adopts a utility function in which government purchases and private consumption are perfect substitutes, which also neutralizes the public sector's activities.

where r_t denotes the equilibrium real interest rate in period t, often called the natural rate of interest in recent studies on monetary policy rules, such as Woodford (1999a) among many others.

2.2 Government's Budget Constraint

We assume that the government issues zero-coupon nominal bonds, each of which pays one yen when it matures. Let $B_{t,t+j}$ denote the face value of bonds at the end of period t that will come due in period $t+j$, and $Q_{t,t+j}$ denote the nominal market price in period t of a bond that matures in period $t+j$.

The accounting identity for a consolidated body consisting of the government and the central bank can be expressed as

$$B_{t-1,t} - \sum_{j=1}^{\infty} Q_{t,t+j} \left[B_{t,t+j} - B_{t-1,t+j} \right] - \left[M_t - M_{t-1} \right] = P_t s_t, \tag{2.3}$$

where M_t is the nominal value of the base money outstanding at the end of period t, P_t is the price level, and s_t is the real primary surplus, which is defined as tax revenues less government expenditures. The first term on the left-hand side represents the amount of repayment for bonds that mature in period t. This is partly financed by the primary surplus in period t, but the government and the central bank must issue new liabilities to the extent that the amount of bond repayment exceeds the primary surplus. This is captured by the second and the third terms on the left-hand side of (2.3). The term $B_{t,t+j} - B_{t-1,t+j}$ represents the change from the previous period in the face value of bonds that mature in period $t+j$, namely, an amount of new issue in period t. These new bonds are issued at the market price in period t, $Q_{t,t+j}$. The third term represents nominal seigniorage.

Nominal bond prices must satisfy:

$$Q_{t,t+j} = P_t E_t \left[D_{t,t+j} \frac{1}{P_{t+j}} \right]. \tag{2.4}$$

Iterating forward (2.3), substituting (2.4), and imposing the transversality condition yields an intertemporal budget constraint of the form

$$\frac{\sum_{j=0}^{\infty} Q_{t,t+j} B_{t-1,t+j} + M_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} D_{t,t+j} \left(s_{t+j} + \sigma_{t+j} \right), \tag{2.5}$$

where σ_t represents the real value of seigniorage in period t. The real seigniorage is defined by

$$\sigma_t \equiv (1 - Q_{t,t+1}) \frac{M_t}{P_t} = (1 - Q_{t,t+1}) V(Q_{t,t+1}), \tag{2.6}$$

where $V(Q_{t,t+1})$ denotes the real money demand.[11] The left-hand side of (2.5) represents the amount of liability outstanding in period t (including the base money), while the right-hand side represents the present value of all future surpluses (including the seigniorage). Equation (2.5) simply says that these two must coincide in equilibrium.

Nominal bond prices are determined in the following way. We suppose that a central bank makes a credible commitment regarding the future path of short-term nominal interest rates, and that bond prices are determined by the expectations theory of the term structure. That is,

$$Q_{t,t+j} = E_t \left[\frac{1}{(1+i_t)(1+i_{t+1}) \times \cdots \times (1+i_{t+j-1})} \right],$$

where i_t denotes the one-period nominal interest rate. According to equation (2.6), the real seigniorage is also determined by the central bank's commitment regarding the path of short-term nominal interest rates. Then, under the assumption that the stream of the primary surplus is not affected by any change in the price level, i.e., the assumption of active fiscal policy, equation (2.5) can be read as an equation that determines the price level P_t.

2.3 Price Level Equation

We are interested in how the equilibrium price level determined in this way would change in response to changes in the exogenous variables, especially changes in expectations about the future path of the natural rate of interest. As a preliminary step, this subsection linearizes equation (2.5) to obtain a simple price level equation. Following the methodology adopted by Cochrane (2001) and Woodford (1996) among others, we first specify a baseline path of the variables, and then take a log-linear approximation of the variables around the baseline.

[11] Utility maximization using a standard money-in-the-utility-function framework, in combination with the assumption of additive separability between goods consumption and real money balances, yields a consumption Euler equation, such as (2.1), as well as a money demand equation in which money demand depends on short-term nominal interest rate and income. The functional form we assume here, $V(Q_{t,t+1})$, can be interpreted as a simplified version of the money demand equation obtained in this way. The analysis in this paper will not change much, even if we use a money demand equation in which money demand depends on income as well as on short-term nominal interest rates.

The baseline path is specified as follows. With respect to the maturity structure of government debt, we assume

$$\frac{B^*_{t-1,t+j}}{B^*_{t+j-1,t+j}} = \theta^j \leq 1 \qquad \text{for} \quad j = 1, 2, \cdots, \tag{2.7}$$

where θ is a parameter satisfying $0 \leq \theta \leq 1$. We use $*$ to indicate the baseline path of a variable. The term $B^*_{t-1,t+j}$ represents the face value of bonds at the end of period $t-1$ that mature in period $t+j$, and $B^*_{t+j-1,t+j}$ represents the face value of the same type of bonds just before redemption in period $t+j$. Equation (2.7) simply states that the government issues additional bonds, which mature in period $t+j$, at a rate θ in each period between t and $t+j-1$. It also says that, as long as θ is strictly less than unity, the ratio on the left-hand side of the equation declines as j increases, which means that the amount of bonds that mature in the remote future is small relative to the total amount of redemption. Note that $\theta = 0$ corresponds to the case in which all bonds mature in one period, while $\theta = 1$ corresponds to the case in which additional bonds are not issued at all.

With respect to s^*_t, P^*_t, $Q^*_{t,t+j}$, and $D^*_{t,t+j}$, we assume the following

$$s^*_t = s^* > 0; \quad P^*_t = P^*; \quad Q^*_{t,t+j} = \beta^j; \quad D^*_{t,t+j} = \beta^j.$$

Note that the inflation rate is assumed to be zero on the baseline path. It follows immediately from these assumptions that

$$V^*_t = V(\beta); \quad M^*_t = V(\beta)P^*; \quad \sigma^*_t = \sigma^* \equiv (1-\beta)V(\beta) > 0.$$

Finally, by substituting the above assumptions into equation (2.5), we obtain the government's intertemporal budget constraint on the baseline

$$\frac{B^*_{t-1,t}}{P^*_t} = \frac{1-\beta\theta}{1-\beta} s^*. \tag{2.8}$$

Note that variables related to the central bank's activities do not appear in this equation. This is a direct reflection of the assumption that the inflation rate is zero on the baseline. Under this assumption, the central bank's liability coincides each period with the present value of the seigniorage stream (i.e., $M^*_{t-1}/P^*_t = \sum_{j=0}^{\infty} \beta^j \sigma^*_{t+j}$), so that there are no transfers between the central bank and the government. Also, note that equation (2.8), together with (2.7), indicates that the amount of redemption is constant over time. Then, the average maturity of bonds is also constant over time, which is given by $1/(1-\theta)^2$ $(= 1 \times \theta^0 + 2 \times \theta^1 + 3 \times \theta^2 + \cdots)$.

Taking derivatives of (2.5) around the baseline path, we obtain

$$\sum_{j=0}^{\infty} \frac{Q^*_{t,t+j} B^*_{t-1,t+j}}{P^*_t} \left[\hat{Q}_{t,t+j} - \hat{P}_t + \hat{B}_{t-1,t+j}\right] + \frac{P^*_{t-1}}{P^*_t} V^*_{t-1} \left[\hat{V}_{t-1} + \hat{P}_{t-1} - \hat{P}_t\right]$$
$$= \sum_{j=0}^{\infty} \left\{ D^*_{t,t+j}(s^*_{t+j} + \sigma^*_{t+j}) E_t \hat{D}_{t,t+j} + D^*_{t,t+j}\left[s^*_{t+j} E_t \hat{s}_{t+j} + \sigma^*_{t+j} E_t \hat{\sigma}_{t+j}\right]\right\}, \qquad (2.9)$$

where a variable with a hat represents the proportional deviation of the variable from its value on the baseline path. For example, $\hat{P}_t$ is defined as $\hat{P}_t \equiv \ln P_t - \ln P^*_t$. Substituting the above specifications about the baseline path into equation (2.9), we obtain

$$\begin{aligned} \hat{P}_t - \omega \hat{P}_{t-1} &= (1-\beta\theta)(1-\omega)\left[\hat{Q}_t + \hat{B}_{t-1}\right] \\ &- (1-\beta) E_t \hat{D}_t + \omega \hat{V}_{t-1} \\ &- (1-\beta) \sum_{j=0}^{\infty} \beta^j E_t \ \hat{y}_{t+j}, \end{aligned} \qquad (2.10)$$

where ω and $\hat{y}_{t+j}$ are defined by $\omega \equiv \sigma^*/(s^* + \sigma^*)$ and $\hat{y}_{t+j} \equiv (1-\omega)\hat{s}_{t+j} + \omega \hat{\sigma}_{t+j}$. Also, $\hat{B}_{t-1}$, $\hat{Q}_t$, and $\hat{D}_t$ are defined as

$$\hat{B}_{t-1} \equiv \sum_{j=0}^{\infty} (\beta\theta)^j \hat{B}_{t-1,t+j}; \quad \hat{Q}_t \equiv \sum_{j=0}^{\infty} (\beta\theta)^j \hat{Q}_{t,t+j}; \quad \hat{D}_t \equiv \sum_{j=0}^{\infty} \beta^j \hat{D}_{t,t+j}. \qquad (2.11)$$

$\hat{B}_{t-1}$ and $\hat{Q}_t$ can be interpreted as a nominal debt aggregate, and an index of nominal bond prices.

