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Sustainable Transmission Planning in Imperfectly Competitive Electricity Industries: Balancing economic efficiency and environmental outcomes

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Sustainable Transmission Planning in Imperfectly Competitive Electricity Industries: Balancing economic efficiency and environmental outcomes[1]

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Abstract

We explore the role of a transmission system operator (TSO) that builds a transmission line to accommodate renewable energy while attempting to lower emissions. A TSO in a deregulated electricity industry can only indirectly influence outcomes through its choice of the transmission line capacity. Via a bi-level model, we show that this results in less transmission capacity and with limited emissions control in a perfectly competitive industry *vis-a-vis* a benchmark centrally planned system. Surprisingly, a carbon tax on industry leads to a perfect alignment of incentives and maximized social welfare only under perfect competition. By contrast, a carbon tax actually lowers social welfare under a Cournot oligopoly as the resulting reduction in consumption facilitates the further exercise of market power.

Keywords: Transmission planning, Electricity market, Environmental policy, Market power, Bi-level modeling

JEL classification: Q48, Q53, Q58, L13, L94

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1 Introduction

The challenge of integrating renewable energy (RE) resources to mitigate climate change is exacerbated by the deregulated nature of the electric power industry. Over the past thirty years, power companies have evolved from being state-regulated and vertically integrated entities to facing lighter regulation while competing for market share (Wilson, 2002). In spite of this evolution, transmission-expansion decisions, which largely exhibit natural monopoly characteristics, have remained under the auspices of regulated planners, viz., transmission system operators (TSOs).

In order to assess the impact of transmission expansion within a market environment, several models have been posited in the OR literature. Focusing on uncertainty, Anderson et al. (2007) use a so-called market distribution function to assess the impact of an interconnection, while Fleten et al. (2011) take a real options approach to determine the optimal investment timing and sizing for a cable between Germany and Norway. A limitation of these papers is that they do not account for the behavior of market participants in response to the interconnection addition. By contrast, Borenstein et al. (2000) use a game-theoretic approach to demonstrate the potential for enhanced competition from even modest transmission expansion in a two-node Cournot oligopoly model. Likewise, Sauma and Oren (2009) examine the impact of financial transmission rights (FTRs) to align electricity producers' incentives with socially desirable ones. In a tri-level model with transmission planning, generation expansion, and market operations, Sauma and Oren (2006) illustrate the complexity faced by a TSO in obtaining politically feasible transmission-expansion plans. However, these papers are focused on economically efficient transmission expansion without consideration of environmental factors.

In the past decade, as environmental concerns have risen to the fore, the TSO's task has become more complex as it must anticipate RE penetration in an effort to provide the complementary transmission expansion. For example, it is legally binding for European Union (EU) member states to obtain 20% of their energy from RE resources by the year 2020 (European Commission, 2009), and California has an ambitious target for an 80%

reduction in greenhouse gas (GHG) emissions relative to 1990 levels by the year 2050 (Executive Order B-30-15). As a consequence of this dramatically shifted landscape, the TSO is called upon to coordinate transmission investment with anticipated RE adoption by power companies. In effect, transmission capacity, along with curbs in consumption and greater reliance on RE resources, is to serve as another tool for environmental control (Vara, 2016). Yet, Hobbs (2012) cautions against the use of proxy measures, e.g., those of the EU, because they may obfuscate economic tradeoffs. Carrying on in this spirit, bi-level models by Baringo and Conejo (2013) and Maurovich-Horvat et al. (2015) tackle transmission expansion with RE adoption in a market setting. The latter, in particular, demonstrates the somewhat unexpected results that may arise when power companies have market power: their withholding of capacity could actually incentivize TSOs to build transmission lines that connect RE-rich regions with load centers.

The environmental economics literature explicitly examines the efficiency of environmental policies in presence of pre-existing distortions, e.g., market power. For example, Barnett (1980) demonstrates that in the extreme case of a monopoly, a pollution tax may worsen social welfare. Intuitively, under imperfect competition, there are two potential distortions: underproduction and pollution. Since a pollution tax based on marginal externality damages, e.g., a Pigouvian tax, aims to curb consumption by internalizing externalities, it may cause imperfectly competitive firms to reduce their output further. Therefore, the distortion from underproduction could increase by more than the decrease in the externality. Requate (2005) further notes that it would be optimal not only to reduce the pollution tax rate below that of the marginal damage from externality but also to set a *negative* tax to incentivize more production.

We explore this interaction between economic efficiency and the cost of damage from emissions by explicitly considering the transmission line as an environmental instrument. Our intention is to develop a framework for assessing the welfare implications of using transmission capacity as a tool to regulate emissions. Toward that end, we address the problem of a welfare-maximizing TSO that internalizes the cost of damage from carbon emissions in a deregulated electricity industry. We use a bi-level approach in which the TSO at the upper level anticipates the possibly strategic behavior of power producers that make generation investment decisions at the lower level. Our analytical approach, which is rare in the bi-level modeling literature, obtains closed-form solutions to the transmission-expansion problem under three settings: central planning (CP), perfect competition (PC), and Cournot oligopoly (CO). We show that relative to the first-best outcome under CP, transmission capacity is curtailed under PC because it is impossible to pass on the cost of damage from emissions to industry in the absence of a carbon tax. As a recourse, the TSO uses the transmission line to curb thermal generation in favor of RE output. However, under PC, a carbon tax rate that is fully proportional to the marginal damage rate from emissions results in a first-best solution. By contrast, imposing such a carbon tax under the CO setting may actually worsen social welfare even though RE generation increases because the resulting curb on already choked consumption further deteriorates social welfare. Hence, we uncover how this finding from environmental economics carries over to a physical transmission network.

The rest of the paper is structured as follows. Section 2 lays out our modeling assumptions, while Section 3 solves the model under three market structures without a carbon tax. Next, Section 4 examines the impact of a carbon tax on market outcomes and social welfare. The main insights are illustrated via numerical examples in Section 5, while Section 6 summarizes our contributions and offers directions for future research. All proofs of propositions are in the Appendix.

2 Assumptions

We assume that there are two nodes, $j = N, S$, in a power system that are initially disconnected. At each node, there is one profit-maximizing generator with linear long-run cost functions $c_j(y_j) = C_j y_j$ such that $C_N > C_S$ (in \$/MW) with generator N (S) representing RE (thermal power). The inverse demand function at each node is linear in total consumption, i.e., $p_j(x_j) = A_j - B_j x_j$, where $A_S > A_N$ (in \$/MW) to reflect the fact that node S is a load center. There is a welfare-maximizing TSO at the upper level that builds a transmission line of capacity k (in MW) with levelized construction cost C_T (in \$/MW). Since only the TSO is regulated, it alone faces the cost of damage from emissions, $\frac{1}{2}Dy_S^2$, where $D \geq 0$ (in \$/MWh^2) is the damage cost parameter.

At the lower level, each generator makes its production decision, $y_j \geq 0$ (in MW), while an independent system operator (ISO) that is distinct from the TSO clears the markets in order to maximize the change in social welfare, i.e., transmits electricity from node S to node N ($t > 0$, in MW) or *vice versa* ($t < 0$) while taking the generators' production decisions as fixed (Sauma and Oren, 2006). Thus, consumption at nodes N and S (in MW) is $x_N = y_N + t$ and $x_S = y_S - t$, respectively. Additionally, we assume that $B_j > 0$ (in \$/MWh^2) and $A_S > A_N > C_N > C_T > C_S > 0$ to reflect that (i) node S is the load center ($A_S > A_N$), (ii) each node's demand may be locally satisfied ($A_j > C_j$), (iii) RE is more expensive than thermal power ($C_N > C_S$), and (iv) transmission capacity is not prohibitively expensive ($C_N > C_T > C_S$). All decisions are for a representative time period without uncertainty in demand and RE output. We explore the following market settings:

Central planning (CP) This benchmark setting has a central planner controlling all generation and transmission capacity to maximize social welfare considering damage from emissions. This results in a single-level quadratic program (QP).

Perfect Competition (PC) Generation decisions at the lower level are made by price-taking RE and thermal power sectors who take the transmission capacity and flows as given and maximize their profits. The ISO handles transmission flows in order to maximize the change in social welfare (Sauma and Oren, 2006). At the upper level, the TSO builds transmission capacity to maximize social welfare constrained by the lower-level mixed complementarity problem (MCP). This bi-level program may be re-cast as a mathematical program with equilibrium constraints (MPEC) if each lower-level problem is convex and may, thus, be replaced by its Karush-Kuhn-Tucker (KKT) conditions (Gabriel et al., 2012).

Cournot Oligopoly (CO) This is the same as PC except that each power sector behaves *à la* Cournot, i.e., it is able to raise the electricity price above the levelized cost, whereas the ISO remains as a price taker.

Since we will examine three market structures, we denote $\bar{\cdot}$, $\hat{\cdot}$, and $\cdot^*$ as the optimal values for decision variables in CP, PC, and CO settings, respectively.

3 Mathematical Model and Analytical Results

3.1 Central Planning

The central planner selects generation, transmission capacity, and net imports in order to maximize social welfare by solving the following QP problem:

$$\begin{aligned}
\max_{y_N \geq 0, y_S \geq 0, t, k \geq 0} \quad & \left[A_S\left(y_S - t\right) - \frac{1}{2}B_S\left(y_S - t\right)^2\right] + \left[A_N\left(y_N + t\right) - \frac{1}{2}B_N\left(y_N + t\right)^2\right] \\
& - \sum_j C_j y_j - C_T k - \frac{1}{2}D y_S^2 && (1) \\
\text{s.t.} \quad & (\lambda^-) - k \leq t \leq k\,(\lambda^+) && (2) \\
& y_N + t \geq 0\,(\beta_N) && (3) \\
& y_S - t \geq 0\,(\beta_S) && (4)
\end{aligned}$$

The objective function in (1) comprises consumer surplus, producer surplus, cost of transmission construction, and damage from emissions. Inequalities (2) constrain the net import at each node by the size of the transmission line. Equations (3) and (4) ensure that the total consumption at each node is non-negative. Finally, the term in the parentheses next to each constraint is the corresponding dual variable.

The KKT conditions for problem (1)–(4) are:

$$0 \le y_S \perp -[A_S - B_S(y_S - t)] + C_S - \beta_S + Dy_S \ge 0 \tag{5}$$

$$0 \le y_N \perp -[A_N - B_N(y_N + t)] + C_N - \beta_N \ge 0 \tag{6}$$

$$0 \le k \perp C_T - \lambda^+ - \lambda^- \ge 0 \tag{7}$$

$$-A_S + B_S(y_S - t) + A_N - B_N(y_N + t) - \lambda^+ + \lambda^- + \beta_N - \beta_S = 0 \text{ with } t \text{ u.r.s.} \tag{8}$$

$$0 \le \lambda^+ \perp k - t \ge 0 \tag{9}$$

$$0 \le \lambda^- \perp k + t \ge 0 \tag{10}$$

$$0 \le \beta_N \perp y_N + t \ge 0 \tag{11}$$

$$0 \le \beta_S \perp y_S - t \ge 0 \tag{12}$$

Without loss of generality, we assume that consumption at each node is strictly positive, i.e., $\beta_N = 0$ and $\beta_S = 0$. In order to solve the system in (5)–(12), we consider four cases that arise as the value of D is increased from zero with analytical results summarized in Table 1.[1]

CP Case 1 Electricity is generated only at node S and transmitted to node N.

