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## Wealth Inequality, or $r-g$ , in the Economic Growth Model<sup>1</sup>

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### Abstract

We investigate a simple continuous-time overlapping generations model with a neoclassical production function and technological progress. We demonstrate that the degree of wealth inequality is positively related to the difference between the real interest rate  $r$  and the growth rate of income per capita  $g$ , and if  $g$  falls, the  $r-g$  gap widens and inequality worsens. We also argue that a wealth tax reduces the wealth inequality. All of these results are consistent with the famous predictions advanced by Thomas Piketty in *Capital in the Twenty-First Century* (2014). We next investigate consumption tax and find that it enhances capital accumulation and reduces  $r-g$ , and thus wealth inequality.

*Keywords:* Overlapping generations, Inequality, Pareto distribution, Wealth tax

*JEL classification:* E5

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# 1 Introduction

In his influential book *Capital in the Twenty-First Century*, Thomas Piketty predicts that the gap between the rate of return on capital  $r$  and the per capita income growth rate  $g$ , crucially affects the distribution of wealth. The gap  $r - g$  represents the degree of capital income deviation from labor income (if it is not consumed). He argues that if  $r$  exceeds  $g$ , inherited wealth will grow faster than labor income and consequently wealth distribution will become highly concentrated. He also argues that if  $g$  is low, the gap  $r - g$  widens and the wealth inequality worsens. In terms of economic policy, Piketty recommends a wealth tax to reduce wealth inequality. His claims are validated through the theory by Piketty and Zucman (2015); they construct a two-period overlapping generations (OLG) model in which the individuals are heterogeneous in their preferences toward wealth. They demonstrate that in a steady state wealth inequality is an increasing function of the term  $\frac{1+r}{1+g}$ .

Recently, however, some authors have criticized Piketty's claims by using popular economic growth models that are extensively studied in macroeconomics. Krusell and Smith (2014) use the representative agent Ramsey model and show that if the period utility function is logarithmic, then  $r - g$  equals the discount factor and then it does not rise when the economic growth rate slows down. Mankiw (2015) obtains a similar formula in a neoclassical growth model with capitalists and workers.

To our knowledge, Jones (2015) is the first to study a dynamic general equilibrium model with heterogeneous agents and to doubt the importance of  $r - g$  or a wealth tax in wealth inequality. He constructs a continuous-time OLG model with an AK production function, the details of which are in Jones (2014). The stationary wealth distribution is Pareto in his model. Jones represents the wealth inequality measure as a function of the population growth rate,  $r - g$  and the wealth tax rate. However, he finds that in the general equilibrium where the interest rate is determined endogenously, the inequality measure depends only on the population growth rate. Thus, the gap  $r - g$  and wealth tax are *independent* of the wealth inequality. Moll (2014) considers a similar model with capitalists and workers and obtains the same conclusions.

In this paper we argue that the conclusions of Jones and Moll are not robust by using a continuous time OLG model with a neoclassical production function. The set-up is very close to the models advanced by Jones (2015) as well as Blanchard (1985). Our model only differs from Jones in the curvature of the production function. We first obtain the wealth distribution which is the generalized Pareto distribution. Then, we show that the degree of wealth inequality is positively related to  $r - g$ , and, if the per capita income growth rate  $g$  slows down, the gap widens and the inequality worsens. Finally, we show that a wealth tax is useful in reducing wealth inequality. Therefore, our results support Piketty's claims.

Although the conclusions mentioned above are similar to those of Piketty and Zucman (2015), our model crucially differs from theirs on the source of agent heterogeneity. Piketty and Zucman (2015) assume that individuals differ in their preferences toward wealth inheritance, whereas in our model, individuals differ only in their life spans. We show that  $r - g$  affects wealth inequality in the popular Blanchard-type OLG models that are found in many macroeconomics textbooks including Barro and Sala-i-Martin (2004), Bertola et al. (2005), and Acemoglu (2009).