We log-linearize (2.3) in the same way, substitute it into (2.10) to eliminate $\hat{B}$, and finally obtain a price level equation of the form[12]

$$\begin{aligned} \hat{P}_t - \hat{P}_{t-1} &= (1-\beta\theta)(1-\omega)\left[\hat{Q}_t - (\beta\theta)^{-1}\hat{Q}_{t-1}\right] \\ &- (1-\beta)\left[E_t \hat{D}_t - \beta^{-1} E_{t-1} \hat{D}_{t-1}\right] \\ &- \omega \ \hat{Q}_{t-1,t} \\ &- (1-\beta) \sum_{j=0}^{\infty} \beta^j (E_t - E_{t-1}) \ \hat{y}_{t+j}. \end{aligned} \qquad (2.12)$$

We can decompose $\hat{P}_t - \hat{P}_{t-1}$ into two parts. By taking innovations of equation (2.12), we

[12]See Appendix A for more details on the derivation.

obtain

$$
\begin{aligned}
(E_t - E_{t-1})\hat{P}_t &= (1-\beta\theta)(1-\omega)(E_t - E_{t-1})\hat{Q}_t \\
&- (1-\beta)(E_t - E_{t-1})\hat{D}_t \\
&- (1-\beta)\sum_{j=0}^{\infty}\beta^j (E_t - E_{t-1})\ \hat{y}_{t+j}.
\end{aligned} \tag{2.13}
$$

Subtracting (2.13) from (2.12) and substituting (2.11) yields

$$
E_{t-1}\hat{P}_t - \hat{P}_{t-1} = -\left[\hat{Q}_{t-1,t} - \hat{D}_{t-1,t}\right], \tag{2.14}
$$

which is simply a version of the Fisher equation. Equation (2.12) looks similar to Cochrane's (2001) price level equation (equation (19), p.81), but differs in that $\hat{B}_{t-1}$ does not appear in our equation and $\hat{Q}_t$ does not appear in his equation. Cochrane (2001) is mainly interested in the effects of the government's debt policy (i.e., the government's commitment regarding future debt sales and redemptions) upon the equilibrium price level. Therefore, $\hat{B}_{t-1}$ plays a major role in determining the price level, whereas bond prices are determined only in an implicit manner. In contrast, the focus of this paper is on the effects of monetary policy commitment on the equilibrium price level, so that bond prices play a major role in our setting, and debt policy is now behind the scenes.

According to equation (2.13), a change in expectations about the future path of the natural rate of interest would lead to a change in the equilibrium price level. For example, a decline in the expected natural rate of interest would produce a downward pressure on the current price level through an increase in $(E_t - E_{t-1})\hat{D}_t$. In this situation, a central bank could neutralize this pressure, at least partially, by announcing the lowering of the future path of short-term nominal interest rates. There are two transmission channels. First, current bond prices rise in response to the announcement, which would increase the current price level through the first term on the right-hand side of (2.13). Second, a decline in short-term nominal interest rates reduces real seigniorage revenues. To see this, we log-linearize equation (2.6) to obtain

$$
\hat{\sigma}_{t+j} = -\beta(1-\beta)^{-1}\left[1-\beta^{-1}(1-\beta)\eta\right]\hat{Q}_{t+j,t+j+1}, \tag{2.15}
$$

where η is the elasticity of the real money demand with respect to $\hat{Q}_{t,t+1}$, satisfying $0 < \eta < \beta(1-\beta)^{-1}$.[13] Lowering the nominal interest rate reduces the real seigniorage in each period,

[13]This is equivalent to assuming that the economy is located on the upward-sloping side of the inflation-tax Laffer curve.

thereby increasing the price level through the third term on the right-hand side of (2.13).

It is important to note that the efficacy of the first channel depends on the maturity structure of government debt. More specifically, the first term on the right-hand side of (2.13),

$$(1-\omega)(1-\beta\theta)\sum_{j=0}^{\infty}(\beta\theta)^j(E_t - E_{t-1})\hat{Q}_{t,t+j},$$

increases monotonically with θ, given $\{(E_t - E_{t-1})\hat{Q}_{t,t+j}\}_{j=0}^{\infty}$. In the extreme case of $\theta = 0$ (i.e., government debt consists solely of one-period bonds), the coefficients on $\hat{Q}_{t,t+j}$ are all equal to zero. Because all bonds are one-period bonds, changes in expectations about the future path of short-term nominal interest rates have no impact on bond prices, and consequently no impact on the price level.

3 Comparative Statics

3.1 Non-negativity Constraint on Nominal Interest Rates

As we have seen in the previous section, the government's budget constraint is an important element in determining the price level. The other important constraint we will consider is the non-negativity constraint on nominal interest rates.

The marginal utility of real money balances could be negative if the real balances held by a consumer exceed the satiation level. If this applies to all consumers, the marginal utility of money balances at the aggregate level, and the nominal interest rate, are both negative. However, as pointed out by Woodford (1990), this possibility could be ruled out by assuming the existence of at least one consumer having a zero cost of holding additional money balances. This consumer would be able to earn a profit by borrowing an infinite amount of money at the negative interest rate and holding it at a zero interest rate. This is the situation in which zero becomes the lower bound of nominal interest rates.

The non-negativity constraint can be expressed as $Q_{t,t+j} \leq 1$, using the notation in the previous section. This can be converted into a constraint on $\hat{Q}_{t,t+j}$,

$$\hat{Q}_{t,t+j} \leq \ln 1 - \ln Q^*_{t,t+j} = -j\ln\beta \approx j\beta^{-1}(1-\beta), \tag{3.1}$$

and a constraint on $\hat{Q}_t$,

$$\hat{Q}_t \leq \frac{\theta(1-\beta)}{(1-\beta\theta)^2}. \tag{3.2}$$

3.2 When Does the Non-negativity Constraint Become Binding?

Suppose that consumers are informed at the beginning of period t that the natural rate of interest will fall substantially in the current and future periods, and they accordingly revise their expectations. We consider how the equilibrium price level in period t would respond to the change in expectations, and to what extent a central bank could neutralize the shock.

Under the assumption that all variables are on the baseline before the new information is revealed in period t, equation (2.12) reduces to

$$\hat{P}_t = (1-\beta\theta)(1-\omega)\hat{Q}_t - (1-\beta)E_t\hat{D}_t - (1-\beta)\sum_{j=0}^{\infty}\beta^j E_t\hat{y}_{t+j}. \tag{3.3}$$

Since $E_t\hat{D}_t$ is positive, the news produces a downward pressure on the current price level $\hat{P}_t$. A central bank's announcement of lowering the future path of short-term nominal interest rates could neutralize this downward pressure to some extent. Using (3.2), an upper bound of the first term on the right-hand side of (3.3) is given by $\theta(1-\beta)(1-\beta\theta)^{-1}(1-\omega)$. Similarly, (3.1) and (2.15) provide an upper bound of the real seigniorage term on the right-hand side of (3.3). Given that the government does not respond to the shock ($\hat{s}_{t+j} = 0$), if parameter values satisfy

$$\theta(1-\beta)(1-\beta\theta)^{-1}(1-\omega) + [1-\beta^{-1}(1-\beta)\eta]\omega - (1-\beta)E_t\hat{D}_t \geq 0, \tag{3.4}$$

a central bank's announcement alone can completely eliminate the downward pressure on the current price level. Otherwise, the non-negativity constraint on nominal interest rates becomes binding.

