

# Optimal monetary policy when asset markets are incomplete

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(very preliminary)

## Abstract

This paper considers the properties of an optimal monetary policy when households are subject to counter-cyclical uninsured income shocks. We develop a tractable incomplete-markets model with Calvo price setting. In our model the welfare cost of business cycles is large when the variance of income shocks is counter-cyclical. Nevertheless, the optimal monetary policy is very similar to the optimal policy that emerges in the representative agent framework and calls for nearly complete stabilization of the price-level.

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# 1 Introduction

Recent empirical studies have found that individuals face highly persistent idiosyncratic income shocks and that the variance of these shocks is countercyclical (Storesletten, Telmer, Yaron, 2004; Meghir and Pistaferri, 2004). If asset markets are incomplete and these income shocks are not insurable, then the welfare cost of business cycles can be very large as shown, for instance, by Krebs (2003) and De Santis (2007). In this paper, we investigate how monetary policy should be conducted in the presence of persistent and countercyclical idiosyncratic risk.

We consider a New Keynesian model with Calvo-style price rigidity, augmented with uninsured idiosyncratic income shocks.<sup>1</sup> As shown, for instance, by Schmitt-Grohé and Uribe (2007), the optimal monetary policy obtained in the standard New Keynesian model calls for (nearly) complete inflation stabilization. That is, in the context of the classical trade-off between inflation and output fluctuations, the monetary authority should almost exclusively focus on the stabilization of inflation. This result, however, is produced in the representative-agent framework, in which the welfare cost of business cycles is negligible, as is originally shown by Lucas (1987). Our model with persistent and countercyclical idiosyncratic risk yields a sizable cost of business cycles, and it is interesting to see how this alters the trade-off between inflation and output stabilization.

Our model builds on the exchange economy studied by Constantinides and Duffie (1996) and extends it to a production economy with endogenous labor supply. We assume that the labor productivity of each individual follows a geometric random walk, and there are no insurance markets for that risk. Assuming such idiosyncratic shocks would in general require that the wealth distribution, an infinite-dimensional object, be included as a state variable, which causes a “curse of dimensionality” problem. In order to maintain tractability, we assume that the return to savings of each individual is also subject to idiosyncratic risk. Under these assumptions we can show that the no-trade theorem of Constantinides and Duffie (1996) extends to a production economy with endogenous labor supply.<sup>2</sup>

We also demonstrate that there exists a representative-agent economy with preference shocks that yields the same aggregate quantities and prices in equilibrium as the original

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<sup>1</sup>There is large literature on New Keynesian models. Useful overviews are provided by, for instance, Woodford (2003) and Galí (2008).

<sup>2</sup>Papers that apply the no-trade result of Constantinides and Duffie (1996) include, among others, Saito (1998), Krebs (2003, 2007), De Santis (2007). Angeletos (2007) considers a model with idiosyncratic shocks to the return to individual savings. None of these consider endogenous labor-leisure decision.

heterogeneous-agents economy with incomplete markets.<sup>3</sup> We show that an increase in the variance of idiosyncratic income shocks in our incomplete-markets economy has the same effect as an increase (resp. decrease) in the discount factor in the corresponding representative-agent economy if the elasticity of intertemporal substitution of consumption is less (resp. greater) than unity.

We then embed this incomplete-markets model into an otherwise standard New Keynesian model with monopolistic competition and the Calvo price setting. Calvo price setting makes profit maximization of each firm an intertemporal problem. When the financial markets are incomplete, shareholders, in general, do not agree on how to value future dividends.<sup>4</sup> In the context of the Calvo model, this would imply that when a firm obtains an opportunity to adjust the price of its product, its shareholders do not agree upon what price it should charge. Fortunately, however, under our assumptions, we establish that there is no disagreement problem and that all shareholders value future dividends in the same way.

We consider two kinds of aggregate shocks: a shock to the level of aggregate productivity and a shock to the variance of idiosyncratic income shocks. We are particularly interested in the case where the variance of idiosyncratic shocks is negatively correlated with the shock to the aggregate productivity level, so that idiosyncratic risk is countercyclical. For the productivity shock process, we consider two laws of motion, a geometric random walk, and a stationary, autoregressive process. We start by showing that the welfare cost of business cycles can be large in our economy. Consistent with the previous finding on the exchange economy (De Santis, 2007), with countercyclical idiosyncratic risk, the welfare cost of business cycles can be sizable (around 10% permanent decline in consumption) with a reasonable coefficient of relative risk aversion, regardless of whether the aggregate productivity shock is permanent or temporary.

We then use our model to answer the following question. How does such a large welfare cost of business cycles affect the trade-off between output and inflation stabilization? To examine this question, we compare two policy regimes: the Ramsey regime and the inflation-targeting regime. In the Ramsey regime, the monetary authority sets (with commitment) the state-contingent path of the inflation rate so as to maximize the average utility level. In the inflation-targeting regime, the monetary authority sets the inflation rate at zero at all times. Schmitt-Grohé and Uribe (2007) show in a representative-agent economy that the equilibrium obtained under the Ramsey regime is nearly identical to

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<sup>3</sup>For a general discussion on the correspondence between incomplete-markets economies and representative-agent economies, see Nakajima (2005).

<sup>4</sup>For the overview on the theory of incomplete markets, see, for instance, Magill and Quinzii (1996).

the equilibrium obtained under the inflation-targeting regime. Thus, the question here is how much the equilibria under these two policy regimes would differ in the presence of countercyclical idiosyncratic risk. We find that a similar result arises in our model. Optimal monetary policy is essentially given by complete price stabilization even with countercyclical idiosyncratic risk.

The rest of the paper is organized as follows. In Section 2, we describe our heterogeneous-agents economy with incomplete markets, and then construct a corresponding representative-agent economy which yields the same equilibrium as the original economy. In Section 3, we present our numerical results. In Section 4, we conclude.

## 2 The model economy

In this section we describe our model economy. It is a version of the neoclassical growth model with uninsurable idiosyncratic income shocks studied in Braun and Nakajima (2008), augmented with monopolistic competition and nominal price rigidities as in Calvo (1983). For simplicity we consider a cashless economy as in Woodford (2003).

### 2.1 Individuals

The economy is populated by a continuum of individuals of unit measure, indexed by  $i \in [0, 1]$ . They are subject to both idiosyncratic and aggregate shocks. We assume that idiosyncratic shocks are independent across individuals, and a law of large numbers applies. All individuals are assumed to be identical ex ante, that is, prior to period 0.

Individuals consume and invest a composite good, which is produced by a continuum of differentiated products, indexed by  $j \in [0, 1]$ . If the supply of each variety is given by  $Y_{j,t}$ , for  $j \in [0, 1]$ , the aggregate amount of the composite good,  $Y_t$ , is given by

$$Y_t = \left( \int_0^1 Y_{j,t}^{1-\frac{1}{\zeta}} dj \right)^{\frac{1}{1-\frac{1}{\zeta}}} \quad (1)$$

where  $\zeta > 1$  denotes the elasticity of substitution across different varieties. This composite good is used for consumption and investment:

$$Y_t = C_t + I_t \quad (2)$$

where  $C_t$  and  $I_t$  denote the aggregate amounts of consumption and investment in period  $t$ , respectively. Let  $P_{j,t}$  denote the price of variety  $j$  in period  $t$ . It then follows from cost minimization that the demand for each variety is given by

$$Y_{j,t} = \left( \frac{P_{j,t}}{P_t} \right)^{-\zeta} Y_t \quad (3)$$

where  $P_t$  is the price index defined by

$$P_t = \left( \int_0^1 P_{j,t}^{1-\zeta} dj \right)^{\frac{1}{1-\zeta}} \quad (4)$$

Preferences of each individual are described by the utility function defined over stochastic processes of consumption and leisure:

$$u_{i,0} = E_0^i \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\gamma} \left[ c_{i,t}^\theta (1-l_{i,t})^{1-\theta} \right]^{1-\gamma} \quad (5)$$

where  $\beta$  is a subjective discount factor,  $c_{i,t}$  is individual  $i$ 's amount of consumption of the composite good in period  $t$ , and  $l_{i,t}$  is her labor supply in period  $t$ . We use  $E_t^i$  to denote the expectation operator conditional on the history of idiosyncratic shocks to individual  $i$  up to and including period  $t$  as well as the history of aggregate shocks over the same time period. The expectation operator conditional on the history of aggregate shocks up to and including period  $t$  is denoted by  $E_t$ . For later reference, we define  $\gamma_c$  as

$$\gamma_c \equiv 1 - \theta(1 - \gamma) \quad (6)$$

Thus,  $1/\gamma_c$  is the intertemporal elasticity of substitution of consumption with a constant level of leisure.

