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## A Larger Country Sets a Lower Optimal Tariff<sup>1</sup>

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### Abstract

We develop a new optimal tariff theory which is consistent with the fact that a larger country sets a lower tariff. In our dynamic Dornbusch-Fischer-Samuelson Ricardian model, the long-run welfare effects of a rise in a country's tariff consist of the revenue, distortionary, and growth effects. Based on this welfare decomposition, we obtain two main results. First, the optimal tariff of a country is positive. Second, a country's marginal net benefit of deviating from free trade is usually decreasing in its absolute advantage parameter, implying that a larger (i.e., more technologically advanced) country sets a lower optimal tariff.

*Keywords:* Optimal tariff, Dornbusch-Fischer-Samuelson model, Ricardian model, Absolute advantage, Endogenous growth

*JEL classification:* F13, F43

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# 1 Introduction

It is widely believed among trade economists that an optimal tariff for a large country is positive, and that a larger country sets a higher optimal tariff. Based on two-country, two-good trade models, Kennan and Riezman (1988) and Syropoulos (2002) verify the latter statement, and even show that a sufficiently larger country can win a tariff war in that its welfare under the Nash equilibrium of a tariff setting game is higher than under global free trade.<sup>1</sup> More recently, the optimal tariff problem is reconsidered in the Dornbusch-Fischer-Samuelson (1977) (DFS henceforth) Ricardian model with a continuum of goods: Opp (2010) and Costinot et al. (2015) confirm that the optimal tariffs are positive and uniform across imported goods, provided that export taxes are unavailable. Moreover, Opp (2010) demonstrates that a country's uniform optimal tariff is increasing in its "productivity adjusted size" including its absolute advantage parameter and labor endowment. This tempts us to conclude that the beginning two statements are theoretically robust in a wide class of models.<sup>2</sup>

In fact, things go the other way. Fig. 1 indicates the tariff rates (applied, simple mean, all products (%)) of high-, middle-, and low-income countries for four periods: 1997, 2002, 2007, and 2012 (source: World Development Indicators).<sup>3</sup> In 1997, the low-income countries had the highest mean tariff of 21.91%, followed by the middle- (13.78%) and high-income countries (4.64%). Although all three income groups tended to reduce their tariffs over time, the ranking remained stable. In 2012, the mean tariffs of the low-, middle- and high-income countries were 11.51%, 8.15%, and 3.91%, respectively. This means that an economically larger country tends to set a lower tariff in contrast to the existing optimal tariff theory. Broda et al. (2008) try to resolve this puzzle from an empirical perspective by using data on highly disaggregated (i.e., four-digit Harmonized System) product categories for fifteen countries which set their tariffs freely before joining the WTO from 1990s to early 2000s. They find that the actual tariffs follow the optimal tariff formula, that is, tariffs are higher for products whose estimated inverse export supply elasticities are large. However, they do not report direct evidence that countries with larger GDP tend to set higher tariffs as the existing theory suggests. How can we explain the fact that a larger country sets a lower tariff? The purpose of this paper is to develop a new optimal tariff theory which is consistent with the data.

We depart from the DFS Ricardian optimal tariff model of Opp (2010) in one respect: economic growth. Recent well-designed empirical research (e.g., Wacziarg and Welch, 2008; Estevadeordal and Taylor, 2013) shows that trade liberalization does indeed raise economic growth, thereby overcoming Rodriguez and Rodrik's (2000) concern for robustness. If this is true, then a welfare-maximizing country may be less willing to set a high tariff. To address this point, we incorporate import tariffs into the framework developed by Naito (2012), who combines the multi-country AK endogenous growth model of Acemoglu and Ventura (2002) with the DFS Ricardian model to study the dynamic effects of changes in iceberg trade costs. By doing this, we can derive a country's dynamic optimal tariff, which is directly comparable to its static version corresponding to Opp (2010).

In our dynamic DFS Ricardian model, a rise in a country's tariff: (i) increases its tariff revenue relative to its capital income (revenue effect); (ii) decreases both its import share and rate of return to capital (distortionary effect); and (iii) lowers the balanced growth rate (growth effect). The revenue, distortionary,

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<sup>1</sup>Kennan and Riezman (1988) explicitly solve for the Nash equilibrium tariffs and welfare in terms of endowments of goods in the pure exchange model. Syropoulos (2002) analyzes the relationship between the Nash tariffs and the relative labor endowment of two countries in the standard trade model.

<sup>2</sup>Felbermayr et al. (2013) also derive the positive relationship between a country's relative labor endowment and its optimal tariff in an asymmetric two-country version of the Melitz (2003) model with monopolistic competition and heterogeneous firms.

<sup>3</sup>High-income countries are those whose 2015 GNI per capita were no less than US\$ 12,476. Low-income countries are those whose 2015 GNI per capita were no more than US\$ 1,025. The other countries are middle-income countries.

and growth effects on the country's long-run welfare are positive, nonpositive (zero in free trade), and negative, respectively. Based on this welfare decomposition, we obtain two main results. First, the optimal tariff of a country is positive. This is because, evaluating the three long-run welfare effects at free trade, the distortionary effect is zero whereas the growth effect is smaller than the revenue effect. Even if the growth effect pulls down a country's optimal tariff, the former is not large enough to say that the latter can be zero. Second, a country's marginal net benefit of deviating from free trade is usually decreasing in its absolute advantage parameter. An increase in a country's absolute advantage parameter directly decreases its own import share but increases that of the partner country. Both of them increase the size of the growth effect relative to the revenue effect, thereby reducing the country's incentive to deviate from free trade. This implies that a country's optimal tariff will be decreasing in its absolute advantage parameter. Numerical experiments, with benchmark parameter values calibrated to reproduce the actual weighted average growth rate and the relative GDP between the EU and the USA, confirm this analytical prediction for a wide domain of absolute as well as comparative advantage parameters.<sup>4</sup> Our theory demonstrates that a larger (i.e., more technologically advanced) country sets a lower optimal tariff in line with Fig. 1.<sup>5</sup>

The rest of this paper is organized as follows. Section 2 sets up the model. Section 3 examines the long-run effects of tariff changes. Section 4 derives the relationship between a country's absolute advantage and its dynamic optimal tariff under some specifications. Section 5 concludes.

## 2 The model

### 2.1 Setup

Our model is the same as Naito (2012), except that each country's iceberg trade cost for imports is replaced by its import tariff. Suppose that the world consists of two countries. In each country  $j (= 1, 2)$ , a single final good for consumption and investment is produced from a continuum of intermediate goods  $i (\in [0, 1])$ . On the other hand, each variety  $i$  of intermediate good is produced from capital. Constant returns to scale and perfect competition prevail in all sectors. Only the intermediate goods are tradable, whereas both the final good and capital are nontradable.

The representative household in country  $j$  maximizes its overall utility  $U_j = \int_0^\infty \ln C_{jt} \exp(-\rho_j t) dt$ , subject to its budget constraint:

$$p_{jt}^Y (C_{jt} + \dot{K}_{jt} + \delta_j K_{jt}) = r_{jt} K_{jt} + T_{jt}; \dot{K}_{jt} \equiv dK_{jt}/dt, \quad (1)$$

where  $t (\in [0, \infty))$  is time,  $C_j$  is consumption,  $\rho_j$  is the subjective discount rate,  $p_j^Y$  is the price of the final good,  $K_j$  is the supply of capital,  $\delta_j$  is the depreciation rate of capital,  $r_j$  is the rental rate of capital, and  $T_j$  is the lump-sum transfer from the government in country  $j$ . The time subscript is omitted whenever no confusion arises. Dynamic optimization implies the Euler equation  $\gamma_{C_j} \equiv \dot{C}_j/C_j = r_j/p_j^Y - \delta_j - \rho_j$ .

