Firm-to-firm Trade in Sticky Production Networks

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A growing literature studies:

- input-output networks between production units (e.g. firms, industries)
- how these networks matter for aggregate effects of production unit shocks

However, leading theories take fundamental network structure as exogenous:

- typically, network = technological I-O matrix between sectors
- hence, an exogenous technology that does not respond to shocks

How important is the dynamic adjustment of firm production networks for the aggregate effects of firm-level shocks?
Methodology

- Develop a **dynamic structural model** of trade between **heterogeneous firms** with **endogenous network** of firm-level linkages
  
  - CES roundabout production + monopolistic competition → incentive to form more links
  - fixed cost per active relationship → only “good enough” relationships desirable
  - random opportunities to create and destroy links → relationship formation gradual and forward-looking

- Estimate model using data on trading relationships between US firms
- Simulate model to study importance of **production network structure and dynamics** for aggregate effects of shocks to firm-level productivity and demand

- study endogenous formation of firm-to-firm trade networks


- allow for richer relationship heterogeneity
- simultaneously model intensive/extensive margins of traded


- model full network instead of one tier of buyers/sellers

Related to broader literature on social and economic network formation

- solve for transition dynamics without resorting to myopic agents
Outline

- Description of model:
  - (static) given network of relationships, how much do firms trade?
  - (dynamic) which relationships do firms choose to form?
- Data and model estimation:
  - data sources
  - estimation strategy
  - stylized facts and model fit
- Counterfactual exercises and results
Basic Environment

- Exogenous unit continuum of firms producing differentiated goods
- Firms heterogeneous over states $\chi \equiv (\phi, \delta)$
  - $\phi$: **fundamental productivity** (labor input more productive)
  - $\delta$: **fundamental quality** (household prefers product more)
- Exogenous distribution function $F_\chi$ and support $S_\chi \subset \mathbb{R}^2_+$
- Representative household supplies $L$ units of labor inelastically, with preferences:
  \[
  U = \left[ \int_{S_\chi} [\delta x_H(\chi)]^{\frac{\sigma - 1}{\sigma}} dF_\chi(\chi) \right]^{\frac{\sigma}{\sigma - 1}}
  \]
- Conditional on prices, household demand $x_H(\chi)$ is greater for firms with higher $\delta$
Firm-to-firm trade characterized by **production network**

Network fully specified by **matching function** $m$

- $m(\chi, \chi') = \text{probability that } \chi\text{-firm buys from } \chi'\text{-firm}$

Production CES in **labor** and **supplier inputs**, given matching function:

$$X(\chi) = \left[ \phi l(\chi) \frac{\sigma - 1}{\sigma} + \int_{\chi'} m(\chi, \chi') \left[ x(\chi, \chi') \right] \frac{\sigma - 1}{\sigma} dF_{\chi}(\chi') \right]^{\frac{\sigma}{\sigma - 1}}$$

Conditional on prices, firms with higher $\phi$ have lower marginal cost $\eta(\chi)$
Market structure: **monopolistic competition**

Continuum of sellers for each buyer ⇒ all firms charge CES markup \( \mu = \frac{\sigma}{\sigma - 1} \)

Given network, how much do firms buy and sell?
Sourcing and selling decisions

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- Given network, how much do firms buy and sell?

\[
\chi \leftrightarrow \chi'
\]

- firm-to-firm trade depends on fundamental \((\phi, \delta)\) of buyer/seller...
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- Aggregate state variable = entire network?
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- aggregate state variable $= \text{entire network}$?

- Solution: characterize firms in terms of **network productivity and quality**

\[
\Phi (\chi) \equiv \eta (\chi)^{1-\sigma} \\
\Delta (\chi) \equiv \frac{1}{\Delta H} X (\chi) \eta (\chi)^{\sigma}
\]

(inverse marginal cost)

(intermediate demand shifter)
Firm Network Characteristics

- Firm’s network characteristics depend on fundamental characteristics and network characteristics of suppliers/customers through matching function

$$\Phi(\chi) = \phi^{\sigma - 1} + \mu^{1 - \sigma} \int_{S_\chi} m(\chi, \chi') \Phi(\chi') dF_\chi(\chi')$$

$$\Delta(\chi) = \mu^{-\sigma} \delta^{\sigma - 1} + \mu^{-\sigma} \int_{S_\chi} m(\chi', \chi) \Delta(\chi') dF_\chi(\chi')$$

