Network-Motivated Lending Decision

(tentative title)

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  - Sekine/Kobayashi/Saita 2003: the higher debt ratio had positive impact on the loan supply in the construction sector or the real estate sector in the 1990s.
  - Peek/Rosengren 2005: Main banks increased lending to under-performing borrowers. It was prominent among those on the verge of the regulatory capital adequacy.
  - Caballero/Hoshi/Kashyap 2008 : Classify the loans at the rate less than the estimated lower bound of the usual loan, such as the prime rate, into "zombie lending". The sectors with many zombies (retail, wholesale, construction, real estate, service), exhibited the lower TFP, employment, and capital growth.

- Mechanism of the forbearance lending.
    - Dewatripont/Maskin 1996, Berglöf/Roland 1998: Excess investment in the relationship specific monitoring ability by a relational lender.
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    - Kanki 2002 : The lack of the legal practice of equitable subordination.
- Does not seem to be sufficient.
    - Forbearance lending was prevalent among under-performing large publicly traded companies while it is not among small private companies (Hosono 2008; Ogawa 2008; Sakai et al 2010).
    - Threat of a possible legal liability (special breach of trust, shareholder derivative lawsuit).

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- Public bailout of under-performing giant companies: GM, Chrysler, Peugeot/Citroen, Renault (2009), Daiei (2003).

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" 'The attitude of the Prime Minister Office toward Daiei has switched all of a sudden in the mid last December. The government's strong will to avoid its bankruptcy was felt' (Top executives of main banks for Daiei). [...] The bankruptcy of Daiei damages seriously a numerous suppliers and the regional economy."

("Document Daiei Sai Shuppatsu," Jan 19, Nippon Keizai Shinbun)

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- The payoff from the bailing out the hub company through the interest income from peripheral firms may exceed the cost of the bail-out.
- If this is the case, the bail-out is the optimal response for the bank.

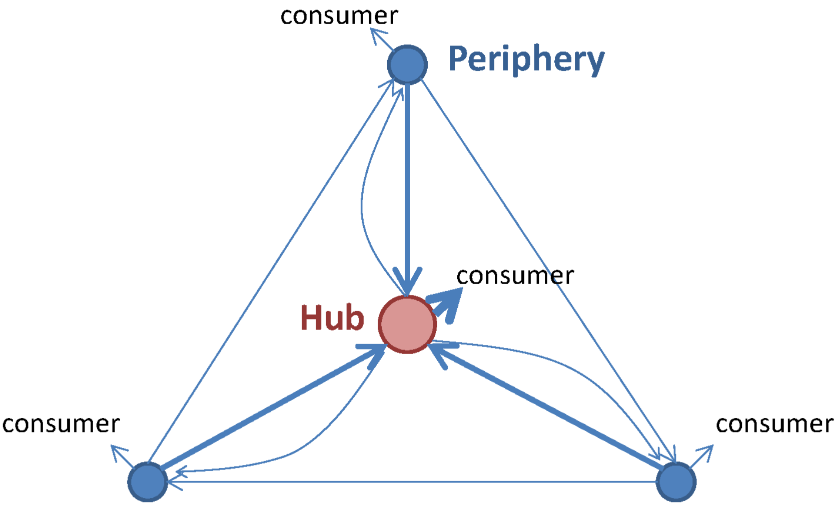

Figure : Supply Chain Network 1. Keeping the Hub Open.

Each node: firm. Arrow: direction of sales. Thickness: amounts of sales.

