Real Estate Production and Structure Depreciation

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Little Consensus on the Real Estate Production Function

Returns to scale

- Urban agglomerations ⇒ increasing returns in aggregate
  [Fujita and Thisse, 2002]
- Little evidence about real estate
  - Most studies assume constant returns
    e.g., [Epple et al., 2010, Ahlfeldt and McMillen, 2014]
  - [Combes et al., 2015] find decreasing returns for new houses in France.

The land-structure substitution

- Critical for the form and size of cities
- Conflicting results
  - Early studies report small elasticities (0.4-0.6)
    [McDonald, 1981] for a review
  - Recent studies report large elasticities (≥ 1)
    e.g., [Epple et al., 2010, Ahlfeldt and McMillen, 2014, Combes et al., 2015]

Production Function for Durable Assets

Production functions are usually defiend for new assets: $f_0\left(S_{tructure}, L_{and}\right)$.

This is a reduced-form function regarding the discounted sum of real service flows:

$$f_0\left(S_0, L\right) = E_0 \int_{t=0}^{\infty} f_s\left(S_t, L\right) e^{-rt} dt,$$

f_s is the service production function and r is the real discount rate.

Thus, the production function for seasoned assets is analogously defined:

$$f_u\left(S_u, L\right) = E_u \int_{t=u}^{\infty} f_s\left(S_t, L\right) e^{-rt} dt$$

provided that the depreciated structure S_u is estimated.

Structure Depreciation Is Important

Macroeconomics: A key parameter for growth and fluctuations [Greenwood and Hercowitz, 1991, Davis and Nieuwerburgh, 2015].

Particularly for Japan's high saving rate [Hayashi, 1986, Dekle and Summers, 1991, Imrohoroglu et al., 2006], etc.

Real estate investments: Impacts the appreciation and income returns

Large depreciation ⟶ Large user cost ⟶ Large rent to price ratio

Housing economics: Impacts the expenditure share of housing and CO_2 emissions

However, there is a wide range of estimates:

Residential: Japan (1%, 5%, 9%, and 15%), U.S. (1%-2%)

Commercial: Japan (6%-7%), U.S. (2%-7%)

This Study

	Production	Depreciation
Theoretical Analysis	Model of real estate production with homogeneous land and structures	
Empirical Analysis	Hedonic regressions to estimate parameters by controlling for heterogeneity	
		Correction of survivorship biases
Data	Japan (Tokyo & Outside Tokyo) Residential & Commercial	
	U.S. (Centre County, PA) Residential	

Objective #1

To analyze the production function for both new and seasoned real estate

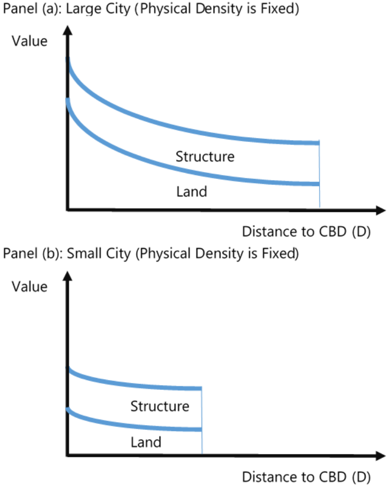
- Theoretical Results
  - Returns to scale determine the total share of land and structure values.
  - The elasticity of land-structure substitution determines the dynamics of the share of structure value.
- Empirical Results
  - Returns are approximately constant in Japan but decreasing in the U.S.
  - Land and structure are substitutes in both countries.
  - The structure value share: 30%-40% (Japan); 50%-70% (U.S.)
  - The land value share: 60%-70% (Japan); 10% (U.S.)

Objective #2

To demonstrate variations in the property-level depreciation rates

- Urban economic theory (Alonso-Muth-Mills) predicts variations in structure value share
- Theoretical Results: Structure value share $\propto$ Property depreciation rate
- Empirical Results: Property depreciation rate is large if a property is
  - located away from the CBD
  - located in a small city
  - new
  - dense

$\longrightarrow$ Variations in real estate returns



Objective #3

To develop and empirically demonstrate new methods of correcting for biases in the estimation of structure depreciation rates

Hedonic Analysis

- Adjust for the structure value share and survivorship
- Empirical results
  - 6.4%-7.0% for residential properties in Japan
  - 9.1%-10.2% for commercial properties in Japan
  - 1.5% for residential properties in the U.S.

