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Abstract

Access to external finance is a major obstacle for small and young firms. Thus, providing subsidized credit to small and young firms is a widely used policy option across countries. We study the impact of such targeted policies on aggregate output and productivity and highlight indirect general equilibrium effects. To do so, we build a model of heterogeneous firms with endogenous entry and exit, wherein each firm may be subject to a forward-looking collateral constraint for external borrowing. Subsidized credit alleviates credit constraints facing small and young firms, which helps them achieve an efficient and larger scale of production. This direct effect, however, is either reinforced or offset by indirect general equilibrium effects. Factor prices increase as subsidized firms demand more capital and labor. As a result, higher production costs induce more unproductive incumbents to exit, while replacing them selectively with productive entrants. This cleansing effect reinforces the direct effect by enhancing the aggregate productivity. However, the number of firms in operation decreases in equilibrium, and this, in turn, depresses the aggregate productivity.

Keywords: Firm dynamics, Misallocation, Financial frictions, Firm size and age

JEL classification: E22, G32, O16

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[1] This study is conducted as a part of the Project "Dynamics of Inter-organizational Network and Geography" undertaken at the Research Institute of Economy, Trade and Industry (RIETI). This paper is a revision of our earlier work, previously circulated as "Aggregate Implications of Firm Dynamics: Financing Small versus Young Firms." We thank seminar participants at Senshu University, RIETI, Korea University, and SNU IRFE, and conference participants at the International Conference on Growth, Trade, and Dynamics, and Asian and European Meeting of the Econometric Society.

1 Introduction

Government policies that attempt to alleviate credit constraints faced by small and young firms are widely adopted across countries; providing subsidized credit is a prime example of this.[1] Despite the popularity of such targeted industrial policies, quantitative studies on their macroeconomic effects are scarce. While a policy benefits targeted firms directly, it may also harm untargeted firms through changes in prices of goods and production factors. Therefore, a well-intended industrial policy may distort macroeconomic outcomes. In this paper, we show that the unintended consequences of a targeted policy may overshadow its direct benefits in a general equilibrium setting.

The recent literature has documented the large contribution of start-up firms to job creation in the United States. In general, young firms start small and grow fast; however, they also tend to fail at greater rates. They are especially vulnerable to negative shocks such as recessions and natural disasters. Moreover, credit constraints compound such adverse shocks; these factors, among others, lead to the observed high exit rates among young firms.[2] Given its pervasiveness, we focus on a subsidized credit policy that alleviates difficulties in external borrowing for small and young firms, as in Buera, Moll, and Shin (2013). In particular, we construct a general equilibrium model with heterogeneous firms and calibrate it against both aggregate and microeconomic data. In doing so, we examine the overall effect of targeted credit subsidy policies on an aggregate economy and decompose it into two components: *a direct effect* from credit subsidies and *indirect general equilibrium effects*.

Our model builds on a standard heterogeneous firm model.[3] The model has three key ingredients that allow us to capture a realistic cross-sectional distribution of firms and their dynamics, as observed in the data. First, we employ a bounded Pareto distribution for firm-level productivity. It is well-known that the empirical distribution of firm size in employment is highly skewed. That is, small firms dominate the business population, while large firms account for the largest fraction of aggregate employment.[4] Our specification of

[1] Beginning in January 2011, the Obama Administration promoted a set of entrepreneur-focused policies, including the Jumpstart Our Business Startups (JOBS) Act. The White House (2012) claims that these policies allow *"Main Street small businesses and high-growth enterprises to raise capital from investors more efficiently, allowing small and young firms across the country to grow and hire faster."* The targets of the policy initiatives include finance, education, red tape, innovation, and market opportunities for entrepreneurs.

[2] For job creation by firm age, see Haltiwanger, Jarmin, and Miranda (2016). Collier et al. (2016) discuss the role of credit constraints.

[3] See Hopenhayn (1992) and Jovanovic (1982) for the seminal works.

[4] In 2012, the share of employment at large firms was 51.6 percent while the employment share of small firms was 16.7 percent in the United States (Statistics of U.S. Businesses, 2012).

firm productivity successfully replicates the empirical firm size distribution in our model economy, coupled with the decreasing-returns-to-scale production technology. Second, we allow endogenous entry and exit by firms. This element is essential to reproduce the empirical patterns of firm dynamics, including substantially lower survival rates among young firms. In addition, it allows us to examine the importance of extensive margins in affecting aggregate productivity. Third, forward-looking collateral constraints are added, in the spirit of Kiyotaki and Moore (1997). This, together with the second ingredient, leads to an empirically consistent age distribution of firms.[5] The presence of this collateral constraint hinders immediate firm growth upon entry in the model.[6] This helps us to have a substantial number of small firms in the economy. Moreover, we can also generate the right pattern of firm lifecycle dynamics; newborn firms gradually build up capital stock over time by relying on debt issuance.

We use this model to study the aggregate implications of targeted credit subsidy policies in a general equilibrium environment. Credit subsidies in our model are lump-sum cash transfers from households to targeted firms. Thus, none of the policy effects that we study in this paper arises from direct policy distortions (e.g., tax distortions). We show that there are four effects, one direct and three indirect, following a targeted credit subsidy in our model. The first one is a direct effect that emerges from the fact that small and young firms can achieve an efficient scale of production by receiving a subsidy. This direct effect stems from alleviating capital misallocation across firms due to credit constraints. In line with the findings in previous studies (e.g., Buera, Kaboski, and Shin (2011), Khan and Thomas (2013), Midrigan and Xu (2014)), we show the quantitative importance of misallocation in explaining aggregate productivity. The second effect is an indirect general equilibrium effect. Increased demand for capital and labor—from the recipients of the subsidy—will raise factor prices. Higher factor prices depress the scale of production of untargeted firms. The third effect is also indirect. Increased factor prices, in equilibrium, lead more unproductive incumbents to exit, which are replaced by productive potential entrants (*cleansing effects*).[7] The last effect is that there are fewer firms in operation because of higher costs of production, which in turn depresses

[5] The importance of firm age has been studied in the following seminal papers: Davis, Haltiwanger, and Schuh (1996), Dunne, Roberts, and Samuelson (1989), and Evans (1987). More recently, Fort et al. (2013) emphasized the age dimension of firm heterogeneity during the Great Recession. Hsieh and Klenow (2014) examined the lifecycle of plants in Mexico and India and the resource misallocation.

[6] See Bahaj, Foulis and Pinter (2016) for microeconometric evidence on the importance of collateral constraints on firm investment.

[7] We examine the cleansing effect of industrial policies, while the business cycle literature has investigated the cleansing effect of recessions. See Osotimehin and Pappada (2015) for a recent application.

the aggregate productivity.[8]

Having established these results, we show the importance of the following two pre-requisites for a useful policy evaluation of targeted credit subsidies. First, equilibrium analysis is essential to evaluate the macroeconomic implications of such policies. We show that, in partial equilibrium with real wages and interest rates fixed, the aggregate impact of the targeted policies is substantially larger than that generated from the model in a general equilibrium environment. Second, we highlight the importance of reproducing the key aspects of firm-level data. In particular, the model's performance in reproducing the skewed empirical firm size distribution is vital to quantifying the aggregate outcomes of targeted credit subsidies.

This is true for designing and implementing such targeted policies in practice. To see this point, we further explore the aggregate implications of policies that differ in their respective targets, one for small firms (*size-dependent policy*) and one for young firms (*age-dependent policy*). In each policy experiment, we analyze the associated impacts on firm size and age distributions, lifecycle patterns of firm dynamics, and the relationship between firm dynamics and aggregate outcomes, including output, capital stock, and productivity.

Our main finding is that targeting young firms will be the most efficient productivity-enhancing policy. Targeting all small firms, instead, is relatively less efficient, as they obtain little productivity gains in aggregate, even though the mass of targeted firms is large. Because our credit subsidy on its own is costless, this result is predominantly driven by the fact that capital misallocation is more pervasive among all young firms in our model, rather than among small firms. Particularly, the following features in the model environment deliver our quantitative results. First, most startups in our economy are born with a relatively high leverage ratio compared to that of incumbents. Second, the firm-level productivity process is persistent, implying that a young financially constrained startup needs to maintain its high leverage ratio for a while. The firm then accumulates capital stock until it reaches the efficient scale which is consistent with prices and its own productivity. Third, and related to the second feature, if firms survive long enough and become grown-up, they are less likely financially constrained, regardless of firm size. At this stage, firm size is largely determined by productivity rather than by credit limit. Among old-small firms in our model economy, some firms are small just because their productivity is low, not because they experience financing difficulties. Therefore, targeting

[8] Under the decreasing-returns-to-scale, the number of production units in a model is positively associated with the level of measured total factor productivity, holding aggregate factor inputs fixed. See Buera, Kaboski, and Shin (2011) and Khan, Senga and Thomas (2016) for similar channels at work.

all small firms may not deliver the desired outcome of enhancing aggregate productivity. Instead, because young firms tend to be financially constrained and, more importantly, some of them are in fact highly productive but without sufficient capital, targeting young firms leads to more efficient improvement of aggregate productivity level.

One contribution of our paper is to provide a general equilibrium analysis of the macroeconomic effects of industrial policies. Our general equilibrium model is simple yet rich enough to replicate cross-sectional features of firm dynamics, as seen in the Business Dynamics Statistics (BDS). Our model builds on a family of the standard theory of firm dynamics such as that of Hopenhayn (1992).[9] We investigate firm dynamics as in Cooley and Quadrini (2002), although we consider different specifications of financial frictions. They study non-contingent debt contracts between firms and lenders and show that their model can explain firm dynamics conditional on firm age and size simultaneously. Instead, we match our model with the empirical firm size and age distributions and explore the quantitative implications of industrial policies in aggregate.

Behind our results from policy experiments, there are two main mechanisms at work on the allocative efficiency of resources, one across incumbents (e.g., Restuccia and Rogerson (2008)), and one across entering and exiting firms.[10] The role of financial frictions in generating resource misallocation and its aggregate implications have been studied by Khan and Thomas (2013), Buera and Moll (2015), Buera, Kaboski, and Shin (2011), and Midrigan and Xu (2014), among others.

In this paper, our focus is on the role of policies in alleviating micro-level distortions. Previous studies, in contrast, have looked at policies that distort the allocation of resources. Guner, Ventura, and Xu (2008) considered a model with size-contingent regulations and used their calibrated version of the model to show sizable welfare losses. Gourio and Roys (2014) also studied the aggregate implication of a French policy that distorts the size distribution of firms, showing the important role of such policy distortion in explaining aggregate productivity.[11]

While we focus on targeted credit subsidy policies, there are a variety of government support schemes for smaller businesses. Hence, the argument made in this paper is also relevant to many policy debates around the world. For example, the US Small Business Administration (SBA) provides funding for entrepreneurs that want to start or expand

[9] See Clementi and Palazzo (2016), Sedlacek and Sterk (2016), and Siemer (2016), among others.

[10] Seminal empirical works on the allocative efficiency include Hsieh and Klenow (2009) and Bartelsman, Haltiwanger, and Scarpetta (2013).

[11] See Garicano, Lelarge, and Van Reenen (2016) and Braguinsky, Branstetter, and Regateiro (2011) for econometric applications of size-dependent policies.

small businesses under Advantage Loans 7(a). The volume of loan guarantees in this program has been steadily growing in recent years (USD 24 billion in 2016, compared to USD 14 billion in 2007).

Government-backed loans for SMEs are more pervasive in Japan. The Moratorium Act came into effect in 2009, at the onset of the Great Recession, which allowed small and medium-size firms to suspend repayment of principal and interest backed by government guarantees. Under this scheme, rescheduled loans have grown markedly.[12] Despite the popularity of targeted policies across countries, quantitative studies on the macroeconomic effects of such policies are relatively scarce in the literature.[13] In this regard, we provide simple but informative results from our quantitative policy experiments.

