An Eaton-Kortum model of trade and growth

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Waseda U
Revival of Ricardian trade model

Dornbusch, Fischer, and Samuelson (1977, AER):
- two-country, continuum-good Ricardian model
- extensive margins of trade (numbers or fractions of traded varieties)

Eaton and Kortum (2002, EMA):
- extend DFS to $N(\geq 2)$ countries
- examine effects of various forms of trade liberalization on EM under cross-country asymmetries (unlike Melitz (2003, EMA))

static formulation overlooks:

$$(\text{trade cost } \downarrow \rightarrow) \text{ growth } \uparrow \rightarrow \text{ EM of exports } \uparrow$$

e.g., Hummels and Klenow (2005), Broda and Weinstein (2006), Kehoe and Ruhl (2009)

Why not extend Eaton-Kortum dynamically?
Extending Eaton-Kortum dynamically

Acemoglu and Ventura (2002, QJE):
- multi-country AK model: capital → tradable intermediate → final
- growth ↑ → relative rental (∝ ToT) ↓ → growth ↓ → convergence
- intermediates are differentiated → trade pattern is fixed, not evolving

Naito (2012, JIE):
- DFS × Acemoglu-Ventura
- unilateral trade liberalization → growth, welfare, & extensive margins

this paper:
- Eaton-Kortum × Acemoglu-Ventura
- various forms of liberalization → growth, welfare, & extensive margins
  (incl. preferential trade agreement)
Main results in three-country case

analytical results:

1. A permanent fall in any trade cost raises the balanced growth rate
   \[ \tau_{12} \] (trade cost for country 1 to buy each variety from country 2) \downarrow
   \rightarrow 1’s growth potential \uparrow
   \rightarrow 1's rental rates relative to 2 & 3 \downarrow
   \rightarrow 2 & 3's ToT against 1 \uparrow

2. Trade liberalization increases the liberalizing countries’ long-run fractions of exported varieties to all destinations
   \[ \tau_{12} \downarrow \]
   \rightarrow 1’s rental rates relative to 2 & 3 \downarrow
   \rightarrow it’s cheaper for 3 (or 2) to import from 1 than from 2 (or 3)

numerical results:

long-run effects \neq short-run effects (static Eaton-Kortum)

balanced growth \neq market clearing
Overview (N = 3)
Households

budget constraint:

\[ p_{jt}^Y (C_{jt} + \dot{K}_{jt}) = r_{jt} K_{jt}; \dot{K}_{jt} \equiv dK_{jt} / dt. \]  

(1)

w/ log utility, Euler equation:

\[ \dot{K}_{jt} / K_{jt} = \dot{C}_{jt} / C_{jt} = r_{jt} / p_{jt}^Y - \rho_j \forall t \in [0, \infty). \]  

(2)
Final good firms

unit cost function (= intermediate good price index):

\[ q_j(\{p_j(i)\}_{i=0}^1) = B_j^{-1}(\int_0^1 p_j(i)^{1-\sigma_j} di)^{1/(1-\sigma_j)}; \sigma_j > 1. \]  \hspace{1cm} (3)

profit maximization ⇒ zero profit:

\[ p_j^* = q_j. \]  \hspace{1cm} (4)
Intermediate good firms: price distributions

\(A_j\): random variable for country \(j\)’s unit capital requirement, i.i.d. across \(i\)
capital productivity \(1/A_j\) follows a Fréchet distribution:

\[
F_j(z) \equiv \Pr(1/A_j \leq z) \equiv \exp(-b_j z^{-\theta}); \quad b_j > 0, \theta > 1.
\]

distrib. of unit cost \(P_{nj} = \tau_{nj} r_j A_j\) & demand price \(P_n = \min\{\{P_{nj}\}_{j=1}^N\}:

\[
G_{nj}(p) \equiv \Pr(P_{nj} \leq p) = 1 - \exp(-p^\theta b_j (\tau_{nj} r_j)^{-\theta}),
G_n(p) \equiv \Pr(P_n \leq p) = 1 - \exp(-p^\theta \Phi_n); \quad \Phi_n \equiv \sum_{j=1}^N b_j (\tau_{nj} r_j)^{-\theta}.
\]

properties of \(G_{nj}(p)\) and \(G_n(p)\):