The representative final good firm in country  $j$  maximizes its profit, subject to its production function  $Y_j = Z_j (\int_0^1 x_j(i)^{(\sigma_j-1)/\sigma_j} di)^{\sigma_j/(\sigma_j-1)}$ ;  $\sigma_j > 1$ , where  $Y_j$  is the supply of the final good,  $Z_j$  is the productivity of the final good,  $x_j(i)$  is the demand for variety  $i$ ,  $\sigma_j$  is the elasticity of substitution between any two

<sup>4</sup>The graph of  $\partial U_1/\partial \ln t_1$  against  $t_1$ , where  $U_1$  is country 1's long-run welfare, and  $t_1$  is one plus country 1's ad valorem tariff rate, is indeed downward-sloping around the benchmark parameter values. This means that the second-order condition is satisfied, and that a decrease in  $\partial U_1/\partial \ln t_1|_{t_1=1}$ , country 1's marginal net benefit of deviating from free trade, caused by an increase in country 1's absolute advantage parameter, decreases country 1's optimal tariff.

<sup>5</sup>Our simulations show that a country with a relatively larger absolute advantage parameter has a relatively larger GDP in the long run, so it is indeed a larger country.

varieties. Cost minimization implies that  $\int_0^1 p_j(i)x_j(i)di = P_j Y_j$ , where  $P_j \equiv Z_j^{-1}(\int_0^1 p_j(i)^{1-\sigma_j} di)^{1/(1-\sigma_j)}$  is the price index of intermediate goods, and  $p_j(i)$  is the demand price of variety  $i$ . The first-order condition for profit maximization, implying zero profit, is given by:

$$p_j^Y = P_j. \quad (2)$$

The representative intermediate good firm producing variety  $i$  in country  $j$  maximizes its profit, subject to its production function  $x(i) = K^x(i)/a_j(i)$ , where  $x(i)$  is the supply of variety  $i$ ,  $K^x(i)$  is the demand for capital from the firm, and  $a_j(i)$  is the unit capital requirement for variety  $i$ . Suppose that the relative productivity of capital for variety  $i$  in country 1 to country 2 is distributed as  $A(i) \equiv a_2(i)/a_1(i)$ ;  $A'(i) < 0$ , meaning that the varieties of intermediate goods are sorted in the descending order of country 1's relative capital productivity. Let  $t_j (\geq 1)$  denote one plus country  $j$ 's ad valorem tariff rate, which is assumed to be uniform across imported varieties based on the uniformity result of Opp (2010) and Costinot et al. (2015). The representative final good firm in country 1 buys variety  $i_1$  domestically if and only if  $r_1 a_1(i_1) \leq t_1 r_2 a_2(i_1)$ , or  $r_1/(t_1 r_2) \leq A(i_1)$ . Under the assumed productivity distribution, all varieties  $i_1 \in [0, I_1]$  are produced in country 1, where their supply prices  $p(i_1)$  and the cutoff variety  $I_1$  are given by:

$$p(i_1) = r_1 a_1(i_1), i_1 \in [0, I_1]; \quad (3)$$

$$r_1/(t_1 r_2) = A(I_1) \Leftrightarrow I_1 = A^{-1}(r_1/(t_1 r_2)) \equiv I_1(t_1 r_2/r_1); I_1'(t_1 r_2/r_1) > 0. \quad (4)$$

Similarly, the representative final good firm in country 2 buys variety  $i_2$  domestically if and only if  $t_2 r_1 a_1(i_2) \geq r_2 a_2(i_2)$ , or  $t_2 r_1/r_2 \geq A(i_2)$ . Then it follows that:

$$p(i_2) = r_2 a_2(i_2), i_2 \in [I_2, 1]; \quad (5)$$

$$t_2 r_1/r_2 = A(I_2) \Leftrightarrow I_2 = A^{-1}(t_2 r_1/r_2) \equiv I_2(t_2 r_1/r_2); I_2'(t_2 r_1/r_2) < 0. \quad (6)$$

For country 1, all varieties in  $[I_1, 1]$  are not produced domestically but imported from country 2 due to its relatively low productivity. Of produced varieties in  $[0, I_1]$ , only varieties in its left-hand subset  $[0, I_2]$  with relatively high productivity are even exported to country 2, whereas the remaining varieties in  $[I_2, I_1]$  become nontraded.

The government in country  $j$  imposes the import tariff and transfers the resulting revenue to the representative household in country  $j$  in each period. Each country's government budget constraint is:

$$T_1 = \int_{I_1}^1 (t_1 - 1)p(i_2)x_1(i_2)di_2, T_2 = \int_0^{I_2} (t_2 - 1)p(i_1)x_2(i_1)di_1. \quad (7)$$

The demand prices of intermediate goods are related to their supply prices in the following way:

$$p_j(i_k) = \begin{cases} t_j p(i_k), & k \neq j; \\ p(i_k), & k = j. \end{cases} \quad (8)$$

The market-clearing conditions for the final good, capital, and the exported and nontraded intermediate goods in country 1 are given by:

$$Y_1 = C_1 + \dot{K}_1 + \delta_1 K_1, \quad (9)$$

$$K_1 = \int_0^{I_1} K^x(i_1) di_1, \quad (10)$$

$$x(i_1) = x_1(i_1) + x_2(i_1), i_1 \in [0, I_2], \quad (11)$$

$$x(i_1) = x_1(i_1), i_1 \in [I_2, I_1]. \quad (12)$$

Similar conditions apply to country 2.

## 2.2 Dynamic system

Let  $c_j \equiv C_j/K_j$  and  $\kappa \equiv K_1/K_2$  denote the consumption/capital ratio in country  $j$  and the relative supply of capital in country 1 to country 2, respectively, and let capital in country 2 be the numeraire:  $r_2 \equiv 1$ . Then our model is reduced to the following four-dimensional dynamic system (see Appendix A for derivations):

$$\dot{c}_1/c_1 = 1/q_1(t_1/r_1) - \delta_1 - \rho_1 - (\eta_1(t_1, \beta_1(t_1/r_1))/q_1(t_1/r_1) - \delta_1 - c_1), \quad (13)$$

$$\dot{c}_2/c_2 = 1/q_2(t_2/r_1) - \delta_2 - \rho_2 - (\eta_2(t_2, \beta_2(t_2/r_1))/q_2(t_2/r_1) - \delta_2 - c_2), \quad (14)$$

$$\dot{\kappa}/\kappa = \eta_1(t_1, \beta_1(t_1/r_1))/q_1(t_1/r_1) - \delta_1 - c_1 - (\eta_2(t_2, \beta_2(t_2/r_1))/q_2(t_2/r_1) - \delta_2 - c_2), \quad (15)$$

$$\kappa = (\zeta_2(t_2, \beta_2(t_2/r_1))/\zeta_1(t_1, \beta_1(t_1/r_1)))/r_1. \quad (16)$$

Eqs. (13), (14), and (15) correspond to  $\dot{c}_1/c_1 = \dot{C}_1/C_1 - \dot{K}_1/K_1$ ,  $\dot{c}_2/c_2 = \dot{C}_2/C_2 - \dot{K}_2/K_2$ , and  $\dot{\kappa}/\kappa = \dot{K}_1/K_1 - \dot{K}_2/K_2$ , respectively. Eq. (16) comes from country 1's capital market clearing condition (10), which is equivalent to its zero balance of trade from Walras' law. There are several functions to be explained. First,  $q_j(t_j r_k/r_j) \equiv Q_j(t_j r_k/r_j, 1)$ , where  $Q_j(t_j r_k, r_j)$  is a simplified version of country  $j$ 's price index of intermediate goods defined as:

$$Q_j(t_j r_k, r_j) \equiv \tilde{Q}_j(t_j r_k, r_j, I_j(t_j r_k/r_j)); \quad (17)$$

$$\tilde{Q}_1(t_1 r_2, r_1, I_1) \equiv Z_1^{-1} [(t_1 r_2)^{1-\sigma_1} \int_{I_1}^1 a_2(i_2)^{1-\sigma_1} di_2 + r_1^{1-\sigma_1} \int_0^{I_1} a_1(i_1)^{1-\sigma_1} di_1]^{1/(1-\sigma_1)},$$

$$\tilde{Q}_2(t_2 r_1, r_2, I_2) \equiv Z_2^{-1} [(t_2 r_1)^{1-\sigma_2} \int_0^{I_2} a_1(i_1)^{1-\sigma_2} di_1 + r_2^{1-\sigma_2} \int_{I_2}^1 a_2(i_2)^{1-\sigma_2} di_2]^{1/(1-\sigma_2)}.$$