We present our theory in Section 2. We calibrate the theoretical model in Section 3. We then examine the steady state of the benchmark model in Section 4. In Section 5, we analyze the impact of industrial policies, and Section 6 concludes the paper.

2 Model

Time is discrete in infinite horizon. There is a large number of heterogeneous firms producing a homogeneous good. Firms are subject to persistent shocks to individual productivity and face collateral constraints. They, together with endogenous entry and exit, yield substantial heterogeneity in production. Households are identical each other and infinitely-lived.

In the following sub-sections, we present our model economy in detail and examine the optimization problem of firms, followed by the household problem and the definition of a recursive competitive equilibrium. We then characterize the firm-level decisions in a tractable way to render our counterfactual exercises feasible. In particular, we summarize two firm-level states, capital and borrowing, by using a single-state variable defined as cash-on-hand.

[12] The Bill on Provisional Measures for the Facilitation of Financing to Small and Medium-Sized Enterprises (Moratorium Act) came into effect on 4 December 2009. The Moratorium Act requires banks to *"endeavor to take steps to alleviate the burden of debt, upon receipt of a request for the postponement of repayment of debt by a Small and Medium Sized Business,"* and the total loans rescheduled amount to approximately 8 trillion dollars in 2012.

[13] Craig, Jackson, and Thomson (2007) use panel data from SBA backed loans and find a positive impact on regional economic growth.

2.1 Firms

2.1.1 Production and Financial Friction

The economy consists of a continuum of firms. Each firm owns its predetermined capital stock, k, and hires labor, n. The production technology is described by $y = z\epsilon F(k,n)$, where $F(\cdot)$ exhibits the property of decreasing-returns-to-scale (DRS). A firm's productivity is determined by two components, z and ϵ. z is the aggregate level of productivity common across firms and ϵ represents firm-specific idiosyncratic productivity. We assume that z follows a Markov chain such that $z \in \mathbf{Z} \equiv \{z_1, z_2, \ldots, z_{N_z}\}$, $\pi^z_{fg} \equiv Pr(z' = z_g | z = z_f) \geq 0$ and $\sum_{g=1}^{N_z} \pi^z_{fg} = 1$ for each $f = 1, 2, \ldots, N_z$, where primes denote future values. Similarly, $\epsilon \in \mathbf{E} \equiv \{\epsilon_1, \epsilon_2, \ldots, \epsilon_{N_\epsilon}\}$ independently follows a Markov chain with $\pi^\epsilon_{ij} \equiv Pr(\epsilon' = \epsilon_j | \epsilon = \epsilon_i) \geq 0$ and $\sum_{j=1}^{N_\epsilon} \pi^\epsilon_{ij} = 1$ for each $i = 1, 2, \ldots, N_\epsilon$.

In this model, not all firms are able to finance their desired investment due to financial frictions. Each firm faces an individual borrowing limit for obtaining one-period discount debt at price q. This borrowing constraint restricts the amount of newly determined current debt level, b', not to exceed a firm's collateral. This is based on the idea of limited enforceability of financial contracts, and we assume that the firm's future period capital, k', serves as collateral for current borrowing. Therefore, for a firm choosing k' in the current period, the collateral constraint is given by $b' \leq \theta k'$, where the financial parameter, θ, captures the financial frictions at the economy-wide level. Notice that θ is assumed to be common across firms, but the borrowing decision depends endogenously on each firm's state. When θ is close to the real interest rate, $\frac{1}{q}$, the financial market allows firms to invest at their desired level. Note that our specification of collateral constraint is forward-looking in the spirit of Kiyotaki and Moore (1997). In the presence of collateral constraints, firms gradually accumulate capital. Once they achieve a capital level consistent with their expected productivity, they can also accumulate its financial savings, $b' < 0$. This implies that a borrowing constraint is binding for some but not all firms. In this way, the model reproduces the observed heterogeneity in the reliance on external borrowing.

2.1.2 Entry and Exit

To model endogenous entry and exit by firms, we follow the standard approach in the literature. One distinctive feature of our model, however, is the presence of financial frictions at the firm level. Therefore, individual states of productivity, capital, and borrowing jointly affect the exit decisions made by firms. Together with endogenous entry decisions

by potential entrants, this leads to realistic firm dynamics in our model economy.

At the beginning of each period, firms need to pay $\xi_o > 0$ in units of output in order to operate in this or any future period.[14] This fixed cost of operation creates a binary decision of endogenous exit. If a firm does not pay this cost, it has to exit the economy permanently with a value of zero. In addition to the endogenous exit, we impose an exogenous exit with a fixed probability, $\pi_d \in (0, 1)$, after production in each period. This assumption, coupled with financial frictions, prevents firms from growing immediately. Exiting firms must forfeit current flow profit and liquidate its remaining capital; thus, only the firms continuing to the next period make intertemporal decisions about investment and borrowing.

We further assume that there is a fixed measure, M^e, of potential entrants in each period. Before deciding its entry, a potential entrant randomly draws its initial productivity, ϵ_0, and capital, k_0, respectively from a time-invariant distribution, while the initial debt level, b_0, is assumed to be common across potential entrants.[15] When a potential entrant decides to enter, it needs to pay a fixed entry cost, $\xi_e > 0$, in units of output. In this way, we are able to set up a simple binary problem of endogenous entry. Note that firm entry takes place at the end of each period, and the actual entrants start operating in the next period.

2.1.3 Timing and Firm Distribution

At the beginning of a period, a firm is identified by its individual state vector, (k, b, ϵ); the predetermined capital, $k \in \mathbf{K} \subset \mathbb{R}_+$; the amount of debt carried from the previous period, $b \in \mathbf{B} \subset \mathbb{R}$; and the current period idiosyncratic productivity level, $\epsilon \in \mathbf{E}$. We summarize the distribution of firms by a probability measure, $\mu(k, b, \epsilon)$, which is defined on a Borel algebra $\mathcal{S} \equiv \mathbf{K} \times \mathbf{B} \times \mathbf{E}$. We use $s \equiv (z, \theta)$ to denote the aggregate exogenous state with its transition probability, $\pi^s_{lm} \geq 0$. In this way, the aggregate state of the economy is described by (s, μ), and the evolution of μ follows a mapping Γ such that $\mu' = \Gamma(s, \mu)$.

After the exogenous shocks are realized, s and ϵ, at the beginning of a period, a firm decides whether to remain in the economy by paying the fixed operation cost, ξ_o. If it decides to exit, the firm immediately disappears. If it remains, on the other hand, the firm is then informed of its exogenous exit, which takes place at the end of the period. Given

[14] The payment takes place after exogenous productivity shocks are realized but before a firm's exogenous exit status is known. This timing is different from that of Hopenhayn (1992).

[15] We assume that k_0 is drawn from a uniform distribution, $U(0, \bar{k}_0)$. $\bar{k}_0$ is calibrated to reproduce the relative size of entrants to incumbents in the BDS data.

the aggregate and individual states, $(k, b, \epsilon; s, \mu)$, the firm maximizes the sum of its current and future expected discounted dividends. After production, the firm pays its wage bill and repays the existing debt b. Conditional on its survival to the next period, the firm undertakes the intertemporal decisions on investment, i, and borrowing, b', while deciding the current period dividend payments, D, to its shareholders. The capital accumulation of each firm is standard, $i = k' - (1-\delta)k$, with $\delta \in (0, 1)$. In the meantime, potential entrants draw their initial capital and productivity and decide whether to enter by paying the entry cost ξ_e. Entering firms are identified by (k_0, b_0, ϵ_0) and they make their decisions starting from the next period. Markets are perfectly competitive, so firms take the wage rate, $w(s, \mu)$, and the discount debt price, $q(s, \mu)$, as given.

2.1.4 Firm's Problem

Let $v^0(k, b, \epsilon_i; s_l, \mu)$ be the value of a firm at the beginning of the current period, before making its decision to pay the operation cost ξ_o. Accordingly, define $v^1(k, b, \epsilon_i; s_l, \mu)$ as the firm's value of operating, after the endogenous exit decision, but before the exogenous exit status is known. Finally, if a firm survives to the next period, its continuation value is given by $v(k, b, \epsilon_i; s_l, \mu)$. Then the firm's optimization problem can be recursively defined by using v^0, v^1, and v.

$$v^0(k, b, \epsilon_i; s_l, \mu) = \max\left\{0, v^1(k, b, \epsilon_i; s_l, \mu)\right\} \tag{1}$$

$$\begin{aligned} v^1(k, b, \epsilon_i; s_l, \mu) = \pi_d \cdot \max_n \left[z_l \epsilon_i F(k, n) - w(s_l, \mu)n + (1-\delta)k - b - \xi_o\right] \\ + (1 - \pi_d) \cdot v(k, b, \epsilon_i; s_l, \mu) \end{aligned} \tag{2}$$

With the current realization of (s_l, ϵ_i), the firm makes a binary decision of exit over the values of zero *(endogenous exit)*, and $v^1(k, b, \epsilon_i; s_l, \mu)$ *(stay)*, in equation (1). After paying ξ_o, the firm takes a binary expectation over the possibility of exogenous exit given by π_d in equation (2). In the case of exogenous exit, the firm maximizes its liquidation value at the end of the period without considering dynamics decisions. Given the current aggregate state, (s_l, μ), and $\mu' = \Gamma(s_l, \mu)$, the value of continuing to the next period is then given

by $v(k, b, \epsilon_i; s_l, \mu)$ as below.

$$v(k, b, \epsilon_i; s_l, \mu) = \max_{n,k',b',D} \Big[D + \sum_{m=1}^{N_s} \pi^s_{lm} d_m(s_l, \mu) \sum_{j=1}^{N_\epsilon} \pi^\epsilon_{ij} v^0(k', b', \epsilon_j; s_m, \mu') \Big] \tag{3}$$

subject to

$$0 \leq D = z_l \epsilon_i F(k, n) - w(s_l, \mu) n + (1 - \delta) k - b - \xi_o - k' + q(s_l, \mu) b'$$
$$b' \leq \theta k'$$
$$\mu' = \Gamma(s_l, \mu)$$

In (3), the firm optimally chooses its labor demand, n, future capital, k', and new debt level, b', to maximize the sum of the firm's current dividends, D, and the beginning-of-the-period value, $v^0(k', b', \epsilon_j; s_m, \mu')$, in the next period. The current period dividends are the residual in the firm's budget constraint, and we restrict the dividends to be non-negative. The firm's future expected value is discounted by using the stochastic discount factor, $d_m(s_l, \mu)$. The financial friction is introduced as a collateral constraint for the firm's new borrowing in the above maximization problem.

$$v^e(k_0, b_0, \epsilon_0; s_l, \mu) = \max \big\{ 0, -\xi_e + \sum_{m=1}^{N_s} \pi^s_{lm} d_m(s_l, \mu) v^0(k_0, b_0, \epsilon_0; s_m, \mu') \big\} \tag{4}$$

Lastly, we define the value of a potential entrant with (k_0, b_0, ϵ_0) as v^e in (4) above. The potential entrant makes a binary decision to pay or not to pay the entry cost ξ_e. Once it enters, the firm starts operation in the next period given its initial capital and productivity.

