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If confidence is restored through Bayesian learning by economic agents based on observations of government actions, then the inaction of the government (forbearance) impedes Bayesian learning. Since it seems plausible to assume that economic agents face Knightian uncertainty (i.e., a non-unique prior) after a financial crisis, I use the framework of uncertainty aversion developed by Gilboa and Schmeidler (1989). The Bayesian update rule for a non-unique prior is proposed and used for the analysis of confidence building. The model shows that the forbearance policy hinders the convergence of a non-unique prior and delays the economic recovery.

JEL Classification: E22, E23, E61.

Keywords: Confidence, non-unique prior, Bayesian learning, forbearance policy.

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1 Introduction

Many countries have experienced financial crises. We have the stylized fact that a quick policy response (e.g., resolving nonperforming loans, recapitalizing the banking sector, reorganizing failed firms) is followed by a quick recovery of economic growth.

For example, Bergoeing, Kehoe, Kehoe, and Soto (2002) compare the quick and sustained recovery of Chile and the long stagnation of Mexico after the external debt crises at the beginning of the 1980s. They show that although both macroeconomic policies and the international trade environment were favorable for Mexico, Chile recovered at a higher level and with long-lasting economic growth. They argue that what caused the different outcomes are (1) the different policy reactions to the banking sectors and (2) the difference in the efficiency of bankruptcy procedures. Chile undertook quick banking reforms using 35 % of one year's GDP as the costs during 1982-86, while Mexico nationalized banks and allocated credit discretionarily at below-market rates for a long time. The Chilean bankruptcy procedure became quite efficient by the time of the 1982 bankruptcy reform law, while Mexico had an obsolete and inefficient bankruptcy law from 1943 in place until 2000. Bergoeing et. al (2002) conclude that these differences in banking reform and bankruptcy procedures caused the differences in economic growth subsequent to the debt crises.

Another episode is the bursts of asset-price bubbles in Sweden and Japan in the early 1990s. Both Sweden and Japan experienced declines of prices in their real estate markets at the beginning of the 1990s. Sweden quickly disposed of nonperforming loans and recapitalized the banking sector in 1992-1994, while Japan forbore the resolution of their nonperforming loans problem until 1997. The asset prices in Sweden picked up in 1994 and have continued to rise, while the asset prices in Japan have continued to fall for more than a decade.

The stylized fact that quick reform is followed by quick economic recovery is usually explained as follows: forbearance (i.e., an implicit subsidy to the inefficient sectors) causes the inefficient allocation of economic resources; the resources are absorbed into the inefficient sectors and the productive sectors cannot get enough resources for their activities. Therefore, the macroeconomic inefficiency is usually explained as being caused by a *shortage of supply* of economic resources in the productive sector.

A puzzle One puzzle is the case of the Japanese economy. Although forbearance lending to *de facto* insolvent firms has been widespread among the Japanese banks, they still have a huge number of deposits that they cannot help but invest in Japanese Government Bonds. This fact indicates that the Japanese banks have had enough money to lend to any borrowers who are potentially productive. Thus Japanese banks must have been unable to find productive borrowers during the 1990s. In other words, Japanese firms seem to have ceased undertaking productive projects after the collapse of the asset-price bubble at the beginning of the 1990s. We need to clarify why the Japanese corporate sector did not undertake productive projects although there was enough money for their activities.

Our explanation in this paper is that the forbearance of economic reform impedes rebuilding the confidence that is lost during a financial crisis. In a financial crisis, losses emerge (due to, e.g., asset-price declines or the devaluation of domestic currency) that are unexpected beforehand and should be finalized and borne by banks and firms. If the government expects that the asset prices (or domestic currency) will pick up following the spontaneous economic recovery, then it rationally chooses to postpone the reckoning to avoid the social and political costs of a rush of bankruptcies. Suppose, however, that the economic recovery necessitates an increase of high-risk-high-return investments, and that investments will increase only if the peoples' confidence is restored, while the confidence people have is their shared belief in the rigidity and fairness of bankruptcy procedures. Suppose that people's confidence is revised by the Bayesian rule based on observations of the government's actions toward the failed firms and banks. In this case, if the government chooses forbearance, confidence may not be restored and business investments may stagnate. (If the government postpones the bankruptcies, peoples' beliefs in bankruptcy procedures is not restored.)