The fact that country  $j$ 's gross rate of return to capital  $r_j/p_j^Y = 1/(Q_j(t_j r_k, r_j)/r_j) = 1/q_j(t_j r_k/r_j)$  is decreasing in  $t_j r_k/r_j$  implies that country  $j$ 's consumption grows faster, the lower its import tariff is and/or the higher its relative rental rate is. Second,  $\beta_j(t_j r_k/r_j)$  is country  $j$ 's expenditure share of imported varieties ( $\int_{I_1}^1 p_1(i_2)x_1(i_2)di_2/(P_1Y_1)$  and  $\int_0^{I_2} p_2(i_1)x_2(i_1)di_1/(P_2Y_2)$  for countries 1 and 2, respectively), where:

$$\begin{aligned}
\beta_j(t_j r_k / r_j) &\equiv \tilde{\beta}_j(t_j r_k / r_j, I_j(t_j r_k / r_j)); \\
\tilde{\beta}_1(t_1 r_2 / r_1, I_1) &\equiv (Z_1 Q_1(1, r_1 / (t_1 r_2)))^{\sigma_1 - 1} \int_{I_1}^1 a_2(i_2)^{1 - \sigma_1} di_2, \\
\tilde{\beta}_2(t_2 r_1 / r_2, I_2) &\equiv (Z_2 Q_2(1, r_2 / (t_2 r_1)))^{\sigma_2 - 1} \int_0^{I_2} a_1(i_1)^{1 - \sigma_2} di_1.
\end{aligned} \tag{18}$$

Eq. (18), together with Eqs. (4), (6), and (17), means that a fall in country  $j$ 's import tariff and/or a rise in its relative rental rate increases its import share both at the intensive margin (i.e., by increasing the value of imports of the existing varieties) and extensive margin (i.e., by expanding the set of imported varieties). Third,  $\eta_j(t_j, \beta_j) \equiv t_j / [t_j - (t_j - 1)\beta_j]$  is equal to the ratio of country  $j$ 's total income including the tariff revenue to its capital income. It is increasing in both  $t_j$  and  $\beta_j$ , and takes the value of unity at  $t_j = 1$ . Fourth,  $\zeta_j(t_j, \beta_j) \equiv \beta_j / [t_j - (t_j - 1)\beta_j]$  is interpreted as the ratio of country  $j$ 's value of imports evaluated at the world prices to its capital income because  $\zeta_1 r_1 K_1 = \zeta_2 r_2 K_2$  implied from Eq. (16) shows country 1's (and also country 2's) zero balance of trade. The function  $\zeta_j(t_j, \beta_j)$  is decreasing in  $t_j$  but increasing in  $\beta_j$ , and takes the value of  $\beta_j$  at  $t_j = 1$ .

A balanced growth path (BGP) is defined as a path along which all variables grow at constant rates. In our model, a BGP is characterized by Eqs. (13), (14), (15), (16), and  $\dot{c}_1/c_1 = \dot{c}_2/c_2 = \dot{\kappa}/\kappa = 0$ . From Eqs. (13), (14), and (15), country 1's rental rate is implicitly determined by:

$$1/q_1(t_1/r_1^*) - \delta_1 - \rho_1 = 1/q_2(t_2 r_1^*) - \delta_2 - \rho_2, \tag{19}$$

where an asterisk over a variable represents a BGP. Then Eqs. (13), (14), and (16) give  $c_1^*$ ,  $c_2^*$ , and  $\kappa^*$ , respectively. Since the left- and right-hand sides of Eq. (19) are increasing and decreasing in  $r_1$ , respectively,  $r_1^*$  is unique if exists. We assume that a BGP exists (implying uniqueness) and is saddle-path stable (see Appendix B for stability).

### 3 Long-run effects of tariff changes

#### 3.1 Balanced growth rate

From now on, we focus only on the long-run effects of tariff changes. As long as we consider small-scale policy changes, a period of transition from an old to a new BGP will be short, so the short-run effects are negligible. This approach is also taken by Chen and Lu (2013), who characterize the optimal tax incidence in their endogenous growth model with physical and human capital.

The rate of change in  $r_1^*$  is solved as (see Appendix C for derivation):

$$dr_1^*/r_1^* = [(\beta_1^*/q_1^*)/(\beta_1^*/q_1^* + \beta_2^*/q_2^*)]dt_1/t_1 - [(\beta_2^*/q_2^*)/(\beta_1^*/q_1^* + \beta_2^*/q_2^*)]dt_2/t_2. \tag{20}$$

A rise in country 1's tariff rate, *ceteris paribus*, lowers its growth rate of consumption in the left-hand side of Eq. (19). For country 1 to catch up with country 2, the former's relative rental rate should rise. The amount of change in the balanced growth rate is obtained as (see Appendix C for derivation):

$$d\gamma_{C1}^* = d\gamma_{C2}^* = -[(\beta_1^*/q_1^*)(\beta_2^*/q_2^*)/(\beta_1^*/q_1^* + \beta_2^*/q_2^*)](dt_1/t_1 + dt_2/t_2). \tag{21}$$

Eq. (21) means that a rise in any tariff rate always lowers the balanced growth rate. This is because, as shown in Eq. (20), a rise in each country's tariff rate can raise its relative rental rate by less than the rate of its tariff rise.

### 3.2 Long-run welfare

Suppose that the world is on a BGP from the initial period on. Then country  $j$ 's consumption in period  $t$  is expressed as  $C_{jt} = K_{j0}c_j^* \exp(\gamma_{C_j}^* t)$ . Substituting this into country  $j$ 's overall utility, the latter is rewritten as  $U_j = (1/\rho_j)(\ln K_{j0} + \ln c_j^* + \gamma_{C_j}^*/\rho_j)$ , which serves as a measure of its long-run welfare.

Since we are interested in an optimal tariff of a country given a tariff of the other country, we focus on country 1's welfare. The welfare effect of its own tariff change is given by (see Appendix C for derivation):

$$\begin{aligned} \partial U_1 / \partial \ln t_1 &= (1/\rho_1) \{ (1/c_1^*) [(\eta_1^*/q_1^*)\zeta_1^* + C_r^{1*} (\beta_2^*/q_2^*) / (\beta_1^*/q_1^* + \beta_2^*/q_2^*)] \\ &\quad - (1/\rho_1) (\beta_1^*/q_1^*) (\beta_2^*/q_2^*) / (\beta_1^*/q_1^* + \beta_2^*/q_2^*) \}; \\ C_r^{1*} &\equiv \partial(\dot{c}_1/c_1) / \partial \ln r_1|_* \equiv -(1/q_1^*) [\eta_1^* \zeta_1^* (t_1 - 1) B_1^* + \beta_1^* (\eta_1^* - 1)], \\ B_j^* &\equiv -d \ln \beta_j / d \ln (t_j r_k / r_j)|_* > 0 \Rightarrow C_r^{1*} \leq 0. \end{aligned} \quad (22)$$

In the right-hand side of Eq. (22), the first and second lines correspond to changes in  $\ln c_1^*$  and  $\gamma_{C_1}^*/\rho_1$ , respectively. The latter, which can be called the growth effect, is clearly negative as discussed above. For the former, it is convenient to express  $c_1^*$  as  $c_1^* = \rho_1 + (\eta_1(t_1, \beta_1(t_1/r_1^*)) - 1)/q_1(t_1/r_1^*)$  from Eq. (13). A rise in  $t_1$  directly increases  $\eta_1^*$ , which increases  $c_1^*$ . On the other hand, the resulting increase in  $t_1/r_1^*$  decreases  $\beta_1^*$  but increases  $q_1^*$  (i.e., decreases  $1/q_1^* = (r_1/p_1^Y)^*$ ), both of which decreases  $c_1^*$  unless  $t_1 = 1$  and hence  $\eta_1^* = 1$  at the old BGP. These two effects on  $c_1^*$  can be called the revenue effect and the distortionary effect, respectively. Two things can be pointed out from Eq. (22). First, the last term suggests that consideration of endogenous growth pulls down a country's optimal tariff. Second, in the absence of tariff revenue, the positive revenue effect would vanish, so the optimal trade cost would be zero as in Naito (2012).