2.2 Households and Equilibrium

2.2.1 Representative Household

We assume that there is a unit measure of identical households in the economy. In each period, the households earn their labor income by supplying a fraction of time endowment. The period utility is given by $U(C, 1 - N)$, and the households discount the future utility by a subjective discount factor, $\beta \in (0, 1)$. The representative household holds a comprehensive portfolio of assets; firm shares with measure λ and non-contingent discount bonds ϕ. The household maximizes the lifetime expected discounted utility by choosing the quantities of aggregate consumption demand, C^h, and labor supply, N^h,

while adjusting the asset portfolio in each period. The household value, V^h, is defined as below.

$$V^h(\lambda, \phi; s_l, \mu) = \max_{C^h, N^h, \lambda', \phi'} \left[U(C^h, 1 - N^h) + \beta \sum_{m=1}^{N_s} \pi^s_{lm} V^h(\lambda', \phi'; s_m, \mu') \right] \tag{5}$$

subject to

$$\begin{aligned} & C^h + q(s_l, \mu)\phi' + \int_{\mathcal{S}} \rho_1(k', b', \epsilon'; s_l, \mu)\lambda'(d[k \times b \times \epsilon]) \\ & \quad \leq w(s_l, \mu)N^h + \phi + \int_{\mathcal{S}} \rho_0(k, b, \epsilon; s_l, \mu)\lambda(d[k \times b \times \epsilon]) \\ & \mu' = \Gamma(s_l, \mu) \end{aligned}$$

We apply the conventional notation for the stock prices. In (5), $\rho_1(k', b', \epsilon'; s_l, \mu)$ denotes the ex-dividend prices of firm shares in the current period, and $\rho_0(k, b, \epsilon; s_l, \mu)$ is the dividend-inclusive values from the current shareholding, λ. Let $\Phi^h(\lambda, \phi; s, \mu)$ be the household's decision for bond holding and $\Lambda^h(k', b', \epsilon', \lambda, \phi; s, \mu)$ be the choice of firm shareholding corresponding to future state (k', b', ϵ').

2.2.2 Recursive Competitive Equilibrium and Prices

We define a recursive competitive equilibrium of the economy. For simplicity, we denote the distribution of producing firms as μ^p, the measure of actual entrants as μ^e, and the measure of endogenously exiting firms as μ^{ex}. Whenever possible, we suppress the arguments of functions in the definition below.

A *recursive competitive equilibrium* is a set of functions including prices $\left(w, q, (d_m)_{m=1}^{N_s}, \rho_0, \rho_1\right)$, quantities $\left(N, K, B, D, C^h, N^h, \Phi^h, \Lambda^h\right)$, and values $\left(v^0, v^1, v, v^e, V^h\right)$ that solve the optimization problem and clear markets; the associated policy functions are consistent with the aggregate law of motion as in the following conditions.

1. v^0, v^1, and v solve equations (1)–(3), and (N, K, B, D) are the associated policy functions for firms.
2. V^h solves (5), and $(C^h, N^h, \Phi^h, \Lambda^h)$ are the associated policy functions for households.
3. The labor market clears, $N^h = \int_{\mathcal{S}} N(k, \epsilon; s, \mu) \cdot \mu^p(d[k \times b \times \epsilon])$.

4. The goods market clears.

$$C^h = \Bigg(\int_{\mathcal{S}} \Big[z\epsilon F(k,N) - (1-\pi_d)\big(K(k,b,\epsilon;s,\mu) - (1-\delta)k\big) + \pi_d(1-\delta)k - \xi_o \Big] \cdot \mu^p(d[k \times b \times \epsilon]) \Bigg) + \int_{U(0,\bar{k}_0)} (k_0 - \xi_e) \cdot \mu^e(d[k_0 \times b_0]) - \int_{\mathcal{S}} k \cdot \mu^{ex}(d[k \times b \times \epsilon])$$

5. The law of motion for the firm distribution, μ, is consistent with the value functions, (v^0, v^1, v, v^e), and the measures, (μ^p, μ^e, μ^{ex}), and with the associated policy functions, where Γ defines the mapping from μ to μ' given π_d, $K(k,b,\epsilon;s,\mu)$, and $B(k,b,\epsilon;s,\mu)$.

As noted by Khan and Thomas (2013), it is convenient to modify the firm's value functions, (v^0, v^1, v, v^e), by using the equilibrium prices at the market clearing conditions in the above. Let C and N be the market clearing quantities for aggregate consumption and labor supply from the perspective of households. Then the equilibrium value of output goods can be expressed by using the marginal utility of consumption, $D_1U(C, 1-N)$. Likewise, the real wage, $w(s,\mu)$, is equal to the marginal rate of substitution between leisure and consumption. Next, the inverse of the discount bond price, q^{-1}, equals the expected gross real interest rate. Lastly, the stochastic discount factor, $d_m(s_l,\mu)$, is equal to the household's intertemporal marginal rate of substitution across states. The following equations summarize the equilibrium prices in terms of marginal utilities.

$$\begin{aligned} w(s,\mu) &= \frac{D_2U(C,1-N)}{D_1U(C,1-N)} \\ q(s,\mu) &= \frac{\beta \sum_{m=1}^{N_s} \pi^s_{lm} D_1U(C'_m, 1-N'_m)}{D_1U(C,1-N)} \\ d_m(s,\mu) &= \frac{\beta D_1U(C'_m, 1-N'_m)}{D_1U(C,1-N)} \end{aligned}$$

By using the above price expressions, we solve the equilibrium allocations directly from the firm's problem in a consistent manner with the household's optimal decisions. Let $p(s,\mu) \equiv D_1U(C,1-N)$ denote the marginal utility of consumption at C and N. Then firms value their current output and dividends by $p(s,\mu)$, and the value functions, (v^0, v^1, v, v^e), can be re-written in terms of $p(s,\mu)$ without carrying the stochastic discount factor. In particular, we define new value functions, (V^0, V^1, V, V^e), by respectively multiplying $p(s,\mu)$ by the original values. Then the following equations (6)–(9) newly

summarize the firm's problem.

$$V^0(k, b, \epsilon_i; s_l, \mu) = \max\left\{0, V^1(k, b, \epsilon_i; s_l, \mu)\right\} \tag{6}$$

$$\begin{aligned} V^1(k, b, \epsilon_i; s_l, \mu) = \pi_d \cdot \max_n p(s_l, \mu)\left[z_l \epsilon_i F(k, n) - w(s_l, \mu)n + (1-\delta)k - b - \xi_o\right] \\ + (1 - \pi_d) \cdot V(k, b, \epsilon_i; s_l, \mu) \end{aligned} \tag{7}$$

$$V(k, b, \epsilon_i; s_l, \mu) = \max_{n, k', b', D}\left[p(s_l, \mu)D + \beta \sum_{m=1}^{N_s}\sum_{j=1}^{N_\epsilon} \pi^s_{lm}\pi^\epsilon_{ij} V^0(k', b', \epsilon_j; s_m, \mu')\right] \tag{8}$$

subject to

$$\begin{aligned} &0 \leq D = z_l \epsilon_i F(k, n) - w(s_l, \mu)n + (1-\delta)k - b - \xi_o - k' + q(s_l, \mu)b' \\ &b' \leq \theta k' \\ &\mu' = \Gamma(s_l, \mu) \end{aligned}$$

$$V^e(k_0, b_0, \epsilon_0; s_l, \mu) = \max\left\{0, -p(s_l, \mu)\xi_e + \beta \sum_{m=1}^{N_s} \pi^s_{lm} V^0(k_0, b_0, \epsilon_0; s_m, \mu')\right\} \tag{9}$$

For the remainder of this paper, we occasionally suppress the aggregate state (s, μ) in the price functions and the decision rules for simplicity.

2.3 Characterizing Firm-Level Decisions

2.3.1 Firm Types

To characterize the firm-level decision rules in the model, it is convenient to first define a subset of firms in the distribution whose decisions are not affected by the collateral constraints in any possible future state. In particular, we follow the approach of Khan and Thomas (2013) to distinguish firm types in our model economy. Note that this distinction is solely for deriving all the intertemporal decisions made by firms and that a firm may alter its type depending on the state variables.

We define a firm as *unconstrained* when it has already accumulated sufficient wealth such that it never experiences binding borrowing constraints in any possible future state. In this case, all the unconstrained firm's Lagrangian multipliers on its borrowing constraints become zero, and the firm is indifferent between paying dividends and retained earnings. On the other hand, the remaining firms in the distribution are defined as *con-*

strained. Constrained firms may or may not experience binding borrowing constraints in the current period; thus, they choose to pay zero dividends in the current period as the shadow value of retained earnings is greater than that of paying dividends.

First, we derive the optimal static labor choice, which applies to all firms. Labor choices are frictionless and a firm with (k, ϵ) chooses $n = N^w(k, \epsilon; s, \mu)$, which solves the static labor condition, $z\epsilon D_2 F(k, n) = w(s, \mu)$.

Next, we derive the choice of future capital, k', by the unconstrained firms. The collateral constraint is irrelevant for this type of firm, so we can easily derive their optimal level of $k' = K^w(\epsilon; s, \mu)$ as follows. Let $\Pi^w(k, \epsilon; s, \mu) \equiv z\epsilon F(k, N^w) - wN^w$ be the current earnings of a firm with its optimal labor hiring N^w. Given the Markov property for the assumed stochastic processes and the absence of capital adjustment costs, K^w is the solution to the following problem.

$$\max_{k'} \Big[-p(s_l, \mu)k' + \beta \sum_{m=1}^{N_s} \pi^s_{lm} p(s_m, \mu') \sum_{j=1}^{N_\epsilon} \pi^\epsilon_{ij} \big(\Pi^w(k', \epsilon_j; s_m, \mu') + (1-\delta)k' \big) \Big]$$

With the policy functions N^w and K^w, as in Khan and Thomas (2013), the *minimum savings policy*, $b' = B^w(\epsilon; s, \mu)$, is recursively defined by the following two equations.

$$B^w(\epsilon_i; s_l, \mu) = \min_{(s_m, \epsilon_j)_{j,m}} \Big(\tilde{B}\big(K^w(\epsilon_i, s_l, \mu), \epsilon_j; s_m, \mu'\big) \Big) \tag{10}$$

$$\begin{aligned} \tilde{B}(k, \epsilon_i; s_l, \mu) = {} & z_l \epsilon_i F(k, N^w) - wN^w - \xi_o + (1-\delta)k - K^w(\epsilon_i) \\ & + q \min \big\{ B^w(\epsilon_i; s_l, \mu), \theta K^w(\epsilon_i) \big\} \end{aligned} \tag{11}$$

$\tilde{B}(K^w, \epsilon_j; s_m, \mu')$ in (10) denotes the maximum level of debt (or the minimum level of saving) that an unconstrained firm can hold at the beginning of the next period in which the exogenous state (s_m, ϵ_j) is realized. Having chosen the unconstrained choice of capital, K^w, at the current period, the firm will remain unconstrained in the subsequent periods by definition. The minimum savings policy, B^w, ensures that the firm's debt never exceeds this threshold level, $\tilde{B}$, given all possible realizations of (s, ϵ). Moreover, the threshold function can be retrieved by using B^w and K^w at the current period state with $(k, \epsilon_i; s_l, \mu)$, as in (11). Notice that the minimum operator in (11) reflects the collateral constraint in the firm's problem. In (10), B^w again is determined to have the unconstrained firms unaffected by the constraint over any future path of (s, ϵ).

