Firm-to-firm Trade in Sticky Production Networks

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Motivation

- A growing literature studies:
  - input-output networks between production units (e.g. firms, industries)
  - how these networks matter for aggregate effects of production unit shocks
- However, leading theories take fundamental network structure as exogenous:
  - typically, network = technological I-O matrix between sectors
  - hence, an exogenous technology that does not respond to shocks
- How important is the dynamic adjustment of firm production networks for the aggregate effects of firm-level shocks?

Methodology

- Develop a **dynamic structural model** of trade between **heterogeneous firms** with **endogenous network** of firm-level linkages

CES roundabout production + monopolistic competition	$\longrightarrow$	incentive to form more links
fixed cost per active relationship	$\longrightarrow$	only "good enough" relationships desirable
random opportunities to create and destroy links	$\longrightarrow$	relationship formation gradual and forward-looking

- Estimate model using data on trading relationships between US firms
- Simulate model to study importance of **production network structure and dynamics** for aggregate effects of shocks to firm-level productivity and demand

Relation to Literature

- **Network structure, micro shocks, and macro effects:** Acemoglu et al (2012), Magerman et al (2016), Taschereau-Dumouchel (2017), Baqaee (2017), Baqaee and Farhi (2017), Grassi (2017)
  - study endogenous formation of firm-to-firm trade networks
- **Dynamics of firm-level trade networks:** Atalay et al (2011), Oberfield (2015), Chaney (2014, 2015)
  - allow for richer relationship heterogeneity
  - simultaneously model intensive/extensive margins of traded
- **Buyer-seller matches:** Bernard, Moxnes, and Saito (2015), Bernard, Moxnes, and Ulltveit-Moe (2015), Eaton, Jinkins, Tybout, and Xu (2015), Krolikowski and McCallum (2016), Monarch and Schmidt-Eisenlohr (2016)
  - model full network instead of one tier of buyers/sellers
- Related to broader literature on **social and economic network formation**
  - solve for transition dynamics without resorting to myopic agents

Outline

- Description of model:
    - (static) given network of relationships, how much do firms trade?
    - (dynamic) which relationships do firms choose to form?
- Data and model estimation:
    - data sources
    - estimation strategy
    - stylized facts and model fit
- Counterfactual exercises and results

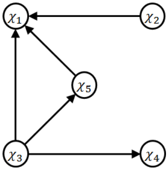
Basic Environment

- Exogenous unit continuum of firms producing differentiated goods
- Firms heterogeneous over states $\chi \equiv (\phi, \delta)$
    - ϕ: **fundamental productivity** (labor input more productive)
    - δ: **fundamental quality** (household prefers product more)
    - exogenous distribution function F_χ and support $S_\chi \subset \mathbb{R}_+^2$
- Representative household supplies L units of labor inelastically, with preferences:

$$U = \left[\int_{S_\chi} \left[\delta x_H(\chi) \right]^{\frac{\sigma-1}{\sigma}} dF_\chi(\chi) \right]^{\frac{\sigma}{\sigma-1}}$$

- Conditional on prices, household demand $x_H(\chi)$ is greater for firms with higher δ

Production Network



- Firm-to-firm trade characterized by **production network**
- Network fully specified by **matching function** m
  - $m\left(\chi, \chi'\right) =$ probability that χ-firm buys from χ'-firm
- Production CES in labor and supplier inputs, given matching function:

$$X(\chi) = \left[\left[\phi l(\chi)\right]^{\frac{\sigma-1}{\sigma}} + \int_{S_\chi} m\left(\chi, \chi'\right) \left[x\left(\chi, \chi'\right)\right]^{\frac{\sigma-1}{\sigma}} dF_\chi\left(\chi'\right) \right]^{\frac{\sigma}{\sigma-1}}$$

- Conditional on prices, firms with higher ϕ have lower marginal cost $\eta(\chi)$

Sourcing and selling decisions

- Market structure: **monopolistic competition**
- Continuum of sellers for each buyer $\Rightarrow$ all firms charge CES markup $\mu = \frac{\sigma}{\sigma - 1}$
- Given network, how much do firms buy and sell?

$$\chi \longleftarrow \chi'$$

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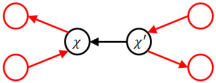

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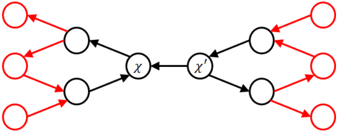

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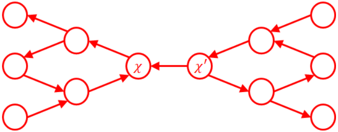

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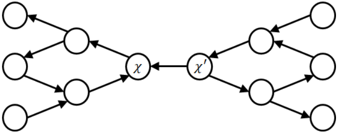