To see whether country 1's optimal tariff is zero or not, we evaluate Eq. (22) at  $t_1 = 1$ . Since  $\eta_1^* = 1$ ,  $\zeta_1^* = \beta_1^*$ ,  $c_1^* = \rho_1$ , and  $C_r^{1*} = 0$ , we have:

$$\partial U_1 / \partial \ln t_1|_{t_1=1} = (1/\rho_1^2) V_1^*; V_1^* \equiv (\beta_1^*/q_1^*) [1 - (\beta_2^*/q_2^*) / (\beta_1^*/q_1^* + \beta_2^*/q_2^*)] \equiv (\beta_1^*/q_1^*)^2 / (\beta_1^*/q_1^* + \beta_2^*/q_2^*) > 0. \quad (23)$$

Eq. (23) represents country 1's marginal net benefit of deviating from free trade. It is proportional to  $V_1^*$ , which consists of a common term  $(\beta_1^*/q_1^*)$  multiplied by two terms in the square brackets. The first and second terms come from the revenue and growth effects, respectively. Since the former is larger than the latter at  $t_1 = 1$ , we obtain the first main result:

**Proposition 1** *The optimal tariff of a country is positive.*

Starting from free trade, a large country can always raise its welfare by raising its tariff. Put the other way around, gradual tariff reduction from a high value by a country at first continues to raise its welfare as Naito (2012, section 5.1) conjectures, but eventually its tariff reaches a positive critical point, that is, the optimal tariff. It is the revenue effect that distinguishes our result from Naito (2012).



Another observation from Eq. (23) is that openness  $\beta_j^*$  matters for country 1's incentive to deviate from free trade. When country 1 is more open (i.e.,  $\beta_1^*$  increases), the common term ( $\beta_1^*/q_1^*$ ) increases whereas the growth effect relatively decreases, both of which induce country 1 to deviate further from free trade. On the other hand, when country 2 is more open (i.e.,  $\beta_2^*$  increases), the growth effect relatively increases, which reduces country 1's incentive to deviate from free trade. The former suggests that, if a larger country is more closed in terms of its import share, then its optimal tariff can be lower unlike the existing optimal tariff models. We explore this possibility in the next section.

## 4 Absolute advantage and the dynamic optimal tariff

Having confirmed that the optimal tariff of a country is positive even in our model, we next see if a larger (i.e., more technologically advanced) country sets a lower optimal tariff. Country 1's optimal tariff is determined by equating Eq. (22) to zero. To proceed further, we specify some functional forms following Opp (2010). First, each country's final good production function is Cobb-Douglas:  $\sigma_j \rightarrow 1$ . Second, each country's unit capital requirement, and hence  $A(i)$ , are log-linear in  $i$ :<sup>6</sup>

$$\begin{aligned} a_1(i) &= \exp(-a_{01} + b_1 i); b_1 > 0, \\ a_2(i) &= \exp(a_{02} - b_2 i); b_2 > 0, \\ A(i) &= \exp(a - bi); a \equiv a_{01} + a_{02}, b \equiv b_1 + b_2 > 0. \end{aligned}$$

Under these specifications,  $a_1(i)$  is increasing, whereas  $a_2(i)$  is decreasing, in  $i$ . The larger  $a_{01}$  is, the lower the graph of  $a_1(i)$  is overall. The larger  $b_1$  is, the faster  $a_1(i)$  increases with  $i$ . The former measures country 1's absolute advantage, whereas the latter captures country 1's comparative advantage across varieties.  $a_{02}$  and  $b_2$  for country 2 can be similarly interpreted, with the opposite effects on  $a_2(i)$ . Finally,  $a$  and  $b$  summarize the two countries' absolute and comparative advantages. Then functions  $q_j(t_j r_k / r_j)$  and  $\beta_j(t_j r_k / r_j)$  are simplified to:

$$\begin{aligned} q_1(t_1/r_1) &= Z_1^{-1} \exp(-[(\ln(t_1/r_1))^2 - 2(b-a)\ln(t_1/r_1) + a^2 - b(2a_{02} - b_2)]/(2b)), \\ q_2(t_2 r_1) &= Z_2^{-1} \exp(-[(\ln(t_2 r_1))^2 - 2a\ln(t_2 r_1) + a^2 - b(2a_{02} - b_2)]/(2b)), \\ \beta_1(t_1/r_1) &= (b - a - \ln(t_1/r_1))/b = 1 - I_1(t_1/r_1), \\ \beta_2(t_2 r_1) &= (a - \ln(t_2 r_1))/b = I_2(t_2 r_1). \end{aligned}$$

The following analytical result provides a prediction for the optimal tariff (see Appendix D for proof):

**Proposition 2**  $V_1 \equiv (\beta_1/q_1)^2/(\beta_1/q_1 + \beta_2/q_2)$  in Eq. (23) is decreasing in  $a_{01}$  if  $b \leq 4$ .

As  $a_{01}$  increases, ceteris paribus, country 1 gets more closed whereas country 2 gets more open (i.e.,  $\beta_1$  decreases whereas  $\beta_2$  increases). This always increases  $\beta_2/q_2$ , whereas it decreases  $\beta_1/q_1$  if  $b \leq 4$ .<sup>7</sup> In this

<sup>6</sup>Opp (2010, Eq. (32)) instead uses  $A(i) = \exp(\mu - \gamma(i - 1/2))$ , which means that  $A(1/2) = \exp(\mu)$ . By letting  $a \equiv \mu + \gamma/2$  and  $b \equiv \gamma$ , this is equivalent to our specification.

<sup>7</sup> $A(i) = \exp(a - bi)$  implies that  $A(0)/A(1) = \exp(a)/\exp(a - b) = \exp(b)$ . The fact that  $\exp(4) \approx 54.598$  means that country 1's most productive variety should be less than 54.598 times as productive relative to country 2 as its least productive variety.

case, Eq. (23), country 1's marginal net benefit of deviating from free trade, decreases. This indicates that country 1's optimal tariff will be decreasing in its absolute advantage parameter.

To confirm this prediction, we run some numerical experiments. Let the EU and the USA, the two largest economies in the world, be countries 1 and 2, respectively. We first calibrate the old BGP as follows. We use Eqs. (13), (14), (15), (16), and  $\dot{c}_1/c_1 = \dot{c}_2/c_2 = \dot{\kappa}/\kappa = 0$ , together with the actual weighted average growth rate  $\gamma_{C2}^* = 0.0204015$  and the relative GDP  $r_1^*\kappa^* = 1.14501$  from the World Development Indicators, to solve for  $r_1^*, c_1^*, c_2^*, \kappa^*, a_{01}$ , and  $b_1$ , given the actual tariffs  $t_1 = 1.02238, t_2 = 1.03265$  from WDI, and other parameters:  $\rho_1 = \rho_2 = 0.02, \delta_1 = \delta_2 = 0.05, K_{20} = 100, Z_1 = Z_2 = 0.07, a_{02} = 0.5, b_2 = 1$ .<sup>8</sup> All calculations are done with Mathematica 10. The values of main endogenous variables at the old BGP are reported in the first line of panel (b) of Table 1. Country 1's absolute and comparative advantage parameters are calibrated as  $a_{01} = 0.51025, b_1 = 0.896509$ . Our model reproduces the target data  $\gamma_{C2}^* = 0.0204015$  and  $r_1^*\kappa^* = 1.14501$ .