2.3.2 Cash-on-Hand and Decision Rules

The incumbent firm's problem in (8) is a challenging object because of the occasionally binding constraints for D and b'. In addition, notice that a firm's individual state vector includes two endogenous variables, (k, b), with each having a continuous support. However, levels of k and b of firms do not separately determine the choices of k' and b'. This is due to the absence of any real adjustment costs in the model, and therefore we can collapse these two continuous individual state variables into a newly defined variable called *cash-on-hand*. We define the cash-on-hand, $m(k, b, \epsilon)$, of a firm with (k, b, ϵ) by using the optimal labor demand N^w.

$$m(k, b, \epsilon; s, \mu) \equiv z\epsilon F(k, N^w) - wN^w + (1-\delta)k - b$$

Notice that the decisions of k' and b' made by continuing incumbents determine the level of cash-on-hand in the future period, $m(k', b', \epsilon'; s', \mu')$, along with the realization of (ϵ', s'). By using this new state variable, we rewrite the incumbent firm's problem in (6)–(8) using the following values W^0, W^1, and W.

$$W^0(m, \epsilon_i; s_l, \mu) = \max\left\{0, W^1(m, \epsilon_i; s_l, \mu)\right\} \tag{12}$$

$$W^1(m, \epsilon_i; s_l, \mu) = \pi_d \cdot p(s_l, \mu) \cdot (m - \xi_o) + (1 - \pi_d) \cdot W(m, \epsilon_i; s_l, \mu) \tag{13}$$

$$W(m, \epsilon_i; s_l, \mu) = \max_{m', k', b', D} \left[p(s_l, \mu)D + \beta \sum_{m=1}^{N_s} \sum_{j=1}^{N_\epsilon} \pi^s_{lm} \pi^\epsilon_{ij} W^0(m'_{jm}, \epsilon_j; s_m; \mu') \right] \tag{14}$$

subject to

$$\begin{aligned}
&0 \leq D = m - \xi_o - k' + q(s_l, \mu)b' \\
&b' \leq \theta k' \\
&m'_{jm} \equiv m(k', b', \epsilon_j; s_m, \mu') \\
&\quad = z_m \epsilon_j F(k', N^w(k', \epsilon_j)) - w(s_m, \mu')N^w(k', \epsilon_j) + (1-\delta)k' - b' \\
&\mu' = \Gamma(s_l, \mu)
\end{aligned}$$

Given the unconstrained policies K^w and B^w, we define a threshold level of m for each ϵ that distinguishes the unconstrained firms. From the firm's budget constraint in (14), an unconstrained firm pays the current dividends, $D^w = m - \xi_o - K^w + qB^w \geq 0$. It follows that this firm's cash-on-hand is greater than or equal to a certain threshold level,

$\tilde{m}(\epsilon; s, \mu) \equiv \xi_o + K^w(\epsilon; s, \mu) - q(s, \mu)B^w(\epsilon; s, \mu)$. Therefore, any firm with $m(k, b, \epsilon; s, \mu) < \tilde{m}(\epsilon; s, \mu)$ can be identified as constrained.

Recall that, in any given period, some constrained firms experience currently binding borrowing constraints while the others do not. We call the latter constrained firms *Type-1*, and the firms with currently binding constraints *Type-2*. Notice that Type-1 firms can still adopt the unconstrained capital policy, K^w, but not the minimum savings policy, B^w. The debt policy of Type-1 firms can be easily determined by the zero-dividend policy after substituting $k' = K^w$ in the budget constraint. On the other hand, Type-2 firms can only invest to the extent allowed by their borrowing limits. This constrained choice of capital is derived from the level of cash-on-hand, m, held by Type-2 firms. Specifically, $D = 0$ implies that a Type-2 firm's decision of k' and b' must be feasible, given the firm's available cash-on-hand after paying the fixed operation cost. Given m, we define the upper bound of a Type-2 firm's capital choice as $\bar{K}(m) \equiv \frac{m - \xi_o}{1 - q\theta}$, from the binding collateral constraint. Firms with more cash-on-hand, therefore, can relax this upper bound until they can choose the unconstrained capital policy, K^w. Notice also that the upper bound, $\bar{K}(m)$, approaches infinity as the financial parameter θ becomes closer to q^{-1}, which illustrates the case of perfect credit markets. Below, we summarize the decision rules of k' and b' by firm type given (m, ϵ).

- Firms with $m \geq \tilde{m}(\epsilon; s, \mu)$ are *unconstrained* and therefore adopt $K^w(\epsilon)$ and $B^w(\epsilon)$.
- For *constrained* firms with $m < \tilde{m}(\epsilon; s, \mu)$, the upper bound for k' is $\bar{K}(m) \equiv \frac{m - \xi_o}{1 - q\theta}$.
  - Firms with $K^w(\epsilon) \leq \bar{K}(m)$ are *Type-1* and adopt $k' = K^w(\epsilon)$ and $b' = \frac{1}{q}\big(K^w(\epsilon) + \xi_o - m\big)$.
  - Firms with $K^w(\epsilon) > \bar{K}(m)$ are *Type-2* and adopt $k' = \bar{K}(m)$ and $b' = \frac{1}{q}\big(\bar{K}(m) + \xi_o - m\big)$.

3 Model Parameters

We parameterize the model and quantitatively investigate the macroeconomic implications of targeted industrial policies. The model is annual, and we set the model parameter values of our benchmark economy to jointly match the key macroeconomic moments and the rich firm-level heterogeneity observed in the US data. Table 1 summarizes the calibrated parameter values.

The period utility function of the representative household is standard as in the literature. We specifically assume the functional form of indivisible labor following Rogerson

(1988), $U(C, 1-N) = \log C + \psi(1-N)$. The decreasing-returns-to-scale (DRS) production function is in Cobb-Douglas form, $F(k, n) = k^{\alpha} n^{\nu}$ with $\alpha, \nu > 0$, and $\alpha + \nu < 1$.

The value of the household subjective discount factor, β, is set to imply the long-run real interest rate of 4 percent. The preference parameter of labor disutility, ψ, is set to match the average hours worked at 0.33. The annual depreciation rate, δ, is chosen to match the average aggregate investment to capital ratio of the postwar US economy. The curvature of capital input in the production function, α, is set to imply the average capital to output ratio of 2.3. The other production parameter, ν, is set to have the average share of labor income of 0.6, following Cooley and Prescott (1995). For the time series of aggregate investment, output, and private capital stock, we use the Fixed Asset Tables and National Income and Product Accounts (NIPA) from the Bureau of Economic Analysis (BEA).

We jointly determine the rest of model parameter values $(\theta, \xi_o, \xi_e, \pi_d, \epsilon_m, \epsilon_M, \gamma_\epsilon, \rho_\epsilon)$, to replicate the aggregate debt to asset ratio, the average exit rate of firms, and the size and age distributions of firms. In addition, we maintain the number of producing firms at unit measure at the steady state of the model. The financial parameter in the collateral constraint, θ, is set to be 0.9, to be consistent with the average debt to asset ratio of the non-farm, non-financial businesses in the Flow of Funds from 1954 to 2007. The period operation cost, ξ_o, and the fixed entry cost, ξ_e, are set to imply about 10 percent of the total firm exit rate, along with the exogenous exit rate, π_d. We use the data tables from the BDS to calculate the average total exit rate of private firms in the United States. The BDS also reports the empirical firm size and age distributions in detailed size and age groups, which we divide into 6 employment size bins and 11 age bins.

We assume that the idiosyncratic productivity, ϵ, is drawn from a time-invariant distribution, $G(\epsilon; \epsilon_m, \epsilon_M, \gamma_\epsilon)$, which is a bounded Pareto distribution.[16] In each period, a firm in our model economy retains its previous level of individual productivity with a fixed probability of ρ_ϵ. We set $\rho_\epsilon = 0.75$ to be consistent with the evidence on the average persistence of firm-level productivity in the data.[17] The bounds of ϵ support, (ϵ_m, ϵ_M), and the shape parameter, γ_ϵ, of the Pareto distribution are chosen to have both the employment share and the population share for each firm size bin aligned with the corresponding average values reported in the BDS from 1977 to 2007. We discretize ϵ using 11 values in our numerical applications.

[16] Jo (2015) shows that the bounded Pareto distribution requires a minimal set of parameters to replicate the empirical firm size distribution.

[17] Our choice of the persistence value is consistent with the estimates in Foster, Haltiwanger, and Syverson (2008).

Table 1 : Calibration

Model Parameters				
	Value	Description and Target Value		Model
β	0.96	Annual gross real interest rate	1.04	$1/q = 1.04$
ν	0.60	Labor share of income	0.60	
ψ	2.408	Average hours worked	0.33	$N = 0.3337$
δ	0.069	Investmet to capital ratio (BEA)	0.069	$I/K = 0.069$
α	0.272	Capital to output ratio (BEA)	2.30	$K/Y = 2.2443$
Calibrated to jointly match the following data moments, firm size and age distributions				
θ	0.90	Debt to Asset ratio (Flow of Funds)	0.36	$D/A = 0.3401$
π_d	0.0874	Average firm exit rate (BDS)	0.11-0.13	Exit rate = 0.1005
ξ_o	0.01382	Steady state measure of firms	1.0	$\int \mu \cdot d\mu = 1.0069$
ξ_e	0.1522			
ρ_ϵ	0.75			
ϵ_m	0.43			
ϵ_M	1.15			
γ_ϵ	4.16			

4 Results: Steady State

4.1 Firm Heterogeneity and Decisions

We begin by describing the stationary distribution of firms in the model. Figure 1 shows the entire firm distribution over capital and debt levels, (k, b), at the steady state. It displays all three types of firms that we considered in the previous section: unconstrained, Type-1, Type-2 firms. The unconstrained firms are located at each endpoint-mass of negative debt levels of the figure. These negative debt levels represent the minimum savings policy, $B^w(\epsilon)$, taken by this type of firm. The constrained firms are distributed across capital and debt, and the majority are relatively small with positive leverages.

The above firm types are more easily distinguishable when we look at the snapshot of decision rules on capital, k', debt, b', and dividends, D, as functions of a firm's cash-on-hand level, m. Figure 2 shows these firm-level decision rules at a given productivity value, ϵ_8. In the figure, the level of cash-on-hand, $m(k, b, \epsilon)$, is on the horizontal axis, and we add the two vertical lines to distinguish the firm types. The vertical line near $m = 56$ represents the threshold for being unconstrained, $\tilde{m}(\epsilon)$, and the one near $m = 2$ is the threshold for Type-1 firms. Starting from the right-hand side of the figure, when a firm has survived and accumulated sufficient wealth over time such that $m \geq \tilde{m}(\epsilon)$, it is considered unconstrained. The firm then adopts the unconstrained choices of capital, $K^w(\epsilon)$, and debt, $B^w(\epsilon)$, and starts paying positive dividends. Constrained firms with

m less than the threshold value, on the other hand, follow the zero-dividend policy to accumulate their internal savings toward being unconstrained. Type-1 firms between the two thresholds can still adopt the optimal level of capital, $K^w(\epsilon)$, while gradually reducing debt as their m increases. Lastly, Type-2 firms with small m are only able to invest up to their borrowing limits, so their choice of capital is constrained with positive borrowing.

As discussed earlier, the level of cash-on-hand is critical in determining a firm's optimal decisions of investment and borrowing. Upon entry, young firms in our model start relatively small and then gradually accumulate their cash-on-hand over time. These lifecycle dynamics of firms are illustrated in Figure 3. At age 0, an average firm in our model economy is financially constrained because it is short on collateral for external financing. Thus, the firm keeps raising its external debt level until age 4, and then gradually de-leverages once it can finance the optimal level of investment for $K^w(\epsilon)$ around age 6. In addition, firms still accumulate financial savings even after age 18, so that they become unconstrained and pay positive dividends. This is represented by the hump-shaped leverage curve in the lower panel of the figure. Any policy targeting firms in a specific age group will therefore shift the average lifecycle dynamics in Figure 3, which eventually re-shapes the entire firm distribution in the model economy. Further, such policy will also affect the entry and exit margins of firm decision by affecting the equilibrium prices.

Next, we compare the model-generated firm size and age distributions with those observed in the BDS database. As is evident in Figure 4, our model economy exhibits substantial firm-level heterogeneity that is endogenously determined by the combination of persistent shocks, financial frictions, and firm dynamics. In the upper panel of the figure, we divide firm employment size into 6 bins and report the average population share in each bin from 1977 to 2007 in the BDS. As is well known in the literature, the empirical firm size distribution is highly skewed, where more than 88 percent of firms hire fewer than 20 employees in a given year. We almost perfectly replicate this lower tail of firm size distribution in our benchmark economy by using the Pareto-distributed ϵ process. By the nature of our collateral constraint, small firms in the size distribution are more likely to be financially constrained when their productivity is expected to rise, while the accumulated cash-on-hand is insufficient to cover their investment. In the lower panel of Figure 4, we report the age distribution of firms generated from the model. As observed in the data, the firm age distribution in our model preserves the decreasing population shares as firms become mature.[18] These realistic firm size and age distributions in the benchmark economy serve as the foundation for our policy exercises aimed at small and

[18] We use the average population shares between 1993 and 2007, due to the censored firms in the BDS.

young firms. Therefore, replicating the observed firm heterogeneity is necessary before quantitatively examining the aggregate consequences of such policies.

