Direct Evidence for Synchronization in International Business Cycle

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Outline

- Limit Cycle in Macro Economic System
- Evidence of the Synchronization:
  Frequency Entrainment and Partial Phase Locking
- Common and Individual Shocks
- Coupled Limit Cycle Oscillator Model
  Origin of Synchronization: International Trade

Background

- The synchronization in international business cycle attracts economists and physicist as an example of the self-organization in the time domain*.

* P. R. Krugman, “The Self-Organizing Economy”, Cambridge, Mass., and Oxford: Blackwell Publishers (1996).

- Synchronization of the business cycle has been discussed using correlation coefficients between GDP time series**. However more definitive discussion using a suitable quantity describing the business cycle is needed.

** J. H. Stock and M. W. Watson, “Understanding Changes in International Business Cycle Dynamics”, Journal of the European Economic Association, 3 (5) pp.968-1006 (2005).

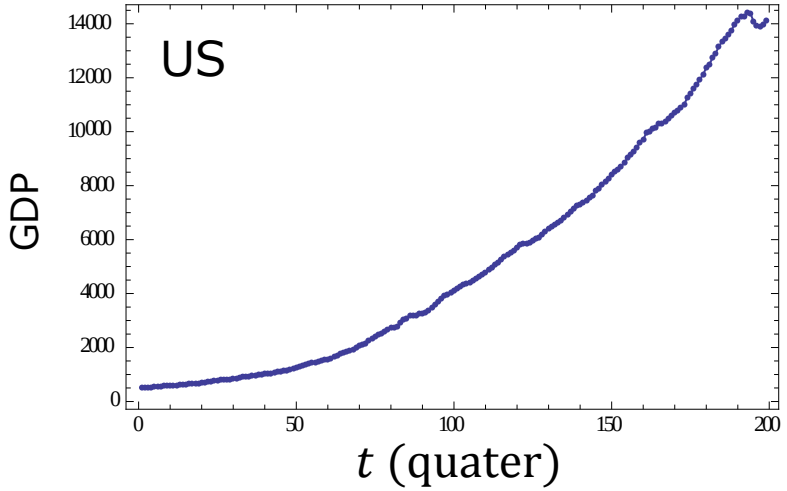
Purpose

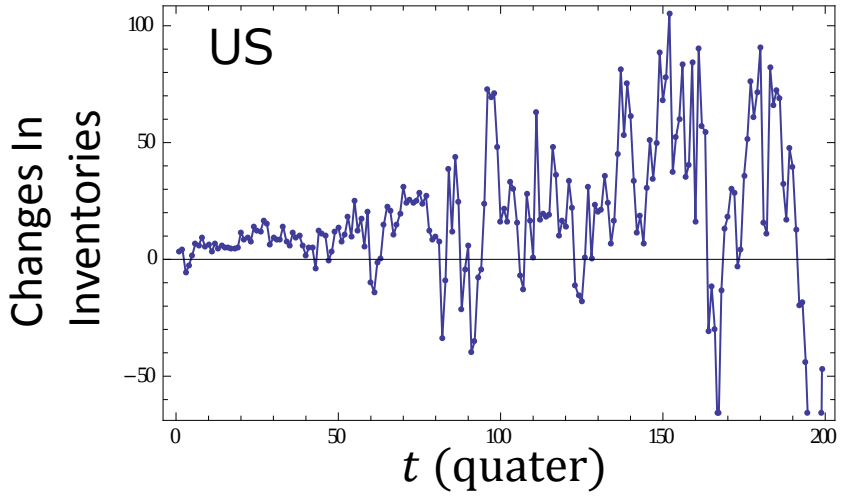
The quarterly GDP time series is available during the last 50 years for Australia, Canada, France, UK, Italy, and US. For these 6 countries we study international business cycle in order to answer the following questions:

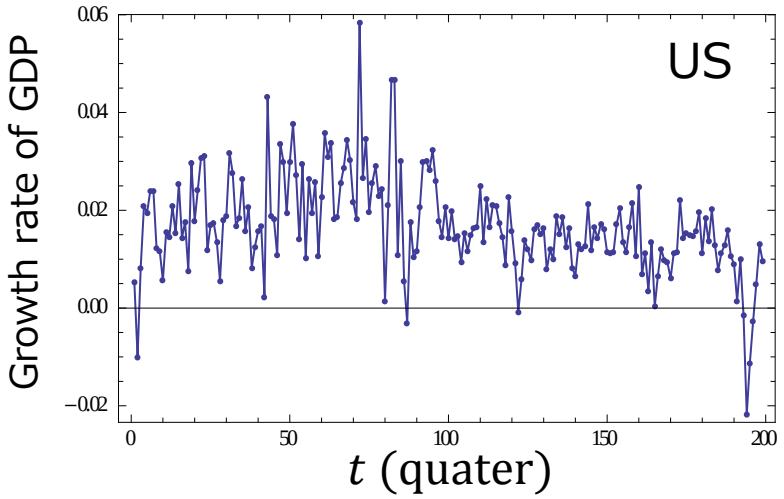
- What is the direct evidence for the synchronization?
- What is the origin of the synchronization?
- What is an appropriate model of the business cycle?