- Decoupled contraction mappings in $\Phi(\cdot)$ and $\Delta(\cdot)$ ⇒ easily solved

- Firm network characteristics determine all variables of interest:

  - firm revenue: $R(\chi) \propto \Delta(\chi) \Phi(\chi)$
  - firm profit: $\Pi(\chi) \propto \Delta(\chi) \Phi(\chi)$
  - firm-to-firm sales: $r(\chi, \chi') \propto \Delta(\chi) \Phi(\chi')$
  - firm-to-firm profit: $\pi(\chi, \chi') \propto \Delta(\chi) \Phi(\chi')$
Now ask: which relationships do firms choose to form?
- discrete time
- linear household preferences

CES production technology generates incentives to form links:
- constant marginal cost \( \Rightarrow \) more customers desirable
- finite, positive substitution elasticity \( \Rightarrow \) more suppliers desirable

Counterbalance incentives with two kinds of frictions in relationship formation

Relationship **reset shocks (exogenous chance)**:
- \( 1 - \nu \) opportunity to activate/terminate relationship each period
- reset shocks independent across all firm pairs and time

Relationship **cost shocks (endogenous choice)**:
- active relationship requires \( f_t = \psi \xi_t \) units of labor
- \( \xi_t \) iid across relationships and time, CDF \( F_{\xi} \) with unit mean
- zero serial correlation in \( \xi_t \) for tractability, persistence built in through \( \nu \)
Dynamic Network Formation

inactive $\chi - \chi'$ relationship at date $t - 1$
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  - with probability $1 - \nu$, reset shock received: firms select based on cost $\xi_t$
    - with probability $a_t (\chi, \chi')$, relationship activated
    - with probability $1 - a_t (\chi, \chi')$, relationship rejected
Dynamic Network Formation

- $a_t$ denotes **acceptance function**: probability that a relationship is voluntarily selected given chance to reset relationship
- Law of motion for matching function:
  
  $$m_t = m_{t-1} + (1 - \nu) a_t (1 - m_{t-1}) - (1 - \nu)(1 - a_t) m_{t-1}$$

- In steady-state, $m(\chi, \chi') = a(\chi, \chi')$
Given the chance to reset a relationship, when do firms choose to do so?

Assume that selling firm pays full share of relationship cost:

- optimal pricing is the same as before
- buying firm is always agreeable to any trading relationship

Static variable profit earned by a $\chi'$-firm from selling to $\chi$-firm at date $t$:

$$\pi_t (\chi, \chi') \propto \Delta_t (\chi) \Phi_t (\chi')$$

Acceptance function with myopic firms:

$$\tilde{a}_t (\chi, \chi') = \Pr \left[ \pi_t (\chi, \chi') \geq \psi \xi_t \right] = F_{\xi} \left[ \frac{\pi_t (\chi, \chi')}{\psi} \right]$$

But stickiness of relationships makes acceptance decisions forward-looking.
Value of selling:

\[ V_t^+ \left( \chi, \chi' | \xi_t \right) = \pi_t \left( \chi, \chi' \right) - \psi \xi_t + \beta \nu \mathbb{E}_t \left[ V_{t+1}^+ \left( \chi, \chi' | \xi_{t+1} \right) \right] + \beta (1 - \nu) \mathbb{E}_t \left[ V_{t+1}^O \left( \chi, \chi' | \xi_{t+1} \right) \right] \]

- static profit
- stuck-in value
- reset option value

Value of not selling:

\[ V_t^- \left( \chi, \chi' \right) = \beta \nu V_{t+1}^- \left( \chi, \chi' \right) + \beta (1 - \nu) \mathbb{E}_t \left[ V_{t+1}^O \left( \chi, \chi' | \xi_{t+1} \right) \right] \]

