Weekly Hedonic House Price Indices and the Rolling Time Dummy Method: An Application to Sydney and Tokyo

Robert J. Hill¹, Michael Scholz¹ and Chihiro Shimizu²

¹ Department of Economics, University of Graz, Universitätsstrasse 15/F4, 8010 Graz, Austria, robert.hill@uni-graz.at, michael.scholz@uni-graz.at

² Nihon University, Japan, shimizu.chihiro@nihon-u.ac.jp

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Abstract:
The rolling-time-dummy (RTD) hedonic method is well suited to computing house price indexes on a weekly basis. However, it is not clear what is the optimal window length or way of linking the most recent period to earlier periods. Also, it is not clear how much these decisions matter empirically. Using data for Sydney and Tokyo we show that weekly RTD indexes are highly sensitive to the choice of window length, and quite sensitive to the linking method. By contrast, quarterly indexes are much less sensitive to the choice of method. We use this insight to construct a criterion for determining the optimal RTD window length and linking method that uses a quarterly hedonic price index as the point of reference. We apply this method to data for Sydney and Tokyo. For Sydney we find that the optimal window length is 21 weeks, irrespective of whether the reference quarterly index is computed using the hedonic imputation or time-dummy methods. However, for Tokyo no clear result is obtained. We also find that it is possible to improve on the standard linking method employed by the RTD method.

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1 Introduction

The rolling time dummy (RTD) method is a relatively simple and robust method for constructing hedonic price indexes. Since it was first proposed by Shimizu, Takatsuji, Ono and Nishimura (2010), it has become popular with national statistical institutes (NSIs) in Europe. For example, it is used by the NSIs of Ireland, Luxembourg, Cyprus, and Malta to construct the official House Price Index (HPI) in these countries. These HPIs are computed on a quarterly frequency.

One of the attractions of the RTD method is that it tends to perform well with smaller data sets (which is why it is especially smaller countries that use it). For example, the smaller the data set, the longer the window can be made to ensure there are enough data to estimate the hedonic model. For these same reasons it is also potentially well suited for computing higher frequency house price indexes, such as weekly indexes.

This paper attempts to assess the performance of weekly RTD house price indexes. More specifically, the paper focuses on the following issues. First, it considers how sensitive weekly indexes are to the choice of window length. We allow the window length for weekly indexes to vary between 2 and 53 weeks. The sensitivity of the house price index to the choice of window length is assessed using detailed micro data sets for Tokyo and Sydney. We find that the weekly indexes are highly sensitive to the choice of window length.

Second, we consider a slight generalization of the RTD method. We show that when the window contains \( k + 1 \) periods, there are \( k \) distinct ways of computing the price index for the most recent period from the estimated hedonic model, depending on which of the earlier periods in the window it is linked with. Thus far the literature has focused on just one of these. Again using Tokyo and Sydney data we assess the sensitivity of the results to the way the RTD method is specified. The results are quite sensitive to the choice of linking method, although less so than to the choice of window length.

Third, we show that quarterly indexes are much less sensitive to the choice of method than weekly indexes. This insight provides the basis for a novel criterion for discriminating between competing RTD indexes. Using this criterion we get clear results for Sydney regarding the optimal window length. The results are less clear for Tokyo. Using the same criterion we also show that it is possible to improve on the standard
linking approach used by the RTD method.

2 The Rolling Time Dummy (RTD) Method

Consider the standard version of the RTD method with a window length of \( k+1 \) periods, as defined in Shimizu et al. (2010). Supposing that the first period in the window is period \( t \), the first step is to estimate a semilog hedonic model as follows:

\[
\ln p_h = \sum_{c=1}^{C} \beta_c Z_{hc} + \sum_{s=t+1}^{t+k} \delta_s D_{hs} + \varepsilon_{hs},
\]

where \( h \) indexes the housing transactions in the data set, \( p_h \) the transaction price, and \( c \) indexes the set of available characteristics of the transacted dwellings. The characteristics of the dwellings are given by the \( Z_{hc} \) matrix, while \( D_{hs} \) is a matrix of dummy variables that equals 1 when \( s \) is the period in which the dwelling sold, and zero otherwise.