Age of demolished buildings

- Adjust for survivorship and the past construction volume
- Empirical results (Japan)
  - Median life span is
    - 30-35 years for residential
    - 20-30 years for commercial
  - Mean depreciation rate is
    - 6.2% for residential
    - 9.2% for industrial
    - 11.7% for office
    - 14.8% for retail
    - 17.2% for hotel

Model

Property Value

$$V_{t,u} = P_t^H H_u = P_t^H \left[\alpha(E_u S)^{\frac{\theta-1}{\theta}} + (1-\alpha)L^{\frac{\theta-1}{\theta}}\right]^{\frac{\eta\theta}{\theta-1}},$$

P_t^H: Unit price (latent variable)

H_u: Effective quantity of property (latent variable)

S: Quantity of structure (floor s.f.)

E_u: Effectiveness of structure at age u

- Depreciation: $d \ln E_u / du < 0$

L: Quantity of homogeneous land (lot s.f.)

α: Relative weight on the effective structure

θ: Elasticity of substitution between structure and land

η: Returns to scale

Shares of Structure and Land Value

Consider a home seller's hypothetical problem:

$$\max_{S,L} \Pi \equiv V_{t,u} - P_t^{ES} E_u S - P_t^L L,$$

where P_t^{ES} and P_t^L are shadow factor prices.
FOC (Factor demand optimality condition):

$$\frac{P_t^{ES} E_u S}{V_{t,u}} (\equiv s_{t,u}) = \eta \left[\frac{\alpha \left(E_u S\right)^{\left(1-\frac{1}{\theta}\right)}}{\alpha \left(E_u S\right)^{\left(1-\frac{1}{\theta}\right)} + (1-\alpha) L^{\left(1-\frac{1}{\theta}\right)}} \right],$$

$$\frac{P_t^L L}{V_{t,u}} (\equiv l_{t,u}) = \eta \left[\frac{(1-\alpha) L^{\left(1-\frac{1}{\theta}\right)}}{\alpha \left(E_u S\right)^{\left(1-\frac{1}{\theta}\right)} + (1-\alpha) L^{\left(1-\frac{1}{\theta}\right)}} \right].$$

Supply of $E_u S$ and L is inelastic at the time of sale.

Results

Result 1

$$s_{t,u} + l_{t,u} = \eta$$

Returns to scale are constant ($\eta = 1$), decreasing ($\eta < 1$), or increasing ($\eta > 1$).

Result 2

$$-\frac{\partial \ln V_{t,u}}{\partial u} = \delta_u s_{t,u}$$

Property depreciation rate is proportional to the structure value share.

Result 3

$$\frac{\partial s_{t,u}}{\partial u} = \frac{(1-\theta)\,\delta_u s_{t,u} l_{t,u}}{\theta\eta}$$

Structure value share is decreasing (increasing) if $(1-\theta)\delta_u < 0$ (> 0).

Data and Empirical Strategy

Samples

- Prices of residential properties
  - 13,803 obs. in Centre County (MLS, 1996-2015)
  - 12,624 obs. in Tokyo
  - 53,938 obs. outside Tokyo (MLIT transaction prices, 2005-2007)
- Prices of commercial properties
  - 2,184 obs. in Tokyo
  - 7,413 obs. outside Tokyo (MLIT transaction prices, 2005-2007)
- Age of Demolished Properties in Japan
  - 1,351 residential properties
  - 30,837 commercial properties (Annual Survery on Capital Expenditures and Disposals, 2005-2014)

Hedonic Regression

$$
\begin{aligned}
\ln V_{ijt} = {} & a_0 + f(A_i, \ln S_i, \ln L_i, D_i) \\
& + a_2 \ln S_i + a_3 (\ln S_i)^2 + a_4 \ln L_i + a_5 (\ln L_i)^2 + a_6 D_i + a_7 D_i^2 + a_8 D_i^3 \\
& + a_9 \ln S_i \times \ln L_i + a_{10} \ln S_i \times D_i + a_{11} \ln L_i \times D_i \\
& + X_i b + N_j + Q_t + \epsilon_{it}
\end{aligned}
$$

V_{ijt}: Price of property i in district j in period t

S_i: Floor area

L_i: Lot size

D_i: Distance

$f(A_i, \ln S_i, \ln L_i, D_i)$: Functions of building age A_i

N_j: Location fixed effects

Q_t: Time fixed effects

X_i: Property characteristics; building style, site shape, etc.