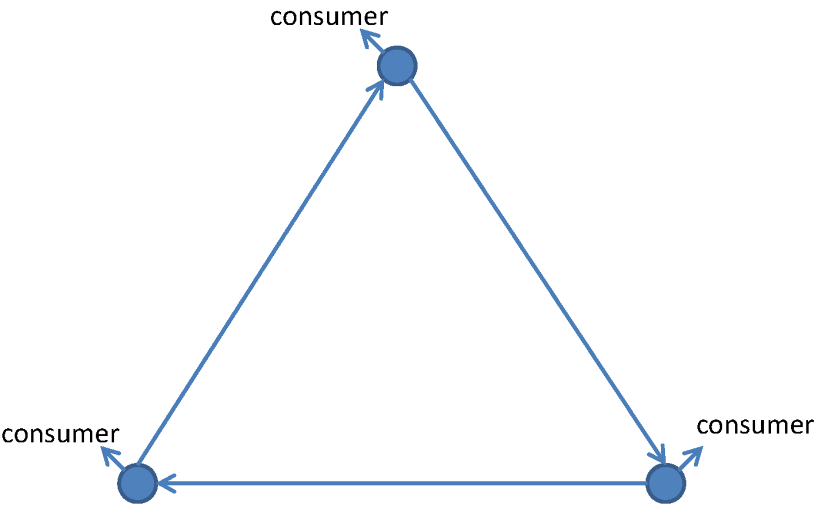

Figure : Supply Chain Network 2. Closing the Hub.

Outline

1. Baseline Model of the Supply Chain Network
2. Influence vector / Leontief multiplier
3. Network-Motivated Forbearance and Loan Market Structure

Model: Overview

- One period. Perfect foresight.
- Monopolistic competition by n firms. Dixit-Stiglitz type CES utility and production function. Demand consists of the final demand and the intermediate demand.

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- Extend the multi-sector model by Acemoglu et al 2011, Horvath 2000, and Long/Plosser 1983 into the monopolistic competition case.

Model: Household

- Household h's (countable, symmetric, $h = 1, \cdots, H$) utility is:

$$U_h = \left(\sum_{j=1}^{n} \omega_j c_{hj}^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}}, \quad \theta > 1,\ 0 \leq \omega_j \leq 1,\ \sum_{j=1}^{n} \omega_j = 1. \qquad (1)$$

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- The budget constraint of household h:

$$\sum_{j=1}^{n} c_{hj} p_j \leq R_h. \qquad (2)$$

  - p_j: Price of product j.
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- Each household is endowed with a fixed amount of numeraire κ_h in the real term.

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The production function of Firm i (countable, $i = 1, \cdots, n$), which produce product i, is

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- Supply-chain network: w_{ij} (fixed) is the *technological importance* of the input j for the production of firm i. $0 \leq w_{ij} \leq 1$, $\forall\, i, j$. $0 < w_{i0}$, $\forall i$. $w_{ii} = 0$ (no self-input), and $\sum_{i=1}^{n} w_{ij} \leq 1$.

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- Every firm knows which companies will operate when it decides the input basket and its price.
- Total demand for firm i:

$$x_i = \sum_{j=1}^{n} x_{ji} + \sum_{h=1}^{H} c_{hi}, \qquad \text{for each product } i. \tag{4}$$

Model: Financial market.

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* In any case, households and banks have an outside opportunity that can yield **the real risk-free rate** of return ρ (> 0). They are all protected by the limited liability.

Model: Two alternative assumptions about income

1. **General equilibrium**
   - The non-intermediate input is labor.
   - The nominal income R = (labor income) + (return from investment into firms, banks, or the outside opportunity) + (residual profit of firms).
   - Assume $\sum_h \kappa_h > \sum_i F_i$ (agg. numeraire > agg. fixed costs).
   - A part of households owns a firm.
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   - A part of households owns a firm.
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2. **Partial equilibrium case**
   - Nominal income R is exogenously fixed.
   - The non-intermediate input is a basket of inputs from outside of the network.

Product market: Product demand

1. Household h determines its demand by solving the problem;

$$\max_{\{c_{hj}\}_{j=1}^{n}} \left(\sum_{j=1}^{n} \omega_j c_{hj}^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}}, \text{s.t.} \quad \sum_{j=1}^{n} c_{hj} p_j \leq R_h.$$

Final demand for the product of firm i.

$$c_i = \frac{R}{P_c} \left(\frac{p_c \omega_i}{p_i} \right)^{\theta}, \text{ where } p_c \equiv \left(\sum_{j=1}^{n} \omega_j^{\theta} p_j^{1-\theta} \right)^{\frac{1}{1-\theta}}, \; R \equiv \sum_{h=1}^{H} R_h. \tag{5}$$

p_c is the consumer price index (CPI).