- \(b_j \uparrow, \tau_{nj} r_j \downarrow \rightarrow G_{nj}(p) \uparrow\): \(P_{nj}\) tends to be lower
- \(\theta \uparrow \rightarrow 1/A_j\) is less variable \(\rightarrow \tau_{nj} r_j\) matters relatively more
- \(G_n(p) \geq G_{nn}(p)\): trade makes lower \(P_n\) more likely
Three important properties (Eaton-Kortum)

1. Probability that $n$ buys a variety from $j$ is:

$$\pi_{nj}(\{\tau_{nk}r_k\}_{k=1}^N) \equiv b_j(\tau_{nj}r_j)^{-\theta} / \left[\sum_{k=1}^N b_k(\tau_{nk}r_k)^{-\theta}\right].$$  \hspace{1cm} (6)

2. Conditional distribution of $P_{nj}$, given that $n$ buys a variety from $j$, is the same as $G_n(p) \forall j$

$$\therefore \Pr(P_{nj} \leq \min\{\{P_{nk}\}_{k\neq j}\}, P_{nj} \leq p) = \pi_{nj} G_n(p).$$

3. Intermediate good price index function (3) for $n$ is rewritten as:

$$Q_n(\{\tau_{nj}r_j\}_{j=1}^N) \equiv c_n\left[\sum_{j=1}^N b_j(\tau_{nj}r_j)^{-\theta}\right]^{-1/\theta};$$  \hspace{1cm} (7)

$$= \Phi_n$$

$$c_n \equiv B_n^{-1} \Gamma(1 + (1 - \sigma_n) / \theta)^{1/(1-\sigma_n)}.$$
Implications of three properties

- $\pi_{nj}$: fraction of varieties $n$ buys from $j$
  - $\therefore$ probability $\pi_{nj}$ applies to a large number of varieties in $[0,1]$
- $\pi_{nj}$ is homogeneous of degree zero;
- $Q_n$ is homogeneous of degree one, in $\{\tau_{nj} r_j\}_{j=1}^N$
- $\pi_{nj}$: cost share of varieties $n$ buys from $j$

\[
\therefore \int_{l_{nj}} p_n(i_j) x_n(i_j) \frac{d_{ij}}{(Q_n Y_n)} = \pi_{nj}(\{\tau_{nk} r_k\}_{k=1}^N).
\]  

(13)

$\Rightarrow$ all adjustments in the cost shares occur at the extensive margins
Dynamic system

dynamic system \((r_N \equiv 1, \kappa_j \equiv K_j / K_N)\):

\[
\dot{\kappa}_j = \kappa_j (\gamma_j (\{\tau_{jn} r_n / r_j\}_{n=1}^N) - \gamma_N (\{\tau_{Nn} r_n\}_{n=1}^N)), j = 1, ..., N - 1; \quad (14)
\]

\[
\gamma_j(\cdot) \equiv \dot{C}_j / C_j = 1 / Q_j (\{\tau_{jn} r_n / r_j\}_{n=1}^N) - \rho_j,
\]

\[
\kappa_j = \sum_{n=1}^N \pi_{nj} (\{\tau_{nk} r_k / r_n\}_{k=1}^N) \kappa_n / (r_j / r_n), j = 1, ..., N - 1. \quad (15)
\]

(14): growth rate of \(\kappa_j = \) growth rate in \(j \) — growth rate in \(N\)

(15): capital market-clearing condition in \(j\) relative to \(N\)

\(= \) labor market-clearing condition in Eaton and Kortum (2002))

ceteris paribus effects:

- \(\tau_{jn} \downarrow, r_j / r_n \uparrow \rightarrow \gamma_j \uparrow\)
- \(\tau_{nj} \downarrow, r_n / r_j \uparrow \rightarrow \pi_{nj} \uparrow\)
- \(\tau_{nk} \downarrow, r_n / r_k \uparrow \forall k \neq j \rightarrow \pi_{nj} \downarrow\)
Three-country model

\[ \dot{k}_1 = \kappa_1(\gamma_1(1, \tau_{12} r_2 / r_1, \tau_{13} / r_1) - \gamma_3(\tau_{31} r_1, \tau_{32} r_2, 1)), \quad (19) \]
\[ \dot{k}_2 = \kappa_2(\gamma_2(\tau_{21} r_1 / r_2, 1, \tau_{23} / r_2) - \gamma_3(\tau_{31} r_1, \tau_{32} r_2, 1)), \quad (20) \]
\[ k_1 = \pi_{11}(1, \tau_{12} r_2 / r_1, \tau_{13} / r_1)k_1 + \pi_{21}(\tau_{21} r_1 / r_2, 1, \tau_{23} / r_2)k_2 / (r_1 / r_2) + \pi_{31}(\tau_{31} r_1, \tau_{32} r_2, 1) / r_1, \quad (21) \]
\[ k_2 = \pi_{12}(1, \tau_{12} r_2 / r_1, \tau_{13} / r_1)k_1 / (r_2 / r_1) + \pi_{22}(\tau_{21} r_1 / r_2, 1, \tau_{23} / r_2)k_2 + \pi_{32}(\tau_{31} r_1, \tau_{32} r_2, 1) / r_2. \quad (22) \]

w/ \( k_{1t}, k_{2t} \) predetermined, (21), (22): \( r_{1t}, r_{2t} \rightarrow (19), (20): \dot{k}_{1t}, \dot{k}_{2t} \)

BGP:

\[ 0 = \gamma_1(1, \tau_{12} r_2^* / r_1^*, \tau_{13} / r_1^*) - \gamma_3(\tau_{31} r_1^*, \tau_{32} r_2^*, 1) \iff r_1^* = R_1(r_2^*), \quad (23) \]
\[ 0 = \gamma_2(\tau_{21} r_1^* / r_2^*, 1, \tau_{23} / r_2^*) - \gamma_3(\tau_{31} r_1^*, \tau_{32} r_2^*, 1) \iff r_2^* = R_2(r_1^*). \quad (24) \]

and then, (21), (22): \( k_1^*, k_2^* \)
Rental rates at the BGP

\[ r_1 = R_1(r_2) \]
\[ r_2 = R_2(r_1) \]

Determination of \( r^*_1 \) and \( r^*_2 \).

\[ |\gamma_1 \downarrow| > |\gamma_3 \downarrow| \iff \gamma_1 - \gamma_3 \downarrow \]

\(|\gamma_1 \downarrow| > |\gamma_3 \downarrow| \iff \gamma_1 - \gamma_3 \downarrow\)
Transitional dynamics

(19), (20), (21), (22):

\[
\begin{align*}
\dot{\kappa}_1 / \kappa_1 &= d\gamma_1 - d\gamma_3 = a_{11} dr_1 / r_1 + a_{12} dr_2 / r_2, \\
\dot{\kappa}_2 / \kappa_2 &= d\gamma_2 - d\gamma_3 = a_{21} dr_1 / r_1 + a_{22} dr_2 / r_2, \\
\dot{r}_1 / r_1 &= (r_1 \kappa_1 / c) e_{11} d\kappa_1 / \kappa_1 + (r_2 \kappa_2 / c) e_{12} d\kappa_2 / \kappa_2, \\
\dot{r}_2 / r_2 &= (r_1 \kappa_1 / c) e_{21} d\kappa_1 / \kappa_1 + (r_2 \kappa_2 / c) e_{22} d\kappa_2 / \kappa_2.
\end{align*}
\]

(27'), (28'), (29), (30)

e.g., \( \kappa_{10} < \kappa_1^* \), \( \kappa_{20} < \kappa_2^* \)
\[\Rightarrow \begin{align*}
& r_{10} > r_1^*, r_{20} > r_2^* (\because e_{11} < 0, e_{22} < 0) \\
& \gamma_{10} - \gamma_{30} > 0, \gamma_{20} - \gamma_{30} > 0 (\because a_{11} > 0, a_{22} > 0) \\
& \kappa_{1t} \uparrow, \kappa_{2t} \uparrow, r_{1t} \downarrow, r_{2t} \downarrow
\end{align*}\]

\[\therefore \text{BGP is locally stable iff these "own effects" outweigh "cross effects"}\]
4.1 Balanced growth rate: changes in rental rates at the BGP

Changes in rental rates at the BGP

Fig. 1. Rental rates at the balanced growth path: $a_{12} < 0, a_{21} < 0$.