Data

- We analyze the quarterly GDP time series (OECD Quarterly National Accounts, QNA) for Australia, Canada, France, Italy, UK, and US from 1960/2Q to 2010/1Q to study the synchronization in international Business Cycle.

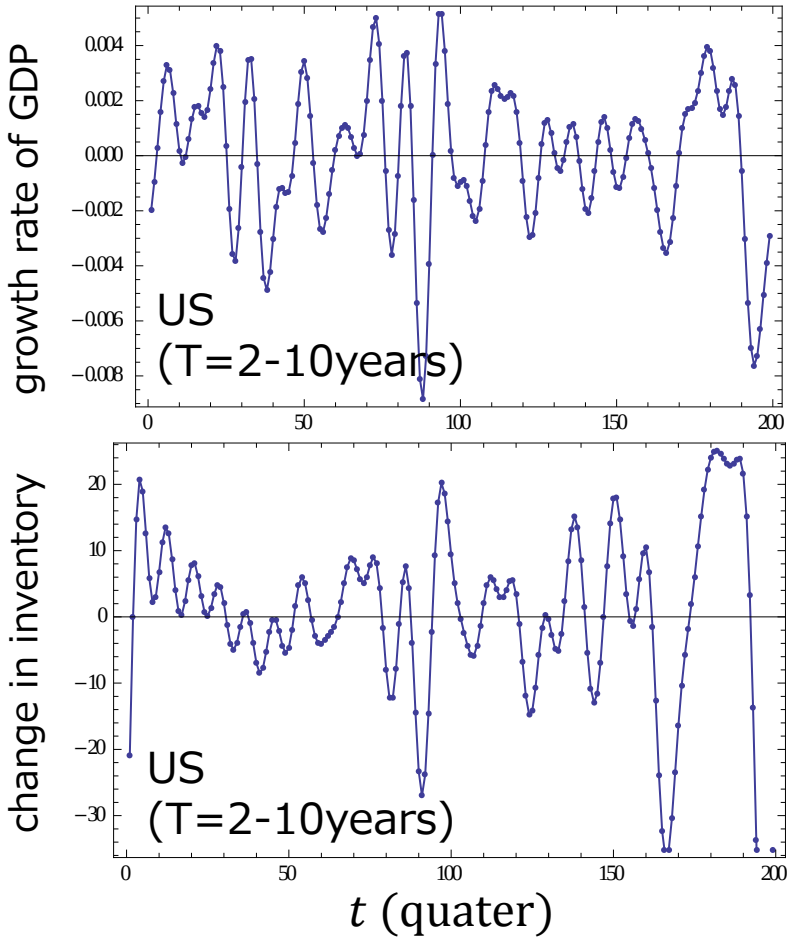


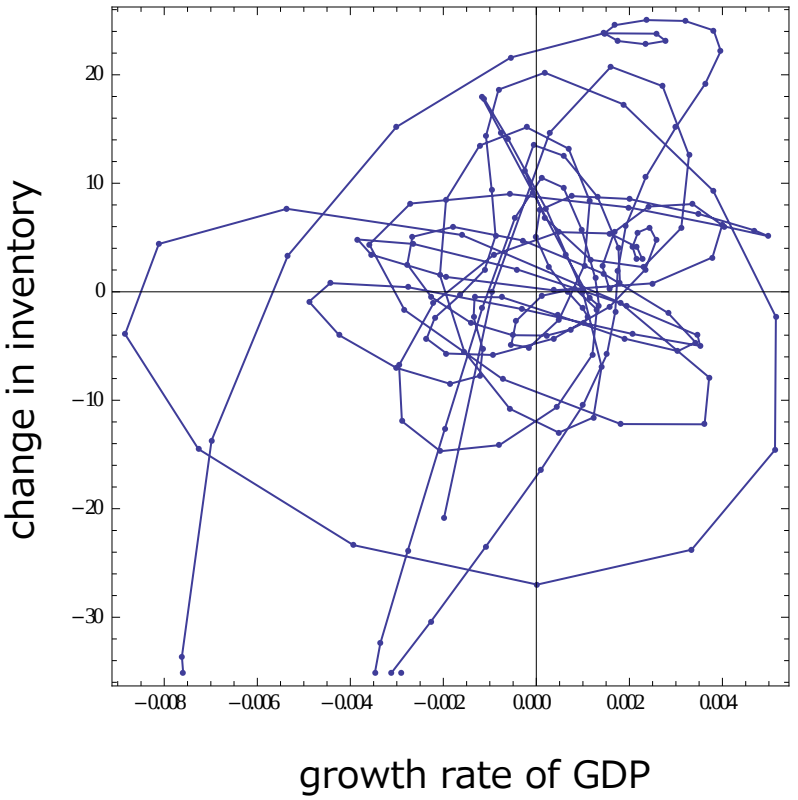




GDP and Inventory

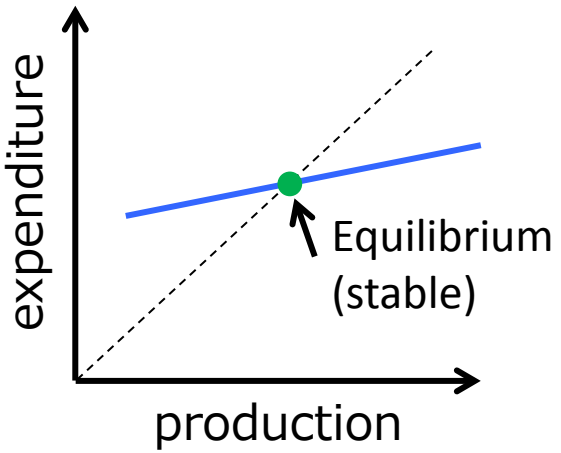
- The business cycle is due to the inventory adjustment. The existence of a limit cycle is suggested.



