Weekly Hedonic House Price Indices and the Rolling Time Dummy Method: An Application to Sydney and Tokyo

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Overview

Weekly hedonic indexes are harder to compute than lower frequency (e.g. monthly or quarterly) indexes, since there are less data points.

The rolling time dummy (RTD) method of Shimizu, Takatsuji, Ono and Nishimura (2010) is well suited to computing weekly indexes, since the hedonic formula is computed over a rolling window consisting of a number of weeks.

Including multiple weeks increases the number of data points over which the hedonic model is estimated. The rolling window ensures that the characteristic shadow prices maintain market relevance.

Nevertheless there are some important unresolved questions relating to the RTD method.
Important Questions

(i) How much difference does it make how many weeks are included in the rolling window?

(ii) How can one determine the optimal length of the rolling window?  
A longer window provides more data points.  
A shorter window provides more market relevance.

(iii) In the RTD method there are potentially many different ways of linking the current period with the earlier periods. How much difference does the linking method make?

(iv) How can one determine the optimal linking method?
The Rolling Time Dummy Method

The RTD method begins by estimating the following hedonic model:

$$\ln p_h = \sum_{c=1}^{C} \beta_c Z_{hc} + \sum_{s=t+1}^{t+k} \delta_s D_{hs} + \varepsilon_{hs}$$

The change in the price index from period $t+k-1$ to period $t+k$ is then calculated as follows:

$$\frac{P_{t+k}}{P_{t+k-1}} = \frac{\exp(\hat{\delta}_{t+k}^t)}{\exp(\hat{\delta}_{t+k-1}^t)}.$$ 

A superscript $t$ is included on the estimated $\delta$ coefficients to indicate that they are obtained from the hedonic model with period $t$ as the base.
The window is then rolled forward one period and the hedonic model is reestimated. The change in house prices from period \( t + k \) to period \( t + k + 1 \) is now computed as follows:

\[
\frac{P_{t+k+1}}{P_{t+k}} = \frac{\exp(\hat{\delta}_{t+k+1})}{\exp(\hat{\delta}_{t+k})}.
\]

The price index over multiple periods is the computed by chaining these bilateral comparisons together as follows:

\[
\frac{P_{t+k+1}}{P_t} = \left[ \frac{\exp(\hat{\delta}_{t+1})}{\exp(\hat{\delta}_{t})} \right] \left[ \frac{\exp(\hat{\delta}_{t+2})}{\exp(\hat{\delta}_{t+1})} \right] \times \ldots \times \left[ \frac{\exp(\hat{\delta}_{t+k+1})}{\exp(\hat{\delta}_{t+k})} \right].
\]
Alternative Linking Methods

The price change between periods $t + k - 1$ to period $t + k$ could instead be calculated as follows:

$$\frac{P_{t+k}}{P_{t+k-1}} = \left( \frac{P_{t+k-2}}{P_{t+k-1}} \right) \frac{\exp(\hat{\delta}^t_{t+k})}{\exp(\hat{\delta}^t_{t+k-2})},$$

where as has been noted above both $P_{t+k-1}$ and $P_{t+k-2}$ are already fixed by the time the data for period $t + k$ becomes available. Another alternative is the following:

$$\frac{P_{t+k}}{P_{t+k-1}} = \left( \frac{P_{t+k-3}}{P_{t+k-1}} \right) \frac{\exp(\hat{\delta}^t_{t+k})}{\exp(\hat{\delta}^t_{t+k-3})}. $$
More generally for $j \leq k$,

$$\frac{P_{t+k}}{P_{t+k-1}} = \left( \frac{P_{t+k-j}}{P_{t+k-1}} \right) \frac{\exp(\hat{\delta}_{t+k})}{\exp(\hat{\delta}_{t+k-j})}.$$ 

In other words, given a window length of $k + 1$ periods, there are $k$ distinct ways of linking period $t + k$ with the earlier periods. Each will give a different answer.

Another possibility is to take an average of these $k$ sets of results as follows:

$$\frac{P_{t+k}}{P_{t+k-1}} = \prod_{j=1}^{k} \left[ \left( \frac{P_{t-j}}{P_{t-1}} \right) \left( \frac{\exp(\hat{d}_{t})}{\exp(\hat{d}_{t-j})} \right) \right]^{1/k}.$$ 

An average could also be taken over a subset of periods.
A Criterion for Determining Optimal Window Length

The greater robustness of quarterly indexes is a property we can exploit to discriminate between competing weekly RTD indexes.

Let \( t = 1, \ldots, T \) index the quarters and \( v = 1, \ldots, V \) the 13 weeks in a quarter.

A quarterly price index \( P_{t,t+1}^w \) is obtained from a weekly price index as follows:

\[
P_{t,t+1}^w = \prod_{v=1}^{13} \left( \frac{P_{t+1,v}}{P_{t,v}} \right)^{1/13}.
\]

Each element is a price index calculated at a quarterly frequency.

