
# Buyer-Supplier Networks and Aggregate Volatility: Evidence from Firm Level Data

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# Goldman Quantifies Adverse Impact From Japanese Earthquake: Up To 1% Of Q2 GDP, Higher Inflation

"Our auto analysts expect roughly a 10% decline in North American vehicle production in Q2, overwhelmingly due to a shortage of MCU supply. (To put this in perspective with the financial crisis, US vehicle unit production fell at slightly faster rates in the third and fourth quarter of 2008, and three times as rapidly in the first quarter of 2009.)"

"Reasonable parameters suggest **a potential impact on Q2 annualized real GDP growth from one-quarter point to as much as a full point**. Although there could be some additional impact in other sectors of the economy, this seems likely to be quite small."

Speech by Ben S. Bernanke at the International Monetary Conference, Atlanta, Georgia, on June 7, 2011

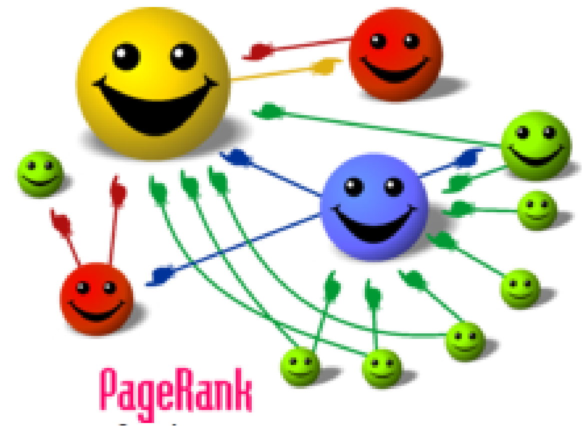
U.S. economic growth so far this year looks to have been somewhat slower than expected. Aggregate output increased at only 1.8 percent at an annual rate in the first quarter, and **supply chain disruptions associated with the earthquake and tsunami in Japan are hampering economic activity this quarter**. A number of indicators also suggest some loss of momentum in the labor market in recent weeks.

# Network Origin of Aggregate Volatility

- Inequality across firms/sectors in terms of "Importance" of firms in a buyer-supplier network
  - Dupor (1999) shows that, without inequality, idiosyncratic shocks are canceled out with each other due to LLN, so that their impact on aggregate volatility decays quickly with the number of firms (at the rate of $\sqrt{N}$).
  - Acemoglu et al. (2010, 2011, 2012) and Carvalho (2008) derive some conditions about the structure of networks to deliver low convergence rates. One of the necessary conditions is that the number of customer links follows a power law distribution with a tail exponent lower than 2.

- Acemoglu et al (2010, 2011, 2012) provides some empirical evidence on the structure of US trade network among sectors using IO data. Foerster et al (2011) also provides evidence on the propagation of sectoral shocks through the US IO network.
  - Trade occurs not between sectors but between firms. The definition of sectors is, in some sense, arbitrary. More importantly, empirical evidence from sectoral data may overestimate the role of networks because it does not fully account the possibility of substitution of partners. For example, an automobile firm may switch to a new steel firm from its old partner firm which is in trouble.
  - Evidence on the structure of trade networks is only suggestive. There is not much direct evidence on the propagation of shocks through networks.

# Empirical questions to be addressed by this paper




PageRank is defined as follows:

*We assume page A has pages T1...Tn which point to it (i.e., are citations). The parameter d is a damping factor which can be set between 0 and 1. We usually set d to 0.85. There are more details about d in the next section. Also C(A) is defined as the number of links going out of page A. The PageRank of a page A is given as follows:*

$$PR(A) = (1-d) + d\,(PR(T1)/C(T1) + \ldots + PR(Tn)/C(Tn))$$

*Note that the PageRanks form a probability distribution over web pages, so the sum of all web pages' PageRanks will be one.*

Brin, Sergey and Lawrence Page "The Anatomy of a Large-Scale Hypertextual Web Search Engine," Comput. Networks ISDN Systems 33, 107–117, 1998.

# Equivalence of Leontief and PageRank Models

$$\mathbf{x} = (1-\alpha)\Gamma^{'}\mathbf{x} + \mathbf{f}$$

Leontief, Wassily "Quantitative Input and Output Relations in the Economic System of the United States," Review of Economics and Statistics, 1936.

Assumption 1: Final demand is equal across firms

Assumption 2: Supplier link is of the same size

$$\mathbf{x} = (1-\alpha)\hat{\Gamma}^{'}\mathbf{x} + \frac{\alpha}{n}\mathbf{1}$$

Brin, Sergey and Lawrence Page "The Anatomy of a Large-Scale Hypertextual Web Search Engine," Comput. Networks ISDN Systems 33, 107–117, 1998.

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 1/10 \\ 0 & 0 & 0 & 1/10 \\ 1/10 & 3/10 & 0 & 8/10 \\ 9/10 & 7/10 & 10/10 & 0 \end{pmatrix} \begin{pmatrix} (1-\alpha)x_1 \\ (1-\alpha)x_2 \\ (1-\alpha)x_3 \\ (1-\alpha)x_4 \end{pmatrix} + \begin{pmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{pmatrix}$$

Firm 1 purchases from firm 3 by 1/10, and from firm 4 by 9/10

where $\sum f_i = \alpha \sum x_i$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 1/3 \\ 0 & 0 & 0 & 1/3 \\ 1/2 & 1/2 & 0 & 1/3 \\ 1/2 & 1/2 & 1 & 0 \end{pmatrix} \begin{pmatrix} (1-\alpha)x_1 \\ (1-\alpha)x_2 \\ (1-\alpha)x_3 \\ (1-\alpha)x_4 \end{pmatrix} + \begin{pmatrix} \alpha \sum x_i/4 \\ \alpha \sum x_i/4 \\ \alpha \sum x_i/4 \\ \alpha \sum x_i/4 \end{pmatrix}$$

Firm 1 purchases evenly from firm 3 and from firm 4

# Granular Hypothesis vs. Network Hypothesis
## Two hypotheses are identical under the two assumptions

**Granular Hypothesis**

Gabaix (2010, Proposition 2)

- **Firm sales** follows a power law with an exponent of $\mu$.
- The SD of the growth rate of individual firm is $\sigma$ (identical across firms)

**Network Hypothesis**

Acemoglu et al (2010, 2011, 2012, Corollary 1)

- **Page rank** follows a power law with an exponent of $\mu$.
- The SD of the growth rate of individual firm is $\sigma$ (identical across firms)

The SD of GDP decays with the number of firms, *N*, but the convergence rate depends on the value of $\mu$.

The SD of GDP converges at $\ln N$ for $\mu = 1$

The SD of GDP converges at $N^{1-1/\mu}$ for $1 < \mu < 2$

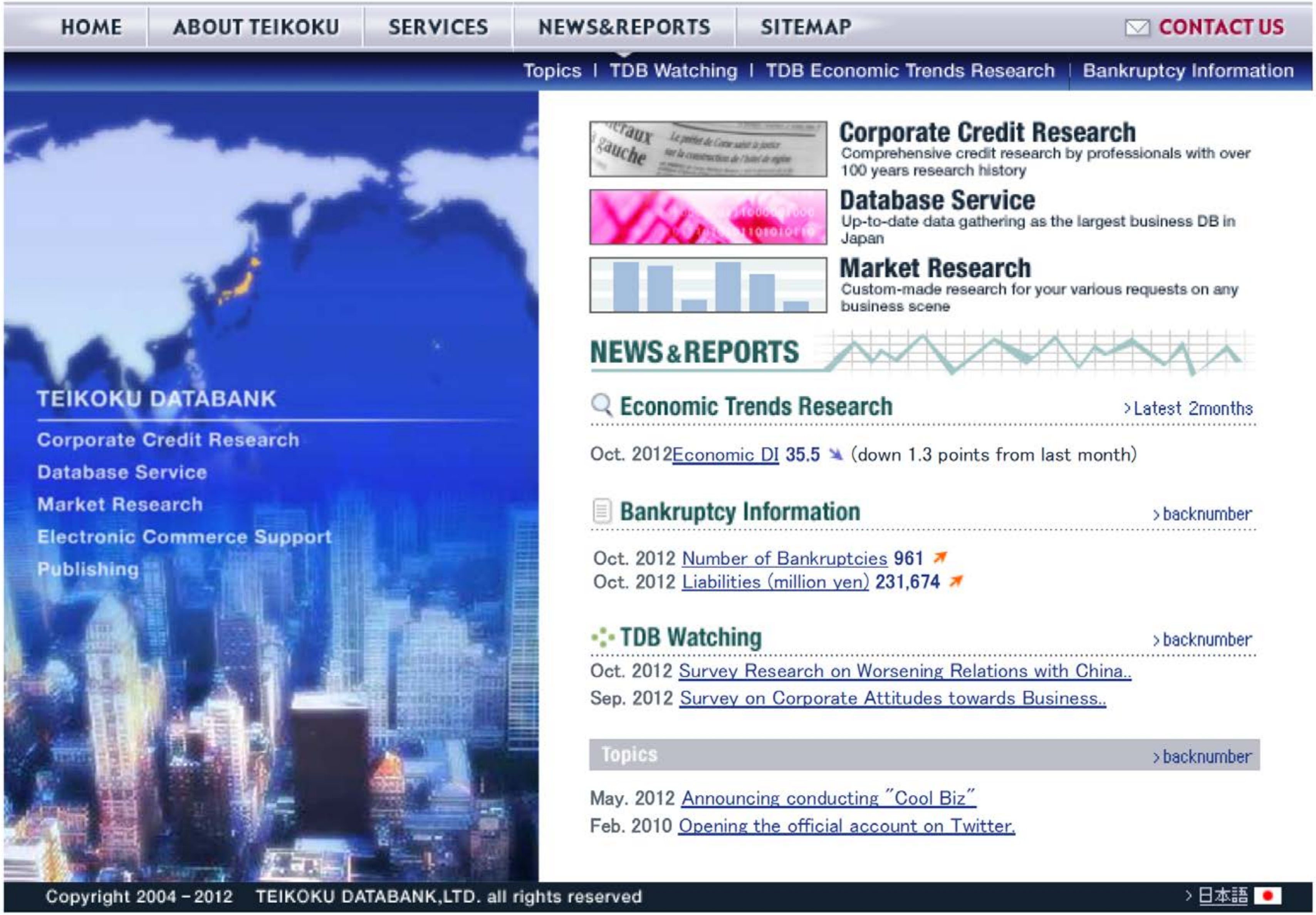
The SD of GDP converges at $N^{1/2}$ for $\mu \geq 2$