The idiosyncratic risk faced by individual  $i$  is represented by a geometric random walk  $\{\eta_{i,t}\}$ :

$$\ln \eta_{i,t} = \ln \eta_{i,t-1} + \sigma_{\eta,t} \epsilon_{\eta,i,t} - \frac{\sigma_{\eta,t}^2}{2} \quad (7)$$

where  $\epsilon_{\eta,i,t}$  is  $N(0, 1)$  and i.i.d. across individuals and over time. The standard deviation,  $\sigma_{\eta,t}$ , is allowed to fluctuate over time, in a way whose evolution is specified below. All agents are assumed to start with the same initial realization of  $\eta$ , i.e.,  $\eta_{i,-1} = \eta_{-1}$ , for all  $i$ . The process  $\{\eta_{i,t}\}$  affects individual  $i$ 's income in two ways. First,  $\eta_{i,t}$  represents the productivity of individual  $i$ 's labor (her efficiency units of labor). Thus, if  $w_t$  is the real wage rate per efficiency unit of labor, the labor income of individual  $i$  in period  $t$  is given by  $w_t \eta_{i,t} l_{i,t}$ . If the idiosyncratic risk only affects individuals' labor income, then the distribution of wealth would have to be included in the vector of aggregate state variables, which would make the numerical evaluation of optimal monetary policy very costly to undertake. We circumvent this problem in the following way. Following Braun and Nakajima (2008), we assume that the rate of return to individuals' savings is also subject to idiosyncratic risk,  $\eta_{i,t}$ .

Suppose that claims to the ownership of physical capital and the ownership of firms are traded separately. We abstract from government bonds. Let  $q_{j,t}$  be the period- $t$  price of a share in firm  $j \in [0, 1]$ , and  $e_{i,j,t}$  be the share in firm  $j$  held by individual  $i$  at the end of period  $t$ . Below we look for an equilibrium in which all individuals choose the same portfolio weights, and hence they hold equal shares of all firms, that is,  $e_{i,j,t} = e_{i,t}$  for all  $j \in [0, 1]$ . Let  $s_{i,t}$  be the value of stocks held by individual  $i$ :  $s_{i,t} \equiv \int_0^1 q_{j,t} e_{i,j,t} dj = e_{i,t} \int_0^1 q_{j,t} dj$ , and let  $R_{s,t}$  be the gross rate of return on equities:  $R_{s,t} \equiv \int_0^1 (q_{j,t} + d_{j,t}) dj / \int_0^1 q_{j,t-1} dj$ . Then, without idiosyncratic shocks to the return on savings, the flow budget constraint for each individual would be given by

$$c_{i,t} + k_{i,t} + s_{i,t} = R_{k,t}k_{i,t-1} + R_{s,t}s_{i,t-1} + \eta_{i,t}w_t l_{i,t}$$

where  $k_{i,t}$  is the amount of physical capital obtained by individual  $i$  in period  $t$ , and  $R_{k,t}$  is the gross rate of return on physical capital, that is,

$$R_{k,t} = 1 - \delta + r_{k,t} \tag{8}$$

where  $r_{k,t}$  is the rental rate of capital and  $\delta$  is its depreciation rate. Instead we will assume that the return to savings is also subject to the idiosyncratic risk, so that the flow budget constraint becomes

$$c_{i,t} + k_{i,t} + s_{i,t} = \frac{\eta_{i,t}}{\eta_{i,t-1}} (R_{k,t}k_{i,t-1} + R_{s,t}s_{i,t-1}) + \eta_{i,t}w_t l_{i,t} \tag{9}$$

Since individuals are identical ex ante,

$$k_{i,-1} = K_{-1}, \quad \text{and} \quad s_{i,-1} = S_{-1} \tag{10}$$

for all  $i \in [0, 1]$ . To rule out Ponzi schemes, we impose  $k_{i,t} \geq 0$  and  $s_{i,t} \geq 0$ . These last two constraints will not bind in equilibrium.

In equation (9),  $\eta_{i,t}/\eta_{i,t-1}$  is an idiosyncratic shock to the return on savings. This assumption is purely a technical requirement that makes it possible for us to extend the result obtained by Constantinides and Duffie (1996) in an exchange economy to our production economy. Under this assumption “permanent income” of individual  $i$ , which is defined as the sum of human and financial wealth, is proportional to  $\eta_{i,t}$ . Under our assumption shocks to  $\eta_{i,t}$  magnify the effect of idiosyncratic risk on wealth as compared to the specification where they affect labor income only. As we shall see below, however, our main finding is that the presence of idiosyncratic shocks does not matter much for the conduct of monetary policy. Therefore, this feature of our model strengthens our conclusions.

At date 0, each individual chooses a contingent plan  $\{c_{i,t}, l_{i,t}, k_{i,t}, s_{i,t}\}$  so as to maximize her utility (5) subject to the sequence of flow budget constraints (9) and the short-selling constraint on  $\{k_{i,t}, s_{i,t}\}$ . The Lagrangian for the household's problem is

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{1}{1-\gamma} \left[ c_{i,t}^\theta (1-l_{i,t})^{1-\theta} \right]^{1-\gamma} + \lambda_{i,t} \left[ \frac{\eta_{i,t}}{\eta_{i,t-1}} (R_{k,t} k_{i,t-1} + R_{s,t} s_{i,t-1}) + \eta_{i,t} w_t l_{i,t} - c_{i,t} - k_{i,t} - s_{i,t} \right] \right\}$$

Then the first-order conditions are

$$\theta c_{i,t}^{-\gamma\theta} (1-l_{i,t})^{(1-\theta)(1-\gamma)} = \lambda_{i,t} \quad (11)$$

$$\frac{1-\theta}{\theta} \frac{c_{i,t}}{1-l_{i,t}} = w_t \eta_{i,t} \quad (12)$$

$$\lambda_{i,t} = \beta E_t^i \lambda_{i,t+1} \frac{\eta_{i,t+1}}{\eta_{i,t}} R_{k,t+1} \quad (13)$$

$$\lambda_{i,t} = \beta E_t^i \lambda_{i,t+1} \frac{\eta_{i,t+1}}{\eta_{i,t}} R_{s,t+1} \quad (14)$$

and the flow budget constraint (9). The transversality conditions for  $k_{i,t}$  and  $s_{i,t}$  are given respectively as

$$\lim_{t \rightarrow \infty} E_0^i \beta^t \lambda_{i,t} k_{i,t} = 0 \quad (15)$$

$$\lim_{t \rightarrow \infty} E_0^i \beta^t \lambda_{i,t} s_{i,t} = 0 \quad (16)$$

Given a vector stochastic process  $\{R_{k,t}, R_{s,t}, w_t\}$ , a solution to the utility maximization problem of each individual is a state-contingent plan  $\{c_{i,t}, l_{i,t}, k_{i,t}, s_{i,t}, \lambda_{i,t}\}$  that satisfies the first-order conditions (9)-(14), as well as the transversality conditions (15)-(16) and the initial conditions (10).