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  - aggregate state variable = entire network?
- Solution: characterize firms in terms of **network productivity and quality**

$$\begin{aligned} \Phi(\chi) &\equiv \eta(\chi)^{1-\sigma} && \text{(inverse marginal cost)} \\ \Delta(\chi) &\equiv \frac{1}{\Delta_H} X(\chi)\, \eta(\chi)^{\sigma} && \text{(intermediate demand shifter)} \end{aligned}$$

Firm Network Characteristics

- Firm's network characteristics depend on fundamental characteristics and network characteristics of suppliers/customers through matching function

$$\Phi(\chi) = \phi^{\sigma-1} + \mu^{1-\sigma} \int_{S_\chi} m\left(\chi, \chi'\right) \Phi\left(\chi'\right) dF_\chi\left(\chi'\right)$$

$$\Delta(\chi) = \mu^{-\sigma} \delta^{\sigma-1} + \mu^{-\sigma} \int_{S_\chi} m\left(\chi', \chi\right) \Delta\left(\chi'\right) dF_\chi\left(\chi'\right)$$

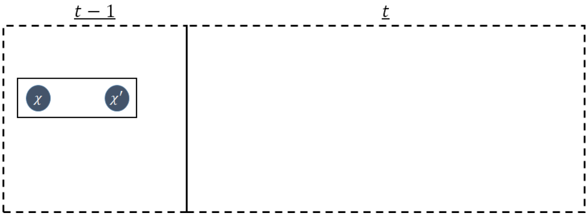
- Decoupled contraction mappings in $\Phi(\cdot)$ and $\Delta(\cdot)$ $\Rightarrow$ easily solved
- Firm network characteristics determine all variables of interest:

$$\begin{aligned}
\text{firm revenue:} \quad & R(\chi) && \propto \Delta(\chi)\,\Phi(\chi) \\
\text{firm profit:} \quad & \Pi(\chi) && \propto \Delta(\chi)\,\Phi(\chi) \\
\text{firm-to-firm sales:} \quad & r\left(\chi, \chi'\right) && \propto \Delta(\chi)\,\Phi\left(\chi'\right) \\
\text{firm-to-firm profit:} \quad & \pi\left(\chi, \chi'\right) && \propto \Delta(\chi)\,\Phi\left(\chi'\right)
\end{aligned}$$

Dynamic Network Formation

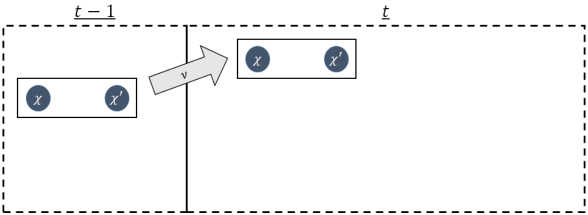
- Now ask: which relationships do firms choose to form?
  - discrete time
  - linear household preferences
- CES production technology generates incentives to form links:
  - constant marginal cost $\Rightarrow$ more customers desirable
  - finite, positive substitution elasticity $\Rightarrow$ more suppliers desirable
- Counterbalance incentives with two kinds of frictions in relationship formation
- Relationship **reset shocks (exogenous chance)**:
  - $1-\nu$ opportunity to activate/terminate relationship each period
  - reset shocks independent across all firm pairs and time
- Relationship **cost shocks (endogenous choice)**:
  - active relationship requires $f_t = \psi\xi_t$ units of labor
  - ξ_t *iid* across relationships and time, CDF F_ξ with unit mean
  - zero serial correlation in ξ_t for tractability, persistence built in through ν

Dynamic Network Formation



- Inactive $\chi - \chi'$ relationship at date $t-1$

Dynamic Network Formation



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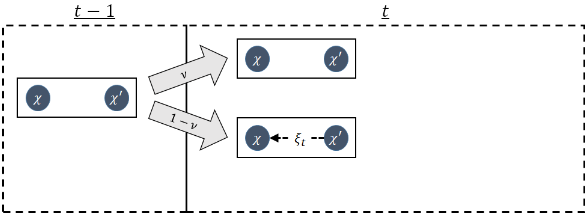

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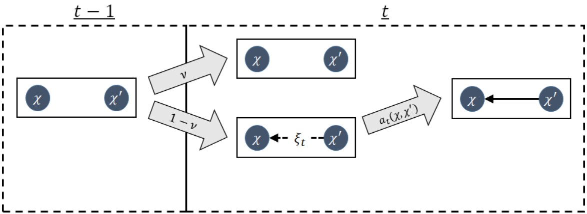

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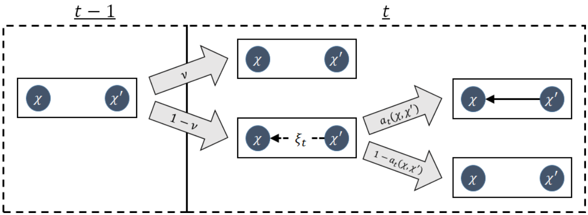
Dynamic Network Formation