In terms of economic policy, Jones (2015) only examines wealth tax. Here we compare consumption tax with wealth tax in terms of steady state wealth inequality. We show that consumption tax reduce  $\bar{r} - g$  and also wealth inequality, but the mechanism is different. Wealth tax reduces the net of tax rate of return on capital  $\bar{r}$  and lowers wealth inequality. It decreases the steady state level of capital and raises the interest rate. On the other hand, consumption tax enhances capital accumulation and reduces the rate of return on capital. Therefore, consumption tax can reduce wealth inequality without reducing the steady state output, while wealth tax reduces both inequality and output.

This paper is related to some recent literature on wealth inequality in dynamic general equilibrium models. Benhabib et al. (2011) explicitly characterize wealth distribution in an OLG model in which the agents receive idiosyncratic shocks on their investment and age. Nirei and Aoki (2015) investigate a neoclassical growth model in which individuals are subject to idiosyncratic investment shocks and borrowing constraints, and

demonstrate that wealth is distributed according to Pareto. However, they do not study the relationship between  $r - g$  and inequality. Moreover, Nirei and Aoki (2015) argue that, wealth distribution becomes flatter when the economic growth rate is low, and their conclusion is contrary to Piketty's prediction.

This paper is organized as follows. Section 2 describes the basic structure of the model. Section 3 investigates how the degree of inequality is related to  $r - g$ . Section 4 compares consumption tax with wealth tax. Section 4 concludes the paper.

## 2 The Model

In this section, we provide an overview of our model.

### 2.1 Preferences

Time is continuous. In each period, a continuum of individuals is born. The number of newborns at date  $t$  is  $B_t = B_0 e^{nt}$ , where  $n > 0$  and  $B_0 > 0$ . Death follows the Poisson process with an arrival rate  $d$ . The population of agents born on date  $s$  (henceforth cohort  $s$ ) is  $L_{t,s} = de^{-d(t-s)} B_s$  in period  $t$ . In accordance with Jones (2014), the total population  $L_t = \int_{-\infty}^t L_{t,s} ds$  evolves according to

$$\dot{L}_t = B_t - dL_t,$$

and in steady state,  $\frac{\dot{L}_t}{L_t} = n$  and  $L_t = \frac{B_t}{n+d}$ .

An agent supplies one unit of labor in the labor market and receives wage income in each period. Cohort  $s$  maximizes the following expected intertemporal utility:

$$U = \int_s^{\infty} e^{-(\rho+d)(t-s)} \ln C_{t,s} dt,$$

subject to the following budget constraint

$$\dot{Z}_{t,s} = (r_t - \tau)Z_{t,s} + W_t - C_{t,s}, \tag{1}$$

where  $\rho$  is the discount factor,  $C_{t,s}$  is the consumption level of cohort  $s$  in period  $t$ ,  $Z_{t,s}$  is the asset holdings of cohort  $s$  in period  $t$ ,  $r_t$  is the real interest rate in period  $t$ ,  $\tau$  is the

linear capital income tax, and  $W_t$  is the wage income in period  $t$ . Here, we follow Jones (2015) and assume that the tax revenue is discarded. We let  $Z_t = \int_{-\infty}^t L_{t,s} Z_{t,s} ds$  denote the total wealth at time  $t$ . Individuals equally inherit the assets of the agents who die. Then, the initial asset level of cohort  $s$  is  $Z_{s,s} = \frac{dZ_s}{B_s} = \theta \frac{Z_s}{L_s}$  with  $\theta = \frac{d}{n+d}$ .

Following Piketty and Zucman (2015), we let  $\bar{r}_t = r_t - \tau$  denote the rate of return (net-of-tax) on capital. The human wealth is defined as  $H_t = \int_t^\infty e^{-\bar{R}_{x,t}} W_x dx$ , where  $\bar{R}_{x,t} = \int_t^x \bar{r}_z dz$  is the compound interest rate (net-of-tax). It evolves according to

$$\dot{H}_t = \bar{r}_t H_t - W_t.$$

As the utility function is logarithmic, the consumption of cohort  $s$  at time  $t$  is given by

$$C_{t,s} = (\rho + d)(Z_{t,s} + H_t). \quad (2)$$

In the competitive equilibrium, total wealth is equal to total capital, which means that  $Z_t = K_t$ . Therefore, the aggregate consumption at time  $t$ ,  $C_t = \int_{-\infty}^t L_{t,s} C_{t,s} ds$  is

$$C_t = (\rho + d) \int_{-\infty}^t L_{t,s} (Z_{t,s} + H_t) ds = (\rho + d)(K_t + L_t H_t). \quad (3)$$

