Optimal monetary policy when asset markets are incomplete

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Outline

1. Introduction
2. Model
   - Individuals
   - Aggregation
   - Firms
   - Aggregate shocks
   - Government
3. Results
   - Permanent productivity shock
   - Temporary productivity shock
4. Conclusion
Inflation-output tradeoff in the representative-agent framework

- In the standard sticky price model, the optimal monetary policy is approximately given by complete inflation stabilization.
  - Schmitt-Grohé and Uribe (2007), etc.

- Concerning the output-inflation tradeoff, the monetary authority should place exclusive weight on the inflation stabilization.

- The welfare cost of business cycles is nil in the representative-agent framework used in the standard New Keynesian model.
Uninsured idiosyncratic shocks

- Idiosyncratic income shocks are very persistent and their variance fluctuate countercyclically.
  - Storesletten, Telmer and Yaron (2004), Meghir and Pistaferri (2004), etc.
- The existence of such idiosyncratic shocks may generate a large welfare-cost of business cycles.
  - Krebs (2003), De Santis (2007), etc.
- How does it affect optimal monetary policy? In particular, how does it change the weight the monetary authority should place on the inflation stabilization?
This paper

- Individuals face uninsured idiosyncratic income shocks with countercyclical variance.
- The model is otherwise standard new Keynesian model with:
  - monopolistic competition;
  - Calvo price setting;
  - capital accumulation.
- Consider optimal monetary policy (Ramsey policy).
Main findings

- Countercyclical idiosyncratic risk can generate a very large welfare-cost of business cycles.
- But it does not affect the inflation-output tradeoff much.
  - The optimal monetary policy is essentially characterized as complete price-level stabilization.
  - Thus, the monetary authority should place almost exclusive weight on the stabilization of inflation.
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Composite good

- $Y_t = \text{aggregate output of a composite good:}$

$$Y_t = \left( \int_0^1 Y_{j,t}^{1-\frac{1}{\zeta}} \, dj \right)^{\frac{1}{1-\frac{1}{\zeta}}}$$

which can be consumed or invested:

$$Y_t = C_t + I_t$$

- $P_t = \text{price index:}$

$$P_t = \left( \int_0^1 P_{j,t}^{1-\zeta} \, dj \right)^{\frac{1}{1-\zeta}}$$
Preferences of individuals

- A continuum of ex-ante identical individuals.
- Preferences:
  \[
  u_{i,0} = E^i_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{1 - \gamma} \left[ c_{i,t}^{\theta} (1 - l_{i,t})^{1-\theta} \right]^{1-\gamma}
  \]

- Let \( 1/\gamma_c = \) elasticity of intertemporal substitution of consumption for a fixed level of leisure:
  \[
  \gamma_c \equiv 1 - \theta(1 - \gamma)
  \]
In general, with uninsured idiosyncratic shocks, the wealth distribution, an infinite-dimensional object, must be included in the state variable.

We circumvent this problem by assuming that

- idiosyncratic shocks follow random walk processes;
- idiosyncratic shocks affect both labor and capital income.
Idiosyncratic shocks

Random walk with countercyclical variance

\[ \eta_{i,t} = \text{the idiosyncratic shock for individual } i: \]

\[ \ln \eta_{i,t} = \ln \eta_{i,t-1} + \sigma_{\eta,t} \epsilon_{\eta,i,t} - \frac{\sigma_{\eta,t}^2}{2} \]

where

- \( \epsilon_{\eta,i,t} \) is i.i.d., and \( N(0, 1) \).
- \( \sigma_{\eta,t} \) = variance of innovations to idiosyncratic shocks, which is assumed to fluctuate countercyclically.
Assume that $\eta_{i,t}$ affects $i$’s income in two ways.

- $\eta_{i,t}$ equals the productivity of individual $i$’s labor.
- $\eta_{i,t}$ also affects the return to savings of individual $i$.

The flow budget constraint of $i$ is given by

$$c_{i,t} + k_{i,t} + s_{i,t} = \frac{\eta_{i,t}}{\eta_{i,t-1}} \left( R_{k,t} k_{i,t-1} + R_{s,t} s_{i,t-1} \right) + \eta_{i,t} w_t l_{i,t}$$

where $k_{i,t} = \text{physical capital}$ and $s_{i,t} = \text{value of shares}$.
The assumption that $\eta_{i,t}$ also operates as a shock to the return to individual savings is artificial, but ...