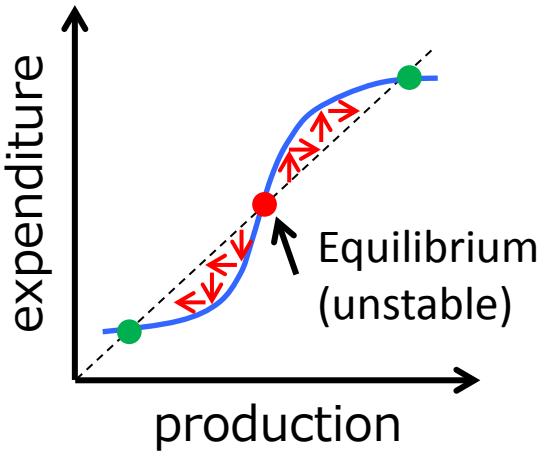


Limit Cycle (Kaldor 1940)

Production=
investment+
consumption

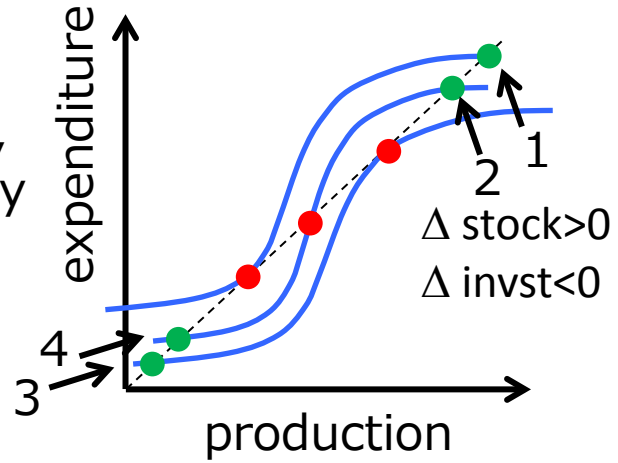


(a) Linear expenditure function

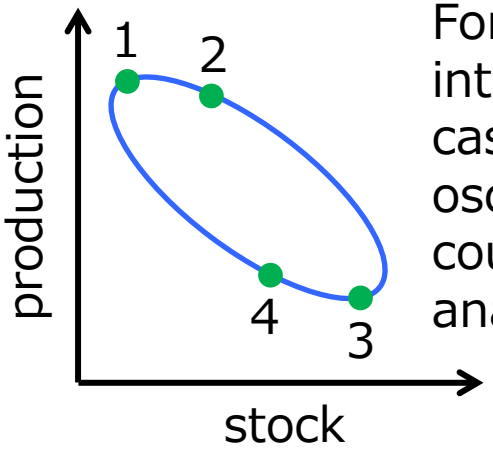


(b) S-shape expenditure function

Stock=
fixed asset,
or inventory



(c) expenditure as a function of stock



(d) Limit cycle

For the international case, limit cycle oscillators will be coupled. Let us analyze data.

Hilbert Transformation

- A complex time series is obtained by adopting the time series $y(t)$ as a imaginary part. Consequently a phase time series $\theta(t)$ is obtained.

$$z(t) = x(t) + iy(t) = A(t)exp[i\theta(t)]$$

$$y(t) = H[x(t)] = \frac{1}{\pi} PV \int_{-\infty}^{\infty} \frac{x(s)}{t-s} ds$$

(PV: the Cauchy principal value)

- Example:

$$x(t) = cos(\omega t)$$

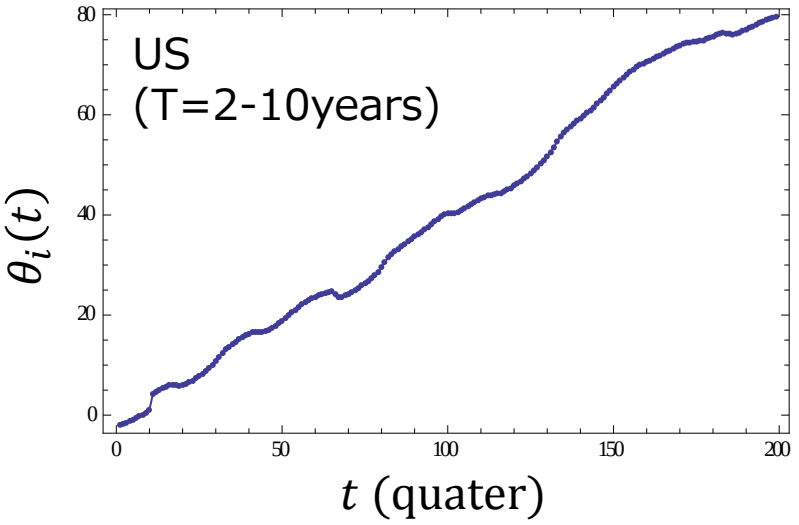
$$y(t) = H[cos(\omega t)] = sin(\omega t)$$