$r_1 = R_1(r_2)$

$r_2 = R_2(r_1)$
4.1 Balanced growth rate: changes in rental rates at the BGP

Changes in rental rates at the BGP

Fig. 1. Rental rates at the balanced growth path: \(a_{12} < 0, a_{21} < 0\).

\[
\begin{align*}
    r_1 &= R_1(r_2), \\
    r_2 &= R_2(r_1) \\
    r_1 &= R_1(r_2; \tau_{12}') \\
    \tau_{12} &\downarrow
\end{align*}
\]
Proposition 2

For all $j, n = 1, 2, 3, n \neq j$, a permanent fall in $\tau_{jn}$ raises the balanced growth rate.

intuition:

$\tau_{12} \downarrow$

$\rightarrow \gamma_1 \uparrow$

$\rightarrow r_1 \downarrow, \ r_1 / r_2 \downarrow: \ 3 \& 2$’s ToT against $1 \uparrow$

$\rightarrow \gamma_3 \uparrow, \ \gamma_2 \uparrow$
Proposition 3

A permanent fall in \( \tau_{12} \) increases \( \pi_{12}^*, \pi_{21}^*, \) and \( \pi_{31}^* \), whereas it decreases \( \pi_{13}^* \).

Intuitions:

- \( \tau_{12} \downarrow \rightarrow \pi_{12} \uparrow, \pi_{13} \downarrow \)
- \( \tau_{12} \downarrow \rightarrow r_1 \downarrow, r_1/r_2 \downarrow \rightarrow \pi_{21} \uparrow, \pi_{31} \uparrow \)
Bilateral trade liberalization

Proposition 4

Permanent falls in $\tau_{12}$ and $\tau_{21}$, with $r_1^*/r_2^*$ unchanged, increase $\pi_{12}^*, \pi_{21}^*, \pi_{31}^*$, and $\pi_{32}^*$, whereas they decrease $\pi_{11}^*, \pi_{13}^*, \pi_{22}^*, \pi_{23}^*$, and $\pi_{33}^*$.

intuitions:

- $\tau_{12} \downarrow \rightarrow |\pi_{12} \uparrow| > |\pi_{13} \downarrow| \rightarrow \pi_{11} \downarrow$
- $\tau_{21} \downarrow \rightarrow |\pi_{21} \uparrow| > |\pi_{23} \downarrow| \rightarrow \pi_{22} \downarrow$
- $\tau_{12} \downarrow, \tau_{21} \downarrow \rightarrow r_1 \downarrow, r_2 \downarrow \rightarrow \pi_{31} \uparrow, \pi_{32} \uparrow \rightarrow \pi_{33} \downarrow$
### Benchmark case

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**Table:** $\tau_{12} = \tau_{13} = \tau_{21} = \tau_{23} = \tau_{31} = \tau_{32} = 2$, $b_1 = b_2 = b_3 = 1$
Calibrated case: CJK ($j=1$), NAFTA ($j=2$), EU ($j=3$)

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Table: $\tau_{12} = 2.95638$, $\tau_{13} = 6.45658$, $\tau_{21} = 2.91636$, $\tau_{23} = 6.53986$, $\tau_{31} = 1.16897$, $\tau_{32} = 1.21261$, $b_1 = 0.119177$, $b_2 = 0.117$, $b_3 = 0.111751$
Concluding remarks

policy implications:

- trade liberalization, be it unilateral, bilateral, or multilateral, raises global growth
  Romalis (2007), Estevadeordal and Taylor (2008), and Wacziarg and Welch (2008)
- import promotion acts as export promotion at the extensive margins

possible extensions:

- $b_j$ is increasing in $\kappa_j$ (as externalities): qualitatively unchanged
- import tariffs: unchanged if long-run welfare gains are dominant
- $N > 3$: qualitatively unchanged, quantitatively weaker