If the government recognizes that peoples' confidence is revised depending on observations of the government's action, it will choose not to procrastinate in situations where confidence matters. If the government perceives that a change in confidence is an exogenous event, then it may choose procrastination, leading the economy into a long-lasting stagnation.

Uncertainty associated with the Financial Crisis In order to formalize the above intuition on confidence rebuilding as a theoretical model, we can utilize the Bayesian learning mechanism in the spirit of Barro (1986). The unique characteristic of the expectation problem after a financial crisis is that we need to analyze the expectations of economic agents on *unprecedented events*. For example, land prices in Japan had continued to rise for more than 100 years with a few exceptional years until the beginning of the 1990s. The continuous decline of land prices over the decade of the 1990s was unprecedented in Japan. The economic institutions or business customs in Japan had been formed on the premise that land prices never fall. How to deal with the losses from the land price decline was an unprecedented problem for the Japanese economy. Japan had a legal and social system of bankruptcy procedures that worked well until the beginning of the 1990s. The continuous decline of land prices, however, changed the fundamental environment of bankruptcy practices, and consequently increased the uncertainty concerning the outcome of bankruptcy procedures.

Currency crises in developing countries may introduce a similar uncertainty into domestic economies. Before the crises, there were no economic institutions in those countries to cope with business and banking failures associated with currency devaluations under large external debts. Business failures due to external debt problems are usually unprecedented in the crisis countries. The bankruptcy systems in those countries do not seem to function very well in solving the defaults that are caused by unprecedented external debt problems.

A straightforward method of modeling the situation when economic agents face unprecedented events is to use the model of Knightian Uncertainty. The problem of Knightian Uncertainty can be modeled as uncertainty aversion under a non-unique prior (Gilboa and Schmeidler [1989] [1993], Casadesus-Masanell, Klibanoff and Ozdenoren [2000]) or a non-additive prior (Schmeidler [1989], Dow and Werlang [1992]). In the following model, I assume that economic agents face a non-unique prior concerning the unprecedented events after a financial crisis; the uncertainty-averse agents maximize the minimum expected utility under a non-unique prior.[1]

2 Model

The economy is the infinite horizon economy where time discretely extends from zero to infinity: $t = 0, 1, 2, \cdots$. The economy consists of many representative agents (firms) and a benevolent government. The number of firms is M ($\gg 1$). The firms maximize the expected value of the discounted sum of the consumption flow:

$$\max E_0 \left[\sum_{t=1}^{\infty} \beta^t c_t \right] \tag{1}$$

subject to a budget constraint that is specified below, where $E_0[\cdot]$ is the expected value as of time 0 and c_t is the consumption at time t. Each firm owns one unit of land and m units of consumer goods at the beginning of time 0. The land is non-depletable. This economy is a one-sector economy where the land yields the consumption goods.

We assume that firms and the government are risk-neutral and uncertainty-averse. According to Gilboa and Schmeidler (1989), this assumption can be formalized as follows:

Assumption 1 *When firms and the government face a unique prior, they maximize the expected value of their objective functions. When they face a non-unique prior, they maximize the minimum expected value of their objective functions.*

In this economy, the risk-free rate of interest r is determined by $r = \beta^{-1} - 1$, since firms (= representative agents) maximize (1). If the market rate of interest is greater than $\beta^{-1} - 1$, firms invest all consumption goods, while if the market rate is less than $\beta^{-1} - 1$, they consume all goods instantaneously.

[1]Technically speaking, we may be able to formalize the intuition on confidence rebuilding as the conventional model of expected utility maximization of risk-averse agents under a unique prior. A simpler exposition is one advantage of the following model.