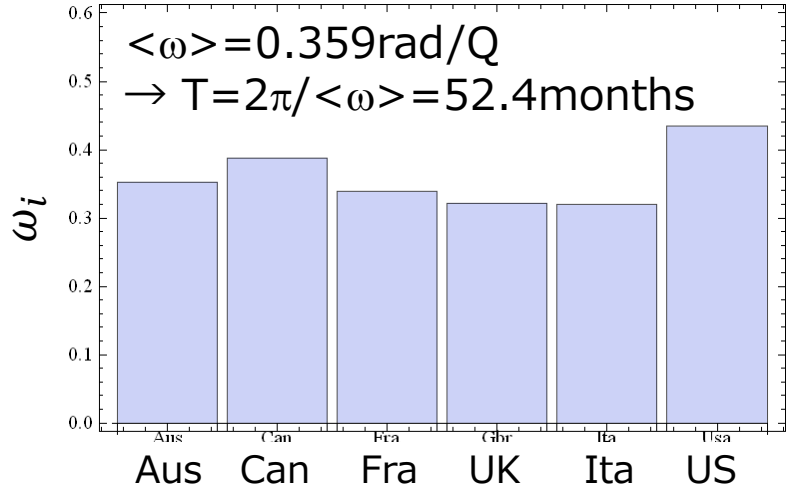
$$z(t) = x(t) + iy(t) = cos(\omega t) + i\, sin(\omega t) = A(t)exp[i\theta(t)]$$

D. Gabor (1945) "Theory of Communication, Part 1"
C.W.J. Granger and M. Hatanaka (1964) "Spectral Analysis of Economic Time Series"

Frequency Entrainment

- The angular frequencies are estimated using the GDP growth rate and the Hilbert transformation of the GDP growth rate.
- The angular frequency ω_i and the intercept $\tilde{\theta}_i$ are estimated by fitting the time-series of the phase $\theta_i(t)$ using $\theta_i(t) = \omega_i t + \tilde{\theta}_i$, where i indicates the country. The estimated angular frequencies ω_i for all the 6 countries are plotted.
- The estimated angular frequencies using the Hilbert transformation are almost identical for these 6 countries.



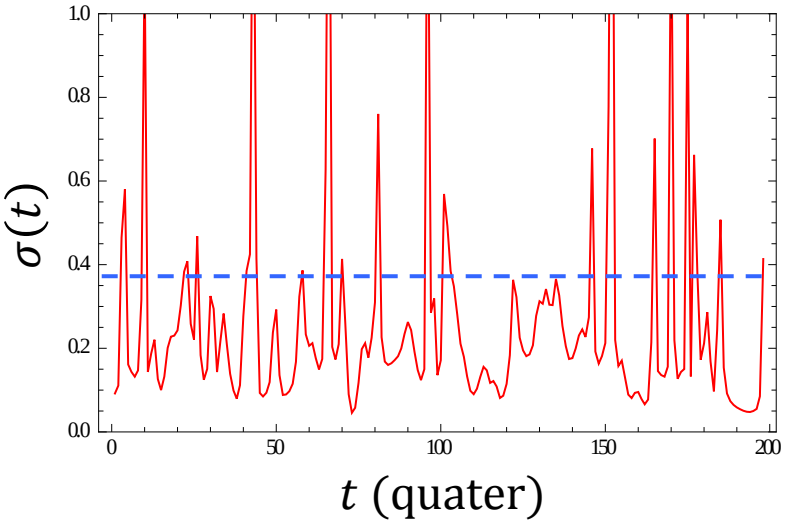


Partial Phase Locking

- The condition of the partial phase locking $\sigma(t) \ll \omega_i$ is satisfied.

$$\sigma(t) = \left[\frac{1}{N}\sum_{i=1}^{N}\left\{\frac{d}{dt}(\theta_i(t)-\omega_i t)-\mu(t)\right\}^2\right]^{1/2}$$

$$\mu(t) = \frac{1}{N}\sum_{i=1}^{N}\frac{d}{dt}(\theta_i(t)-\omega_i t)$$



- The frequency entrainment and the phase locking are the direct evidence of the synchronization in the international business cycle.