CP Case 2 Electricity is generated at both nodes N and S with some power flow from S to N.

CP Case 3 The two nodes are separated with no power flow. From (9)–(10), we obtain $\bar{\lambda}^+(k) > 0$ and $\bar{\lambda}^-(k) > 0$. KKT condition (7) indicates that $C_T > \bar{\lambda}^+(k) + \bar{\lambda}^-(k)$.

CP Case 4 Electricity is generated at both nodes, but it flows from N to S.

Using the solutions[2] to the QP, we assess the impact of the damage cost parameter, D, on capacity decisions (Proposition 1) and the cost of damage from emissions (Proposition 2):

Proposition 1. *Under a solution for CP,* ceteris paribus *increases in D result in:*

[1] We can show that the case in which $k > 0$, $y_N > 0$, $y_S = 0$, and $t < 0$ does not occur. At the optimum, $-[A_S - B_S(y_S - t)] + C_S + Dy_S = C_S - C_N - C_T < 0$ holds with regard to (5), which is not feasible.

[2] We let $\bar{y}_j \equiv \bar{y}_j(\bar{k}), j = N, S$.

Case	$t(k)$	$\lambda^+(k)$	$\lambda^-(k)$	$y_S(k)$	$y_N(k)$	k
CP 1	k	C_T	0	$\frac{A_S-C_S+B_Sk}{B_S+D}$	0	$\frac{D(A_N-A_S-C_T)+B_S(A_N-C_S-C_T)}{(B_S+B_N)D+B_NB_S}$
CP 2	k	C_T	0	$\frac{A_S-C_S+B_Sk}{B_S+D}$	$\frac{A_N-C_N-B_Nk}{B_N}$	$\frac{(C_N-C_T)(B_S+D)-B_SC_S-A_SD}{B_SD}$
CP 3	0	>0	>0	$\frac{A_S-C_S}{B_S+D}$	$\frac{A_N-C_N}{B_N}$	0
CP 4	$-k$	0	C_T	$\frac{A_S-C_S-B_Sk}{B_S+D}$	$\frac{A_N-C_N+B_Nk}{B_N}$	$\frac{-(C_N+C_T)(B_S+D)+B_SC_S+A_SD}{B_SD}$
PC 1	k	$A_N-C_S-B_Nk$	0	$\frac{A_S-C_S+B_Sk}{B_S}$	0	$\frac{D(C_S-A_S)+B_S(A_N-C_S-C_T)}{B_S(D+B_N)}$
PC 2	k	C_N-C_S	0	$\frac{A_S-C_S+B_Sk}{B_S}$	$\frac{A_N-C_N-B_Nk}{B_N}$	$\frac{D(C_S-A_S)+B_S(C_N-C_S-C_T)}{B_SD}$
PC 3	0	>0	>0	$\frac{A_S-C_S}{B_S}$	$\frac{A_N-C_N}{B_N}$	0
CO 1	k	$\frac{2A_N-A_S-C_S-k(2B_N+B_S)}{2}$	0	$\frac{A_S-C_S+B_Sk}{2B_S}$	0	$\frac{4A_N-A_S-3C_S-4C_T-\frac{D}{B_S}(A_S-C_S)}{B_S+4B_N+D}$
CO 2	k	$\frac{A_N-A_S+C_N-C_S-k(B_N+B_S)}{2}$	0	$\frac{A_S-C_S+B_Sk}{2B_S}$	$\frac{A_N-C_N-B_Nk}{2B_N}$	$\frac{A_N+3C_N-A_S-3C_S-4C_T-\frac{D}{B_S}(A_S-C_S)}{B_S+B_N+D}$
CO 3	0	>0	>0	$\frac{A_S-C_S}{2B_S}$	$\frac{A_N-C_N}{2B_N}$	0
CO 4	$-k$	0	$\frac{C_S-C_N+A_S-A_N-k(B_N+B_S)}{2}$	$\frac{A_S-C_S-B_Sk}{2B_S}$	$\frac{A_N-C_N+B_Nk}{2B_N}$	$\frac{A_S+3C_S-A_N-3C_N-4C_T+\frac{D}{B_S}(A_S-C_S)}{B_S+B_N+D}$

Table 1: Summary of analytical results

Case 1: $\frac{\partial \bar{k}}{\partial D} < 0$, $\frac{\partial \bar{y}_S}{\partial D} < 0$.

Case 2: $\frac{\partial \bar{k}}{\partial D} < 0$, $\frac{\partial \bar{y}_S}{\partial D} < 0$, $\frac{\partial \bar{y}_N}{\partial D} > 0$.

Case 3: $\frac{\partial \bar{y}_S}{\partial D} < 0$, $\frac{\partial \bar{y}_N}{\partial D} = 0$.

Case 4: $\frac{\partial \bar{k}}{\partial D} > 0$, $\frac{\partial \bar{y}_S}{\partial D} < 0$, $\frac{\partial \bar{y}_N}{\partial D} > 0$.

Proposition 2. *Under a solution for CP,* ceteris paribus *increases in D result in decreases in the damage cost from emissions in Cases 2 and 4.*

Intuitively, for low D, all of the demand is satisfied via the cheapest generation source at node S (Case 1). As D increases, however, the levelized social cost of thermal generation increases, and it is optimal to internalize the cost of damage from emissions by curbing consumption and, consequently, both production and transmission capacity, which is akin to a Pigouvian tax. Further increases in D lead the levelized costs of RE and thermal generation to be equated once the cost of damage from emissions is internalized (Case 2). Any marginal increase in D within Case 2 will lead to subsequent reduction in thermal generation and transmission capacity but not in consumption at node N as RE generation will increase to compensate for the shortfall in net imports, thereby maintaining the price at node N at the long-run marginal cost of RE. For a high enough value of D, it may even be optimal not to build the line at all (Case 3) or to reverse the direction of prevalent transmission flows (Case 4). In the latter case, RE generation from node N is sent to node S, which becomes a net importer. A marginal increase in D leads to further displacement of thermal generation by RE with a supporting increase in the transmission capacity. Finally, Proposition 2 indicates that the cost of damage from emissions decreases in Cases 2 and 4 only as the fraction of RE in the two-node system increases. By contrast, the results for Cases 1 and 3 are ambiguous, depending on the relation among B_N, B_S, and D.

3.2 Perfect Competition

In a deregulated setting, each generator maximizes its own profit at the lower level by selecting the level of output while treating the transmission capacity set by the TSO at the upper level as a fixed quantity (13)–(14). Each producer acts as a price taker under perfect competition. Also, the ISO at the lower level manages net imports in order to maximize the change in social welfare assuming that y_j and k are fixed (15).

$$\max_{y_S \geq 0} \quad p_S y_S - C_S y_S \tag{13}$$

$$\max_{y_N \geq 0} \quad p_N y_N - C_N y_N \tag{14}$$

$$\begin{aligned} \max_t \quad & [A_N - B_N y_N] t - [A_S - B_S y_S] t - \frac{1}{2}(B_N + B_S) t^2 \\ \text{s.t.} \quad & \text{Equations (2)–(4)} \end{aligned} \tag{15}$$

Since the lower-level problems are all convex, they may be replaced by their KKT conditions, which means that equilibrium problems (13)–(15) become the following MCP:

$$0 \leq y_S \perp -[A_S - B_S(y_S - t)] + C_S \geq 0 \tag{16}$$

$$0 \leq y_N \perp -[A_N - B_N(y_N + t)] + C_N \geq 0 \tag{17}$$

$$-A_N + B_N y_N + A_S - B_S y_S + B_N t + B_S t + \lambda^+ - \lambda^- - \beta_N + \beta_S = 0 \text{ with } t \text{ u.r.s.} \tag{18}$$

$$\text{Equations (9)–(12)}$$

Thus, the TSO's bi-level problem of choosing the optimal transmission capacity, k, in order to maximize social welfare while constrained by lower-level problems may be re-cast as the following MPEC:

$$\begin{aligned} \max_{\{k \geq 0\} \cup \Omega \cup \Xi} \quad & \text{Equation (1)} \\ \text{s.t.} \quad & \text{Equations (9)–(12), (16)–(17), (18)} \end{aligned} \tag{19}$$

where $\Omega \equiv \left\{\hat{y}_N(k) \geq 0, \hat{y}_S(k) \geq 0, \hat{t}(k)\right\}$ and $\Xi \equiv \left\{\hat{\lambda}^+(k) \geq 0, \hat{\lambda}^-(k) \geq 0, \hat{\beta}_N(k) \geq 0, \hat{\beta}_S(k) \geq 0\right\}$. We assume that $\hat{\beta}_N(k) = 0$ and $\hat{\beta}_S(k) = 0$ for the rest of analysis. Substitution of the solutions to the lower-level MCP allows the replacement of sets Ω and Ξ, thereby leading to the following unconstrained non-linear programming (NLP) problem:

$$\begin{aligned} \max_{k \geq 0} \quad & A_S\left(\hat{y}_S(k) - \hat{t}(k)\right) - \frac{1}{2}B_S\left(\hat{y}_S(k) - \hat{t}(k)\right)^2 - C_S\hat{y}_S(k) + A_N\left(\hat{y}_N(k) + \hat{t}(k)\right) \\ & -\frac{1}{2}B_N\left(\hat{y}_N(k) + \hat{t}(k)\right)^2 - C_N\hat{y}_N(k) - C_T k - \frac{1}{2}D\hat{y}_S(k)^2 \end{aligned} \tag{20}$$

As in Section 3.1, we have multiple solution cases[3] depending on the value of D:

PC Case 1 The lower-level solutions can now be substituted into NLP (20) to yield an unconstrained QP problem parameterized on k. Using the fact that $\hat{y}_S'(k) = 1$, the first-order necessary condition (FONC) for the QP yields:

$$-C_S - C_T - D\hat{y}_S(k) + A_N - B_N k = 0 \tag{21}$$

Since (20) is a QP, the resulting solution is a global optimum and, for $D = 0$, the PC solution coincides with that of the central planner.

PC Case 2 Substitution of lower-level solutions into (20) again results in a QP. Using the facts that $\hat{y}_S'(k) = 1$ and $\hat{y}_N'(k) = -1$, the FONC to (20) yields:

$$-C_S + C_N - C_T - D\hat{y}_S(k) = 0 \tag{22}$$

PC Case 3 The nodes are disconnected with $\hat{\lambda}^+(k) > 0$ and $\hat{\lambda}^-(k) > 0$ from (9)–(10).

[3] We can show that a possible Case 4 in which $\hat{k} > 0$, $\hat{y}_N > 0$, $\hat{y}_S > 0$, and $\hat{t} < 0$ is not feasible because $\hat{\lambda}^- = C_S - C_N < 0$ holds from (18), which violates $\hat{\lambda}^- > 0$.

Based on the solutions for the three cases, we can now perform comparative statics with respect to D analogous to those in Propositions 1 and 2:

Proposition 3. *Under a solution for PC,* ceteris paribus *increases in D result in:*

Case 1: $\frac{\partial \hat{k}}{\partial D} < 0$, $\frac{\partial \hat{y}_S}{\partial D} < 0$.

Case 2: $\frac{\partial \hat{k}}{\partial D} < 0$, $\frac{\partial \hat{y}_S}{\partial D} < 0$, $\frac{\partial \hat{y}_N}{\partial D} > 0$.