The change in the price index from period \( t+k-1 \) to period \( t+k \) is then calculated as follows:

\[
\frac{P_{t+k}}{P_{t+k-1}} = \frac{\exp(\hat{\delta}_{t+k})}{\exp(\hat{\delta}_{t+k-1})}.
\]

A superscript \( t \) is included on the estimated \( \delta \) coefficients to indicate that they are obtained from the hedonic model with period \( t \) as the base. As can be seen from (2), the hedonic model with period \( t \) as the base is only used to compute the change in house prices from period \( t+k-1 \) to period \( t+k \). The window is then rolled forward one period and the hedonic model is reestimated. The change in house prices from period \( t+k \) to period \( t+k+1 \) is now computed as follows:

\[
\frac{P_{t+k+1}}{P_{t+k}} = \frac{\exp(\hat{\delta}_{t+k+1})}{\exp(\hat{\delta}_{t+k})},
\]

where now the base period in the hedonic model is period \( t+1 \). The price index over multiple period is the computed by chaining these bilateral comparisons together as follows:

\[
\frac{P_{t+k+1}}{P_{t}} = \left[ \frac{\exp(\hat{\delta}_{t+1})}{\exp(\hat{\delta}_{t})} \right] \left[ \frac{\exp(\hat{\delta}_{t+2})}{\exp(\hat{\delta}_{t+1})} \right] \times \cdots \times \left[ \frac{\exp(\hat{\delta}_{t+k+1})}{\exp(\hat{\delta}_{t+k})} \right].
\]
An important feature of the RTD method is that once a price change $P_{t+k}/P_{t+k-1}$ has been computed, it is never revised. Hence when data for a new period $t+k+1$ becomes available, the price indexes $P_t, P_{t+1}, \ldots, P_{t+k}$ are already fixed. The sole objective when estimating the hedonic model inclusive of data from period $t+k+1$ is to compute $P_{t+k+1}$, irrespective of how many periods are included in the rolling window in the hedonic model.

### 3 Linking Variants on the Rolling Time Dummy (RTD) Method

Instead of always focusing on the last two estimated $\delta$ coefficients in each hedonic model, an alternative would be to take the last and third last coefficients. In this case the price change between periods $t+k-1$ to period $t+k$ could be calculated as follows:

$$\frac{P_{t+k}}{P_{t+k-1}} = \left( \frac{P_{t+k-2}}{P_{t+k-1}} \right) \frac{\exp(\hat{\delta}_{t+k})}{\exp(\hat{\delta}_{t+k-2})},$$

where as has been noted above both $P_{t+k-1}$ and $P_{t+k-2}$ are already fixed by the time the data for period $t+k$ becomes available. Another alternative is the following:

$$\frac{P_{t+k}}{P_{t+k-1}} = \left( \frac{P_{t+k-3}}{P_{t+k-1}} \right) \frac{\exp(\hat{\delta}_{t+k})}{\exp(\hat{\delta}_{t+k-3})},$$

and more generally,

$$\frac{P_{t+k}}{P_{t+k-1}} = \left( \frac{P_{t+k-j}}{P_{t+k-1}} \right) \frac{\exp(\hat{\delta}_{t+k})}{\exp(\hat{\delta}_{t+k-j})},$$

where $j \leq k$. In other words, given a window length of $k+1$ periods, there are $k$ distinct ways of linking period $t+k$ with the earlier periods. Each will give a different answer, and one cannot say ex ante that one is better than another. Another possibility is to take an average of these $k$ sets of results as follows:

$$\frac{P_{t+k}}{P_{t+k-1}} = \left[ \prod_{j=1}^{k} \left( \frac{P_{t-j}}{P_{t-1}} \right) \frac{\exp(d_{t-j})}{\exp(d_{t-j})} \right]^{1/k}.$$  

An average could also be taken over a subset of periods. For example, supposing $k$ is an even integer, we could compute separate averages over the first $k/2$ sets of results, and the last $k/2$ sets of results.
4  Links Between the Rolling Time Dummy (RTD) Method and the Scanner Data Literature

Still to be written

5  The Hedonic Imputation Method

The hedonic imputation method is an alternative to the RTD method (see Diewert 2011 and Hill 2013). We use the hedonic imputation method here as a reference index for assessing the performance of different versions of the RTD method.