Focuses

Structure value share: $s_{t,u} = \partial \ln V_{ijt} / \partial \ln S_i$

Land value share: $l_{t,u} = \partial \ln V_{ijt} / \partial \ln L_i$

Returns to scale: $\eta = s_{t,u} + l_{t,u}$

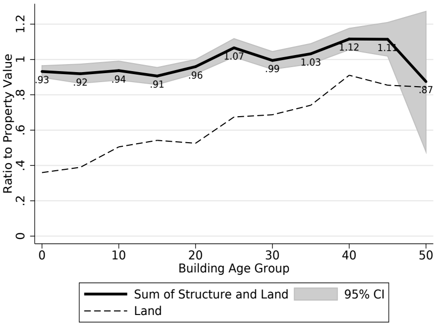
Elasticity of substitution: $sgn\left((1-\theta)\delta_u\right) = sgn\left(\partial s_{t,u} / \partial u\right)$

Property depreciation rate: $\delta_u s_{t,u} = \partial f / \partial A_i$

- non-parametric $f(A_i)$
- linear: $f = a_1 A_i$
- pairwise linear: $\sum_g a_{1,g} A_i \mathbb{I}_g + a_{1,s} A_i \ln S_i + a_{1,l} A_i \ln L_i + a_{1,d} A_i D_i$
- step: $\sum_g a_{1,g} \mathbb{I}_g + a_{1,g,s} \mathbb{I}_g \times \ln S_i + a_{1,g,l} \mathbb{I}_g \times \ln L_i + a_{1,g,d} \mathbb{I}_g \times D_i$

Empirical Results

Returns to Scale and Substitution (Residential, Japan)



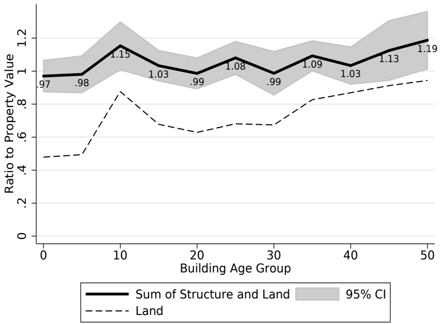
(a) Tokyo

(b) Outside Tokyo

Finding 1: $s_{t,u} + l_{t,u} = \eta \approx 1$ (constant returns)

Finding 2: $\frac{\partial s_{t,u}}{\partial u} < 0 \Rightarrow \theta > 1$ (gross substitution)

Returns to Scale and Substitution (Commercial, Japan)



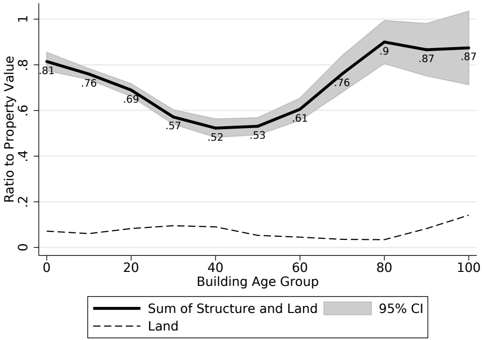
(c) Tokyo

(d) Outside Tokyo

Finding 1: $s_{t,u} + l_{t,u} = \eta \approx 1$ (constant returns)

Finding 2: $\frac{\partial s_{t,u}}{\partial u} < 0 \Rightarrow \theta > 1$ (gross substitution)

Returns to Scale and Substitution (Residential, Centre County)



Finding 1:

1. $\eta < 1$ (decreasing returns)
2. $s_{t,u} + l_{t,u} < 1$ ("dark matter")

Finding 2:

$\frac{\partial s_{t,u}}{\partial u} < 0$ for $u \leq 40$.

Implication:

$\delta_u > 0$ and $\theta > 1$ (substitutes)

Finding 3:

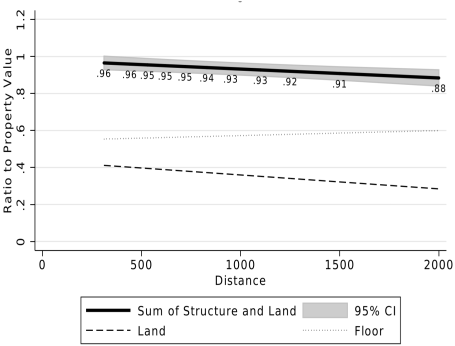
$\frac{\partial s_{t,u}}{\partial u} > 0$ for $u > 40$.