Production Technology There are two production technologies: S(safe) and R(risky). If the firm chooses Technology S, then one unit of its land yields y_L (> 0) units of consumer goods. The land is the only input for Technology S. If the firm chooses Technology R, it must provide m units of consumer goods as input to one unit of its land. Then, the one unit of land yields y_H with probability p and yields nothing with probability $1 - p$. We assume the following restriction on Technology R:

Assumption 2 *A firm cannot use the consumer goods that it already owns as the input for its own production activity. A firm must borrow input m from another firm. Firms cannot make any strategic contract that is contingent on output. The only contract that firms can make is a debt contract between two firms with a fixed repayment.*

This assumption prohibits the exchange of inputs (m) between two firms.[2] We assume that the parameter values satisfy the following condition:

$$py_H > (1 + r)^2 m \text{ and } y_L < r(1 + r)m. \tag{2}$$

This condition implies that the per capita output of Technology R is larger than that of Technology S:

$$py_H - (1 + r)m > y_L. \tag{3}$$

We assume that the land market is established so that a firm can sell (any fraction of) its land at market price Q_t in order to repay a debt when the firm undertakes Technology R and then fails.

The unit price of land Q_t is determined as follows. If Technology R is dominant in this economy, the price of land Q_H is determined by the expected value of its cash-flow: $py_H - (1 + r)m$. Therefore,

$$Q_H = \beta \cdot (py_H - (1 + r)m) + \beta^2 \cdot (py_H - (1 + r)m) + \cdots = \frac{py_H - (1 + r)m}{r}. \tag{4}$$

[2]It is justified as follows. Suppose there is a technological constraint on Technology R that firm i must use the consumer goods of firm $i + 1$ ($i = 1, 2, \cdots, M$) where firm $M + 1 \equiv$ firm 1. In this case, firms cannot make a contract for the arrangement of exchange if they cannot make a contract among three or more firms.

If Technology S is dominant in this economy, the price of land Q_L becomes

$$Q_L = \beta y_L + \beta^2 y_L + \cdots = \frac{y_L}{r}. \tag{5}$$

Thus condition (2) implies

$$Q_L < (1+r)m < Q_H. \tag{6}$$

This condition implies the following: If the land price is Q_H, then the firm can make repayment ($(1+r)m$) by selling its land when it undertakes Technology R and fails; and if the land price is Q_L, then the firm cannot repay the debt by selling its land when it undertakes Technology R and fails.

Default and Bankruptcy Suppose that debt Δ remains unpaid even after the debtor sells her land. We assume for simplicity of argument that each defaulter has sufficient private wealth to repay the unpaid debt when the judgment of the bankruptcy court orders repayment. But information asymmetry between debtors and creditors makes it impossible for the creditors to capture the debtors' private properties without a lawsuit.

Assumption 3 *If the government undertakes a bankruptcy procedure, the debtor is forced to repay Δ with probability θ and the creditor is forced to bear Δ as a loss with probability $1-\theta$. The value of θ is unknown to firms and the government at time 0. Before observing the bankruptcy results, firms and the government have a closed set of probability distributions C as a non-unique prior on θ:*

$$C = \{\textit{Probability distribution with p.d.f. } \pi(\theta;\alpha,\beta) \mid \alpha \geq 1, \beta \geq 1, \alpha+\beta=\gamma\}, \tag{7}$$

$$\textit{where} \quad \pi(\theta;\alpha,\beta) = \frac{\theta^{\alpha-1}(1-\theta)^{\beta-1}}{\int_0^1 x^{\alpha-1}(1-x)^{\beta-1}dx}, \tag{8}$$

and $\gamma(\gg 1)$ is a positive number.

Therefore, firms and the government perceive θ as a realized value of a random variable, whose probability distribution is unknown. We also assume the following for parameter values.

Assumption 4

$$y_L > py_H - (1+r)m - \frac{(1-p)(\gamma-2)\{(1+r)m - Q_L\}}{p\gamma + 1 - p}. \tag{9}$$

Timetable The events occur in the following order at time t and $t+1$.