The hedonic imputation approach estimates a separate hedonic model for each period:

\[ y_t = Z_t \beta_t + \varepsilon_t. \]  \hspace{1cm} (9)

The hedonic model is then used to impute prices for individual houses. For example, let \( \hat{p}_{t+1,h}(z_{t,h}) \) denote the imputed price in period \( t + 1 \) of a house sold in period \( t \). This price is imputed by substituting the characteristics of house \( h \) sold in period \( t \), \( z_{t,h} \), into the estimated hedonic model of period \( t + 1 \) as follows:

\[ \hat{p}_{t+1,h}(z_{t,h}) = \exp \left( \sum_{c=1}^{C} \hat{\beta}_{c,t+1} z_{c,t,h} \right). \]  \hspace{1cm} (10)

With these imputed prices it is now possible to construct a matched sample, thus allowing standard price index formulas to be used.

Paasche – Type Imputation :  
\[ P_{t,t+1}^{PI} = \prod_{h=1}^{H_{t+1}} \left[ \left( \frac{\hat{p}_{t+1,h}(z_{t,h})}{\hat{p}_{t,h}(z_{t+1,h})} \right)^{1/H_{t+1}} \right] \]  \hspace{1cm} (11)

Laspeyres – Type Imputation :  
\[ P_{t,t+1}^{LI} = \prod_{h=1}^{H_{t}} \left[ \left( \frac{\hat{p}_{t+1,h}(z_{t,h})}{\hat{p}_{t,h}(z_{t,h})} \right)^{1/H_{t}} \right] \]  \hspace{1cm} (12)

Törnqvist – Type Imputation :  
\[ P_{t,t+1}^{TI} = \sqrt{P_{t,t+1}^{PI} \times P_{t,t+1}^{LI}} \]  \hspace{1cm} (13)

In a comparison between periods \( t \) and \( t + 1 \), the Laspeyres-type index focuses on houses that sold in the earlier period \( t \) while the Paasche-type index focuses on houses that
sold in the later period $t + 1$. These price indexes give equal weight to each house sold. By taking the geometric mean of Paasche and Laspeyres, the Törnqvist-type index gives equal weight to both periods. The Paasche, Laspeyres and Törnqvist-type indexes above are of the double imputation variety meaning that both prices in each price relative are imputed. A single imputation approach by contrast imputes only one price in each pair (since the actual price is always available for one of the two periods being compared). There has been some discussion in the literature over the relative merits of the two approaches (see for example de Haan 2004, and Hill and Melser 2008). Empirically we try both approaches. The resulting price indexes are virtually indistinguishable. Hence to simplify the presentation we focus here only on double imputation price indexes. The hedonic imputation method is flexible in that it allows the characteristic shadow prices to evolve over time.

In the context of weekly indexes, the hedonic imputation method is unlikely to work well since the sample sizes in many weeks may be small to justify estimating a separate hedonic model each week. However, in our context, quarterly hedonic imputation will provide a useful benchmark for our weekly RTD indexes.

6 The Time Dummy Method

We also use the time-dummy index as a reference for assessing the performance of RTD weekly indexes. The time-dummy method is the limiting case of the RTD method where the window length is the same as the number of periods in the comparison. This means that it is only necessary to estimate the hedonic model once, as follows:

$$\ln p_h = \sum_{c=1}^{C} \beta_c Z_{hc} + \sum_{t=1}^{T} \delta_t D_{ht} + \varepsilon_{ht},$$

(14)

The price index for period $t$ relative to period 1 is then simply calculated as follows:

$$\frac{P_t}{P_1} = \exp(\delta_t).$$

(15)
7 A Performance Criterion for Weekly Indexes Derived from Quarterly Indexes

We propose a criterion here for determining the optimal window and linking method in weekly RTD methods, by comparing them with reference quarterly hedonic indexes. We consider two reference quarterly hedonic indexes: these are the hedonic imputation method and the time-dummy method described above. We focus on these two methods because they are quite different (i.e., one reestimates the hedonic model every quarter while the other does not reestimate at all), and to try and avoid biasing the results by using a quarterly RTD index as a reference index. Empirically we find that the hedonic imputation and time-dummy methods approximate each other closely. By contrast for weekly RTD indexes, if we allow the window length to vary between 2 and 53 weeks, the range of possible results becomes much larger (see section 9).