Case 3: $\frac{\partial \hat{y}_S}{\partial D} = 0$, $\frac{\partial \hat{y}_N}{\partial D} = 0$.

Proposition 4. *Under a solution for PC,* ceteris paribus *increases in D result in a decrease and an increase in the damage cost from emissions in Cases 2 and 3, respectively.*

Except for Case 3 with $\frac{\partial \hat{y}_S}{\partial D} = 0$, the qualitative findings in Proposition 3 reflect those in Proposition 1 even though the optimal solutions are different. The reason for this difference is that the damage cost is not borne by producers, which means that a marginal increase in D does not impact the producers' output. Unlike the CP setting, direct control over consumption is not possible under PC. Therefore, for Case 3, an increase in the damage cost parameter, D, leads to an increase in the damage cost (Proposition 4). However, the result for Case 1 is ambiguous, depending on the relation among B_N, B_S, and D.

3.3 Cournot Oligopoly

When generators behave as Cournot oligopolists at the lower level, they are able to raise prices above long-run marginal costs. At the lower level, we have an MCP with each generator's KKT conditions for profit maximization (16)–(17) and the ISO's welfare-maximizing net imports, viz., (9)–(12) and (18). Thus, only (16) and (17) are adjusted as follows:

$$\begin{aligned} \max_{y_S \geq 0} \quad & [A_S - B_S(y_S - t)]\, y_S - C_S y_S \\ \Rightarrow \qquad & 0 \leq y_S \perp -[A_S - B_S(2y_S - t)] + C_S \geq 0 \end{aligned} \tag{23}$$

$$\begin{aligned} &\max_{y_N \geq 0} \quad [A_N - B_N(y_N + t)]\, y_N - C_N y_N \\ &\Rightarrow \quad 0 \leq y_N \perp -[A_N - B_N(2y_N + t)] + C_N \geq 0 \end{aligned} \tag{24}$$

As before, the lower-level problems may be replaced by their KKT conditions. Thus, the TSO's bi-level transmission-investment problem may again be rendered as an MPEC:

$$\begin{aligned} &\max_{\{k \geq 0\} \cup \Omega \cup \Xi} \quad \text{Equation (1)} \\ &\text{s.t.} \quad \text{Equations (9)–(12), (18), (23)–(24)} \end{aligned} \tag{25}$$

where $\Omega \equiv \left\{ y_N^*(k) \geq 0, y_S^*(k) \geq 0, t^*(k) \right\}$ and $\Xi \equiv \left\{ \lambda^{+,*}(k) \geq 0, \lambda^{-,*}(k) \geq 0, \beta_N^*(k) \geq 0, \beta_S^*(k) \geq 0 \right\}$. Again, assuming $\beta_N^*(k) = 0$ and $\beta_S^*(k) = 0$, substitution of the solutions to the lower-level MCP allows the replacement of sets Ω and Ξ, thereby leading to the following unconstrained NLP problem:

$$\begin{aligned} \max_{k \geq 0} \quad & A_S\left(y_S^*(k) - t^*(k)\right) - \frac{1}{2} B_S \left(y_S^*(k) - t^*(k)\right)^2 - C_S y_S^*(k) + A_N\left(y_N^*(k) + t^*(k)\right) \\ & - \frac{1}{2} B_N \left(y_N^*(k) + t^*(k)\right)^2 - C_N y_N^*(k) - C_T k - \frac{1}{2} D y_S^*(k)^2 \end{aligned} \tag{26}$$

Variation of the D parameter leads to four solution cases[4] as follows:

CO Case 1 Substitution of lower-level solutions and the fact that $y_S^{*\prime}(k) = \frac{1}{2}$ into (26) yields the FONC for the QP:

$$-\frac{1}{2} A_S + \frac{1}{2} B_S \left(y_S^*(k) - k\right) - \frac{1}{2} C_S - C_T - \frac{1}{2} D y_S^*(k) + A_N - B_N k = 0 \tag{27}$$

CO Case 2 Again, substitution of lower-level solutions into problem (26) leads to a QP. Using the facts that $y_S^{*\prime}(k) = \frac{1}{2}$ and $y_N^{*\prime}(k) = -\frac{1}{2}$, the FONC for the resulting QP yields:

$$-\frac{A_S}{2} + \frac{B_S\left(y_S^*(k) - k\right)}{2} + \frac{A_N}{2} - \frac{B_N\left(y_N^*(k) + k\right)}{2} - \frac{C_S}{2} + \frac{C_N}{2} - C_T - \frac{D y_S^*(k)}{2} = 0 \tag{28}$$

[4]We can show that a potential Case 5 in which $k > 0$, $y_N > 0$, $y_S = 0$, and $t < 0$ does not occur. At the optimum, $-[A_S - B_S(2y_S - t)] + C_S = \frac{B_N(C_S - A_S) + B_S[(C_S - A_N) + 3(C_S - C_N) - 4C_T]}{4B_S + B_N} < 0$ holds with regard to (23). Hence, this case is not feasible.

CO Case 3 Once the nodes are disconnected, $\lambda^{+,*}(k) > 0$ and $\lambda^{-,*}(k) > 0$ from (9)–(10).

CO Case 4 Substitution of lower-level solutions into problem (26) and using the facts that $y_S^{*\prime}(k) = -\frac{1}{2}$ and $y_N^{*\prime}(k) = \frac{1}{2}$, the FONC to the resulting QP yields:

$$\frac{A_S}{2} - \frac{B_S\left(y_S^*(k)+k\right)}{2} - \frac{A_N}{2} + \frac{B_N\left(y_N^*(k)-k\right)}{2} + \frac{C_S}{2} - \frac{C_N}{2} - C_T + \frac{Dy_S^*(k)}{2} = 0 \quad (29)$$

Analogous to Propositions 3 and 4, we perform comparative statics with respect to D:

Proposition 5. *Under a solution for CO,* ceteris paribus *increases in D result in:*

Case 1: $\frac{\partial k^*}{\partial D} < 0$, $\frac{\partial y_S^*}{\partial D} < 0$.

Case 2: $\frac{\partial k^*}{\partial D} < 0$, $\frac{\partial y_S^*}{\partial D} < 0$, $\frac{\partial y_N^*}{\partial D} > 0$.

Case 3: $\frac{\partial y_S^*}{\partial D} = 0$, $\frac{\partial y_N^*}{\partial D} = 0$.

Case 4: $\frac{\partial k^*}{\partial D} > 0$, $\frac{\partial y_S^*}{\partial D} < 0$, $\frac{\partial y_N^*}{\partial D} > 0$.

Proposition 6. *Under a solution for CO,* ceteris paribus *increases in D result in an increase in the damage cost from emissions in Case 3.*

Although the solutions are different from the CP and PC cases, the comparative statics mirror the findings in the previous settings. Here, results for Cases 1, 2, and 4 are ambiguous, depending on the relation among B_N, B_S, and D. Finally, the damage cost parameter unambiguously decreases the maximized social welfare for all solutions:

Proposition 7. *Under solutions for CP, PC, and CO,* ceteris paribus *increases in D result in decreases in social welfare.*

4 Impact of a Carbon Tax

We now examine the effect of a carbon tax introduced in a deregulated industry, i.e., in the lower-level problem. Suppose that under the carbon tax system, thermal generator S incurs the full or partial cost of damage from emissions, $\frac{1}{2}EDy_S^2$, where $0 < E \leq 1$ is the carbon tax rate. The tax burden may be partial due to political reasons. If $E = 1$ ($E = 0$), then the thermal generator incurs the entire damage cost (faces no environmental regulation as in the previous sections).

4.1 Perfect Competition with Carbon Tax

At the lower level, the MCP that consists of price-taking generator S's KKT conditions for profit maximization is re-written as follows:

$$\begin{aligned} & \max_{y_S \geq 0} \quad p_S y_S - C_S y_S - \frac{1}{2} E D y_S^2 \\ \Rightarrow \quad & 0 \leq y_S \perp -[A_S - B_S(y_S - t)] + C_S + E D y_S \geq 0 \end{aligned} \tag{30}$$

As before, the lower-level problems may be replaced by their KKT conditions. Thus, the TSO's problem is again an MPEC to select the transmission capacity, k, in order to maximize social welfare:

$$\begin{aligned} \max_{\{k \geq 0\} \cup \Omega \cup \Xi} \quad & \text{Equation (1)} \\ \text{s.t.} \quad & \text{Equations (9)–(12), (17), (18), (30)} \end{aligned} \tag{31}$$

We can show that this problem can be solved as in Section 3.2 and yields the same first-best results as under CP if the thermal generator bears full cost of damage from emissions.[5]

Proposition 8. *If a full carbon tax, i.e., $E = 1$, is imposed under PC, then the TSO's problem is equivalent to that under CP.*

Therefore, a regulatory authority could use a full carbon tax to obtain the same outcome as in CP, i.e., make the thermal generator as concerned about emissions damage

[5] Other environmental policy instruments include tradable emission permits. If both the permit and output markets are perfectly competitive, a tradable permit system would yield the optimal allocation of resources as under a full carbon tax by internalizing externalities. Another policy instrument is a feed-in tariff (FIT), which encourages deployment of renewable energy by creating a long-term financial incentive. The FIT scheme decreases the long-run marginal cost of the renewable generator, thereby making the thermal generator disadvantageous. Thus, FITs would yield outcomes similar to those under a carbon tax to some extent. However, note that the FIT scheme would introduce an economic distortion of resource allocation.

as a social planner. However, a full carbon tax is usually difficult to implement due to political reasons. With a partial carbon tax under PC, i.e., $0 < E < 1$, the outcome deviates from that of CP and approaches the result of PC without any environmental regulation in Section 3.2. In a similar manner as in Section 3.2, it is easy to show that Cases 1 to 3 can occur, although we do not repeat the analysis here. We only note that Case 4 in which $\hat{k} > 0$, $\hat{y}_N > 0$, $\hat{y}_S > 0$, and $\hat{t} < 0$ can occur for $E > 0$. This can be verified by deriving $\hat{\lambda}^-(k) = \frac{E^2 DC_T + B_S[(1-E)(C_S - C_N) + EC_T]}{B_S + E^2 D}$. Although $C_S - C_N$ is negative, $\hat{\lambda}^-(k)$ can be positive if E is sufficiently large, thereby leading to Case 4.

The subsequent analysis shows the impact of increasing the tax burden, i.e., raising E, on the transmission capacity. We focus on cases in which the optimal transmission capacity is positive.