Without this assumption, the wealth distribution would have to be included as a state variable.

With this assumption, the effect of the presence of idiosyncratic shocks would be overemphasized.

Our finding is that the tradeoff faced by the monetary authority is little affected by the presence of idiosyncratic shocks.

Hence, dropping this assumption would strengthen our result.
Associated representative-agent problem

- Consider a representative-agent’s utility maximization problem:

$$\max U_0 = E_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\gamma} \nu_t \left[ C_t^\theta (1 - L_t)^{1-\theta} \right]^{1-\gamma}$$

subject to

$$C_t + K_t + S_t = R_{k,t} K_{t-1} + R_{s,t} S_{t-1} + w_t L_t$$

- Here, $\nu_t$ is a preference shock defined by

$$\nu_t \equiv \exp \left[ \frac{1}{2} \gamma_c (\gamma_c - 1) \sum_{s=0}^{t} \sigma_{\eta,s}^2 \right]$$

$$= E_t[\eta_{i,t}^{1-\gamma_c}]$$
Suppose that \( \{ C_t^*, L_t^*, K_t^*, S_t^* \}_{t=0}^\infty \) is a solution to the representative agent’s problem. For each \( i \in [0, 1] \), let

\[
\begin{align*}
  c_{i,t}^* &= \eta_{i,t} C_t^* \\
  l_{i,t}^* &= L_t^* \\
  k_{i,t}^* &= \eta_{i,t} K_t^* \\
  s_{i,t}^* &= \eta_{i,t} S_t^*
\end{align*}
\]

Then \( \{ c_{i,t}^*, l_{i,t}^*, k_{i,t}^*, s_{i,t}^* \}_{t=0}^\infty \) is a solution to the problem of individual \( i \).
Remark 1

The utility of the representative agent is indeed the cross-sectional average of individual utility:

\[ U_0 = E_0[u_{i,0}] \]
Remark 2

How idiosyncratic shocks affect the aggregate economy can be understood by looking at the “effective discount factor”:

$$\tilde{\beta}_{t,t+1} \equiv \beta \frac{\nu_{t+1}}{\nu_t}$$

$$= \beta \exp \left[ \frac{1}{2} \gamma_c (\gamma_c - 1) \sigma_{\tilde{\eta},t+1}^2 \right]$$

Thus

$$\uparrow \sigma_{\tilde{\eta},t+1} \quad \implies \quad \begin{cases} 
\uparrow \tilde{\beta}_{t,t+1} & \text{if } \gamma_c > 1 \\
\downarrow \tilde{\beta}_{t,t+1} & \text{if } \gamma_c < 1 
\end{cases}$$
Remark 3

- The SDF used by individual $i$ is

\[
\frac{\beta \lambda_{i,t+1}}{\lambda_{i,t}} = \frac{\lambda_{t+1}}{\lambda_t} \left( \frac{\eta_{i,t+1}}{\eta_{i,t}} \right)^{-\gamma_c}
\]

\[
= \beta \frac{\lambda_{t+1}}{\lambda_t} \exp \left( -\gamma_c \sigma_{\eta,t+1} \varepsilon_{\eta,i,t+1} + \frac{\gamma_c}{2} \sigma_{\eta,t+1}^2 \right)
\]

- It follows that individuals agree on the present value of the profit stream of each firm.

- In particular, they agree with the representative agent, whose SDF is given by $\beta \frac{\lambda_{t+1} \nu_{t+1}}{\lambda_t \nu_t}$.
Firms

- Standard model with monopolistic competition and Calvo price setting.
- Production technology of firm $j$:

$$Y_{j,t} = z_t^{1-\alpha} K_j^{\alpha} L_j^{1-\alpha} - \Phi_t$$

where $z_t$ is aggregate productivity shock, and $\Phi_t$ is a fixed cost of production.
- Demand for variety $j$:

$$Y_{j,t} = \left( \frac{P_{j,t}}{P_t} \right)^{-\xi} Y_t$$

$1 - \xi$ = probability of arriving an opportunity to change the price of each variety.
Aggregate shocks

Productivity shock is either permanent or temporary.