By taking the geometric mean of these 13 quarterly frequency price indices, we obtain a quarterly equivalent of the original weekly index.
The quarterly version of the weekly index can be compared with a reference quarterly index using one of Diewert’s (2002, 2009) relative price dissimilarity metrics:

$$X_1 = \frac{1}{T-1} \sum_{t=1}^{T-1} \left[ \left( \frac{P_{w}^{t,t+1}}{P_{quart}^{t,t+1}} \right) + \left( \frac{P_{quart}^{t,t+1}}{P_{w}^{t,t+1}} \right) - 2 \right],$$

$$X_2 = \frac{1}{T-1} \sum_{t=1}^{T-1} \left\{ \left[ \left( \frac{P_{w}^{t,t+1}}{P_{quart}^{t,t+1}} \right) - 1 \right]^2 + \left[ \left( \frac{P_{quart}^{t,t+1}}{P_{w}^{t,t+1}} \right) - 1 \right]^2 \right\}.$$

The smaller the value of the $X$ metric, the more similar are the two indexes.

Given a reference quarterly index, we can then vary the length of the RTD rolling window and observe how it affects the $X$ metric.
How robust is the optimal window length to the choice of reference quarterly index?

If it is robust, then we can prefer whichever RTD weekly window length generates the smallest $X$ metric.

This window length is optimal in the sense that it generates weekly price indexes that, when converted into quarterly form, are most consistent with reference quarterly indexes.

How robust is the optimal window length to the frequency of the reference index? We should redo our results using monthly, biannual or annual indexes as the reference.

Holding the window length fixed at 53 weeks, we can also observe how changing the RTD linking method affects the $X$ metric. Again, we prefer the linking method with the smallest $X$ metric.
The Data Sets

(i) Sydney (2002-2014)

We focus on houses. We have 433,202 observations with no missing characteristics.

\[ \text{log(transaction price)} \]
number of bedrooms,
number of bathrooms,
land area,
house type (detached, or semi),
postcode.
(ii) Tokyo (1986-2016)

We focus on apartments. We have 242,233 observations with no missing characteristics.

\[ \log(\text{asking price}) \]
floor area,
age,
time to nearest station,
time to Tokyo central station (included as a quadratic),
city code,
ward dummy.
Sensitivity of Results to Hedonic Approach

How sensitive are RTD results to the window length (between 2 and 53 weeks)?

How sensitive are RTD results to the linking method (holding window length fixed at 53 weeks)?

How similar are the hedonic imputation and time-dummy methods at a quarterly frequency?
Figure 1: The Impact of Varying the Window Length on Weekly RTD House Price Indexes for Sydney
Figure 2: The Impact of Varying the Window Length on Weekly RTD House Price Indexes for Tokyo
Figure 3: The Impact of Varying the Linking Method on Weekly RTD House Price Indexes with a 53 Week Window for Sydney
Figure 4: The Impact of Varying the Linking Method on Weekly RTD House Price Indexes with a 53 Week Window for Tokyo
Figure 5: Quarterly Hedonic Imputation and Time-Dummy House Price Indexes for Sydney
Figure 6: Quarterly Hedonic Imputation and Time-Dummy House Price Indexes for Tokyo
Optimal window length

21 weeks is optimal for Sydney for both reference quarterly indexes.

18 weeks is optimal for Tokyo according to the hedonic imputation benchmark.

Longer than 53 weeks is optimal according to time-dummy method.

But the fit with hedonic imputation reference index is a lot better. Holding shadow prices fixed for 30 years (and through a major crash) may not be ideal. Hence we think the quarterly hedonic imputation reference index is more credible.

Puzzle: How can the results for the $X$ criterion be so different when the reference quarterly indexes are so similar?
Figure 7: Performance of Alternative Window Lengths with the Quarterly Hedonic Imputation Price Index as the Reference: Sydney
Figure 8: Performance of Alternative Window Lengths with the Quarterly Time-Dummy Price Index as the Reference: Sydney
Figure 9: Performance of Alternative Window Lengths with the Quarterly Hedonic Imputation Price Index as the Reference: Tokyo

Reference Index: quarterly HDI

Performance criterion

window length

Performance criterion
Figure 10: Performance of Alternative Window Lengths with the Quarterly Time-Dummy Price Index as the Reference: Tokyo
Optimal RTD Linking

There are 52 distinct ways of linking the current week to earlier weeks, when the window length is 53 weeks. One can also take averages of subsets of these 52 distinct estimates.

For Sydney the optimal link for week $t$ is week $t - 16$ for the hedonic imputation reference index, and $t - 13$ for time-dummy reference index.

These distinct links perform better than taking the geometric mean.

Taking a geometric mean over links $t - 1$ through to say $t - 20$ should beat any distinct link. What is the optimal number of weeks to average over? We need to check this.

For Tokyo, the geometric mean beats any distinct link. Of the distinct links, $t - 12$ is best according to both reference quarterly indexes.
Figure 11: Performance of Alternative Window Lengths with the Quarterly Hedonic Imputation Price Index as the Reference: Sydney
Figure 12: Performance of Alternative Window Lengths with the Quarterly Time-Dummy Price Index as the Reference: Sydney
Figure 13: Performance of Alternative Window Lengths with the Quarterly Hedonic Imputation Price Index as the Reference: Tokyo
Figure 14: Performance of Alternative Window Lengths with the Quarterly Time-Dummy Price Index as the Reference: Tokyo
What Do We Do Next?

What happens when we change the frequency of the reference index to monthly, biannual or annual?

We will look at links between RTD house price indexes and the literature on constructing price indexes for scanner data.

Optimal averaging for RTD linking.