## 2.2 Production

There are many identical firms. The production function  $F$  has constant returns to scale and is given by  $F(K_t, A_t L_t)$ , where  $K_t$  is the capital,  $A_t$  is the technology level, and  $L_t$  is the labor supply. The growth rate of  $A_t$ ,  $\frac{\dot{A}_t}{A_t}$  is exogenous and equals  $g$ . We let  $k_t = \frac{K_t}{A_t L_t}$  denote the capital per efficiency unit of labor at time  $t$ . The production function per efficiency unit of labor  $f(k) = F(k, 1)$  satisfies  $f(0) = 0$ ,  $f' > 0$ ,  $f'' < 0$  and  $f'(0) = +\infty$ .

Factor markets are perfectly competitive, and the equilibrium wage rate in period  $t$  is  $W_t = A_t \{f(k_t) - k_t f'(k_t)\}$  and the capital rental rate in period  $t$  is  $r_t = f'(k_t)$ . As the total tax revenue  $\tau K_t$  is thrown away, the resource constraint is

$$\dot{K}_t = F(K_t, A_t L_t) - C_t - \tau K_t. \quad (4)$$

Eq. (4) is re-expressed as

$$\dot{k}_t = f(k_t) - c_t - (g_t + n_t + \tau)k_t, \quad (5)$$

where  $c_t = \frac{C_t}{A_t L_t}$  denotes the consumption per efficiency unit of labor.

Let  $h_t = \frac{H_t}{A_t}$  and  $w_t = \frac{W_t}{A_t}$ . From Eq. (3), we have  $c_t = (\rho + d)(k_t + h_t)$ . As  $h_t$  evolves according to  $\dot{h}_t = (\bar{r}_t - g)h_t - w_t$ , we have

$$\dot{c}_t = (\bar{r}_t - g - \rho - d)c_t - (\rho + d)nk_t. \quad (6)$$

The path of  $(c_t, k_t)$  is determined by Eqs. (5) and (6).

### 2.3 Balanced growth path

We now focus on the balanced growth path (BGP), where  $c_t = c$ ,  $k_t = k$ ,  $w_t = w$ , and  $r_t = r$  are all constant, and the growth rate of output per capita,  $A_t f(k_t)$  is  $g$ . Eqs. (5) and (6) imply that the stationary allocation  $(c, k)$  is determined by

$$c = \frac{n(\rho + d)k}{\bar{r} - g - \rho - d}, \quad (7)$$

$$c = f(k) - (g + \tau + n)k, \quad (8)$$

where  $\bar{r} = f'(k) - \tau$ . When  $n$  and  $d$  are zero, our model coincides with the Ramsey model, and the steady state interest rate is  $\bar{r} = \rho + g$ . Thus,  $\bar{r} - g$  coincides with the discount factor, as Krusell and Smith (2014) have pointed out.

## 3 Wealth inequality and $r - g$

This section follows Jones (2015) and obtains the wealth distribution along the BGP.

### 3.1 The gap $r - g$ in the steady state

We first characterize  $\bar{r} - g$ . Here we focus on the Cobb-Douglas production function  $f(k) = k^\alpha$ , where  $\alpha \in (0, 1)$ . In this case,  $r = \alpha k^{\alpha-1}$ . We let  $c_1(k)$  and  $c_2(k)$  denote the right-hand side of Eqs. (7) and (8), respectively. The two functions are expressed as

$$c_1(k) = n(\rho + d) \frac{k}{\alpha k^{\alpha-1} - g - \tau - \rho - d}, \quad (9)$$