# Empirical questions addressed by this paper

- Is PageRank distribution with a heavy tail?
  - Acemoglu et al (2010, 2011, 2012) shows that idiosyncratic shocks matter if the influence vector (another name of PageRank vector) has elements of unequal size, implying that the distribution of PageRank across firms has a heavy tail.
- How is PageRank of a firm related with its sales?
  - Does a firm with large PageRank have large sale? If PageRank of a firm and its sales are independent, it implies that the granular hypothesis and the linkage hypothesis are not related. However, if there is an exact one-to-one relationship between PageRank and sales, the two hypothesis is not indistinguishable.
- Are growth correlations across firms higher for neighbor firms?
  - The linkage hypothesis implies that the growth rates of firms are highly correlated if their locations are close on the network.

# Data and Some Facts

TEIKOKU DATABANK



# Two types of information on customer-supplier relationships

- The dataset contains **the total number of relationships** a firm has with other firms.
  - customers (i.e., the set of firms to which a firm sells its products)
  - suppliers (i.e., the set of firms from which a firm purchases raw materials and intermediate products)
  - owners (i.e., the set of firms by which a firm is owned).

- The dataset records **the list of core partners** (i.e. customers, suppliers, and owners) for a firm, with their IDs.
  - The list is not exhaustive and the length of the list cannot exceed thirty firms. For some firms, typically large firms, with more than thirty partners, only a part of their lists of partners is recorded, with the most important one on the top of the list, and the second important one on the next line and so on.

- A distinctive feature of the dataset is that it records information on linkages for three different years (i.e. 2008, 2009, and 2010), so that it allows us to investigate not only the structure of a customer-supplier network at a particular point in time, but also its evolution over time.

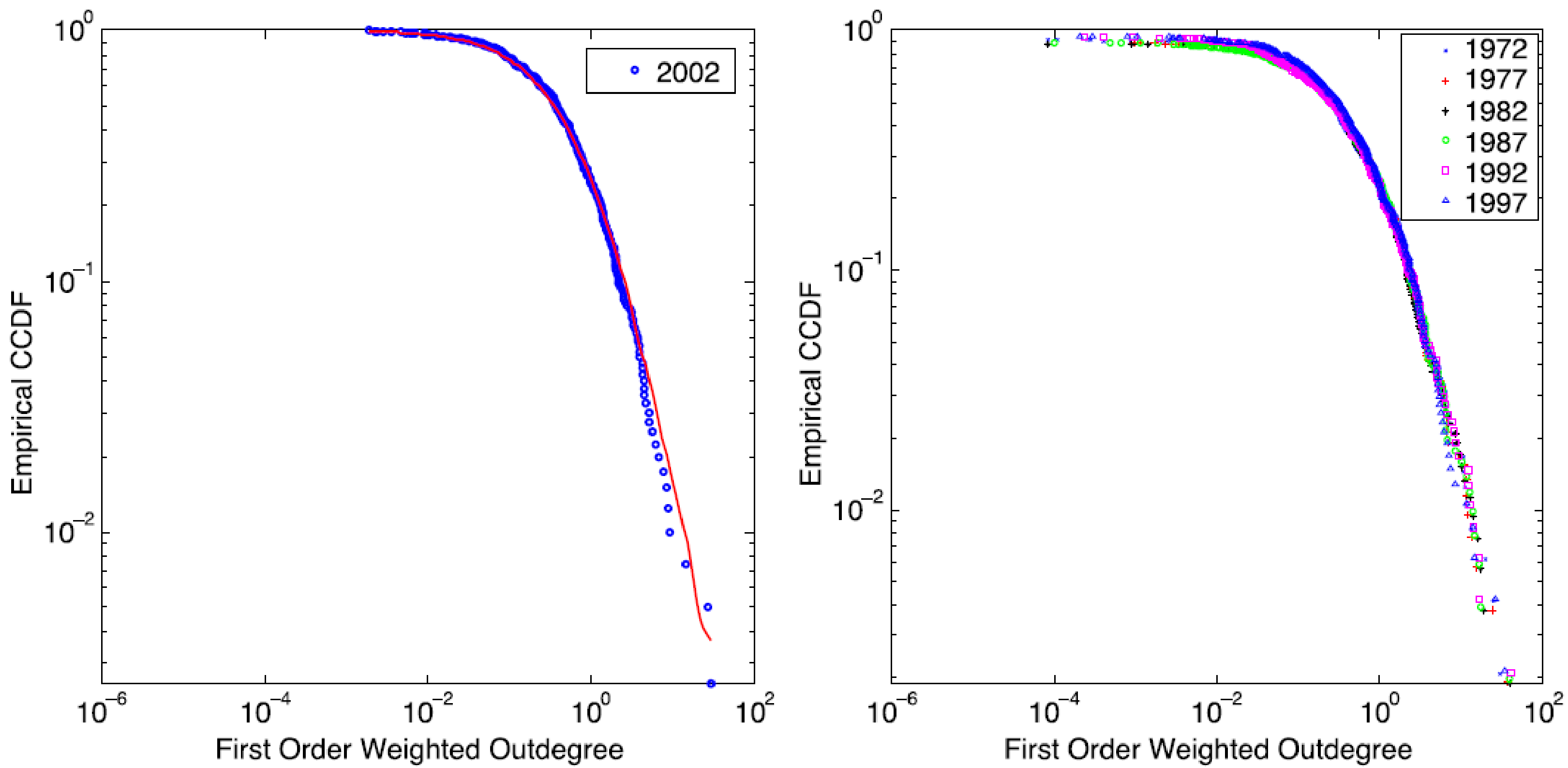



FIGURE 8.—Empirical counter-cumulative distribution function of first-order degrees.

Source: Acemoglu, Daron, Vasco M. Carvalho, Asuman Ozdaglar, and Alireza Tahbaz-Salehi (2012), "The Network Origins of Aggregate Fluctuations," Econometrica, Vol. 80, No. 5 (September, 2012), 1977–2016.

## 得意先

**主要得意先** List of core customers

| 品　目 | | 得意先名（ＴＤＢ企業コード） | | 所在地 | 取引シェア（%） |
|---|---|---|---|---|---|
| 切削工具 | ◎ | ダイヤモンド工業株式会社 | (400628014) | 愛知県名古屋市中区 | 10 |
| | ◎ | 株式会社青山自動車 | (400671445) | 愛知県名古屋市中区 | 10 |
| | | 株式会社大水エンジニアリング | (986004628) | 東京都大田区 | |
| | | 小野川モーターテクノ株式会社 | (986002436) | 東京都大田区 | |
| | | 株式会社松元精密工具 | (985551569) | 東京都墨田区 | |
| | | 川津自動車工業株式会社 | (985526207) | 東京都江戸川区 | |
| | | 千葉精密工具株式会社 | (987001999) | 東京都品川区 | |

**得意先概数：** 300社 Total number of customers

## 仕入先および外注先

**主要仕入先および外注先（支払先）** List of core suppliers

| 品　目 | | 仕入先名（ＴＤＢ企業コード） | | 所在地 | 取引シェア（%） |
|---|---|---|---|---|---|
| 鋼材 | ◎ | 日進鋼機株式会社 | (400491170) | 愛知県名古屋市瑞穂区 | 40 |
| | | 日吉鋼材株式会社 | (985714431) | 東京都千代田区 | |
| | | 株式会社八木下鉄鋼所 | (985542603) | 東京都目黒区 | |
| タングステン・コバルトなど | ◎ | 株式会社開進 | (986054352) | 東京都千代田区 | 30 |
| | | ダイヤモンド工業株式会社 | (400628014) | 愛知県名古屋市中区 | |
| 外　注 | | 株式会社小谷栄工舎 | (400893158) | 愛知県名古屋市中区 | |
| | | 株式会社藤木製作所 | (400859657) | 愛知県名古屋市東区 | |
| | | 日本機工 | | 茨城県土浦市 | |