## 2.2 Aggregation

Nakajima (2005) establishes that incomplete-markets economies can be aggregated into representative-agent economies with stochastic shocks to the utility function. That is also true for our economy and, in addition, the utility function of the corresponding representative agent has an explicit form. The result here extends a previous result of Braun and Nakajima (2008) to an economy with monopolistic competition and staggered price setting.

Consider a representative agent with preferences defined by the utility function:

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\gamma} \nu_t \left[ C_t^\theta (1-L_t)^{1-\theta} \right]^{1-\gamma} \quad (17)$$

where  $C_t$  is the amount of consumption of the composite good defined in (1) in period  $t$ , and  $L$  is the amount of labor supply in period  $t$ . Here,  $\nu_t$  is the preference shock to the representative agent's utility in period  $t$  defined by

$$\nu_t \equiv \exp \left[ \frac{1}{2} \gamma_c (\gamma_c - 1) \sum_{s=0}^t \sigma_{\eta,s}^2 \right] \quad (18)$$

where  $\gamma_c$  is defined in (6), and  $\sigma_{\eta,t}$  is the standard deviation of the idiosyncratic shocks in period  $t$ , as in (7). Note that  $\nu_t$  is the cross-sectional average of  $\eta_{i,t}^{1-\gamma_c}$ :

$$\nu_t = E_t[\eta_{i,t}^{1-\gamma_c}]$$

where  $E_t$  denotes the expectation operator conditional on the history of aggregate shocks up to and including period  $t$ .

Suppose that the representative agent faces the following flow budget constraint:

$$C_t + K_t + S_t = R_{k,t}K_{t-1} + R_{s,t}S_{t-1} + w_tL_t \quad (19)$$

and initial conditions  $K_{-1}, S_{-1} > 0$ . Here  $K_t$  and  $S_t$  are the amount of physical capital and the value of stocks held by the representative agent in period  $t$ . We assume the short-selling constraints:  $K_t, S_t \geq 0$ . These two constraints do not bind in equilibrium. Given prices and the initial condition, the representative agent chooses a contingent plan  $\{C_t, L_t, K_t, S_t\}$  so as to maximize the lifetime utility  $U_0$  in (17) subject to the sequence of flow budget constraints (19) and the short-selling constraints. Let us form the Lagrangian as

$$\begin{aligned} \mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \nu_t \left\{ \frac{1}{1-\gamma} \left[ C_t^\theta (1-L_t)^{1-\theta} \right]^{1-\gamma} \right. \\ \left. + \lambda_t [R_{k,t}K_{t-1} + R_{s,t}S_{t-1} + w_tL_t - C_t - K_t - S_t] \right\} \end{aligned}$$

The first-order conditions are given by

$$\theta C_t^{-\gamma_c} (1-L_t)^{(1-\theta)(1-\gamma)} = \lambda_t \quad (20)$$

$$\frac{1-\theta}{\theta} \frac{C_t}{1-L_t} = w_t \quad (21)$$

$$\lambda_t = E_t \beta \frac{\nu_{t+1}}{\nu_t} \lambda_{t+1} R_{k,t+1} \quad (22)$$

$$\lambda_t = E_t \beta \frac{\nu_{t+1}}{\nu_t} \lambda_{t+1} R_{s,t+1} \quad (23)$$

and the flow budget constraint (19). The transversality condition for  $K_t$  and  $S_t$  are, respectively,

$$E_0 \beta^t \nu_t \lambda_t K_t = 0 \quad (24)$$

$$E_0 \beta^t \nu_t \lambda_t S_t = 0 \quad (25)$$



Given the initial conditions  $K_{-1}$  and  $S_{-1}$ , a solution to the utility maximization problem of the representative agent is given by  $\{C_t, L_t, K_t, S_t, \lambda_t\}$  that satisfies the first-order conditions (19)-(23), as well as the transversality conditions (24)-(25). The next proposition establishes the relationship between the solution to the utility maximization problem of the representative agent, and the solution to the utility maximization problem of each individual described in the previous subsection.

**Proposition 1.** *Given stochastic processes  $\{R_{k,t}, R_{s,t}, w_t, \sigma_{\eta,t}\}$  and initial conditions  $\{K_{-1}, S_{-1}\}$ , consider the utility maximization problem of individual  $i$  described in the previous subsection and the utility maximization problem of the representative agent described in this subsection. Suppose that  $\{C_t^*, L_t^*, K_t^*, S_t^*, \lambda_t^*\}_{t=0}^\infty$  is a solution to the representative agent's problem. For each  $i \in [0, 1]$ , let  $c_{i,t}^* = \eta_{i,t} C_t^*$ ,  $l_{i,t}^* = L_t^*$ ,  $k_{i,t}^* = \eta_{i,t} K_t^*$ ,  $s_{i,t}^* = \eta_{i,t} S_t^*$ , and  $\lambda_{i,t}^* = \eta_{i,t}^{-\gamma_c} \lambda_t^*$ . Then  $\{c_{i,t}^*, l_{i,t}^*, k_{i,t}^*, s_{i,t}^*, \lambda_{i,t}^*\}_{t=0}^\infty$  is a solution to the problem of individual  $i$ .*

*Proof.* TO BE ADDED. □

In what follows we derive the equilibrium conditions for our incomplete-markets economy using the first-order conditions for the representative agent, (19)-(23) and the transversality conditions (24)-(25). Note that in equilibrium the utility of the representative agent (17) equals the cross-sectional average of individual utility (5):

$$\begin{aligned}
E_0[u_{i,0}] &= E_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\gamma} c_{i,t}^{1-\gamma_c} (1-l_{i,t})^{(1-\theta)(1-\gamma)} \\
&= E_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\gamma} \eta_{i,t}^{1-\gamma_c} C_t^{1-\gamma_c} (1-L_t)^{(1-\theta)(1-\gamma)} \\
&= E_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\gamma} E_t[\eta_{i,t}^{1-\gamma_c}] C_t^{1-\gamma_c} (1-L_t)^{(1-\theta)(1-\gamma)} \\
&= E_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\gamma} \nu_t C_t^{1-\gamma_c} (1-L_t)^{(1-\theta)(1-\gamma)} \\
&= U_0
\end{aligned}$$

To see how the size of idiosyncratic shocks,  $\sigma_{\eta,t}$ , affects the economy, define the “effective discount factor” between periods  $t$  and  $t+1$ ,  $\tilde{\beta}_{t,t+1}$ , as

$$\begin{aligned}
\tilde{\beta}_{t,t+1} &\equiv \beta \frac{\nu_{t+1}}{\nu_t} \\
&= \beta \exp \left[ \frac{1}{2} \gamma_c (\gamma_c - 1) \sigma_{\eta,t+1}^2 \right]
\end{aligned} \tag{26}$$

where the second equality follows from (18). This expression illustrates that the presence of idiosyncratic shocks ( $\sigma_{\eta,t} > 0$ ) makes the effective discount factor higher if  $\gamma_c > 1$  and

lower if  $\gamma_c < 1$ . Moreover, cyclical fluctuations in the variance of idiosyncratic shocks,  $\sigma_{\eta,t}^2$ , induce cyclical variations in the effective discount factor  $\tilde{\beta}_{t,t+1}$ .

A special feature of our economy is that, in spite of market incompleteness, there is agreement among individuals on the present value of future dividends of each firm. To see this, note that the stochastic discount factor used by individual  $i$  is

$$\begin{aligned}\beta \frac{\lambda_{i,t+1}}{\lambda_{i,t}} &= \beta \frac{\lambda_{t+1}}{\lambda_t} \left( \frac{\eta_{i,t+1}}{\eta_{i,t}} \right)^{-\gamma_c} \\ &= \beta \frac{\lambda_{t+1}}{\lambda_t} \exp \left( -\gamma_c \sigma_{\eta,t+1} \epsilon_{\eta,i,t+1} + \frac{\gamma_c}{2} \sigma_{\eta,t+1}^2 \right)\end{aligned}$$

Since  $\epsilon_{\eta,i,t+1}$  is i.i.d. across individuals and independent of the stochastic shocks faced by each firm, all individuals evaluate a given future payoff in the same way. In particular, we can use the stochastic discount factor of the representative agent,  $\beta \lambda_{t+1} \nu_{t+1} / (\lambda_t \nu_t)$ , to value future dividend streams of firms.