Proposition 9. *Under a solution for PC with $E \in (0,1)$, a more stringent carbon tax leads to an increase (decrease) in the transmission capacity, i.e., $\frac{\partial \hat{k}}{\partial E} > 0$ ($\frac{\partial \hat{k}}{\partial E} < 0$), for Cases 1 and 2 (Case 4).*

Proposition 9 suggests that the optimal transmission line under PC is smaller than (larger than) that under CP for Cases 1 and 2 (Case 4). Intuitively, consumption may be restricted under CP as D increases in Cases 1 and 2 by internalizing the cost of damage from emissions, thereby increasing prices. As a consequence, there is less need for transmission capacity to take power from node S to node N. However, such recourse is unavailable to a TSO operating in a decentralized electricity industry. Instead, it responds optimally to higher D in Cases 1 and 2 by directly reducing the transmission capacity to below the optimal level under CP. With the carbon tax, total consumption at node N is not affected because the RE production cost effectively acts as a ceiling on the price. Thus, $\hat{y}_N + \hat{t}$ is unaffected, but increasing the carbon tax reduces $\hat{y}_N$ as more electricity is imported from node S. Conversely, at node S, increasing the carbon tax increases the marginal cost of production at node S, which causes consumption, $\hat{y}_S - \hat{t}$, to decrease. In effect, a higher tax frees up thermal capacity to supply more of the demand at node N, which also increases $\hat{k}$. Meanwhile, in Case 4, the prevalent power flow has reversed as an exorbitant cost of damage from emissions has been imposed on the thermal producer at node S. While a partial carbon tax curbs thermal output at node S, it restricts neither it nor the consumption at node S to the socially optimal level as under CP. This means that a larger transmission line is needed with a partial carbon tax under PC in comparison to CP in order to meet the excess quantity demanded at node S via RE generation from node N. Hence, an increase in the carbon tax corrects fully for this externality, thereby causing the optimal transmission capacity to decrease to the CP level.

4.2 Cournot Oligopoly with Carbon Tax

At the lower level, Cournot generator S's KKT condition for profit maximization is re-written as follows:

$$\begin{aligned} \max_{y_S \geq 0} \quad & [A_S - B_S(y_S - t)]\,y_S - C_S y_S - \frac{1}{2} E D y_S^2 \\ \Rightarrow \quad & 0 \leq y_S \perp -[A_S - B_S(2y_S - t)] + C_S + EDy_S \geq 0 \end{aligned} \tag{32}$$

The TSO's problem is again an MPEC to choose the transmission capacity, k, in order to maximize social welfare:

$$\begin{aligned} \max_{\{k \geq 0\} \cup \Omega \cup \Xi} \quad & \text{Equation (1)} \\ \text{s.t.} \quad & \text{Equations (9)–(12), (18), (24), (32)} \end{aligned} \tag{33}$$

In a similar way as in Section 3.3, we can show that Cases 1 to 4 can occur, although the analysis is not repeated here. We instead focus on the impact of the tax burden on the transmission capacity. We again deal with the cases in which the transmission capacity is positive at the optimum.

Proposition 10. *Under a solution for CO with $E \in (0,1)$, if $E > 1 - \frac{B_S}{D}$, then a more stringent carbon tax leads to a decrease (increase) in the transmission capacity, i.e., $\frac{\partial k^*}{\partial E} < 0$ ($\frac{\partial k^*}{\partial E} > 0$), for Cases 1 and 2 (Case 4), and* vice versa.

In order to understand the intuition for Proposition 10, first note that $E > 1 - \frac{B_S}{D}$

is equivalent to the condition $B_S y_S^* > (1-E)Dy_S^*$. The left-hand (right-hand) side of the inequality refers to the impact on the social welfare from a marginal increase in production at node S in terms of the reduced potential for market power (enhanced marginal cost of damage from emissions). Now, if E were infinitesimally increased, then it would *reduce* production at node S, thereby leading to a marginal reduction in the cost of damage from emissions and a marginal increase in the potential for the exertion of market power. When $B_S y_S^* > (1-E)Dy_S^*$, the latter more than offsets the former, i.e., the gains from lower emissions as a result of a higher carbon tax are outweighed by the loss in welfare from the more potent effects of market power. Since maximized social welfare is bound to decrease from a higher carbon tax, it must be the case that $E > 1 - \frac{B_S}{D}$ corresponds to a situation in which the carbon tax is higher than the optimal level.

When power is transmitted from node S to N as in Cases 1 and 2, an infinitesimal increase in the carbon tax is undesirable as it will only lower maximized social welfare by curbing production at node S. Since node S was already supplying consumers at node N, a further reduction in thermal generation will be to the detriment of consumers at node S. As a countervailing measure, the TSO will optimally reduce the size of the transmission line in order to limit the damage to consumer surplus (and maximized social welfare overall). Indeed, a smaller transmission capacity will restrict the power flow from node S to N. By contrast, in Case 4, power is optimally transmitted from node N to S since the cost of damage from emissions is exceptionally high. Nevertheless, the carbon tax is already too high, and any further increases to it will lead to a further decrease in consumption at node S, which will lower social welfare as the loss in consumer surplus will more than offset the lower cost of damage from emissions. Again, as a countervailing measure, the TSO increases the capacity of the transmission line to bring more RE from node N to S.

4.3 Implications for Social Welfare

In the environmental economics literature, it is known that the imposition of a Pigouvian tax may actually worsen social welfare in the presence of a pre-existing distortion, e.g., imperfect competition (Barnett, 1980; Requate, 2005). Since a producer with market power withholds production in order to raise its own profit, an emissions tax may actually be to the detriment of society even though it mitigates the externality. Thus, the literature concludes that the optimal emission tax rate should be lower than the marginal damage when market power exists, i.e., $E < 1$ in our setting. In fact, even a negative tax rate may be desirable when the alleviation from the withholding distortion outweighs the negative externality of emissions. However, this work typically does not consider network effects associated with the transmission line as we do here. Indeed, in the absence of direct control over production or taxation decisions, the TSO's only recourse is to vary the size of the line in balancing economic and environmental objectives.

We now formalize how the TSO uses the transmission line as an alternative environmental control mechanism demonstrating the impact of the tax rate, E, on social welfare. Note that since E is a parameter and outside the control of the TSO, its only role is to determine the size of the transmission line. We let $s\hat{w}(E)$ and $sw^*(E)$ denote the maximized social welfare under PC and CO, respectively, for a given level of carbon tax.

Proposition 11. *Under PC with a carbon tax of $E \in (0,1)$, a more stringent carbon tax leads to an increase in maximized social welfare, i.e., $\frac{\partial s\hat{w}}{\partial E} > 0$.*

Proposition 12. *Under CO with a carbon tax with $E \in (0,1)$, if $E > 1 - \frac{B_S}{D}$, then a more stringent carbon tax leads to a decrease in maximized social welfare, i.e., $\frac{\partial sw^*}{\partial E} < 0$. On the contrary, if $E < 1 - \frac{B_S}{D}$, then a more stringent carbon tax leads to an increase in maximized social welfare, i.e., $\frac{\partial sw^*}{\partial E} > 0$.*

The intuition for Proposition 11 is related to the findings of Propositions 8 and 9. Under PC without a carbon tax, there is over-generation from the renewable plant at node N as the cost of damage from emissions is not internalized. Consequently, high consumption at node S prevents the export of relatively cheap energy to node N. For this reason, the optimal transmission line under PC without a carbon tax in Cases 1 and 2 is smaller than the socially optimal one under CP. Once the carbon tax is imposed and starts to increase from zero, consumers at node S receive the price signal to reduce quantity demanded, which liberates relatively cheap thermal generation at node S for export to node N with a concomitant increase (decrease) in transmission capacity (RE generation). In Case 4, a partial carbon tax does not curb consumption at node S sufficiently even as thermal generation is reduced. Thus, the shortfall between consumption and local generation needs to be made up via imports from RE generation at node N, which necessitates a larger transmission line than is socially optimal. Again, as the carbon tax increases, this distortion is corrected: consumption and thermal generation at node S are curbed along with both renewable generation at node N and transmission capacity. Hence, as the carbon tax rate approaches unity, the PC solution approaches the socially optimal one under CP.

If industry is imperfectly competitive, however, then simply setting $E = 1$ will not suffice as indicated in Proposition 12. While this finding reflects results established in the environmental economics literature, it is, nevertheless, novel in that the transmission capacity also functions as an instrument for regulating emissions at node S, reducing consumption at node S, and increasing RE output at node N. The intuition here is related to the discussion of Proposition 10 as there are two consequences of increasing the carbon tax rate: (i) an enhanced potential for the thermal producer at node S to exert market power and (ii) a mitigation of the cost of damage from emissions. Since these two effects have opposing impacts on social welfare, the TSO is in a quandary. As a result, unlike in the PC case, simply setting the carbon tax rate to unity to cover full damage may actually *worsen* maximized social welfare if effect (i) more than offsets (ii), which is precisely the situation captured by $E > 1 - \frac{B_S}{D}$. Therefore, although the TSO optimally adjusts transmission capacity in response to a marginal increase in an already high carbon tax rate, e.g., reducing (increasing) capacity in Cases 1 and 2 (Case 4) to limit the extent of the consumer surplus loss at node S, maximized social welfare decreases. By contrast, if the carbon tax rate is relatively low, i.e., the marginal impact of the cost of damage from emissions outweighs the potential welfare loss from an enhanced capability to exercise market power, then a higher carbon tax will improve social welfare.

We, therefore, summarize this section with the following results:

Proposition 13. *Under PC, a full carbon tax rate, $E = 1$, is optimal.*

Proposition 14. *Under CO, the optimal carbon tax rate is strictly less than one. If $B_S < D$ ($B_S \geq D$), then the optimal tax rate is $E = 1 - \frac{B_S}{D}$ ($E = 0$).*

5 Numerical Examples

In this section, we use numerical examples to illustrate the key analytical findings. We use the parameter values $C_S = 20$, $C_T = 25$, $C_N = 80$, $A_S = 400$, $A_N = 200$, $B_S = 1$, $B_N = 1$, and $D \in [0, 0.5]$. Furthermore, we also investigate the impact of a carbon tax by allowing for $E \in [0, 1]$.

5.1 Results without a Carbon Tax

For a relatively low damage cost and no carbon tax, Figure 1 demonstrates that the CP and PC solutions are similar: power is produced only at node S and transmitted to node N, which corresponds to Case 1. However, apart from $D = 0$, the two solutions are not identical. Indeed, Figure 2 demonstrates that because it internalizes the cost of damage from emissions, CP leads to a higher power price at node S than does PC. Meanwhile, the price at node N under CP is exactly $\bar{p}_S + C_T$, i.e., \$25/MW higher than the price at node S (Equation (8)). In effect, CP curbs consumption at node S such that the marginal social cost equals the marginal benefit from consumption, i.e., $\bar{p}_S = C_S + D\bar{y}_S$ (Equation (5)), thereby freeing up resources to meet demand at node N. As D increases, both consumption and thermal generation at node S are curbed further along with the capacity of the transmission line. By contrast, the PC setting is unable to internalize the cost of damage from emissions without a carbon tax, i.e., $\hat{p}_S = C_S$ (Equation (16)), and leads to "too much" consumption at node S (Siddiqui et al., 2016). Therefore, the TSO's only recourse under PC is to use the transmission line as an instrument for regulating emissions by investing in less capacity. While such a response is clearly sub-optimal from a welfare perspective as it leads to a higher cost of damage from emissions (Figure 3a), it may be thought of as a second-best solution.