◎印主力

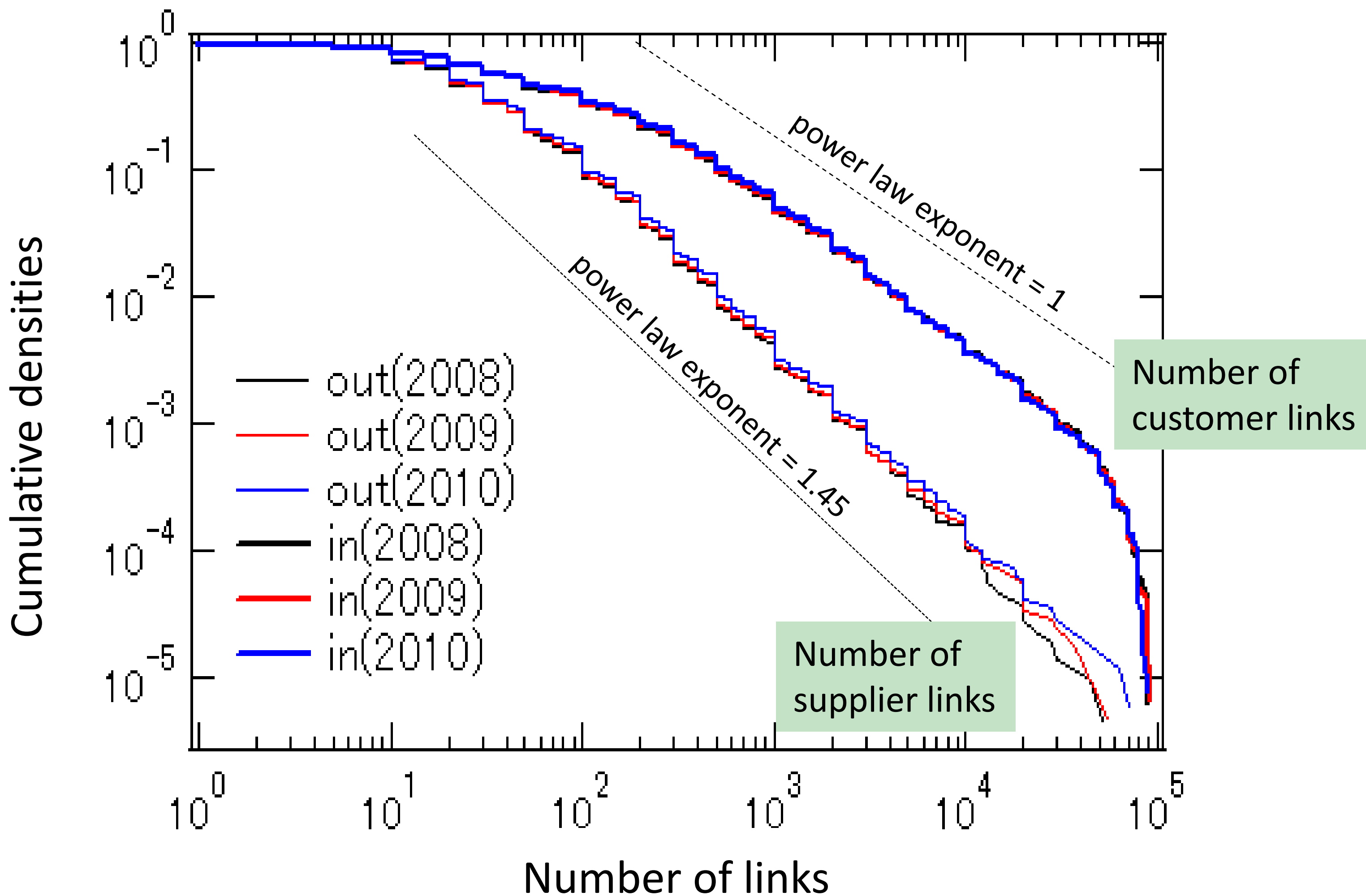
**仕入先概数：** 70社 Total number of suppliers

# Descriptive Statistics on Customer and Supplier Linkages

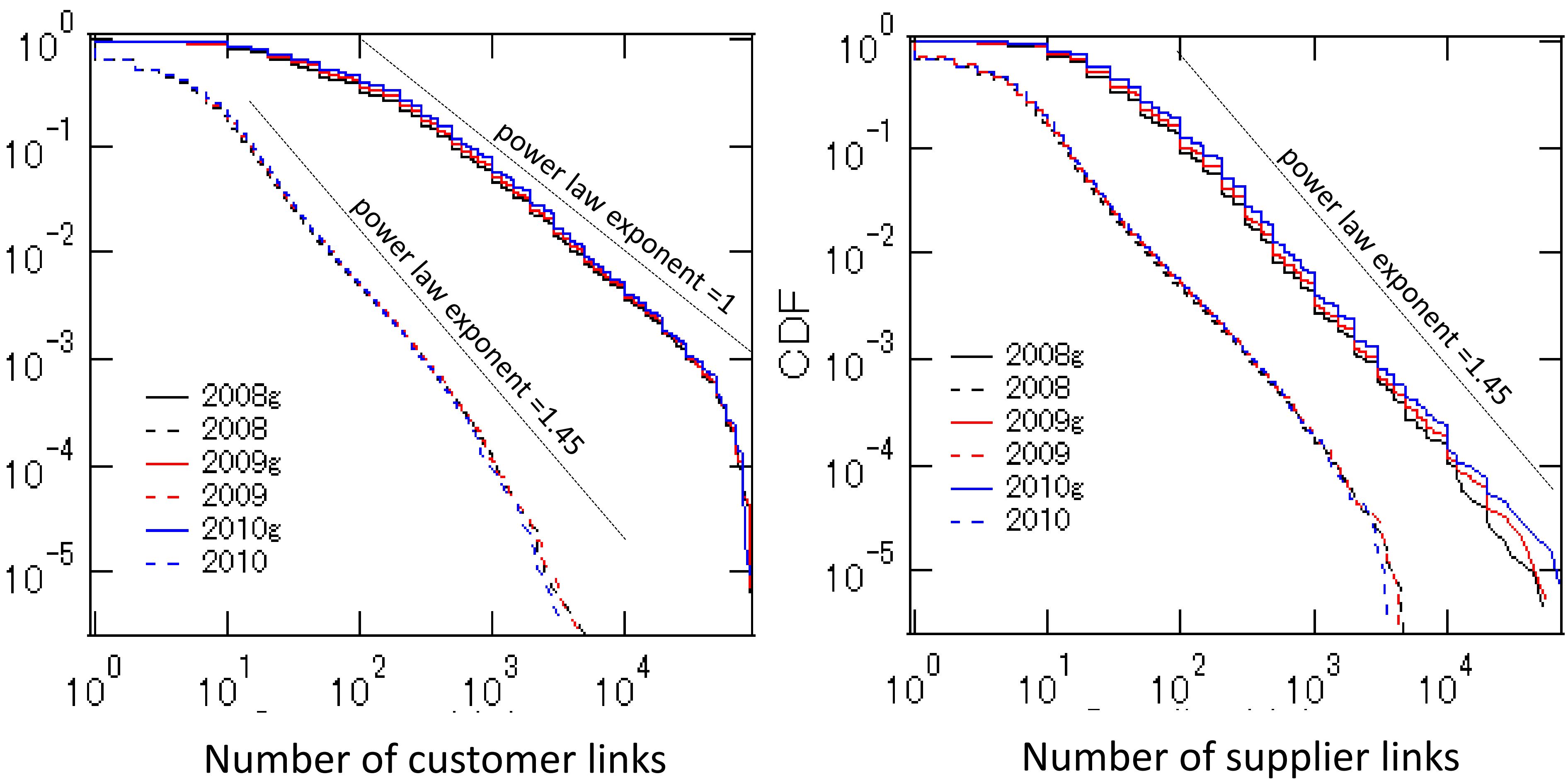
| **Customer Linkage** | 2008 | 2009 | 2010 |
| --- | --- | --- | --- |
| Number of firms | 160,512 | 155,813 | 129,216 |
| Number of links per firm | | | |
| Mean | 339 | 343 | 350 |
| Median | 50 | 50 | 50 |
| Std. Dev. | 2,107 | 2,089 | 2,062 |
| Max | 90,200 | 90,504 | 90,000 |
| Min | 0 | 0 | 0 |

| **Supplier Linkage** | 2008 | 2009 | 2010 |
| --- | --- | --- | --- |
| Number of firms | 215,567 | 208,467 | 172,149 |
| Number of links per firm | | | |
| Mean | 56 | 58 | 63 |
| Median | 20 | 20 | 20 |
| Std. Dev. | 281 | 314 | 372 |
| Max | 52,100 | 55,100 | 70,000 |
| Min | 0 | 0 | 0 |