### 2.3 Firms

The production side of our economy is standard in the New Keynesian literature and similar to the one considered by Schmitt-Grohé and Uribe (2007). Each differentiated product is produced by a single firm in a monopolistically competitive environment. Firm  $j \in [0, 1]$  has the production technology:

$$Y_{j,t} = z_t^{1-\alpha} K_{j,t}^\alpha L_{j,t}^{1-\alpha} - \Phi_t \quad (27)$$

where  $z_t$  is the aggregate productivity shock,  $K_{j,t}$  is the physical capital used by firm  $j$  in period  $t$ ,  $L_{j,t}$  is its labor input, and  $\Phi_t$  is the fixed cost of production. The market clearing conditions for capital and labor are

$$\int_0^1 K_{j,t} dj = K_{t-1}, \quad \text{and} \quad \int_0^1 L_{j,t} dj = L_t$$

Here, note that the stock of capital available for the production in period  $t$  is  $K_{t-1}$ . The processes for  $z_t$  and  $\Phi_t$  are specified in the next subsection.

Consider the cost minimization problem of firm  $j$ :

$$\min_{K_{j,t}, L_{j,t}} w_t L_{j,t} + r_t K_{j,t}$$

subject to

$$z_t^{1-\alpha} K_{j,t}^\alpha L_{j,t}^{1-\alpha} - \Phi_t = Y_{j,t}$$

Let  $mc_{j,t}$  be the Lagrange multiplier, which will be interpreted as the marginal cost of production of firm  $j$ . Then the first-order conditions read

$$\begin{aligned} w_t &= mc_{j,t}(1 - \alpha)z_t^{1-\alpha}K_{j,t}^\alpha L_{j,t}^{-\alpha} \\ r_t &= mc_{j,t}\alpha z_t^{1-\alpha}K_{j,t}^{\alpha-1}L_{j,t}^{1-\alpha} \end{aligned}$$

It follows that all firms choose the same capital-labor ratio:

$$\frac{K_{j,t}}{L_{j,t}} = \frac{\alpha}{1 - \alpha} \frac{w_t}{r_t}$$

and that the marginal cost is identical for all firms:

$$\begin{aligned} mc_{j,t} &= \alpha^{-\alpha}(1 - \alpha)^{-1+\alpha}z_t^{\alpha-1}w_t^{1-\alpha}r_t^\alpha \\ &\equiv mc_t \end{aligned}$$

The first-order conditions for the cost-minimization problem of firm  $j$  can now be rewritten as

$$w_t = mc_t(1 - \alpha)z_t^{1-\alpha}K_{t-1}^\alpha L_t^{-\alpha} \quad (28)$$

$$r_t = mc_t\alpha z_t^{1-\alpha}K_{t-1}^{\alpha-1}L_t^{1-\alpha} \quad (29)$$

Firm  $j$ 's profit in period  $t$  is then given as

$$\begin{aligned} \frac{P_{j,t}}{P_t}Y_{j,t} - w_tL_{j,t} - r_tK_{j,t} &= \frac{P_{j,t}}{P_t}Y_{j,t} - mc_t(Y_{j,t} + \Phi_t) \\ &= \left(\frac{P_{j,t}}{P_t}\right)^{1-\zeta} Y_t - mc_t \left\{ \left(\frac{P_{j,t}}{P_t}\right)^{-\zeta} Y_t + \Phi_t \right\} \end{aligned}$$

The price of each variety is adjusted in a sluggish way as in Calvo (1983) and Yun (1996). For each firm, the opportunity to change the price of its product arrives with probability  $1 - \xi$  in each period. This random event occurs independently across firms (it is also independent of all other stochastic shocks in our economy). Without such an opportunity, a firm must charge the same price as in the previous period. Suppose that firm  $j$  obtains an opportunity to change its price in period  $t$ . It chooses  $P_{j,t}$  to maximize the present discounted value of profits:

$$\max_{P_{j,t}} E_t \sum_{s=0}^{\infty} \beta^s \frac{\lambda_{t+s}\nu_{t+s}}{\lambda_t\nu_t} \xi^s \left[ \left(\frac{P_{j,t}}{P_{t+s}}\right)^{1-\zeta} Y_{t+s} - mc_{t+s} \left\{ \left(\frac{P_{j,t}}{P_{t+s}}\right)^{-\zeta} Y_{t+s} + \Phi_{t+s} \right\} \right]$$

where  $\beta^s \lambda_{t+s}\nu_{t+s}/(\lambda_t\nu_t)$  is the stochastic discount factor used to evaluate (real) payoffs in period  $t + s$  in units of consumption in period  $t$ .

All firms with the opportunity to change their prices will choose the same price, so denote it by  $\tilde{P}_t$ . Then the first-order condition for the above profit-maximization problem is given by

$$E_t \sum_{s=0}^{\infty} (\xi\beta)^s \frac{\lambda_{t+s} \nu_{t+s}}{\lambda_t \nu_t} \left\{ (1-\zeta) \tilde{P}_t^{-\zeta} P_{t+s}^{\zeta-1} Y_{t+s} + \zeta \text{mc}_{t+s} \tilde{P}_t^{-\zeta-1} P_{t+s}^{\zeta} Y_{t+s} \right\} = 0$$

Define  $\tilde{\nu}_{t+s}$  as

$$\tilde{\nu}_{t+s} \equiv \frac{\nu_{t+s}}{\nu_t} = \exp \left\{ \frac{1}{2} \gamma_c (\gamma_c - 1) \sum_{u=t+1}^{t+s} \sigma_{\eta,u}^2 \right\}$$

Then, after some algebra, we can reexpress the first-order condition for  $\tilde{P}_t$  as

$$x_t^1 = \frac{\zeta - 1}{\zeta} \tilde{p}_t x_t^2 \quad (30)$$

where

$$\begin{aligned} \tilde{p}_t &\equiv \frac{\tilde{P}_t}{P_t} \\ x_t^1 &\equiv E_t \sum_{s=0}^{\infty} (\xi\beta)^s \lambda_{t+s} \tilde{\nu}_{t+s} \left( \frac{P_{t+s}}{P_t} \right)^{\zeta} Y_{t+s} \text{mc}_{t+s} \\ x_t^2 &\equiv E_t \sum_{s=0}^{\infty} (\xi\beta)^s \lambda_{t+s} \tilde{\nu}_{t+s} \left( \frac{P_{t+s}}{P_t} \right)^{\zeta-1} Y_{t+s} \end{aligned}$$

It is convenient to express  $x_t^1$  and  $x_t^2$  in a recursive fashion:

$$x_t^1 = \lambda_t Y_t \text{mc}_t + \xi \beta E_t \tilde{\nu}_{t+1} \pi_{t+1}^{\zeta} x_{t+1}^1 \quad (31)$$

$$x_t^2 = \lambda_t Y_t + \xi \beta E_t \tilde{\nu}_{t+1} \pi_{t+1}^{\zeta-1} x_{t+1}^2 \quad (32)$$

where  $\pi_{t+1}$  is the gross inflation rate between periods  $t$  and  $t+1$ :

$$\pi_{t+1} \equiv \frac{P_{t+1}}{P_t}$$

Since all firms that adjust their prices in a given period choose the same new price,  $\tilde{P}_t$ , equation (4) implies that the price index,  $P_t$ , evolves as

$$P_t^{1-\zeta} = \xi P_{t-1}^{1-\zeta} + (1-\xi) \tilde{P}_t^{1-\zeta}$$

which can be rewritten as

$$1 = \xi \pi_t^{-1+\zeta} + (1-\xi) \tilde{p}_t^{1-\zeta} \quad (33)$$

To derive the aggregate production function, rewrite the production function of individual firms (27) as

$$z_t^{1-\alpha} K_{j,t}^\alpha L_{j,t}^{1-\alpha} - \Phi_t = \left( \frac{P_{j,t}}{P_t} \right)^{-\zeta} Y_t$$