- stuck-out value
- reset option value

Reset option value:

\[ V_t^O \left( \chi, \chi' | \xi_t \right) = \max \left\{ V_t^+ \left( \chi, \chi' | \xi_t \right), V_t^- \left( \chi, \chi' \right) \right\} \]
Selling premium equals EPV of profits before relationship can be reset:

\[ V_t^+ (\chi, \chi' | \xi_t) - V_t^- (\chi, \chi') = E_t \left[ \sum_{s=0}^{\infty} (\beta \nu)^s \left[ \pi_{t+s} (\chi, \chi') - \psi \xi_{t+s} \right] \right] \]

Acceptance function with forward-looking firms:

\[ a_t (\chi, \chi') = F_\xi \left[ 1 + \sum_{s=0}^{\infty} (\beta \nu)^s \left( \frac{\pi_{t+s} (\chi, \chi')}{\psi} - 1 \right) \right] \]

Need to solve for future path of profit functions:

- guess number of periods \( T \) before convergence to post-shock steady-state
- guess \( \{\pi_{t+s}\}_{s=1}^{T} \) and solve static equilibrium at each date
- iterate on guess of \( \{\pi_{t+s}\}_{s=1}^{T} \) until convergence
- increment \( T \) until \( \pi_{t+T} \) is close enough to post-shock steady-state \( \pi \)
Relationship Selection

- In steady-state:
  \[ a(\chi, \chi') = F_\xi \left[ \frac{\pi(\chi, \chi') - \beta \nu \psi}{(1 - \beta \nu) \psi} \right] \]

- Firms with better network characteristics are more likely to trade.

- Forward-looking firm decisions imply:
  - temporarily unprofitable relationships may be activated if \( \pi(\chi, \chi') > \psi \)
  - temporarily profitable relationships may not be activated if \( \pi(\chi, \chi') < \psi \)
  - firm pairs will never trade in steady-state if \( \pi(\chi, \chi') < \beta \nu \psi \)

- Model closed using labor market clearing condition.
Model Properties and Predictions

- Existence and uniqueness
  - static market equilibrium is unique (contraction mapping theorem)
  - uniqueness of dynamic market equilibrium harder to prove, but no numerical counterexample found in simulations

- Efficiency
  - static market equilibrium is inefficient: double marginalization
  - in dynamic setting, additional source of “network externality”

- Model generates analytic predictions about:
  - firm-level revenue and degree distributions
  - assortativity of matching between firms
  - dynamics of relationships

- Take these predictions to data in order to discipline parameters of the model
Data Sources

- **Compustat data**
  - publicly-listed firms in the US
  - records of firms’ major customers (>10% revenue)
  - panel data from 1979-2008, >100,000 firm-year observations

- **Capital IQ data**
  - private and public firms
  - relationships recorded from multiple sources (publications, news reports)
  - select all firms in continental US with recorded relationship data and positive average revenue from 2003-2007
  - ~9,000 firms accounting for 54.3% of total non-farm US business revenue
Estimation Procedure

- **Parametric assumptions**
  - \( \log(\phi, \delta) \): uncorrelated Gaussian RVs with common variance \( \nu^2 \) and zero mean (scale invariance)
  - \( \xi_t \): Weibull RV with unit mean and shape \( s_\xi \)

- **7 model parameters**
  - not estimated: \( L = 1, \beta = .95, \sigma = 4 \)
  - estimated via simulated method of moments: \( \nu, \psi, s_\xi, \nu \)

- **Targeted moments**
  - firm size distribution (\( \nu \))
  - relationship retention/creation rates (\( s_\xi, \nu \))
  - 70% labor share (\( \psi \))
Model Fit

- Objective function contour plots

- Targeted moments:
  - size firm size distribution
  - dynamics relationship retention/creation rates

- Untargeted moments:
  - degree firm degree distribution
  - size-degree firm size-degree joint distribution
  - matching firm matching distributions
Use model to study aggregate effects of firm-level supply/demand shocks

- start from model steady-state at estimated parameter values
- group firms according to deciles of firm size
- solve for predicted effects of 1-s.d. shock to $\phi$ or $\delta$ for each firm group

Focus on aggregate welfare effects and role of network structure and dynamics:

- **CF1** relationship heterogeneity
- **CF2** supply chain heterogeneity
- **CF3** relationship dynamics
Conclusion