The indirect utility / the social welfare in GE: R/p_c.

2. Firm i decides its input combination of intermediate goods and labor to minimize the production cost.

$$\min_{\{x_j\}_{j=0}^n} \quad \textstyle\sum_{j=0}^n p_j x_{ij}, \text{s.t.,} \quad x_i = \left(\sum_{j=0}^n w_{ij} x_{ij}^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}}.$$

The intermediate demand for firm i.

$$\sum_{j=1}^n x_{ji} = \sum_{j=1}^n \left(\frac{w_{ji} p^j}{p_i} \right)^{\theta} x_j, \text{ where } p^i \equiv \left(\sum_{j=0}^n w_{ij}^{\theta} p_j^{1-\theta} \right)^{\frac{1}{1-\theta}}. \tag{6}$$

p^i is the producer price index (PPI) for firm i. It is also noteworthy that

$$\sum_{j=0}^n p_j x_{ij} = p^i x_i.$$

Product market: Total sales of firm i.

$$p_i x_i = p_i c_i + p_i^{1-\theta} \sum_{j=1}^{n} \frac{\left(w_{ji} p^j\right)^\theta}{p_j} \cdot p_j x_j.$$

In the vector expression; the total sales are

$$\mathbf{s} = \mathbf{f} + \mathbf{Q}\mathbf{s}, \tag{7}$$

$$\begin{aligned} \text{where} \quad & \mathbf{s} \equiv (p_1 x_1, p_2 x_2, \cdots, p_n x_n)', \\ & \mathbf{f} \equiv (p_1 c_i, p_2 c_2, \cdots, p_n c_n)', \\ & (i,j) \text{ element of } \mathbf{Q} : q_{ij} \equiv p_i^{1-\theta} \left(w_{ji} p^j\right)^\theta / p_j. \end{aligned}$$

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By the assumptions w.r.t. w_{ij} and the definition of p^i, $\mathbf{I_n} - \mathbf{Q}$ is invertible.

$$\begin{aligned} \mathbf{s} &= (\mathbf{I_n} - \mathbf{Q})^{-1} \mathbf{f}, \\ &= \sum_{k=0}^{\infty} \mathbf{Q}^k \mathbf{f}. \end{aligned} \tag{8}$$

Product market: Profit maximization

Aggregate demand externality: Firm i assumes that $\frac{\partial p_c}{\partial p_i} = 0$ and $\frac{\partial p^j}{\partial p_i} = 0 \; \forall j$.
The profit maximization problem for firm i is

$$\max_{p_i} \quad (p_i - p^i)x_i.$$

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FOC for the profit max.

$$p_i = \frac{\theta}{\theta - 1} p^i. \tag{9}$$

The profit of firm i is

$$\pi_i = \frac{p_i x_i}{\theta}.$$

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Product market: Equilibrium price

Stacking FOC into a vector after raising both sides to a power of $1-\theta$ gives

Equilibrium price.

$$\mathbf{p}_\theta = \left\{ \mathbf{I_n} - \left(\frac{\theta - 1}{\theta} \right)^{\theta-1} \mathbf{W}_\theta \right\}^{-1} \mathbf{w_{0\theta}} p_0^{1-\theta}, \tag{10}$$

$$\text{where} \quad \mathbf{p}_\theta \equiv ({p_1}^{1-\theta}, {p_2}^{1-\theta}, \cdots, {p_n}^{1-\theta})',$$
$$\mathbf{w_{0\theta}} \equiv (w_{10}^\theta, w_{20}^\theta, \cdots, w_{n0}^\theta)',$$

and the (i,j) element of $\mathbf{W}_\theta$ is equal to w_{ij}^θ.