Using the fact that  $K_{j,t}/L_{j,t}$  is the same for all  $j$ , and integrating both sides of this equation yields

$$\varsigma_t Y_t = z_t^{1-\alpha} K_{t-1}^\alpha L_t^{1-\alpha} - \Phi_t \quad (34)$$

where  $\varsigma_t \leq 1$  measures the inefficiency due to price dispersion:

$$\varsigma_t = \int_0^1 \left( \frac{P_{j,t}}{P_t} \right)^{-\zeta} dj$$

The evolution of  $\varsigma_t$  can be written as

$$\varsigma_t = (1 - \xi) \tilde{p}_t^{-\zeta} + \xi \pi_t^\zeta \varsigma_{t-1} \quad (35)$$

## 2.4 Aggregate shocks

The aggregate productivity shock is either permanent or temporary. For the case where the productivity shock is permanent, we assume that  $z_t$  is a geometric random walk:

$$\ln z_t = \ln z_{t-1} + \mu + \sigma_z \epsilon_{z,t} - \frac{\sigma_z^2}{2} \quad (36)$$

and the fixed cost of production,  $\Phi_t$ , grows at the rate  $\mu$ :

$$\Phi_t = \Phi \exp(\mu t) \quad (37)$$

where  $\mu$  and  $\sigma_z$  are constant parameters, and  $\epsilon_{z,t}$  is  $N(0, 1)$  and i.i.d. across periods. For the case where the productivity shock is temporary, we assume that  $z_t$  follows an AR(1) process:

$$\ln z_t = \rho_z \ln z_{t-1} + \sigma_z \epsilon_{z,t} - \frac{\sigma_z^2}{2(1 + \rho_z)} \quad (38)$$

and that the fixed cost is constant:

$$\Phi_t = \Phi \quad (39)$$

In both cases, the constant  $\Phi$  is calibrated so that the aggregate profit is zero in the non-stochastic steady state (balanced growth path) with zero inflation.

The standard deviation of innovations to individual labor productivity,  $\sigma_{\eta,t}$ , is also an aggregate shock. Evidence provided by Storesletten, Telmer and Yaron (2004) and Meghir and Pistaferri (2004) suggests that it fluctuates countercyclically. Krebs (2003) and De Santis (2007) have found that the welfare cost of business cycles can be sizable with countercyclical idiosyncratic risk. Following this literature, we allow  $\sigma_{\eta,t}$  to covary with the aggregate technology shock. Specifically, when the evolution of the aggregate productivity is given by (36), we assume that the variance of idiosyncratic shocks evolves as

$$\sigma_{\eta,t}^2 = \bar{\sigma}_{\eta}^2 + b\sigma_z\epsilon_{z,t} \quad (40)$$

and when  $z_t$  follows the temporary process given by (38), we assume that

$$\sigma_{\eta,t}^2 = \bar{\sigma}_{\eta}^2 + b \ln z_t \quad (41)$$

An important difference between these two specifications is that  $\sigma_{\eta,t}^2$  is serially correlated in (41) but not in (40).

## 2.5 Government

Government policy is very simple in our economy. First we abstract from fiscal policy: the government does not consume, and there are no government bonds nor taxes. We assume that the monetary authority can directly control the inflation rate. Thus, monetary policy is specified as a state contingent path of the inflation rate,  $\{\pi_t\}_{t=0}^{\infty}$ . We consider two regimes for the monetary policy. The first regime is “inflation targeting,” where the monetary authority sets the inflation rate to zero at all times and in all contingencies, that is,  $\pi_t = 1$ , for all  $t$ . The second regime is “Ramsey,” where the monetary authority precommits to the state-contingent path of the inflation rate so as to maximize the average utility of individuals  $U_0 = E_0[u_{i,0}]$ .

## 3 Numerical results

In this section we analyze how the presence of idiosyncratic shocks affects the conduct of monetary policy. We are particularly interested in the case where the idiosyncratic risk,  $\sigma_{\eta,t}$ , fluctuates countercyclically. We show that even though countercyclical idiosyncratic risk makes the welfare cost of business cycles sizable, properties of the optimal monetary policy are little affected by the presence of idiosyncratic shocks.

The parameter values of our model are calibrated as follows. One period in the model corresponds to a quarter. The share of capital is  $\alpha = 0.36$ , and the depreciation rate is

$\delta = 0.02$ . These are taken from Boldrin, Christiano and Fisher (2001). The probability of price adjustment is set to 0.2, i.e.,  $\xi = 0.8$  and the elasticity of substitution across different varieties of products is  $\zeta = 5$ , following Schmitt-Grohé and Uribe (2007). The fixed cost of production,  $\bar{\Phi}$ , is set so that the profit of each firm at the non-stochastic steady state under optimal monetary policy is zero. The discount factor  $\beta$  is chosen so that the real interest rate at the non-stochastic steady state is four percent a year. For the preference parameter, we consider two values for  $\gamma_c$ , 0.7 and 2. For each value of  $\gamma_c$ , another preference parameter  $\theta$  is set so that the labor supply at the stochastic steady state is one third (then,  $\gamma$  is determined as  $\gamma = 1 - (1 - \gamma_c)/\theta$ ). For the case of permanent productivity shock (36), we follow Boldrin, Christiano and Fisher (2001) and set  $\mu = 0.004$ , and  $\sigma_z = 0.018$ . For the case of temporary productivity shock (38), we follow Schmitt-Grohé and Uribe (2007)<sup>5</sup> and set  $\rho_z = 0.8556$  and  $\sigma_z = 0.0064/(1 - \alpha)$ . For the idiosyncratic shock process, we follow De Santis (2007) and set  $\bar{\sigma}_\eta = 0.1/2$  and  $b = 0$  or  $b = -0.8$ . As it turns out, as long as we adjust  $\beta$  so as to make the steady state interest rate equals to a fixed rate (i.e., four percent a year), the value of  $\sigma_\eta$  does not matter. When  $b = 0$ , the idiosyncratic risk is acyclical; when  $b = -0.8$ , it is countercyclical. De Santis (2007) chooses  $b = -0.8$  based on the evidence provided by Storesletten, Telmer and Yaron (2004).

In what follows, we compare dynamics of different versions of our model economy, which differ in terms of the risk aversion parameter,  $\gamma_c \in \{0.7, 2\}$ ; the cyclicity of the idiosyncratic risk,  $b \in \{0, -0.8\}$ ; the persistence of the aggregate productivity shock, (36) and (38); or the policy regime, the Ramsey and the inflation-targeting regimes. In addition, for each values of  $\gamma_c$  and  $b$ , and for each process for  $z_t$ , we compute two normative measures.

The first one is the welfare cost of business cycles as originally estimated by Lucas (1987). Specifically, we consider the real-business-cycle version of our model, in which there are no nominal rigidities, and compare the economy with positive aggregate shocks,  $\sigma_z > 0$ , and the economy without aggregate shocks,  $\sigma_z = 0$ . In both cases we assume that there are idiosyncratic shocks,  $\bar{\sigma}_\eta > 0$ . We also assume that both economies are at the non-stochastic steady state prior to date 0 and compare the welfare conditional on the state vector at  $t = -1$ .<sup>6</sup> Let  $X_t$  denote the vector of the state variables, and let  $\bar{X}$  denote its value at the non-stochastic steady state. Further, let  $\{C_t^{\text{rbc}}, L_t^{\text{rbc}}\}$  denote the equilibrium process of aggregate consumption and labor supply in the RBC version of our

<sup>5</sup>Note that the productivity level  $z_t$  in Schmitt-Grohé and Uribe (2007) corresponds to our  $z_t^{1-\alpha}$ , so that their standard deviation must be adjusted by  $1/(1 - \alpha)$ .