$$c_2(k) = k^\alpha - (g + \tau + n)k. \quad (10)$$

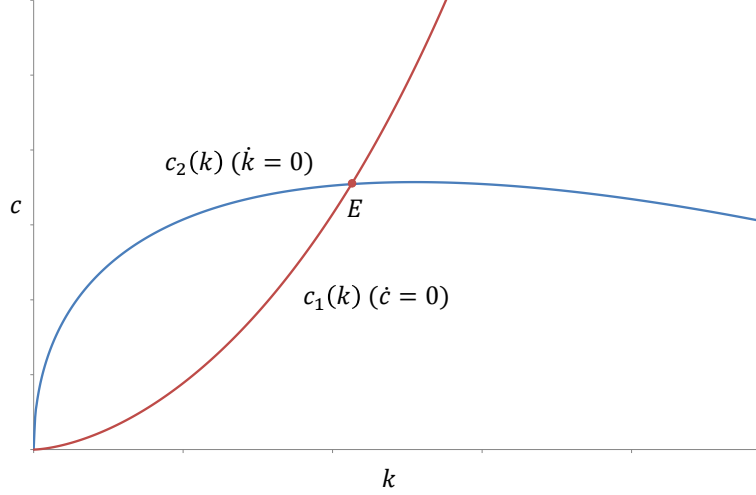


Figure 1: Stationary allocation

The function  $c_1(k)$  is increasing, convex, and satisfies  $c_1(0) = 0$  and  $\lim_{k \rightarrow k^m} c_1(k) = +\infty$  with  $k^m = (\frac{\alpha}{g+\rho+\tau})^{1/(1-\alpha)}$ . Similarly, the function  $c_2(k)$  is concave and satisfies  $c_2(0) = 0$  and  $c_2'(0) = +\infty$ . As Figure 1 shows, the curves  $c = c_1(k)$  and  $c = c_2(k)$  have a unique intersection. We have the following proposition on  $\bar{r} - g$ .

**Proposition 1** *Along the BGP,  $\bar{r} - g$  solves the following quadratic equation on  $x$ :*

$$\{x + (1 - \alpha)(g + \tau) - \alpha n\}(x - \rho - d) = \alpha n(\rho + d). \quad (11)$$

*The gap  $r - g$  is a strictly decreasing function of  $g$  and  $\tau$ .*

**Proof.** See the Appendix. ■

Proposition 1 shows that when the economic growth rate slows down, the gap  $r - g$  rate widens. This is consistent with Piketty's prediction.

### 3.2 Wealth distribution and $r - g$

From Eqs. (1) and (2), the assets of cohort  $s$  evolve according to

$$\dot{Z}_{t,s} = (\bar{r}_t - \rho - d)Z_{t,s} + W_t - (\rho + d)H_t.$$

If we let  $z_{t,s} = Z_{t,s}/A_t$ , then  $z_{s,s} = \theta k_s$  and

$$\dot{z}_{t,s} = (\bar{r} - g - \rho - d)z_{t,s} + w_t - (\rho + d)h_t.$$



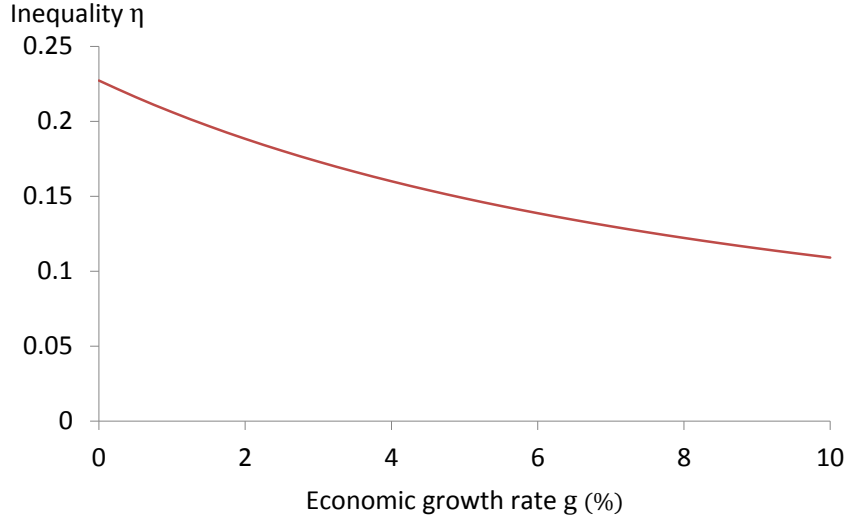


Figure 2: Inequality and growth

Along the BGP, where  $k$ ,  $w$ , and  $h$  are constant and  $(\bar{r} - g)h = w$ , we have

$$z_{t,s} = e^{(\bar{r}-g-\rho-d)(t-s)} (h + \theta k) - h.$$

At time  $t$ , the relative population of cohort  $s$  is  $\frac{L_{t,s}}{L_t} = (d+n)e^{-(d+n)(t-s)}$ . Therefore

$$\Pr(Z_{t,s} \geq x) = \left( \frac{h + x/A_t}{h + \theta k} \right)^{-\frac{d+n}{\bar{r}-g-\rho-d}}. \quad (12)$$