Note $p_c^* = (\omega_\theta' \mathbf{p}_\theta)^{\frac{1}{1-\theta}}$, where $\omega_\theta = (\omega_1^\theta, \omega_2^\theta, \cdots, \omega_n^\theta)'$ and p_0 is the exogenous price of the non-intermediate input.
Hereafter, focus on this eq p.

Product market: Aggregate sales and the influence vector.

The aggregate sales of all firms is

$$\begin{aligned} \mathbf{1}'\mathbf{s} &= \mathbf{1}'(\mathbf{I_n} - \mathbf{Q})^{-1}\mathbf{f} \\ &= \mathbf{v}'\mathbf{f}, \end{aligned} \tag{11}$$

where

Influence vector / Leontief multiplier

$$\mathbf{v} \equiv \mathbf{1}'(\mathbf{I_n} - \mathbf{Q})^{-1} \tag{12}$$

$$= \mathbf{1}'\sum_{k=0}^{\infty} \mathbf{Q}^k. \tag{13}$$

Decreases in the non-intermediate-input (input 0) dependence.

Social welfare in the GE.

Social welfare in GE
= Aggregate real income of households
= (agg. firm profit) + (agg. wage income) + (agg. r from outside op.).
(Note: aggregate real income = agg. indirect utility = social welfare).

$$\begin{aligned} \frac{R}{p_c} &= \frac{\sum_{i=1}^{n} p_i x_i}{p_c \theta} + (\kappa - \sum_{i=1}^{n} F_i)(1+\rho) + \frac{p_0 \sum_{i=1}^{n} x_{i0}}{p_c} \\ &= \frac{(1+\rho)(\kappa - \sum_{i=1}^{n} F_i)}{1 - p_c^{\theta-1}(\mathbf{1}'/\theta + \mathbf{q_0}')(\mathbf{I_n} - \mathbf{Q})^{-1}\mathbf{p}_\omega^\theta}, \end{aligned} \tag{14}$$

where κ (agg. endowed numeraire) $> \sum_i F_i$,
$\mathbf{p}_\omega^\theta$ is the column vector of which ith element is $p_i^{1-\theta}\omega_i^\theta$,
and $\mathbf{q_0}$ is the column vector of which ith element is $p_0^{1-\theta} w_{i0}^\theta {p^i}^\theta / p_i$.

Social welfare in the PE.

- Income R is exogenously given.
- The input $j = 0$ is the basket of input from the outside of the network.

The social welfare:

$$\frac{R}{p_c} - (1+\rho)\sum_{i=1}^{n} F_i - \frac{p_0 \sum_{j=1}^{n} x_{j0}}{p_c}. \tag{15}$$

The aggregate real profit of firms within the network:

$$\begin{aligned}
&\frac{\sum_{i=1}^{n}(p_i x_i - p^i x_i)}{p_c} - (1+\rho)\sum_{i=1}^{n} F_i \\
&= \frac{\sum_{i=1}^{n} p_i c_i + \sum_{i=1}^{n} p_i \sum_{j=1}^{n} x_{ji} - \sum_{j=0}^{n} p_j \sum_{i=1}^{n} x_{ij}}{p_c} - (1+\rho)\sum_{i=1}^{n} F_i \\
&= \frac{R}{p_c} - \frac{p_0 \sum_{i=1}^{n} x_{i0}}{p_c} - (1+\rho)\sum_{i=1}^{n} F_i.
\end{aligned}$$

Social welfare = aggregate real profit of firms.

Financial market 1: Centralized market

- Households and investors outside of the economy can invest the numeraire only through a monopolistic bank.
- The bank is owned by the households and all of its profit is shared to households.
- Firms can borrow only from the bank. It cannot borrow directly from investors inside or outside of the economy.

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Financial market 1: Centralized market

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- Monopolistic bank decides the optimal combination of firms to be financed so as to max total real economic profit of these firms.
- (Alternative possible assumption) the bank decides to maximize the social welfare; i.e., the total real economic profit of firms plus the real wage income.