Lastly, we look at the endogenous exit decision by firms at the steady state of the model. Figure 5 shows the exit choice (=1) over capital and debt at the ergodic distribution of ϵ. Consistent with conventional knowledge, a firm in the model decides to exit when it accumulates relatively larger debt than its existing capital stock. This is because the future expected value of such over-leveraged firm falls below 0, and the firm thus finds it better to exit before paying the fixed operation cost at the beginning of a period. Young firms with both low productivity and small wealth are easily tempted to this outside option of exit, so we are also able to capture the disproportionately high exit rates among those firms.

5 Results: Targeted Policies

5.1 Overview of Counterfactual Exercises

We next examine the aggregate consequences of credit subsidies. Our counterfactual policy exercise is to compare the pre-intervention equilibrium described above and the post-intervention equilibrium, which is the new equilibrium reached by the economy after implementation of credit subsidy policies. Policy implementation works as follows. The government selects a group of target firms and provides them credit subsidies. If a firm faces binding collateral constraints, this credit subsidy policy helps it to achieve its optimal level of capital stock, by setting $\theta = q^{-1} > \theta_{ss}$.[19] We assume that the government can exactly identify firms with binding borrowing constraints to help only such firms. Therefore, the actual number of subsidized firms is a subset of each targeted group.[20] We consider two different target groups, small firms (*size-dependent policy*) and young firms (*age-dependent policy*). The *size-dependent policy* selects small-medium sized firms (SME), defined as those hiring fewer than 500 employees.[21] The *age-dependent policy*

[19] This implies that the government subsidizes the additionally required investment for the targeted firms such that they can obtain an efficient scale of production at a given level of productivity.

[20] This is a different approach from that taken by Guner, Ventura, and Xu (2008). They investigate the effects of distortionary taxes on factor input uses. Once a firm falls into the targeted group in their model, its size is always restricted by such policies. In our approach, however, only firms with binding constraints are subsidized. In each exercise, we maintain the model parameter values at the benchmark economy except θ, and then find new equilibrium allocations.

[21] Our definitions of small and young firms are consistent with Fort et al. (2013). In addition, the firms approved for business loans by the US Small Business Administration mostly hire more than 500 employees in most industries.

selects firms with age between 0 and 4. The rest of the firms that are not given credit subsidies may be constrained by the borrowing limits with θ at the benchmark economy.

We assume that subsidized capital is financed by lump-sum cash transfers from households. Therefore, no resource costs arise from policy implementation. While we abstract from potentially important distortions such as tax distortions that are caused by policymakers, this framework allows us to highlight the unintended effects through general equilibrium price movements.[22] Importantly, the total amount of cash transfers will be determined in equilibrium as policies reshape the distribution of firms and change the level of unconstrained capital, $K^w(\epsilon)$. To compute the total cash transfers under a certain policy, we isolate the firms with binding borrowing constraints and then calculate the gap between those firms' efficient investment, i, and the constrained level of investment without the policy, i^B.[23] The aggregate cash transfer, rt, is computed by aggregating the gap between i and i^B:

$$rt(p) = \int_{\mu^B} (i - i^B) d\mu^B,$$

where μ^B denotes the measure of subsidized firms with binding borrowing constraints on $\mu(k, b, \epsilon)$.

In relative to the benchmark economy, we measure the aggregate gains in measured total factor productivity, ΔTFP, under each credit subsidy policy. We also consider the productivity gain normalized by the total amount of cash transfers, $\frac{\Delta TFP}{rt}$, to compare the effectiveness of different targeted policies.

5.2 Aggregate Results

This sub-section presents our counterfactual analysis of three different targeted policies. We examine the aggregate effects of each credit subsidy policy and explain the direct effect and the indirect general equilibrium effects. As in Table 2, all policies improve aggregate outcomes in the economy. Specifically, the age-dependent policy raises aggregate output by 2.5 percent from the benchmark case. Aggregate output under the size-dependent policy that targets small-medium sized firms (SMEs) increases by 3.3 percent from the benchmark case. The last column of Table 2 shows the impact of the size-dependent policy that targets only small firms with 20 or fewer employees. This policy increases aggregate output by 2.6 percent, implying that the gains from credit subsidy

[22] See Buera, Moll, and Shin (2013) for long-run consequences of taxes and subsidies.

[23] Note that the market clearing prices in turn affect the firm distribution in our model economy when θ is adjusted. In particular, changes in $K^w(\epsilon)$ and $\bar{K}(m)$ alter the composition of firm types in equilibrium among unconstrained, Type-1, and Type-2 firms.

policies mainly come from the smallest firms in the economy.

Table 2 : Aggregate Results from Policy Experiments

Policy Counterfactual: Aggregates				
	Benchmark	Age-dependent (age 0 to 4)	Size-dependent (SMEs)	Size-dependent (small only)
C	0.2103	0.2147 (2.11)	0.2157 (2.54)	0.2150 (2.23)
K	0.6322	0.6675 (5.58)	0.6817 (7.83)	0.6688 (5.79)
Y	0.2817	0.2887 (2.49)	0.2911 (3.34)	0.2890 (2.59)
N	0.3337	0.3350 (0.39)	0.3363 (0.78)	0.3349 (0.36)
D/A	0.3401	0.3664	0.3692	0.3658
firms (μ)	1.006848	0.978711	0.982922	0.981148
endo. exit rate	0.013172	0.020803	0.024077	0.020207
rt	–	0.136042	0.175993	0.129888
μ subsidized	–	0.041864	0.089157	0.073906
$\Delta TFP(\%)$	(0.616511)	0.746149	0.769246	0.815537
$\Delta TFP/rt$	–	0.033814	0.026947	0.038709
avg. rt	–	3.249618	1.973967	1.757476
$\Delta TFP_{\mu}(\%)$	–	1.112317	1.079934	1.149754

Note: We use (α, ν) values at the benchmark to compute the measured TFP under each policy. Values in parentheses are percentage changes from their benchmark values.

These aggregate improvements are largely driven by the reduced resource misallocation across firms. In our model, collateral constraints prevent small but productive firms from undertaking investment at their desired level. Insufficient capital in those firms reduces aggregate total factor productivity (TFP). Therefore, providing credit subsidies to those firms undoes capital misallocation, which in turn leads to increased aggregate productivity. In Table 2, we report these gains in measured TFP (ΔTFP) following each targeted policy, and the productivity improvement ranges from 0.75 to 0.82 percent. These gains are obtained by subsidizing firms with binding borrowing constraints, which accounts for about 4 to 9 percent of the entire population.

The number of firms in production falls following each policy implementation, as will be clear below. Because the size of these falls is different among the policies, we calculate the productivity gains per firm, ΔTFP_{μ}, to control for the differences in the number of firms in each policy regime. The productivity gain per firm is 1.11 percent under the age-dependent policy; 1.07 percent under the size-dependent policy that targets SMEs, and 1.14 percent under the size-dependent policy that targets only small firms. This also confirms that the marginal productivity gains are larger for the policy that targets firms in the smallest size category.

We also normalize the productivity gains by the size of credit transfers from households, ($\Delta TFP/rt$). As in Table 2, the size-dependent policy that targets only small firms leads to the largest productivity gain. The age-dependent policy is less effective in improving TFP per each unit of credit transfer. These results shed light on the importance of targeting the group of firms whose marginal gains are large in achieving aggregate productivity improvement in practice.[24]

In addition to the direct effects from undoing capital misallocation, providing credit subsidies has indirect effects on the general equilibrium as follows. The first effect arises from entry and exit margins. In labor markets, due to the boosted aggregate productivity, higher labor input by the subsidized firms leads to an increase in the equilibrium wage rate. The rise in factor price puts more pressure on firms with low profitability to exit. This mechanism is represented by the higher endogenous exit rates under policies in Table 2, ranging from 2 to 2.4 percent in each period. Moreover, the higher wage rate also affects the entry decision by potential firms, which strengthens the selection among them. The potential entrants observe higher profitability under each policy that relaxes borrowing constraints, but the increased cost of production prevents firms with low productivity from entering. As will be discussed in the next sub-section on firm dynamics, the selection effect on the entry decision dominates, and hence the average productivity of actual entrants increases in our policy experiments. The persistence of idiosyncratic firm productivity implies these entrants will maintain a higher productivity level during their early lifecycle stages, which further improves the aggregate TFP. Therefore, our policies targeting only incumbents adjust firm entry and exit margins actively through the general equilibrium channel. This accelerates *the cleansing effect of young firms replacing less productive incumbents*.[25] This yields additional gains from such policies in our model economy.

The second effect, which is a result of the first effect, is that the number of firms falls, which in turn depresses the aggregate productivity. Due to decreasing returns to scale production technology at the firm level, the number of producing firms is a non-trivial factor that affects measured aggregate total factor productivity. Therefore, fewer firms in operation following targeted policies dampens the aggregate gains in productivity. This negative impact on TFP in each policy can be represented by the difference of the relative gains in TFP (ΔTFP) and the average productivity per firm (ΔTFP_μ) in Table 2. In

[24] The information asymmetry between firms and the government, for example, may further distort credit allocations. We do not consider this case, but the age-dependent policy may partly overcome this issue, given firm registry data.

[25] This is a different perspective from that in the literature on creative destruction, in Caballero and Hammour (1994). In that study, the cleansing effect of recessions comes from replacing low productive-outdated production units with innovative ones.

every counterfactual policy experiment that we report in Table 2, the total number of firms decreases by 1.7 to 2.1 percent from the benchmark economy.

In sum, we are able to identify both direct and indirect effects of credit subsidy policies targeting small and young firms. As expected, a targeted policy directly resolves the resource misallocation faced by small or young firms. However, such policy also has unintended effects that indirectly emerge from the equilibrium price channel. In particular, the increased price under each policy has three effects at the firm level that in turn affects the aggregate results. First, it reduces the efficient scales of production for all firms. Next, the rise in price also adjusts the firm entry and exit margins to determine the relative size of the cleansing effect resulting from each policy. The last effect comes from the change in the total number of firms, which lowers the aggregate productivity gains from credit subsidy policies. Together, these direct and indirect effects of a credit subsidy policy reshape firm dynamics and determine the aggregate consequences in our model economy.

5.3 Firm Dynamics

The credit subsidy policies have disparate impact on different firms. Clearly, the impact of the age-dependent policy on young firms is different from its impact on old firms. In this sub-section, we examine how firm dynamics are affected by each credit subsidy policy by looking at the average size and productivity of firms for each age group and tracking them over the lifecycle.

Figure 6 compares the average firm dynamics under each different policy. The upper-left panel of the figure shows the typical growth pattern of entrants in our benchmark economy (*blue line*). First, firms, on average, start small without any policy intervention. The relative size of entrants is about one-fourth of that of mature firms at age 20. Upon entry, entrants start accumulating capital stock and gradually become larger as they age. As they become older, selection forces unproductive firms to exit from the economy. Hence, as shown in the upper-right panel of Figure 6, the average productivity of firms increases over time as they age. Specifically, the productivity of age-0 firms in our benchmark economy is about 10 percent lower than that of firms at age 20.