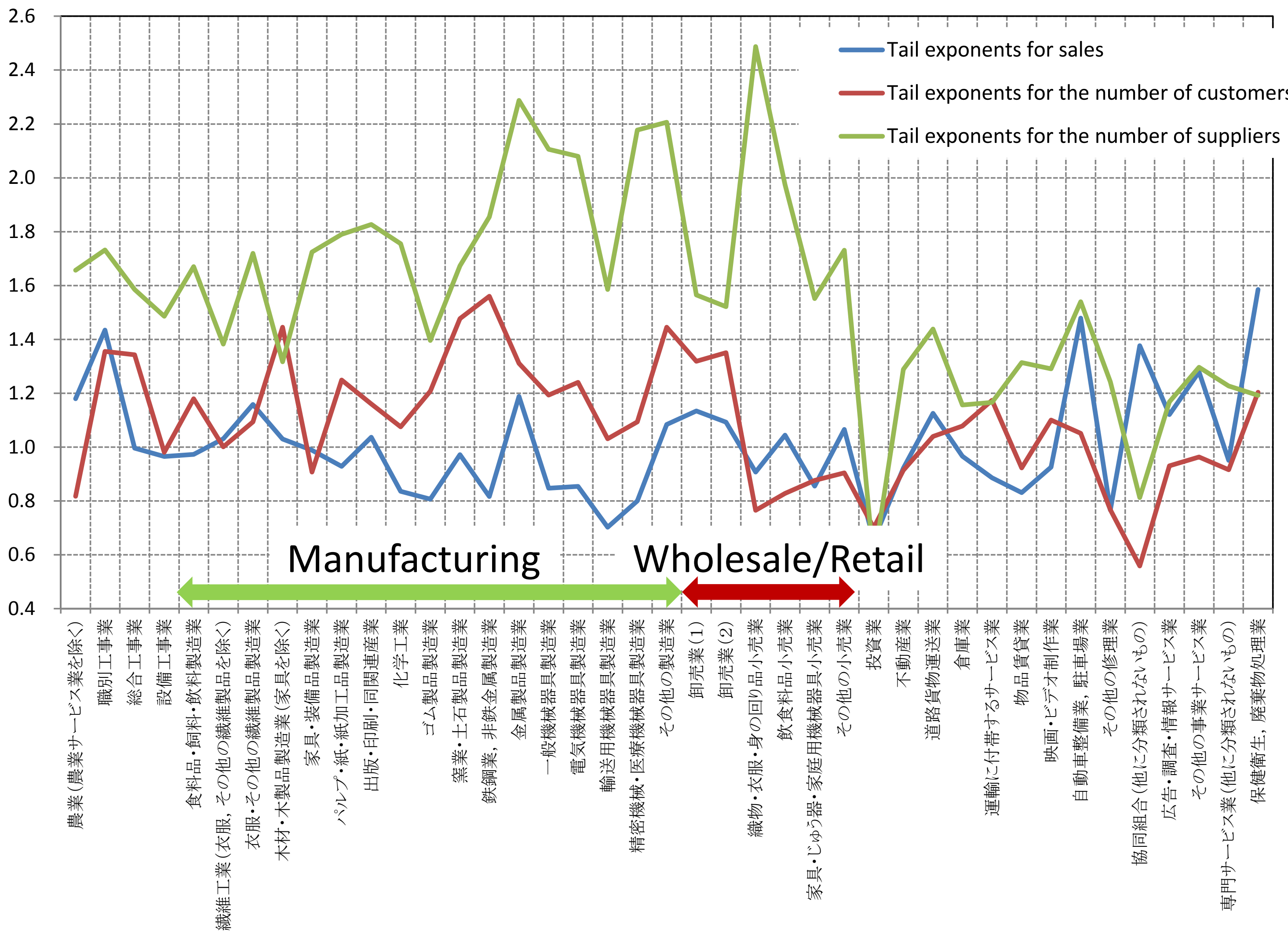
# Cumulative distributions of customer and supplier links estimated using the total number of partners



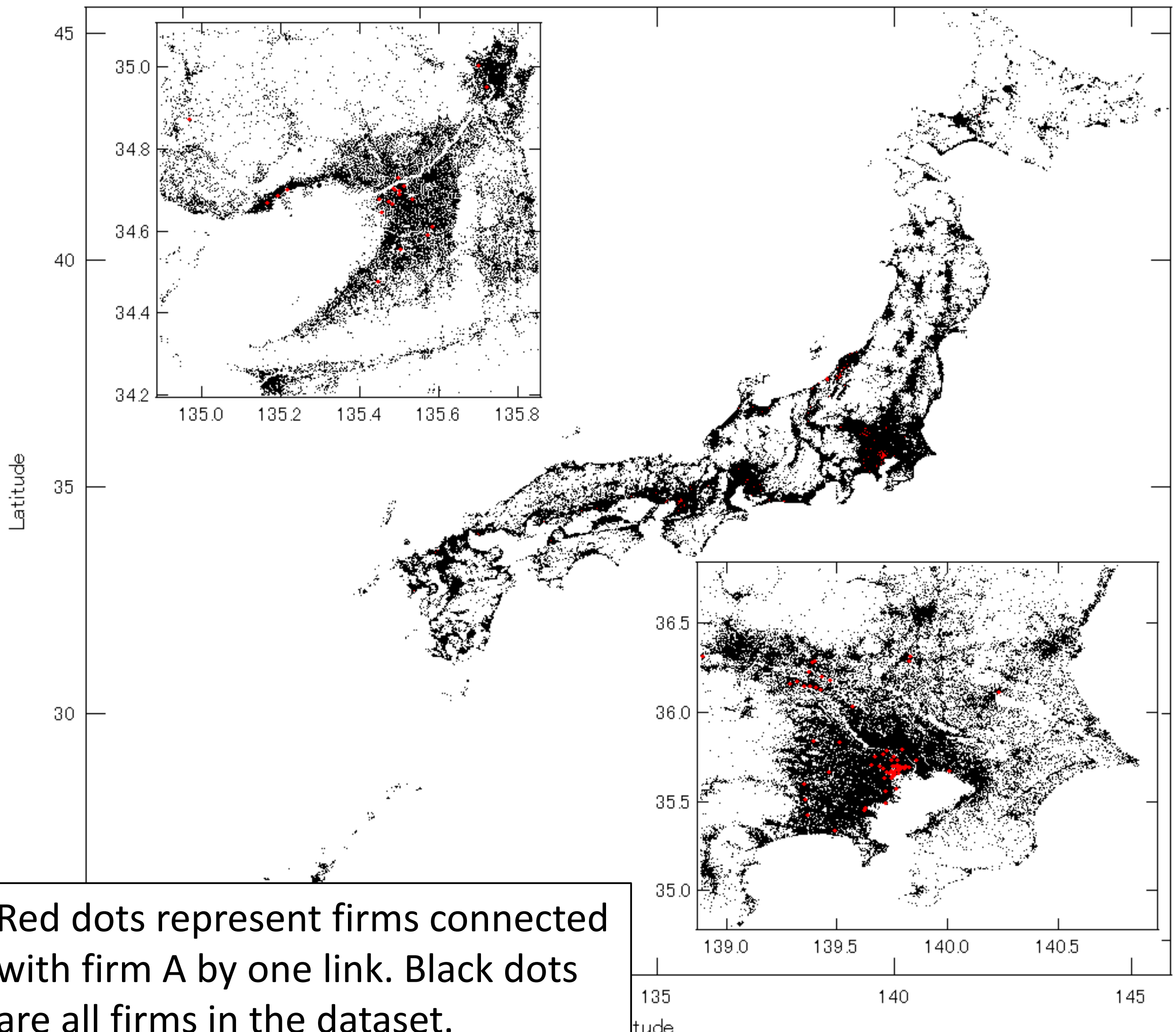
# Cumulative distributions of customer and supplier links estimated using the list of core partners