- Inactive $\chi - \chi'$ relationship at date $t-1$
  - with probability ν, no reset shock received: relationship remains inactive
  - with probability $1-\nu$, reset shock received: firms select based on cost ξ_t
    - with probability $a_t\left(\chi, \chi'\right)$, relationship activated
    - with probability $1 - a_t\left(\chi, \chi'\right)$, relationship rejected

Dynamic Network Formation

- a_t denotes **acceptance function**: probability that a relationship is voluntarily selected given chance to reset relationship
- Law of motion for matching function:

$$
\begin{aligned}
m_t &= \underbrace{m_{t-1}}_{\text{existing relationships}} + \underbrace{(1-\nu)\, a_t\, (1-m_{t-1})}_{\text{newly created relationships}} - \underbrace{(1-\nu)\,(1-a_t)\, m_{t-1}}_{\text{terminated relationships}} \\
&= \nu m_{t-1} + (1-\nu)\, a_t
\end{aligned}
$$

- In steady-state, $m\left(\chi,\chi^{'}\right) = a\left(\chi,\chi^{'}\right)$

Relationship Selection

- Given the chance to reset a relationship, when do firms choose to do so?
- Assume that selling firm pays full share of relationship cost:
  - optimal pricing is the same as before
  - buying firm is always agreeable to any trading relationship
- Static variable profit earned by a χ'-firm from selling to χ-firm at date t:

$$\pi_t\left(\chi,\chi'\right) \propto \Delta_t\left(\chi\right)\Phi_t\left(\chi'\right)$$

- Acceptance function with myopic firms:

$$\tilde{a}_t\left(\chi,\chi'\right) = \Pr\left[\pi_t\left(\chi,\chi'\right) \geq \psi\xi_t\right] = F_\xi\left[\frac{\pi_t\left(\chi,\chi'\right)}{\psi}\right]$$

- But stickiness of relationships makes acceptance decisions forward-looking

Relationship Selection

- Value of selling:

$$V_t^+\left(\chi,\chi^{'}|\xi_t\right)=\underbrace{\pi_t\left(\chi,\chi^{'}\right)-\psi\xi_t}_{\text{static profit}}+\underbrace{\beta\nu\mathbb{E}_t\left[V_{t+1}^+\left(\chi,\chi^{'}|\xi_{t+1}\right)\right]}_{\text{stuck-in value}}+\underbrace{\beta\left(1-\nu\right)\mathbb{E}_t\left[V_{t+1}^O\left(\chi,\chi^{'}|\xi_{t+1}\right)\right]}_{\text{reset option value}}$$

- Value of not selling:

$$V_t^-\left(\chi,\chi^{'}\right)=\underbrace{\beta\nu V_{t+1}^-\left(\chi,\chi^{'}\right)}_{\text{stuck-out value}}+\underbrace{\beta\left(1-\nu\right)\mathbb{E}_t\left[V_{t+1}^O\left(\chi,\chi^{'}|\xi_{t+1}\right)\right]}_{\text{reset option value}}$$

- Reset option value:

$$V_t^O\left(\chi,\chi^{'}|\xi_t\right)=\max\left\{V_t^+\left(\chi,\chi^{'}|\xi_t\right),V_t^-\left(\chi,\chi^{'}\right)\right\}$$

Relationship Selection

- Selling premium equals EPV of profits before relationship can be reset:

$$V_t^+\left(\chi,\chi^{'}|\xi_t\right)-V_t^-\left(\chi,\chi^{'}\right)=\mathbb{E}_t\left[\sum_{s=0}^{\infty}(\beta\nu)^s\left[\pi_{t+s}\left(\chi,\chi^{'}\right)-\psi\xi_{t+s}\right]\right]$$

- Acceptance function with forward-looking firms:

$$a_t\left(\chi,\chi^{'}\right)=F_\xi\left[1+\sum_{s=0}^{\infty}(\beta\nu)^s\left[\frac{\pi_{t+s}\left(\chi,\chi^{'}\right)}{\psi}-1\right]\right]$$

- Need to solve for future path of profit functions dynamic algorithm
  - guess number of periods T before convergence to post-shock steady-state
  - guess $\{\pi_{t+s}\}_{s=1}^{T}$ and solve static equilibrium at each date
  - iterate on guess of $\{\pi_{t+s}\}_{s=1}^{T}$ until convergence
  - increment T until π_{t+T} is close enough to post-shock steady-state π

Relationship Selection

- In steady-state:

$$a\left(\chi,\chi^{'}\right)=F_{\xi}\left[\frac{\pi\left(\chi,\chi^{'}\right)-\beta\nu\psi}{\left(1-\beta\nu\right)\psi}\right]$$