Definition (Forbearance)

We say a bank undertakes the **forbearance** if it extends a loan to firm z despite that its real economic profit is negative; namely,

$$\frac{x_z p_z}{\theta p_c} - (1+\rho)F_z < 0. \tag{16}$$

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- Define an indicator variable, $e_i = 1$ if firm i can finance F_i or zero otherwise.
- Network-motivated forbearance for firm z occurs if
  - firm z is financed at the bank optimum, despite that
  - (16) holds.

  The first statement means

$$\sum_{i=1}^{n} e_i^* \left(\frac{x_i^* p_i^*}{\theta p_c^*} - (1+\rho)F_i \right) \; > \; \sum_{i=1}^{n} \tilde{e}_i \left(\frac{\tilde{x}_i \tilde{p}_i}{\theta \tilde{p}_c} - (1+\rho)F_i \right).$$

where $*$ indicates values at the bank profit max $\{e_i\}_{i=1}^n$, the tilde indicates values at the constrained maximum s.t. $e_z = 0$.

Rearranging the last inequality and combining with (16) at $e_i = e_i^*$ give

$$\underbrace{\frac{v_z^* p_z^* c_z^* - p_z^* x_z^*}{\theta p_c^*}}_{\text{Leontief effect}} + \underbrace{\sum_{i \neq z} \left[e_i^* \left(\frac{v_i^* p_i^* c_i^*}{\theta p_c^*} - (1+\rho) F_i \right) - \tilde{e}_i \left(\frac{\tilde{v}_i \tilde{p}_i \tilde{c}_i}{\theta \tilde{p}_c} - (1+\rho) F_i \right) \right]}_{\text{business-stealing/influence-enhancing/entry effect by firm } z}$$

$$> \underbrace{-\left(\frac{p_z^* x_z^*}{\theta p_c^*} - (1+\rho) F_z \right)}_{\text{cost to support firm } z} > 0. \quad (17)$$

Proposition (Network-Motivated Forbearance, 追い貸し)

The monopolistic bank can maximize its profit by undertaking the forbearance for firm z if the inequality (17) holds.

- Firm z obtains a loan at the rate lower than ρ:

$$\frac{x_z^* p_z^*}{\theta p_c^* F_z} - 1 < \rho. \tag{18}$$

Caballero et al (2008): negative spread is the indicator of forbearance.

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- Loss from supporting Firm z have to be recouped from peripheral firms. If they are symmetric, premium required to cover it is

$$\frac{(1+\rho)F_z - \frac{x_z^* p_z^*}{\theta p_c^*}}{(n-1)F_i}, \quad i \neq z. \tag{19}$$

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- It is likely if
  1. Firm z is more "central" than the others; $v_z > v_i (i \neq z)$.
  2. Number of peripheral firms is larger.

In PE, the network-motivated forbearance is welfare-improving.
In GE, it is welfare-improving if

$$\sum_{i=1}^{n} e_i^* \left\{ \left(\frac{1}{\theta} + q_{0z}^* \right) \frac{v_i^* p_i^* c_i^*}{p_c^*} - (1+\rho) F_i \right\} > \sum_{i=1}^{n} \tilde{e}_i \left\{ \left(\frac{1}{\theta} + \tilde{q}_{0z} \right) \frac{\tilde{v}_i \tilde{p}_i \tilde{c}_i}{\tilde{p_c}} - (1+\rho) F_i \right\}, \tag{20}$$

where q_{0i} is the i the element of the vector $\mathbf{q_0}$ (see the note of Eq. 14); i.e., $p_0^{1-\theta} w_{i0}^{\theta} p^{i^{\theta}} / p_i$.
(Note) $q_{0z}^* \cdot \frac{p_z^* x_z^*}{p_c^*}$ is the real wage income from firm z.

Proposition (Welfare impact of the network-motivated forbearance)

In the partial equilibrium, the network-motivated forbearance is welfare-improving. In the general equilibrium, it is welfare-improving if the condition (20) is satisfied.