The individual wealth is distributed according to the generalized Pareto distribution, and the Pareto inequality measure  $\eta$  which is the inverse of the exponent in Eq. (12) is

$$\eta = \frac{\bar{r} - g - \rho - d}{d + n}. \quad (13)$$

If  $\bar{r} - g$  widens, so inequality definitely worsens. As shown in Proposition 1, the reduction of  $g$  increases  $\bar{r} - g$ , and subsequently raises  $\eta$ . Similarly, the increase of  $\tau$  reduces the gap, which in turn reduces  $\eta$ . Thus, we have the following proposition.

**Proposition 2** *A slowdown in the per capita income growth rate raises the inequality measure  $\eta$ , and wealth tax reduces  $\eta$ .*

Figure 2 shows a negative relationship between the wealth inequality and  $g$  when  $\alpha = 1/3$ ,  $\rho = 0.05$ ,  $n = 0.01$  and  $d = 0.05$ . The parameters for  $\alpha$ ,  $\rho$ , and  $d$  are from

Mankiw and Weinzierl (2006). As the economic growth rate declines from 10% to zero, the Pareto inequality measure almost doubles from 0.12 to 0.22.

Our result differs from Jones (2015) who demonstrated that the inequality index is independent of  $r - g$  and  $\tau$ . The production function he uses is an  $AK$  type, with the wage income and human wealth both equal to zero. In this case, Eq. (3) becomes  $C = (\rho + d)K$ , and Eq. (4) is expressed as  $\dot{K} = (A - \rho - d - \tau)K$ . Thus the per capita income growth rate is  $A - \rho - d - \tau - n \equiv \hat{g}$ . The interest rate is  $r = A$ , and the inequality measure is  $\eta = \frac{\bar{r} - \hat{g} - \rho - d}{d + n} = \frac{n}{d + n}$ , which is unrelated to  $r - g$  and  $\tau$ . As this paper illustrates, Jones' conclusion crucially depends on the linearity of the production function.

## 4 Robustness

In this section, we show that our result on the relevance of  $r - g$  and  $\tau$  on inequality continues to hold in a three cases: 1) the individuals buy annuities; and 2) the tax revenue is rebated lump-sum.

### 4.1 Use of annuities

In the previous section, we assumed that the wealth of the people who die is re-distributed to the newborns. Here we consider a model where the individuals purchases annuities as in Blanchard (1985). Annuity markets are competitive and the insurance company pays to the individual with financial wealth  $Z$  by  $dZ$  units. The agent born at date  $s$  is now subject to

$$\dot{Z}_{t,s} = (\bar{r} + d)Z_{t,s} + W_t - C_{t,s}.$$

The initial asset level  $Z_{s,s}$  is zero. Here the rate of return on asset is  $\bar{r} + d$ , while in the original model it is  $\bar{r}$ . The individual consumption function is the same as before, but the human wealth now evolves according to  $\dot{H}_t = (\bar{r} + d)H_t - W_t$  or equivalently

$$\dot{h} = (r + d - g)h - w.$$

The resource constraint is the same as before, and thus  $\dot{c} = (\bar{r} - g - \rho)c - (\rho + d)(d + n)k$ .

The steady state allocation  $(c, k)$  are determined by

$$\begin{aligned} c &= (n + d)(\rho + d)\frac{k}{\bar{r} - g - \rho}, \\ c &= f(k) - (g + n + \tau)k. \end{aligned}$$

If we let  $z_{t,s} = Z_{t,s}/A$ , it evolves according to

$$\dot{z}_{t,s} = (r - g - \rho - \tau)z_{t,s} + w - (\rho + d)h.$$

Along the BGP,  $z_{t,s} = (e^{(r-g-\rho-\tau)(t-s)} - 1)h$ . Then

$$\Pr(Z_{t,s} \geq x) = \left(1 + \frac{x}{he^{gt}}\right)^{-\frac{d}{\bar{r}-g-\rho}}$$

and the Pareto inequality measure is  $\hat{\eta} = \frac{\bar{r}-g-\rho}{d}$ , which is very close to the previous one.