Time t:

(1) Firms choose whether they will use Technology S or R.

(2) If firms choose Technology R, they borrow m from other firms. (Firms choose whether to lend m to one another.)

Time $t+1$

(3) Output is produced. Output is destroyed with probability $1-p$ if Technology R is chosen.

(4) Firms repay the creditors by selling outputs and land if necessary.

(5) If firms default, the government can choose either to undertake bankruptcy procedures or to give subsidies to keep them alive.

2.1 Stationary Equilibria

There are two stationary equilibria in this economy in which the true value of θ is never revealed. Technology R is dominant in one equilibrium, and Technology S is dominant in the other.

Optimal Equilibrium The optimal equilibrium where the aggregate output that is proportional to the expected value of per capita output is maximized is described as follows. The asset price is Q_H. The profit maximizing firms always choose Technology R. If they fail with probability $(1-p)$, then they sell their land to other firms in the land market, and repay all debt to their creditors. The per capita output is py_H and per capita consumption is $py_H - m$ in each period. This is an equilibrium where agents face risk but no uncertainty. Thus agents maximize their profits under a unique prior on production failure $(1-p)$. Since debtors never default when htey fails, the non-unique prior C does not evolve in this equilibrium.

Suboptimal Equilibrium Suppose that the land price is Q_L. In this case, unpaid debt Δ may remain when a firm chose Technology R and failed. Since firms have non-unique priors on the distribution of θ, they choose their actions in order to maximize the minimum expected profits. The minimum expected profit for the creditor is $p(1 + r_l)m + (1-p)Q_L + (1-p)\min_{\theta \in C} E[\theta]\{(1+r_l)m - Q_L\}$ where r_l is the interest rate of the loan. From the property of the beta distribution, we have $E[\theta] = \frac{\alpha}{\gamma}$ for $\theta \sim \pi(\theta; \alpha, \beta)$. Therefore, $\min_{\theta \in C} E[\theta] = \frac{1}{\gamma}$. Since the expected return $(1 + r_l)$ must be equal to the return at the market rate $(1 + r)$, the repayment $(1 + r_l)m$ must satisfy

$$(1 + r_l)m = \frac{(1 + r)m\gamma - (1 - p)(\gamma - 1)Q_L}{p\gamma + 1 - p}. \tag{10}$$

On the other hand, the minimum expected profit for a debtor (π_F) is

$$\pi_F = \min_{\theta \in C}[py_H - \{p + (1 - p)E[\theta]\}(1 + r_l)m - (1 - p)(1 - E[\theta])Q_L],$$

which is minimized when $E[\theta]$ is maximized. Since $\max_{\theta \in C} E[\theta] = \frac{\gamma-1}{\gamma}$,

$$\pi_F = py_H - \frac{\{(1 + r)m\gamma - (1 - p)(\gamma - 1)Q_L\}\{p\gamma + (1 - p)(\gamma - 1)\}}{(p\gamma + 1 - p)\gamma} - \frac{1 - p}{\gamma}Q_L.$$

Assumption 4 implies that the minimum expected profit for a debtor who adopts Technology R is smaller than for a firm that adopts Technology S.

In this case, firms always choose Technology S and the land price stays at Q_L. This is a suboptimal equilibrium where the level of aggregate output is low (y_L) and θ is never revealed since there is no default and no bankruptcy in this equilibrium.

2.2 Financial Crisis — Emergence of Uncertainty

In the stationary equilibria where the asset price is constant (Q_H or Q_L) for all t, there is no default and thus θ is never revealed. In this subsection, we examine the case where the asset price changes by an exogenous macroeconomic shock.

Suppose that this economy had been initially at the optimal equilibrium and that it was suddenly hit by a financial crisis at time τ. The financial crisis consists of the following three events: (a) outputs are destroyed for N firms ($1 \ll N \leq M$), (b) the land price suddenly fell from Q_H to Q_L, and (c) pessimism prevailed that the land price would stay at Q_L from time τ onward. As a result, the N firms defaulted on their debt obligations at τ since all firms had chosen Technology R at time $\tau - 1$. We assume the following for parameter N.