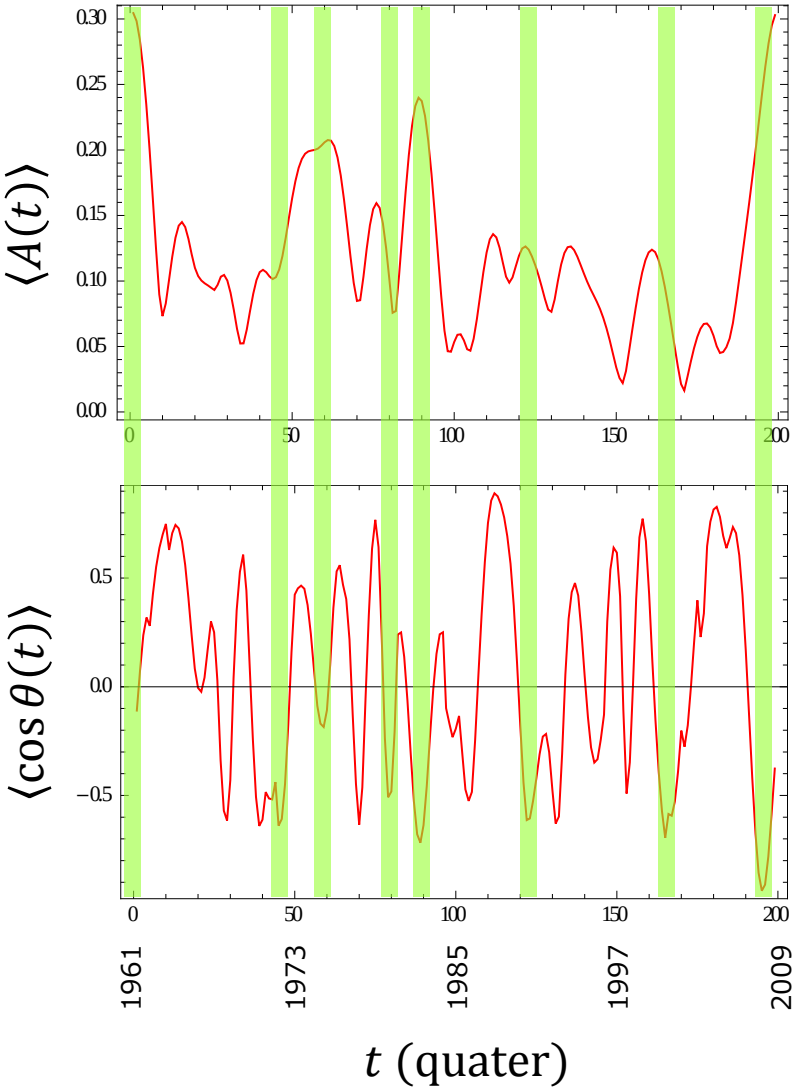
Amplitude, Phase, and Recessions

- Phase time series identifies the 8 recessions after 1960. phase time series is more sensitive to the recessions compared with the amplitude.

$$\langle A(t)\rangle = \frac{1}{N}\sum_{i=1}^{N} A_i(t) = \frac{1}{N}\sum_{i=1}^{N} \frac{x_i(t)}{\cos\theta_i(t)}$$

$$\langle \cos\theta(t)\rangle = \frac{1}{N}\sum_{i=1}^{N} \cos\theta_i(t)$$

- NBER Business Cycles (Trough)
  - (1) 1961/1Q
  - (2) 1970/4Q
  - (3) 1975/1Q
  - (4) 1980/3Q
  - (5) 1982/4Q
  - (6) 1991/1Q
  - (7) 2001/4Q
  - (8) 2009/2Q



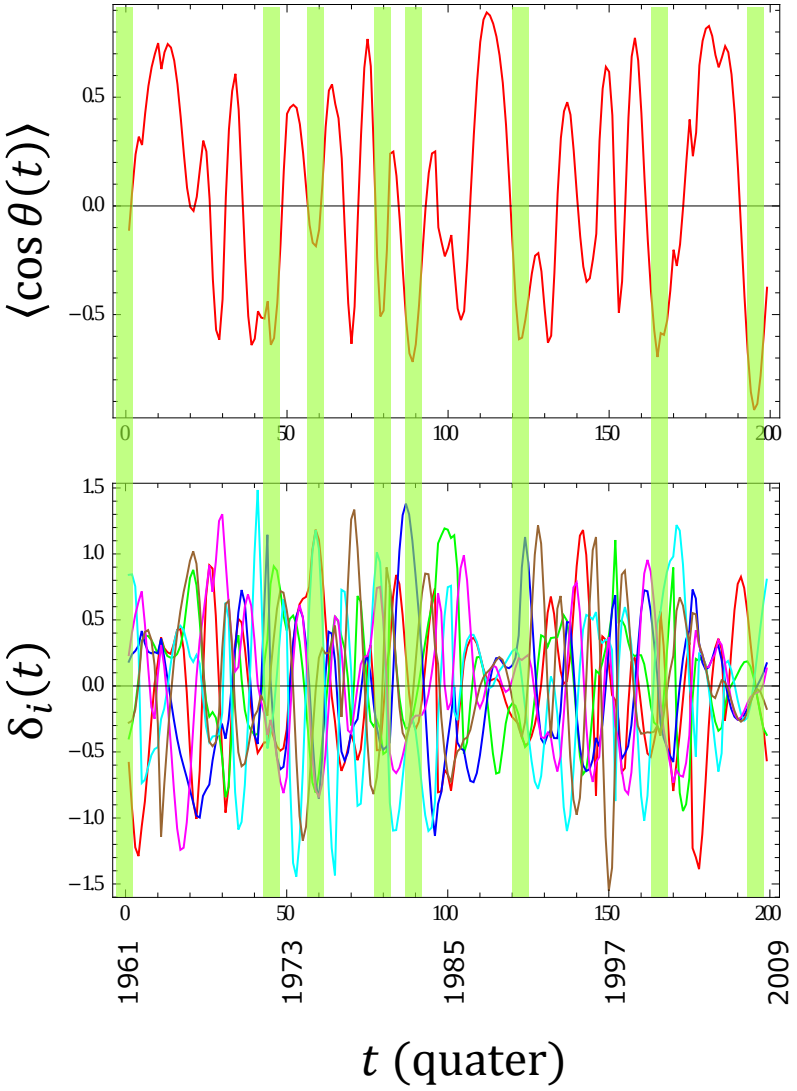
Common and Individual Shocks

- Economic shocks fall into the general classification of common shocks $\langle \cos\theta(t) \rangle$ and individual shocks $\delta_i(t)$.