To understand the condition (3.4), it is important to note that the basic economic mechanism behind deflation in period t is arbitrage between government bonds (including the base money) and alternative investment opportunities. A decline of the natural rate of interest in the current and future periods raises the real discount factor, thereby increasing the present value of the surplus stream even if the surplus stream itself does not change. This makes government bonds more attractive to consumers/investors. In a normal situation in which the level of nominal interest rates is not close to zero, such an increase in demand for government bonds leads to a rise in nominal bond prices. A sufficient rise in nominal bond prices can restore equilibrium by increasing the liability side of the government's balance sheet in accordance with an increase in its asset side. However, if the level of nominal interest rates is very close to zero, bond prices cannot rise sufficiently because of the non-negativity constraint of nominal interest rates. In other words, the lower bound on nominal interest rates creates an upper bound on bond prices. In this situation, equilibrium can be restored only when the current price level falls, thereby raising real bond prices sufficiently.

We observe the following implications of (3.4). First, (3.4) is easier satisfied when θ is greater. This is because a longer maturity of government debt increases the response of nominal bond prices to a given change in future nominal interest rates. Thus, even a small decline in future interest rates has a large upward impact on the current price level. As a result, the longer the maturity of government debt, the less likely that the non-negativity constraint is to be binding. In other words, keeping the maturity of government debt long during peacetime (i.e., on the baseline) is an effective way of insulating against the risk of falling into a liquidity trap. It is important to note that this is closely related to Summers's (1991) proposal of setting the target rate of inflation at a higher level. Its basic idea is to retain a margin for a nominal interest rate cut as insurance against future adverse shocks.[14] By contrast, long-term government debt functions as a device to amplify changes in nominal interest rates, and the nominal interest rate cut needed to avoid a liquidity trap is minimized by this leverage effect.

Second, a negative natural rate of interest is not a necessary condition for the non-negativity constraint on nominal interest rates to be binding. That is, the term $E_t\hat{D}_t$ can take a large positive value if the natural rate of interest is expected to be above zero, but below the baseline path, for a sufficiently long time. Then, under some parameter values (particularly when θ is close to zero), the left-hand side of (3.4) can be negative, so that the downward pressure on the current price level is not completely eliminated by the central bank's commitment. In this sense, the non-negativity constraint is binding even though the natural rate of interest stays above zero. This finding contrasts with previous studies on liquidity traps, including those of Krugman (1998, 2000) and Woodford (1999c), which consider a special case in which the non-negativity constraint becomes binding just because the natural rate of interest falls below zero.

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[14] Summers's proposal can be described as follows. Recall that (3.4) is obtained under the assumption that the baseline inflation rate is zero. If it is π^* rather than zero, the condition corresponding to (3.4) becomes $\left\{\theta(1-\beta)(1-\beta\theta)^{-1}(1-\omega)+[1-\beta^{-1}(1-\beta)\eta]\omega\right\}\left[1+\beta(1-\beta)^{-1}\pi^*\right]-(1-\beta)E_t\hat{D}_t \geq 0$, which is more easily satisfied when π^* takes a large positive value. When the baseline inflation rate is higher, the nominal interest rate is higher as well. Therefore, the non-negativity constraint is less likely to be binding.

3.3 "Deflation Now" or "Deflation Later"

We rewrite equation (3.3) as[15]

$$\hat{P}_t + \sum_{j=0}^{\infty} \beta^{j+1} \lambda_j E_t \hat{\pi}_{t+j+1} = -\sum_{j=0}^{\infty} \beta^{j+1}(1-\lambda_j) E_t \hat{D}_{t+j,t+j+1} - (1-\beta)(1-\omega) \sum_{j=0}^{\infty} \beta^j E_t \hat{s}_{t+j}, \tag{3.5}$$

where $\hat{\pi}_{t+j}$ ($\equiv \hat{P}_{t+j} - \hat{P}_{t+j-1}$) is the inflation rate, λ_j is a parameter defined by $\lambda_j \equiv (1-\omega)\theta^{j+1} + \left[1-\beta^{-1}(1-\beta)\eta\right]\omega$, and satisfying $0 \leq \lambda_j \leq 1$. This equation can be seen as imposing an intertemporal restriction on the inflation rate in each period. For example, suppose that the natural rate of interest declines substantially, and thus the first term on the right-hand side of (3.5) takes a negative number. Furthermore, suppose that the condition (3.4) is satisfied. Given that $\hat{P}_t$ equals zero, and that $\hat{s}_{t+j}$ are zero as well, equation (3.5) implies that the second term on the left-hand side must be negative; that is, deflation must take place some time in and after period $t+1$. In other words, as long as the condition (3.4) is satisfied, a central bank is able to stop deflation in period t, but it inevitably creates another deflation in the future. In this sense, deflation in period t is not removed by the monetary policy, but just postponed to future periods. Note that this tradeoff between current and future deflation becomes weaker when λ_j is close to unity.[16/17]

As a prescription to the Japanese liquidity trap, Krugman (1998, 2000) recommends that the BOJ raise the private sector's expectation of the future price level by announcing that it would never adhere to price stability and instead would conduct "irresponsible" monetary policy in the future.[18] In our terminology, the path of the price level Krugman proposes can be expressed as: $\hat{P}_t = 0$; and $\hat{\pi}_{t+j} \geq 0$ for $j = 1, \cdots$. Given this path, the left-hand side of (3.5) is positive, while the first term on the right-hand side is negative. Thus, the path is

[15] See Appendix B for more details on the derivation.

[16] Observe that the first term on the right-hand side of (3.5) becomes smaller in absolute value when λ_j is close to unity. Observe also that λ_j is equal to unity if $\theta = 1$ and $\omega = 0$, or $\theta = 1$ and $\eta = 0$

[17] Suppose $\hat{D}_{t+j,t+j+1} \equiv 0$ (no change in the natural rate of interest) and $\omega = 0$ (no seigniorage revenue) in equation (3.5). Then, according to the equation, if the surplus stream $\{\hat{s}_{t+j}\}$ declines, the inflation rate must be positive now or later. This tradeoff between current and future inflation is one of the main findings of Cochrane (2001), and can be seen as an extension of Sargent and Wallace's (1981) "unpleasant monetarist arithmetic" to an economy with long-term nominal government bonds. See Daniel (2001) for another example of the tradeoff in the context of currency crises.

[18] According to Krugman, the natural rate of interest is below zero in Japan; the Fisher equation implies that the future price level should be high relative to the current one; then, a sufficient rise in the future price level is needed to avoid a fall in the current price level.

consistent with the government's budget constraint only when the second term on the right-hand side takes a positive value. That is, the government needs to commit itself to reducing the future stream of the primary surplus to keep consistency with its budget constraint. Put differently, raising future inflation ($\hat{\pi}_{t+j} > 0$) without changing the stream of future primary surplus inevitably leads to a fall in the current price level ($\hat{P}_t < 0$). In this sense, targeting a higher future inflation is not an appropriate policy option unless supplemented by sufficient fiscal adjustment.

3.4 The Role of Fiscal Policy

It is straightforward to calculate the surplus stream required to achieve price stability when the natural rate of interest declines substantially. Under the assumption that the natural rate of interest stays above zero, we substitute $\hat{P}_t = 0$ and $\hat{\pi}_{t+j} = 0$ into (3.5) to obtain[19]

$$(1-\beta)\sum_{j=0}^{\infty}\beta^j E_t \hat{s}_{t+j} = -(1-\omega)^{-1}\sum_{j=0}^{\infty}\beta^{j+1}(1-\lambda_j)E_t\hat{D}_{t+j,t+j+1}. \tag{3.6}$$

In words, the central bank targets zero inflation rate as usual (not "irresponsible" as recommended by Krugman (1998, 2000)), and the government reduces the primary surplus. Such a well-organized coordination between the central bank and the government makes it possible to achieve price stability even in the presence of the zero bound on nominal interest rates. Recall that the current price level falls just because government bonds are too attractive to investors. Reducing the primary surplus weakens the attractiveness of those government bonds, and thereby successfully stops deflation in the current period.

The above discussion shows that fiscal policy plays an important role when the non-negativity constraint prevents a central bank from implementing monetary easing sufficient to offset an adverse shock to the natural rate of interest. This is consistent with the findings of previous studies on the role of fiscal policy when the economy is in a liquidity trap.[20]

[19] Note that $\hat{\pi}_{t+j} = 0$ cannot be achieved if the natural rate of interest is below zero in period $t+j$. See the next section for more on the optimal fiscal policy when the natural rate of interest declines below zero.