We have the following proposition.

**Proposition 3** *The slowdown in the economic growth rate worsens wealth inequality when the individuals buy annuities.*

**Proof.** See the Appendix. ■

## 4.2 Lump-sum transfer

So far we have assumed that the wealth tax is thrown away. In this section, we consider a case where the tax revenue is rebated via lump-sum. The agent born at date  $s$  maximizes the utility  $\int_0^\infty e^{-(\rho+d)(t-s)} \ln C_{t,s} dt$  subject to

$$\dot{Z}_{t,s} = (r - \tau)Z_{t,s} + S_t + W_t - C_{t,s}.$$

where  $S_t$  is the lump-sum government subsidy. In the competitive equilibrium, the government budget constraint is

$$S_t L_t = \tau K_t.$$

Here the human wealth includes the lump-sum transfer  $H_t^* = \int_t^\infty e^{-R_x(x-t)}(W_x + S_x)dx$ .

It evolves according to

$$\dot{H}_t^* = (r - \tau)H_t^* - W_t - \tau \frac{K_t}{L_t}.$$

The consumption function is given by  $C_{t,s} = (\rho + d)(Z_{t,s} + H_t^*)$ . Therefore the aggregate consumption function is  $C_t = (\rho + d)(Z_t + L_t H_t^*)$ . If we let  $h_t^* = H_t^*/A_t$ , it satisfies

$$\dot{h}_t^* = (r - \tau - g)h_t^* - w_t - \tau k_t.$$

The tax revenue is no longer thrown away and then the resource constraint implies  $\dot{k} = f(k) - c - (g + n)k$ . One can easily check that the technology-adjusted consumption function  $c = (\rho + d)(k + h^*)$  evolves according to (6). Along the BGP, the stationary allocation  $(c, k)$  is determined by (7) and

$$c = f(k) - (g + n)k.$$

The only difference is that the second equation does not include  $\tau$ . Thus we have the following proposition.

**Proposition 4** *Capital income tax reduces inequality even when the tax is revenue is rebated lump-sum.*

**Proof.** See the Appendix. ■

## 5 Use of consumption tax

So far we have investigated only wealth tax. In this section, we consider a linear consumption tax and check how it affects wealth inequality. Cohort  $s$  maximizes the following expected intertemporal utility  $U$  subject to the following budget constraint

$$\dot{Z}_{t,s} = r_t Z_{t,s} + W_t - (1 + \tau)C_{t,s} + S_t. \quad (14)$$

Here  $\tau$  is a linear consumption tax, and  $S_t$  is the lump-sum government subsidy. In the competitive equilibrium, the budget constraints are written as

$$S_t L_t = \tau C_t.$$

As the utility function is logarithmic, the consumption of cohort  $s$  at time  $t$  is given by

$$(1 + \tau)C_{t,s} = (\rho + d)(Z_{t,s} + H_t), \quad (15)$$

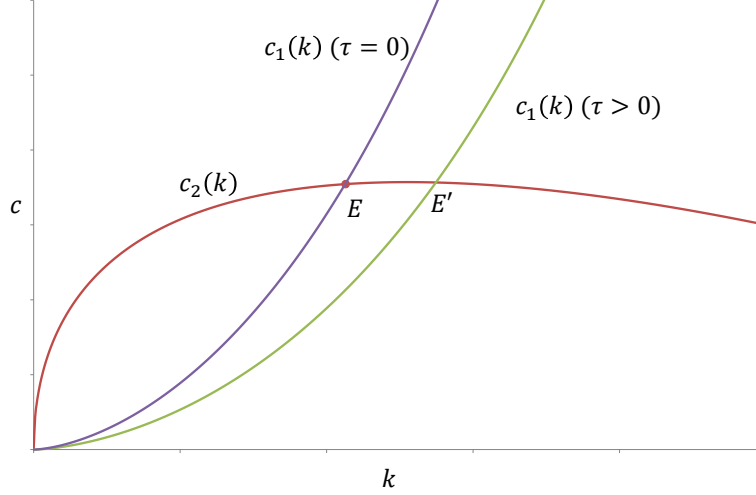


Figure 3: consumption tax and inequality

where the human wealth evolves according to  $\dot{H}_t = r_t H_t - (W_t + S_t)$ . With simple algebra, we get