Assumption 5 *The number of the defaulters N is large enough to satisfy*

$$y_L < py_H - (1+r)m - \frac{(1-p)(\gamma-2)\{(1+r)m - Q_L\}}{(N+\gamma)p + 1 - p}.$$

Condition (3) guarantees the existence of N that satisfies Assumption 5.

Suppose that the government undertakes bankruptcy proceedings of v_t firms at time $t(\geq \tau)$. The variable $v_t \in \{0, 1, 2, \cdots, N\}$ is the choice variable for the government. The judgment of the bankruptcy proceeding is determined in the period that it starts: Suppose that the debtor firms are forced to repay Δ in s_t cases and the creditors are forced to bear Δ as a loss in the other $(v_t - s_t)$ cases. The number s_t is a random variable that obeys the binomial distribution $b(v_t, \theta)$.

In the following argument, we assume that agents update their non-unique priors on θ by Bayesian learning observing the results of bankruptcy procedures. The Bayesian update rule of a non-unique prior is argued in Gilboa and Schmeidler(1993). At time $t(\geq \tau)$, firms and the government update their priors on θ by the Bayesian rule observing the sequence $\{v_{t'}, s_{t'}\}_{t'=\tau}^{t}$.

We can use the Bayesian update rule of the beta distribution (Morris[1996]) for updating C. Observing $\{v_{t'}, s_{t'}\}_{t'=\tau}^{t}$ at time t, firms and the government update the prior C to $C(S_t, V_t)$ where $S_t = \sum_{t'=\tau}^{t} s_{t'}$ and $V_t = \sum_{t'=\tau}^{t} v_{t'}$ by the Bayesian rule.

$$C(S_t, V_t) = \{ \text{ Probability distribution with p.d.f. } \xi(\theta|S_t, V_t, \alpha, \beta) \mid \alpha \geq 1, \beta \geq 1, \alpha+\beta = \gamma\}, \tag{11}$$

where

$$\xi(\theta|s, v, \alpha, \beta) = \frac{\theta^s (1-\theta)^{v-s} \pi(\theta; \alpha, \beta)}{\int_0^1 x^s (1-x)^{v-s} \pi(x; \alpha, \beta) dx}, \tag{12}$$

and $\pi(\theta; \alpha, \beta) \in C$. The property of the beta distribution (see, for example, Hartigan [1993], pp.76-78) implies that $\theta \sim \xi(\theta|S_t, V_t, \alpha, \beta)$ satisfies

$$E[\theta|S_t, V_t] = \frac{S_t + \alpha}{V_t + \gamma}, \text{ and } V[\theta|S_t, V_t] = \frac{(S_t + \alpha)(V_t - S_t + \beta)}{(V_t + \gamma + 1)(V_t + \gamma)^2}.$$

Thus

$$\lim_{V_t \to \infty} V[\theta | S_t, V_t] = 0 \text{ for all } \alpha \text{ and } \beta \text{ that satisfy } \alpha \geq 1, \beta \geq 1, \alpha + \beta = \gamma.$$

The law of large numbers implies that $\frac{S_t}{V_t}$ converges to θ^* where θ^* is the true value of θ. Thus, we have the following for the random variable $\theta \sim \xi(\theta | S_t, V_t, \alpha, \beta) \in C(S_t, V_t)$

$$\lim_{V_t \to \infty} E[\theta | S_t, V_t] = \lim_{V_t \to \infty} \frac{S_t + \alpha}{V_t + \gamma} = \theta^*.$$

Therefore, the prior $C(S_t, V_t)$ converges to the point distribution that $\Pr\{\theta = \theta^*\} = 1$ as V_t goes to infinity. In this sense, firms and the government can learn the true value θ^* by Bayesian learning based on the observations of bankruptcies if there are a sufficient number of defaults at time τ.