Starting from the old BGP, country 1's optimal tariff is calculated as  $t_1^U = 1.23915$ , or 23.9%, in the second line of panel (b). For comparison, country 1's optimal tariff in the static version of our model, where  $\dot{K}_j + \delta_j K_j = 0$  in Eqs. (1) and (9), is calculated as  $t_1^c = 1.68694$ , or 68.7%.<sup>9</sup> Even at the benchmark case, where the two countries are similar in terms of economic size at the old BGP, the value of the dynamic optimal tariff is much more realistic than the static one.

Fig. 2 displays the relationships between  $a_{01}, t_1^U$ , and  $t_1^c$ , with  $b_1 = 0.896509$  fixed. As expected, for a wide domain of  $a_{01}$  around  $a_{01} = 0.51025$ , the graph of  $t_1^U$  is downward sloping, whereas that of  $t_1^c$  is upward sloping just like Opp (2010, Proposition 3).<sup>10</sup> This is confirmed in Table 1: as  $a_{01}$  increases from panel (c) ( $a_{01} = 0.31025$ ) to (b) ( $a_{01} = 0.51025$ ) to (a) ( $a_{01} = 0.71025$ ),  $\beta_1^*$  decreases whereas  $\beta_2^*$  increases, implying from Eq. (16) that country 1 becomes relatively larger (i.e.,  $r_1^*\kappa^*$  increases) at the old BGP. During this process, country 1's optimal tariff decreases from 53.8% to 13.0%.

It is also shown numerically that  $t_1^U$  as well as  $t_1^c$  is increasing in  $b_1$ . This is because an increase in  $b_1$  means that country 1's relative productivity decreases with  $i$  more steeply, so it tends to import more fraction of varieties. This implies that  $t_1^U$  falls as  $a_{01}$  gets larger and/or  $b_1$  gets smaller. Fig. 3 depicts some contours of  $t_1^U$  in the  $(a_{01}, b_1)$  plane for  $a_{01} \in [0.51025 - 0.2, 0.51025 + 0.2]$  and  $b_1 \in [0.896509 - 0.2, 0.896509 + 0.2]$ . The value of  $t_1^U$  falls as one moves to the right and/or down, and it falls below 10% near the southeast corner: for  $a_{01} = 0.71025$  and  $b_1 = 0.696509$ , we have  $t_1^U = 1.0776$ , or only 7.76%.

## 5 Concluding remarks

In spite of the fact that a larger country tends to set a lower tariff, the existing optimal tariff models have predicted the opposite. By incorporating endogenous growth based on capital accumulation into the DFS Ricardian model, we show that the optimal tariff of a country is positive but decreasing in its absolute advantage parameter. This enables us to explain the above fact within the optimal tariff framework.

Although we focus on an optimal tariff of a country taking the partner country's tariff as given, our analysis can easily be extended to a tariff war game. In the normal case where each reaction curve is downward sloping in the  $(t_1, t_2)$  plane and country 2's reaction curve crosses country 1's reaction curve from below, an increase in country 1's absolute advantage parameter pulls its reaction curve inward, thereby

<sup>8</sup>Data on  $\gamma_{C2}^*, r_1^*\kappa^*, t_1$ , and  $t_2$  are averaged over twenty years during 1996-2015.  $\rho_1 = \rho_2 = 0.02$  and  $\delta_1 = \delta_2 = 0.05$  are borrowed from Barro and Sala-i-Martin (2004). The other parameter values are arbitrarily chosen.

<sup>9</sup>Appendix E shows that  $t_1^c$  is positive. In calculating  $t_1^c$ ,  $\kappa$  is determined as its old BGP value, by substituting the old BGP value of  $r_1^*$  from Eq. (19) into Eq. (16).

<sup>10</sup>The graph of  $t_1^U$  turns upward sloping as  $a_{01}$  decreases to around  $a_{01} = 0.26$ . On the other hand,  $t_1^c$  takes a complex value as  $a_{01}$  increases to around  $a_{01} = 0.8$ .

decreasing  $t_1$  and hence increasing  $t_2$  in the Nash equilibrium. Even in this case, the negative relationship between a country's absolute advantage parameter and its Nash tariff will be unchanged.

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## Appendix A. Derivations of Eqs. (13) to (16)

Using Eqs. (3), (4), (5), (6), and (8), country  $j$ 's price index of intermediate goods  $P_j \equiv Z_j^{-1}(\int_0^1 p_j(i)^{1-\sigma_j} di)^{1/(1-\sigma_j)}$  is rewritten as Eq. (17). From Eqs. (2) and linear homogeneity of (17), country  $j$ 's gross rate of return to capital  $r_j/p_j^Y$  is rewritten as:

$$r_j/p_j^Y = 1/q_j(t_j r_k/r_j); q_j(t_j r_k/r_j) \equiv Q_j(t_j r_k/r_j, 1), \quad (\text{A.1})$$

Substituting Eq. (A.1) into country  $j$ 's Euler equation, its growth rate of consumption is given by:

$$\dot{C}_j/C_j = 1/q_j(t_j r_k/r_j) - \delta_j - \rho_j \equiv \gamma_{Cj}. \quad (\text{A.2})$$

Using Eqs. (3), (4), (5), (6), (8), the derived demand for a variety  $x_j(i) = (\partial P_j / \partial p_j(i)) Y_j = Z_j^{\sigma_j-1} P_j^{\sigma_j} p_j(i)^{-\sigma_j} Y_j$ , and linear homogeneity of Eq. (17), countries' expenditure shares of imported varieties  $\int_{I_1}^1 p_1(i_2) x_1(i_2) di_2 / (P_1 Y_1)$  and  $\int_0^{I_2} p_2(i_1) x_2(i_1) di_1 / (P_2 Y_2)$  are rewritten as Eq. (18). Rewriting Eq. (7) using Eqs. (1), (2), (8), (9) (or its counterpart for country 2), and (18), we obtain  $T_j = [(t_j - 1)/t_j] \beta_j (r_j K_j + T_j)$ , or equivalently,  $(r_j K_j + T_j)/(r_j K_j) = t_j/[t_j - (t_j - 1)\beta_j] \equiv \eta_j(t_j, \beta_j)$ . Dividing Eq. (1) by  $p_j^Y K_j$ , and using Eqs. (18), (A.1), and the definition of  $\eta_j(t_j, \beta_j)$ , the growth rate of capital in country  $j$  is given by:

$$\dot{K}_j/K_j = \eta_j(t_j, \beta_j(t_j r_k/r_j))/q_j(t_j r_k/r_j) - \delta_j - c_j \equiv \gamma_{Kj}. \quad (\text{A.3})$$

From Eqs. (A.2) and (A.3), we immediately obtain Eqs. (13) to (15).

Finally, rewriting Eq. (10) using Eqs. (1), (2), (3), (8), (9), (11), (12), (18), and the definition of  $\eta_j(t_j, \beta_j)$ , we obtain Eq. (16).