Implication:

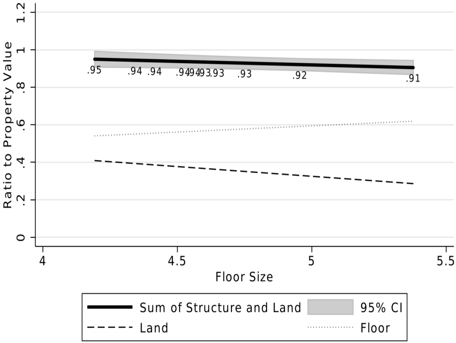
$\delta_u < 0$ and $\theta > 1$ (substitutes)

Variation by Location and Size

Tokyo, Residential, Age 0



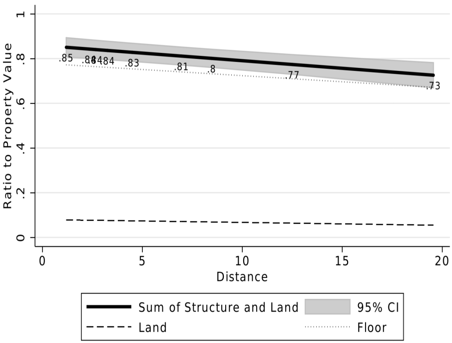
(a) By Distance



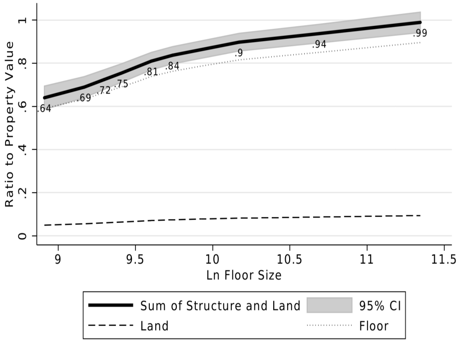
(b) By Floor Area

Variation by Location and Size

Centre County, Residential, Age 0



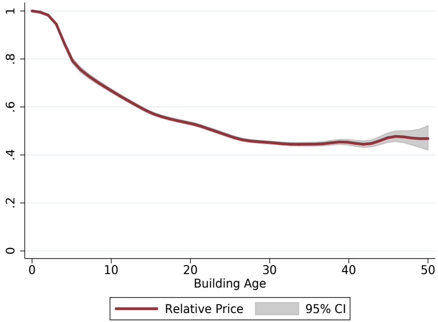
(a) By Distance



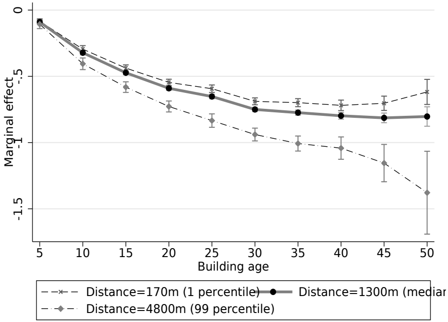
(b) By Floor Area

Property-Level Depreciation (Residential, Outside Tokyo)

Average: 2.3%/year
1-5 years old: 4.4%; 21-25 years old: 2.5%; 41-45 years old: 1.6%

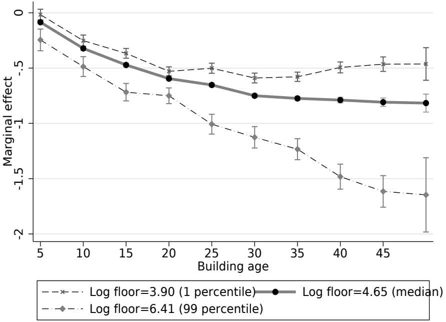


(a) Non-Parametric

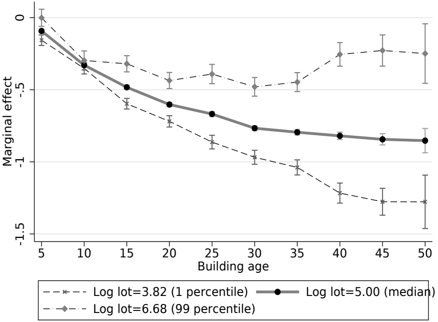


(b) Distance

Property-Level Depreciation (Residential, Outside Tokyo)