# Power Law Exponents for Sales, Indegree, and Outdegree by Industry

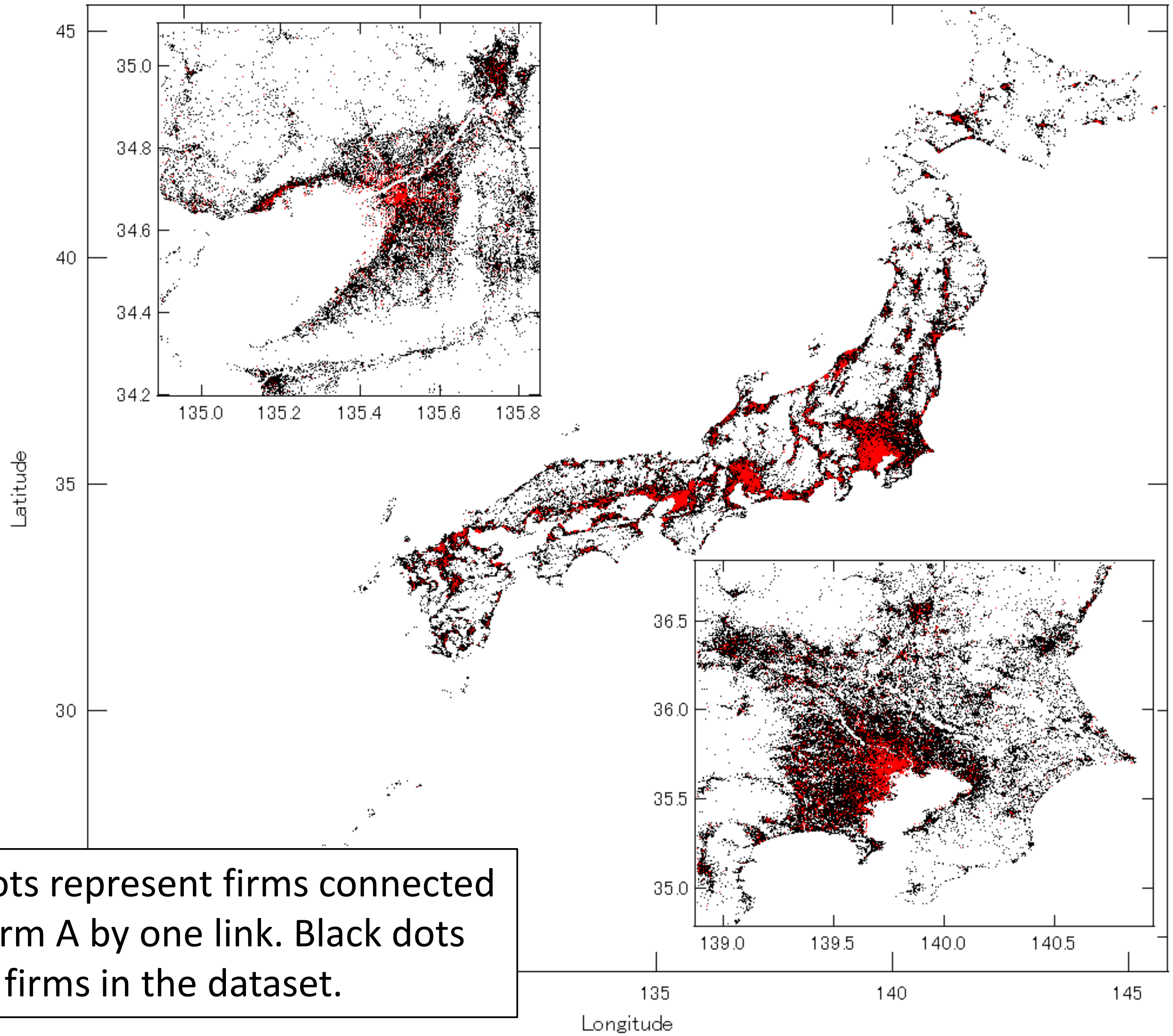


# Firms Connected with Firm A by **One** Link



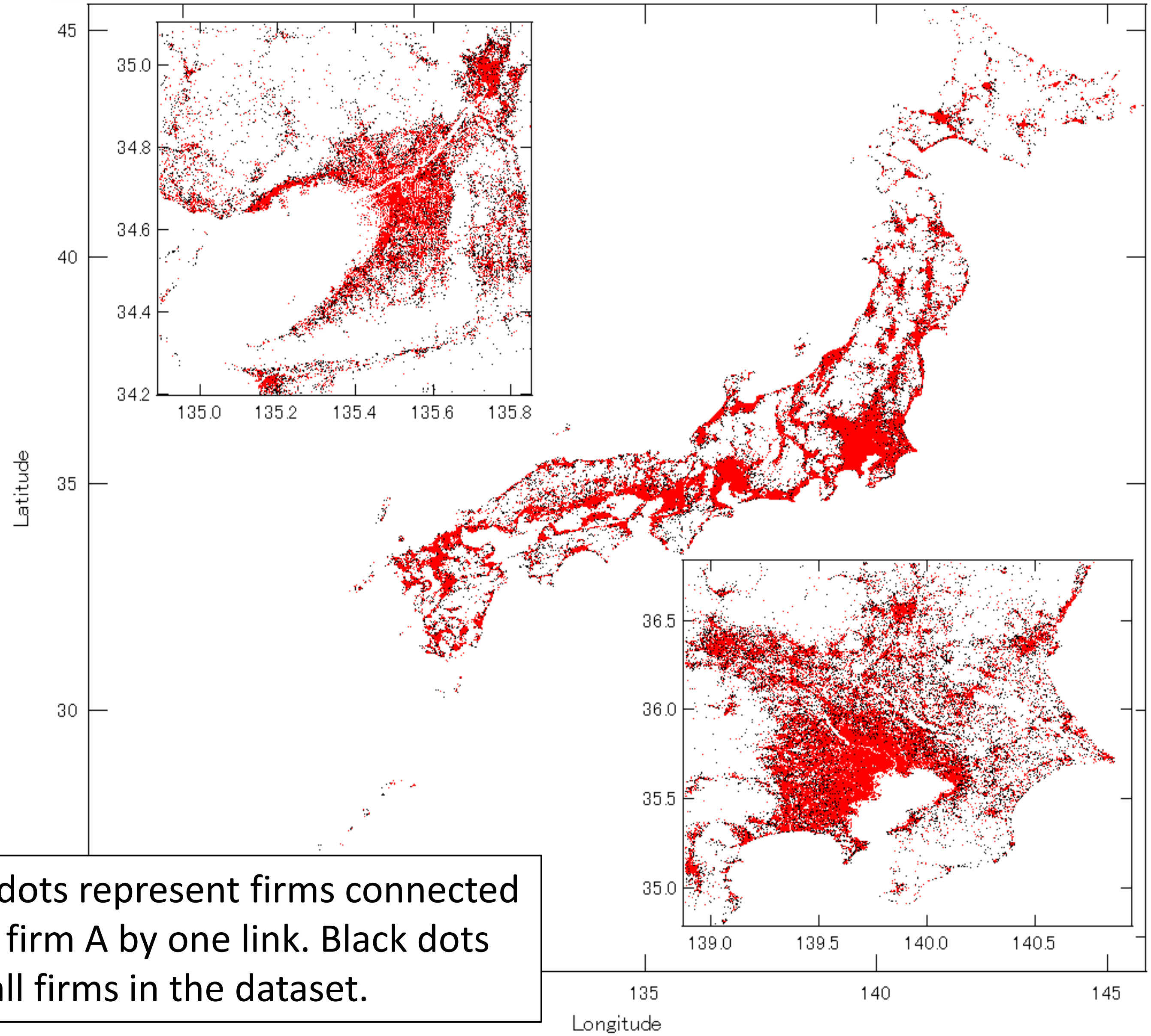

Red dots represent firms connected with firm A by one link. Black dots are all firms in the dataset.

# Firms Connected with Firm A by **Two** Links or Less



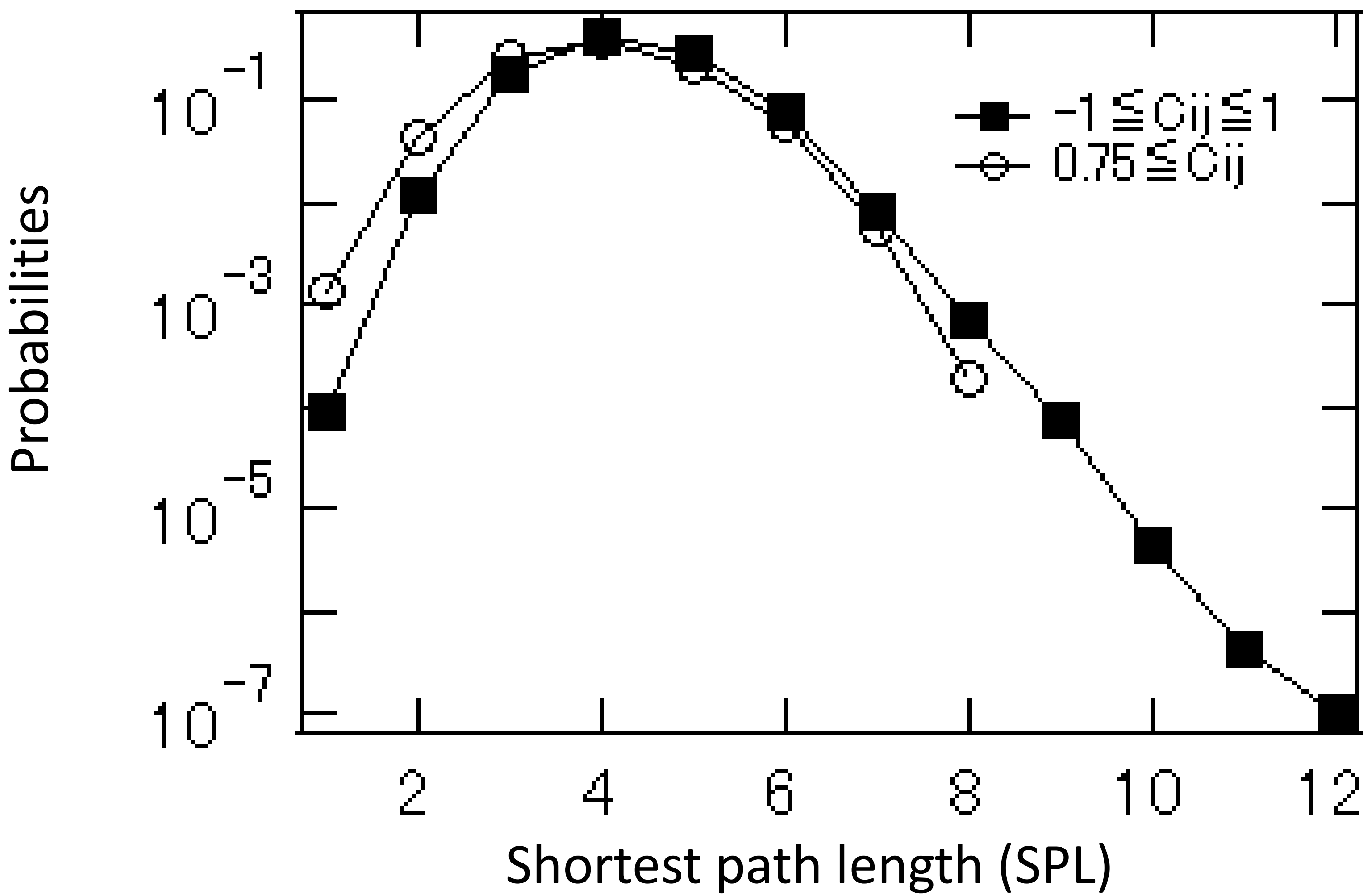

Red dots represent firms connected with firm A by one link. Black dots are all firms in the dataset.