- Firms with better network characteristics are more likely to trade
- Forward-looking firm decisions imply:
  - temporarily unprofitable relationships may be activated if $\pi\left(\chi,\chi^{'}\right)>\psi$
  - temporarily profitable relationships may not be activated if $\pi\left(\chi,\chi^{'}\right)<\psi$
  - firm pairs will never trade in steady-state if $\pi\left(\chi,\chi^{'}\right)<\beta\nu\psi$
- Model closed using labor market clearing condition market clearing

Model Properties and Predictions

- Existence and uniqueness
  - static market equilibrium is unique (contraction mapping theorem)
  - uniqueness of dynamic market equilibrium harder to prove, but no numerical counterexample found in simulations
- Efficiency
  - static market equilibrium is inefficient: double marginalization
  - in dynamic setting, additional source of "network externality" efficiency
- Model generates analytic predictions about:
  - firm-level revenue and degree distributions distributions
  - assortativity of matching between firms matching
  - dynamics of relationships dynamics
- Take these predictions to data in order to discipline parameters of the model

Data Sources

- Compustat data
  - publicly-listed firms in the US
  - records of firms' major customers ($>$10% revenue)
  - panel data from 1979-2008, $>$100,000 firm-year observations
  - relationship data also exploited by Atalay et al (2011), Barrot and Sauvagnat (2016), Taschereau-Dumouchel (2017)
- Capital IQ data
  - private and public firms
  - relationships recorded from multiple sources (publications, news reports)
  - select all firms in continental US with recorded relationship data and positive average revenue from 2003-2007
  - $\sim$9,000 firms accounting for 54.3% of total non-farm US business revenue

Estimation Procedure

- Parametric assumptions
  - $\log(\phi, \delta)$: uncorrleated Gaussian RVs with common variance v^2 and zero mean (scale invariance)
  - ξ_t: Weibull RV with unit mean and shape s_ξ
- 7 model parameters
  - not estimated: $L = 1$, $\beta = .95$, $\sigma = 4$
  - estimated via simulated method of moments: v, ψ, s_ξ, ν
- Targeted moments:
  - firm size distribution (v)
  - relationship retention/creation rates (s_ξ, ν)
  - 70% labor share (ψ)

Model Fit

- Objective function contour plots objfun
- Targeted moments:
  - size firm size distribution
  - dynamics relationship retention/creation rates
- Untargeted moments:
  - degree firm degree distribution
  - size-degree firm size-degree joint distribution
  - matching firm matching distributions

Counterfactual Exercises

- Use model to study aggregate effects of firm-level supply/demand shocks
  - start from model steady-state at estimated parameter values
  - group firms according to deciles of firm size
  - solve for predicted effects of 1-s.d. shock to ϕ or δ for each firm group
- Focus on aggregate welfare effects and role of network structure and dynamics:
  - CF1 relationship heterogeneity
  - CF2 supply chain heterogeneity
  - CF3 relationship dynamics

Conclusion

- New theory of how heterogeneous firms create/destroy trading relationships
- Tractable model with rich relationship heterogeneity and endogenous dynamics
- Simulations highlight role of network structure/dynamics in propagation and aggregation of firm-level supply and demand shocks
- Ongoing research agenda:
  - network adjustment and business cyckes (paper revision)
  - role of networks in labor market outcomes (with Kory Kroft, David Price)
  - network shocks with adjustment costs (with Sungki Hong)
  - market structures that deliver efficient outcomes
  - microfoundations of relationship frictions

Rauch classification of traded products

- US trade:

	2014 US Exports	2014 US Imports
Traded via organized exchanges	12.9%	17.1%
Reference priced	17.0%	12.7%
All others	70.1%	70.2%

- World trade:

Shares of commodity categories in value of total trade (percent)

		1970	1980	1990
Conservative Aggregation	Organized exchange	19.5	27.2	12.6
	Reference priced	24.0	21.3	20.3
	Differentiated	56.5	51.5	67.1
Liberal Aggregation	Organized exchange	24.7	31.7	16.0
	Reference priced	21.8	19.5	19.5
	Differentiated	53.6	48.9	64.6

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Network Formation Literature

- Statistical models: e.g. Erdös-Rényi (1959), Watts-Strogatz (1998), Barabási-Albert (1999), Atalay et al (2011)
- Strategic network formation models: Aumann & Myerson (1988), Myerson (1991), Jackson & Wollinksy (1996), Kranton & Minehart (2001)
- Approach here is combination of chance and choice, similar in spirit to Bala and Goyal (2000), Watts (2001), Jackson and Watts (2002)
  - but within the context of structural trade model
  - can solve for rational expectations dynamics instead of approximate best-response