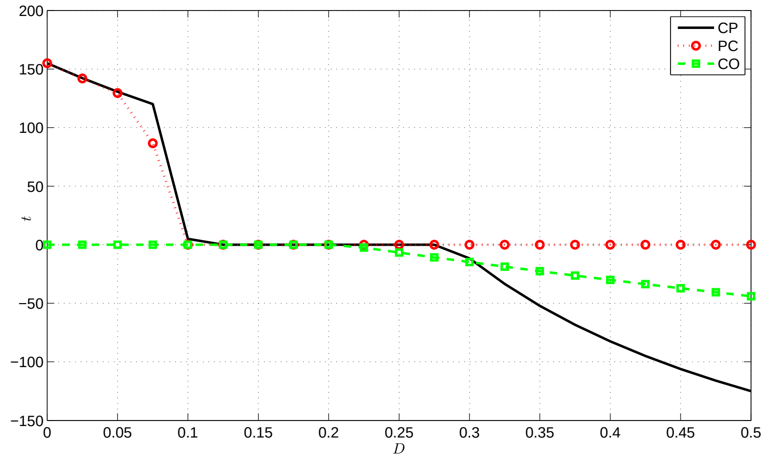


Figure 1: Optimal net import at node N with respect to D

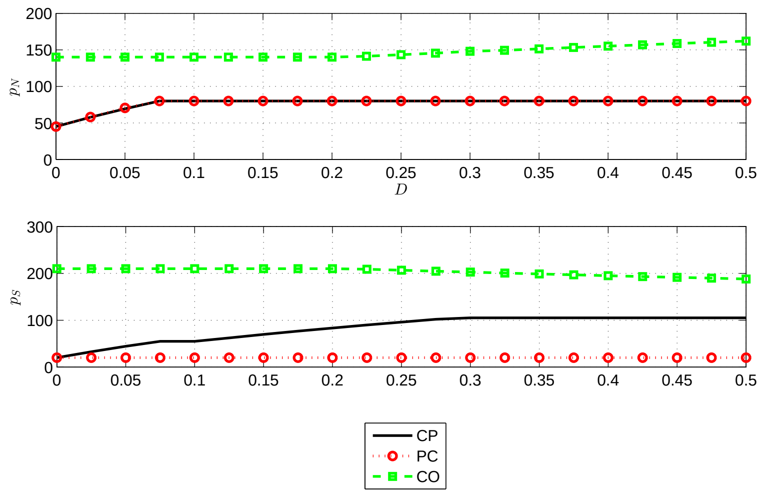
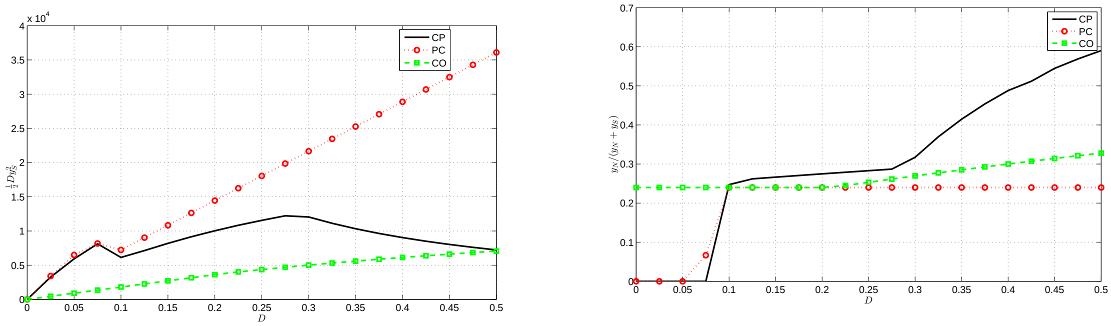


Figure 2: Equilibrium nodal prices with respect to D

The contrast between the CP and PC settings becomes starker as D increases to 0.075. Here, CP continues to rely on thermal generation at node S (albeit in steadily decreasing quantities), whereas PC sees production from the RE generator at node N (Case 2). The rationale for the switch in the PC setting is that the cost of damage from emissions is high enough to induce the TSO to decrease drastically the capacity of the transmission line. This intervention creates scarcity at node N as the economic cost of producing power at node S and transmitting it to node N, i.e., $\hat{p}_S + \hat{\lambda}^+$ (Equation



(a) Equilibrium damage cost with respect to D

(b) Fraction of renewable energy with respect to D

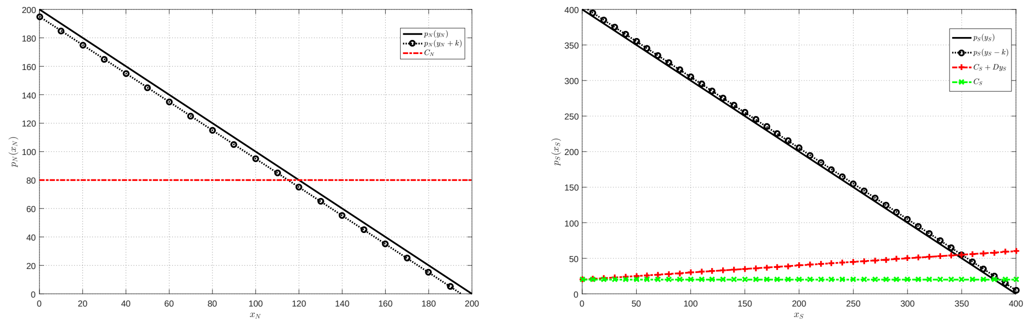
Figure 3: Environmental and RE output outcomes without a carbon tax

(18)), increases to exactly C_N and enables RE generation to enter the market in response. Since the PC setting could not free up thermal generation to supply node N by curbing consumption at node S, it has to rely on RE generation. Under CP, such parity between the two costs does not occur until $D = 0.10$ when $\bar{p}_S + C_T$ is exactly \$80/MW. At this point, the CP setting produces nearly a quarter of the total power from RE sources at node N and builds only a 5 MW line (Figure 3b). Indeed, only when the marginal social cost of producing power from the thermal generator is high enough is it optimal to start relying on RE generation.

Figures 4 and 5 illustrate the producers' incentives at each node under CP and PC, respectively. Figure 4b demonstrates that the optimal thermal generation at node S is indicated by the intersection between the marginal social cost of generation, $C_S + Dy_S$, and the local inverse demand curve shifted by the amount of electricity transmitted to node N, $p_S(y_S - k)$. Thus, both consumption at node S and thermal generation are curbed in line with the cost of damage from emissions, which leads to an equilibrium price at node S in excess of the marginal production cost. A *ceteris paribus* increase in D will further curb thermal production and consumption at node S for a given k. However, as D increases, k will optimally be reduced in order to equate the marginal social cost with the marginal benefit, i.e., causing the shifted inverse demand curve at node S to move closer to the local one and leaving equilibrium consumption unchanged. At node N, the decrease in k due to an infinitesimally larger D will not change the equilibrium price but will result in slightly more RE generation (see Figure 4a). In summary, the optimal transmission capacity under CP is such that the marginal benefit and marginal cost of expansion are equated: the former stems from the reduction in RE production (C_N), whereas the latter is due to the increase in thermal production (C_S), the increase in the transmission capacity (C_T), and increase in the marginal cost of emissions damage ($D\bar{y}_S(k)$). Although thermal generation does not increase like-for-like with transmission expansion, the resulting curb in consumption frees up precisely enough thermal production to transmit to node N to use the added transmission capacity.

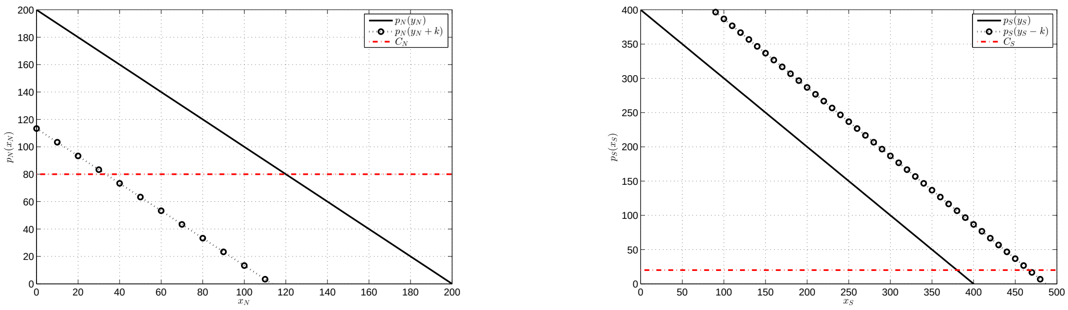
Since industry cannot internalize the cost of damage from emissions under the PC setting, the optimal thermal generation is obtained by simply equating the marginal production cost with the shifted inverse demand curve at node S (see Figure 4b). A *ceteris paribus* increase in D curbs neither thermal generation nor consumption at node S for a given k, which prompts the TSO to reduce the size of the transmission line drastically. Although consumption at node S is unaffected by the reduced transmission capacity, the lower net import at node N reduces thermal generation. Consequently, there is a like-for-like increase in RE generation at node N to replace the curbed thermal production (see Figure 5a). Under PC, the optimal transmission capacity is again determined by setting the marginal benefit of expansion to its marginal cost, i.e., $C_N = C_T + C_S + D\hat{y}_S(k)$. Since it is impossible for the TSO to curb consumption at node S, an infinitesimally larger transmission line leads to a like-for-like increase in thermal output to transmit to node N. Hence, the optimal transmission capacity for a given D is smaller under PC due to its higher marginal cost of expansion, i.e., $D\hat{y}_S(k) > D\bar{y}_S(k)$.

From Case 2, sustained increases in D will make it more expensive to produce power from thermal generation at node S and to transmit it to node N. Once $\bar{p}_S + C_T > C_N$ under CP (and $\hat{p}_S + \hat{\lambda}^+ > C_N$ under PC), it is actually more efficient to cut the transmission line, thereby leading to Case 3. Such autarky persists under CP until the cost of damage from emissions is high enough to warrant building a transmission line to send RE generation to node S, i.e., $\bar{p}_S = C_N + C_T$ (Case 4). However, this reversal of the prevalent power flow does not happen under PC as the price at node S is too



(a) Node N (b) Node S

Figure 4: Power sectors' incentives under CP (Case 2)

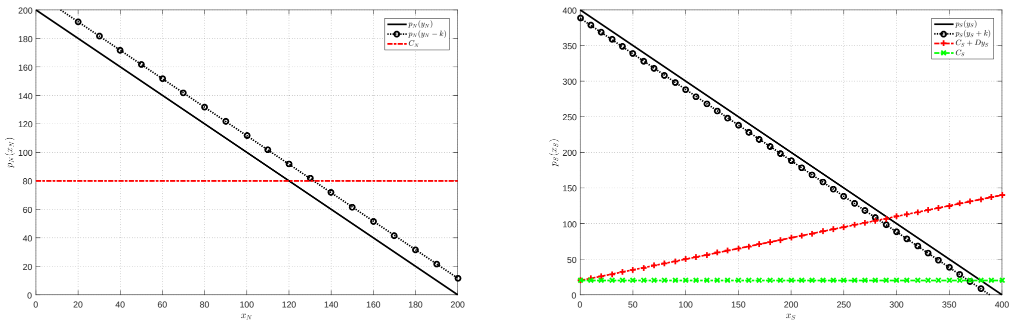


(a) Node N (b) Node S

Figure 5: Power sectors' incentives without a carbon tax under PC (Case 2)

low to entice the RE generator as we showed in Section 3.2 (see Figure 1). Since the CP setting allows for the internalization of the cost of damage from emissions, once thermal generation is curbed to the extent that the price at node S is \$25/MW higher than that at node N, the transmission line is built (see Figure 6b). Here, the local inverse demand at node S is shifted inward by the amount of imported energy, which is then equated with the marginal social cost of generation to yield the optimal thermal output. The optimal size of the transmission line is determined by setting the marginal benefit of expansion (reduction in both the thermal production cost and marginal cost of damage from emissions, $C_S + D\bar{y}_S(k)$) to its marginal cost (due to increased RE output and construction of the line, $C_N + C_T$). Thus, an infinitesimal increase in k leads to a like-for-like increase in RE output without affecting consumption at node N and a less than like-for-like reduction (increase) in thermal output (consumption at node S). For a given k, a *ceteris paribus* increase in D will curb thermal generation and, thus, consumption at node S. In order to prevent an inefficiently large price increase at node