(c) Log Floor Area

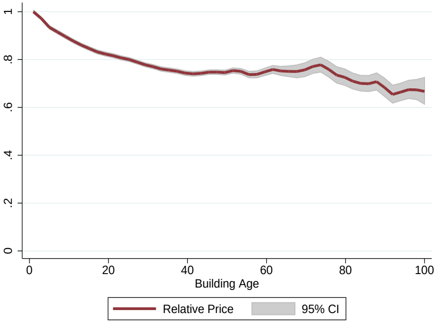


(d) Log Lot Area

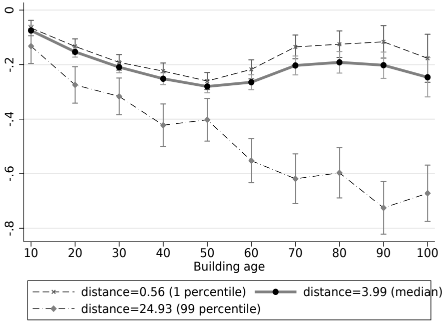
Property-Level Depreciation (Residential, Centre County)

Average: 0.4%/year

1-10 years old: 1.2%; 21-30 years old: 0.7%; 41-50 years old: 0.5%

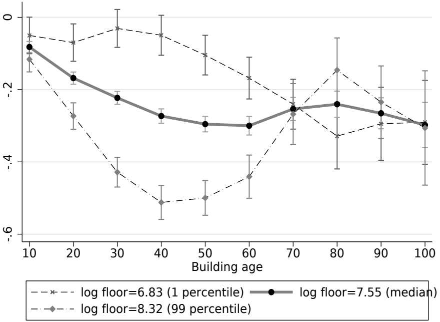


(a) Non-Parametric

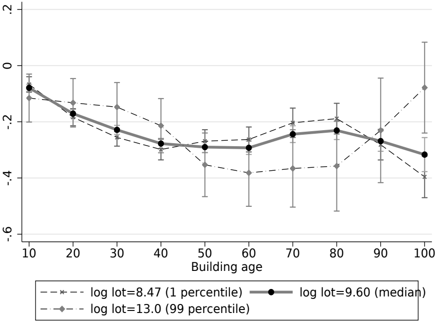


(b) Distance

Property-Level Depreciation (Residential, Centre County, PA



(c) Log Floor Area



(d) Log Lot Area

Magnitude of Variations in Annual Depreciation Rates

	Residential			Commerical	
	Centre County	Tokyo	Outside Tokyo	Tokyo	Outside Tokyo
	(1)	(2)	(3)	(4)	(5)
Distance Measure					
1 percentile	0.0056	0.0130	0.0180	0.0103	0.0163
	(0.0004)	(0.0009)	(0.0005)	(0.0019)	(0.0011)
99 percentile	0.0106	0.0233	0.0260	0.0285	0.0255
	(0.001)	(0.0025)	(0.0011)	(0.0054)	(0.0039)
Difference	0.0050	0.0103	0.0081	0.0183	0.0092
Floor Area					
1 percentile	0.0012	0.0079	0.0123	-0.0012	0.0122
	(0.0007)	(0.0011)	(0.0006)	(0.0037)	(0.0023)
99 percentile	0.0128	0.0334	0.0370	0.0334	0.0251
	(0.0006)	(0.0026)	(0.0014)	(0.0052)	(0.0032)
Difference	0.0116	0.0255	0.0247	0.0347	0.0128
Lot Size					
1 percentile	0.0075	0.0283	0.0304	0.0268	0.0195
	(0.0005)	(0.0015)	(0.0009)	(0.004)	(0.0026)
99 percentile	0.0053	-0.0070	0.0064	-0.0050	0.0150
	(0.0012)	(0.0019)	(0.001)	(0.0057)	(0.0039)
Difference	-0.0021	-0.0353	-0.0240	-0.0318	-0.0045

Survivorship Bias in Structure Depreciation Rate

The structure depreciation rate can be estimated by:

$$\bar{\delta_u} = -\frac{\partial \ln V_{t,u}}{\partial u} \frac{1}{s_{t,u}}.$$

This rate based on the observed properties is biased by survivorship.