- New theory of how heterogeneous firms create/destroy trading relationships
- Tractable model with rich relationship heterogeneity and endogenous dynamics
- Simulations highlight role of network structure/dynamics in propagation and aggregation of firm-level supply and demand shocks
- Ongoing research agenda:
  - network adjustment and business cycles (paper revision)
  - role of networks in labor market outcomes (with Kory Kroft, David Price)
  - network shocks with adjustment costs (with Sungki Hong)
  - market structures that deliver efficient outcomes
  - microfoundations of relationship frictions
Rauch classification of traded products

- **US trade:**

<table>
<thead>
<tr>
<th>Traded via organized exchanges</th>
<th>2014 US Exports</th>
<th>2014 US Imports</th>
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</thead>
<tbody>
<tr>
<td>Reference priced</td>
<td>12.9%</td>
<td>17.1%</td>
</tr>
<tr>
<td>All others</td>
<td>17.0%</td>
<td>12.7%</td>
</tr>
<tr>
<td></td>
<td>70.1%</td>
<td>70.2%</td>
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</table>

- **World trade:**

  Shares of commodity categories in value of total trade (percent)

<table>
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<th>conservative aggregation</th>
<th>1970</th>
<th>1980</th>
<th>1990</th>
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<td>19.5</td>
<td>27.2</td>
<td>12.6</td>
</tr>
<tr>
<td>reference priced</td>
<td>24.0</td>
<td>21.3</td>
<td>20.3</td>
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<tr>
<td>differentiated</td>
<td>56.5</td>
<td>51.5</td>
<td>67.1</td>
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<tr>
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<td>31.7</td>
<td>16.0</td>
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<tr>
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<td>21.8</td>
<td>19.5</td>
<td>19.5</td>
</tr>
<tr>
<td>differentiated</td>
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<td>48.9</td>
<td>64.6</td>
</tr>
</tbody>
</table>
Network Formation Literature

  - but within the context of structural trade model
  - can solve for rational expectations dynamics instead of approximate best-response
Dynamic Algorithm

\[ \pi_t(x, x') \]

- pre-shock steady-state
- post-shock steady-state

\[ t_1, t_2, t_3, t_4, t_5 \]

\[ 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 \]
Market Clearing

- Labor market clearing:

\[ L - L_f = \int_{S_{\chi}} l(\chi) \, dF_\chi(\chi) \]

\[ L_f = f \int_{S_\chi} \int_{S_{\chi'}} \left[ \nu m(\chi, \chi') + (1 - \nu) \bar{\xi}(\chi, \chi') \right] \, dF_\chi(\chi) \, dF_{\chi'}(\chi') \]

\[ \bar{\xi}(\chi, \chi') = \int_{0}^{\xi_{\text{max}}(\chi, \chi')} \xi \, dF_\xi(\xi) \]

\[ \xi_{\text{max}}(\chi, \chi') = \max \left\{ \frac{\pi(\chi, \chi') - \beta \nu f}{(1 - \beta \nu) f}, 0 \right\} \]

- Output market clearing:

\[ X(\chi) = x_H(\chi) + \int_{S_{\chi}} m(\chi', \chi) \times (\chi', \chi) \, dF_{\chi'}(\chi') \]
Using labor market clearing condition, welfare is approximately equal to:

\[ U \approx (L - L_f) \left[ \int_{S_x} \int_{S_x} \left[ \sum_{d=0}^{\infty} \left( \frac{\alpha}{\mu} \right)^{d(\sigma - 1)} m(d) \left( \chi, \chi' \right) \right] \left( \delta \phi' \right)^{\sigma - 1} \right] \frac{1}{\sigma - 1} \]

Welfare is greater when high-\(\delta\) buyers are connected with high-\(\phi\) sellers, both directly and indirectly.