Under the age-dependent credit subsidy policy, the relative size of entrants is almost the same as that of mature firms in Figure 6 (*red line*). This is because the age-dependent policy removes borrowing constraints for firms aged 0 to 4, and this will help them to achieve the optimal scale of production. Until age 5, the entrants keep growing even larger than the mature firms that face borrowing constraints. Thus, the young firms have

the largest borrowing upon entry and then gradually de-leverage over time, conditional on survival (lower-left panel). In addition, from the upper-right panel of Figure 6, the productivity of age-0 firms under the age-dependent policy is larger than that of its counterpart in the benchmark case. It follows that our policy intended for incumbents induces a strong selection among the potential entrants, so that relatively more productive firms actually enter and start operating, which in turn lifts the aggregate productivity. This is mainly through the general equilibrium price feedback on firms' entry decision, and we will further discuss this in the next sub-section. Once firms pass the age of 5, the average size reverts back to the level under borrowing constraints. However, since those firms were able to accumulate more cash-on-hand under this policy, they tend to start saving faster in terms of financial assets after age 5, as illustrated by the steeper slope of de-leveraging over time. This implies that, at age 5, firms are better prepared for idiosyncratic risks due to their lower leverage ratio, which is a relatively sound financial position. Hence, we confirm that our age-dependent policy further supports the young-productive firms by allowing them to survive longer, similar to *selectively protecting infant firms*. This view is shared by Moll (2014), when firm-level productivity shocks are persistent. Moll (2014) distinguishes between the loss from misallocation at the steady state and that during the transition. With persistent productivity shocks, he shows that the loss at the steady state is relatively small, while the economy stays longer in transition. In this regard, our policy exercises consider the possibility of expediting the transition process by allowing firms to self-finance.

In the case of subsidizing SMEs (*green line* in Figure 6), on the other hand, we observe the same typical firm dynamics patterns as in the benchmark. Indeed, the size-dependent credit subsidy policy for SMEs in our counterfactual experiment has only level effects on the average size and productivity by firm age.[26] In particular, the policy helps entrants start relatively large and then allows them to mature faster. This is mainly driven by the higher productivity, on average, in the early stage of a firm's lifecycle. These level effects are mainly due to the nature of our size-dependent policy, which only affects financially constrained SMEs at each point of time in the model. That is, whenever a productive-small firm hits its borrowing limit, regardless of age, the policy effectively finances the firm's optimal investment at a level matching its productivity. The equilibrium price effects on firm entry and exit decisions still exist as in the age-dependent policy, but the selection among potential entrants is relatively less pronounced, which is represented by

[26] We plot the average capital and productivity in Figure 6 as their relative values at each age compared to mature firms at age 20. In absolute values, the results from the size-dependent policy (*green line*) are almost parallel shifts from their counterparts in the benchmark (*blue line*).

the initial average productivity of age-0 firms.

Figure 7 compares the distribution of firm size and age across policy regimes. From our results on firm dynamics in the above, it is evident that both types of targeted credit subsidy policies raise the population share of the smallest firm size group (upper panel) by a similar magnitude. On the other hand, the lower tail of the firm age distribution (lower panel) is largely affected by the type of policy. When the age-dependent policy is implemented, due to the relatively strong selection effect on entry, the number of age-0 firms is slightly lower than the benchmark. In contrast, the share of all young firms increases relatively more under the size-dependent policy.

5.4 The Importance of General Equilibrium

So far, we have conducted our policy counterfactual exercises in a general equilibrium (GE) environment. This involves endogenous changes in the equilibrium price of the model, which is the real wage rate (w). In fact, the equilibrium wage under each policy is higher than that in the benchmark case. This sub-section compares the aggregate results of a targeted policy in GE and partial equilibrium (PE) of the model. In particular, we illustrate the importance of the GE price effect on firm entry and exit margins by comparing the firm dynamics respectively under GE and PE. We only report the results from the age-dependent policy because the main results still hold under different credit subsidy policies.

From Table 3, we recognize that the quantitative effects of a credit subsidy policy can be seriously misleading when the equilibrium price adjustments are not considered. The last column of the table reports the aggregate results from the age-dependent policy when the real wage is fixed. Under this PE exercise, the policy increases the aggregate quantities of the economy by more than 20 percent from the benchmark. In particular, the number of firms increases by about 8 percent, doubling the size of productivity gains in the aggregate: 1.48% under PE and 0.75% under GE. From the last row of the table, we can see that the boosted aggregate productivity under PE largely comes from the increased number of firms that are, on average, less productive than those in GE.

Therefore, Table 3 shows that it is quantitatively important to consider the effects of equilibrium price changes when we evaluate a targeted credit subsidy policy. In PE, we fail to consider important indirect general equilibrium effects as discussed in the previous sub-section: (i) changes in optimal production scale, (ii) firm entry and exit margins, and (iii) equilibrium number of firms. In the following, we investigate each of these channels from the perspective of firm dynamics.

Table 3 : Aggregate Results, GE vs. PE

Policy Counterfactual: Aggregates			
	Benchmark	Age-dependent (GE)	Age-dependent (PE)
C	0.2103	0.2147 (2.11)	–
K	0.6322	0.6675 (5.58)	0.7858 (24.30)
Y	0.2817	0.2887 (2.49)	0.3389 (20.31)
N	0.3337	0.3350 (0.39)	0.4015 (20.32)
D/A	0.3401	0.3664	0.3633
firms (μ)	1.006848	0.978711	1.075427
endo. exit rate	0.013172	0.020803	0.020760
rt	–	0.136042	0.150841
μ subsidized	–	0.041864	0.038734
$\Delta TFP(\%)$	(0.616511)	0.746149	1.483320
$\Delta TFP/rt$	–	0.033814	0.060626
avg. rt	–	3.249618	3.894279
$\Delta TFP_{\mu}(\%)$	–	1.112317	0.630977

Note: We use (α, ν) values at the benchmark to compute the measured TFP under each policy. Values in parentheses are percentage changes from their benchmark values.

Figure 8 compares firm dynamics with and without the equilibrium price adjustments under the age-dependent policy. Clearly, the dynamics of the average firm size and productivity under PE are not much different from those in the GE environment. One noticeable difference, however, is that the levels of capital stock of each age group are larger in PE. With credit subsidies, the equilibrium real wage rate rises and this reduces the optimal capital choice, $K^w(\epsilon)$, and the constrained capital choice, $\bar{K}(m)$. Thus, the average size of firms falls in a GE environment. This is exactly what we previously mentioned as the first indirect effect of the GE price channel.

More importantly, the changes in equilibrium wage affect firm exit and entry decisions, which we distinguished as the second indirect effect above. Other things being equal, the higher real wage rate hurts firms with low productivity and high leverage ratio, as reflected in the higher exit rates in Table 2.

In addition, the rise in equilibrium wage rates strengthens the selection among the potential entrants. Without any change in wage rates, credit subsidies increase the value of entry, V^1, in equation (6) by removing credit constraints. It follows that more firms are willing to pay the fixed entry cost. However, as mentioned above, the rise in equilibrium wage rates accompanied by credit subsidies cuts the profitability of firms and hence lowers the value of entry by reducing the lifetime expected profits. This selects only productive

firms that can afford the higher cost of production. Because those two effects on firm entry operate in opposite directions, the overall effect with the equilibrium price change is ambiguous. Our quantitative experiments in GE show that the latter selection effect dominates the former effect, and therefore, the average productivity of entrants is higher under each credit subsidy policy, as shown in Figure 6.[27]

The last indirect effect is on the number of firms in the economy under GE. Through the adjustments in entry and exit margins, the total number of firms under a targeted policy changes from the benchmark case. From the increased endogenous exit and the selection on firm entry following the credit subsidies, as shown in Table 2 and Figure 6, the equilibrium measure of firms, μ, decreases under GE.[28] Without these adjustments in exit and entry, on the other hand, the total number of firms rises by about 7.5 percent under PE (Table 3). This in turn increases the aggregate gains from the size- and age-dependent policies in a misleading way. Our point here is that the GE environment is crucial in more precisely quantifying and evaluating the macroeconomic impact of a targeted credit subsidy policy for firms' external financing.

5.5 The Importance of Firm Size Distribution

Lastly, we highlight the importance of capturing the empirical firm size distribution in quantitatively evaluating the effects of credit subsidy policies. As shown earlier, our benchmark model economy features both rich firm-level heterogeneity and realistic firm dynamics. In particular, the calibrated model replicates the empirical size distribution of firms, consistent with the BDS database. To address how the results from our counterfactual policy exercises can differ across the underlying heterogeneity in firm size, we now consider an economy *without a realistic firm size distribution*.

As an illustration, we adjust the parameter value for the idiosyncratic firm-level productivity, ϵ_m, from its benchmark value, which gives rise to an inconsistent model distribution with that in the data. In particular, we reduce the population share of the smallest firm size bin by 10 percent by setting the parameter value of ϵ_m to 0.46, instead of 0.4306 as in the benchmark case. The population share of the smallest firm size bin

[27] This is different from what Buera, Moll, and Shin (2013) predict in a similar model environment. They show that credit subsidies improve aggregates in the short run, while distorting the entry into entrepreneurships in the long run due to the fixed subsidy scheme over time. Our policies, in contrast, depend on firm size or age in each period.

[28] See Buera, Shin, and Kaboski (2011) and Khan, Senga, and Thomas (2016) for these negative effects of the decreased number of production units. In a different context, Cagesse (2016) highlights the competition effect among innovating firms with financing constraints.

in this new steady state becomes 0.48 compared to 0.53 in the benchmark case. We then conduct the counterfactual exercises with the size- or age-dependent policy at the new steady state of the economy. Table 4 reports the resulting size distribution.[29] It follows that the number of financially constrained firms that are eligible for the credit subsidies also becomes relatively small.

Table 4 : Firm Size Distribution

Population Shares by Employment Size			
Size Bin	BDS	Benchmark Steady State ($\epsilon_m = 0.4306$)	New Steady State ($\epsilon_m = 0.46$)
1 to 4	0.5506	0.5311	0.4824
5 to 19	0.3342	0.3368	0.3531
20 to 99	0.0964	0.0912	0.1098
100 to 499	0.0153	0.0226	0.0297
500 to 2499	0.0026	0.0093	0.0125
2500+	0.0009	0.0089	0.0125

Table 5 : Aggregate Results, Size Distribution

Policy Counterfactual: Aggregates				
	Benchmark ($\epsilon_m = 0.4306$)	Steady State ($\epsilon_m = 0.46$)	Age-dependent (age 0 to 4)	Size-dependent (SMEs)
C	0.2103	0.2250	0.2290	0.2297
K	0.6322	0.6763	0.7125	0.7261
Y	0.2817	0.2999	0.3065	0.3087
N	0.3337	0.3321	0.3335	0.3349
D/A	0.3401	0.3562	0.3793	0.3801
firms (μ)	1.006848	1.020702	0.987136	0.993577
endo. exit rate	0.013172	0.014039	0.022946	0.025605
rt	–	–	0.141471	0.181857
μ subsidized	–	–	0.053783	0.105320
$\Delta TFP(\%)$	(0.616511)	(0.646275)	0.507452	0.456787
$\Delta TFP/rt$	–	–	0.023182	0.016233
avg. rt	–	–	2.630404	1.726709

Note: We use (α, ν) values at the benchmark to compute the measured TFP under each policy. $\Delta TFP(\%)$ are relative to the steady state value.

With the less-skewed distribution of firms in Table 4, we now examine the aggregate results of the targeted policies under GE. Table 5 shows that the relative gain in the

[29] With $\epsilon_m = 0.4306$ at the benchmark, the implied skewness of the ϵ distribution is 2.18, whereas the value becomes 2.01 when $\epsilon_m = 0.46$. The less-skewed firm productivity distribution, in turn, affects the resulting size distribution in Table 4.

measured TFP under each credit subsidy policy (ΔTFP) is smaller than the corresponding value in Table 2. Specifically, the rise in TFP under the size-dependent policy for SMEs in Table 5 is about 0.45 percent relative to the new steady state with $\epsilon_m = 0.46$ (last column). In contrast, from Table 2, the same policy leads to a relative gain of 0.77 percent with a realistic firm size distribution. Moreover, our efficiency measure of the size-dependent policy, $\frac{\Delta TFP}{rt}$, significantly falls when we do not replicate the empirical size distribution. Therefore, our results illustrate that the micro-level consistency of a macroeconomic model is vital to quantifying the aggregate outcomes of targeted credit policies. This also implies that understanding the firm-level heterogeneity of an economy is essential in designing and implementing such targeted policies in practice.