# Firms Connected with Firm A by **Three** Links or Less

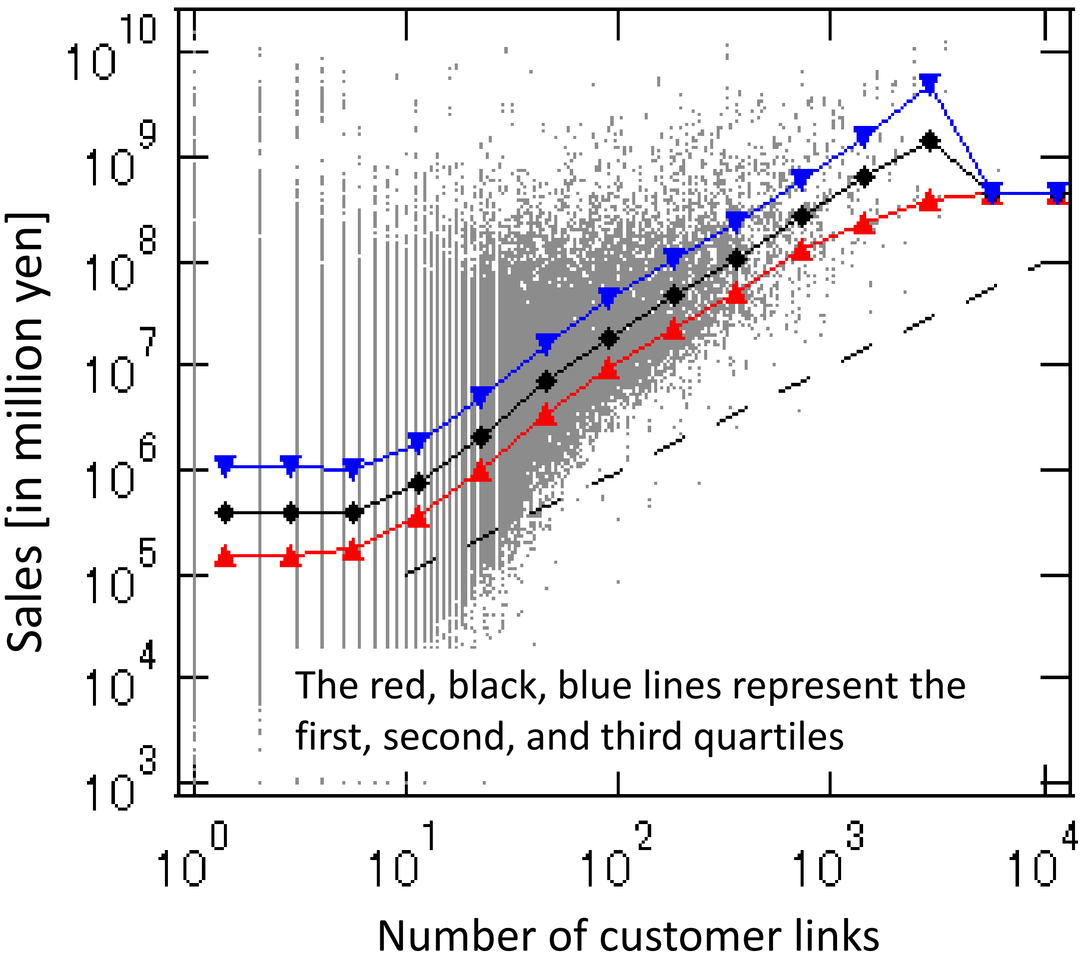



Red dots represent firms connected with firm A by one link. Black dots are all firms in the dataset.

# Shortest path lengths between two firms



# Relationship between sales and the number of customer links calculated using the list of core partners




ln Sales = 1.38 ln Degree

This implies:
When the sales of firm A is higher than the sales of firm B by 10 percent, the contribution of the number of links (i.e. extensive margin) is **7.2 percent** while the contribution of the size of the links (i.e. intensive margin) is **2.8 percent**.

# Turnover of Customer and Supplier Links

| **Customer Links** | Number of Links in the Initial Year | Net Increase | Entry | Survive | Exit |
|---|---|---|---|---|---|
| Between 2008 and 2009 | 867,621 | 29,579 | 93,539 | 803,661 | 63,960 |
| | | (0.034) | (0.108) | (0.926) | (0.074) |
| Between 2009 and 2010 | 777,886 | 24,429 | 78,281 | 724,034 | 53,852 |
| | | (0.031) | (0.101) | (0.931) | (0.069) |
| Between 2008 and 2010 | 767,231 | 43,494 | 140,574 | 670,151 | 97,080 |
| | | (0.057) | (0.183) | (0.873) | (0.127) |

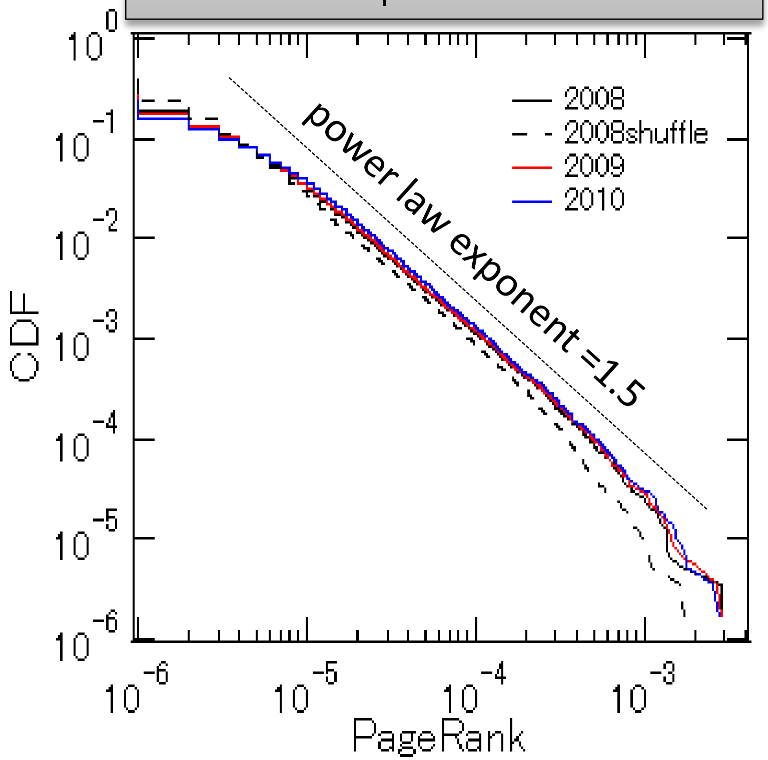
| **Supplier Links** | Number of Links in the Initial Year | Net Increase | Entry | Survive | Exit |
|---|---|---|---|---|---|
| Between 2008 and 2009 | 864,822 | 19,413 | 77,149 | 807,086 | 57,736 |
| | | (0.022) | (0.089) | (0.933) | (0.067) |
| Between 2009 and 2010 | 769,501 | 12,790 | 59,593 | 722,698 | 46,803 |
| | | (0.017) | (0.077) | (0.939) | (0.061) |
| Between 2008 and 2010 | 767,695 | 26,467 | 114,621 | 679,541 | 88,154 |
| | | (0.034) | (0.149) | (0.885) | (0.115) |

# PageRank Distributions

# Cumulative Distributions of PageRanks

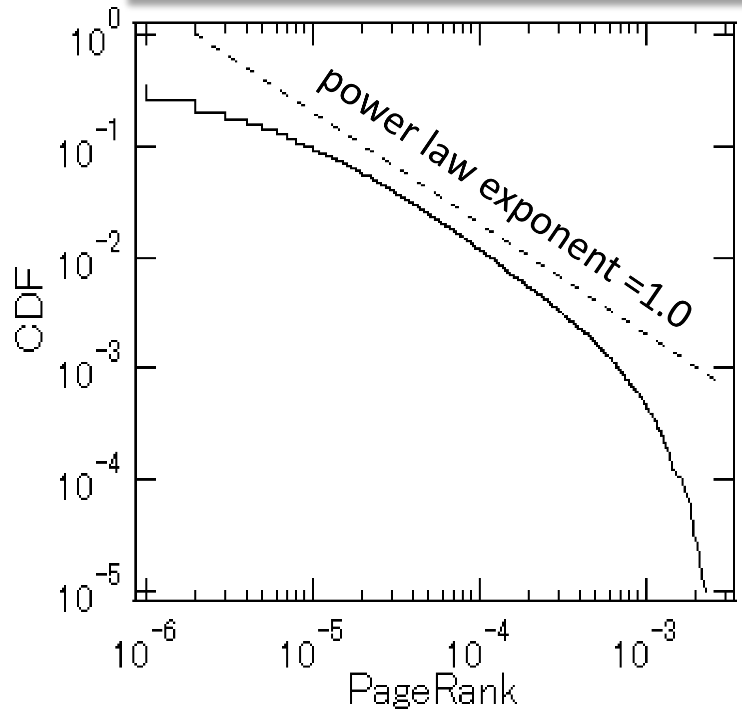
**Network among core partners**

Estimated using the list of main partners




**Network among all partners**

Estimated using the total number of customer/supplier links




- PageRank distributions are close to power law with a tail exponent ranging from 1.0 to 1.5.
- The tail part is less heavy for network among core partners than for network among all partners.
- The estimated tail exponents are almost the same as the tail exponents for the number of customers.