## Appendix B. Stability of dynamic system

To study local dynamics around a BGP, we have to linearize our dynamic system (13) to (16). Totally differentiating Eqs. (17) and (18), and using Eqs. (4), (6), and (18), we obtain:

$$dQ_j/Q_j = \beta_j(dt_j/t_j + dr_k/r_k) + (1 - \beta_j)dr_j/r_j, \quad (\text{B.1})$$

$$d\beta_j/\beta_j = -B_j(dt_j/t_j + dr_k/r_k - dr_j/r_j); \quad (\text{B.2})$$

$$B_1 \equiv (\sigma_1 - 1)(1 - \beta_1) - (I_1 a_2(I_1)^{1-\sigma_1} / \int_{I_1}^1 a_2(i_2)^{1-\sigma_1} di_2) A(I_1)/(A'(I_1)I_1) > 0,$$

$$B_2 \equiv (\sigma_2 - 1)(1 - \beta_2) - (I_2 a_1(I_2)^{1-\sigma_2} / \int_0^{I_2} a_1(i_1)^{1-\sigma_2} di_1) A(I_2)/(A'(I_2)I_2) > 0.$$

Using Eqs. (B.1) and (B.2), the totally differentiated forms of  $q_j$ ,  $\eta_j$ , and  $\zeta_j$  are derived as:

$$dq_j/q_j = dQ_j/Q_j - dr_j/r_j = \beta_j(dt_j/t_j + dr_k/r_k - dr_j/r_j), \quad (\text{B.3})$$

$$d\eta_j/\eta_j = \zeta_j[dt_j/t_j + (t_j - 1)d\beta_j/\beta_j] = \zeta_j[dt_j/t_j - (t_j - 1)B_j(dt_j/t_j + dr_k/r_k - dr_j/r_j)], \quad (\text{B.4})$$

$$d\zeta_j/\zeta_j = \eta_j[d\beta_j/\beta_j - (1 - \beta_j)dt_j/t_j] = -\eta_j[(1 - \beta_j)dt_j/t_j + B_j(dt_j/t_j + dr_k/r_k - dr_j/r_j)]. \quad (\text{B.5})$$

Using Eqs. (B.3), (B.4), and (B.5), Eqs. (13) to (16) are linearized to:

$$\dot{c}_1/c_1 = c_1^*dc_1/c_1 + C_r^{1*}dr_1/r_1; C_r^{1*} \equiv -(1/q_1^*)[\eta_1^*\zeta_1^*(t_1 - 1)B_1^* + \beta_1^*(\eta_1^* - 1)] \leq 0, \quad (\text{B.6})$$

$$\dot{c}_2/c_2 = c_2^*dc_2/c_2 + C_r^{2*}dr_1/r_1; C_r^{2*} \equiv (1/q_2^*)[\eta_2^*\zeta_2^*(t_2 - 1)B_2^* + \beta_2^*(\eta_2^* - 1)] \geq 0, \quad (\text{B.7})$$

$$\dot{\kappa}/\kappa = -c_1^*dc_1/c_1 + c_2^*dc_2/c_2 + K_r^*dr_1/r_1; \quad (\text{B.8})$$

$$K_r^* \equiv (\eta_1^*/q_1^*)[\zeta_1^*(t_1 - 1)B_1^* + \beta_1^*] + (\eta_2^*/q_2^*)[\zeta_2^*(t_2 - 1)B_2^* + \beta_2^*] > 0,$$

$$dr_1/r_1 = R_\kappa^*d\kappa/\kappa; R_\kappa^* \equiv -1/(1 + \eta_1^*B_1^* + \eta_2^*B_2^*) \in (-1, 0). \quad (\text{B.9})$$

Substituting Eq. (B.9) into Eqs. (B.6), (B.7), and (B.8) to eliminate  $dr_1/r_1$ , and noting for example that  $\dot{c}_1/c_1 = d(\ln c_1 - \ln c_1^*)/dt$  and  $dc_1/c_1 = \ln c_1 - \ln c_1^*$ , we obtain the following three-dimensional linearized dynamic system:

$$\begin{bmatrix} d(\ln c_1 - \ln c_1^*)/dt \\ d(\ln c_2 - \ln c_2^*)/dt \\ d(\ln \kappa - \ln \kappa^*)/dt \end{bmatrix} = J^* \begin{bmatrix} \ln c_1 - \ln c_1^* \\ \ln c_2 - \ln c_2^* \\ \ln \kappa - \ln \kappa^* \end{bmatrix}; \quad (\text{B.10})$$

$$J^* \equiv \begin{bmatrix} j_{11}^* & j_{12}^* & j_{13}^* \\ j_{21}^* & j_{22}^* & j_{23}^* \\ j_{31}^* & j_{32}^* & j_{33}^* \end{bmatrix} \equiv \begin{bmatrix} c_1^* & 0 & C_r^{1*}R_\kappa^* \\ 0 & c_2^* & C_r^{2*}R_\kappa^* \\ -c_1^* & c_2^* & K_r^*R_\kappa^* \end{bmatrix}.$$

The characteristic polynomial associated with the Jacobian matrix  $J^*$  is:

$$\varphi(J^*) \equiv \det(\lambda I - J^*) = \lambda^3 - \text{tr}J^* \cdot \lambda^2 + J_2^* \cdot \lambda - \det J^*;$$

$$\text{tr}J^* \equiv j_{11}^* + j_{22}^* + j_{33}^*,$$

$$J_2^* \equiv j_{22}^*j_{33}^* - j_{23}^*j_{32}^* + j_{33}^*j_{11}^* - j_{31}^*j_{13}^* + j_{11}^*j_{22}^* - j_{12}^*j_{21}^*,$$

$$\det J^* \equiv j_{11}^*j_{22}^*j_{33}^* + j_{12}^*j_{23}^*j_{31}^* + j_{13}^*j_{21}^*j_{32}^* - j_{13}^*j_{22}^*j_{31}^* - j_{12}^*j_{21}^*j_{33}^* - j_{11}^*j_{23}^*j_{32}^*.$$

Noting that the linearized dynamic system (B.10) contains two control variables  $c_1$  and  $c_2$  and one state variable  $\kappa$ , it is saddle-path stable if and only if the characteristic equation  $\varphi(J^*) = 0$  has two positive eigenvalues  $\lambda_1$  and  $\lambda_2$  and one negative eigenvalue  $\lambda_3$ . A necessary condition is that  $\det J^* < 0$ , whereas a sufficient condition is that  $\det J^* < 0$  and  $\text{tr}J^* > 0$ . In the present case,  $\det J^*$  is calculated as:

$$\det J^* = c_1^*c_2^*R_\kappa^*(K_r^* + C_r^{1*} - C_r^{2*}) = c_1^*c_2^*R_\kappa^*(\beta_1^*/q_1^* + \beta_2^*/q_2^*) < 0.$$

This implies that (B.10) satisfies the necessary condition for saddle-path stability. On the other hand,  $\text{tr}J^*$  is simply given by:

$$\text{tr}J^* = c_1^* + c_2^* + K_r^* R_\kappa^*.$$

Since  $c_1^* + c_2^* > 0$  but  $K_r^* R_\kappa^* < 0$ , we cannot ensure that the sufficient condition is always satisfied. However, even if  $\text{tr}J^* < 0$ , it is still possible that  $\lambda_1 > 0, \lambda_2 > 0$ , and  $\lambda_3 < 0$ .

## Appendix C. Derivations of Eqs. (20) to (22)

Substituting  $dq_j/q_j$  from Eq. (B.3) into the totally differentiated form of Eq. (19), we have:

$$(1/q_1^*)[-\beta_1^*(dt_1/t_1 - dr_1^*/r_1^*)] = (1/q_2^*)[-\beta_2^*(dt_2/t_2 + dr_1^*/r_1^*)].$$

Solving this for  $dr_1^*/r_1^*$  yields Eq. (20). Substituting this back into either side of the above equation, we obtain Eq. (21).