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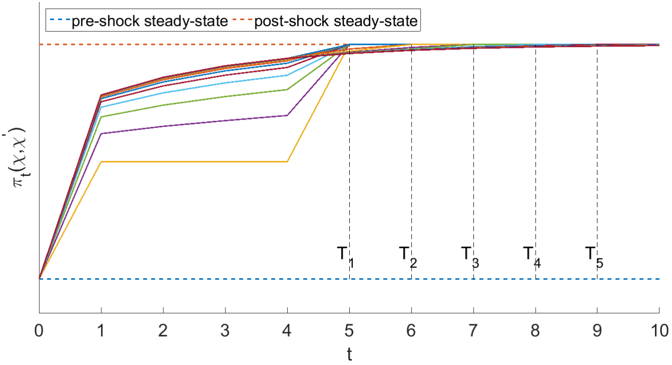
Dynamic Algorithm



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Market Clearing

- Labor market clearing:

$$L - L_f = \int_{S_\chi} l(\chi)\, dF_\chi(\chi)$$

$$L_f = f \int_{S_\chi} \int_{S_\chi} \left[\nu m \left(\chi, \chi' \right) + (1-\nu)\, \bar{\xi} \left(\chi, \chi' \right) \right] dF_\chi(\chi)\, dF_\chi \left(\chi' \right)$$

$$\bar{\xi} \left(\chi, \chi' \right) = \int_0^{\xi_{max}\left(\chi, \chi' \right)} \xi\, dF_\xi(\xi)$$

$$\xi_{max} \left(\chi, \chi' \right) = \max \left\{ \frac{\pi \left(\chi, \chi' \right) - \beta \nu f}{(1 - \beta\nu) f}, 0 \right\}$$

- Output market clearing:

$$X(\chi) = x_H(\chi) + \int_{S_\chi} m \left(\chi', \chi \right) x \left(\chi', \chi \right) dF_\chi(\chi')$$

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Household Welfare

- Using labor market clearing condition, welfare is approximately equal to:

$$U \approx (L - L_f) \left[\int_{S_\chi} \int_{S_\chi} \left[\sum_{d=0}^{\infty} \left(\frac{\alpha}{\mu} \right)^{d(\sigma-1)} m^{(d)} \left(\chi, \chi' \right) \right] \left(\delta \phi' \right)^{\sigma - 1} \right]^{\frac{1}{\sigma-1}}$$

- Welfare is greater when high-δ buyers are connected with high-ϕ sellers, both directly and indirectly
- Welfare cost of additional relationships captured by $L - L_f$

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Efficiency

- Static social value of a relationship in the planner's problem:

$$\frac{dU_t}{d\bar{m}_t\left(\chi,\chi'\right)} = \mathcal{C}_t\left[\pi_t^{SP}\left(\chi,\chi^{'}\right)-\psi\right]$$

- π_t^{SP} is planner's analogue of the profit function, differs only by constant term μ
- C_t is total connectivity in the economy:

$$\mathcal{C}_t \equiv \left[\int_{S_\chi}\int_{S_\chi}\left[\sum_{d=0}^{\infty} m_t^{SP,(d)}\left(\chi,\chi^{'}\right)\right]\left(\delta\phi^{'}\right)^{\sigma-1} dG_\chi\left(\chi\right) dG_\chi\left(\chi^{'}\right)\right]^{\frac{1}{\sigma-1}}$$

Efficiency

- Planner's acceptance function:

$$a_t^{SP}\left(\chi,\chi'\right) = F_\xi\left[1+\sum_{s=0}^{\infty}(\beta\nu)^s\left(\frac{\mathcal{C}_{t+s}}{\mathcal{C}_t}\right)\left[\frac{\pi_{t+s}^{SP}\left(\chi,\chi'\right)}{\psi}-1\right]\right]$$

- Two sources of market equilibrium inefficiency
  - **double marginalization**: lowers private cost-benefit ratio of relationships relative to social ratio ($\pi_t^{SP}/\psi > \pi_t/\psi$)
  - **network externalities**: firms do not internalize effect of creating/destroying relationships on overall network (amplification by factor $\mathcal{C}_t$)

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Firm-level Distributions

- Firm size:

$$R(\chi) = \mu \Delta_H \Delta(\chi) \Phi(\chi)$$
$$I(\chi) = (\mu - 1) \Delta_H \Delta(\chi) \Phi(\chi) + I_f(\chi)$$

- In- and out-degrees:

$$M_S(\chi) = \int_{S_\chi} m\left(\chi, \chi'\right) dF_\chi\left(\chi'\right)$$
$$M_C(\chi) = \int_{S_\chi} m\left(\chi', \chi\right) dF_\chi\left(\chi'\right)$$

- Firms with better fundamental characteristics are larger and more connected
- Two dimensions of heterogeneity $\Rightarrow$ imperfect correlation between firm size and degree