Suppose the initial depreciation rate is uniformly distributed on $\left[\delta^L, \delta^H\right]$, and building i is demolished when $\ln P^{Si}_{t,u} S - \ln P^{Si}_{t,0} S \leq \zeta$.

The mean depreciation rate for the surviving structures is:

$$\bar{\delta_u} = \begin{cases} \dfrac{\delta^H + \delta^L}{2} & \text{if } u < -\frac{\zeta}{\delta^H} \\ \dfrac{-\frac{\zeta}{u} + \delta^L}{2} & \text{if } u \in \left(-\frac{\zeta}{\delta^H}, -\frac{\zeta}{\delta^L}\right) \end{cases}$$

Bias-Corrected Rate of Structure Depreciation (Residential, Tokyo)

Age Group	Property Depreciation Rate	Structure Value Ratio	Structure Depreciation Rate without Correction	Survival Rate	Structure Depreciation Rate with Correction
5	0.031	0.519	0.058	1.000	**0.058**
10	0.021	0.476	0.049	1.000	0.049
15	0.020	0.433	0.056	1.000	0.056
20	0.019	0.390	0.045	0.866	0.052
25	0.016	0.347	0.042	0.674	0.058
30	0.014	0.304	0.045	0.551	0.063
35	0.014	0.261	0.047	0.466	0.065
40	0.013	0.218	0.062	0.404	0.071
45	0.013	0.175	0.050	0.357	0.066
50	0.011	0.132	0.086	0.319	0.077

Assumptions: $\zeta = \ln 0.2, \delta^L = 0.005, \delta^H = 0.111$

Bias-Corrected Rate of Structure Depreciation (Residential, Outside Tokyo)

Age Group	Property Depreciation Rate	Structure Value Ratio	Structure Depreciation Rate without Correction	Survival Rate	Structure Depreciation Rate with Correction
5	0.044	0.718	0.067	1.000	**0.067**
10	0.036	0.671	0.054	1.000	0.054
15	0.033	0.625	0.055	1.000	0.055
20	0.029	0.579	0.044	0.738	0.061
25	0.025	0.532	0.046	0.574	0.069
30	0.024	0.486	0.044	0.469	0.071
35	0.021	0.440	0.042	0.397	0.071
40	0.018	0.393	0.049	0.344	0.074
45	0.016	0.347	0.054	0.304	0.075
50	0.014	0.301	0.049	0.272	0.073

Assumptions: $\zeta = \ln 0.2, \delta^L = 0.005, \delta^H = 0.130$

Bias-Corrected Rate of Structure Depreciation (Commercial, Tokyo)

Age Group	Property Depreciation Rate	Structure Value Ratio	Structure Depreciation Rate without Correction	Survival Rate	Structure Depreciation Rate with Correction
5	0.053	0.429	0.108	1.000	**0.108**
10	0.036	0.403	0.129	1.000	0.129
15	0.031	0.377	0.086	0.728	0.107
20	0.022	0.351	0.062	0.520	0.102
25	0.021	0.324	0.052	0.405	0.101
30	0.019	0.298	0.062	0.331	0.106
35	0.016	0.272	0.059	0.280	0.105
40	0.010	0.245	0.061	0.243	0.105
45	0.009	0.219	0.041	0.214	0.101
50	0.008	0.193	0.034	0.192	0.100

Assumptions: $\zeta = \ln 0.2, \delta^L = 0.02, \delta^H = 0.197$

Bias-Corrected Rate of Structure Depreciation (Commercial, Outside Tokyo)

Age Group	Property Depreciation Rate	Structure Value Ratio	Structure Depreciation Rate without Correction	Survival Rate	Structure Depreciation Rate with Correction
5	0.048	0.477	0.098	1.000	**0.098**
10	0.028	0.456	0.058	1.000	0.058
15	0.029	0.436	0.071	0.824	0.086
20	0.025	0.415	0.058	0.589	0.089
25	0.024	0.395	0.054	0.458	0.092
30	0.023	0.375	0.056	0.375	0.094
35	0.020	0.354	0.066	0.317	0.098
40	0.016	0.334	0.047	0.275	0.092
45	0.015	0.313	0.049	0.243	0.093
50	0.012	0.293	0.042	0.217	0.091

Assumptions: $\zeta = \ln 0.2, \delta^L = 0.02, \delta^H = 0.176$

Bias-Corrected Rate of Structure Depreciation (Centre County, PA)