Slow convergence of the priors impedes economic recovery Given the prior $C(S_t, V_t)$, a firm calculates its expected profits in the case where it lends m to another firm that chooses Technology R with the contracted repayment D. The creditor's expected profit is $\{p + (1-p)E[\theta | S_t, V_t]\}D + (1-p)(1 - E[\theta | S_t, V_t])Q_L$ where $E[\theta | S_t, V_t]$ is the expected value of θ with the p.d.f. $\xi(\theta | S_t, V_t, \alpha, \beta)$. The minimum expected profit for a creditor under the prior $C(S_t, V_t)$ is $\left\{p + (1-p)\theta^C\right\} D + (1-p)(1-\theta^C)Q_L$ where $\theta^C = \min_{\theta \in C(S_t, V_t)} E[\theta | S_t, V_t] = \frac{S_t + 1}{V_t + \gamma}$, which is attained when $\alpha = 1$ and $\beta = \gamma - 1$. The competition and arbitrage in the financial market guarantee that the creditor will offer D so that the minimum expected return is equal to the return at the risk-free rate: $(1+r)m$. Therefore,

$$D = \frac{(1+r)m - (1-p)(1-\theta^C)Q_L}{p + (1-p)\theta^C}. \tag{13}$$

If a firm borrows m from a creditor and promises to repay D, and chooses Technology R, then the expected profit for the debtor becomes $py_H - \{p + (1-p)E[\theta | S_t, V_t]\}D - (1-p)(1 - E[\theta | S_t, V_t])Q_L$. The debtor's minimum expected profit $\pi_F(S_t, V_t)$ is

$$\pi_F(S_t, V_t) = py_H - \frac{\left\{(1+r)m - (1-p)(1-\theta^C)Q_L\right\}\left(p + (1-p)\theta^D\right)}{p + (1-p)\theta^C} - (1-p)(1-\theta^D)Q_L \tag{14}$$

where $\theta^D = \max_{\theta \in C(S_t,V_t)} E[\theta|S_t,V_t] = \frac{S_t+\gamma-1}{V_t+\gamma}$, that is attained when $\alpha = \gamma - 1$ and $\beta = 1$. It is easily calculated that

$$\pi_F(S_t,V_t) = py_H - (1+r)m - \frac{(1-p)(\gamma-2)\{(1+r)m - Q_L\}}{(V_t+\gamma)p + (S_t+1)(1-p)}. \tag{15}$$

Assumption 4 implies that $\pi_F(S_t,V_t)$ is smaller than y_L when V_t is small. Therefore, all firms choose Technology S when S_t and V_t are small. Assumption 5 implies that $\pi(S_t,V_t)$ is larger than y_L for V_t that is close to N. Thus all firms choose Technology R for large V_t that is sufficiently close to N. In the extreme case where the government undertakes bankruptcy procedures for all N firms at time τ, all firms choose Technology R from time $\tau+1$ onward, even if the firms believe that the asset price is Q_L; in this case, the equilibrium asset price is revised to Q_H from $\tau+1$ onward, as a result of the firms choosing Technology R.

$\pi_F(S_t,V_t)$ is increasing in V_t because the gain from Bayesian learning increases as V_t increases. Meanwhile, equation (15) shows that $\pi_F(S_t,V_t)$ is increasing in S_t. Therefore, if the true value of θ, i.e., $\theta^* = \lim_{V_t\to\infty} \frac{S_t}{V_t}$, is larger, then the debtor's expected profits from Technolgy R tend to be larger. This fact is understood as follows: if θ^* is larger, then the creditor will recover the unpaid debt Δ with a higher probability in a bankruptcy procedure; therefore, the risk premium for bankruptcy that the creditor demands ex ante becomes smaller; the gain for the debtor from the lower interest rate overwhelms his loss from the higher probability with that he will lose Δ in the bankruptcy procedure; therefore, $\pi_F(S_t,V_t)$ tends to increase as S_t (or θ^*) increases.