<sup>6</sup>In this sense, our welfare cost measures are the conditional welfare cost, as opposed to the unconditional one. Schmitt-Grohé and Uribe (2007) discuss a related issue.

economy, and let  $\{\bar{C}, \bar{L}\}$  denote their values in the steady state. Then, define the lifetime utility evaluated at period  $t = -1$  by

$$V(\bar{X}, \sigma_z; \text{rbc}) \equiv E_{-1} \sum_{t=0}^{\infty} \beta^t \nu_t \frac{1}{1-\gamma} \left[ (C_t^{\text{rbc}})^\theta (1 - L_t^{\text{rbc}})^{1-\theta} \right]^{1-\gamma}$$

where  $\nu_t$  is given by (18). The corresponding value for the non-stochastic economy is given by

$$V(\bar{X}, 0; \text{rbc}) = \sum_{t=0}^{\infty} \beta^t \bar{\nu}_t \frac{1}{1-\gamma} \left[ (\bar{C})^\theta (1 - \bar{L})^{1-\theta} \right]^{1-\gamma}$$

where  $\bar{\nu}_t$  is defined by

$$\bar{\nu}_t \equiv \exp \left[ \frac{1}{2} \gamma_c (\gamma_c - 1) \bar{\sigma}_\eta^2 t \right]$$

The welfare cost of business cycles is defined by  $\Delta_{\text{bc}}$  that solves

$$\sum_{t=0}^{\infty} \beta^t \bar{\nu}_t \frac{1}{1-\gamma} \left[ ((1 - \Delta_{\text{bc}}) \bar{C})^\theta (1 - \bar{L})^{1-\theta} \right]^{1-\gamma} = V(\bar{X}, \sigma_z; \text{rbc})$$

that is,

$$\Delta_{\text{bc}} = 1 - \left\{ \frac{V(\bar{X}, \sigma_z; \text{rbc})}{V(\bar{X}, 0; \text{rbc})} \right\}^{\frac{1}{1-\gamma_c}}$$

The second normative measure is the cost of adopting a non-optimal policy regime (the inflation-targeting regime) as opposed to the optimal policy regime (the Ramsey regime). Somewhat abusing notation, we again use  $\bar{X}$  to denote the non-stochastic steady state under the Ramsey regime. As it turns out, the steady-state inflation rate under the Ramsey regime is zero. Therefore,  $\bar{X}$  is also the non-stochastic steady state associated with the inflation-targeting regime. Suppose that the economy is at the steady state  $\bar{X}$  prior to date 0. Then the welfare cost of the inflation-targeting regime,  $\Delta_{\text{inf}}$ , is given as

$$\Delta_{\text{inf}} = 1 - \left\{ \frac{V(\bar{X}, \sigma_z; \text{inf})}{V(\bar{X}, \sigma_z; \text{ram})} \right\}^{\frac{1}{1-\gamma_c}}$$

where  $V(\bar{X}, \sigma_z; \text{inf})$  and  $V(\bar{X}, \sigma_z; \text{ram})$  are the lifetime utility associated with the inflation-targeting and Ramsey regimes, respectively.

### 3.1 The case with permanent productivity shock

Let us first look at the case where the aggregate productivity level  $z_t$  follows the process given by (36) and the variance of idiosyncratic shocks follows the process given by (40).



Then how do cyclical fluctuations in  $\sigma_{\eta,t}$  affect the economy? Recall that, given our aggregation result, the idiosyncratic risk affects the aggregate dynamics through its effect on  $\nu_t$ , and hence, through its effect on the effective discount factor,  $\tilde{\beta}_{t,t+1}$ , which is defined in (26). When the processes for  $z_t$  and  $\sigma_{\eta,t}$  are given, respectively, by (36) and (40), the effective discount factor becomes

$$\ln \tilde{\beta}_{t,t+1} = \ln \bar{\beta} + \frac{1}{2} \gamma_c (\gamma_c - 1) b \sigma_z \epsilon_{z,t+1}$$

where

$$\bar{\beta} \equiv \beta \exp \left[ \frac{1}{2} \gamma_c (\gamma_c - 1) \bar{\sigma}_\eta^2 \right]$$

Since  $\epsilon_{z,t+1}$  is i.i.d. and standard normal, the effective discount factor in this case is also i.i.d. and it is log-normal:

$$\ln \tilde{\beta}_{t,t+1} \sim N \left( \ln \bar{\beta}, \left[ \frac{1}{2} \gamma_c (\gamma_c - 1) b \sigma_z \right]^2 \right) \quad (42)$$

Thus, under the specification given by (36) and (40), the effect of cyclical idiosyncratic risk is to make the effective discount factor an i.i.d. random variable with the distribution given by (42).

Table 1 shows the welfare cost of business cycles,  $\Delta_{bc}$ , for  $\gamma_c = 0.7, 2$  and for  $b = 0, -0.8$ . When the risk aversion is relatively low,  $\gamma_c = 0.7$ , the welfare cost of business cycles is negative, that is, the expected utility is higher when  $\sigma_z > 0$  than when  $\sigma_z = 0$ . This result is consistent with the finding by Cho and Cooley (2005).<sup>7</sup> Furthermore, in this case, making the idiosyncratic risk countercyclical decreases the welfare cost of business cycles (that is, it increases the welfare gain of business cycles). On the other hand, when the relative risk aversion is higher,  $\gamma_c = 2$ , the welfare cost of business cycles is positive and is magnified by cyclical fluctuations in  $\sigma_{\eta,t}$ . Indeed, when  $\gamma_c = 2$  and  $b = -0.8$ , the welfare cost of business cycles is about 7.3 percent of consumption, which is a sizable amount.

Figures 1-4 show impulse response functions to a one-standard deviation shock to the productivity growth under the policy regimes and for  $\gamma_c = 0.7, 2$  and  $b = 0, -0.8$ . These figures show that, regardless of the policy regime, changing  $b$  does not affect the impulse response functions. In other words, changing  $\tilde{\beta}_{t,t+1}$  from a constant to an i.i.d. random variable does not change the impulse response functions. In addition, for each value of  $\gamma_c$  and  $b$ , the impulse response functions are the same between the two policy regimes.

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<sup>7</sup>Note that our welfare measure is conditional on the initial state variable. It turns out that the unconditional welfare cost of business cycles is positive for  $\gamma_c = 0.7$ .

Turning back to Table 1, we see that the welfare cost of adopting the inflation-targeting regime is negligible for all values of  $\gamma_c$  and  $b$  considered here. Even when  $\gamma_c = 2$  and  $b = -0.8$ , it is only 0.0006 percent (recall that the welfare cost of business cycles is 7.3 percent for that case). Thus we conclude that, under permanent productivity shocks, cyclical fluctuations in the idiosyncratic risk do not affect how the monetary policy should be conducted, even if it makes the welfare cost of business cycles very large.

### 3.2 The case with temporary productivity shock

Now consider the case where the productivity shock follows the process given by (38), and the variance of idiosyncratic shocks follows the process given by (41). This specification differs from the specification in the previous subsection in two important ways. First, the productivity process (38) is stationary. Second, since  $\ln z_t$  is autocorrelated, so is  $\sigma_{\eta,t}$ . This introduces predictable variability to the idiosyncratic risk, and thus, to the effective discount factor, which was i.i.d. in the previous subsection. Specifically, the effective discount factor is now given by

$$\ln \tilde{\beta}_{t,t+1} = \ln \bar{\beta} + \frac{1}{2} \gamma_c (\gamma_c - 1) b \ln z_{t+1}$$

Its conditional expectation then becomes

$$E_t[\ln \tilde{\beta}_{t,t+1}] = \ln \bar{\beta} + \frac{1}{2} \gamma_c (\gamma_c - 1) b \left( \rho_z \ln z_t - \frac{\sigma_z^2}{2(1 + \rho_z)} \right)$$

which fluctuates over time. Indeed, when  $\gamma_c < 1$  and  $b < 0$ , the productivity shock today increases  $z_t$  as well as the expected value of the effective discount factor,  $E_t[\ln \tilde{\beta}_{t,t+1}]$ . On the other hand, when  $\gamma_c > 1$  and  $b < 0$ , the shock increasing  $z_t$  decreases  $E_t[\ln \tilde{\beta}_{t,t+1}]$ .