Welfare cost of additional relationships captured by \(L - L_f\)
Static social value of a relationship in the planner’s problem:

\[
\frac{dU_t}{d\bar{m}_t(\chi, \chi')} = C_t \left[ \pi_{t}^{SP}(\chi, \chi') - \psi \right]
\]

- \( \pi_{t}^{SP} \) is planner’s analogue of the profit function, differs only by constant term \( \mu \)
- \( C_t \) is total connectivity in the economy:

\[
C_t \equiv \left[ \int_{S_X} \int_{S_X} \left[ \sum_{d=0}^{\infty} m_{t}^{SP,(d)}(\chi, \chi') \right] \left( \delta \phi' \right)^{\sigma-1} dG_X(\chi) dG_X(\chi') \right]^{\frac{1}{\sigma-1}}
\]
Efficiency

- Planner’s acceptance function:

\[
a_t^{SP} (\chi, \chi') = F_\xi \left[ 1 + \sum_{s=0}^{\infty} (\beta \nu)^s \left( \frac{C_{t+s}}{C_t} \right) \left( \frac{\pi_{t+s}^{SP} (\chi, \chi')}{\psi} - 1 \right) \right]
\]

- Two sources of market equilibrium inefficiency
  - **double marginalization**: lowers private cost-benefit ratio of relationships relative to social ratio \(\pi_{t}^{SP} / \psi > \pi_t / \psi\)
  - **network externalities**: firms do not internalize effect of creating/destroying relationships on overall network (amplification by factor \(C_t\))
Firm-level Distributions

- Firm size:

\[ R(\chi) = \mu \Delta_H \Delta(\chi) \Phi(\chi) \]
\[ I(\chi) = (\mu - 1) \Delta_H \Delta(\chi) \Phi(\chi) + I_f(\chi) \]

- In- and out-degrees:

\[ M_S(\chi) = \int_{S_\chi} m(\chi, \chi') dF_\chi(\chi') \]
\[ M_C(\chi) = \int_{S_\chi} m(\chi', \chi) dF_\chi(\chi') \]

- Firms with better fundamental characteristics are larger and more connected
- Two dimensions of heterogeneity ⇒ imperfect correlation between firm size and degree
Matching Assortativity

Matching between $\chi$-buyer and $\chi'$-seller depends only on $\Delta_H \Delta (\chi) \Phi (\chi')$:

$$m (\chi, \chi') = \bar{m} \left[ \Delta_H \Delta (\chi) \Phi (\chi') \right] \equiv F_\xi \left[ \frac{\Delta_H \Delta (\chi) \Phi (\chi') - \beta \nu \psi}{(1 - \beta \nu) \psi} \right]$$

Assortativity, e.g. average supplier revenue:

$$\bar{R}_S (\chi) = \frac{\int_{S_\chi} \bar{m} \left[ \Delta_H \Delta (\chi) \Phi (\chi') \right] R (\chi') \, dF_\chi (\chi')}{\int_{S_\chi} \bar{m} \left[ \Delta_H \Delta (\chi) \Phi (\chi') \right] \, dF_\chi (\chi')}$$

Assortativity depends on elasticity $\epsilon_\xi$ of $F_\xi$, e.g. in special case with $\delta = \text{constant}$ and $\nu = 0$:

- $\epsilon_\xi > 0 \Rightarrow \frac{d\bar{R}_S (\phi)}{d\phi} > 0$
- $\epsilon_\xi < 0 \Rightarrow \frac{d\bar{R}_S (\phi)}{d\phi} < 0$
- $\epsilon_\xi = 0 \Rightarrow \frac{d\bar{R}_S (\phi)}{d\phi} = 0$
Relationship Dynamics

- Relationship retention rate, e.g. with suppliers:

\[ \rho_{S}^{\text{ret}} (\chi) = \frac{\nu M_{S} (\chi) + (1 - \nu) \int_{S \chi} m (\chi, \chi') a (\chi, \chi') dF_{\chi} (\chi')} {M_{S} (\chi)} \]

- Relationship creation rate, e.g. with suppliers:

\[ \rho_{S}^{\text{new}} (\chi) = \frac{(1 - \nu) \int_{S \chi} [1 - m (\chi, \chi')] a (\chi, \chi') dF_{\chi} (\chi')} {M_{S} (\chi)} \]

- Larger firms have greater relationship retention rates, lower relationship creation rates
Model Fit - Revenue Distribution

![Graph showing model fit to revenue distribution](image)

- **Data** (red dots)
- **Model** (blue line)

Quantile axis ranges from 0 to 1, and the log normalized revenue axis ranges from -15 to 10.
Model Fit - Relationship Dynamics