2.3 Optimal choice for the government

The government chooses the schedule of bankruptcy proceedings $\{v_t\}_{t=\tau}^{\infty}$ for the N defaulters. Note that no new default occurs from time $\tau+1$ onward, since firms choose Technology S as long as $\pi_F(S_t,V_t) < y_L$, and the asset price becomes Q_H as $\pi_F(S_t,V_t)$ exceeds y_L and all firms swich to Technology R. Unless the government undertakes the bankruptcy proceedings of all the N defaulters at time τ, some of the bankruptcies will be postponed, or some firms will be bailed out by a (implicit) subsidy from the government. The government chooses v_t to minimize the discounted present value of the flow of social costs of bankruptcies and forbearance.

The objective function of the government For simplicity, we assume that v_t bankruptcies at time t cause an instantaneous social cost at time t. Let $\Phi(v_t)$ denote the instantaneous cost of bankruptcies. $\Phi(v_t)$ is a convex and increasing function of v_t.

There are two costs of the forbearance policy. One is the implicit subsidy that the government should pay to keep the non-viable firms afloat or to bail them out completely. (For example, we can consider the case where the implicit subsidy $r\Delta$ is necessary for a defaulter to continue his interest payment at time t.) For simplicity, we assume that $\Psi(N - V_t)$ denotes the instantaneous subsidy at time t for the forbearance policy for $N - V_t$ defaulters where $V_t = \sum_{t'=\tau}^{t} v_{t'}$. The function $\Psi(\cdot)$ is a weakly convex and increasing function.

The other cost of forbearance is caused by the slow convergence of expectations $C(S_t, V_t)$. Technology S is dominant at time τ and the dominant technology jumps to Technology R at the time when $C(S_t, V_t)$ converges to a certain extent. Since the speed of convergence of $C(S_t, V_t)$ is increasing in V_t, we can consider the opportunity cost ($\delta y \equiv \beta(py_H - y_L) - m$) due to the distorted choice of production technology as the cost of forbearance.

A problem for the government The problem for the government is to maximize social welfare $\{W(V_t, S_t)\}_{t=\tau}^{\infty}$ by choosing v_t for $t(\geq \tau)$. $W(V_t, S_t)$ is defined by the following Bellman equation:

$$W(V_t, S_t) = \max_{v} \left\{ -X(v) + \min_{\theta \in C(S_t, V_t)} E \left[- \sum_{s \in \Lambda_t} \Theta(s, v)\delta y + \beta \sum_{s=0}^{v} \Theta(s, v) W(V_t + v, S_t + s) \right] \right\} \tag{16}$$

where $X(v) = \Phi(v) + \Psi(N - V_t - v)$, $\delta y = \beta(py_H - y_L) - m$, $\Lambda_t = \{s : \pi_F(S_t + s, V_t + v) < y_L\}$, and $\Theta(s, v) = \frac{v!}{s!(v-s)!}\theta^s(1-\theta)^{v-s}$. Note that firms choose Technology R or S for period $t+1$ after the government chooses v and s is realized (See Timetable). It is obvious from (15) that there exists an integer $s(V_t + v, S_t)$ that is no greater than v such that

$$\Lambda_t = \{0, 1, 2, \cdots, s(V_t + v, S_t)\}. \tag{17}$$

The solution to this problem $\{v_t\}_{t=\tau}^{\infty}$ is the socially optimal bankruptcy schedule for the firms that defaulted at time τ. It is easily shown that the solution exists. See Appendix for the proof. In this optimal problem, the government takes into account that government action (v_t) affects the timing of confidence recovery, since the probability of confidence recovery is $1 - \sum_{s \in \Lambda_t} \Theta(s, v)$ for a given θ.

But in reality, the government, when facing unprecedented events, may regard the recovery of confidence as an *exogenous* event to its own action. If the government assumes that the dominant technology jumps from Technology S to Technology R at time t with an exogenous probability $(1 - q_t)$, then the problem for the government becomes the following Bellman equation:

$$W'(V_t, S_t) = \max_{v} \left\{ -X(v) + \beta \min_{\theta \in C(S_t, V_t)} E\left[\sum_{s=0}^{v} \Theta(s, v) W'(V_t + v, S_t + s) \right] \right\}, \tag{18}$$

since the gain from technology change (δy) is perceived by the government as exogenous to the government actions. In this case, the government underestimates the cost of a forbearance policy after a financial crisis.