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Matching Assortativity

- Matching between χ-buyer and $\chi^{'}$-seller depends only on $\Delta_H \Delta(\chi) \Phi\left(\chi^{'}\right)$:

$$m\left(\chi, \chi^{'}\right) = \tilde{m}\left[\Delta_H \Delta(\chi) \Phi\left(\chi^{'}\right)\right] \equiv F_{\xi}\left[\frac{\Delta_H \Delta(\chi) \Phi\left(\chi^{'}\right) - \beta\nu\psi}{(1-\beta\nu)\psi}\right]$$

- Assortativity, e.g. average supplier revenue:

$$\bar{R}_S(\chi) = \frac{\int_{S_\chi} \tilde{m}\left[\Delta_H \Delta(\chi) \Phi\left(\chi^{'}\right)\right] R\left(\chi^{'}\right) dF_\chi\left(\chi^{'}\right)}{\int_{S_\chi} \tilde{m}\left[\Delta_H \Delta(\chi) \Phi\left(\chi^{'}\right)\right] dF_\chi\left(\chi^{'}\right)}$$

- Assortativity depends on elasticity ϵ_ξ of F_ξ, e.g. in special case with $\delta = \text{constant}$ and $\nu = 0$:
  - $\epsilon_\xi > 0 \Rightarrow \frac{d\bar{R}_S(\phi)}{d\phi} > 0$
  - $\epsilon_\xi < 0 \Rightarrow \frac{d\bar{R}_S(\phi)}{d\phi} < 0$
  - $\epsilon_\xi = 0 \Rightarrow \frac{d\bar{R}_S(\phi)}{d\phi} = 0$

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Relationship Dynamics

- Relationship retention rate, e.g. with suppliers:

$$\rho_S^{ret}(\chi) = \frac{\nu M_S(\chi) + (1-\nu)\int_{S_\chi} m\left(\chi, \chi'\right) a\left(\chi, \chi'\right) dF_\chi\left(\chi'\right)}{M_S(\chi)}$$

- Relationship creation rate, e.g. with suppliers:

$$\rho_S^{new}(\chi) = \frac{(1-\nu)\int_{S_\chi} \left[1 - m\left(\chi, \chi'\right)\right] a\left(\chi, \chi'\right) dF_\chi\left(\chi'\right)}{M_S(\chi)}$$

- Larger firms have greater relationship retention rates, lower relationship creation rates

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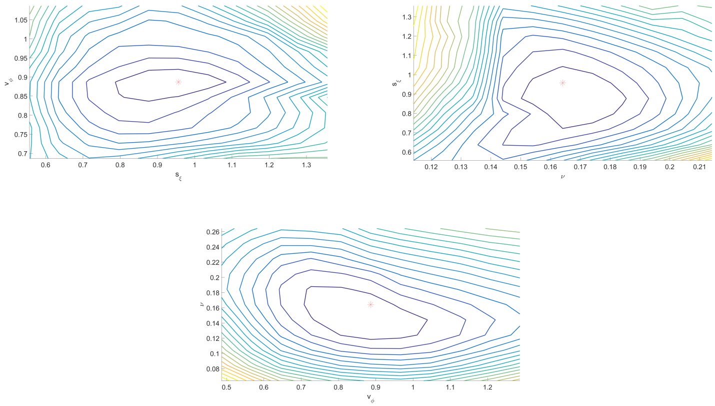
Objective Function



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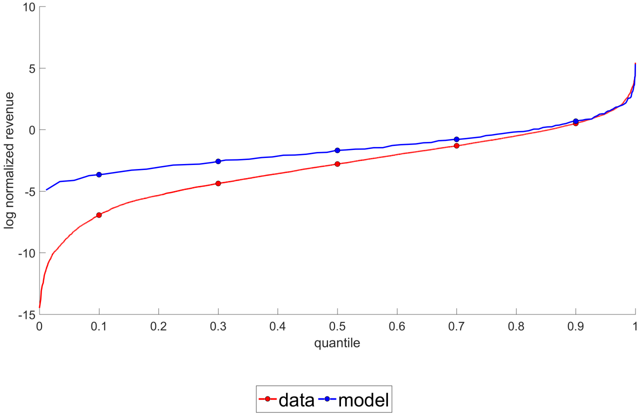
Model Fit - Revenue Distribution



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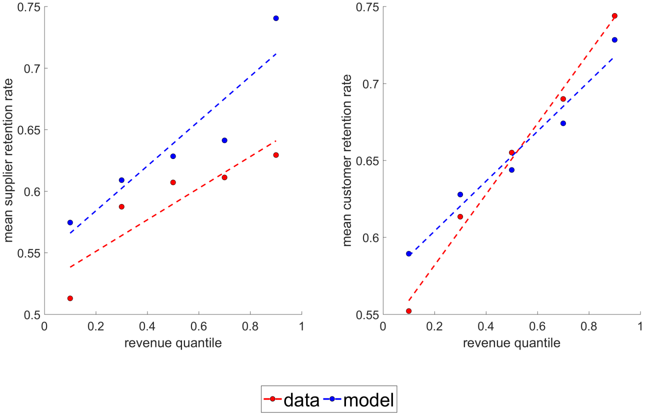
Model Fit - Relationship Dynamics