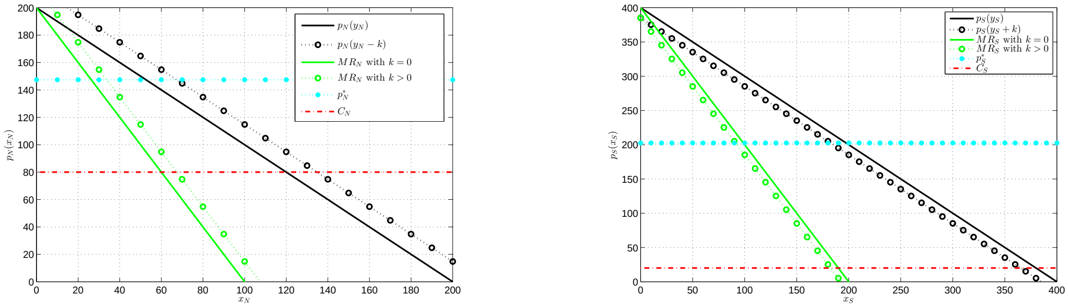
S, transmission capacity will be increased. Hence, RE generation will increase without affecting consumption at node N (Figure 6a) as the surplus power will be transmitted to node S to replace the curbed thermal generation without affecting consumption there.



(a) Node N (b) Node S

Figure 6: Power sectors' incentives under CP (Case 4)

As Figure 1 indicates, Case 4 can occur under CO even without a carbon tax for sufficiently high D. The intuition for its observation is summarized by Figure 7 in which the marginal production costs are equated to the marginal revenues of the producers at each node. Thus, the equilibrium price is always higher than the marginal production cost. Since node S has higher local demand than node N, it will have a higher equilibrium price due to the exercise of market power by the thermal generator. Thus, alleviation of this inefficient curb on consumption via a larger transmission line will enter into the TSO's calculus. From the perspective of the TSO, the optimal size of the transmission line depends on the marginal benefit of expansion (reduction in not only thermal production and damage costs, $\frac{C_S}{2} + \frac{Dy_S^*(k)}{2}$, but also choke on consumption, $\frac{p_S^*}{2}$) and its marginal cost (comprising not only the cost of transmission and RE production, $C_T + \frac{C_N}{2}$, but also loss in consumption at node N, $\frac{p_N^*}{2}$). In effect, as D increases, the TSO will expand the transmission line to mitigate two types of inefficiencies at node S: (i) cost of damage from emissions and (ii) exertion of market power suppressing consumption at node S. This twin utilization of the transmission capacity also enables the TSO to attain a higher maximized social welfare under CO than under PC (Figure 8).



(a) Node N (b) Node S

Figure 7: Power sectors' incentives without a carbon tax under CO (Case 4)

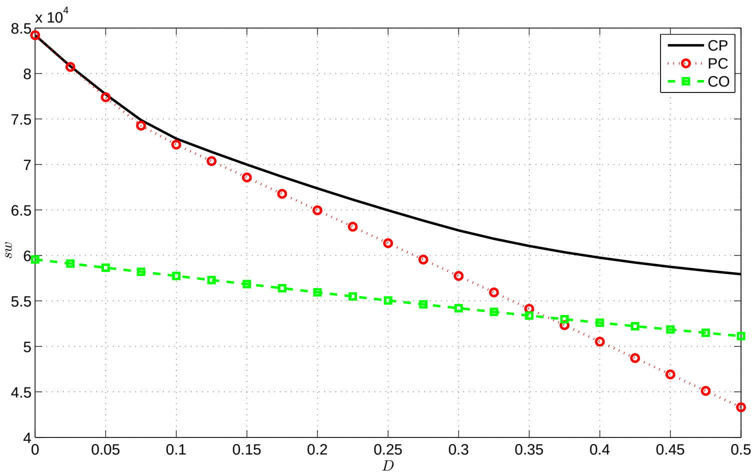
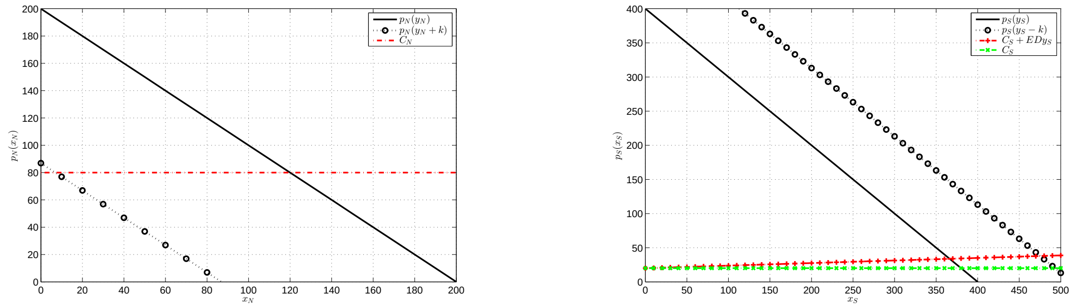


Figure 8: Maximized social welfare with respect to D

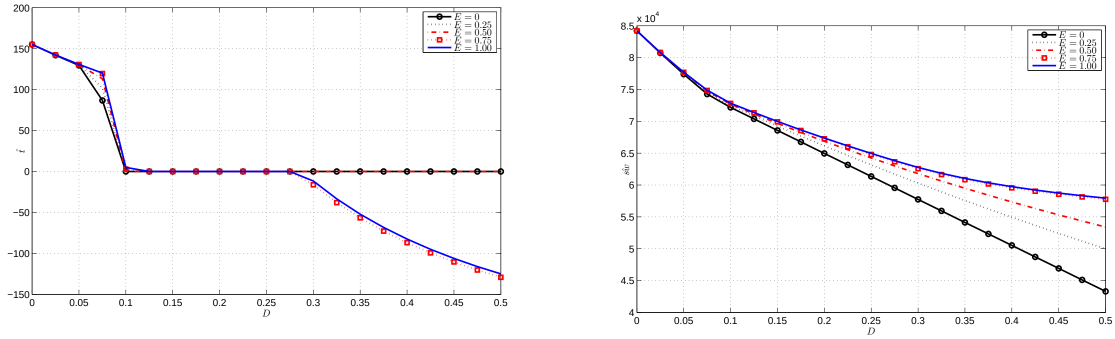
5.2 Results with a Carbon Tax

With a carbon tax, e.g., $E = 0.50$ in Case 2, total consumption at node N under PC is not affected because the RE production cost is an upper limit on the price. Thus, $\hat{y}_N(k)+k$ is unchanged, but increasing the carbon tax reduces RE output, $\hat{y}_N(k)$, as more electricity is imported from node S (see Figure 9a). Conversely, at node S, increasing the carbon tax increases the marginal cost of production at node S, which causes consumption, $\hat{y}_S(k)-k$, to decrease. In effect, a higher tax enables the TSO to free up thermal capacity to supply more of the demand at node N, which also increases k (see Figure 9b). As illustrated in Figure 10a, the optimal transmission line capacity increases with the carbon tax because the TSO's marginal cost of capacity expansion decreases as an infinitesimal increase in k would lead to a less than proportional increase in thermal output due to curbing of consumption at node S (see Proposition 9). Since the carbon tax is able to align private incentives with social ones, social welfare increases as the economically inefficient RE overproduction at node N is curtailed without affecting consumption at node N (see Proposition 11). Hence, even though there is the cost of increasing transmission capacity, lower consumption at node S, and slightly higher emissions, society is better overall (see Figure 10).



(a) Node N (b) Node S

Figure 9: Power sectors' incentives with a $E = 0.50$ carbon tax under PC (Case 2)



(a) Optimal net import at node N with respect to D (b) Maximized social welfare with respect to D

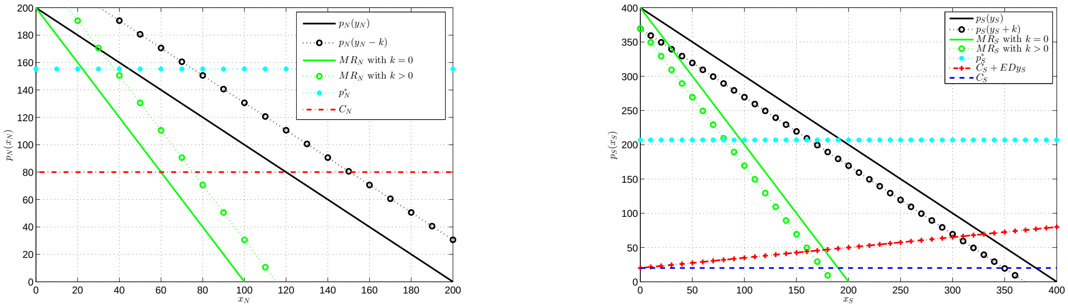
Figure 10: Outcomes under PC setting with a carbon tax

In assessing the impact of a $E = 0.50$ carbon tax on the CO setting, we focus on Case 4. As in a situation without a carbon tax, we determine optimal RE production by setting net marginal revenue at node N equal to the RE production cost. Increasing the carbon tax will cause RE generation to increase along with the price at node N, but the total consumption at node N will decrease as more electricity is sent to node S (see Figure 11a). Intuitively, by internalizing the externality from emissions, consumers at node S obtain the signal to reduce consumption and to pay a higher price (see

Figure 11b). This incentivizes more RE production, which is reinforced by the TSO's increasing transmission capacity to send more electricity to node S. As in Section 5.1, the TSO compares the marginal benefit of an infinitesimal increase in k with its marginal cost. Although the latter is still equal to the cost of augmenting transmission and RE production, $C_T + \frac{C_N}{2}$, and the value of lost consumption at node N, $\frac{p_N^*}{2}$, the former is higher due to the imposition of a carbon tax. The marginal benefit from increasing k is related to two factors: increased consumption at node S and less thermal generation. First, the marginal value of alleviating the choke on consumption via an infinitesimally larger transmission line increases with the carbon tax because consumption at node S was already low from internalizing the cost of damage from emissions. Second, the marginal benefit from reduction in thermal production and damage costs via an infinitesimally larger transmission line decreases with the carbon tax because there is less thermal output to mitigate. Since the increase in the marginal benefit from higher consumption at node S outweighs the decrease in marginal benefit from reduction in thermal output, the marginal benefit of expanding transmission capacity increases overall in the presence of the carbon tax. Therefore, k increases with E as shown in Proposition 10 and illustrated by Figure 12a for Case 4. Although the TSO's best response to the imposition of a carbon tax is to increase the transmission capacity, maximized social welfare is actually lower than without a tax as long as $E > 1 - \frac{B_S}{D}$ (see Figure 12b). As mentioned in the discussion of Proposition 12, the CO setting presents two opposing challenges for the TSO to address: the exertion of market power (especially at node S) and the cost of damage from emissions. If the former is relatively large, then the use of a carbon tax will merely enhance the scope for the exercise of market power.