The greater sensitivity of weekly indexes to the choice of hedonic method makes them a more interesting focus of analysis than quarterly indexes. Furthermore, the greater robustness of quarterly indexes is a property we can exploit to discriminate between competing weekly RTD indexes.

The first step of our criterion for assessing the performance of alternative weekly RTD indexes is to construct a quarterly index from each weekly index. This can be done in the following way. Let $t = 1, \ldots, T$ index the quarters in the data set, and $v = 1, \ldots, V$ the 13 weeks in a quarter. A quarterly price index $P^w_{t,t+1}$ is obtained from a weekly price index as follows:

$$P^w_{t,t+1} = \prod_{v=1}^{13} \left( \frac{P_{t+1,v}}{P_{t,v}} \right)^{1/13},$$

where $P_{t,v}$ denotes the level of the weekly price index in quarter $t$, week $v$. Each element $P_{t+1,v}/P_{t,v}$ in (16) is a price index comparing a particular week with another week one quarter later. In other words, each of these elements is a price index calculated at a quarterly frequency. A total of 13 such indexes can be computed in each quarter. By taking the geometric mean of these 13 quarterly frequency price indices, we obtain an overall quarterly price index, which can be interpreted as the quarterly equivalent of the original weekly index.
The index above in (16) is unweighted. It gives equal weight to all weeks irrespective of how the transactions are distributed across weeks. A weighted variant that weights according to the number of transactions each week is the following:

\[ P_{w,t,t+1}^w = \prod_{v=1}^{13} \left( \frac{P_{t+1,v}^w}{P_{t,v}^w} \right)^{s_{t,v}/S_t}, \]

where \( s_{t,v} = S_{t,v}/S_t \), with \( S_{t,v} \) denoting the number of transactions in week \((t, v)\), and \( S_t \) the total number of transactions in quarter \(t\). Empirically we find that the weighted and unweighted results are very similar. Hence in what follows we focus only on the unweighted case represented in (16).

Once the quarterly version of the weekly index has been constructed, its performance can be measured by comparing it with a reference quarterly index. Here we make the comparison using two alternative metrics proposed by Diewert (2002, 2009).

\[
X_1 = \frac{1}{T-1} \sum_{t=1}^{T-1} \left[ \left( \frac{P_{w,t+1}}{P_{t,t+1}} \right)^{P_{quart}} + \left( \frac{P_{w,t+1}}{P_{t,t+1}} \right)^{P_{quart}} - 2 \right],
\]

\[
X_2 = \frac{1}{T-1} \sum_{t=1}^{T-1} \left\{ \left[ \left( \frac{P_{w,t+1}}{P_{t,t+1}} \right)^{P_{quart}} - 1 \right]^2 + \left[ \left( \frac{P_{w,t+1}}{P_{t,t+1}} \right)^{P_{quart}} - 1 \right]^2 \right\}.
\]

The smaller the value of the \( X \) metric, the more similar are the two indexes. The ordinal rankings generated by the two \( X \) metrics are almost identical. Hence in our empirical analysis we only present results for \( X_1 \).

Given a reference quarterly index, we can then vary the length of the RTD rolling window and observe how it affects the \( X \) metric. We prefer whichever window length generates the smallest \( X \) metric. An important question then is how robust is the optimal window length to the choice of reference quarterly index. If it is reasonably robust, then the selected window length is optimal in the sense that it generates a weekly RTD index that is the most consistent with our reference quarterly indexes. Similarly, holding the window length fixed at 53 weeks, we can observe how changing the RTD linking method affects the \( X \) metric. Again, we prefer the linking method with the smallest \( X \) metric.
8 The Data Sets

8.1 The Sydney data set and hedonic model

We use a data set obtained from Australian Property Monitors that consists of prices and characteristics of houses sold in Sydney (Australia) for the years 2001–2014. For each house we have the following characteristics: the actual sale price, time of sale, postcode, property type (i.e., detached or semi), number of bedrooms, number of bathrooms, land area, exact address, longitude and latitude. (We exclude all townhouses from our analysis since the corresponding land area is for the whole strata and not for the individual townhouse itself.)