Totally differentiating Eq. (13) with  $\dot{c}_1/c_1 = 0$ , and using Eqs. (20), (B.3), and (B.4), we obtain:

$$dc_1^* = (\eta_1^*/q_1^*)\zeta_1^* dt_1/t_1 + C_r^{1*}[(\beta_2^*/q_2^*)/(\beta_1^*/q_1^* + \beta_2^*/q_2^*)](dt_1/t_1 + dt_2/t_2), \quad (\text{C.1})$$

where  $C_r^{1*}(\leq 0)$  is defined in Eq. (B.6). Substituting Eqs. (21) and (C.1) into the totally differentiated form of country 1's long-run welfare measure  $dU_1 = (1/\rho_1)(dc_1^*/c_1^* + d\gamma_{C1}^*/\rho_1)$ , the latter is rewritten as:

$$\begin{aligned} dU_1 = & (1/\rho_1)\{(1/c_1^*)\{(\eta_1^*/q_1^*)\zeta_1^* dt_1/t_1 + C_r^{1*}[(\beta_2^*/q_2^*)/(\beta_1^*/q_1^* + \beta_2^*/q_2^*)](dt_1/t_1 + dt_2/t_2)\} \\ & + (1/\rho_1)\{-(\beta_1^*/q_1^*)(\beta_2^*/q_2^*)/(\beta_1^*/q_1^* + \beta_2^*/q_2^*)\}(dt_1/t_1 + dt_2/t_2)\}. \end{aligned}$$

This immediately implies Eq. (22).

## Appendix D. Proof of Proposition 2

Differentiating the natural log of  $q_1(t_1/r_1), q_2(t_2 r_1), \beta_1(t_1/r_1)$ , and  $\beta_2(t_2 r_1)$  with respect to  $a_{01}$  gives:

$$\begin{aligned} \partial \ln q_1 / \partial a_{01} &= -(\ln(t_1/r_1) + a)/b = -I_1 = -(1 - \beta_1) < 0, \\ \partial \ln q_2 / \partial a_{01} &= -(-\ln(t_2 r_1) + a)/b = -I_2 = -\beta_2 < 0, \\ \partial \ln \beta_1 / \partial a_{01} &= -1/(b - a - \ln(t_1/r_1)) = -1/(b\beta_1) < 0, \\ \partial \ln \beta_2 / \partial a_{01} &= 1/(a - \ln(t_2 r_1)) = 1/(b\beta_2) > 0, \end{aligned}$$

which immediately imply that:

$$\begin{aligned} \partial \ln(\beta_1/q_1) / \partial a_{01} &= \partial \ln \beta_1 / \partial a_{01} - \partial \ln q_1 / \partial a_{01} = -1/(b\beta_1) + 1 - \beta_1, \\ \partial \ln(\beta_2/q_2) / \partial a_{01} &= \partial \ln \beta_2 / \partial a_{01} - \partial \ln q_2 / \partial a_{01} = 1/(b\beta_2) + \beta_2 > 0. \end{aligned}$$

On the other hand, totally differentiating  $\ln V_1 = 2 \ln(\beta_1/q_1) - \ln(\beta_1/q_1 + \beta_2/q_2)$ , we have:

$$d \ln V_1 = [(\beta_1/q_1 + 2\beta_2/q_2)/(\beta_1/q_1 + \beta_2/q_2)]d \ln(\beta_1/q_1) - [(\beta_2/q_2)/(\beta_1/q_1 + \beta_2/q_2)]d \ln(\beta_2/q_2).$$

Combining these results, we obtain:

$$\partial \ln V_1 / \partial a_{01} = \{(\beta_1/q_1 + 2\beta_2/q_2)[-1/(b\beta_1) + 1 - \beta_1] - (\beta_2/q_2)[1/(b\beta_2) + \beta_2]\}/(\beta_1/q_1 + \beta_2/q_2).$$

This implies that  $\partial \ln V_1 / \partial a_{01} < 0$  if  $-1/(b\beta_1) + 1 - \beta_1 < 0$ .

Let  $f(\beta_1) \equiv -1/(b\beta_1) + 1 - \beta_1$  defined on  $\beta_1 \in [0, 1]$ . The function has the following properties:

$$\begin{aligned} f(0) &= -\infty < 0, f(1) = -1/b < 0, \\ f'(\beta_1) &= 1/(b\beta_1^2) - 1, f'(0) = \infty > 0, f'(1) = 1/b - 1 < 0 \Leftrightarrow b > 1, \\ f''(\beta_1) &= -2/(b\beta_1^3) < 0. \end{aligned}$$

Suppose first that  $b \leq 1$ . Then, since  $f'(\beta_1) \geq f'(1) \geq 0 \forall \beta_1 \in [0, 1]$ , we have  $f(\beta_1) \leq f(1) < 0 \forall \beta_1 \in [0, 1]$ , satisfying the sufficient condition for  $\partial \ln V_1 / \partial a_{01} < 0$ .

Consider next that  $b > 1$ . Then, solving the first-order condition  $f'(\beta_1) = 1/(b\beta_1^2) - 1 = 0$  gives  $\beta_1 = b^{-1/2} \equiv \hat{\beta}_1 \in (0, 1)$ , and the resulting maximum value is given by  $f(\hat{\beta}_1) = 1 - 2b^{-1/2}$ . If  $f(\hat{\beta}_1) \leq 0 \Leftrightarrow b \leq 4$ , then  $f(\beta_1) \leq f(\hat{\beta}_1) \leq 0 \forall \beta_1 \in [0, 1]$ , and hence  $\partial \ln V_1 / \partial a_{01} < 0$ .

## Appendix E. The optimal tariff in the static model

Consider the static version of our model, which is basically the same as DFS (1977), Opp (2010), and Costinot et al. (2015). Without the investment term  $\dot{K}_j + \delta_j K_j$  in the household budget constraint (1), country  $j$ 's consumption/capital ratio is simply expressed as  $c_j = \eta_j(t_j, \beta_j(t_j r_k / r_j)) / q_j(t_j r_k / r_j)$ , serving as its welfare measure. Since  $c_1, c_2$ , and  $\kappa$  do not change over time, only Eq. (16) applies in determining an equilibrium. Totally differentiating Eq. (16) with  $d\kappa = 0$ , and using Eq. (B.5), we obtain:

$$dr_1/r_1 = R_\kappa[-\eta_1(1 - \beta_1 + B_1)dt_1/t_1 + \eta_2(1 - \beta_2 + B_2)dt_2/t_2],$$

where  $R_\kappa(\in (-1, 0))$  is defined in Eq. (B.9). This implies that:

$$(dr_1/r_1)/(dt_1/t_1) = -R_\kappa \eta_1(1 - \beta_1 + B_1) = \eta_1(1 - \beta_1 + B_1)/(1 + \eta_1 B_1 + \eta_2 B_2).$$

We immediately know that  $(dr_1/r_1)/(dt_1/t_1) > 0$ . Moreover, since  $\eta_1(1 - \beta_1 + B_1) = t_1(1 - \beta_1)/[t_1(1 - \beta_1) + \beta_1] + \eta_1 B_1 < 1 + \eta_1 B_1 < 1 + \eta_1 B_1 + \eta_2 B_2$ , we have  $(dr_1/r_1)/(dt_1/t_1) < 1$ .

Using Eqs. (B.3) and (B.4), we obtain:

$$dc_j/c_j = d\eta_j/\eta_j - dq_j/q_j = \zeta_j dt_j/t_j - [\zeta_j(t_j - 1)B_j + \beta_j](dt_j/t_j + dr_k/r_k - dr_j/r_j).$$

This immediately implies that:

$$\partial \ln c_1 / \partial \ln t_1 = \zeta_1 - [\zeta_1(t_1 - 1)B_1 + \beta_1][1 - (dr_1/r_1)/(dt_1/t_1)]. \quad (\text{E.1})$$

Just like the first line of Eq. (22), the first term in the right-hand side of Eq. (E.1) shows the positive revenue effect, whereas the second term represents the distortionary effect, which is negative because  $(dr_1/r_1)/(dt_1/t_1) \in (0, 1)$ . Evaluating Eq. (E.1) at  $t_1 = 1$ , we have:

$$\partial \ln c_1 / \partial \ln t_1|_{t_1=1} = \beta_1(1 - \beta_1 + B_1)/(1 + B_1 + B_2) > 0. \quad (\text{E.2})$$

Therefore, the optimal tariff of a country is positive even in the static version of our model. Finally, equating Eq. (E.1) to zero, we obtain country 1's static optimal tariff  $t_1^c$ .