$$c = \frac{1}{1 + \tau} \frac{n(\rho + d)k}{r - g - \rho - d}, \quad (16)$$

$$c = f(k) - (g + n)k, \quad (17)$$

As is clear from Figure 3, consumption tax shifts  $\dot{c} = 0$  curve to the right and consequently the equilibrium level of steady state capital increases. Therefore the rate of return on capital  $r(k)$  and also the gap  $r(k) - g$  declines. Thus we have the following proposition.

**Proposition 5** *Consumption tax reduces wealth inequality.*

Although both wealth tax and consumption tax reduce  $\bar{r} - g$  and also wealth inequality, the mechanism is different. Wealth tax reduces the net of tax rate of return on capital  $\bar{r}$  and lowers wealth inequality. It then decreases the steady state level of capital. On the other hand, consumption tax discourages consumption and enhances saving. Thus it accumulates more capital and reduces the rate of return on capital. Therefore, consumption tax can reduce wealth inequality without reducing the steady state output, while wealth tax reduces both inequality and output.

## 6 Conclusion

In this paper, we investigate a continuous-time OLG model with capital accumulation in which agents are subject to the constant death probability, and demonstrate that the gap  $r - g$  and wealth tax are closely related to wealth inequality. All of these results are consistent with the famous predictions advanced by Thomas Piketty in *Capital in the Twenty-First Century* (2014). In terms of economic policy, we find that consumption tax is better than wealth tax in terms of reducing inequality, because consumption tax does not reduce the steady state capital and also consumption. As a future study, we would like to incorporate heterogeneity in human capital accumulation.

# Appendix

The Appendix provides proofs for propositions.

## A Proof of Proposition 1

As  $f(k)/k = k^{\alpha-1} = r/\alpha$ , Eqs. (9) and (10) imply that if  $c_1(k) = c_2(k)$ , then

$$(r/\alpha - g - \tau - n)(\bar{r} - g - \rho) = nd(\rho + d).$$

Therefore,  $\bar{r} - g$  solves Eq. (11). In Eq. (11), if  $g$  and  $x (= \bar{r} - g)$  simultaneously increase, the left-hand side increases, while the right hand side is constant. This is impossible, and therefore,  $\frac{d(\bar{r}-g)}{dg} < 0$ . We can argue the same point on  $\tau$ . ■

## B Proof of Proposition 3

The steady state capital per efficiency unit of labor satisfies

$$f(k) - (g + n + \tau)k = (n + d)(\rho + d)\frac{k}{\bar{r} - g - \rho}.$$

As  $f(k)/k = r/\alpha$ , the gap  $x = \bar{r} - g$  solves the following quadratic equation:

$$\{x + (1 - \alpha)(g + \tau) - \alpha n\}(x - \rho) = \alpha(n + d)(\rho + d). \quad (18)$$

The left hand side is an increasing function of  $g$  and  $x$  while the right hand side is independent of  $g$  and  $x$ . Thus  $dx/dg < 0$ . The inequality measure  $\hat{\eta} = (x - \rho)/d$  is an increasing function of  $x$  and then  $d\hat{\eta}/dg < 0$ . ■

## C Proof of Proposition 4

The net-of-tax rate of returns on capital  $\bar{r}$  solves

$$(\bar{r} + (1 - \alpha)\tau - \alpha g)(\bar{r} + \tau - \rho - g) = \alpha d(\rho + d).$$

The left hand side is an increasing function of  $\bar{r}$  and  $\tau$ , while the right hand side is independent of  $\bar{r}$  and  $\tau$ . Thus  $d\bar{r}/d\tau < 0$  and then  $d\hat{\eta}/d\tau < 0$ . ■

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