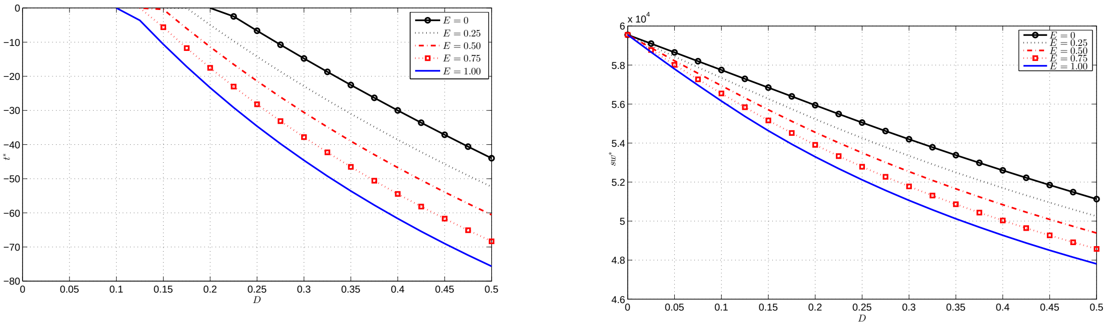
6 Conclusions

The proliferation of RE resources as a means to mitigate climate change will typically require an augmentation of the transmission network. Consequently, expanding transmission capacity in order to balance economic and environmental objectives poses



(a) Node N (b) Node S

Figure 11: Power sectors' incentives with a $E = 0.50$ carbon tax under CO (Case 4)



(a) Optimal net import at node N with respect to D (b) Maximized social welfare with respect to D

Figure 12: Outcomes under CO setting with a carbon tax

an opportunity as well as a significant challenge for a regulated TSO. In particular, a TSO must anticipate the response of profit-maximizing power companies that invest in generation capacity when making its socially optimal transmission expansion. While the OR literature has addressed such anticipative transmission-expansion problems, research is still limited on the implications of using the transmission line as an instrument for regulating an environmental externality, which arguably is likely to play an increasing role with ambitious RE targets set forth by various countries and regions.

In parallel with the environmental economics literature, we show how curbs on consumption, greater reliance on RE resources, and enhanced transmission capacity are all intertwined measures for limiting the cost of damage from emissions. However, it is only in an idealized CP setting that the socially optimal balance among the three is attained. Indeed, in a deregulated industry, the TSO can only indirectly influence consumption and production decisions through its sizing of the transmission line. We explore this dilemma via a bi-level modeling framework that we solve analytically in both PC and CO settings. Our main insights are that the solution under PC inhibits the TSO's ability to incentivize a curb in consumption in line with the cost of damage from emissions. As a countervailing measure, the TSO constructs a smaller transmission line to limit production from the thermal generator and relies to a greater extent on RE output. Thus, although this result appears to be in line with certain policy targets, e.g., the EU's 20-20-20 energy and climate package, it is actually socially inefficient because there is "too much" RE generation instead of curbing consumption. Next, under a CO setting, the exercise of market power by producers provides the impetus for a reversal of the prevalent power flow as RE generation becomes a net exporter provided that the damage cost parameter is high enough. Somewhat counterintuitively, a CO setting provides greater scope for the TSO to maintain social welfare as it is able to use the transmission capacity to alleviate both economic and environmental distortions by routing more RE output. Therefore, for an extremely high damage cost parameter, the maximized social welfare under CO is even higher than under PC.

In order to correct for such externalities, it is appealing to deploy a carbon tax on thermal generation. Under PC, we prove that such a measure results in perfect alignment of incentives: an increase in the carbon tax while holding the transmission capacity fixed reduces thermal generation and lowers consumption enough to liberate thermal capacity to be used in place of socially inefficient RE output. Recognizing this change in incentives, the TSO is able to increase the transmission capacity to the socially optimal level as in CP. Yet, a carbon tax under CO may worsen outcomes if consumption is already restricted by the exercise of market power. Indeed, while the carbon tax leads the TSO to increase the transmission capacity to supplant thermal generation by RE output, the resulting loss in consumer surplus at both nodes leads to a net reduction in social welfare. Although this undesirable effect of a carbon tax in a setting with market power is known in the environmental economics literature, we have demonstrated that may also arise when there is recourse to a transmission line as a mitigation measure. Therefore, proxies for climate control should be taken with caution, especially in the presence of market power.

For future work, the model could be expanded to handle more realistic features such as uncertainty in RE output and storage. Due to the complexity and range of strategies available to market players in such a setting, a numerical solution approach may be advisable. On the other hand, allowing for distinct generation capacity-expansion and operational decisions would lead to a tri-level model that would be interesting to solve analytically (Murphy and Smeers, 2010).

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Appendix: Proofs of Propositions

We denote $F = C_N - C_S - C_T$ and $G = C_N - C_S + C_T$, where $G > 0$ as $C_N > C_S$.

Proof of Proposition 1

Case 1: At the optimum, $-[A_N - B_N(y_N + t)] + C_N = \frac{B_N B_S F - D[B_S(A_N - C_N) + B_N(A_S - C_N + C_T)]}{(B_S + B_N)D + B_N B_S}$ with regard to KKT condition (6). This needs to be positive for Case 1 with $\bar{y}_N = 0$. The denominator and the expression in the square bracket in the numerator are positive, noting that $A_S > A_N > C_N > C_S$. Thus, F needs to be positive for Case 1 to occur, i.e., $C_N > C_S + C_T$. We next calculate the derivative of each variable with respect to D. We obtain $\frac{\partial \bar{k}}{\partial D} = -\frac{B_S[B_N(A_S - C_S) + B_S(A_N - C_N + F)]}{[(B_S + B_N)D + B_N B_S]^2} < 0$ and $\frac{\partial \bar{y}_S}{\partial D} = -\frac{(B_S + B_N)[B_N(A_S - C_S) + B_S(A_N - C_N + F)]}{[(B_S + B_N)D + B_N B_S]^2} < 0$.

Case 2: At the optimum, $\bar{y}_S = \frac{F}{D}$, which needs to be positive for Case 2. Thus, F needs to be positive for Case 2 to occur. We obtain $\frac{\partial \bar{k}}{\partial D} = -\frac{F}{D^2} < 0$, $\frac{\partial \bar{y}_S}{\partial D} = -\frac{F}{D^2} < 0$, and $\frac{\partial \bar{y}_N}{\partial D} = \frac{F}{D^2} > 0$.

Case 3: Simple calculation yields $\frac{\partial \bar{y}_S}{\partial D} = -\frac{A_S - C_S}{(B_S + D)^2} < 0$ and $\frac{\partial \bar{y}_N}{\partial D} = 0$.

Case 4: We have $\frac{\partial \bar{k}}{\partial D} = \frac{G}{D^2} > 0$, $\frac{\partial \bar{y}_S}{\partial D} = -\frac{G}{D^2} < 0$, and $\frac{\partial \bar{y}_N}{\partial D} = \frac{G}{D^2} > 0$.

□

Proof of Proposition 2 Differentiating the damage cost $\frac{1}{2}D\bar{y}_S^2$ with respect to D at the optimum yields $-\frac{F^2}{2D^2} < 0$ and $-\frac{G^2}{2D^2} < 0$ in Cases 2 and 4, respectively. □

Proof of Proposition 3

Case 1: At the optimum, $-[A_N - B_N(y_N + t)] + C_N = \frac{B_N B_S F - D[B_S(A_N - C_N) + B_N(A_S - C_S)]}{B_S(D + B_N)}$ with regard to KKT condition (17). This needs to be positive for Case 1 with $\hat{y}_N = 0$. The denominator and the expression in the square bracket in the numerator are positive. Thus, F needs to be positive for Case 1 to occur. We next calculate the derivative of each variable with respect to D. We obtain $\frac{\partial \hat{k}}{\partial D} = -\frac{B_N(A_S - C_S) + B_S(A_N - C_N + F)}{B_S(D + B_N)^2} < 0$ and $\frac{\partial \hat{y}_S}{\partial D} = \hat{y}_S'(k)\frac{\partial \hat{k}}{\partial D} = \frac{\partial \hat{k}}{\partial D} < 0$.

Case 2: At the optimum, $\hat{y}_S = \frac{F}{D}$, which needs to be positive for Case 2. Thus, F needs to be positive for Case 2 to occur. We obtain $\frac{\partial \hat{k}}{\partial D} = -\frac{F}{D^2} < 0$, $\frac{\partial \hat{y}_S}{\partial D} = \hat{y}_S'(k)\frac{\partial \hat{k}}{\partial D} = \frac{\partial \hat{k}}{\partial D} < 0$, and $\frac{\partial \hat{y}_N}{\partial D} = \hat{y}_N'(k)\frac{\partial \hat{k}}{\partial D} = -\frac{\partial \hat{k}}{\partial D} > 0$.

Case 3: Simple calculation yields $\frac{\partial \hat{y}_S}{\partial D} = \frac{\partial \hat{y}_N}{\partial D} = 0$.

$\square$

Proof of Proposition 4 Differentiating the damage cost $\frac{1}{2}D\hat{y}_S^2$ with respect to D at the optimum yields $-\frac{F^2}{2D^2} < 0$ and $\frac{(A_S - C_S)^2}{2B_S^2} > 0$ in Cases 2 and 3, respectively. $\square$

Proof of Proposition 5

Case 1: With regard to KKT condition (24), $-[A_N - B_N(2y_N + t)] + C_N = \frac{H - D[B_S(A_N - C_N) + B_N(A_S - C_S)]}{B_S(B_S + 4B_N + D)}$ at the optimum, which needs to be positive for Case 1 with $y_N^* = 0$. The denominator and the expression in the square bracket in the numerator are positive. $H = B_S[B_S(C_N - A_N) + B_N(C_N - A_S - C_T) + 3B_N F]$, where the expressions in the first two parentheses are negative. Thus, F needs to be positive for Case 1 to occur. We next calculate the derivative of each variable with respect to D. We obtain $\frac{\partial k^*}{\partial D} = -\frac{4[B_N(A_S - C_S) + B_S(A_N - C_N + F)]}{B_S(B_S + 4B_N + D)^2} < 0$ and $\frac{\partial y_S^*}{\partial D} = y_S^{*\prime}(k)\frac{\partial k^*}{\partial D} = \frac{1}{2}\frac{\partial k^*}{\partial D} < 0$.

Case 2: From Table 1, $k^* = \frac{B_S(A_N - A_S - C_T + 3F) - D(A_S - C_S)}{B_S(B_S + B_N + D)}$ at the optimum, which needs to be positive for Case 2. The denominator is positive. In the numerator, $A_N - A_S - C_T < 0$ and $A_S - C_S > 0$. Thus, F needs to be positive for Case 2 to occur. We obtain $\frac{\partial k^*}{\partial D} = -\frac{B_S(A_N - C_N + 4F) + B_N(A_S - C_S)}{B_S(B_S + B_N + D)^2} < 0$, $\frac{\partial y_S^*}{\partial D} = y_S^{*\prime}(k)\frac{\partial k^*}{\partial D} = \frac{1}{2}\frac{\partial k^*}{\partial D} < 0$, and $\frac{\partial y_N^*}{\partial D} = y_N^{*\prime}(k)\frac{\partial k^*}{\partial D} = -\frac{1}{2}\frac{\partial k^*}{\partial D} > 0$.