Table 2 shows the welfare costs of business cycles,  $\Delta_{bc}$ , for  $\gamma_c = 0.7, 2$  and for  $b = 0, -0.8$ . As opposed to the case of permanent shocks in the previous subsection, when  $b = 0$ ,  $\Delta_{bc}$  is negative for the both values of  $\gamma_c$ . In addition, its absolute value is much smaller. As in the permanent-shock case, countercyclical idiosyncratic risk increases the welfare gain of business cycles for  $\gamma_c = 0.7$ , and magnifies the welfare cost of business cycles when  $\gamma_c = 2$ . When  $\gamma_c = 2$  and  $b = -0.8$ , the welfare cost of business cycles is sizable (12.2 percent), even though the productivity process is stationary.

Figures 5-8 show impulse response functions to a one-standard deviation shock. In contrast to the previous subsection, now the impulse response functions under  $b = 0$  and  $b = -0.8$  differ significantly. When  $\gamma_c < 1$ , countercyclical idiosyncratic risk tends to magnify the effect of a productivity shock: the responses of output, investment, and labor are all greater when  $b = -0.8$  than when  $b = 0$ . This is because when  $\gamma_c < 1$ , a

current productivity increase tends to increase the discount factor between the current and the next periods, which tends to increase the investment demand and the labor supply. The opposite would happen when  $\gamma_c > 1$ , where a productivity increase in the current period tends to reduce the effective discount factor between the current and the next periods, which tends to lower investment and labor supply. Thus, now the cyclical nature of the idiosyncratic risk affects how the aggregate economy responds to a productivity shock. But, as these figures show, again, the difference in the impulse response functions between the two policy regimes is minimal. And as Table 2, the difference is negligible from the viewpoint of welfare. The welfare cost of adopting the inflation-targeting regime remains to be very small:  $\Delta_{\text{inf}}$  is merely 0.0024 percent for  $\gamma_c = 2$  and  $b = -0.8$ , even though  $\Delta_{\text{bc}} = 12.2$  percent in that case.

To summarize, with countercyclical idiosyncratic shocks, the welfare cost of business cycles can be sizable, and also, it may amplify or dampen the responses of the aggregate variables to a productivity shock, depending on the value of  $\gamma_c$  (inverse of the elasticity of intertemporal substitution of consumption). However, it does not affect how monetary policy should be conducted. Even with countercyclical idiosyncratic shocks, the optimal monetary policy is essentially given by the one that keeps the inflation rate at zero.

## 4 Conclusion

In this paper we have developed a New Keynesian model with uninsured idiosyncratic income shocks, and analyzed the optimal monetary policy. We are particularly interested in the case where the variance of idiosyncratic income shocks fluctuate countercyclically over time. Our calibration exercise shows that, although the existence of such idiosyncratic income shocks implies a large welfare cost of business cycles, it does not affect much how monetary policy should be conducted. Specifically, the optimal monetary policy remains to be very close to the complete price-level stabilization even in the presence of countercyclical idiosyncratic shocks.

Note that our assumption that idiosyncratic shocks hit both labor and capital income tends to overemphasize the effect of idiosyncratic shocks. In a model where idiosyncratic shocks only affect the labor income, the optimal conduct of monetary policy would be even less affected by the presence of countercyclical idiosyncratic risk.

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$\gamma_c$	0.7	0.7	2	2
$b$	0	-0.8	0	-0.8
$\Delta_{bc}$ (%)	-0.8191	-1.2983	2.0938	7.3301
$\Delta_{inf}$ (%)	0.0000	0.0000	0.0002	0.0006

Table 1: Welfare measures with permanent technology shocks

$\gamma_c$	0.7	0.7	2	2
$b$	0	-0.8	0	-0.8
$\Delta_{bc}$ (%)	-0.0171	-0.6191	-0.0073	12.2258
$\Delta_{inf}$ (%)	0.0000	0.0001	0.0000	0.0024

Table 2: Welfare measures with temporary technology shocks

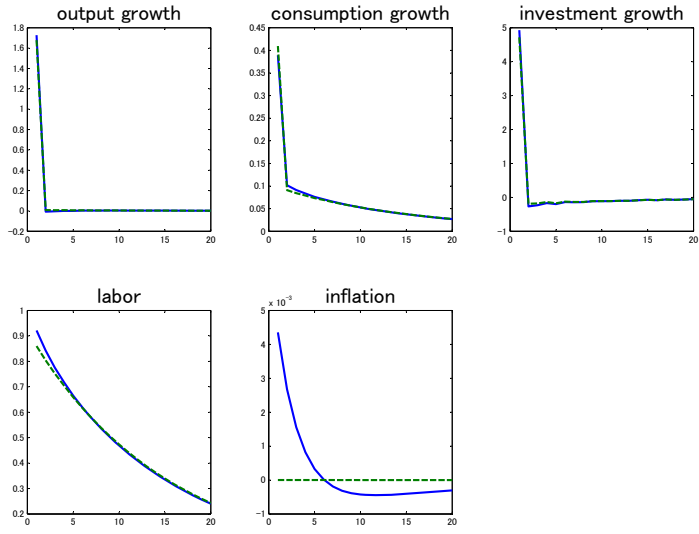


Figure 1: Impulse responses to a permanent productivity shock when  $\gamma_c = 0.7$  and  $b = 0$ . Solid lines = Ramsey policy; dashed lines = inflation targeting.

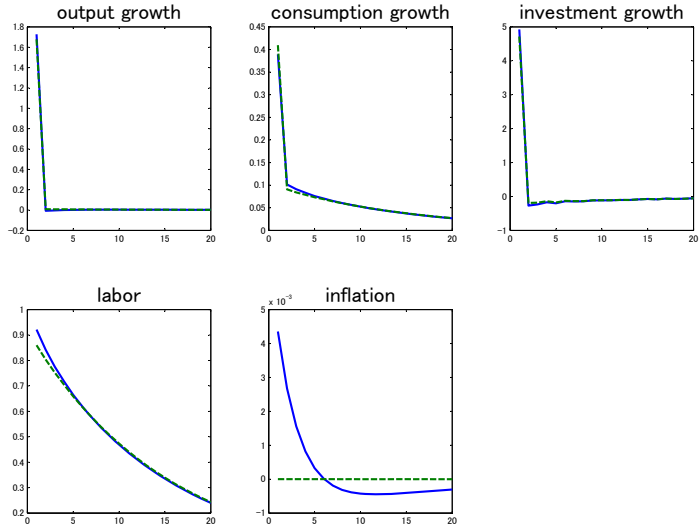


Figure 2: Impulse responses to a permanent productivity shock when  $\gamma_c = 0.7$  and  $b = -0.8$ . Solid lines = Ramsey policy; dashed lines = inflation targeting.