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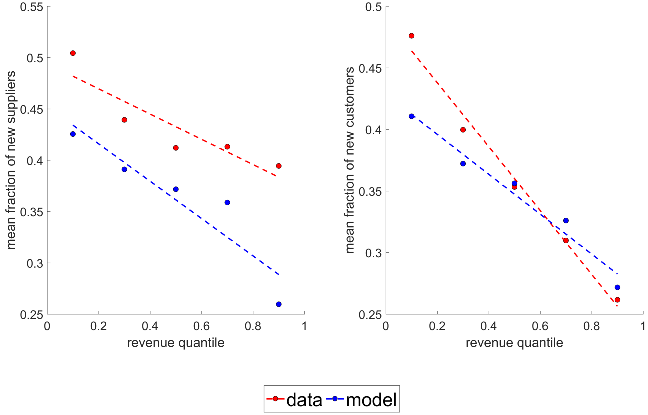
Model Fit - Relationship Dynamics



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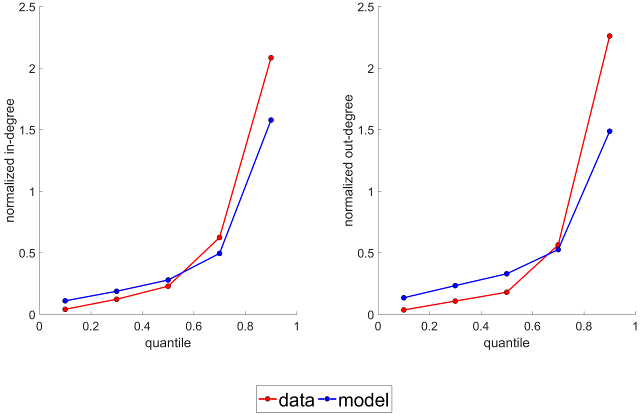
Model Fit - Degree Distributions



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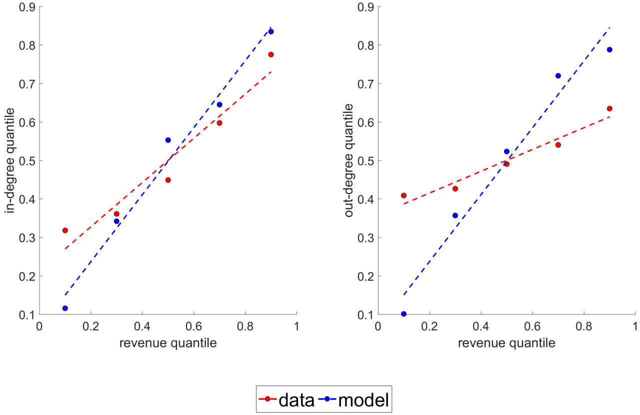
Model Fit - Size-Degree Distributions



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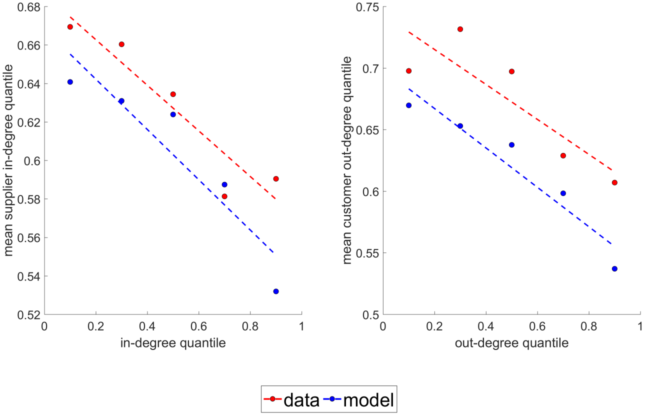
Model Fit - Matching



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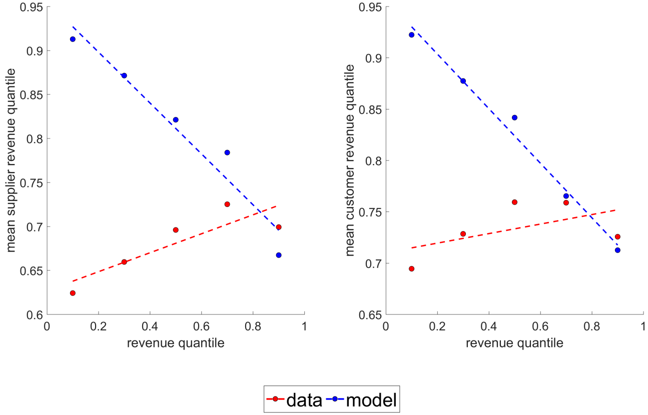
Model Fit - Matching