6 Concluding Remarks

In this paper, we studied a general equilibrium model of heterogeneous producers with collateral constraints and endogenous entry and exit. By employing a bounded Pareto distribution for firm-level productivity, our model replicates cross-sectional features of firm dynamics, as seen in the BDS. We use this model to quantify the macroeconomic implications of targeted credit subsidy policies. We show that aggregate outcomes of such policies critically hinge on equilibrium price changes. In particular, we show that targeted credit subsidy policies have indirect general equilibrium effects, which in turn may undermine the direct benefit that subsidized firms can receive. In addition to the importance of taking into account general equilibrium price movements, we also show the significance of capturing empirical regularities associated with firm dynamics for a macroeconomic analysis of industrial policies. Specifically, we find that with empirically inconsistent firm size distribution at the micro level, the size of the effect of policies on macroeconomic outcomes will be different. Our results critically highlight the role of indirect general equilibrium effects of targeted credit subsidy policies in affecting micro-level firm dynamics with a strong selection process. This also reshapes firm size and age distributions.

Because small and young firms account for a substantial share of employment in the economy, policies that target small and young firms are popular among policymakers. However, research on the aggregate implications of such industrial policies is scarce. We believe that our results serve as a cautionary tale about the possibility of misleading public debate if such debate lacks consideration of general equilibrium price movements and micro-level consistency.

Enhancing aggregate productivity in the economy appears to be immediately relevant for many countries. In particular, a variety of targeted industrial policies have been implemented in European and Asian countries. The prime example includes the Moratorium Act in Japan as mentioned above. Quantitative investigations of this specific example using our framework and micro-level company financial data may be useful. This paper, however, did not consider any policy distortion itself, which is also potentially important. This research is left for future study.

Figure 1

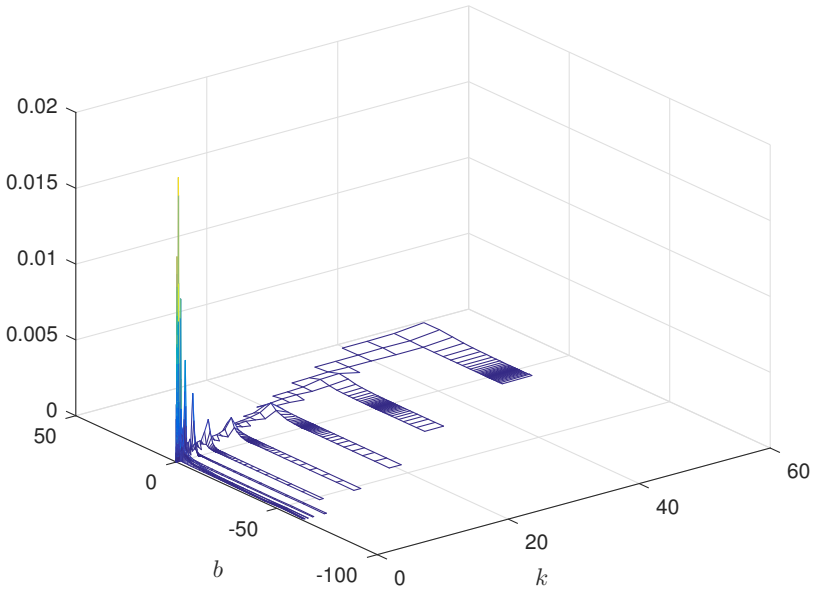


Figure 2

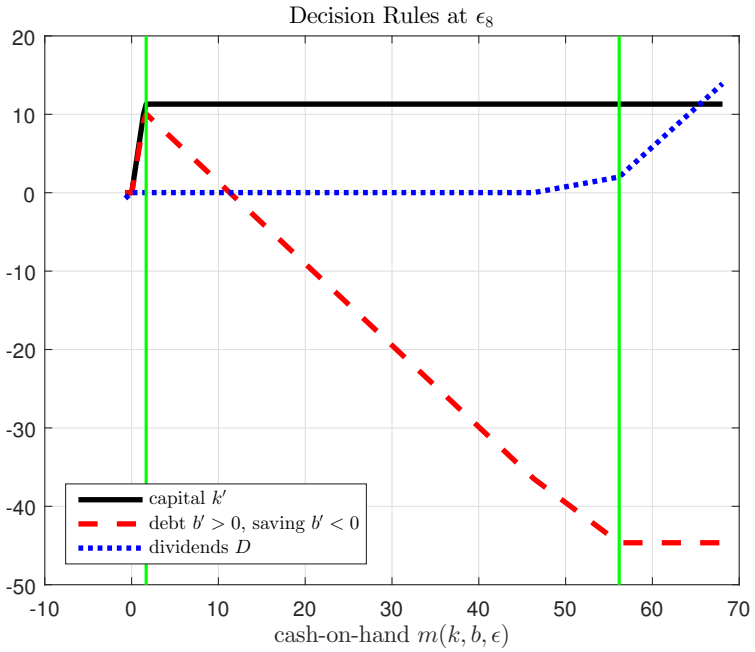


Figure 3

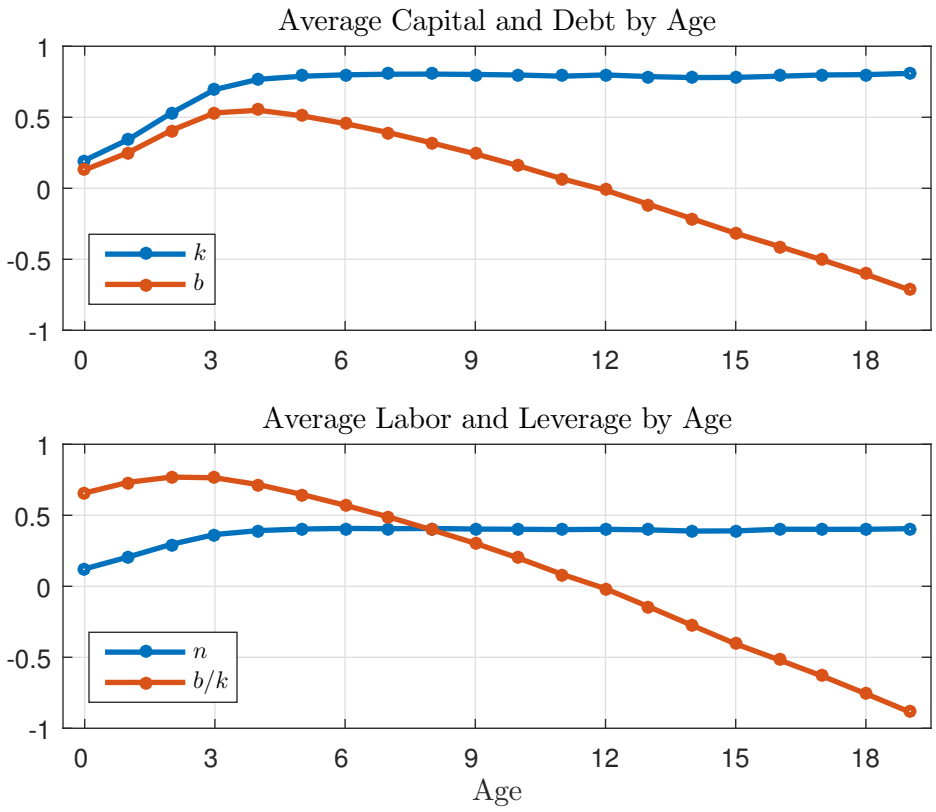
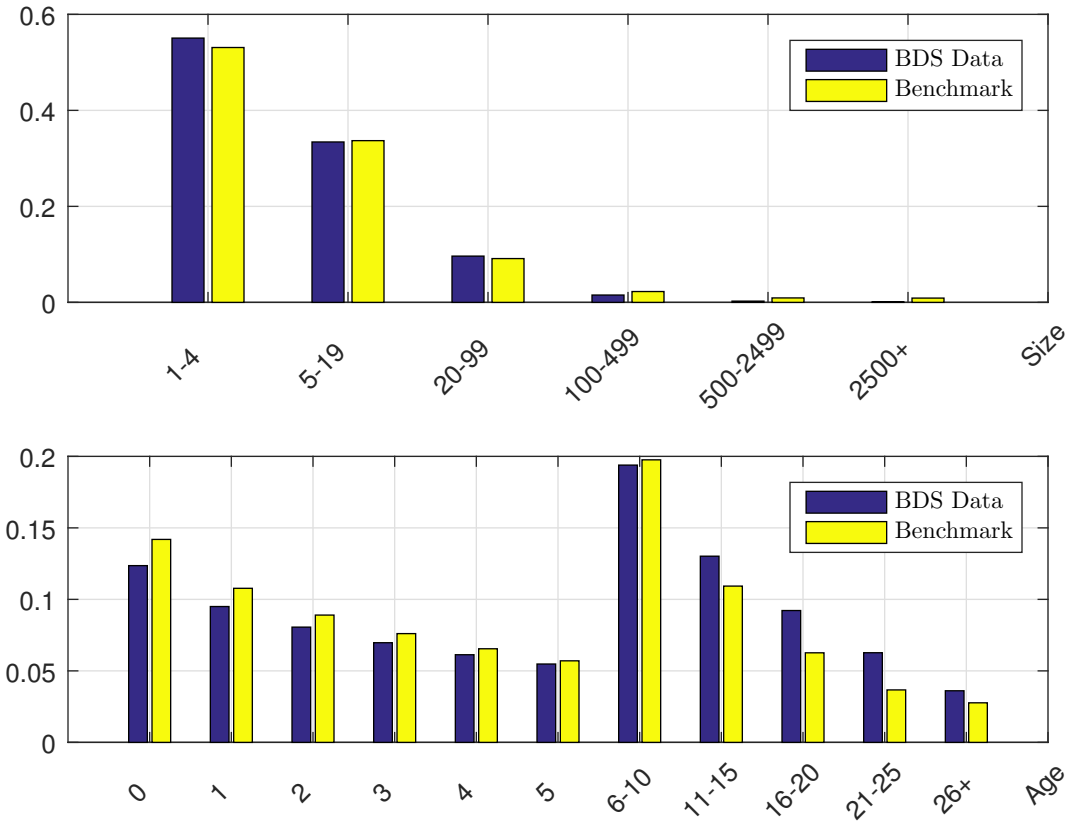
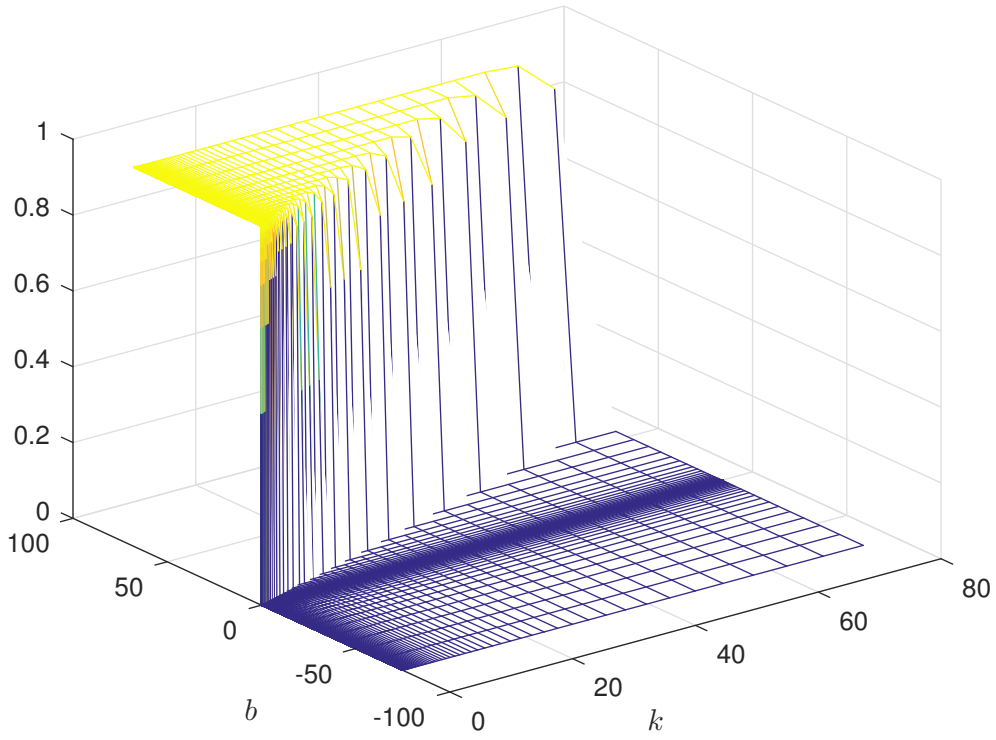

Average Capital and Debt by Age
1
0.5
0
-0.5
-1
0
3
6
9
12
15
18
k
b
Average Labor and Leverage by Age
1
0.5
0
-0.5
-1
0
3
6
9
12
15
18
n
b/k
Age

Figure 4



Note: The sample period for the average firm size distribution is from 1977 to 2007 in the Business Dynamics Statistics (BDS) database. For the average age distribution, we use the data tables from 1993 to 2007.