- A firm is expected to make profit by itself, but the monopolistic bank does not finance the fixed cost of the firm.
- This case emerges when the business stealing effect (Mankiw and Whinston 1986) dominates the Leontief externality from the entry of the firm.

$$\frac{v_z^* p_z^* c_z^* - p_z^* x_z^*}{\theta p_c^*} + \sum_{i \neq z} \left[e_i^* \left(\frac{v_i^* p_i^* c_i^*}{\theta p_c^*} - (1+\rho) F_i \right) - \tilde{e}_i \left(\frac{\tilde{v}_i \tilde{p}_i \tilde{c}_i}{\theta \tilde{p}_c} - (1+\rho) F_i \right) \right]$$
$$< - \left(\frac{p_z^* x_z^*}{\theta p_c^*} - (1+\rho) F_z \right) \leq 0. \quad (21)$$

Proposition (Closure of a profitable firm, 貸し剥し)

The monopolistic bank can maximize its profit by refusing financing and closing firm z if the inequality (21) holds.

Financial Market 2: Decentralized market

- Each household can quote a loan rate and lend directly to firms. They cannot communicate with each other.
- Alternative setup: every household deposits their numeraire to several banks. Banks cannot communicate with each other.

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- Simultaneous ascending auction (Milgrom 2000) of divisible goods: Bidders simultaneously bids in the simultaneous multiple English auctions. A firm finance from those who offer the cheapest rate. A competitive equilibrium does not necessarily exist due to the complementarity resulting from the Leontief effect.

Possibility of Forbearance in the Decentralized Case

1. If they can earn a monopolistic return from the peripheral firms; potentially because of information advantage over the competitors including outsiders (Sharpe 1990, Rajan 1992, Dinç 2000, Boot and Thakor 2000), the forbearance can emerge.

Possibility of Forbearance in the Decentralized Case

1. If they can earn a monopolistic return from the peripheral firms; potentially because of information advantage over the competitors including outsiders (Sharpe 1990, Rajan 1992, Dinç 2000, Boot and Thakor 2000), the forbearance can emerge.
2. Tacit coalition to achieve the forbearance is possible under a certain condition.
    - Each bank holds a share of the portfolio that the monopolistic bank would have.
    - A failure to extend a loan to firm z by a single bank can lead to the closure of firm z.
    - This may bring the domino closure.
    - Keeping the coalition could be profitable than deviating it.

Numerical example: network-motivated forbearance.

- Monopolistic bank.
- 4 firms. Firm 1 is the hub. We call it Firm z. The other firms are symmetric.
- Three cases on the dependence on each input $\mathbf{W}$ ((i,j) element is w_{ij} $(i,j = 1, \cdots, 4)$), and $\mathbf{w}_0 = (w_{10}, \cdots, w_{40})'$. Baseline is

$$\mathbf{W} = \begin{pmatrix} 0 & .3 & .3 & .3 \\ .1 & 0 & .1 & 0 \\ .1 & 0 & 0 & .1 \\ .1 & .1 & 0 & 0 \end{pmatrix} \quad \mathbf{w_0} = \begin{pmatrix} .1 \\ .8 \\ .8 \\ .8 \end{pmatrix}.$$

- The importance of each product for the utility.

$$\omega = (.7,\ .1,\ .1,\ .1)'.$$

- Fixed cost of each firm.

$$\mathbf{F} = (2,\ .1,\ .1,\ .1)'.$$

- Aggregate nominal income (exogenous): $R = 100$.
- Return from the outside opportunity: $\rho = 0.01$.
- Price of the non-intermediate input: $p_0 = 1$.
- Constant elasticity of substitution: $\theta = 7, 8, 9$ (7-17, 2001 Financial Statement Statistics of Corporations by Industry. MoF).