[20] See, for example, Woodford (2001) and Benhabib et al. (2002) for a recent analysis on the role of fiscal policy in avoiding a liquidity trap.

4 Optimal Policy Responses

So far, we have treated monetary policy only in an implicit manner. This section discusses how the path of nominal interest rates announced by a central bank determines the path of the price level. We first discuss the case in which a central bank commits itself to a simple policy rule, and then proceed to computing the optimal path of nominal interest rates by solving an intertemporal loss minimization problem.

4.1 Simple Monetary Policy Rules

The Taylor rule, advocated by Taylor (1993), requires a central bank to equalize the short-term nominal interest rate with the level of the natural rate of interest plus the target rate of inflation, which is assumed to be zero in this paper. That is,

$$i_{t+j} = \max \left\{ r_{t+j},\ 0 \right\}.$$

Or, equivalently,

$$\hat{Q}_{t+j,t+j+1} = \min \left\{ \hat{D}_{t+j,t+j+1},\ \beta^{-1}(1-\beta) \right\}. \tag{4.1}$$

To see what will happen under this rule, we rewrite equation (3.3) as[21]

$$\hat{P}_t = \sum_{j=0}^{\infty} \beta^{j+1} \left[\lambda_j \hat{Q}_{t+j,t+j+1} - E_t \hat{D}_{t+j,t+j+1} \right] - (1-\beta)(1-\omega) \sum_{j=0}^{\infty} \beta^j E_t \hat{s}_{t+j}. \tag{4.2}$$

Since, under this rule, $\hat{Q}_{t+j,t+j+1} \leq E_t \hat{D}_{t+j,t+j+1}$ for $j = 0, 1, 2, \cdots$, equation (4.2) implies that $\hat{P}_t$ must be negative as long as $\lambda_j \neq 1$ and $\hat{s}_{t+j} = 0$.

Reifschneider and Williams (2000) modified the Taylor rule by specifying the way monetary policy should be conducted in the presence of a zero bound on nominal interest rates. The augmented Taylor rule proposed by them can be expressed in our setting as

$$\hat{Q}_{t+j,t+j+1} = \min \left\{ \lambda_j^{-1} \hat{D}_{t+j,t+j+1} - \beta^{-1} Z_{t+j-1},\ \beta^{-1}(1-\beta) \right\}, \tag{4.3}$$

where Z_{t+j} is defined by

$$Z_{t+j} \equiv \beta^{-1} Z_{t+j-1} + \left[\hat{Q}_{t+j,t+j+1} - \lambda_j^{-1} \hat{D}_{t+j,t+j+1} \right]; \quad Z_{t-1} \equiv 0, \tag{4.4}$$

[21]See Appendix B for more details on the derivation.

and represents the cumulative sum of the deviations of the actual level of the short-term nominal interest rate from a desired level. When the deviation of the natural rate of interest from the baseline is substantially large (i.e., $\hat{D}_{t+j,t+j+1} > \lambda_j \beta^{-1}(1-\beta)$) in period $t+j$, the expression in the bracket of (4.4) takes a negative value. Therefore, during these periods, Z_{t+j} monotonically decreases starting from zero. Then, equation (4.3) instructs a central bank not to terminate its ZIRP until a backlog of past deviations, measured by Z_{t+j}, completely vanishes. This is what Woodford (1999) calls "monetary policy inertia". Note that, given the expected path of the natural rate of interest, the augmented Taylor rule instructs a longer continuation of a ZIRP when the maturity of government debt is shorter.

It can be shown that a central bank achieves $\hat{P}_t = 0$ under the augmented Taylor rule if parameter values satisfy (3.4). Let $t + J^{ATR}$ denote the final period of ZIRP under the augmented Taylor rule. Observe first that J^{ATR} takes a finite value if and only if Z_{t+j} becomes positive when ZIRP is continued forever. That is, $\sum_{j=0}^{\infty} \beta^j [\beta^{-1}(1-\beta) - \lambda_j^{-1} E_t \hat{D}_{t+j,t+j+1}] \geq 0$ must be satisfied. This restriction on parameter values is the same as (3.4). Then, by the definition of J^{ATR}, $Z_{t+J^{ATR}}$ must equal zero. That is,

$$\sum_{j=0}^{t+J^{ATR}} \beta^j \left[\hat{Q}_{t+j,t+j+1} - \lambda_j^{-1} E_t \hat{D}_{t+j,t+j+1} \right] = 0,$$

which implies $\hat{P}_t = 0$. Note that monetary policy inertia plays an important role in stopping deflation in period t. That is, a central bank's commitment to continuing its ZIRP for a longer time creates an expectation that the price level will be declining in the future, which produces an upward pressure on the current price level through the government's budget constraint. This expectational channel differs from the one studied by previous researchers, including Woodford (1999), Reifschneider and Williams (2000) and Jung et al. (2001), all of whom adopt the assumption of passive fiscal policy.

4.2 Optimal Monetary Policy

The tradeoff between current and future deflation raises a question about the optimal timing of deflation. This and following subsections solve a central bank's intertemporal loss minimization problem in order to address this question. This subsection deals with a situation in which the central bank is solely responsible for price stability, and the next subsection extends the analysis to a case in which the central bank and the government make a coordinated policy decision for price stability.

Suppose that, at the beginning of period 0, a new piece of information about the future path of the natural rate of interest is revealed, and that consumers have a new expectation that $\hat{D}_{j,j+1}$ will evolve over time, following an autoregressive process

$$\hat{D}_{j,j+1} = D\mu^j \qquad \text{for} \quad j = 0, 1, 2, \cdots , \tag{4.5}$$

where μ is a parameter satisfying $0 < \mu < 1$, and D is a positive parameter. Note that the natural rate of interest is expected to stay above zero if $D < \beta^{-1}(1-\beta)$, while it is expected to decline below zero if $D > \beta^{-1}(1-\beta)$. In the latter case, we assume that the natural rate of interest is expected to be below zero between period 0 and period J, where J is defined by $\hat{D}_{J,J+1} = \beta^{-1}(1-\beta)$.

Given an adverse shock to the natural rate of interest, a central bank chooses a path of short-term nominal interest rates to minimize

$$E_0 \sum_{j=0}^{\infty} \beta^j \hat{\pi}_j^2,$$

and commits itself to it.

The optimization problem is represented by a Lagrangian of the form

$$\begin{aligned} E_0 \sum_{j=0}^{\infty} \beta^j \Big\{ \hat{\pi}_j^2 \ &- \ 2\phi_{1j} \Big[\hat{\pi}_j - (1-\beta\theta)(1-\omega)(\hat{Q}_j - (\beta\theta)^{-1}\hat{Q}_{j-1}) \\ &+ \ (1-\beta)(E_j\hat{D}_j - \beta^{-1}E_{j-1}\hat{D}_{j-1}) + \omega\hat{Q}_{j-1,j} + (1-\beta)\sum_{k=0}^{\infty}\beta^k(E_j - E_{j-1})\hat{y}_{j+k} \Big] \\ &- \ 2\phi_{2j} \Big[(1-\beta\theta)\hat{Q}_j - \beta\theta(1-\beta\theta)E_j\hat{Q}_{j+1} - \beta\theta\hat{Q}_{j,j+1} \Big] \Big\}, \end{aligned} \tag{4.6}$$

where ϕ_{1j} and ϕ_{2j} are the Lagrange multipliers associated with the price level equation (equation (2.12)) and the definition of $\hat{Q}_j$ (equation (2.11)), respectively. We differentiate the Lagrangian with respect to $\hat{\pi}_j$, $\hat{Q}_j$, and $\hat{Q}_{j,j+1}$ to obtain the first-order conditions:

$$\hat{\pi}_j - \phi_{1j} = 0 \tag{4.7}$$

$$(1-\omega)\phi_{1j} - \theta^{-1}(1-\omega)\phi_{1j+1} - \phi_{2j} + \theta\phi_{2j-1} = 0 \tag{4.8}$$

$$\Big[\hat{Q}_{j,j+1} - \beta^{-1}(1-\beta)\Big]\Big\{\theta\phi_{2j} - \omega\phi_{1j+1} + \omega\Big[1 - \beta^{-1}(1-\beta)\eta\Big]\phi_{10}\Big\} = 0 \tag{4.9}$$

$$\hat{Q}_{j,j+1} - \beta^{-1}(1-\beta) \le 0 \tag{4.10}$$

$$\theta\phi_{2j} - \omega\phi_{1j+1} + \omega\Big[1 - \beta^{-1}(1-\beta)\eta\Big]\phi_{10} \le 0 \tag{4.11}$$

for $j = 0, 1, 2, \cdots$. Equations (4.7) and (4.8) are obtained by differentiating the Lagrangian with respect to $\hat{\pi}_j$ and $\hat{Q}_j$, respectively. Equations (4.9), (4.10), and (4.11) are the Kuhn-Tucker conditions relating to the non-negativity constraint on the nominal interest rate. If the non-negativity constraint is not binding, $\theta\phi_{2j} - \omega\phi_{1j+1} + \omega\left[1 - \beta^{-1}(1-\beta)\eta\right]\phi_{10} = 0$ holds. On the other hand, if the constraint is binding, $\theta\phi_{2j} - \omega\phi_{1j+1} + \omega\left[1 - \beta^{-1}(1-\beta)\eta\right]\phi_{10} < 0$ holds. Note that we treat the stream of the primary surplus as exogenously given ($\hat{s}_j = 0$).