Age Group	Property Depreciation Rate	Structure Value Ratio	Structure Depreciation Rate without Correction	Survival Rate	Structure Depreciation Rate with Correction
10	0.012	0.699	0.018	1.000	**0.018**
20	0.010	0.607	0.016	1.000	0.016
30	0.007	0.476	0.015	1.000	0.015
40	0.006	0.433	0.014	1.000	0.014
50	0.005	0.478	0.010	1.000	0.010
60	0.004	0.560	0.007	1.000	0.007
70	0.003	0.725	0.004	1.000	0.004
80	0.002	0.866	0.002	1.000	0.002
90	0.002	0.783	0.003	1.000	0.003
100	0.002	0.733	0.002	0.948	0.003

Assumptions: $\zeta = \ln 0.2$, $\delta^L = 0.009$, $\delta^H = 0.027$

Bias in Demolition Data

The demolition sample overrepresents short-lived structures

$\longrightarrow$ The average life is too short and the implied depreciation rate is too large.

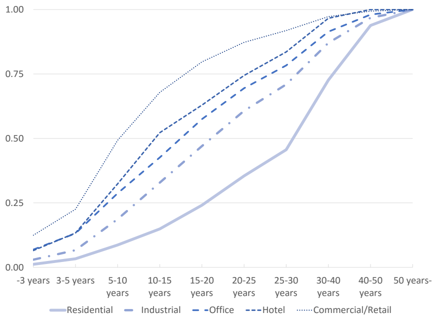
The frequency of demolition is inversely proportional to life span $u(\delta)$. Thus, the pdf of depreciation rate can be adjusted by:

$$g^*(\delta) \equiv \frac{g(\delta)u(\delta)}{\int_{\delta^L}^{\delta^H} g(\theta)u(\theta)d\theta}$$

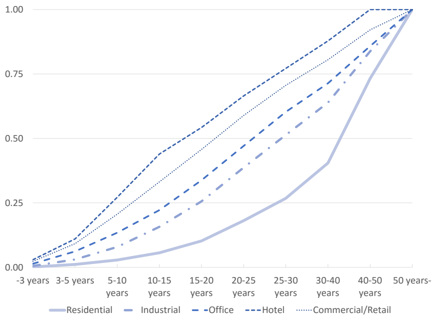
Furthermore, the effect of past construction volume on the distribution can be corrected by:

$$g^{**}(\delta) \equiv \frac{g(\delta)u(\delta)C_{u(\delta)}^{-1}}{\int_{\delta^L}^{\delta^H} g(\theta)u(\theta)C_{u(\theta)}^{-1}d\theta}.$$

Cumulative Distribution of Building Age at Demolition

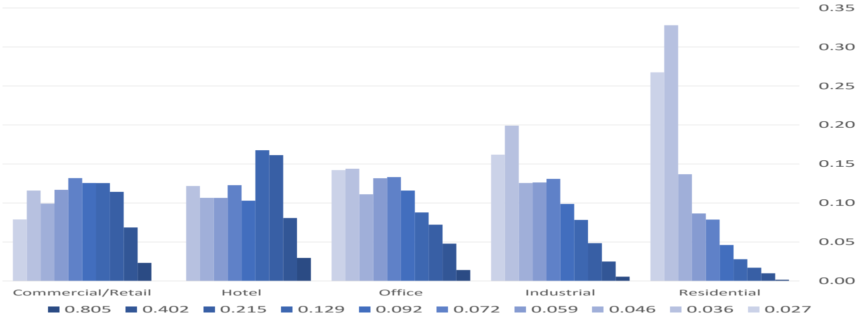


(a) Unadjusted



(b) Adjusted for Frequency and Construction Volume

Distribution of Depreciation Rates



Mean depreciation rate

Retail	Hotel	Office	Industrial	Residential
14.8%	17.2%	11.7%	9.2%	6.2%

Conclusion

Importance

- Returns to scale and substitution $\longrightarrow$ Urban and regional economics
- Variation in depreciation rates $\longrightarrow$ Real estate investments
- The structure depreciation rate $\longrightarrow$ Macroeconomics

Findings

- Returns to scale are constant in Japan but decreasing in the U.S.
- Land and structures are gross substitutes
- Property depreciation rate is larger for newer and denser properties located further away from the CBD in a smaller city
- The structure depreciation rate is larger
  - in Japan than in the U.S.
  - for commercial properties than for residential properties

References I

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