$$\langle \cos\theta(t) \rangle = \frac{1}{N} \sum_{i=1}^{N} \cos\theta_i(t)$$

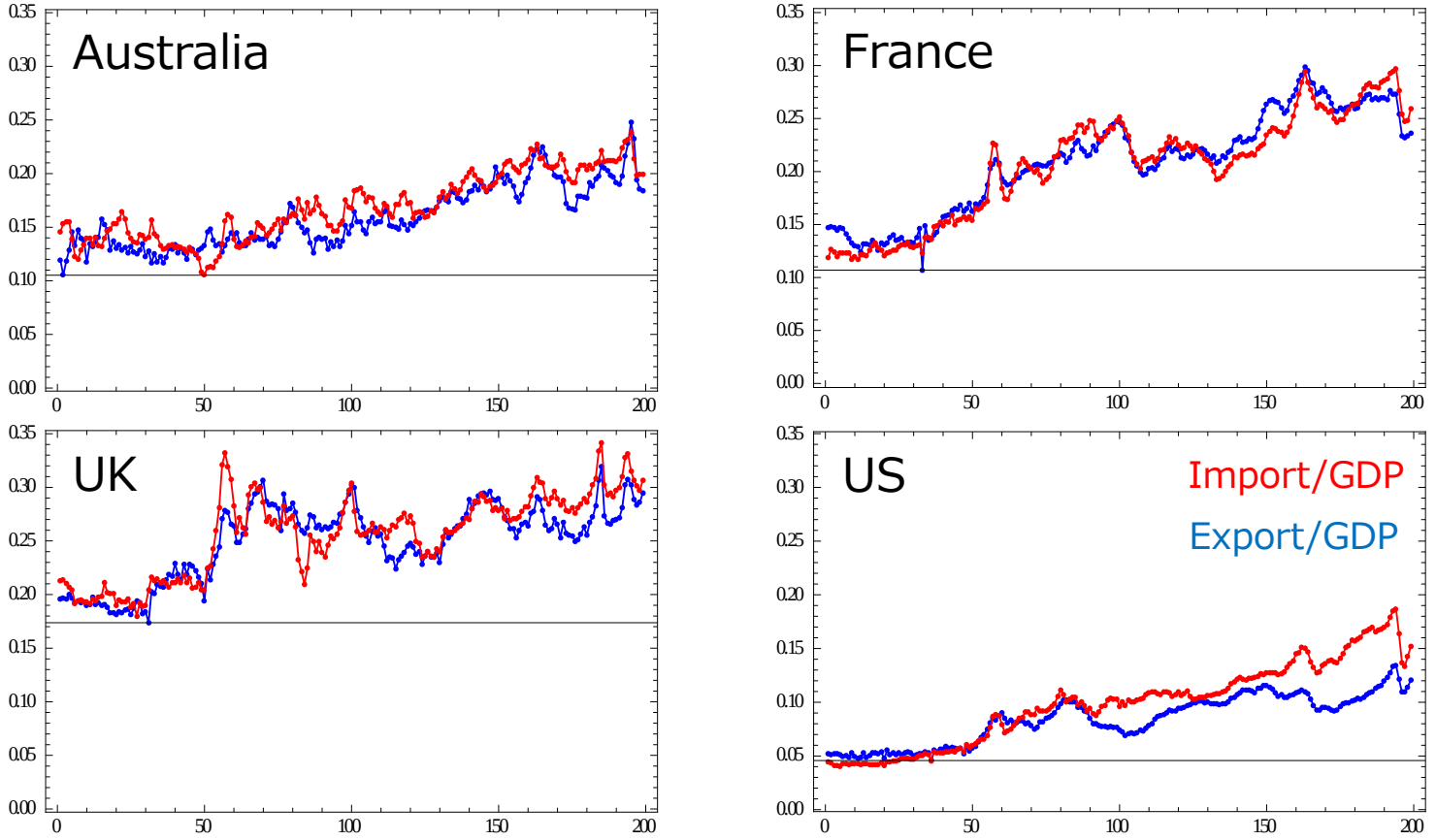
$$\delta_i(t) = \cos\theta_i(t) - \langle \cos\theta(t) \rangle$$

- We have many individual shocks all the time, and many of them seem to occur randomly. More noteworthy are contraction of the individual shocks at the recessions. All countries were exposed to the common shocks.