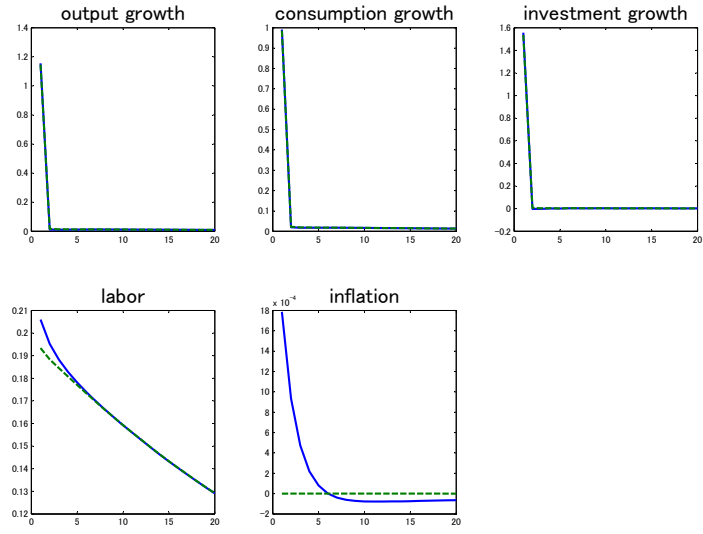


Figure 3: Impulse responses to a permanent productivity shock when  $\gamma_c = 2$  and  $b = 0$ . Solid lines = Ramsey policy; dashed lines = inflation targeting.

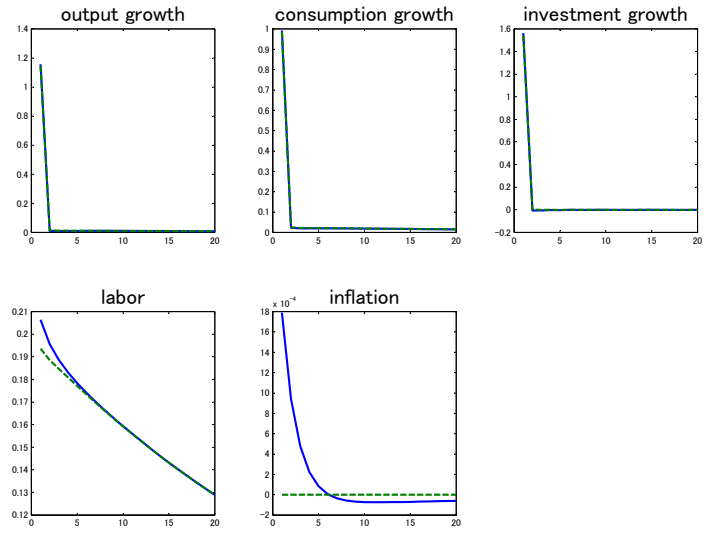


Figure 4: Impulse responses to a permanent productivity shock when  $\gamma_c = 2$  and  $b = -0.8$ . Solid lines = Ramsey policy; dashed lines = inflation targeting.



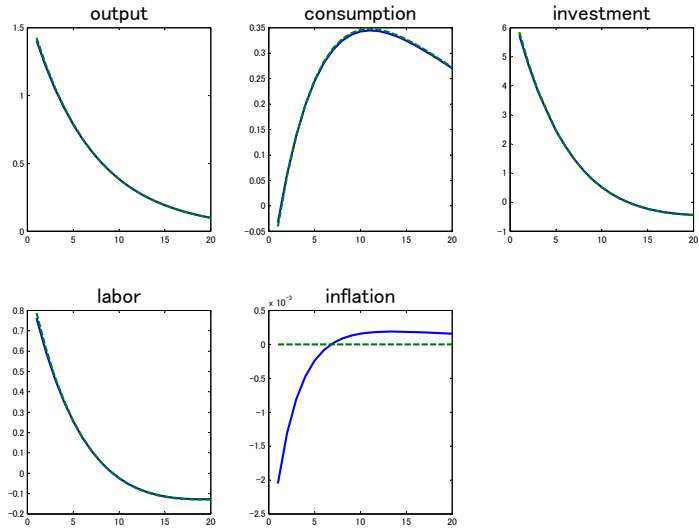


Figure 5: Impulse responses to a temporary productivity shock when  $\gamma_c = 0.7$  and  $b = 0$ . Solid lines = Ramsey policy; dashed lines = inflation targeting.

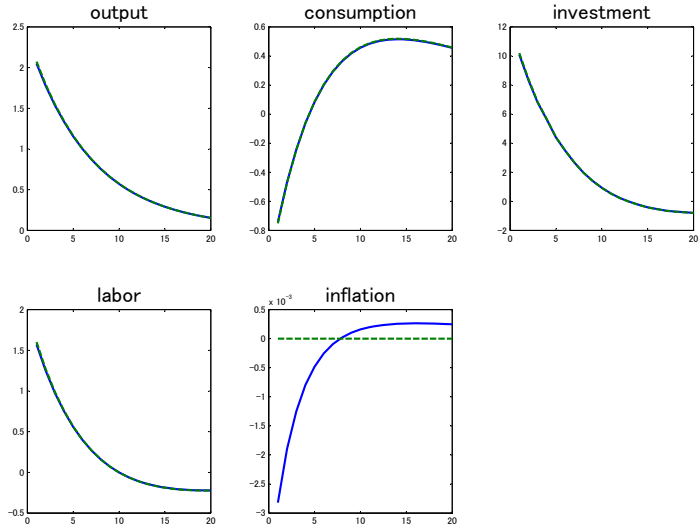


Figure 6: Impulse responses to a temporary productivity shock when  $\gamma_c = 0.7$  and  $b = -0.8$ . Solid lines = Ramsey policy; dashed lines = inflation targeting.

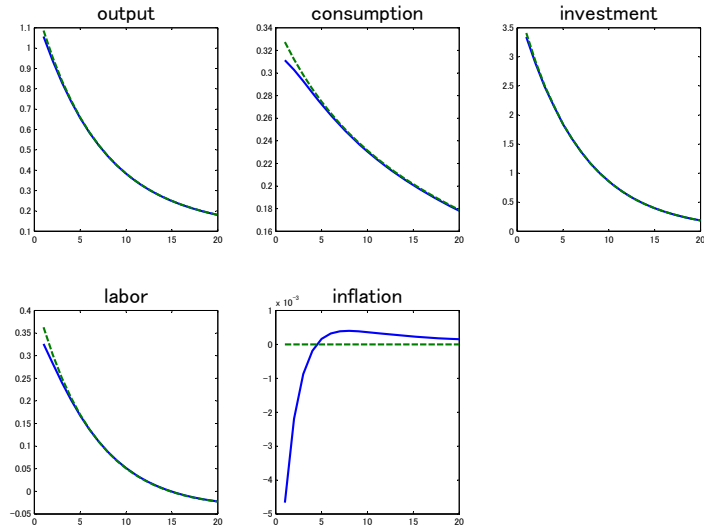


Figure 7: Impulse responses to a temporary productivity shock when  $\gamma_c = 2$  and  $b = 0$ . Solid lines = Ramsey policy; dashed lines = inflation targeting.

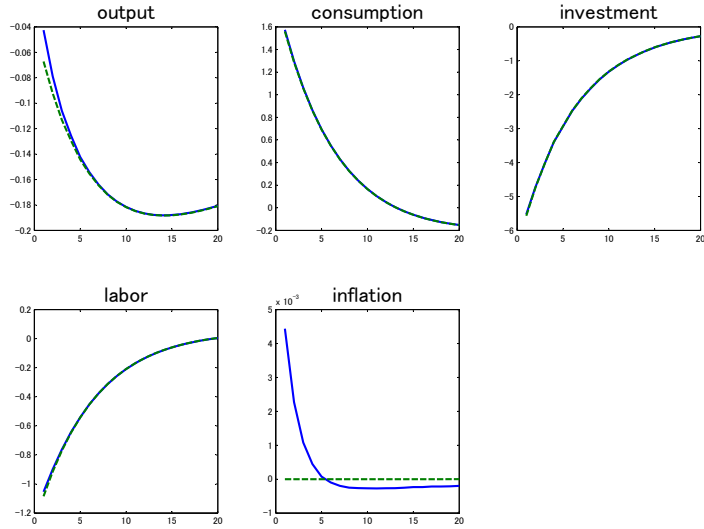


Figure 8: Impulse responses to a temporary productivity shock when  $\gamma_c = 2$  and  $b = -0.8$ . Solid lines = Ramsey policy; dashed lines = inflation targeting.