For a robust analysis it was necessary to remove some outliers. This is because there is a concentration of data entry errors in the tails of the distribution, caused for example by the inclusion of erroneous extra zeroes. These extreme observations can distort the results. Complete data on all our hedonic characteristics are available for 433,202 observations. To simplify the computations we also merged the number of bathrooms and number of bedrooms to broader groups (one, two, and three or more bathrooms; one or two, three, four, five or more bedrooms).

Using weekly periods, the hedonic model for Sydney is estimated with a rolling window ranging between 2 weeks and 53 weeks. The window is then rolled forward one period and the hedonic model re-estimated. Hence in the case of the 2 week window, a total of 711 hedonic models are estimated covering the time interval from January 2003 to December 2014.

The hedonic model estimated for Sydney is semilog, and contains the following five characteristics:

- number of bedrooms,
- number of bathrooms,
- land area,
- house type (detached, or semi),
- postcode.

All these variables with the exception of land area take the form of dummy variables.
8.2 The Tokyo data set and hedonic model

The Tokyo data set consists of 23 wards of the Tokyo metropolitan area (621 square kilometers), and the analysis period is approximately 30 years between January 1986 and June 2016. The data set covers previously-owned condominiums published in Residential Information Weekly (or Shukan Jyutaku Joho in Japanese) published by RECRUIT, Co. This magazine provides information on the characteristics and asking prices of listed properties on a weekly basis. Moreover, Shukan Jutaku Joho provides time-series data on housing prices from the week they were first posted until the week they were removed as a result of successful transactions. We only use the price in the final week because this can be safely regarded as sufficiently close to the contract price.

The available housing characteristics include floor space and age. The convenience of public transportation from each housing location is represented by travel time to the central business district (CBD), and time to the nearest station. Ward dummies (i.e., city codes) and a railway dummy to indicate along which railway/subway line a housing property is located are also available.

The hedonic model for Tokyo is estimated over 242,233 observations. The functional form is semilog. The explanatory variables used here are:

- log of floor area,
- age,
- time to nearest station,
- time to Tokyo central station (included as a quadratic),
- city code,
- ward dummy.

9 Results

9.1 The sensitivity of the results to the choice of window length

The spreads of the weekly RTD hedonic price indexes for Sydney and Tokyo as the window length is varied between 2 and 53 weeks are shown in Figures 1 and 2. It can
be seen that the weekly indexes are quite sensitive to the choice of window length.

**Figure 1:** The Impact of Varying the Window Length on Weekly RTD House Price Indexes for Sydney

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### 9.2 The sensitivity of the results to the choice of linking method

Holding the window length fixed at 53 weeks, the sensitivity of a weekly RTD method to the choice of linking method is shown for Sydney and Tokyo in Figures 3 and 4.
Figure 2: The Impact of Varying the Window Length on Weekly RTD House Price Indexes for Tokyo

respectively. It can be seen that the variation in the RTD price index is smaller than the variation resulting from changing the window length. However, the spread is still significant.
**Figure 3:** The Impact of Varying the Linking Method on Weekly RTD House Price Indexes with a 53 Week Window for Sydney

9.2.1 A quarterly index as a benchmark

The hedonic imputation and time-dummy methods generate very similar quarterly price indexes. The results are shown in Figures 5 and 6. These results indicate that at a quarterly frequency we have quite a good idea of what the right answer is. Hence these quarterly indexes can be used as a benchmark for discriminating between competing
**Figure 4:** The Impact of Varying the Linking Method on Weekly RTD House Price Indexes with a 53 Week Window for Tokyo

weekly indexes.