We assume that the economy is on the baseline before the shock occurs in period 0. Since the first-best outcome is achieved before period 0, the Lagrange multipliers must be equal to zero:

$$\phi_{1-1} = 0\ ; \quad \phi_{2-1} = 0. \tag{4.12}$$

The above conditions, together with (2.12) and (2.11), are the first-order conditions for loss minimization.

We eliminate ϕ_{1j} by substituting (4.7) into (4.8) to obtain:

$$\phi_{2j} = (1-\omega)\left[\theta^j \hat{\pi}_0 - \theta^{-1}\hat{\pi}_{j+1}\right] \qquad \text{for} \quad j = 0, 1, 2, \cdots. \tag{4.13}$$

Given that a shock decays over time as described by (4.5), it is natural to guess that the non-negativity constraint is binding until some period, denoted by period J^* ($J^* \geq -1$), but not thereafter. Then, substituting (4.13) and (4.7) into the Kuhn-Tucker conditions yields

$$\begin{cases} \hat{\pi}_j > \lambda_{j-1}\hat{\pi}_0 & \text{for} \quad j = 1, 2, \cdots, J^*, J^*+1 \\ \hat{\pi}_j = \lambda_{j-1}\hat{\pi}_0 & \text{for} \quad j = J^*+2, J^*+3, \cdots. \end{cases} \tag{4.14}$$

If $\hat{\pi}_0 \geq 0$, equation (4.14) implies that $\hat{\pi}_j$ takes a non-negative value in every period, which contradicts equation (3.5). Therefore, $\hat{\pi}_0$ in (4.14) must be negative.

Note that the optimal solution could be characterized by a ZIRP even in cases where the natural rate of interest stays above zero (i.e., $D_{j,j+1} < 1$ or, equivalently, $\hat{D}_{j,j+1} < \beta^{-1}(1-\beta)$ for $j = 0, 1, 2, \cdots$). To see this, suppose that the non-negativity constraint is not binding in any period. Then (4.14) indicates

$$\hat{\pi}_{j+1} = \lambda_j \hat{\pi}_0 \qquad \text{for} \qquad j = 0, 1, 2, \cdots,$$

which implies that $\hat{\pi}_{j+1} < 0$ for $j = 0, 1, 2, \cdots$. Observe that the rate of deflation in each period becomes greater if the path of the natural rate of interest, $\left\{\hat{D}_{j,j+1}\right\}_{j=0}^{\infty}$ is closer to

$\beta^{-1}(1-\beta)$. Then, if the path of the natural rate of interest is sufficiently close to $\beta^{-1}(1-\beta)$,

$$[\hat{D}_{j,j+1} - \beta^{-1}(1-\beta)] - \hat{\pi}_{j+1} > 0$$

must hold for some period j. This implies

$$\hat{Q}_{j,j+1} - \beta^{-1}(1-\beta) > 0,$$

which contradicts equation (4.10).

The optimal solution has the following features. First, deflation occurs, not only in period 0, but also in and after period $J^* + 2$. In other words, the optimal solution is characterized by "deflation smoothing" in the sense that deflation is spread out over time.[22] This result sharply contrasts with the policy recommendations given to the BOJ by Krugman (1998, 2000) and Bernanke (2000) and others, which advise that a central bank in a liquidity trap should target a positive rate of inflation. Note that these policy recommendations are based on the assumption of passive fiscal policy.

Second, consider a case in which the natural rate of interest declines below zero (i.e., $J \geq 0$). The inflation rate corresponding to the final period of ZIRP, $\hat{\pi}_{J^*+1}$, must be equal to $\lambda_{J^*}\hat{\pi}_0$, so that

$$\hat{D}_{J^*,J^*+1} = \beta^{-1}(1-\beta) + \lambda_{J^*}\hat{\pi}_0$$

must hold. Since $\hat{D}_{J,J+1}$ is equal to $\beta^{-1}(1-\beta)$ by definition, this equation implies $\hat{D}_{J^*,J^*+1} \leq \hat{D}_{J,J+1}$, or equivalently

$$J \leq J^*. \tag{4.15}$$

In other words, the optimal solution is characterized by policy inertia, in the sense that a ZIRP should be continued for a while even after the natural rate of interest returns to a positive level. The degree of inertia depends on various parameter values, particularly on the value of θ. If θ is close to zero (i.e., the average maturity of government debt is close to one-period) the difference between J and J^* is small, and vice versa.

Third, a simple comparison between the optimal solution and the outcome obtained under the augmented Taylor rule shows that

$$J^* \leq J^{ATR}.$$

[22]Note that deflation in and after period $J^* + 2$ implies that the nominal interest rate is set lower than the natural rate of interest even after a ZIRP is terminated.

This can be understood if one recalls that, under the augmented Taylor rule, the nominal interest rate in the post-ZIRP periods is on the baseline; nevertheless, deflation in period 0 is completely eliminated.[23]

4.3 Optimal Policy Mix

We now proceed to the case in which the government coordinates with the central bank to achieve price stability. The primary surplus is an additional control variable, so that we differentiate (4.6) with respect to $(E_0 - E_{-1})\sum_{k=0}^{\infty}\beta^k \hat{s}_k$ to obtain a corresponding first-order condition

$$\phi_{10} = 0.$$

Then, the Kuhn-Tucker conditions, (4.9)-(4.11), reduce to

$$\begin{aligned}\left[\hat{Q}_{j,j+1} - \beta^{-1}(1-\beta)\right]\left[\hat{Q}_{j,j+1} - \hat{D}_{j,j+1}\right] &= 0\\ \hat{Q}_{j,j+1} - \beta^{-1}(1-\beta) &\leq 0\\ \hat{Q}_{j,j+1} - \hat{D}_{j,j+1} &\leq 0\end{aligned}$$

for $j = 0, 1, 2, \cdots$. These conditions imply that the optimal monetary policy rule is given by

$$\hat{Q}_{j,j+1} = \min\left\{\hat{D}_{j,j+1},\ \beta^{-1}(1-\beta)\right\},$$

which is identical to equation (4.1). It is interesting to see that the optimal monetary policy is represented by the simple Taylor rule in this coordination regime, but deviates from it if the central bank is solely responsible for price stability.

On the other hand, the optimal fiscal policy must satisfy

$$(1-\beta)(1-\omega)\sum_{j=0}^{\infty}\beta^j E_0\hat{s}_j = \sum_{j=0}^{\infty}\beta^{j+1}\left[\lambda_j\hat{Q}_{j,j+1} - \hat{D}_{j,j+1}\right]. \tag{4.16}$$

Because $\lambda_j\hat{Q}_{j,j+1} \leq \hat{D}_{j,j+1}$, the right-hand side of (4.16) must be non-positive, which implies that the government must commit itself to reducing the present value of the surplus stream. An interpretation of this result is that the government needs to reduce the primary surplus

[23] It is also seen that if the discount factor β is very close to zero, the optimal solution is close to the outcome under the augmented Taylor rule. In this sense, the augmented Taylor rule can be seen as an approximation to the optimal solution in the special circumstance that a central bank does not care much about future losses resulting from deflation.

in accordance with a decline in the total amount of the economy's endowment (see equation (2.2)). Otherwise, compared with alternative investment opportunities, government bonds are too attractive to investors, and thus create a downward pressure on the current price level.[24]

4.4 Numerical Example

In this subsection we compute the optimal path of the short-term nominal interest rate and the inflation rate, by using the parameter values shown in Table 1. We need to specify parameter values for β, ω, θ, η, D and μ. We interpret a period as a quarter. The value assumed for β implies a rate of time preference of four percent per year. The value for ω comes from assumptions that the ratio of the base money to GDP is 0.07, and that the ratio of the primary surplus to GDP is 0.05.[25] The value assumed for θ implies that the average maturity of government bonds is equal to 25 quarters ($= 1/(1-0.8)^2$). Finally, the value assumed for η is roughly consistent with the one obtained in empirical studies on the money demand function.