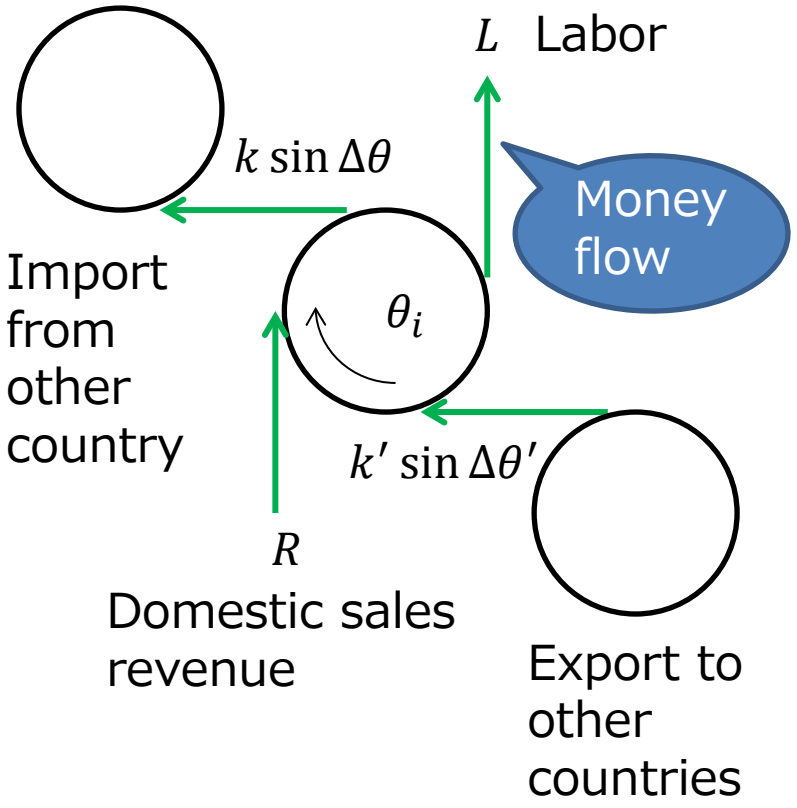


GDP and International Trade

- Cycle of the inventory adjustment may differ from one country to another. Why dose the business cycle synchronize?
- Trade data shows that the import (export) to GDP ratio is high except for US. Trade may affects the cycle and phase of the business cycle.



Coupled Limit-cycle Oscillator Model



If the power is balanced, the oscillator rotates with a constant speed $\dot{\theta}_i$.

- "Power" balance equation (change in "kinetic energy"=summed "power")

$$\frac{d}{dt}\left[\frac{1}{2}\mathrm{I}_i\dot{\theta}_i^{\,2}\right] = R_i - L_i - K_d\dot{\theta}_i^{\,2} + \sum_j k_{ji}\sin\Delta\theta_{ji}$$

- the Kuramoto oscillator ($\ddot{\theta}_i \ll \alpha_i\dot{\theta}_i$)

$$K_d\dot{\theta}_i = R_i - L_i + \sum_j k_{ji}\sin\Delta\theta_{ji}$$

$$\Rightarrow \dot{\theta}_i = Q_i + \sum_j \kappa_{ji}\sin\Delta\theta_{ji}$$

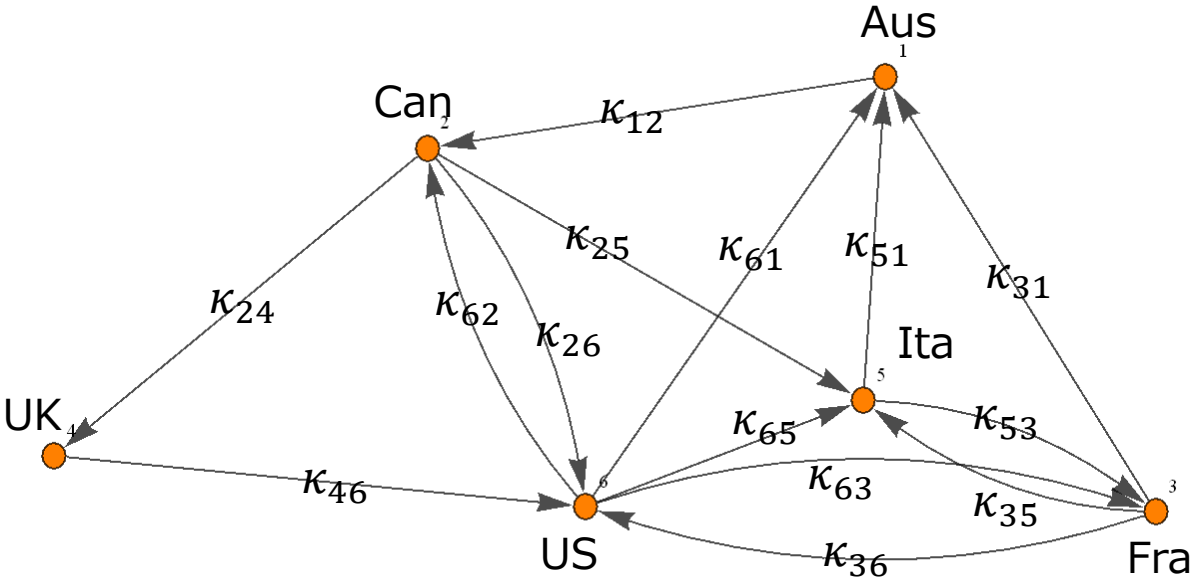
Parameter Estimation

- The coupled limit-cycle oscillator fits the phase time series of the GDP growth rate very well. This means that the origin of the synchronization is interaction due to the international trade.