Numerical Example (Results)

- The optimal portfolio for the monopolistic bank:
  1. to lend to all firms when $(w_{z0}, w_{i0}) = (0.05, 0.85)$ or $(0.1, 0.8)$,
  2. to lend to two out of firms except firm z when $(w_{z0}, w_{i0}) = (0.15, 0.75)$.
- Under the constraint that firm z should not be financed, the constrained optimum for the bank:
  1. to lend to all firms but firm z when $(w_{z0}, w_{i0}) = (0.05, 0.85)$ or $(0.1, 0.8)$,
  2. the same as the unconstrained optimum when $(w_{z0}, w_{i0}) = (0.15, 0.75)$.

Table : Bank profit when lending to all firms.

(Note) Gray cells indicate the cases where the monopolistic bank undertakes the forbearance for firm z.

	w_{z0}	w_{i0}	$\theta = 7$	8	9
(Case 1)	0.05	0.85	1.5622	1.2336	0.9479
(Case 2)	0.10	0.80	1.0695	0.7935	0.5504
(Case 3)	0.15	0.75	0.5594	0.3434	0.1504

Table : Bank profit when firm z is not financed.

	w_{z0}	w_{i0}	$\theta = 7$	8	9
(Case 1)	0.05	0.85	0.6640	0.5711	0.4931
(Case 2)	0.10	0.80	0.5979	0.5126	0.4407
(Case 3)	0.15	0.75	0.5326	0.4546	0.3886

Table : Economic profit of firm z when all are financed.

(Note) Gray cells indicate the cases where the monopolistic bank undertakes the forbearance for firm z.

	w_{z0}	w_{i0}	$\theta = 7$	8	9
(Case 1)	0.05	0.85	0.0613	-0.1276	-0.2902
(Case 2)	0.10	0.80	-0.2054	-0.3632	-0.5012
(Case 3)	0.15	0.75	-0.4345	-0.5721	-0.6918

Table : Influence factor of firm z when all are financed.

	w_{z0}	w_{i0}	$\theta = 7$	8	9
(Case 1)	0.05	0.85	1.8571	1.8750	1.8889
(Case 2)	0.10	0.80	1.8556	1.8743	1.8886
(Case 3)	0.15	0.75	1.7985	1.8315	1.8570

Table : Leontief multiplier effect of firm z

	w_{z0}	w_{i0}	$\theta = 7$	8	9
(Case 1)	0.05	0.85	1.7839	1.6559	1.5376
(Case 2)	0.10	0.80	1.5525	1.4486	1.3496
(Case 3)	0.15	0.75	1.2661	1.2039	1.1383

Table : Business-stealing effect of firm z

	w_{z0}	w_{i0}	$\theta = 7$	8	9
(Case 1)	0.05	0.85	-0.9470	-0.8658	-0.7927
(Case 2)	0.10	0.80	-0.8756	-0.8044	-0.7388
(Case 3)	0.15	0.75	-0.8047	-0.7429	-0.6847

Table : Loan spread of firm z when all firms are financed.

(Note) Gray cells indicate the cases where the monopolistic bank undertakes the forbearance for firm z.

	w_{z0}	w_{i0}	$\theta = 7$	8	9
(Case 1)	0.05	0.85	0.0406	-0.0538	-0.1351
(Case 2)	0.10	0.80	-0.0927	-0.1716	-0.2406
(Case 3)	0.15	0.75	-0.2073	-0.2761	-0.3359

Empirical Implications / Findings

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3. Such a network-motivated forbearance can be welfare-improving.
4. Companies with a higher influence factor is likely to obtain a loan at a lower (even negative) rate and less likely to close.
5. Forbearance to a hub firm is more likely when a bank can earn quasi-rents from peripheral firms; such as those from relational lending or coalition.

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7. The influence factor of each firm is increasing in the dependence on the intermediate input within a network.

Caveats

- **Caveat 1**: We assume that a new link is impossible. If we assume the existence of more efficient potential new entrant as is assumed in Caballero et al 2008, the welfare implication of forbearance is more negative.
- **Caveat 2**: We assume away the usual cost of the monopolistic lender that limits the supply of loanable funds and the size of each firm.
- **Caveat 3**: We assume away the moral hazard problem of the hub company resulting from the anticipated bailout.