Figure 1 shows the paths for $\hat{D}_{j,j+1}$ (upper panel), $\hat{Q}_{j,j+1}$ (middle panel), and $\hat{\pi}_j$ (bottom panel). Here the assumed value for D is 0.009, which implies that the natural rate of interest declines, but still stays above zero.

The dotted lines in the middle and bottom panels represent the path of $\hat{Q}_{j,j+1}$ and $\hat{\pi}_j$, respectively, when the short-term nominal interest rate is determined by the Taylor rule, which is given by equation (4.1). Under the assumption that the natural rate of interest stays above zero, the Taylor rule instructs to equate the short-term nominal interest rate with the natural rate of interest in each period. Given this path of nominal interest rates, the price level falls in period 0 ($\hat{\pi}_0 = -0.05$), but remains completely stable in and after period 1.

The optimal path of $\hat{Q}_{j,j+1}$, which is represented by the solid line in the middle panel,

[24]Note that equation (4.16) does not uniquely determine the path of the primary surplus. There are many paths of the primary surplus that satisfy equation (4.16). This is a simple reflection of our assumption that taxes are collected through lump-sum tax, so that taxation is costless. Incorporating distortionary taxation would be an important step to proceed.

[25]Substituting these two numbers into the definition of ω ($\omega \equiv (1-\beta)V^*/[(1-\beta)V^* + s^*]$), we find that $\omega = (1-0.99)*0.07*4/[(1-0.99)*0.07*4+0.05] = 0.053$. This number is roughly consistent with the value of total transfers from the BOJ to the Ministry of Finance, namely 0.3 percent of nominal GDP during 19@@-19@@.

shows a substantial deviation from the path under the Taylor rule. An important difference is that the short-term nominal interest rate is lowered to zero, and a ZIRP is continued for eight quarters until period 7. Furthermore, the short-term nominal interest rate continues to be below the baseline even after the ZIRP is terminated. Stronger monetary easing in these two respects provides an upward pressure on the price level in period 0 through a rise in nominal bond prices and a reduction in seigniorage revenues. The optimal path of $\hat{\pi}_j$, represented by the solid line in the bottom panel, shows a smaller decline in the price level in period 0 ($\hat{\pi}_0 = -0.03$), as well as deflation in and after period 1.

Figure 2 conducts an exercise similar to Figure 1, but the natural rate of interest is assumed to fall below zero ($D = 0.015$). The upper panel shows that the natural rate of interest is negative in periods 0 to 3. The short-term nominal interest rate instructed by the Taylor rule equals zero in periods 0 to 3, during which the natural rate of interest is below zero, and coincides with the natural rate of interest in and after period 4. Given this path of nominal interest rates, the price level falls in period 0 ($\hat{\pi}_0 = -0.10$), followed by inflation in periods 1 to 4. The inflation rate returns to the baseline in and after period 5.

The optimal path of $\hat{Q}_{j,j+1}$ again shows a substantial deviation from the path under the Taylor rule. That is, a ZIRP is continued ten quarters longer, until period 13, and the short-term nominal interest rate stays below the baseline even after the ZIRP is terminated. Reflecting stronger monetary easing, the optimal path of $\hat{\pi}_j$ shows a smaller decline in the price level in period 0 ($\hat{\pi}_0 = -0.07$).

Table 2 compares the optimal duration of ZIRP, J^*, for different values of θ and D to see how it depends on the maturity structure of government debt. The natural rate of interest is assumed to stay above zero in the last three columns of the table ($D = 0.010,\ 0.012,\ 0.014$), while it is assumed to fall below zero in the other four columns. The last three columns indicate that the optimal duration of a ZIRP monotonically increases with θ. If the maturity of government debt is longer, continuing a ZIRP is more effective in weakening a downward pressure on the price level in period 0, so that the optimal duration of the ZIRP increases with θ. However, in cases when the natural rate of interest stays above zero, we no longer see a monotonic relationship between J^* and θ. In the case of $D = 0.06$, for example, J^* increases with θ until $\theta = 0.70$, but starts to decline thereafter. This is easy to understand if one recalls that the non-negativity constraint is never binding if θ is very close to unity and the natural rate of interest stays above zero.

Figure 3 presents solutions under the coordination regime. The assumed change in the natural rate of interest is the same as in Figure 1; i.e., the natural rate of interest is assumed to decline substantially but remain above zero. Figure 3 compares four cases that differ in terms of the degree of fiscal adjustment: $\hat{s} = 0$ represents the case in which the government does not change the primary surplus at all, as assumed in Figure 1; $\hat{s} = 1/3$ ($\hat{s} = 2/3$) represents the case in which the government adjusts the primary surplus by one-third (two-thirds) of the change indicated by (4.16); and, finally, $\hat{s} = 3/3$ is the case in which the government implements the optimal fiscal policy by adjusting the primary surplus as indicated by (4.16). In all four cases, the central bank is assumed to solve its loss minimization problem given the government's behavior.

The path of $\hat{Q}_{j,j+1}$, which is presented in the upper panel, shows that the optimal duration of ZIRP becomes shorter with the degree of fiscal adjustment, and that the non-negativity constraint on nominal interest rates is not binding at all in the case of $\hat{s} = 3/3$. The path of $\hat{\pi}_j$, presented in the lower panel, shows that fluctuations in the inflation rate becomes smaller with the degree of fiscal adjustment, and that perfect price stability is achieved in the case of $\hat{s} = 3/3$.

5 Conclusion

We have investigated the impact of changes in the natural rate of interest on the price level. An expected decline in the natural rate of interest in the current and future periods increases the present value of the government's primary surplus stream, making government bonds more attractive to investors. Then current real bond prices must rise to restore equilibrium. In a normal situation, a central bank's commitment to lowering short-term nominal interest rates raises current nominal bond prices, thereby restoring equilibrium. If the non-negativity constraint on nominal interest rates is binding, however, such adjustment through nominal bond prices does not necessarily work well. That is, a sufficient rise in nominal bond prices is infeasible under some parameter values. Without a sufficient rise in nominal bond prices, equilibrium can be restored only by a sufficient fall in the current price level. In this sense, the efficacy of monetary policy commitment is limited when the economy is in a liquidity trap.

The analysis in this paper has some implications for Japan's liquidity trap, two of which sharply contrast with the ideas frequently presented in the policy discussion inside and outside the country. First, targeting a higher future price level or a higher future rate of inflation, as recommended by Krugman among many others, leads to a fall in nominal bond prices, thereby creating an additional downward pressure on the current price level. By solving a central bank's loss minimization problem, we have shown that a central bank should target a negative rate of inflation rather than a positive one. Second, the government should commit itself to reducing the primary surplus, because it makes government bonds less attractive to investors, thus contributing to neutralizing the downward pressure on the current price level. From the viewpoint of stopping deflation, the government's balance sheet is problematic not because it lacks soundness, but because it is still healthier than the balance sheets of banks, firms and consumers, which have been substantially deteriorating ever since the bubble burst in the early 1990s.

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A Derivation of equation (2.12)

Equation (2.10) can be expressed as

$$\begin{aligned}(1-\omega L)\hat{P}_t &= (1-\beta\theta)(1-\omega)\left[\hat{Q}_t+\hat{B}_{t-1}\right]-(1-\beta)E_t\hat{D}_t \\ &+ \omega\hat{V}_{t-1}-(1-\beta)\sum_{j=0}^{\infty}\beta^j E_t\ \hat{y}_{t+j},\end{aligned} \tag{A.1}$$

where L is the lag operator. Similarly, we have

$$\begin{aligned}(1-\omega L)\hat{P}_{t+1} &= (1-\beta\theta)(1-\omega)\left[\hat{Q}_{t+1}+\hat{B}_{t}\right]-(1-\beta)E_{t+1}\hat{D}_{t+1} \\ &+ \omega\hat{V}_{t}-(1-\beta)\sum_{j=0}^{\infty}\beta^j E_{t+1}\ \hat{y}_{t+1+j}.\end{aligned} \tag{A.2}$$