Figure 5



Note: We calculate the weighted average exit decision (Exit =1), over the ergodic distribution of ϵ.

Figure 6

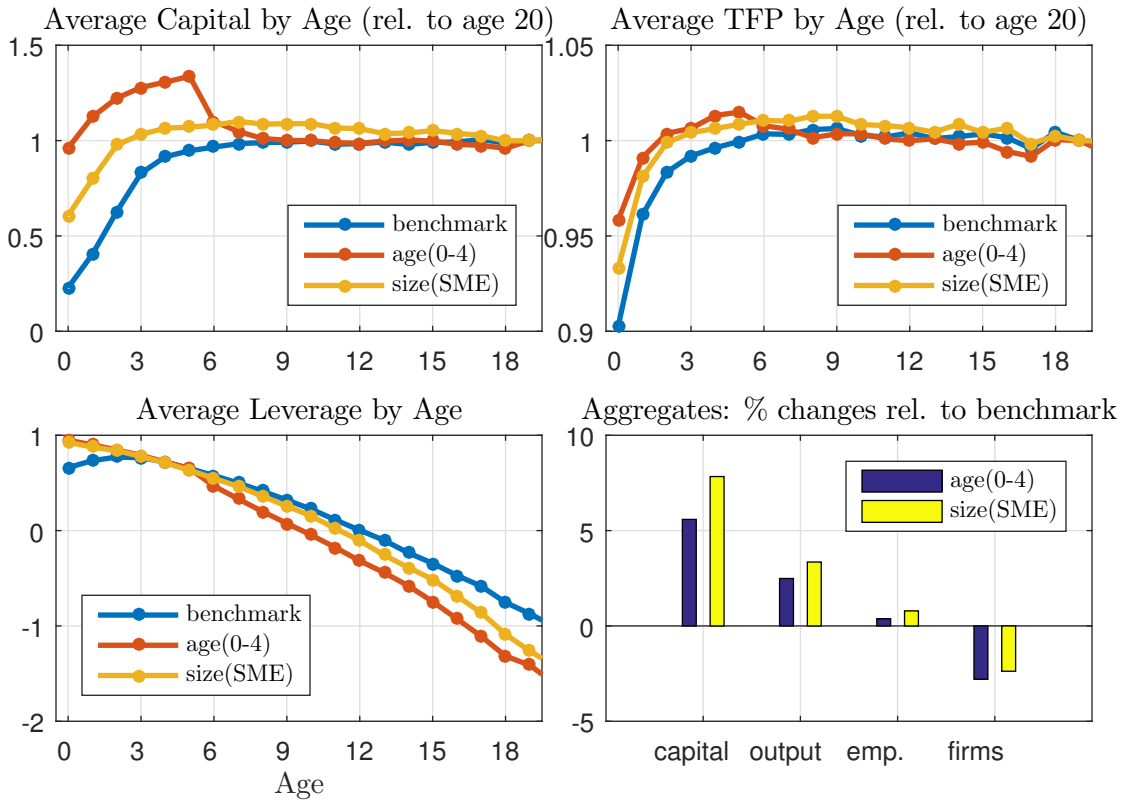

Average Capital by Age (rel. to age 20)
1.5
1
0.5
0
0 3 6 9 12 15 18
benchmark
age(0-4)
size(SME)
Average TFP by Age (rel. to age 20)
1.05
1
0.95
0.9
0 3 6 9 12 15 18
benchmark
age(0-4)
size(SME)
Average Leverage by Age
1
0
-1
-2
0 3 6 9 12 15 18
Age
benchmark
age(0-4)
size(SME)
Aggregates: % changes rel. to benchmark
10
5
0
-5
capital output emp. firms
age(0-4)
size(SME)

Figure 7

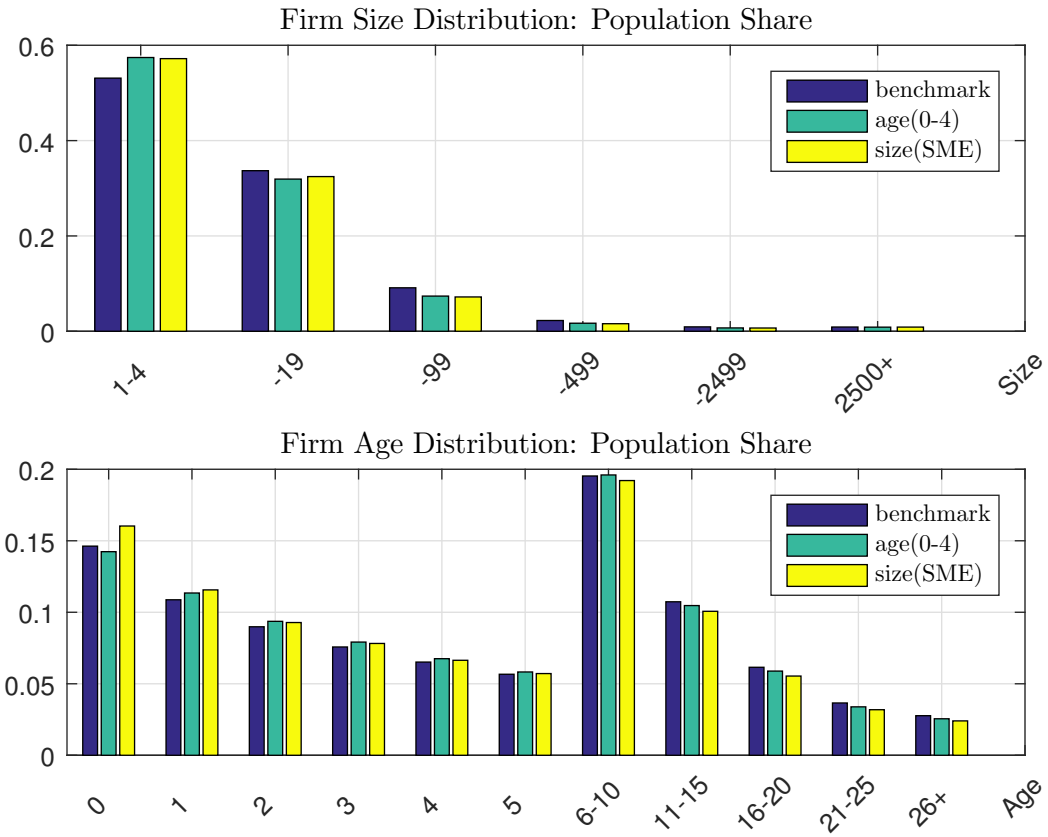
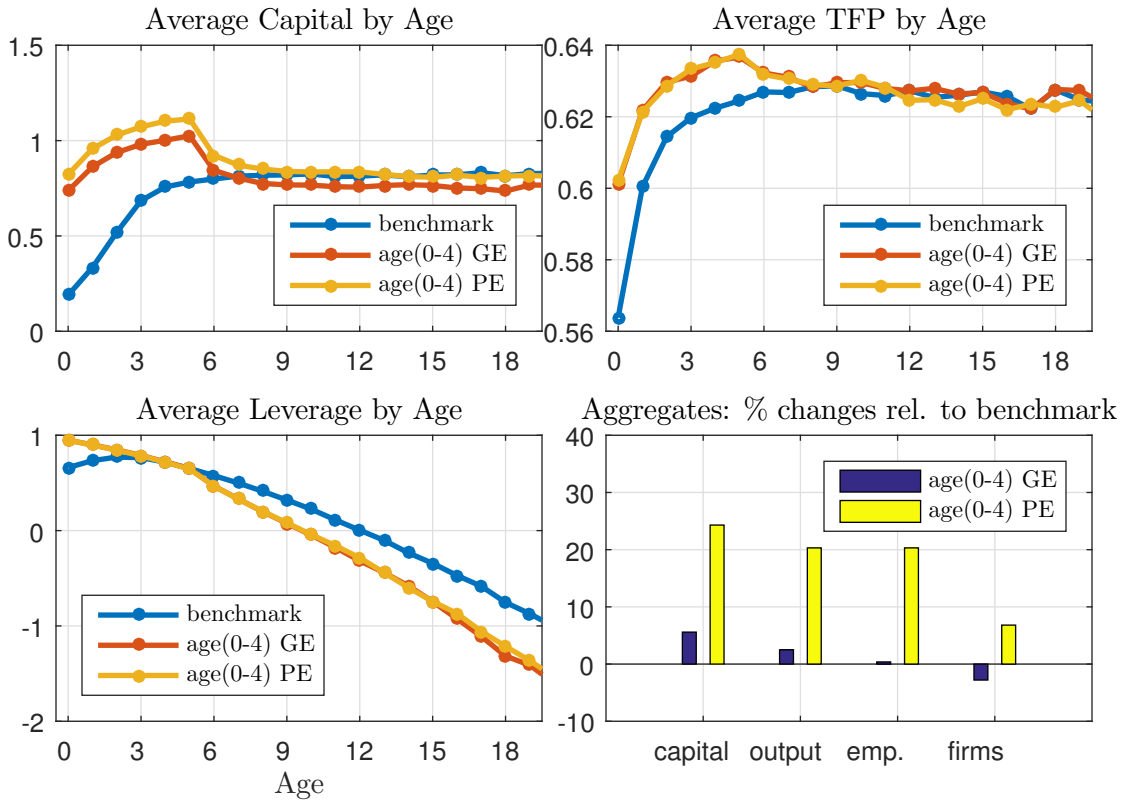

Firm Size Distribution: Population Share
0.6
0.4
0.2
0
benchmark
age(0-4)
size(SME)
1-4
-19
-99
-499
-2499
2500+
Size
Firm Age Distribution: Population Share
0.2
0.15
0.1
0.05
0
benchmark
age(0-4)
size(SME)
0
1
2
3
4
5
6-10
11-15
16-20
21-25
26+
Age

Figure 8


Average Capital by Age
1.5
1
0.5
0
0 3 6 9 12 15 18
benchmark
age(0-4) GE
age(0-4) PE
Average TFP by Age
0.64
0.62
0.6
0.58
0.56
0 3 6 9 12 15 18
benchmark
age(0-4) GE
age(0-4) PE
Average Leverage by Age
1
0
-1
-2
0 3 6 9 12 15 18
Age
benchmark
age(0-4) GE
age(0-4) PE
Aggregates: % changes rel. to benchmark
40
30
20
10
0
-10
capital output emp. firms
age(0-4) GE
age(0-4) PE