# To what extent do idiosyncratic shocks account for aggregate volatility?

Acemoglu et al (2010)

$$\sigma_{\text{macro}} = \sigma_{\text{micro}} \sqrt{\sum_{i=1}^{N} \left( PR_i \Big/ \sum_{i=1}^{N} PR_i \right)^2}$$

For the firms in our dataset, the average of the SDs is 0.4878 ➡ $\sigma_{\text{micro}} = 0.4878$

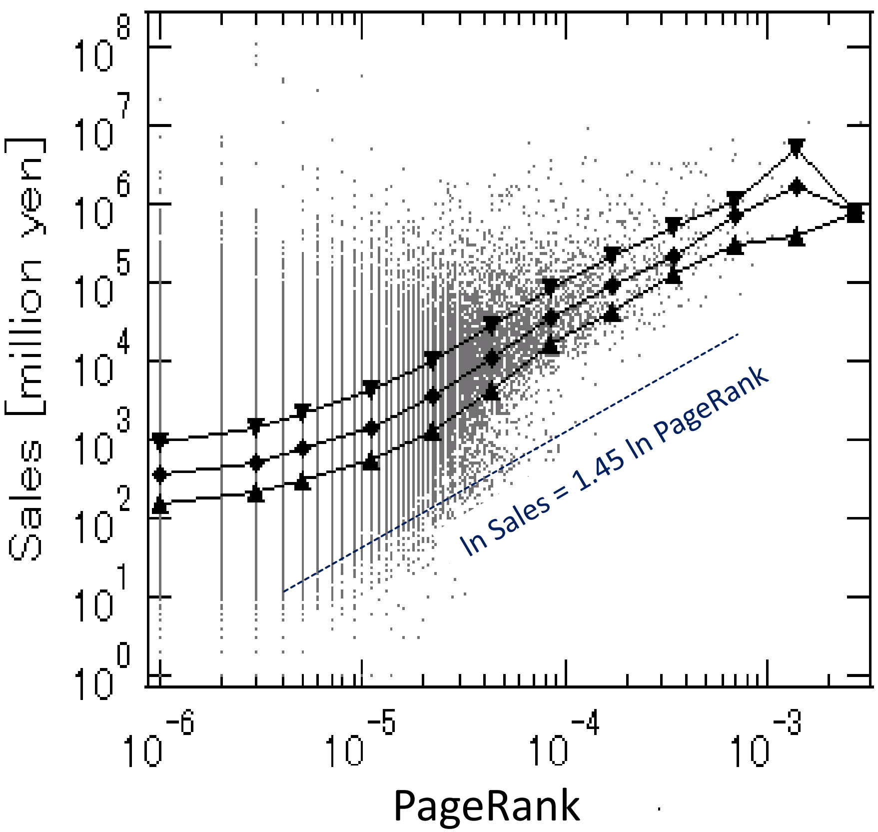
| | PL exponent = 1.0 | | PL exponent = 1.5 | | PL exponent > 2 | |
|---|---|---|---|---|---|---|
| | $\sqrt{\sum_i \left(\frac{PR_i}{\sum_i PR_i}\right)^2}$ | $\sigma_{\text{macro}}$ | $\sqrt{\sum_i \left(\frac{PR_i}{\sum_i PR_i}\right)^2}$ | $\sigma_{\text{macro}}$ | $\sqrt{\sum_i \left(\frac{PR_i}{\sum_i PR_i}\right)^2}$ | $\sigma_{\text{macro}}$ |
| N=10,000 | 0.1310 | 0.0639 | 0.0299 | 0.0146 | 0.0100 | 0.0049 |
| N=100,000 | 0.1061 | 0.0517 | 0.0137 | 0.0067 | 0.0032 | 0.0015 |
| N=1,000,000 | 0.0891 | 0.0435 | 0.0064 | 0.0031 | 0.0010 | 0.0005 |

# PageRank vs. Sales

# PageRank vs. Sales

PageRank is estimated using the list of core partners in 2008

ln Sales = 1.45 ln PageRank




When the sales of firm A is higher than the sales of firm B by 10 percent, PageRank of A is higher than PageRank of B only by 6.9 percent, indicating that there is a close relationship between the two but it is not one-to-one.

This implies that the assumptions adopted in defining PageRank is violated in the data;

(1) final demand may not be equal across firms

(2) the size of links may not be equal across firms

Note: Solid lines indicate the first, second, and third quartiles.

# Granular Hypothesis vs. Network Hypothesis
## Two hypotheses are identical under the two assumptions

**Granular Hypothesis**

Gabaix (2010, Proposition 2)

- **Firm sales** follows a power law with an exponent of $\mu$.
- The SD of the growth rate of individual firm is $\sigma$ (identical across firms)

**Network Hypothesis**

Acemoglu et al (2010, 2011, 2012, Corollary 1)

- **Page rank** follows a power law with an exponent of $\mu$.
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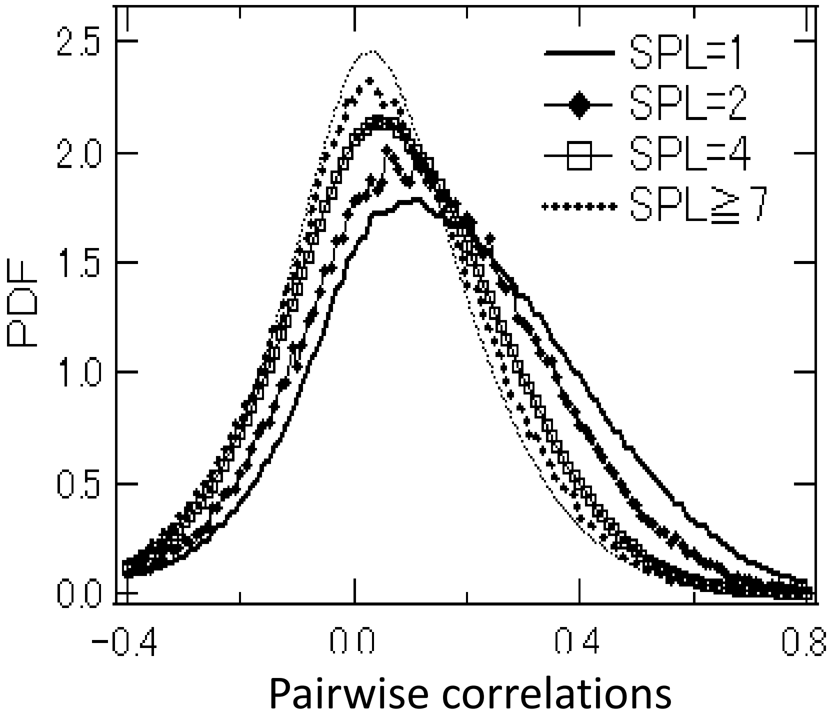
The SD of GDP decays with the number of firms, *N*, but the convergence rate depends on the value of $\mu$.

The SD of GDP converges at $\ln N$ for $\mu = 1$
The SD of GDP converges at $N^{1-1/\mu}$ for $1 < \mu < 2$
The SD of GDP converges at $N^{1/2}$ for $\mu \geq 2$

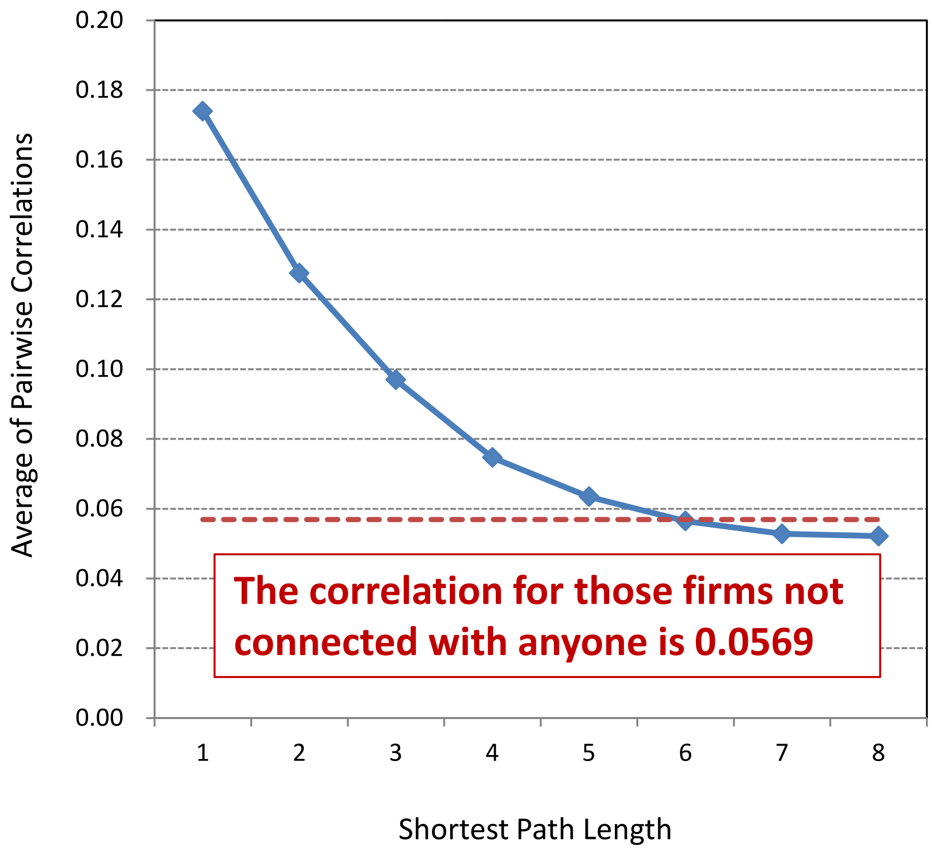
# Growth Correlations of Neighbor Firms

# Pairwise growth correlation across firms conditional on the shortest path lengths

Distributions of pairwise growth correlations




Average growth correlations conditional on the shortest path length




Note: Pairwise growth correlations are calculated for those firms with growth rate data available in 1980 to 2009 (# of OBS=134,067)