Multiplying (A.1) by β^{-1} and subtracting it from (A.2) yields

$$\begin{aligned}(1-\omega L)\left[\hat{P}_{t+1}-\beta^{-1}\hat{P}_t\right] &= (1-\beta\theta)(1-\omega)\left[\hat{Q}_{t+1}-\beta^{-1}\hat{Q}_t\right] \\ &+ (1-\beta\theta)(1-\omega)\left[\hat{B}_{t}-\beta^{-1}\hat{B}_{t-1}\right] \\ &- (1-\beta)\left[\hat{D}_{t+1}-\beta^{-1}\hat{D}_t\right] \\ &+ \omega\left[\hat{V}_t-\beta^{-1}\hat{V}_{t-1}\right] \\ &- (1-\beta)\left[\sum_{j=0}^{\infty}\beta^j E_{t+1}\ \hat{y}_{t+1+j}-\beta^{-1}\sum_{j=0}^{\infty}\beta^j E_t\ \hat{y}_{t+j}\right].\end{aligned} \tag{A.3}$$

Log-linearizing (2.3) yields

$$\begin{aligned}(1-\beta\theta)(1-\omega)\left[\hat{B}_t-\beta^{-1}\hat{B}_{t-1}\right] &= -(1-\beta\theta)(1-\theta)(\beta\theta)^{-1}(1-\omega)\hat{Q}_t \\ &+ \beta^{-1}\omega\left[\hat{V}_{t-1}-\hat{V}_t\right] \\ &- \beta^{-1}(1-\beta)(1-\omega)\hat{s}_t \\ &- \beta^{-1}\left[(1-\beta)(1-\omega)+\omega\right]\hat{P}_t+\beta^{-1}\omega\hat{P}_{t-1}.\end{aligned} \tag{A.4}$$

To eliminate $\hat{B}_t - \beta^{-1}\hat{B}_{t-1}$, we substitute (A.4) into (A.3).

$$\begin{aligned}
\hat{P}_{t+1} - \hat{P}_t &= (1-\beta\theta)(1-\omega)\left[\hat{Q}_{t+1} - (\beta\theta)^{-1}\hat{Q}_t\right] \\
&- (1-\beta)\left[\hat{D}_{t+1} - \beta^{-1}\hat{D}_t\right] \\
&- \beta^{-1}(1-\beta)\omega\ \hat{V}_t \\
&+ \beta^{-1}(1-\beta)\omega\ \hat{\sigma}_t \\
&- \beta^{-1}(1-\beta)\sum_{j=0}^{\infty}\beta^j(E_{t+1}-E_t)\ \hat{y}_{t+j}.
\end{aligned} \tag{A.5}$$

Finally, substituting

$$\hat{\sigma}_t = \hat{V}_t - \beta(1-\beta)^{-1}\hat{Q}_{t,t+1} \tag{A.6}$$

into (A.5) yields equation (2.12).

B Derivation of equations (3.5) and (3.8)

It is useful to rewrite (2.13) in two different ways. The definitions of $\hat{D}_t$ and $\hat{Q}_t$ imply

$$(1-\beta)(E_t - E_{t-1})\hat{D}_t = \sum_{j=0}^{\infty} \beta^{j+1}(E_t - E_{t-1})\hat{D}_{t+j,t+j+1}, \tag{B.1}$$

$$(1-\beta\theta)(E_t - E_{t-1})\hat{Q}_t = \sum_{j=0}^{\infty} (\beta\theta)^{j+1}(E_t - E_{t-1})\hat{Q}_{t+j,t+j+1}. \tag{B.2}$$

We substitute (B.1), (B.2), and (2.15) into (2.13) to obtain

$$\begin{aligned}(E_t - E_{t-1})\hat{P}_t &= \sum_{j=0}^{\infty} \beta^{j+1}(E_t - E_{t-1})\left[\lambda_j \hat{Q}_{t+j,t+j+1} - \hat{D}_{t+j,t+j+1}\right] \\ &- (1-\beta)(1-\omega)\sum_{j=0}^{\infty} \beta^{j}(E_t - E_{t-1})\hat{s}_{t+j},\end{aligned} \tag{B.3}$$

where λ_j is defined as $\lambda_j \equiv (1-\omega)\theta^{j+1} + \left[1 - \beta^{-1}(1-\beta)\eta\right]\omega$. The other useful expression can be obtained by substituting the Fisher equation into (B.3).

$$\begin{aligned}&(E_t - E_{t-1})\hat{P}_t + \sum_{j=0}^{\infty} \beta^{j+1}\lambda_j(E_t - E_{t-1})\hat{\pi}_{t+j+1} \\ = & -\sum_{j=0}^{\infty} \beta^{j+1}(1-\lambda_j)(E_t - E_{t-1})\hat{D}_{t+j,t+j+1} - (1-\beta)(1-\omega)\sum_{j=0}^{\infty} \beta^{j}(E_t - E_{t-1})\hat{s}_{t+j}.\end{aligned} \tag{B.4}$$

Equations (3.5) and (3.8) follow from (B.3) and (B.4), respectively.

Table 1: Parameter values

β	=	0.99
ω	=	0.053
θ	=	0.8
η	=	0.505
D	=	0.05
μ	=	0.7

Table 2: Values of J^*

$D =$	0.002	0.004	0.006	0.008	0.010	0.012	0.014
$\theta =$ 0.30	NB	0	1	2	4	6	9
0.40	NB	0	1	2	4	6	9
0.50	NB	0	2	3	4	6	9
0.60	NB	1	2	4	5	7	9
0.70	NB	1	3	5	7	8	10
0.80	NB	0	2	6	8	10	12
0.90	NB	NB	0	4	8	12	15
1.00	NB	NB	NB	NB	NB	1	3

Note: NB indicates that the non-negativity constraint on nominal interest rates is not binding along the optimal path.