### 9.3 How RTD index performance depends on window length

The $X_1$ metric for each RTD window length for Sydney with the hedonic imputation index as the reference quarterly index is shown in Figure 7. The $X_1$ metric is minimized
**Figure 5:** Quarterly Hedonic Imputation and Time-Dummy House Price Indexes for Sydney

when the RTD window length is 21 weeks. The same answer is obtained when the time-dummy index is used as the reference quarterly index as shown in Figure 8.

For Tokyo, the hedonic imputation index as the reference quarterly index, the $X_1$ metric is minimized when the RTD window length is 18 weeks, as shown in Figure 9. When the time-dummy index is used as the reference quarterly index, the results
for Tokyo are not so clear, as shown in Figure 10. The results suggest that the RTD window length that minimizes the $X_1$ metric may be longer than 53 weeks.

In summary, we get a clear answer for Sydney that the optimal RTD window length is 21 weeks. For Tokyo we do not get a clear result. In spite of the close empirical similarity between the hedonic imputation and time-dummy quarterly indexes as depicted in
Figure 7: Performance of Alternative Window Lengths with the Quarterly Hedonic Imputation Price Index as the Reference: Sydney

Reference Index: quarterly HDI

Figure 6, the $X_1$ metrics obtained from these reference indexes vary considerably.
9.4 How RTD index performance depends on the RTD linking method

Now instead we hold the RTD window length fixed at 53 weeks, and compare the impact on the $X_1$ metric of varying the linking method used by the RTD method. With a 53 week window, there are 52 ways of linking a new period to previous periods. For Sydney,
Figure 9: Performance of Alternative Window Lengths with the Quarterly Hedonic Imputation Price Index as the Reference: Tokyo

Reference Index: quarterly HDI

the $X_1$ metric corresponding to each of these 52 ways of linking is shown in Figure 11 for the case where the quarterly hedonic imputation index is used as the reference. Corresponding results with the quarterly time-dummy index as the reference are shown in 12. In addition to the 52 ways of linking, it is also possible to take averages of combinations of these 52 linking methods. Here we consider only one such averaging
method, which is the geometric mean of all 52 linking methods. The \( X_1 \) metric for this average is represented by the cross in each Figure.

When the hedonic imputation method is used as the reference index the optimal linking method for week \( t \) when it enters the rolling window is to link it to week \( t - 16 \). When the time-dummy method is used as the reference index the optimal link for week
$t$ is with week $t - 13$. In neither case is the $X_1$ metric minimized by the average link. However, it is probable that an average calculated over a subset of the 52 links will outperform any single link. This is an issue we still need to explore. For example, an average over the first 20 links should beat the 16 week and 13 week links in Figures 11 and 12.

**Figure 11:** Performance of Alternative Linking Methods with the Quarterly Hedonic Imputation Price Index as the Reference: Sydney
Figure 12: Performance of Alternative Linking Methods with the Quarterly Time-Dummy Price Index as the Reference: Sydney

The corresponding results for Tokyo are shown in Figures 13 and 14. For Tokyo the geometric average link generates a lower $X_1$ metric than any of the individual links. Again though it is likely that the $X_1$ metric can be further reduced say by taking the average over only the first 20 individual links. Of the individual links, in both Figures 13 and 14, teh $X_1$ metric is minimized by linking week $t$ with week $t - 12$. 
Figure 13: Performance of Alternative Linking Methods with the Quarterly Hedonic Imputation Price Index as the Reference: Tokyo

Discussion on optimal averaging of linking factors is still to be included.
Figure 14: Performance of Alternative Linking Methods with the Quarterly Time-Dummy Price Index as the Reference: Tokyo

![Graph showing performance criterion for linking methods]

10 Conclusion

We have considered two dimensions over which RTD hedonic house price indexes can differ. These are the window length and the method used for linking the current period to earlier periods. We have also proposed a new criterion for discriminating between competing versions of the RTD method.
Empirically using data for Sydney and Tokyo we find that weekly indexes are much more sensitive to the choice of hedonic method than quarterly indexes. This insight provides the basis for our criterion. Using this criterion we find that for Sydney the optimal window length is 21 weeks. For Tokyo we do not get a clear answer. We also find that it is possible to improve on the standard linking method used by the RTD method.

References


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