$$\theta_{i,t+1} = \lambda_i \theta_{i,t} + Q_i + \sum_j \kappa_{ji} \sin \Delta\theta_{ji}$$

	Aus:1	Can:2	Fra:3
λ	0.9968	0.9969	0.9954
Q	0.4128	0.4042	0.3973
R^2	0.9995	0.9997	0.9993

	UK:4	Ita:5	US:6
λ	0.9969	0.9996	0.9982
Q	0.4275	0.3309	0.4506
R^2	0.9993	0.9999	0.9999



The edge between i and j is shown, if the confidence interval of κ_{ij} dose not cross zero.

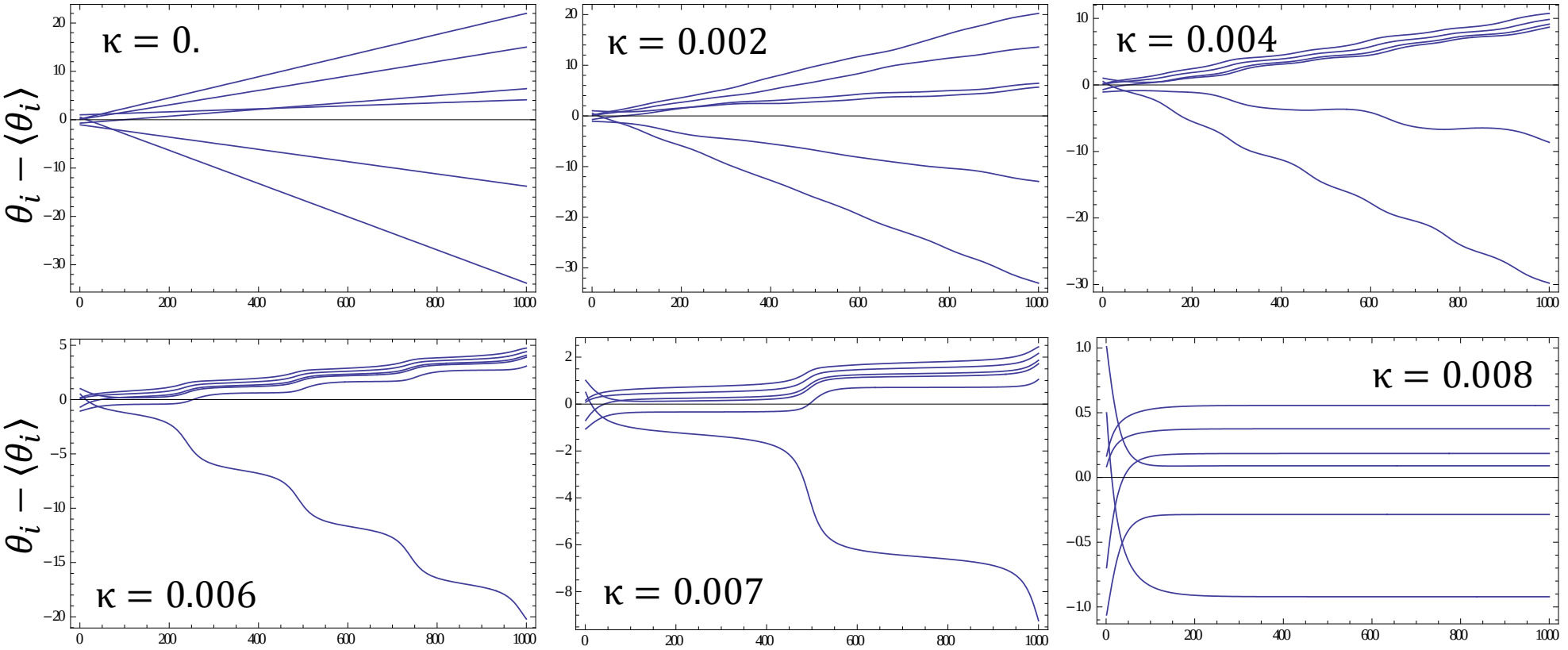
Mechanism of Synchronization

- The synchronization is emerged as a consequence of the interactions.

$$Q_i = UniformRandom[0.35,0.45]$$

$$\theta_i(0) = UniformRandom[-\frac{\pi}{2},\frac{\pi}{2}]$$

$$\frac{d}{dt}\theta_i = Q_i + \sum_j \kappa \sin \Delta\theta_{ji}$$



Summary

We analyzed the quarterly GDP time series for Australia, Canada, France, Italy, UK, and US from 1960/2Q to 2010/1Q to study the synchronization in international Business Cycle. The followings are obtained:

- The frequency entrainment and the phase locking are observed as the direct evidence of the synchronization in the international business cycle.
- The business cycle due to stock adjustment is described using a limit cycle. A coupled limit cycle oscillator model explains the mechanism of synchronization.
- The origin of the synchronization is interaction due to the international trade.