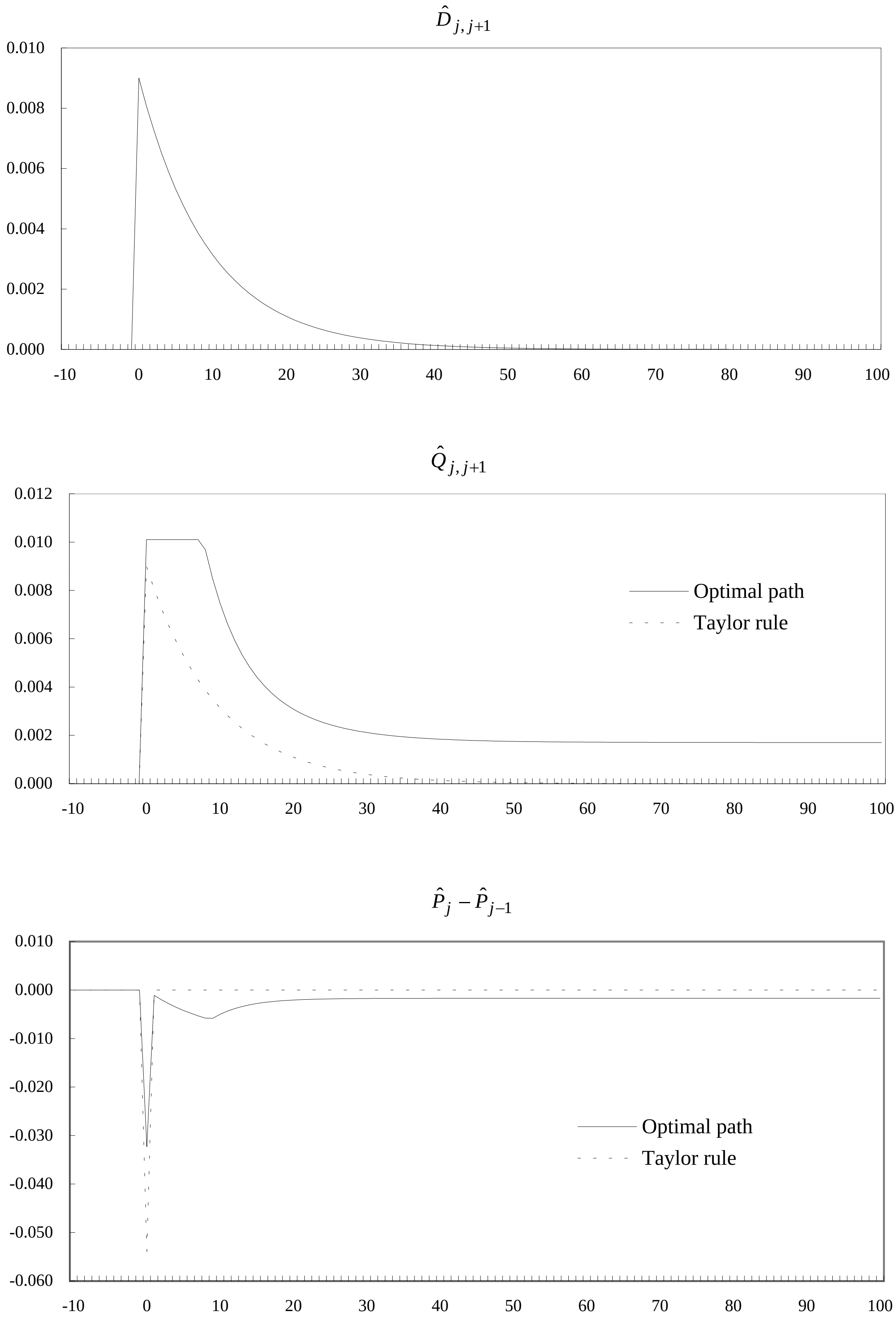

Figure 1
Optimal responses when the natural rate of interest stays above zero

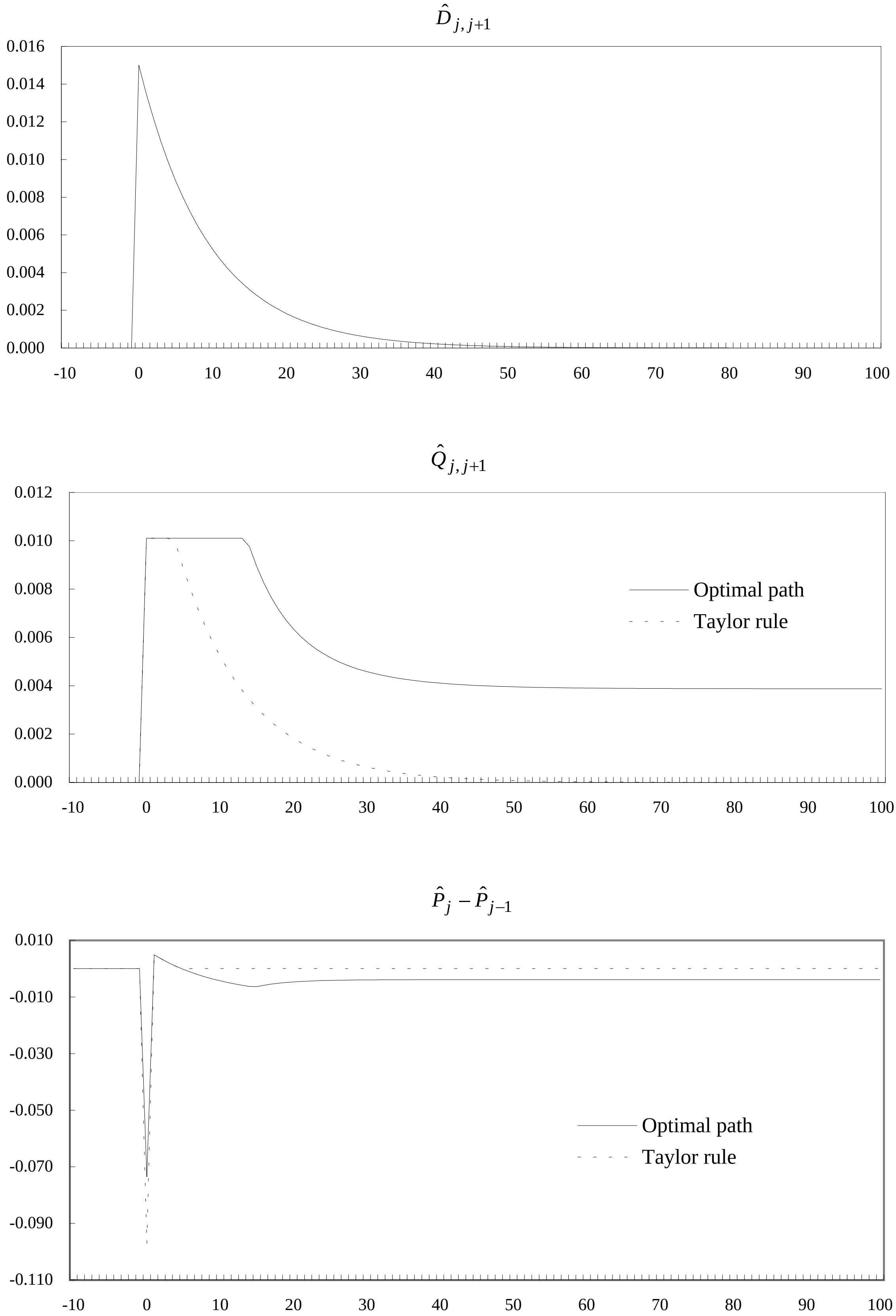

Figure 2
Optimal responses when the natural rate of interest falls below zero

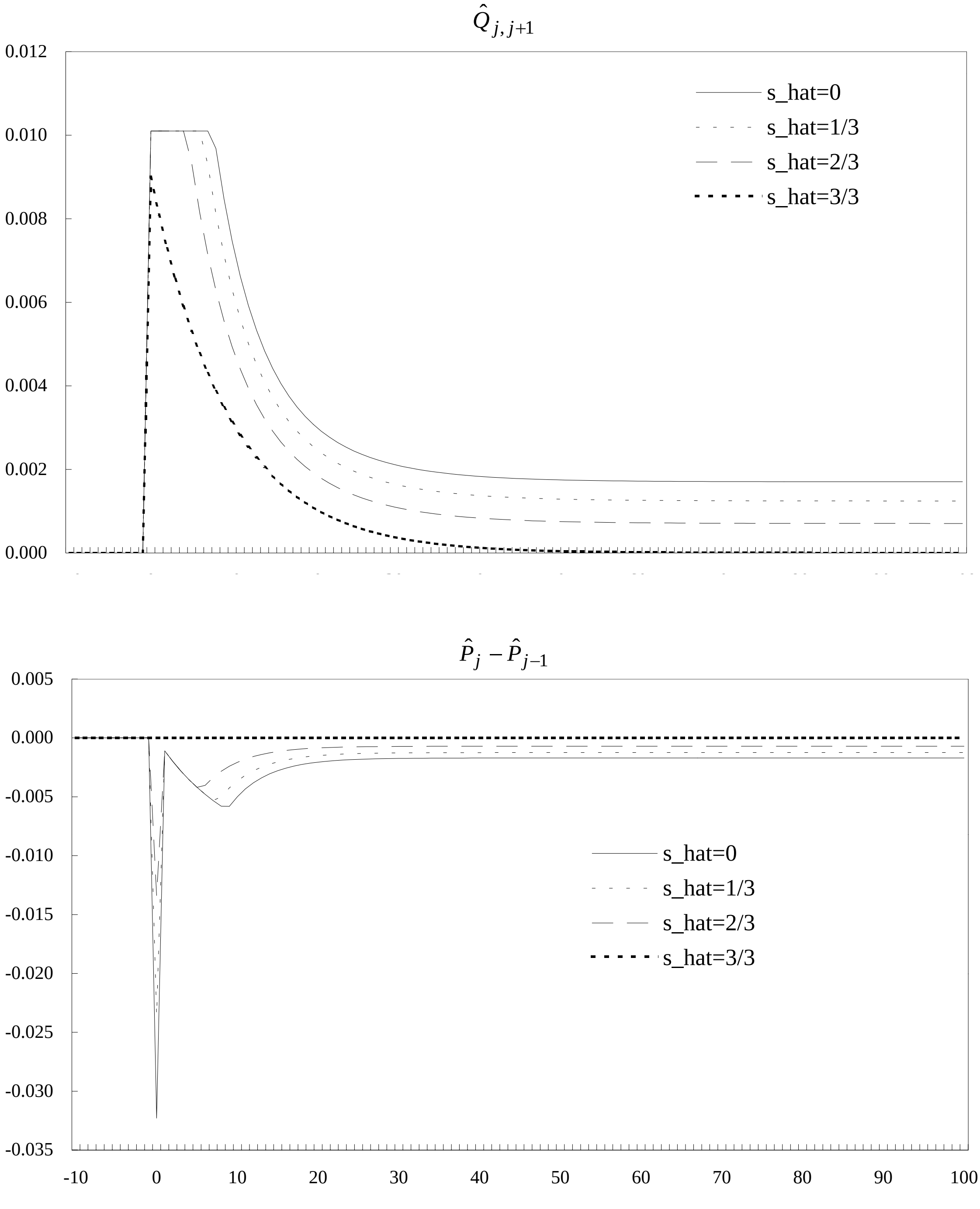

Figure 3 Optimal policy mix