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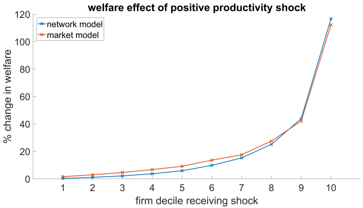
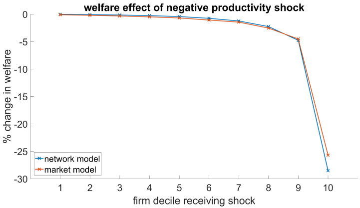
Relationship Heterogeneity

- In model and data, relationships are distributed heterogeneously across firms
- Consider alternative model of production where matching function is $m\left(\chi, \chi^{'}\right) = \bar{m}$ for all $\left\{\chi, \chi^{'}\right\} \in S_{\chi}^{2}$
  - firms identical in connectivity to other firms regardless of characteristics
  - equivalent to assumption that all firms produce using common composite intermediate input ("market model")
- Reestimate parameters of market model using same SMM approach
- Compare effects of supply/demand shocks in network vs. market model

Relationship Heterogeneity





- Market model under-predicts effect of shocks to large firms and over-predicts effects of shocks to small firms
- Large firms are central to the economy not only because they are large, but because they are the most connected

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Supply Chain Heterogeneity

- In model and data, firms occupy different positions in various supply chains
- Structure of model offers simple method of decomposing shock effects into changes along each stage of relevant supply chains
- Consider short-run (fixed m) effects of shock $\phi \rightarrow \hat{\phi}(\phi)$
  - 0^{th}-order effect with no change in intermediate input prices:

$$\hat{\Phi}^{(0)}(\chi) = \hat{\phi}(\phi)^{\sigma-1} + \left(\frac{\alpha}{\mu}\right)^{\sigma-1} \int_{S_\chi} m\left(\chi, \chi'\right) \Phi(\chi)\, dG_\chi\left(\chi'\right)$$

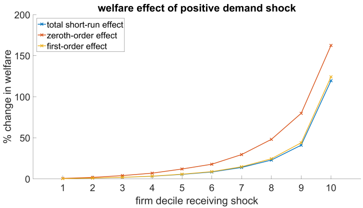
  - 1^{st}-order effect with price changes only by firms directly affected:

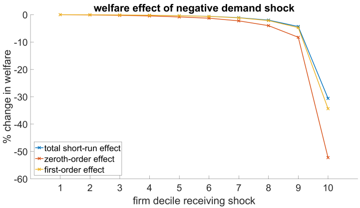
$$\hat{\Phi}^{(1)}(\chi) = \hat{\phi}(\phi)^{\sigma-1} + \left(\frac{\alpha}{\mu}\right)^{\sigma-1} \int_{S_\chi} m\left(\chi, \chi'\right) \hat{\Phi}^{(0)}(\chi)\, dG_\chi\left(\chi'\right)$$

  - n^{th}-order effect with price changes occurring up to n stages downstream:

$$\hat{\Phi}^{(n)}(\chi) = \hat{\phi}(\phi)^{\sigma-1} + \left(\frac{\alpha}{\mu}\right)^{\sigma-1} \int_{S_\chi} m\left(\chi, \chi'\right) \hat{\Phi}^{(n-1)}(\chi)\, dG_\chi\left(\chi'\right)$$

Supply Chain Heterogeneity





- 1^{st}-order effects typically account for over 90% of overall short-run effects
- Suggests that higher-order propagation taking network as fixed is quantitatively unimportant

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Relationship Dynamics

- To what extent does the endogenous response of the production network matter for the aggregate effects of firm-level shocks?
- Compare aggregate welfare effects over different time horizons
  - short-run: holding production network fixed
  - long-run: change in PDV of welfare allowing network to adjust

Relationship Dynamics

- Ratio of long-run to short-run welfare change:

shock		firm size decile									
		1	2	3	4	5	6	7	8	9	10
ϕ	+	3.33	2.22	1.89	1.70	1.55	1.42	1.30	1.20	1.09	0.99
	-	1.47	1.32	1.20	1.12	1.06	1.02	0.99	0.97	0.96	0.84
δ	+	3.63	2.36	2.00	1.77	1.60	1.44	1.31	1.20	1.09	0.99
	-	1.69	1.59	1.37	1.24	1.15	1.08	1.03	0.99	0.97	0.87

- Magnitudes of short- and long-run effects can differ substantially
- Network adjustment has asymmetric implications for large vs. small firm shocks:
  - positive/negative shocks to small firms amplified
  - positive/negative shocks to large firms attenuated

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