Let us consider the extreme example where $\Psi(\cdot) = 0$. In this case, the instantaneous cost of forbearance is zero. Thus the government rationally chooses $v_t = 0$ for all $t (\geq \tau)$ if it regards the jump of the dominant technology as an exogenous event. In the equilibrium, the prior $C(S_t, V_t)$ stays at C for all t, firms choose Technology S for all t, and the equilibrium probability $q_t = 1$ for all t. Therefore, if the government does not recognize that the firms' technology choices are affected by government action, it may choose forbearance to minimize the social cost of bankruptcies although the rebuilding of confidence is hindered by the forbearance policy, and the stagnation continues.

3 Conclusion

We have analyzed a simple model of subsequent stagnation after a financial crisis, in which the government's forbearance policy hinders the Bayesian learning of private agents. The asset prices and outputs stagnate since agents cannot build confidence through learning. If the government endogenizes the effect of its own actions on learning by private agents, it can choose the optimal schedule of reform, i.e., the optimal bankruptcy schedule for those who failed during the financial crisis.

In other words, after an unprecedented economic crisis, the restructuring of failed businesses may promote economic growth through the enhancement of confidence building.

Appendix

The existence of the solution to (16)

Since we assumed the number of defaulters N is a large but finite integer, we can show the existence and uniqueness of the value function $W(V, S)$ for $V = 0, 1, \cdots, N$ and $S = 0, 1, \cdots, V$ by backward induction.

(i) For $V = N$:

In the case when the total number of bankruptcy proceedings already undertaken V is N, there is no more defaulter to go bankrupt. Thus the state variables V and S never changes. Assumption 5 guarantees that $\Lambda_t = \{s : \pi_F(S + s, V + v) < y_L\} = \emptyset$ when $V = N$. Therefore, the Bellman equation (16) implies

$$W(N, S) = 0 \text{ for all } S \in \{0, 1, \cdots, N\}.$$

(ii) Suppose that the values of $W(V, S)$ for $V = k+1, k+2, \cdots, N$ and $S = 0, 1, \cdots, V$ are known. In this case, we can calculate the value $W(k, S)$ for $S = 0, 1, \cdots, k$ as follows. Define $RH(v; k, S)$ by

$$RH(v; k, S) = -X(v) + \min_{\theta \in C(S,k)} E\left[-\sum_{s \in \Lambda_t} \Theta(s, v)\delta y + \beta \sum_{s=0}^{v} \Theta(s, v) W(k + v, S + s)\right].$$

Thus, the Bellman equation (16) is $W(k, S) = \max_v RH(v; k, S)$. The values of $RH(v; k, S)$ for $v = 1, 2, \cdots, N - k$ are calculated since the values of $W(V, S)$ for $V \geq k + 1$ are known. The value of $RH(0; k, S) = -X(0) + \beta W(k, S) = -\Phi(0) - \Psi(N - k) + \beta W(k, S)$

is indeterminate since $W(k,S)$ is what to be determined. If the choice $v=0$ maximizes the right-hand-side of the Bellman equation (16), then it must be the case that $W(k,S)=-\frac{\Phi(0)+\Psi(N-k)}{1-\beta}$. Therefore, we can calculate

$$W(k,S)=\max\left\{-\frac{\Phi(0)+\Psi(N-k)}{1-\beta}, RH(1;k,S), RH(2;k,S),\cdots,RH(N-k;k,S)\right\}.$$

Thus, $W(k,S)$ is uniquely determined and the government chooses the corresponding values of v as the number of the bankruptcy proceedings in current period. Note that the policy v at the state (k,S) may not be unique.

(iii) From (i) and (ii), all values of $W(V,S)$ are uniquely determined by backward induction. (Q.E.D)

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