# Eliminating growth correlations due to common shocks

$$\mathbf{g}_t = \Gamma \mathbf{g}_t + \epsilon_t$$

$\mathbf{g}_t = [g_{1t}, g_{2t}, \ldots, g_{Nt}]'$ : Sales growth rates

$\epsilon_t = [\epsilon_{1t}, \epsilon_{2t}, \ldots, \epsilon_{Nt}]'$ : Productivity shocks

$$\epsilon_t = \underbrace{\Lambda \mathbf{u}_t}_{\text{Common Shocks}} + \underbrace{\mathbf{v}_t}_{\text{Idiosyncratic Shocks}}$$

**Step 1** $\epsilon_t = (I - \Gamma)\,\mathbf{g}_t$

**Step 2** We eliminate a simultaneous pairwise correlation between $\epsilon_i$ and $\epsilon_j$ by randomly exchanging $\epsilon_{it}$ and $\epsilon_{is}$ until the correlations are completely destroyed ("random shuffling"). We denote the uncorrelated new series by $\hat{\epsilon}_t$ .

**Step 3** $\hat{\mathbf{g}}_t \equiv (I - \Gamma)^{-1}\,\hat{\epsilon}_t$

Growth rates for *i* and *j* are not correlated through common shocks but correlated through linkage

**Step 4** We estimate the growth correlation due to common shocks by looking at the correlation for pairs of firms which are not connected through the network.

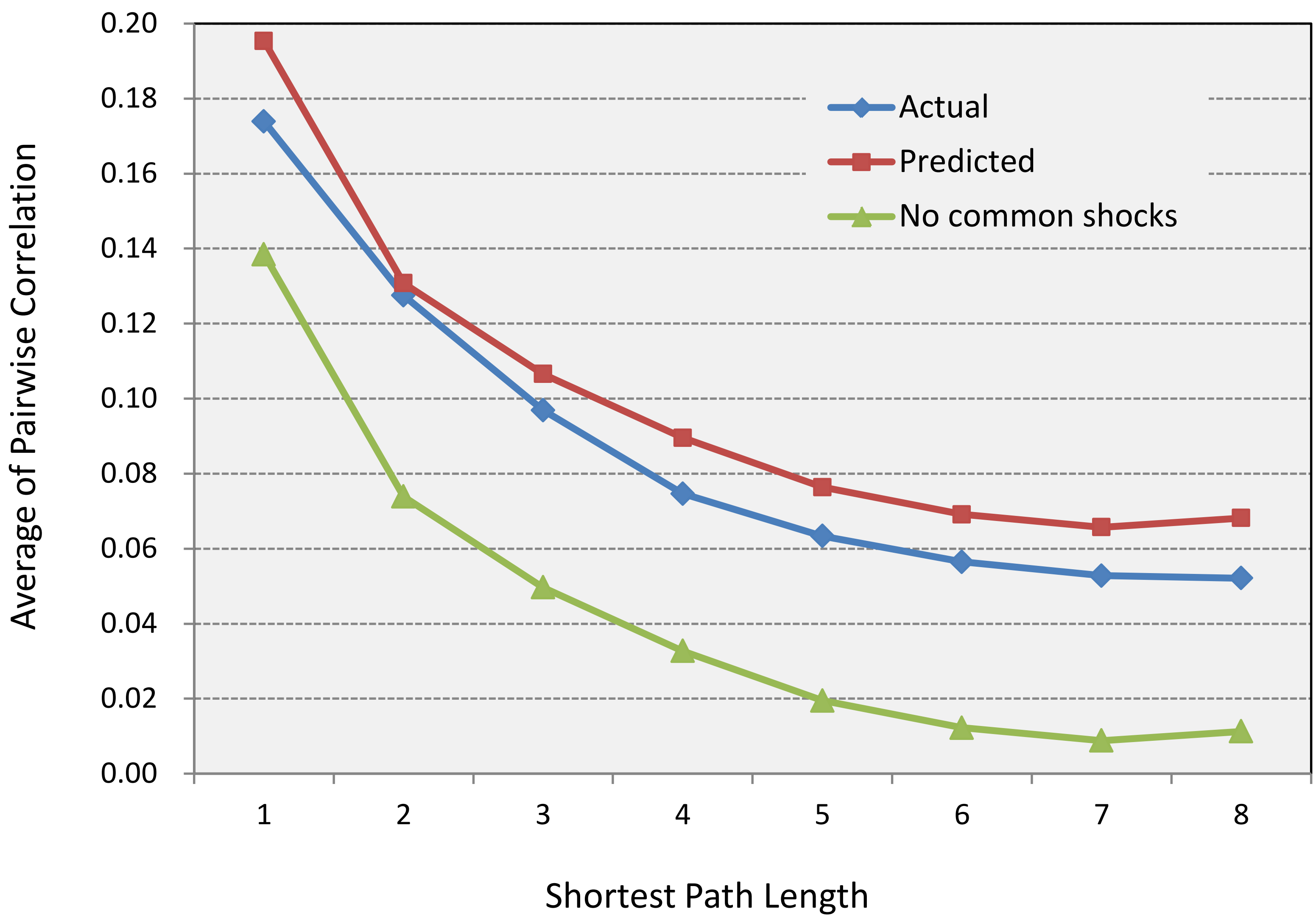
# Pairwise sales growth correlations conditional on SPL

| Shortest Path Length | Actual | Growth correlations calculated for $\hat{g}_t$ (B) | Predicted (A)+(B) |
|---|---|---|---|
| SPL=1 | 0.1740 | 0.1385 | 0.1954 |
| SPL=2 | 0.1275 | 0.0739 | 0.1308 |
| SPL=3 | 0.0969 | 0.0497 | 0.1066 |
| SPL=4 | 0.0746 | 0.0327 | 0.0896 |
| SPL=5 | 0.0634 | 0.0195 | 0.0764 |
| SPL=6 | 0.0565 | 0.0122 | 0.0691 |
| SPL=7 | 0.0528 | 0.0088 | 0.0657 |
| SPL=8 | 0.0521 | 0.0113 | 0.0682 |

| | |
|---|---|
| SPL=∞ Pairs of firms not connected through the network | 0.0569 (A) |

Predicted
= Growth correlations calculated for $\hat{g}_t$
+ correlation due to common shocks (0.0569)

# Disentangling growth correlations due to linkage and growth correlations due to common shocks



# Main Findings of the paper

1. The number of customer links follows a power law distribution with an exponent of one (Zipf's law). The number of supplier links also follows a power law, but the tail exponent is greater (i.e. less heavy tail) compared to the customer link distribution.

2. Firm sales is closely correlated with the number of customer links. When the sales of a firm increases by 10 percent, the contribution of the number of inks (i.e. extensive margin) is 7.2 percent while the contribution of the size of the links (i.e. intensive margin) is 2.8 percent.

3. PageRank follows a power law distribution with the tail exponent ranging from 1.0 to 1.5 (it depends on how it is measured). The tail exponent of 1.0 to 1.5 implies that the impact of idiosyncratic shocks on aggregate volatility decays with the number of firms much more slowly than implied by the law of large number.

4. PageRank is closely correlated with firm sales, but the relationship is not one-to-one. When the sales of firm A is higher than the sales of firm B by 10 percent, PageRank of A is higher than PageRank of B only by 6.9 percent. This implies that inequality in sales may come not only from inequality in intermediate demand, but also from inequality in final demand.

5. Correlations of sales growth between a pair of firms depends negatively on the shortest path length between the two firms. This result remains unchanged even if one eliminates growth correlations due to common shocks. This is a direct evidence that non-trivial portion of aggregate volatility stems from the propagation of idiosyncratic shocks through buyer-supplier networks.

1. Acemoglu, Daron, Vasco M. Carvalho, Asuman Ozdaglar, and Alireza Tahbaz-Salehi (2011), “The Network Origins of Aggregate Fluctuations,” MIT, October 2011.
2. Acemoglu, Daron, Asuman Ozdaglar, and Alireza Tahbaz-Salehi (2010), “Cascades in networks and aggregate volatility,” Working Paper 16516, National Bureau of Economic Research.
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