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	$t_1$	$r_1^*$	$\gamma_{C_2}^*$	$\beta_1^*$	$\beta_2^*$	$c_1^*$	$c_2^*$	$\kappa^*$	$r_1^* \kappa^*$	$U_1$	$U_2$
old BGP	1.02238	0.98737	0.0317344	0.343481	0.627911	0.020771	0.022060	1.85612	1.83268	146.808	118.896
opt tariff	1.13049	1.02223	0.0295736	0.308771	0.609617	0.023680	0.021957	2.07912	2.12533	147.961	113.258

(a)  $a_{01} = 0.71025, b_1 = 0.896509$

	$t_1$	$r_1^*$	$\gamma_{C_2}^*$	$\beta_1^*$	$\beta_2^*$	$c_1^*$	$c_2^*$	$\kappa^*$	$r_1^* \kappa^*$	$U_1$	$U_2$
old BGP	1.02238	0.99320	<u>0.0204015</u>	0.452043	0.519348	0.020904	0.021509	1.15285	<u>1.14501</u>	94.9820	89.2980
opt tariff	1.23915	1.08533	0.0165104	0.397425	0.472573	0.027187	0.021312	1.23226	1.33741	98.3949	79.1103

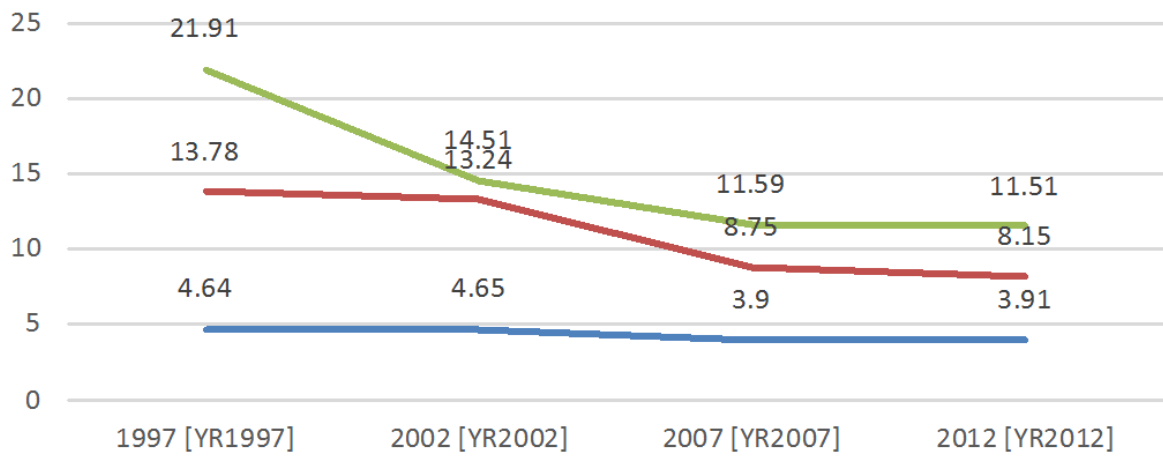
(b)  $a_{01} = 0.51025, b_1 = 0.896509$ (benchmark)

	$t_1$	$r_1^*$	$\gamma_{C_2}^*$	$\beta_1^*$	$\beta_2^*$	$c_1^*$	$c_2^*$	$\kappa^*$	$r_1^* \kappa^*$	$U_1$	$U_2$
old BGP	1.02238	0.99907	0.0121468	0.560606	0.410785	0.021021	0.021081	0.72667	0.72599	51.5487	67.6559
opt tariff	1.53750	1.27570	0.0054788	0.474346	0.281902	0.035005	0.020679	0.58380	0.74475	60.3775	50.0230

(c)  $a_{01} = 0.31025, b = 0.896509$

Table 1: Absolute advantage and the dynamic optimal tariff:  $a_{02} = 0.5, b_2 = 1, t_2 = 1.03265$

Fig. 1. Mean tariffs by income group, 1997-2012



Source: World Development Indicators

— High income HIC    — Middle income MIC    — Low income LIC

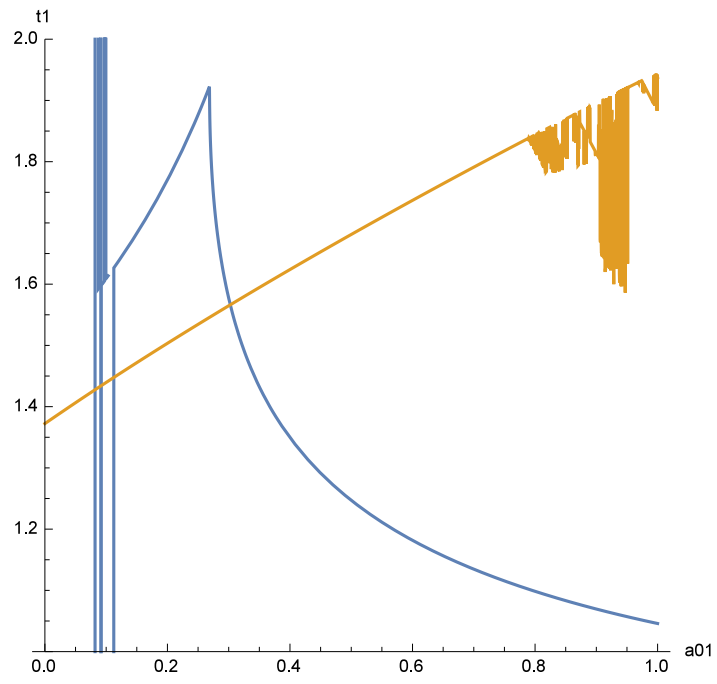


Fig. 2. Absolute advantage and optimal tariffs in the dynamic (blue) and static (yellow) models.

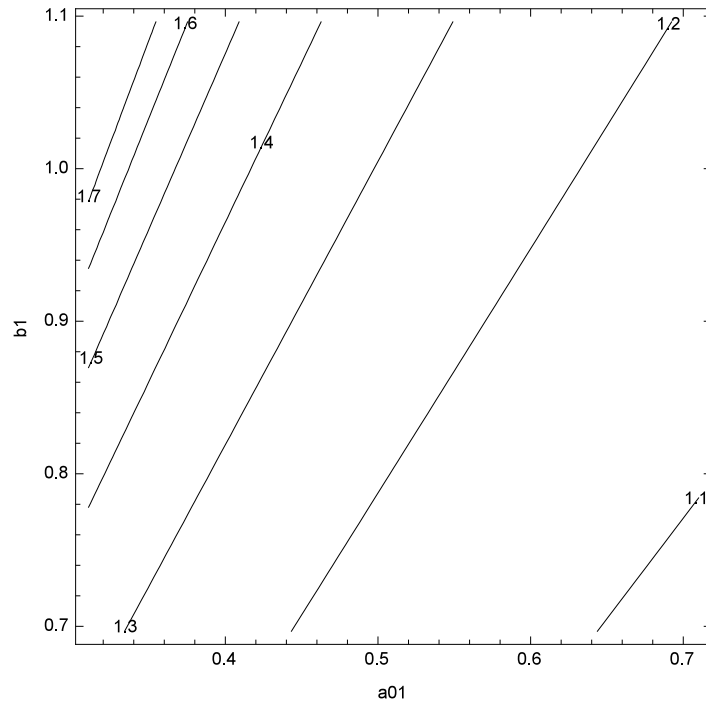


Fig. 3. Contours of optimal tariffs in the dynamic model.