![Graph showing mean supplier and customer retention rate vs revenue quantile. The graph includes two lines, one for data and one for the model, indicating a positive relationship between the variables.](image)
Model Fit - Relationship Dynamics

![Graphs showing the relationship between mean fraction of new suppliers and mean fraction of new customers across different revenue quantiles. The graphs compare data (red dots) with model predictions (blue lines).]
Model Fit - Degree Distributions

The graphs show the normalized in-degree and out-degree distributions for firms, with the data represented by red dots and the model by blue lines. The x-axis represents the quantile, and the y-axis represents the normalized degree.
Model Fit - Size-Degree Distributions

![Graphs showing in-degree and out-degree quantiles against revenue quantiles. The graphs compare data points (red) and model predictions (blue).](image)
Model Fit - Matching

![Graph showing model fit and data comparison](image)

- **data**
- **model**
Model Fit - Matching

[Graphs showing the relationship between mean supplier revenue quantile and revenue quantile, and mean customer revenue quantile and revenue quantile.]

- Red dots: Data
- Blue line: Model
In model and data, relationships are distributed heterogeneously across firms.

Consider alternative model of production where matching function is:

\[ m(\chi, \chi') = \bar{m} \text{ for all } \{\chi, \chi'\} \in S^2_\chi \]

- firms identical in connectivity to other firms regardless of characteristics
- equivalent to assumption that all firms produce using common composite intermediate input (“market model”)

Reestimate parameters of market model using same SMM approach.

Compare effects of supply/demand shocks in network vs. market model.
Market model under-predicts effect of shocks to large firms and over-predicts effects of shocks to small firms

Large firms are central to the economy not only because they are large, but because they are the most connected
Supply Chain Heterogeneity

- In model and data, firms occupy different positions in various supply chains.
- Structure of model offers simple method of decomposing shock effects into changes along each stage of relevant supply chains.
- Consider short-run (fixed $m$) effects of shock $\phi \rightarrow \hat{\phi}(\phi)$.
  - $0^{th}$-order effect with no change in intermediate input prices:
    \[
    \hat{\Phi}^{(0)}(\chi) = \hat{\phi}(\phi)^{\sigma-1} + \left(\frac{\alpha}{\mu}\right)^{\sigma-1} \int_{S_\chi} m(\chi, \chi') \Phi(\chi) \, dG_{\chi}(\chi')
    \]
  - $1^{st}$-order effect with price changes only by firms directly affected:
    \[
    \hat{\Phi}^{(1)}(\chi) = \hat{\phi}(\phi)^{\sigma-1} + \left(\frac{\alpha}{\mu}\right)^{\sigma-1} \int_{S_\chi} m(\chi, \chi') \hat{\Phi}^{(0)}(\chi) \, dG_{\chi}(\chi')
    \]
  - $n^{th}$-order effect with price changes occurring up to $n$ stages downstream:
    \[
    \hat{\Phi}^{(n)}(\chi) = \hat{\phi}(\phi)^{\sigma-1} + \left(\frac{\alpha}{\mu}\right)^{\sigma-1} \int_{S_\chi} m(\chi, \chi') \hat{\Phi}^{(n-1)}(\chi) \, dG_{\chi}(\chi')
    \]
1\textsuperscript{st}-order effects typically account for over 90\% of overall short-run effects. This suggests that higher-order propagation taking network as fixed is quantitatively unimportant.
To what extent does the endogenous response of the production network matter for the aggregate effects of firm-level shocks?

Compare aggregate welfare effects over different time horizons

- short-run: holding production network fixed
- long-run: change in PDV of welfare allowing network to adjust
Relationship Dynamics

- Ratio of long-run to short-run welfare change:

<table>
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<th>shock</th>
<th>firm size decile</th>
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<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>ϕ</td>
<td>+</td>
</tr>
<tr>
<td></td>
<td>-</td>
</tr>
<tr>
<td>δ</td>
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- Magnitudes of short- and long-run effects can differ substantially
- Network adjustment has asymmetric implications for large vs. small firm shocks:
  - positive/negative shocks to small firms amplified
  - positive/negative shocks to large firms attenuated