1. The case of permanent productivity shock:

\[
\ln z_t = \ln z_{t-1} + \mu + \sigma_z \epsilon_z,t - \frac{\sigma_z^2}{2}
\]

\[
\sigma_{\eta,t}^2 = \bar{\sigma}_{\eta}^2 + b \sigma_z \epsilon_z,t
\]
Aggregate shocks

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1. The case of permanent productivity shock:

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\ln z_t = \ln z_{t-1} + \mu + \sigma_z \epsilon_{z,t} - \frac{\sigma_z^2}{2} \\
\sigma_{\eta,t}^2 = \bar{\sigma}_\eta^2 + b \sigma_z \epsilon_{z,t}
\]

2. The case of temporary productivity shock:

\[
\ln z_t = \rho_z \ln z_{t-1} + \sigma_z \epsilon_{z,t} - \frac{\sigma_z^2}{2(1 + \rho_z)} \\
\sigma_{\eta,t}^2 = \bar{\sigma}_\eta^2 + b \ln z_t
\]
Fiscal policy: no taxes, no debt, etc.

Monetary policy: Set the state-contingent path of the inflation rate $\{\pi_t\}$.

Two monetary policy regimes:

1. Ramsey regime: Set $\{\pi_t\}$ so as to maximize the ex ante utility of individuals.
2. Inflation-targeting regime: Set $\pi_t = 1$ at all times.
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Most parameters are calibrated following Boldrin, Christiano and Fisher (2001) and Schmitt-Grohé and Uribe (2007).

We compare the following cases:

- $\gamma_c = 0.7, 2$;
- $b = 0, -0.8$;
- productivity shock is either permanent or temporary;
- the monetary policy regime is either Ramsey or inflation-targeting.
Welfare measures

$\Delta_{bc} =$ welfare cost of business cycles:

$$\sum_{t=0}^{\infty} \beta^t \bar{v}_t \frac{1}{1-\gamma} \left[ (((1 - \Delta_{bc}) \bar{C})^\theta (1 - \bar{L})^{1-\theta}) \right]^{1-\gamma}$$

$$= E_{-1} \sum_{t=0}^{\infty} \beta^t \nu_t \frac{1}{1-\gamma} \left[ (C_{t}^{rbc})^\theta (1 - L_{t}^{rbc})^{1-\theta} \right]^{1-\gamma}$$

$\Delta_{inf} =$ welfare cost of the inflation-targeting regime:

$$E_{-1} \sum_{t=0}^{\infty} \beta^t \nu_t \frac{1}{1-\gamma} \left[ (((1 - \Delta_{inf}) C_{t}^{ram})^\theta (1 - L_{t}^{ram})^{1-\theta}) \right]^{1-\gamma}$$

$$= E_{-1} \sum_{t=0}^{\infty} \beta^t \nu_t \frac{1}{1-\gamma} \left[ (C_{t}^{inf})^\theta (1 - L_{t}^{inf})^{1-\theta} \right]^{1-\gamma}$$
Permanent productivity shock

Welfare costs of business cycles and the inflation-targeting regime

| $\gamma_c$ | 0.7 | 0.7 | 2 | 2 |
| $b$ | 0 | -0.8 | 0 | -0.8 |
| $\Delta_{bc}$ (%) | -0.8191 | -1.2983 | 2.0938 | 7.3301 |
| $\Delta_{inf}$ (%) | 0.0000 | 0.0000 | 0.0002 | 0.0006 |
Optimal monetary policy with incomplete markets

Results

Permanent productivity shock

Permanent productivity shock

Impulse responses when $\gamma_c = 0.7$ and $b = 0$.

Solid lines: Ramsey policy; dashed lines: inflation targeting.
Permanent productivity shock

Impulse responses when $\gamma_c = 0.7$ and $b = -0.8$.

Solid lines: Ramsey policy; dashed lines: inflation targeting.
Permanent productivity shock

Impulse responses when $\gamma_c = 2$ and $b = 0$.

Solid lines: Ramsey policy; dashed lines: inflation targeting.
Permanent productivity shock

Impulse responses when $\gamma_c = 2$ and $b = -0.8$.

Solid lines: Ramsey policy; dashed lines: inflation targeting.
## Temporary productivity shock

Welfare costs of business cycles and the inflation-targeting regime

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Results

Temporary productivity shock

Impulse responses when $\gamma_c = 0.7$ and $b = 0$.

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Conclusion

- We have developed a New Keynesian model with uninsurable idiosyncratic income shocks.
- The welfare cost of business cycles can be very large when the variance of idiosyncratic shocks fluctuates countercyclically.
- Nevertheless, the optimal monetary policy is roughly the same as the zero-inflation policy. The presence of countercyclical idiosyncratic shocks does not affect the inflation-output tradeoff.