Case 3: Simple calculation yields $\frac{\partial y_S^*}{\partial D} = \frac{\partial y_N^*}{\partial D} = 0$.

Case 4: We have $\frac{\partial k^*}{\partial D} = \frac{B_S(A_N - C_N + 4G) + B_N(A_S - C_S)}{B_S(B_S + B_N + D)^2} > 0$, $\frac{\partial y_S^*}{\partial D} = y_S^{*\prime}(k)\frac{\partial k^*}{\partial D} = -\frac{1}{2}\frac{\partial k^*}{\partial D} < 0$, and $\frac{\partial y_N^*}{\partial D} = y_N^{*\prime}(k)\frac{\partial k^*}{\partial D} = \frac{1}{2}\frac{\partial k^*}{\partial D} > 0$.

$\square$

Proof of Proposition 6 Differentiating the damage cost $\frac{1}{2}Dy_S^{*2}$ with respect to D at the optimum yields $\frac{(A_S - C_S)^2}{8B_S^2} > 0$ in Case 3. $\square$

Proof of Proposition 7 By using the envelope theorem, we calculate the partial derivative of the Lagrangian for the QP problem (1)–(4) with respect to D evaluated at the optimum. This gives $-\frac{1}{2}\bar{y}_S^2 < 0$ at the optimum for every case under CP. Thus, the value function (i.e., social welfare here) is decreasing in D under CP. In a similar way using the envelope theorem, we can easily verify $-\frac{1}{2}\hat{y}_S^2 < 0$ and $-\frac{1}{2}y_S^{*2} < 0$ at the optimum for every case under PC and CO, respectively. $\square$

Proof of Proposition 8 The CP welfare-maximization problem, (1)–(4), is equivalent to the following decomposed problem:

$$\begin{array}{ll} \max_{\{k\geq 0\}\cup\Omega\cup\Xi} & \text{Equation (1)} \\ \text{s.t.} & \max\limits_{\Omega} \text{Equation (1)} \\ & \text{s.t. Equations (2)–(4)} \end{array}$$

Since $\max_{\Omega}\{\text{Equation (1)} \mid \text{Equations (2)–(4)}\}$ is a convex programming problem, it can be replaced by its KKT conditions as follows:

$$\begin{array}{ll} \max_{\{k\geq 0\}\cup\Omega\cup\Xi} & \text{Equation (1)} \\ \text{s.t.} & \text{Equations (5)–(6), (8)–(12)} \end{array}$$

With our assumption of $\beta_N = 0$ and $\beta_S = 0$, this is equivalent to the TSO's problem (31) under PC with a full carbon tax, $E = 1$. □

Proof of Proposition 9

Case 1: We calculate $-[A_N - B_N(y_N + t)] + C_N$ at the optimum regarding the KKT condition with respect to y_N. This needs to be positive for Case 1 with $\hat{y}_N = 0$. The denominator is positive, while the numerator is decomposed into negative terms and $B_N B_S F(B_S + 2ED)$. Thus, F needs to be positive for Case 1 to occur. We then calculate $\frac{\partial \hat{k}}{\partial E}$. The denominator is a squared term, while the numerator is $2B_S D^2(1-E)(B_S + ED)[B_N(A_S - C_S) + B_S(A_N - C_N + F)] > 0$. Hence, $\frac{\partial \hat{k}}{\partial E} > 0$.

Case 2: At the optimum, $\hat{y}_S = \frac{(B_S+ED)F}{D(B_S+E^2D)}$, which needs to be positive for Case 2. Thus, F needs to be positive for Case 2 to occur. We then obtain $\frac{\partial \hat{k}}{\partial E} = \frac{2(1-E)(B_S+ED)F}{(B_S+E^2D)^2} > 0$.

Case 4: We have $\frac{\partial \hat{k}}{\partial E} = -\frac{2(1-E)(B_S+ED)G}{(B_S+E^2D)^2} < 0$.

□

Proof of Proposition 10

Case 1: We calculate $-[A_N - B_N(2y_N + t)] + C_N$ at the optimum regarding KKT condition with respect to y_N. This needs to be positive for Case 1 with $y_N^* = 0$. The denominator is positive, while the numerator is decomposed into negative terms and $B_N B_S F(3B_S + 2ED)$. Thus, F needs to be positive for Case 1 to occur. We then calculate $\frac{\partial k^*}{\partial E}$. The denominator is a squared term, and the numerator is $2B_S D[(1-E)D - B_S](2B_S + ED)[B_N(A_S - C_S) + B_S(A_N - C_N + F)]$. Hence, $\frac{\partial k^*}{\partial E} > 0$ if and only if $(1-E)D - B_S > 0$, or $E > 1 - \frac{B_S}{D}$.

Case 2: We calculate k^* at the optimum, which needs to be positive for Case 2. The denominator is positive, while the numerator is decomposed into negative terms and $4B_S F(3B_S + 2ED)$. Thus, F needs to be positive for Case 2 to occur. We then calculate $\frac{\partial k^*}{\partial E}$. The denominator is a squared term. The numerator is $8B_S D[(1-E)D - B_S](2B_S + ED)[B_N(A_S - C_S) + B_S(A_N - C_N) + 4B_S F]$. Hence, $\frac{\partial k^*}{\partial E} > 0$ if and only if $E > 1 - \frac{B_S}{D}$.

Case 4: We calculate $\frac{\partial k^*}{\partial E}$. The denominator is a squared term. The numerator is $-8B_S D[(1-E)D - B_S](2B_S + ED)[B_N(A_S - C_S) + B_S(A_N - C_N) + 4B_S G]$. Hence, $\frac{\partial k^*}{\partial E} < 0$ if and only if $E > 1 - \frac{B_S}{D}$.

$\square$

Proof of Proposition 11 From KKT condition (30), $A_S - B_S(y_S - t) - C_S - EDy_S = 0$ since $\hat{y}_S > 0$ in Cases 1–4. We thus derive $\hat{y}_S(t, E) = \frac{A_S - C_S + B_S t}{B_S + ED}$. For all Cases 1–4, the TSO's problem can be generally expressed as follows using variable t:

$$\begin{aligned} \max_t \quad & A_S(\hat{y}_S(t,E) - t) - \frac{1}{2}B_S(\hat{y}_S(t,E) - t)^2 - C_S\hat{y}_S(t,E) \\ & + A_N(\hat{y}_N(t) + t) - \frac{1}{2}B_N(\hat{y}_N(t) + t)^2 - C_N\hat{y}_N(t) - C_T|t| - \frac{1}{2}D\hat{y}_S(t,E)^2 \end{aligned}$$

where $t = k$ and $\hat{y}_N(t) = 0$ in Case 1, $t = k$ in Case 2, $t = k = 0$ in Case 3, and $t = -k$ in Case 4. We calculate the partial derivative of the objective function with respect to E evaluated at the optimum, i.e., for $k = \hat{k}$ and $t = \hat{t}$. This gives $\frac{\partial \hat{y}_S}{\partial E}\left[A_S - B_S\left(\hat{y}_S - \hat{t}\right) - C_S - D\hat{y}_S\right]$. From $A_S - B_S\left(\hat{y}_S - \hat{t}\right) - C_S - ED\hat{y}_S = 0$, we have $A_S - B_S\left(\hat{y}_S - \hat{t}\right) - C_S - D\hat{y}_S = -(1 - E)D\hat{y}_S < 0$. For Cases 1–3, $\frac{\partial \hat{y}_S}{\partial E} = -\frac{D(A_S - C_S + B_S\hat{t})}{(B_S + ED)^2} = -\frac{D(A_S - C_S + B_S\hat{k})}{(B_S + ED)^2} < 0$. For Case 4,

$\frac{\partial \hat{y}_S}{\partial E} = -\frac{D(A_S - C_S + B_S \hat{t})}{(B_S+ED)^2} = -\frac{D(A_S - C_S - B_S \hat{k})}{(B_S+ED)^2} < 0$ since $A_S - C_S - B_S \hat{k} = (B_S + ED)\hat{y}_S > 0$. Therefore, $\frac{\partial \hat{sw}}{\partial E} > 0$ by the envelope theorem. □

Proof of Proposition 12 From KKT condition (32), $A_S - B_S(2y_S - t) - C_S - EDy_S = 0$ since $y_S^* > 0$ in Cases 1–4. We, thus, derive $y_S^*(t,E) = \frac{A_S - C_S + B_S t}{2B_S + ED}$. For all Cases 1–4, the TSO's problem can be generally expressed as follows using variable t:

$$
\begin{aligned}
\max_t \quad & A_S\left(y_S^*(t,E) - t\right) - \frac{1}{2}B_S\left(y_S^*(t,E) - t\right)^2 - C_S y_S^*(t,E) \\
& + A_N\left(y_N^*(t) + t\right) - \frac{1}{2}B_N\left(y_N^*(t) + t\right)^2 - C_N y_N^*(t) - C_T|t| - \frac{1}{2}D y_S^*(t,E)^2
\end{aligned}
$$

where $t = k$ and $y_N^*(t) = 0$ in Case 1, $t = k$ in Case 2, $t = k = 0$ in Case 3, and $t = -k$ in Case 4. We calculate the partial derivative of the objective function with respect to E evaluated at the optimum, i.e., for $k = k^*$ and $t = t^*$. This gives $\frac{\partial y_S^*}{\partial E}\left[A_S - B_S\left(y_S^* - t^*\right) - C_S - Dy_S^*\right]$. From $A_S - B_S\left(2y_S^* - t^*\right) - C_S - EDy_S^* = 0$, we have $A_S - B_S\left(y_S^* - t\right) - C_S - Dy_S^* = [B_S - (1-E)D]\,y_S^*$. This is positive if and only if $E > 1 - \frac{B_S}{D}$. For Cases 1–3, $\frac{\partial y_S^*}{\partial E} = -\frac{D(A_S - C_S + B_S t^*)}{(2B_S+ED)^2} = -\frac{D(A_S - C_S + B_S k^*)}{(2B_S+ED)^2} < 0$. For Case 4, $\frac{\partial y_S^*}{\partial E} = -\frac{D(A_S - C_S + B_S t^*)}{(2B_S+ED)^2} = -\frac{D(A_S - C_S - B_S k^*)}{(2B_S+ED)^2} < 0$ since $A_S - C_S - B_S k^* = (2B_S + ED)y_S^* > 0$. Therefore, if $E > 1 - \frac{B_S}{D}$, then $\frac{\partial sw^*}{\partial E} < 0$, and *vice versa*, by the envelope theorem. □

Proof of Proposition 13 As shown in Proposition 11, $\hat{sw}(E)$ monotonically increases in E under PC. Hence, $\hat{sw}(E)$ is maximized at $E = 1$ under PC. □

Proof of Proposition 14 As shown in Proposition 12, $sw^*(E)$ increases in E in the range of $E < 1 - \frac{B_S}{D}$, while it decreases in E in the range of $E > 1 - \frac{B_S}{D}$. If $B_S < D$, then $0 < 1 - \frac{B_S}{D} < 1$, and, thus, $sw^*(E)$ is maximized at $E = 1 - \frac{B_S}{D} < 1$. If $B_S \geq D$, then $1 - \frac{B_S}{D} \leq 0$, and, thus, $sw^*(E)$ is maximized at $E = 0$. □